Beslutsblindhet och preferensers (in)transitivitet:
En grafteoretisk metod för att studera preferensstrukturer i realtid

Choice blindness and the (in)transitivity of preferences:
A graph-based method for studying preference structures in real-time

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Choice blindness and the (in)transitivity of preferences: A graph-based method for studying preference structures in real-time

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The axiom of transitivity states that if a person prefers option A to B and B to C, he must also prefer A to C. This simple axiom is often seen as one of the pillars of rational choice, but whether it should be has created a divide in the research community. Some state that there cannot be rational choice or utility without transitivity and that agents who present intransitive preferences allow themself to be exploited. In contrast, others hold the opposite position, that intransitive preferences not only occur but can, in fact, be rational. This paper aims to study whether intransitive choices occur at such a high degree as is suggested in the literature and whether intransitive choices truly reflect intransitive preferences. Taking inspiration from seminal work in decision-making theory and behavioral economics, an experimental method was developed to measure preference transitivity and preference change over time. In this study, two experiments were conducted via a two-alternative forced-choice (2AFC) task in which participants selected the face they found more attractive. During Experiment 1, intransitive choices could be detected and extracted in real-time while dynamically utilizing choice blindness to study its possible impact on intransitive preferences. During Experiment 2, all stimuli combinations were iterated thrice, which allowed the preference graphs to be analyzed in their entirety. To evaluate the intransitive preferences in relation to those with a transitive pattern, we measured preference strength, choice consistency, and how the preferences evolved over time. Our results indicate that although very few indicators of intransitive choice were found, there was a significant decrease in intransitivity over time. Almost no evidence of repeated intransitive choices was found throughout all experimental phases, and even if we found a moderate level of choice blindness, this did not lead to intransitivity in subsequent choices. However, other possibly exploitable preference structures were found.

1 Introduction

Transitive preferences are assumed by many to be one of the core functions and criteria of rational choice (as discussed under transitivity and rationality) on the basis that an agent with intransitive preferences could succumb to exploitable behavior. However, the question of whether transitivity should be considered a canon of rationality remains open. Although being a fundamental assumption of standard economic theories, there is extensive empirical evidence for violations of transitivity. Studies have shown that people make intransitive choices concerning consumer goods (e.g., Guadalupe-Lanas, Cruz-Cardenas, Artola-Jarrín & Palacio-Fierro, 2020), gambles (e.g., Tversky, 1969; Lichtenstein & Slovic, 1971; Lindman, 1971; Loomes, Starmer & Sugden, 1991; Loomes & Taylor, 1991), when assessing candidates (e.g., May, 1954; Tversky, 1969) and in majority vote (Lagerspetz, 1997; Kurrild-Klitgaard, 2001). Preference reversals have even been found in financial decision-making (Muermann et al., 2006; Michenaud & Solnik, 2008). Much of recent research focuses on the distinction between (intransitive) choice and preferences. This study aims to perform a series of experiments in which participants repeat binary decision trials under different conditions to see whether intransitive choice leads to cyclicity or if these are if found, changed to adhere to transitivity. Although probabilistic transitivity will be considered briefly, only repeated violations of transitivity will be considered true intransitive preferences.

To better understand the cognitive function that underlies and governs the decision-making tasks we are routinely faced with, there is a need to study the relationship between choice and preferences and how they relate to the axioms of preference that have long been considered the rules of rationality. To this end, several research questions are proposed:

- To what degree does intransitivity emerge when faced with stated attractiveness preferences?
- Can choice blindness be used to induce intransitive preferences?
- If found, to what degree do participants repeat intransitive choices, indicating truly intransitive preferences?
- Could there be other exploitable preference structures found, similar to the money-pump?
- How do the preferences evolve over time?
- Are there any factors found which predict intransitive choice?

Although none of these questions will be studied exhaustively, a preliminary, explorative study will be performed to provide insight into them.

Preferences

In classical decision theory, preferences are generally considered subjective comparative evaluations. However, comparing and evaluating choices intuitively seems like a straightforward and uncomplicated task. This is especially true if one assumes that preferences are stable and have been constructed ahead of time (i.e., known to the agent). However, in real situations, these become much more intricate than first assumed, often being multidimensional where different attributes may best be evaluated under different circumstances, regarding other attributes, or using different heuristics.

Some decision-making theories assume agents have stable and coherent preferences, and many equate choices with preferences (such as revealed preference theory, e.g., Houthakker, 1950; Samuelson, 1948), inferring latent preferences from the choices made. However, in empirical studies, choices are known to vary, and some theories make the distinction between choice and the agent's actual preferences (such as normative preferences, e.g., Beshears et al., 2008; Noor, 2005). And lately, there has been an interesting distinction of choice variability and structural inconsistencies, “Common sense tells us that the variability of, say, restaurant choice, should not be mistaken for incoherence in, say, food preference.” (Regenwetter & Davis-Stober, 2012). Research further indicates that not only do preferences change over time, but they are also constructed during the decision task (Hoeffler & Ariely, 1999; Payne et al., 1999).

When studying choice behavior and preferences, a series of axioms have been presented to encapsulate rules that dictate preferences. These are sometimes referred to as the axioms of preference. The composition of the axioms differs in the literature, but most, if not all, include the axiom of transitivity (defined below) and the axiom of completeness (also defined below). The axiom of consumer preference usually includes the axiom of continuity which states that the usefulness of a good increases its consumption (Guadalupe-Lanas et al., 2020). Other versions include reflexivity (x is at least as good as x) (Black et al., 2009) and the axiom of independence (Anand, 1987) that states that a preference is independent of context.

As commonly used when discussing preference structures, the notation $\sim$ denotes indifference, $>$ denotes strict preference, and finally, $\gtrsim$ denotes a weak preference (at least as much as). The axiom of transitivity states that if $A \gtrsim B$, $B \gtrsim C$ implies that, $A \gtrsim C$ (von Neumann & Morgenstern, 1947). A person thus has intransitive preferences if he exhibits the following preference order: $A > B$, $B > C$, and finally, $C > A$. Although the axiom of completeness will not be studied in-depth, it is crucial to mention it concerning the axiom of transitivity briefly. Further, the axiom of completeness ($A \gtrsim B \vee B \gtrsim A$) and its relation to rational choice has been criticized (Anand, 1987), there have been compelling arguments for incompleteness (Gustafsson, 2016). An assumption of completeness of preferences will be made in the study. Namely, for every binary decision task, only one item can be selected and at least be weakly preferred by the agent at any time. However, there is no assumption that the preferences are static and unwavering, only that one of two items can be chosen based on some criteria for a single point in time. Another assumption made is comparability, i.e., that all the presented stimuli can be compared by the participant within the studied attribute, in this case, attractiveness which is essential to ensure that indeterminacy will not occur.

In classic economics, it has long been assumed that rational agents evaluate their decision-making according to utility maximization to select the good (or bundle) that will maximize the received utility, equating preferences with utility (Savage, 1954, von Neumann & Morgenstern, 1947). The aforementioned violations of transitivity are thus problematic to the standard economic model of rational choice. If intransitive preference was to occur in natural decision making, then reaching utility maximization would be impossible (Guadalupe-Lanas et al., 2020) since ordering a set of goods to reach a maximized utility, transitivity is implied. A utility function represents such an ordering by a quantifiable measure of utility (often measured by utils, a hypothetical measurement of usefulness or satisfaction); consequently, transitivity is required for utility functions (Haines & Ratchford, 1987). May (1954) exemplifies this with the following example; if one good is preferred to another (aPb), it implies that the utility (u) is greater for the preferred good, i.e., $u(a) > u(b)$. Consequently, if x is preferred to y (xPy) and y is preferred to z (yPz), this implies that the same relations apply for its utility, i.e., $u(x) > u(y)$ and $u(y) > u(z)$, which by transitivity implies that zPx and $u(z) > u(x)$. He further argues that preference and utility (both cardinal and ordinal) are interchangeable. It can then be derived that not only is transitivity necessary for utility, but if transitivity is not found, utility is not justified (May, 1954; Guadalupe-Lanas et al., 2020). Since how a person values a particular good or outcome is highly dependent on the situation and circumstance in which it is being considered. The position of classic economics, which equates preference and utility, is at least, if not faulty, not entirely satisfactory. The variability of choice and preferences causes the idea of rationality and rational choice to succumb to plasticity. The difficulty in assessing rationality and its relation to the axioms of preference is considered further in the section below.

Transitivity and rationality

This paper aims not to argue or study the rationality of intransitive preferences and their role in decision-making. It is, however, important to consider the question of rationality in regards to decision-making when studying preference structures such as transitivity. Since transitivity has long been considered a criterion of rationality, it offers novel aspects to be considered when discussing possible explanations for and the relationship between intransitive choice and preferences.

One of the main arguments (and the foundation of the money-pump argument) of the potential irrationality of intransitivity is that the agent chooses an option to which others are preferred. The fundamental problem with this reasoning is that if genuine intransitive preferences occur, the very idea of said preferences is that every alternative is one to which others are preferred (Andreou, 2015).

As far as the author is aware, little effort has been put into studying the introspective properties of intransitive choice, with a minor exception of Tversky's studies (Tversky, 1969), in which subjects became uneasy (and showed unawareness and disbelief) when presented with their own intransitive choices, which argues that even the participants found their choice to be irrational. Others have also argued that one might (or should) become uncomfortable when confronted with one's intransitive choices (Savage, 1954; Luce & Raiffa, 1957). This becomes less surprising when considering that the axiom of transitivity seems to be deeply rooted in and how we think of the relationship of objects. When using terms such as longer, faster, heavier, or more costly than transitivity is also implied; in fact, when considering all scalar and vector quantities, it becomes a matter of logic and arithmetic. It then seems like preferences should adhere to the same rules of logic.

Intransitive choices can emerge in many situations, e.g., individual choice when inconsistent choice depends on the pair of options presented or different options call for different choice criteria. One such situation where inconsistent choice can emerge is when assessing multiattribute goods, i.e., the attributes which one considers might depend on the context (e.g., item it is being compared to) in which the choice emerges. Several hypothetical examples illustrate intransitive choices that are still to be considered rational since the intransitivity arises from using different choice criteria when evaluating the options; some especially captivating examples are presented by, e.g., Hughes (1980) and Regenwetter et al., (2011).

The discussion of whether transitivity is truly required for rational decision-making rests heavily on the assumption that the agent who stated intransitive preference would necessarily act upon them when presented with the opportunity (Schick, 1986), emphasizing the importance of differentiating between stating preferences and acting upon them (Anand, 1993).

It has also been suggested that there is a correlation between the number of violations of transitivity and rationality (Lee, Amir & Ariely, 2009), although recent findings suggest that transitivity is not an adequate measure for decision quality (Maroiu & Maricuţoiu, 2021).

Heuristics

Economic theories have since their infancy focused on optimal behavior as a basis for rationality. The theories of *Homo economicus* or the *economic man* assume that when an agent is presented with choices, he can, according to a set of stable preferences, obtain the choice that would result in the highest utility gained, i.e., utility maximization (Diaye & Urdanivia, 2009; Simon, 1955; Thaler, 2016). The idea of optimized behavior rests on the assumption of unlimited cognitive abilities, which intuitively seems absurd. The focus on possible limits of our cognitive abilities concerning decision-making and observed behavior lead to a shift in focus, from utility maximization to behavioral economics and heuristics. Much effort has been put into redefining rational decision-making. Seminal work includes, amongst others: satisficing (Simon, 1956), bounded rationality (Simon, 1972), fast and frugal heuristics (e.g., Gigerenzer & Todd, 1999), which can be utilized to decrease our cognitive load.

Haines and Ratchford (1987) argue that rational behavior might even require intransitive preferences when information is costly or hard to come by. A similar argument has been made that consumers gather information until the marginal benefits of said information equals the cost (Haines & Ratchford, 1987). Not only does this increase the difficulty in predicting the preference ordering an agent will present. It has also been hypothesized that agents will not even have a well-defined preference function but rather that the strategy changes with the problem at hand (Haines & Ratchford, 1987). Some empirical evidence seems to strengthen this claim (Wright 1975; Johnson and Meyer 1984).

The seminal works in heuristics presented exemplify the importance of not deeming preferences irrational based on the structure of choices made but rather on the context in which they were made and their outcome. Much of this research on heuristics laid the foundation for some of the models of intransitive choice presented later.

Money-pumps

Frank Ramsey (1931) first presented the basic idea of a money pump in the form of a Dutch book, which is a set of odds that guarantee a loss—stating that a person violating the laws of preference, more specifically transitivity, could have a book made against him that would lead him to lose in any event (Ramsey, 1931). While the Dutch book arguments certainly provide an exciting insight into human decision-making, motivating factors such as the enjoyment of gambling are worrisome since the agent can receive enjoyment or excitement, which becomes troublesome since its possible irrationality cannot rest solely on the monetary incentives. This is, in particular, relevant since Dutch books are deemed irrational due to the guaranteed loss of money (Lehman, 1955).

The money-pump argument is a *reductio ad absurdum* in which an agent with intransitive preferences could be subjected to losing her wealth (Anand 1993; Block & Barnett II, 2012). The core idea of a money-pump lies within the assumption that people who have intransitive preferences, when presented with an opportunity to exchange items, can be exploited by continuously exchanging one good for another in a circular fashion while paying a small amount (Davidson, McKinsey & Suppes 1955, Bar-Hillel & Margalit, 1988). Standard economic theory assumes that a person who prefers one good (or bundle) would pay a small amount of money (premium) to obtain the preferred one. This means that a person with strictly intransitive preferences for some arbitrary goods, A, B, and C, i.e., A > B, B > C, and C > A, would presumably be willing to pay a small amount (ε) for that trade. If this person started with a good C, this would theoretically lead to the following sequence of trades: exchange C for B - ε; exchange B for A - ε; exchange C for A - ε. After these three trades, only one iteration through the preference cycle, the person ends up with C - 3ε, consequently being worse off than if no trades would have occurred, in which case the person would have the same good without losing money. Moreover, since the persons' preferences are cyclical, he would theoretically not end the sequence of trades since he still prefers B - ε over C, and the cycle continues. Although an interesting thought experiment, rationality dictates that the person would eventually catch on to the not so intricate pattern and stop the trades to avoid losing all his money, assuming that the agent prefers more money to less.

Gustafsson and Rabinowicz (2020) add the criterion that the person must be aware of the sequence of trades before they occur for the money-pump argument to be at all plausible since if they were not, then acting upon them can not be deemed irrational.

In economic theory, it is assumed that when money-pumps occur in financial markets, they will get exploited and eventually lose all their money, effectively solving the problem (ceasing to exist) themself. This is the same basic idea as arbitrage (Vishny & Shleifer, 1997) and triangular arbitrage (Aiba, Hatano, Takayasu, Marumo, & Shimizu, 2002), where an exploitable mismatch in pricing ceases to exist by the act of exploitation itself (converging prices). Although the standard definition of the money pump argument includes a repeated sequence of trades in a circular fashion, possibly (or rather probably) due to it being easily deemed irrational since, as mentioned above, it should by any rational agent be easily detected and avoided. The author does, however, propose a modified version of the money pump argument. In which unstable or intransitive preferences can be exploitable if a sequence of trades results in the agent possessing the same good at multiple points in time. This would consequently have the same effect as the generally accepted definition but without identical repeated sequences of trades.

Although the generally accepted definition of a money pump includes (possibly for dramatic effect) monetary incentives, some argue that money pumps need not include monetary exchange (Hansson, 2018). Moreover, the same decision-making structure can lead to similar problems, often a decision made with a limited decision horizon, where the recent decision is based on the immediate rather than the long-term effects. Examples include, amongst others; the torturer's dilemma in the puzzle of the self-torturer (Quinn, 1989), environmental or health, e.g., a smoker´s continued smoking even when knowing the long term detrimental effects it might have, or even time spent on a task based on the "time is money" argument (Hansson, 2018).

Safeguarding against being pumped

Several strategies which can be employed to safeguard against being pumped have been proposed, especially sophistication and being resolute. The strategy of sophistication, championed by Rabinowicz, is when an agent uses foresight and employs backward induction in its decision-making to avoid being exploited when exposed to the standard money pump. Rabinowicz has presented several alternatives to the standard money pump in which the agent with foresight can still be exploited (Gustafsson and Rabinowicz, 2020; Rabinowicz, 2001; Rabinowicz, 2000). On the other hand, a resolute decision-maker would devise a plan of action beforehand and not deviate from said plan (Hansson, 2018, McClennen, 1997), presenting an unwavering determination. Another and quite an obvious strategy to avoid being pumped is simply having strictly transitive preferences. It is, however, questionable whether either of these strategies works in reality since backward induction might be too complex to employ in everyday decision-making (due to the number of attributes and goods often included in a naturalistic problem space) and whether resoluteness will indeed hold when an agent is presented with a preferred good.

Other money-pump structures (arguments) have been presented. E.g., the deluxe money-pump, which applies dominance reasoning (Dougherty, 2013), the weak money pump for incomplete preferences (Peterson, 2014), and a money-pump for acyclic intransitive preferences (Gustafsson, 2010).

Although several strategies have been suggested to save us from exploitable behavior, the question of whether these are applicable in real-life decision making or if some other heuristic takes over which does not aim to optimize but rather to help us determine what is *good enough* remains.

Empirical examples of intransitivity and money-pumps

People have long been known to make intransitive choices, and some even claim that people have predictable intransitive preferences. Tversky (1969) showed this using gambles and choices between student candidates, and Guadalupe-Lanas et al. (2020) found intransitive preferences in both edible and non-edible goods. Intransitivity is not only frequent in individual choice but much of the historic and current focus lies in social choice, with the most prevalent example being the Condorcet paradox, first described by Condorcet in 1785 (Bar-Hillel & Margalit, 1988; Gehrlein, 1983; Kurrild-Klitgaard, 2001) and is probably the most famous example of real-works implications of transitivity. The Condorcet paradox results in intransitivity by aggregating individual choices even when all the individual choice sequences are transitive (Gehrlein, 2002). Posit three voters, who are to state complete pairwise preferences of three candidates A, B, and C. If all three voters have transitive, albeit different preference orders (e.g., A > B > C, B > C > A, and C > A > B) the resulting, aggregated preference sequence can be intransitive. I.e., A > B, B > C, and C > A, i.e., for any selected candidate, another has received more votes. Kenneth Arrow (1951) further generalized these results in Arrow's impossibility theorem, showing that there is no rule for aggregating an always complete and transitive ranking from individual choice (while adhering to specific criteria).

The fundamental idea of money-pumps and pumping behavior lies in the repeated cyclic behavior of an agent. As mentioned previously, many have found intransitive choices, but when tested for repeated cyclicity, most have found that only a few percent of intransitive participants repeated intransitive choices (Birnbaum & Gutierrez, 2007; Birnbaum & Schmidt 2010; Birnbaum, Navarro-Martinez, Ungemach, Stewart & Quispe-Torreblanca, 2016). Only one study was found, which presents a high rate of repeated intransitivities (Butler & Pogrebna, 2018), which relied on stimuli based on the Steinhaus-Trybula paradox and did not present the within-subject rate of presented cycles which is fundamental to the interest of money-pumps, making it hard to evaluate.

The only empirical study of money-pumps the author has been able to find studied household food purchases over the course of two years and found that about 80% of households violated GARP (Generalized axiom of revealed preferences). However, the violations were not severe enough to reject the null hypothesis that the violations were consistent with rational behavior and measurement errors and were deemed insufficient to reject rational behavior by the authors (Echenique et al., 2011).

As is evident from the studies presented, intransitive choice is a relatively common phenomenon. In contrast, there is little evidence of truly intransitive preferences. In addition, the evidence found is usually based on some paradox that makes comparing and assessing the stimuli complicated in some way. Further examples of intransitivity and the relation between choice and preferences are mentioned in the section below.

Theories of rational decision making and possible explanations for intransitive choice

There are two classes of theories of rational decision making, those who do not accept intransitivity, i.e., expected utility, and those who include the possibility of intransitive preferences.

Many models try to find an explanation for intransitive choice or preferences and accept intransitivity under certain conditions. One of which is the lexicographic semiorder in which attributes are considered sequentially by utilizing according to a cut of rule for when the difference of the attribute for the considered good is less than a predetermined value (Tversky, 1969; Manzini & Mariotti, 2012) (of which the priority heuristics are included (Brandstätter, Gigerenzer & Hertwig, 2006). Other models include the additive difference model (Tversky, 1969) and regret theory, which explains preference reversals by the ambition to minimize anticipated regret (Loomes & Sugden, 1982; Bell, 1982, see Bleichrodt & Wakker, 2015 for a review on Regret theory, and Slovic, 1995 for a review on preference reversals). More recent models include the true and error model (TE) (e.g., Birnbaum & Schmidt, 2010; Birnbaum, 2020), the most probable winner (MPW) model (Butler & Pogrebna, 2018; Blavatskyy, 2006), to which the results have been suggested a better fit for the TE model than MPW (Birnbaum, 2020), the mixture model (Regenwetter, Dana & Davis-Stober., 2010; Regenwetter et al., 2011) and the context-sensitive preference model (Müller-Trede, Sheer & McKenzie, 2015).

Like the context-sensitive preference model, many have noted that an agent's choices are not always consistent and that preferences can change over time. The inconsistency of preferences sparked the notion that preferences are best studied using probabilistic methods (Tversky, 1969). It has thus been suggested that true intransitive preferences lie not solely within intransitive choice but rather in violations of stochastic transitivity (ST, which is a stochastic version of transitivity which includes probabilistic instead of discrete values). Different versions of probabilistic measures of transitivity include weak (1), moderate (2), and strong stochastic transitivity (3), henceforth denoted WST, MST, and SST respectively (Cavagnaro & Davis-Stober, 2014) and the triangle inequalities (TI) (4) (Müller-Trede et al., 2015). For formulas 1-4, P_{ab} denotes the probability that a is preferred to b.

$$P_{ab} \geq .5 \ \wedge P_{bc} \geq .5 \ \Rightarrow P_{ac} \geq .5 \qquad (1)$$

$$P_{ab} \geq .5 \ \wedge P_{bc} \geq .5 \ \Rightarrow P_{ac} \geq min\{P_{ab}, P_{bc}\} \qquad (2)$$

$$P_{ab} \geq .5 \ \wedge P_{bc} \geq .5 \ \Rightarrow P_{ac} \geq max\{P_{ab}, P_{bc}\} \qquad (3)$$

$$P_{ab} + P_{bc} - P_{ac} \leq 1 \qquad (4)$$

Violations of stochastic transitivity can be seen as a misleading measure of intransitive preferences since a series of strict transitive preference orders can violate the WST (Müller-Trede et al., 2015; Regenwetter et al., 2010; Regenwetter et al., 2011) and can even be satisfied when aggregating intransitive preferences (Birnbaum, 2020). The triangle inequality can be violated when preferences are completely transitive (Müller-Trede et al., 2015) and even satisfied with strict intransitive preferences (Birnbaum, 2011). That ST can be violated with transitive individual preferences is why it has been argued against having transitive expectations in social choice since it gives rise to problems such as the Condorcet paradox, as discussed previously (Fishburn, 1970). Although violations of ST give important insight into the problems like that of the majority vote of multiple (Jamison, 1975) and even a single person (Notebloom, 1984; Butler & Blavatskyy, 2020) and utility functions or rankings when aggregating choice data, it does not provide much insight into the potential dangers of cyclicity, such as the money-pump argument. Although ST is not of great interest concerning money-pumps, it will be considered briefly in a more general analysis of intransitive choice.

It has also been speculated that truly intransitive preferences cannot occur unless the decision-maker is indifferent between the choices with the argument that if an agent can choose a preferred item from a ternary set when presented simultaneously, he cannot have genuinely intransitive preferences (Butler & Pogrebna, 2018; Tullock, 1964). Or put differently, “That indifference is not transitive is indisputable, and a world in which it was transitive is indeed unthinkable” (Armstrong, 1948). There is further relevance for indifference due to the connection to some models previously mentioned. For example, lexicographic heuristics, since a semi-order is when the differences are small enough to be acted upon as indifference (Luce, 1956).

The author hypothesizes, however, that, even if indifference were to occur amongst one of the choices, e.g., A > B, B > C, and A ~ C, intransitive choice might occur without necessarily being the result of intransitive preferences or due to error, but solely as a result of a vague preference sequence. Intransitivity caused by vagueness or indifference is further supported by studies showing that strong preferences are more common in transitive preferences (Guadalupe-Lanas et al., 2020).

Related to indifference are vague preferences. Vague preferences, which in the literature on intransitivity, are often based on (or at least related to) the sorites paradox of vague predicates and are sometimes called spectrum arguments (Nebel, 2018; Rachels, 1998, Temkin, 1996). Where an agent is presented with two options, each consisting of a positive and negative attribute, each pair of increments is indistinguishable for the negative attribute but distinguishable in the positive, i.e., one is preferred to the alternative and selected. However, larger intervals become distinguishable even for the negative attribute, and the agent consequently ends up at a dispreferred state. Instead, for the purpose of testability, the author will consider ‘vague’ preferences to be those of lowered preference ratings and speculate that this might correlate to increased variability and, subsequently, a higher rate of intransitive choices due to uncertainty.

The author further holds that true intransitive preferences are found only when intransitive preferences are systematically repeated, especially since the study focuses on intransitivity concerning the money-pump argument.

Unfortunately, models based on, e.g., lexicographic heuristics are not applicable to the current dataset (stimuli) since they do not include quantifiable attributes. However, a brief analysis of violations of ST, TI, and some of the effects that have been theorized to cause intransitivities, such as indifference, vague preference, and context-dependent preferences, is performed to study whether there is a correlation between any of these factors and intransitive choice.

Choice blindness

The choice blindness paradigm is a method in which false feedback is used to study the relation and mismatch between intentions and outcomes by manipulating the participants’ choices’ outcomes (Johansson et al., 2005). This is generally done by presenting the non-selected item to the participant. Although our introspective limitations have long been known (Nisbett & Wilson, 1977), the phenomenon of choice blindness was first discovered in 2005 and is seen as an extension to the phenomena of change blindness (Hall & Johansson, 2008).

The choice blindness paradigm has been shown effective in areas such as political opinion (Strandberg et al., 2018; Johansson et al., 2012), financial decision making (McLaughlin & Somerville, 2013), consumer choice in a natural setting (Hall et al., 2010), and risky choice (Johansson et al., 2012). Studies have shown that choice blindness not only induces a momentary change in preferences but that it can lead to lasting changes in, for example, political attitudes (Strandberg et al., 2018)

The motivation for adding choice blindness into this work is two-fold: firstly, as a tool to study the effects of false feedback on transitivity and, secondly, to test whether choice blindness could be used to induce a lasting intransitive preference sequence.

Preference graphs

In order to create a dynamic experimental platform that allows the analysis and extraction of preference sequences in real-time, inspiration are taken from graph theory, and more specifically, the idea that the binary preference-relations and stimuli used can be represented as a preference graph.

Graphs are commonly used in computer science and discrete mathematics to represent data and their relations. A graph, G, is a collection of *vertices* (sometimes referred to as nodes) and *edges* (connections between the vertices), often denoted as G = (V, E). A subgraph G′ = (V′, E′) of a graph G = (V, E) is defined as a graph which is formed from a subset of the vertices and edges of G (i.e., V′ ⊆ V and E′ ⊆ E). A regular graph is a graph where every vertex has the same number of neighbors, a graph is simple when there exists neither self-loops nor parallel edges, and a directed graph is when for any vertex combination, the edge is directed, i.e., the edge (u, v) ≠ (v, u).

To effectively allow a dynamic algorithm that can find cyclic preferences, the stimuli are represented as vertices, and preference-relations are represented as directed edges continuously as the experiment progresses, creating a connected, directed graph.

Generally, three preference relations are used. These are strict preference (aPb), represented by a directed edge, indifference (aIb) represented by an undirected edge, and incomparability (aJb), represented by the absence of an edge (Bouyssou & Vincke, 2010; De La Maza et al., 2018). Since the experimental paradigm employed here is a two-alternative forced-choice, only strict preferences are allowed and subsequently, only directed edges are used in the main analysis. However, indifference can then be inferred from the participants' preference ratings (the lowest rating presented on the Likert scale is not at all preferred); this is further discussed under the method section. In order to perform a speculative analysis of the impact of indifference on repeated cycles, a brief analysis is performed in which preference relations that received "not at all preferred" will be represented by bidirectional edges (Fig. 2.1).

For the sake of readability, the edges are directed in the direction of the preferred item. The preference relation between two stimuli is thus: if A is preferred to B (A > B), then B → A, as illustrated in Figure 1.

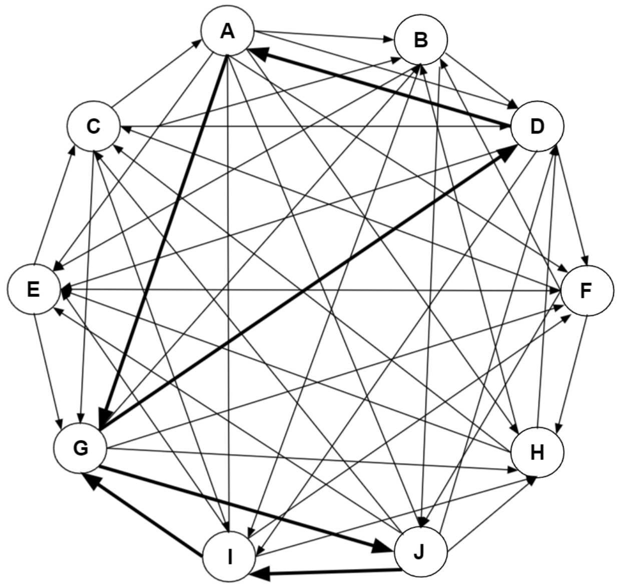

Figure 1: Visualization of a hypothetical preference graph with ten vertices and forty-five edges.

Figure 1 depicts a hypothetical preference graph where several cycles occur, one of which is: {(A, G), (G, D), (D, A)}. A graph similar to that depicted in Figure 1 is the representational output saved by the algorithm after making all comparisons. All preference graphs constructed during the study consist of ten vertices and forty-five edges by the end of each phase. Further, a graphical representation allows efficient algorithms to analyze the data, discussed briefly under *analysis and measure*.

Since every combination of stimuli is presented to the participants, and their preferences are assumed to be complete (one is at least weakly preferred to the other), the result will consequently be a complete, directed, regular, and simple graph. Therefore, every preference graph created by the end of each experimental phase is a tournament, i.e., a digraph containing exactly one edge for every pair of vertices (Guichard, 2016). This is then saved and further analyzed during the post hoc analysis described under *analysis and measures.*

To extend the analysis of potentially exploitable preference structures, both Hamiltonian and intersecting cycles are analyzed. A Hamiltonian cycle is where each vertex in a graph is visited only once with the same start and endpoint; the different Hamiltonian cycles for a subgraph consisting of 4 vertices are depicted in Figure 2, each of which can be rotated, resulting in a total of 6 possible intransitive sequences. Intersecting cycles are analyzed by checking every combination of cycles found in the different phases for intersecting vertices. This is done according to:

$$A \cap B \cap C = \{x: x \in A \; and \; x \in B \; and \; x \in C\}$$

By checking the set cardinality of x, it is possible to check how close the cycles A, B, and C are to each other. If one such cycle combination is found, it might be exploitable in a money-pumping fashion.

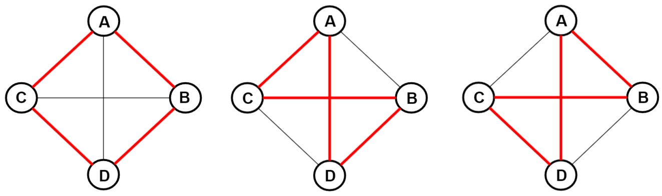

Figure 2: Illustrations of possible Hamiltonian cycles in a subgraph consisting of 4 vertices. The red lines represent the edges included in the cycle, and the thin black lines represent the edges not included.

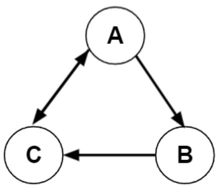

Figure 2.1: Illustration of a subgraph that includes a bidirectional edge.

Hypotheses

The purpose of this thesis is to test and expand the existing work on intransitive preferences and cyclicity, both occurring naturally as well as induced by the use of choice blindness. Building upon and taking inspiration from the classical studies on intransitive preferences presented in the introduction, this study adds false feedback as well as the detection and extraction of preference sequences in real-time. It also uses a more complex problem space, i.e., larger stimuli set with complete preferences and analyzing intransitive sequences of all lengths. To do this, a novel experimental design is presented, which hopes to examine all of these questions. Building upon the findings from Experiment 1, a second experiment was designed, which carried a separate set of hypotheses (H2).

H1.1: By utilizing false feedback, undetected manipulations will, to some degree, change otherwise transitive preference sequences into intransitive.

H1.2: Participants who state intransitive choice will, to some degree, repeat said choices indicating truly intransitive preferences.

H2.1: The rate of repeated cycles will increase for longer cycles due to them being harder to detect.

H2.2: The rate of increased transitivity would be lower in Experiment 2 due to not presenting the intransitive triples in close temporal contiguity.

H2.3: It would be expected that some Hamiltonian or intersecting cycles will occur for a given subgraph for each phase, even if no repeated cycles are found.

H2.4: Preference sequences that include vague preferences or indifference will be repeated to a higher degree since these should be harder to detect.

Although several hypotheses have been stated, the study is still exploratory in nature. It was built in a dynamic fashion with the hope of finding factors that correlate with intransitive choice and its relation to preference strength, choice consistency, probabilistic choice, indifference, and vague preferences.

The case of money-pumps may seem like a thought experiment with absurd consequences, leaving a person without money bewildered by the situation, not knowing what happened. The goal is to study whether cyclicity does occur, albeit to a less extreme extent than that postulated in much of the theoretical literature.

2 Method

The purpose of this study is to further analyze to what extent intransitive choice structures emerge and are repeated in binary choice data, which is the fundamental idea of the money-pump argument.

A problem evident in previous studies is that the stimuli used in most studies known to the author have been either incomplete, hard to understand, uncertain, incomparable, or explicitly designed to elicit intransitive choice by exploiting lexicographic heuristics (Müller-Trede et al., 2015). In some cases, even making the details of the stimuli hard to gauge as a part of the strategy employed (Tversky, 1969).

Two experiments were conducted to build upon the previous work studying intransitive preferences and continue towards a better understanding of cyclicity and money-pumps. *Experiment 1* was designed first and foremost to test whether the choice blindness paradigm could be used to elicit intransitive choice in otherwise transitive preferences. But also to gauge the effect of choice blindness on preference orderings even if they do not result in an intransitive ordering. Secondly, with an explorative approach to study to what degree intransitivity occurs, whether the cycles are repeated and how said preferences change over time. *Experiment 2* was conducted in much the same way as *Experiment 1* but without utilizing choice blindness. During Experiment 2, there are instead three identical phases presenting complete sets of stimuli combinations to see whether the results from Experiment 1 holds without presenting a selection of preference structures in sequence (close temporal contiguity) and without the use of choice blindness. Another reason for the design change was to minimize memory carryover while presenting the participants with complete information and not utilizing distracting trials.

Both experiments are conducted through a two-alternative forced-choice (2AFC) test; the primary empirical paradigm used when studying transitive preferences (e.g., Tversky, 1969; Regenwetter et al., 2010; Regenwetter et al., 2011; Müller-Trede et al., 2015).

By repeating a selection of preference sequences found in Experiment 1 and the entire decision phase of Experiment 2 thrice, the consistency of potential cycles can be studied more accurately than previously. Although some effort has been made in studying subject-specific intransitive preferences (Baillon, Bleichrodt & Cillo, 2015), most use a minimal stimulus set, only analyzing the exact same cycles for att participants. By having a larger stimuli-set than those presented in the literature, the possibility of subject-specific cycles increases, not relying on the same extent of the subject using the same selection strategy and having the same preference structures.

The author is unaware of any empirical studies analyzing repeated intransitive sequences of varying lengths in the fashion presented here. While some (Echenique et al., 2011) opted to limit their analysis due to the computational complexity of analyzing longer cycles, an effort has been made to perform an exhaustive analysis of the preference graphs to understand better what preference structures might lead to money-pumping behavior and would be of interest for future research.

Experiment 1

Participants

31 participants (22 male, 8 female) were recruited (mean age = 30.4 years, SD_{age} = 13.9). One participant was excluded because data was not saved properly to the server.

The participants were recruited using Prolific and paid £5 an hour to participate in the study. The inclusion criteria were that they had an approval rate of 99%, were heterosexual, had English as a fluent language, had access to a computer and mouse, and had not participated in any earlier studies conducted regarding choice blindness to ensure they were unfamiliar with similar tasks.

Stimuli

The stimuli used were two sets of ten pictures of faces (ten female and ten male), and participants were presented with pictures of the opposite sex. Although previous studies show that similarity in the stimuli set used did not affect the detection rate (Johansson et al., 2005), an effort was made to select a set of faces with increased similarity as an effort to elicit a higher rate of intransitive choices. This was in part motivated by some of the similarity models found in the literature (e.g., Buschena & Zilberman, 1999, Leland, 1994).

Materials

The experimental platform was implemented using the jsPsych javascript library (de Leeuw, J. R. 2015). A separate program was developed using Python 3.9 to perform a post hoc analysis to analyze the preference graphs adequately.

Experimental Design

The experiment was designed according to the 2AFC method, consisting of two experimental phases.

During the first phase, the participants were presented with a series of binary choices, of which they were instructed to choose the preferred one. The number of stimuli was fixed at 10, resulting in 45 trials during the first phase (the number of binary combinations for a set of stimuli is calculated using Equation 7).

The procedure of the decision trials was the following: First, a fixation point is shown at the center of the screen, after which a pair of stimuli is shown (Fig. 4A). The stimuli are shown for a duration of 1,5s, after which the back of the cards is presented, and the participants were to choose which one they preferred (Fig. 4B) together with a seven-point Likert scale in which the participant is to indicate by how much the stimuli was preferred (Fig. 4C). Lastly, they were presented with the selected (or not selected in the case of false feedback) image and asked to specify what facial attribute made them choose it, together with the option *"I actually preferred the other face."* (Fig. 4D), which enabled the recording of possible detected manipulations. The option was presented for all trials during the first phase not to make the manipulation obvious; if selected, participants were prompted to motivate the selection in the questionnaire after the experiment had concluded. The total duration of a trial is 6 seconds.

In order to be able to study whether intransitive preferences are repeated, and CB can be utilized to elicit intransitive preferences, a selection of ternary preferences was extracted (during the first phase) to be presented in isolation during the second phase. To do this, three types of preference structures were extracted in real-time throughout the first phase. The preference structures of interest were *intransitive* (Fig. 3A**),** *transitive* (Fig. 3B), and *CB-transitive* (Fig. 3C). *CB-transitive* sequences, or *CB-graphs* as they are termed in the section below, are transitive orderings that would result in an intransitive order if the last presented preference-relation were inverted. The red arrow depicted in the illustration of *CB-graphs* represents the preference-relation whose inversion would lead to a cycle. These are the criteria that need to be met to manipulate a transitive into an intransitive sequence using choice blindness. The process of detecting and extracting preference structures is described in detail in the section below. The motivation for extracting preference structure in real-time for each participant was partly to enable a subject-specific test of repeated intransitivity while maintaining consistent stimuli sets throughout the participants. So that choice blindness could be utilized adequately and within-subject.

During the second phase of the experiment, a selection of preference sequences was presented a second and third time. The trial procedure was almost identical except for the participant not being asked to motivate their choices since false feedback is not utilized. Here, the stack of subgraphs created during the first phase was iterated twice, resulting in six choice trials for each subgraph. This is to see whether the intransitivity of preferences induced or naturally occurring in the first phase leads to cyclicity and to what degree it would be repeated. The number of trials differed amongst the participants due to the experiment's dynamic design and there not being a guaranteed number of preference structures that fulfilled the criteria of Figure 3. The second phase consisted of an average of 108.4 decision trials (SD = 18.4). Transitive sequences were used as a control to better gauge the effect different preference structures have on choice consistency and preference ratings.

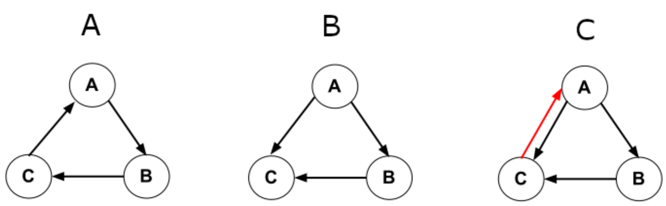

Figure 3: Visualization of the different types of subgraphs used in Experiment 1. A depicts an intransitive graph, i.e., cycle. B depicts a regular transitive subgraph. C depicts a CB-graph, i.e., a subgraph in which the last added edge creates a cycle if it is inverted.

Only subgraphs consisting of three vertices were extracted and presented to increase the consistency of preference orderings studied.

Figure 4: Vizualization of the interactions during the first phase. A, example of a stimuli pair being presented. B, Selection of stimuli. C, preference ratings. D, verification of selected or manipulated stimuli.

Since ten stimuli are used, there can be (although highly unlikely) a maximum of ten CB-trials for each participant. The number of transitive and intransitive subgraphs is then limited to the number of CB-graphs to increase the consistency and ensure an even distribution of subgraph types in the second phase, which means that there can be a maximum of 30 subgraphs in total (the average was a total of 18 subgraphs presented during the second phase per participant). The probability of a CB-graph occurring not being guaranteed lowers the amount of CB-graphs (and consequently the number of transitive and intransitive) used during the second phase. The number of CB-graphs available is lowered further because false feedback shall not occur directly after each other, presenting at least one non-manipulated trial between CB-trials and that the same stimuli cannot be used as false feedback more than once.

The importance of showing the same stimuli during both the first and second phase ensures that no other preference sequences explain eventual cycles or lack thereof. This was further motivated by the hope of capitalizing on reinforced preferences due to repeated presentations of stimuli in order to increase true preferences. By repeating the choices, the participants can reevaluate their preferences and select a different outcome during the second phase, increasing their insight concerning choice consistency and the relation between choices and preferences.

The detection and extraction of preference structures

During the first phase, an algorithm continuously searches the preference graph for the preference sequences of interest, which were then saved in order to be presented during the second phase. Three types of preference sequences were used in the second phase; these are *intransitive* (Fig. 3A**),** *transitive* (Fig. 3B), and what will be called *CB-graphs* throughout the paper (Fig. 3C). There is precisely one edge whose inversion would lead to a cycle for any connected and directed transitive graph with three vertices. There is, however, no guarantee that this would be the last presented stimuli pair (edge), which is a requirement for it to satisfy the conditions of a *CB-graph*. This means that the potential manipulation depends entirely on the ordering of the preferences since no false feedback can be given after the fact, i.e., potential manipulations are attributed to chance and cannot be controlled. To exemplify this, consider the transitive subgraph (Fig. 3B). If the last added edge is (A, C), a CB-graph is obtainable since (C, A) creates a cycle. Nevertheless, suppose the edge added last was instead (B, C). In that case, a CB-graph is not obtainable since (C, B) does not lead to a cycle.

The process of finding additional subgraphs is performed by checking whether the currently presented stimuli pair (vertices) have any mutual *neighbors* (i.e., vertices *adjacent* to them, i.e., vertices connected by a single edge). If such a neighbor is found, so is a subgraph with three vertices. The algorithm then checks the newly found subgraph for a cycle by running a depth-first search (dfs) algorithm. If a cycle is not found, it will check whether inverting the edge (equivalent to testing preferences A < B instead of A > B) creates a cycle, in which case a CB-graph has been found. If such an edge is found, false feedback will be utilized, presenting the non-selected stimuli. Otherwise, it is a regular transitive subgraph. After this, one of the edges to that neighbor is temporarily removed to check for subsequent neighbors (and consequently subgraphs); the algorithm continues this sequence until no more subgraphs are found. In Figure 5, the process of finding additional subgraphs is visualized for the stimuli B and C. The procedure of first looking for intransitive subgraphs was motivated by an effort to decrease the computational load since a cycle cannot become a CB-graph, i.e., eliminating unnecessary operations.

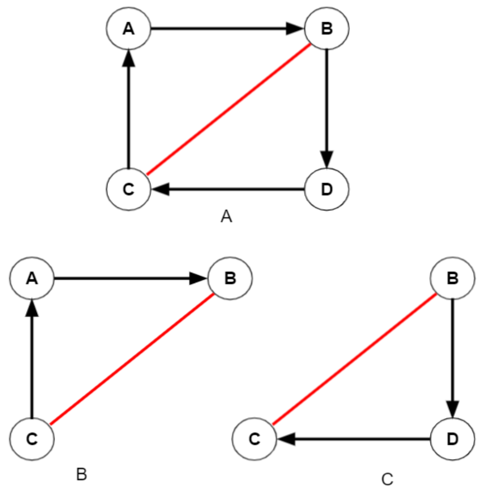

Figure 5: Visualization of the procedure used to find multiple subgraphs consisting of three vertices.

Procedure

The study was conducted via an online experiment. The participants were first presented with an information page, briefly describing the different phases before the experiment. They were told that the purpose of the experiment was to study preferences and attractiveness, but nothing about transitivity or choice blindness. The participants were also informed that the data would be aggregated and anonymized, obtaining informed consent. They were also ensured that they had the right to opt-out of participation at any point during the experiment. After the participants continued to the experiment, a calibration was performed where they dragged a box to the size of a credit card to ensure that the visuals were the same for all participants. The participants also stated their gender since this information was not provided (directly) by prolific. This was done in order to load the correct stimuli set. Before each of the experimental phases, a set of instructions were presented, informing the participants of the stimuli and tasks they will be presented and were prompted to attend the stimuli carefully since they were only going to be shown for a brief period of time. The participants were reassured that the same stimuli would be used in the subsequent phases; this was done to ensure the participants that they would have complete information in subsequent phases so that no eventual repeated cycles would be the result of incomplete information or uncertainty.

After the second phase is completed, a questionnaire is presented to gauge the participants' experience and whether they detected the manipulations. Lastly, an informative text was presented explaining the choice blindness paradigm.

Questions:
1. As you understand it, what was your task during the second phase of the experiment? 2. What do you think about the experiment, was it easy or hard? any other comments? 3. How engaged were you during the task? 4. How focused were you during the task? 5. What was your reason for pressing the 'I actually prefer the other face' button? Please state your reason(s)* 6. Have you ever heard about 'choice blindness'? If so, please describe what you think it is. **

**Question was only displayed if the participant chose the "I actually preferred the other face" choice at any time during the experiment.*

*** Question was only present in the questionnaire during Experiment 1 since Experiment 2 did not utilize choice blindness.*

Table 1: The set of questions presented in the questionnaire after the experimental phases had been completed.

Analysis and measures

As previously mentioned, to retrieve as consistent data as possible, all subgraphs in the second phase of Experiment 1 consisted of the same number of vertices; this part of the data analysis will be performed during run-time. This does, however, leave much to be wanted. To adequately analyze the preferences in their entirety and better understand the structure of the preference graphs, further analysis is performed post hoc due to the computational complexity to perform as the experiment was being run. One such measure is the total number of cycles in the preference graphs. Therefore, a program was written in Python, using the package *NetworkX (Hagberg et al., 2008),* which provides an implementation of Johnson's algorithm (Johnson, 1975) to find all simple cycles, the fastest algorithm for enumerating all cycles (Mateti & Deo, 1976). The number of distinct, free, simple cycles in a complete, directed graph with n vertices is calculated by Equation 5, accounting for rotations. The maximum number of cycles examined during the experiment can be calculated with Equation 6, where n = 10 and r = 3. It is desirable to minimize the number of calculations performed for obvious reasons, especially since all graph data was saved and available for analysis after the experiments had been completed.

$$\sum_{i=3}^{n} \frac{n!}{i!(n-i)!} \times (i-1)! \tag{5}$$

$$\frac{n!}{r!(n-r)!} \tag{6}$$

$$\frac{n(n-1)}{2} \tag{7}$$

$$(n-1)! \tag{9}$$

After the Experiments had been completed, an in-depth analysis of the preferences was conducted. A complete preference graph was first analyzed based on the stated preference from the first phase. Then a new preference graph was created sequentially, where every stated preference during the second phase was iterated through, creating a new preference graph where the current preference relation replaced the preference relation already added to the graph consisting of the same stimuli pair (vertices). This allows for a temporal representation of the evolution of transitivity of the preference graph in its entirety. In total, 3282 complete preference graphs were analyzed. All statistical analysis except for aggregating and formatting data and all graph-theoretical analyses, which were performed using Python, is done with R version 4.1.0 (R Core Team 2021).

Results

Overall, there was a much higher rate of intransitive choices found than expected. Only 3 out of 30 participants had complete acyclic preferences after the first phase, and only a single participant maintained acyclic preferences throughout the entire experiment. To contrast this, the second phase concluded with acyclic preferences for 13 participants.

The occurrence of cycles in the first phase

As can be seen in Figure 6, 194 intransitive triples were found during the first phase. To contrast this, a visualization of the frequencies of all cycles found is presented.

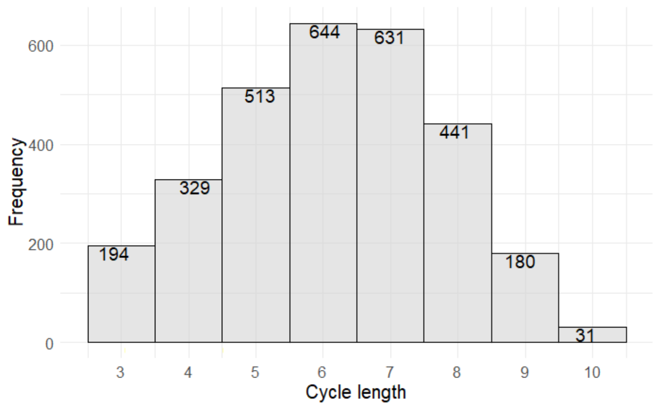

Figure 6: A summary of the frequency of cycle-lengths found in the first phase of Experiment 1.

The detection rate of manipulated trials

The detection rate was relatively high, at 64%. Although the detection rate was not studied in detail, the high detection rate is speculated to be attributed to the fact that the manipulated trials, on average, had a higher preference rating, as can be seen in figure 7, and that all stimuli are shown multiple times. The distributions of preference ratings are not be considered normally distributed as per the Shapiro-Wilks normality test ($w = 0{,}914$, $p < 0{,}001$ for all trials, $w = 0{,}896$, $p < 0{,}001$ for manipulated trials and $w = 0{,}917$, $p < 0{,}001$ for non manipulated trials). Mann-Whitney's U-test shows that the difference between total and non manipulated trials is not significant ($W = 791035$, p-value $= 0.353$) while there is a significant difference between manipulated and non manipulated trials ($W = 132083$, p-value $= 0.001$). However, since the *CB-graphs* are extracted dynamically without any control, this difference is attributed to chance.

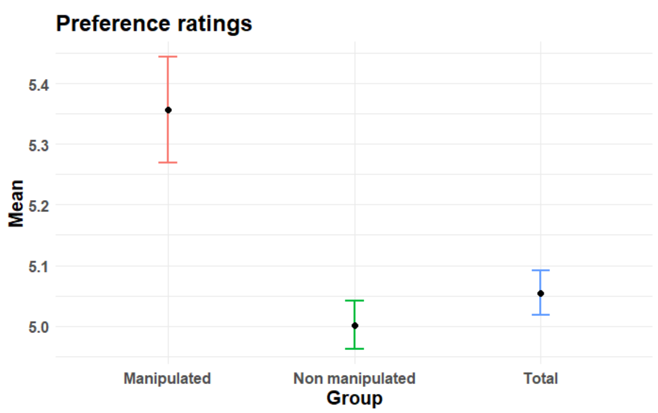

Figure 7: Preference ratings for manipulated, non manipulated, and all trials during the first phase. Graph displaying the mean and standard error.

Effects of choice blindness on preference structures

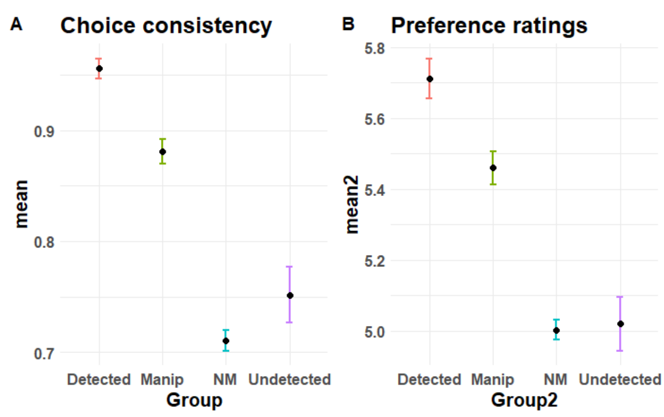

Figure 8: Choice consistency (A) and preference ratings (B) for non manipulated, manipulated, detected, and undetected trials. The graph displays the mean and standard error with choice consistency and preference ratings aggregated for both iterations during the second phase.

As shown in Figure 8, choice blindness did affect the participants' preferences regarding choice consistency and preference ratings. Undetected trials both had a lower choice consistency (75% undetected and 96% for detected), and preference ratings (mean 5.0 out of 7 for undetected and 5.7 out of 7 for detected) than detected and the overall manipulated trials.

Although it was hypothesized that undetected and inconsistent manipulated trials should convert an otherwise transitive preference order into an intransitive one, none did. There were 202 manipulated trials, 23 of which were both undetected and inconsistent during the first iteration during the second phase. However, none resulted in intransitive choices, i.e., for all cases of changed preferences, at least one other preference relation of the triple in question also changed, preserving transitivity, subsequently rejecting hypothesis H1.1.

Preference evolution

As is evident when comparing Equations 5 and 6, the sum of possible cycles compared to the number of possible intransitive triples is staggering, and performing the preference-evolution analysis solely on the sum of all cycles gives a skewed view. In part, a complete graph containing a cycle with more than three vertices must also include at least one cycle consisting of three vertices. Since the number of preference orderings presented during the second phase differed among the participants, a min-max normalization according to Equation 8 was performed to assess the data. The x-axis represents the normalized number of trials, and the y-axis represents the normalized number of cycles (Fig. 9A) or intransitive triples (Fig. 9B) found. As depicted in Figure 9, there was a significant negative correlation of C_{trip} $(F = 117.8\ and\ p < 0.001)$ where C_{trip} is the number of intransitive triples, i.e., cycles consisting of three vertices and C_{all} $(F = 147.7\ and\ p < 0.001)$ where C_{all} are all cycles. A linear regression was performed for both C_{trip} and C_{all}, with the results presented in Table 2. Because the total number of possible cycles can increase faster than exponentially (Johnsson, 1975) as the number of vertices increases, the number of total cycles was also log-transformed with the number of cycles + 1 due to 0-values.

$$x' = \frac{X - min(X)}{max(X) - min(X)} \quad (8)$$

All cycles

	Estimate	Std. Error	t-value	p
Intercept	0.31	0.008	37.19	< 0.001
trial	-0.26	0.019	-12.15	< 0.001

Triples

	Estimate	Std. Error	t-value	p
Intercept	0.195	0.006	32.6	< 0.001
trial	-0.14	0.013199	-10.9	< 0.001

Table 2: Regression table for C_{trip} and C_{all}.

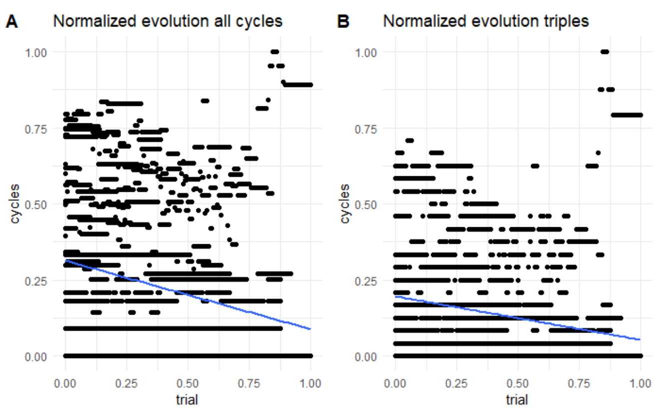

Figure 9: A summary of how the sum of all cycles (A) and intransitive triples (B) changed over time where the entire preference graph was analyzed after each stated preference in the second phase of Experiment 1.

Choice consistency and preference strength

An in-depth analysis of choice consistency was performed, which focused on the ternary preference sequences in their entirety. Detailed analyses of binary choice consistency were omitted because it possibly gives a skewed representation of choice consistency since the focus of this paper is larger preference structures. To exemplify the importance of the distinction, the choice consistency for the binary choices of which the intransitive sequences were constructed (between the first phase and the first iteration of the second phase) was 53%, while the sequence consistency was, as is presented below, 3.6%. It is clear that while binary choice consistency can be of importance in some cases, it is not comparable to sequence consistency since there are a total of forty-five choice combinations, which make up a total of 120 ternary preference sequences. This was done to distinguish choice variability and inconsistent preference structures. Several conditions are considered when analyzing choice consistency for the preference sequences presented during the second phase. Firstly, it is essential to remember that the presented consistency is for the sequences in their entirety, i.e., for a sequence to be consistent, all three binary choices must be repeated. Secondly, all three subgraph types (intransitive, transitive, and CB) were extracted during the first phase. E.g., intransitive orderings are those where cycles were extracted during the first phase, but when consistency is measured during the second phase, these orderings need not be the same, i.e., it can then have changed to a transitive ordering. Firstly, the between phase consistency is measured, i.e., to what degree are the choices made during the first iteration of the second phase consistent with those made during the first phase. Secondly, the sequence consistency during the second phase is measured, i.e., to what degree the choices made are consistent throughout the second phase. As shown in Figure 10, intransitive preference orderings had a significantly lower choice consistency when presented back to the participants during the first iteration. Only 5 out of 138 intransitive orderings were repeated in the first iteration (3.6%), none of which were repeated a second time which is significantly lower than both transitive and CB-graphs. This means that the low between phase consistency for intransitive orderings is attributed to them changing into transitive orderings. The between phase consistency for transitive was 121 out of 202 (60%), and CB was 106 out of 202 (53%). Subsequently, the consistency for the sequences during the second phase was 74% for those that were initially intransitive, 76% for transitive, and 79% for CB-graphs, which is a significant increase for all sequences tested as described below.

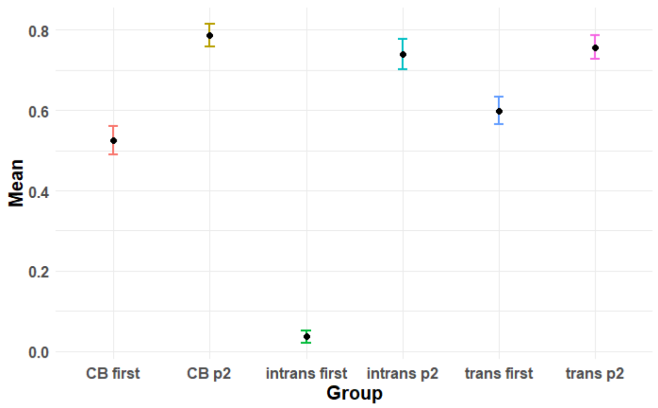

Figure 10: Summary of choice consistency grouped by subgraph type, where first is consistency between the first phase and the first iteration of the second phase and p2 is the consistency between the first and second iteration of the second phase.

According to Wilcoxon signed-rank test with continuity correction, there was a significant difference between the between phase consistency and the consistency during the second phase for all preference sequences (V = 427 and $p < 0.001$ for transitive, V = 688 and $p < 0.001$ for CB, V = 270 and $p < 0.001$ for intransitive). A Wilcoxon rank-sum test indicated no significant difference between the different types of preference orderings during the second phase (W = 21008 and p = 0.4775 for CB and transitive, W = 14607 and p = 0.3045 for CB and intransitive, W = 13683 and p = 0.703 for intransitive and transitive). A Wilcoxon rank-sum test indicated no significant difference between transitive and CB, but it did indicate a significant difference between CB and intransitive and intransitive and transitive orderings between the first phase and first iteration of the second phase (W = 21917 and p = 0.1332 for transitive and CB, W = 7129 and $p < 0.001$ for intransitive and CB, W = 21782 and $p < 0.001$ for transitive and intransitive). Lastly, the preference-order consistency during the second phase, i.e., the stated preference ordering during the first and second iteration, is significantly higher for all categories of preference ordering (although these might have changed from the first phase, especially from intransitive to transitive).

Preliminary results indicate lowered preference ratings for stimuli pairs that were initially part of intransitive triples, which is presented in Figure 11. Also notable is a small, albeit significant negative correlation between the preference rating of a stimuli pair and the number of intransitive subgraphs it is included in according to Pearson's product-moment correlation (cor = -0.221, $p < 0.001$). A test for correlation between the number of intransitive triples and total cycles a stimulus was included in, the number it was selected, and average preference ratings were performed. No significant correlation was found.

This is an extension of the idea that intransitivity is a result of indifference or vague preferences. It is, however, essential to remember that a specific stimuli pair can occur for multiple subgraphs, even of different types. These preliminary results indicate that it might be possible to predict intransitive choice by variables such as preference ratings.

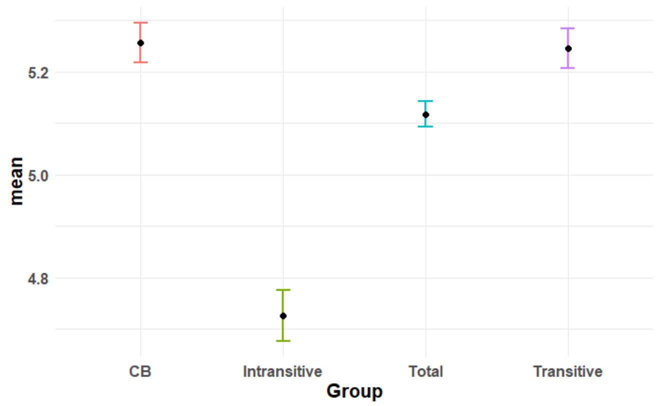

Figure 11: Preference ratings during the second phase for the different subgraph types, CB, intransitive, and transitive, and the total average preference ratings.

Experiment 2

Participants

30 participants (17 male, 13 female) were recruited (mean age = 28.57 years, SD_{age} = 7.3). The prescreen and payment conditions for participation in the study were identical to those of Experiment 1.

Stimuli

The stimuli used were identical to those used during Experiment 1.

Materials

The experimental platform used was a simplified version of that used for Experiment 1. Furthermore, the programs developed using python were based on those used during Experiment 1, with extensions created to perform the analysis needed.

Experimental Design

Experiment 2 was conducted in much the same way as the first phase of *Experiment 1* but without utilizing choice blindness and re-presenting a selection of preference sequences. It consisted of 3 identical phases, consisting of 45 binary decision trials each, where the participants chose the preferred one.

Each decision trial procedure consisted of a series of views shown in order; first, a fixation point was shown, then one of the binary stimuli sets were shown (figure 4A), after which the participants were prompted to select the image which they found the most attractive (figure 4B). The participants were prompted to motivate their choices by stating which amount they preferred the selected face to the non-selected (figure 4C), and lastly, to verify their selection by stating the preferred attribute in the stimuli chosen (Figure 4D). For each phase, the stimuli combinations and order (right or left) were randomized.

Procedure

The procedure was identical to that of Experiment 1, with the exception that no questions or information regarding choice blindness was presented by the end of the experiment.

Analysis and measures

The core analysis (e.g., number of cycles and repeated cycles) was performed similarly to Experiment 1. Due to the simple phase design of Experiment 2 and the fact that complete preference graphs were acquired for each phase, a more rigorous analysis of the preference graphs was possible. The analysis for Experiment 2 included studying cyclicity at three levels in order to perform a broader exploration of money-pumps and other possibly exploitable preference structures. In addition to repeated cycles, which is the fundamental idea of money-pumps, an analysis of Hamiltonian cycles for the same subgraph in the different phases, and the occurrence of repeated stimuli in different or identical cycles (i.e., intersecting cycles) in the different phases. Since the actual pumping behavior stems from the agent paying for the same good, i.e., losing money, iterating through one or multiple goods while being subjected to different cycles would have a similar result as the classic money-pump argument. Intersecting intransitive preferences might thus be harder to detect and subsequently found more frequently. The search for Hamiltonian cycles was, in essence, performed by checking if Hamiltonian cycles exist for a specific subgraph in all three phases of Experiment 2. This will also be performed post hoc due to the computational complexity. The number of subgraphs for each vertice-size $\geqslant 3$ is calculated by Equation 6, and the number of possible distinct Hamiltonian cycles for a cycle of n vertices is calculated with Equation 9.

The analysis of intersecting cycles was performed by checking every combination of cycles found in the three phases for intersecting vertices. The analysis of intersecting and Hamiltonian cycles are described in greater detail under the section *preference graphs*.

For Experiment 2, the preference graphs consisting of the choices made for all 30 participants during the three phases were analyzed, resulting in 90 preference graphs. Furthermore, although some have studied intransitive preferences by aggregating choice data between subjects (e.g., to check for violations of ST and TI), the analysis presented regarding repeated intransitive choices is performed within-subject. The in-depth analysis presented here was not performed on the data collected during Experiment 1 due to it not having consistent, repeated trials in the second phase.

Results

Similar to the results of Experiment 1, few acyclic participants were found as well as an increase in acyclicity as the experiments progressed. 7 out of 30 participants stated acyclic preference graphs for the first phase, only 3 maintained acyclic preference graphs throughout all the phases, and the third phase concluded with 11 acyclic participants.

The occurrence of cycles in phase one

The number of intransitive triples found in the first phase of Experiment 2 (mean = 6.8) did not differ significantly from those found in the first phase of Experiment 1 (mean = 6.5). Although a higher number of total cycles were found (mean = 220.6 for Experiment 2 and mean = 98.8 for Experiment 1), this is misleading due to the sheer amount of possible cycles in a complete graph. The rate of transitive tournaments (participants with completely transitive preference graphs) found after the first phase of Experiment 1 was 10%, and 23.4% for Experiment 2.

Preference-evolution

To better understand the results presented in Figure 9, a comparison of the preference evolution between Experiment 1 and Experiment 2 was performed, shown in Figure 12. However, the difference is lower in Experiment 2 (which might be attributed to the fact that the intransitive sequences were not presented in close temporal contiguity but dispersed over the phases, hypothesized to decrease memory carryover further). This is in line with H2.2: that the rate of decrease in intransitivity would be less than that of Experiment 1, a significant decrease in triples was found. In addition to this, although there is not a significant difference in the number of intransitive triples between the second and third phase of Experiment 2, there is a significant decrease in the total number of cycles, as depicted in figure 13, showing that the rate of transitivity is still increasing.

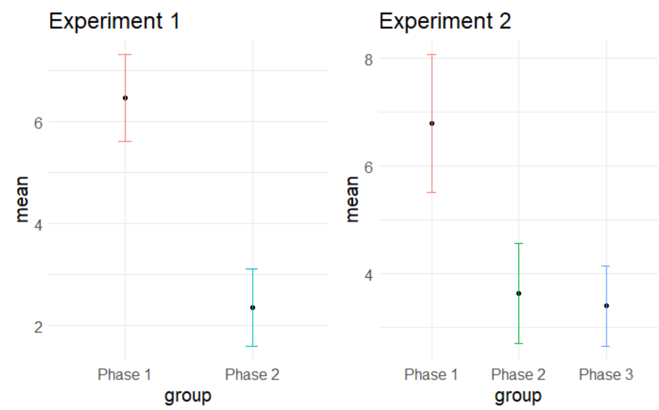

Figure 12: Graph depicting the mean and standard error of the total number of intransitive triples for each phase in Experiment 1 and 2.

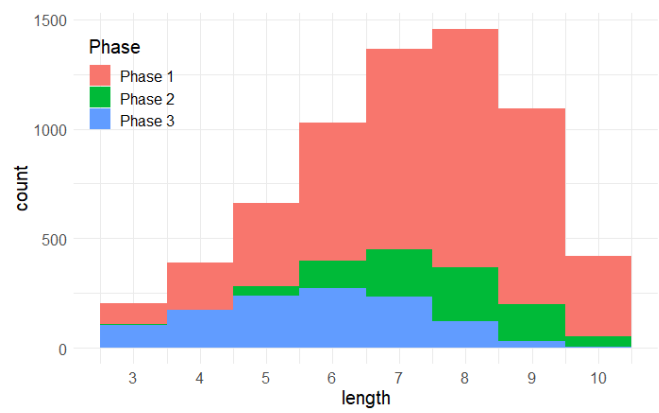

Figure 13: Graph depicting the distribution of cycles of different lengths for each phase of Experiment 2.

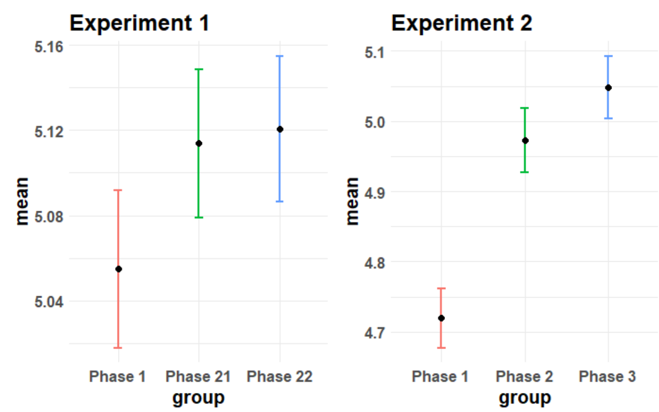

Figure 14: Visualization of the mean and standard error for preference ratings for each phase (as well as each iteration of the second phase) of Experiment 1 and 2.

As shown in Figure 14, the preference ratings are, as mentioned before, not a discrete value assigned to each face, but rather how much they preferred one stimulus over the alternative. The increase in preference rating and the lowered difference of preference ratings suggest that preferences both solidify and stabilize over time as the rate of transitivity increases. Not all binary choice combinations were repeated during Experiment 1, which might at least to some degree count for the lowered difference for Experiment 1 compared to Experiment 2. Taking inspiration from the analysis of the consistency of the preference sequences in their entirety (Fig. 11), an analysis of sequence consistency for all triples was performed, showing similar stabilization across Experiment 2. As is motivated in the section of choice consistency in the results of Experiment 1, binary choice consistency is entirely omitted for Experiment 2 since its focus lies entirely in repeated sequences. Out of the total 3600 triples (120 possible ternary combinations of 10 stimuli for all 30 participants), 46.3% were consistent between the first and second phase, 80.3% were consistent between the second and third phase, and only 40.1% were consistent throughout all three phases. To put the choice consistency into context, in the second phase, only 3% of triples were intransitive; out of these, only 4.6% were repeated (consistent) in the third phase. However, 97% of triples were transitive, out of which 80.5% were repeated (consistent) in the third phase. The sequence consistency of Experiment 2 provides further evidence for stabilization over time.

Indifference, ST, models and predictors of intransitive choice

In table 3, the number of violations of stochastic transitivity and the triangle inequalities is presented. The decrease in violations of ST is to be expected since each level of ST is more restrictive than the last. As discussed in the introduction, the percentage of participants who violated ST, TI, and made intransitive choices, might not be a good indicator of intransitive preferences. However, there was a significant correlation between violations of ST and TI and intransitive choice being made in at least one of the three phases. As per Pearson's product-moment correlation for WST ($t = 30.2$, $df = 3598$, $p < 0.001$, $cor = 0.45$), MST ($t = 15.9$, $df = 3598$, $p < 0.001$, $cor = 0.26$), SST ($t = 8.88$, $df = 3598$, $p < 0.001$, $cor = 0.17$), TI ($t = 15.9$, $df = 3598$, p-value < 0.001, $cor = 0.26$).

Out of the violations presented in table 3, 5.4% of violations of WST, 14.7% of MST and TI, and as many as 35.7% of violations of SST were completely transitive while none, as mentioned before, were completely intransitive. If one were to assess a decision maker's rate of transitivity according to violations of stochastic transitivity or triangle inequalities, it becomes apparent from Table 3 that these occur much less often than violations of transitivity. Moreover, it is also clear that violations of ST and TI do not equate to exploitable preference structure, making it less interesting when considering rationality and, in particular, the money-pump arguments.

	Intransitive	WST	MST	SST	TI
Count	394	98	39	19	39

Table 3: Summary of violations of weak, moderate, and strong stochastic transitivity, violations of the triangle inequalities, as well as the total number of triples that violated the axiom of transitivity in at least one of the three phases aggregated across all participants.

No indifferent triples (intransitive triples where all preference relations received a preference rating of 1 in all three phases) were found. Some significant correlations between a triple containing at least one indifferent preference relation and its being intransitive were found, albeit very small. Further examination and interpretation of indifference was performed by adding a bidirectional edge for all preference ratings of one (Fig. 2.1). This was performed for all three phases of Experiment 2 and checked for repeated cycles, not in preference graphs with added bidirectional edges. Surprisingly, only four such repeated cycles were found, two of length four and two of length 3. These results (together with the fact that no indifferent triples were found) reject H2.4. These results, as well as their speculative nature, are discussed further in the discussion.

Assessing the data through the lens of the context-dependent model (Müller-Trede et al., 2015), one might expect a correlation between longer trial ranges (between the first and last preference relation of the triple being presented) and a triple being intransitive. To test this, a linear regression was performed testing for correlation between the trial range and percentage of intransitive triples created, no statistically significant correlation was found (Est = 0.80, std error = 0.68, t-value = 1.18, p = 0.24). The reasoning behind this is that if the true preference sequences of a DM changes over time, there should be some correlation to longer trial ranges since sequences spanning over longer temporal periods should be affected by the change in preference to a higher degree. However, predicting how or what part of these structures changes is a complicated matter and was not the focus of this study. One possible reason for not finding a correlation between a triple being intransitive and its trial range might be that it changed into another transitive order since for any ternary choice set, there are eight possible sequences of which only two are intransitive. Müller-Trede et al. (2015) further state that “The assumption of limited-capacity memory is also essential to the analysis. If DMs retained perfect memory for previously sampled choice pairs, their posterior models for each attribute should stabilize throughout a sequence of repeated choices.”. Although no assumption is made concerning perfect memory, the participants should retain some memory from earlier phases, allowing for a stabilization in choices made. Which is in line with the decreasing rate of cycles and increasing rate of choice consistency found. This is further strengthened by the results shown in figure 15, i.e., although there is a significant increase in the percentage of closed triples being intransitive as all three phases progress, the rate is halved for the second and third phase (with an almost identical increase), compared to the first phase. With the regression output for percentage of closed triples being intransitive per trial for the first phase: $R^2 = .0157$, F(1348) = 21.55, p < 0.01; second phase: $R^2 = .01$, F(1348) = 13.55, p < 0.01; third phase: $R^2 = .01$, F(1348) = 13.46, p < 0.01.

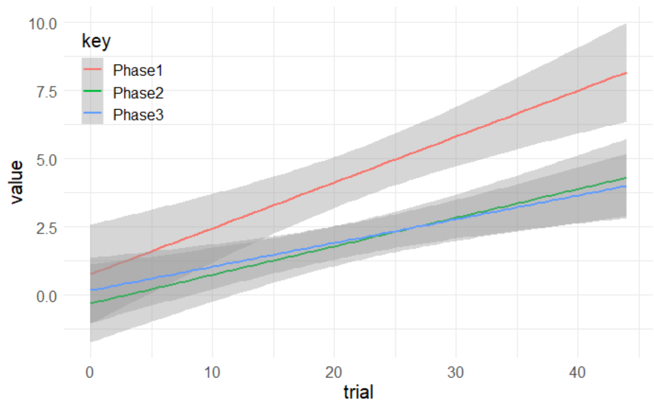

Figure 15: Number of the percentage of closed triples being intransitive for each trial and phase. A slight increase as the experiment progresses and a lower increase for phases 2 and 3.

Significantly lower preference ratings were found for inconsistent choices than those that were consistent between phases. According to Pearson's product-moment correlation there is a positive correlation between consistency and preference ratings (cor = 0.2, p < 0.001) for Experiment 1 and (cor = 0.11, p < 0.001) for Experiment 2. While the distribution of preference ratings is not to be considered normally distributed as per the Shapiro-Wilks normality test (w = 0.875, p < 0.001). This is presented together with the results from Experiment 1 in Figure 18.

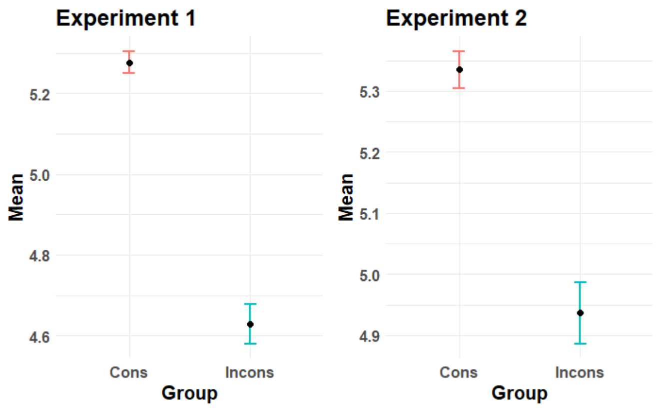

Figure 18: Graph illustrating the mean and standard error of preference ratings for consistent and inconsistent choices made during Experiment 1 and 2.

Repeated, Hamiltonian and intersecting cycles

As with Experiment 1, repeated cycles were analyzed. This is of particular interest since the participants were presented with all stimuli-combinations during all phases of Experiment 2. Motivated by the initial results from Experiment 1, more specifically, the results depicting high frequencies of longer cycles are presented in Figure 6. The author hypothesized that longer cycles might, at least to some degree, elicit repetition to a higher degree than shorter ones according to hypothesis H2.1. This is because shorter cycles could be easier to detect; one might subsequently change one's preferences to gain transitivity.

	Phase 1&2	*Phase 1&3*	*Phase 2&3*	*all*
n = 3	*9*	*1*	*4*	*0*
$3 \leq n \leq 10$	*17*	*2*	*5*	*1*

Table 4: Summary of repeated cycles through the different combinations of phases in Experiment 2 by the same participant for both intransitive triples and all cycles.

	Phase 1	*Phase 2*	*Phase 3*
n = 3	*204*	*109*	*102*
$3 \leq n \leq 10$	*6616*	*2023*	*1174*

Table 5: Summary of the number of cycles found in the different phases of Experiment 2 aggregated across participants. The table shows both intransitive triples and all cycles.

In Experiment 1, none of the participants repeated an intransitive triple during both cycle-iterations during phase 2. In Experiment 2, only one cycle (of length 4) was found in all three phases for the same participant (Tabl. 4). Which, although technically more than repeated intransitive triples, rejects hypothesis H2.1. This is notable since 6412 total cycles and 204 triples were found in the first phase of Experiment 2 (Tab. 5). This results in only 4.4% of intransitive triples being repeated in the second phase of Experiment 2, and none remained throughout the entire experiment. However, it is essential to mention that a single participant repeated 76% of the intransitive triples repeated between the first and second phases and the only cycle found in all three phases.

In *Experiment 2,* only five Hamiltonian cycles of the same subgraph were found in all three phases, all belonging to the same participant. Three were on the same vertice-set of length 6, 2 of length 4, and one of length 5. Notable is the fact that no repeated Hamiltonian triples were found across all three phases. Although some Hamiltonian cycles were found for a specific subgraph throughout the three phases, the number was less than expected, partly rejecting hypothesis H2.3; that if participants have intransitive preferences for a certain set of stimuli, Hamiltonian cycles would to some degree occur for those stimuli set across all phases even if no repeated cycles were found.

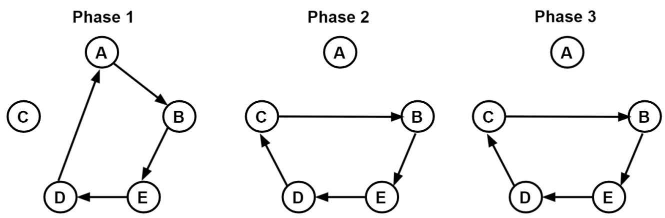

Figure 16: An illustration of one of the cases of multiple repeated stimuli ($|x| = 3$) found for one of the participants, which could possibly be exploited. There is a repeated cycle in the second and third phases, and no preference relation has changed.

There were a total of 14 out of 30 participants (≅ 47%) who stated cyclic preferences in all three phases (not limited to repeated cycles), and intersecting cycles in all three phases of Experiment 2 were found for 12 out of these 14 participants (≅ 85%). The number of combinations (c) of intersecting cycles do, of course, decrease as the cardinality of the intersecting set ($|x|$) increases. To put this into perspective, when $|x| = 1$, $c \cong 2330000$ and $|x| = 6$, $c = 1089$. No intersecting cycles for $|x| > 6$ were found. This is in line with the second part of H2.3. It is also important to remember that intersecting cycles are checked for all cycles found. Consequently, one set of intersecting cycles might include cycles of varying lengths (e.g., one cycle of length 3, one of length 4, and one of length 10). It is, of course, not guaranteed that such a preference structure would be exploitable since changes in preferences might not allow for a transition between cycles if presented in close temporal contiguity (even if all cycles persisted). In figure 16, one such structure is presented. In the second and third phases, the same cycle is repeated, and in the first phase, another 4-cycle containing three of the same stimuli the second and third phases. Moreover, figure 16 shows an example of a potentially exploitable preference structure, where no included preference relation changes during the experiment, i.e., which, if repeated, would allow for a transition between the cycles.

4 Discussion

The primary goal of the present study was twofold. Firstly, to study whether intransitive preferences could be induced by utilizing choice blindness. Secondly, to investigate to what degree intransitive choice occurred and was repeated to assess the realism of money-pumps experimentally. The experiments presented have allowed for a dynamic assessment of the potentials of choice blindness in inducing intransitive preferences, preference strength, preference-order consistency and provided insight into the evolution of preference graphs in their entirety. Additionally, using the graph-theoretical model developed in this paper, it was possible to analyze larger preference structures in their entirety at a detailed level and extract the structures of interest in real-time.

Cyclicity and possibility of money-pumps

The results presented in this paper show that participants did not repeat intransitive choice sequences of any length to a significant degree, rejecting hypotheses H1.2 and H2.1.

The results presented in Figures 12 and 13 indicate that intransitive choices are expected to be found in this type of decision task without necessarily indicating intransitive preferences. When given the opportunity, participants change their preference to adhere to transitivity to a higher degree. The high rate of intransitivities found during the first phase, with only 16.7% when combining results from both experiments of participants having complete transitive preferences, suggests that intransitivity occurs as a normal phenomenon, especially at the beginning of the decision-making process when complete information is not available. In much of the literature presented, the intransitive choice is often taken at face value and seen as evidence of irrationality rather than choices made based on what seems like initially unstable preferences that solidify over time. This is supported by the results presented regarding choice consistency, the evolution of preferences, and repeating intransitive choices, which indicate a process in which the preference structure stabilizes and solidifies over time.

The evidence presented for Hamiltonian and intersecting cycles across the second experiment shows that other exploitable preference structures can exist, at least in larger preference graphs. Even though many instances of intersecting cycles were found, they need not necessarily lead to exploitable behavior if presented in close temporal contiguity since they also need to be able to transition into each other for money-pumping to occur. However, the condition of transitioning was not examined, and it would need to be further analyzed. These results are in line with H2.3 and indicate that exploitable preference structures might occur due to intransitive sequences even when these are not repeated. Moreover, to predict these in the first phase to design a new class of money-pumps might not be an easy task. To find the stimuli that could allow for an exploitable preference structure, one must first perform a more rigorous investigation on whether exploitable behavior can be accurately predicted, which calls for a more complex experimental setup and analysis. However, the cyclicity analysis performed is not exhaustive but instead serves as a proof of concept and to guide future analysis.

Heuristics and optimality

There has been much debate about the dangers of intransitive preference and possible strategies that can be employed to overcome these. Although strategies such as sophistication through backward induction or resoluteness might save the agent from succumbing to the money pump, the question of whether these strategies are used in real situations, and even if they are needed or if there is some other underlying cognitive function that corrects for intransitive choice remains. From the results presented, it seems like money-pumps are not as large of a danger as had been supposed in the literature when intransitive choices occur due to the incredibly low rate of repeated cycles. However, to accurately analyze what strategy is used (if any) or mechanism exists to preserve transitivity requires additional work, discussed briefly under future research.

Choice blindness and inducing preferences

It is clear from the analysis that choice blindness was not successful in inducing intransitive preference orderings, rejecting H1.1. It is, however, interesting that choice blindness successfully changed preferences when undetected, but since these did not result in intransitive sequences, there seems to exist an underlying function preserving transitivity. The results indicate the possibility of a transitivity preserving mechanism; while extremely interesting, it is crucial to consider the preference evolution, and more specifically, the increasing choice consistency over time. Moreover, one must not forget that CB was only used during the first phase when choice consistency was at its lowest. However, if these results were to be consistent in future studies, they could bring new and exciting insight into the effects of choice blindness on a more fundamental level and on the preference structures as a whole and not only on the binary choice in which it was used.

There was also a relatively high rate of detected manipulations of 64.4%. It is not unreasonable to assume that detected manipulations would result in increased awareness of possible future manipulations, resulting in a higher detection rate. As seen in Figure 7**,** manipulated trials had higher preference ratings, which could have been mediated by only utilizing false feedback on trials with lower preference ratings. Another possible contributing factor to the high detection rate is the fact that all stimuli were shown multiple times when it would be optimal to utilize CB when the stimuli are first shown to minimize recollection and reinforced preferences. However, the experiment was designed to have as many CB trials as possible, which might have been a suboptimal approach in hindsight. There was also a significant albeit small negative correlation between when the first manipulation occurred (trial) and the total detection rate (Est = -0.058, SE = 0.025, p = 0.029), shown in Figure 17. The distribution can be considered normally distributed as per the Shapiro-Wilks normality test (w = 0,979, p = 0,814). This, in combination with only manipulating stimuli with lower preference ratings as well as not showing the stimuli multiple times, could be utilized in future work to elicit a lower total detection rate.

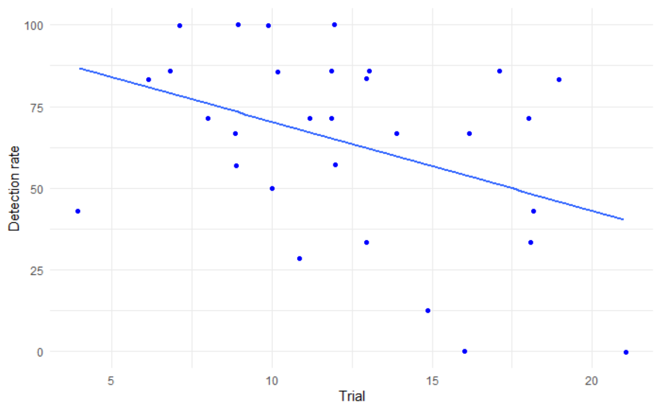

Figure 17: Correlation between the total detection rate and the trial number when the first manipulation occurred.

Preference evolution and stability

The results presented in Figures 9, 10 & 12 show that the transitivity of preferences increases and the preferences stabilize as the participants are subjected to repeated decision trials. This means that during the second phase of the experiments, when the participants had been presented with all the stimuli and had access to complete information, they could state their preferences concerning information that need not have been present when first stating preferences during the first phase and subsequently make more informed decisions, revealing true preferences to a higher degree.

The participants did have a more drastic decrease of cycles in Experiment 1, as expected from H 2.1. This could be explained by the close temporal contiguity of presented intransitive preferences in phase 2, making the cycles more noticeable and easy to "correct."

At first glance, it looks as if the low choice consistency would be evidence of unstable preferences. That the choice consistency is significantly lower for intransitive subgraphs than transitive might be seen as evidence that we are biased towards transitive preference ordering, adjusting for potentially faulty preferences, or reevaluating the preferences to increase its transitive properties. However, compared to the choice consistency between the second and third phases in Experiment 2 and the choice consistency measured for the two iterations of the second phase of Experiment 1 (Figure 10), these results indicate that the lower choice consistency might not necessarily be evidence of unstable preferences but rather choices made with incomplete information, which stabilizes and choice consistency increases over time.

It is also evident from Figures 10 and 11 that the choice consistency and preference ratings are significantly lower for intransitive orderings than transitive ones. This, as well as that inconsistent choices having lower preference ratings than consistent choice (Figure 18), indicate that intransitive choice might be caused by indifference, vagueness, or simply unstable preferences rather than true intransitive preferences. The correlation between lower preference ratings and choice consistency seems to align with the literature (Alós-Ferrer & Garagnani 2021).

Possible explanations and predictors for intransitive choice

As presented in the results, very few indicators of intransitive preferences were found. There were no highly indicative correlations between intransitive choice and indifference, vagueness, trial ranges, preference ratings, or manipulated trials found. All of these results seem to indicate that the intransitive choices found were due to error or heuristic noise. However, a significant difference was found for preference ratings of consistent and inconsistent choices during Experiment 1 and Experiment 2, as shown in Figure 18. These results indicate that intransitivity might result from inconsistent choices even though no significant correlations were found between intransitive triples and lower preference ratings. Since, as mentioned before, there are eight possible sequences for each triple, of which only two are intransitive.

As has been speculated in the introduction and results, indifference might elicit intransitive choice, but the statistical tests have not shown much success. Some liberties were taken when analyzing indifference further. If selecting a preference rating of one “Not at all preferred” truly reflects intransitive choice, it could also be analyzed as such in the graph model. The very low increase in repeated cycles found when adding bidirectional edges (Fig. 2.1) to the preference graphs further rejects hypothesis H2.4; that repeated cycles would increase in preference sequences that include vague preferences or indifference. This is far from an optimal solution and was only added to serve as potential proof of concept.

The lack of significant predictors for intransitive choice might be attributed to the fact that some effort was made in selecting stimuli with increased similarity and, although not conclusive, gives a direction in which future research can be aimed.

Methodologic considerations and limitations

The choice to employ the 2AFC method was motivated primarily due to its simplicity and being the standard method used in simple decision tasks. Although 2FAC is robust, it leaves the study with some problems, the most salient being that true indifference of indeterminacy can not be studied. Although an effort was made to mitigate the problems of indifference, and some effort was made to analyze this, it is still not an accurate measure of indifference. Further, one can not guarantee how the participants perceive the Likert scale of preference ratings. Although this did not impede the study significantly, it is something to consider for future work. Another factor limiting repeated choice tasks is the decrease in maximum complexity which can be examined (Lowenstein, 1999). Much of the suggestions for future research presented in the next section are technical in nature and motivated by increasing the complexity of future behavioral experiments.

Overall, the method devised and implemented and its implementation to test the choice blindness paradigm is considered a success since it was able to test choice blindness for more complex preference structures than before. However, this came with some limitations that, as have been discussed before, might have affected the results negatively. These include the stimuli being shown multiple times (a total of ten possible presentations before a manipulation), manipulated stimuli having a higher preference rating on average, and a correlation between lower detection rate and trial number of the first manipulation. Because of the platform's dynamic nature, none of these factors was controlled for in advance.

Another limitation is the lack of introspective information gained, which might explain the close to the nonexistence of repeated intransitive sequences. This would primarily be valuable to gauge whether participants are aware of the intransitive choice made and, if they are, why they made said choices adhere to transitivity. Were the changes made due to preference change or because the intransitive choices made seemed irrational and they felt compelled to “correct” them like Savage (1954), and Luce and Raiffa (1957) suggested? Nevertheless, to adequately gauge whether this is due to an inherent mechanism for preserving transitivity, true preference change, or conscious actions to extinguish choices that might seem irrational is a subject for future research.

As with most online decision tasks, further examination in a controlled setting under natural conditions would be preferable to truly assess the realism of repeated cycles, as there has been some evidence that transitivity increases when performed under supervision (Zoltan and Tarjan, 2016).

Future research

The method and results presented in this paper are by no means exhaustive and require further examination. The author hopes this paper serves as a gateway to study further the evolution of preferences and exploitable preference structures similar to the classic money-pump. Similar experiments using sequential preference choices and preference graphs will be performed to acquire a quantifiable measurement of money-pumping.

Other studies have relied on interviews and recollection by the participants on the rationality or reason for intransitivity (e.g., Tversky, 1969). It would also be interesting to follow this line, studying the phenomenology and introspective properties of intransitive choice as discussed in the previous section.

Using a graphical representation of preferences allows for a robust methodologic tool for studying even larger preferences structures in their entirety. Further development to the graph-based method developed and used in this paper would allow for more intricate and sophisticated studies of preference graphs. Some potential improvements include support for genuine indifference, indeterminacy, and sparse graphs (graphs that are not complete). Adding further support for indifference and indeterminacy to the graph model in the form of mixed graphs (including both directed and undirected edges and the absence of an edge) would allow for a more comprehensive range of preference patterns that could be studied, e.g., PPI and PII preferences and *the small-bonus approach* (Gustafsson, 2010). The inclusion of sparse graphs would allow for a larger stimulus set to be studied, which is not feasible when only considering complete graphs. Considering there are 45 combinations when ten stimuli are used, consequently, if 20 stimuli were used instead, the number of combinations would increase to 190. In other words, larger stimuli sets would quickly become unreasonable to use if all possible combinations are presented multiple times. Optimizing the algorithms used would also make it feasible to do much more complex analysis in real-time, not relying on post hoc analysis to the same degree.

As one of the primary purposes of this study was to study whether it would be possible to induce intransitive preferences by utilizing choice blindness, it would be of great interest to continue in that path. Although no evidence of induced intransitivity was found, choice blindness did affect the choices made. Furthermore, the fact that the preference structures affected by choice blindness (accepted manipulations) preserved transitivity indicates that choice blindness might not have just affected the choices at hand but changed the preference structure as a whole, which would be of great interest to study further. Moreover, although the experimental platform seems robust enough to study intransitivity and cyclicity adequately, choice blindness may be best studied using a more straightforward experimental design, with more control of which stimuli should be manipulated due to the limitations discussed in the previous section. As noted in previous studies, participants' behavior changed after detection was made (Johansson et al., 2005), which is partly why no manipulations were made during the second phase. However, it might have been advantageous not to have subsequent manipulations after a detection had been made since detections could increase awareness and suspicion.

The preliminary results presented regarding Hamiltonian and intersecting cycles allow for a more sophisticated method of predicting exploitable preference structures to be developed. The evidence presented here indicates that a dynamic algorithm for exploiting these could easily be created if similar results were found when monetary incentives are present.

The most obvious way forward in the study of money pumps would be simple valuation tasks, including different goods and services while implementing monetary incentives to test further the strength of the axiom of transitivity in settings where money-pumping would be feasible.

To adequately analyze the data collected and the relation between choice and preferences, it would be of great value to analyze it according to the models of intransitive choice discussed in the introduction, such as the true and error model. Some models, like those based on lexicographic heuristics, would require other stimuli.

Further analysis is required to acquire the predictive power (if possible) of finding true intransitive preferences.

5 Conclusion

The experiments presented did not find any evidence for stable intransitive preferences, neither naturally occurring nor induced. Evidence is also provided that intransitivity decreases over time as preferences stabilize and preference ratings increase; even when an effort is made to decrease memory carryover, there is an extremely low rate of repeated cycles. Not a single participant repeated any intransitive triple thrice; this combined with the fact that no instances of undetected manipulation which caused a preference reversal resulted in an intransitive sequence. This suggests that preferences are not created in isolation but rather in relation to each other.

All of the findings presented are consistent with underlying transitive preferences and that the intransitive choices found are rather due to error or heuristic noise.

6 Acknowledgements

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7 References

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