Least-square Monte Carlo based option pricing of European and Bermudan stock index options

In the financial markets, there is a never-ending development of asset prices. Thus, for traders to make a profit from them, there exist a need to develop superior methods to determine these instruments' value. One asset that is in particular complex to price is options.

Options are contracts that give the holder the right to either buy or sell its underlying asset for a predetermined price at one or more future dates. In the former case, the contract is called a call option, while it is called a put option when the latter terms apply. There are several approaches to price these contracts, one of which is through Monte Carlo-based methods. The idea is to simulate trajectories of the underlying asset price and estimate the option price based on these paths. At first, the method was applicable for only European options, which are options with one date of possible exercise, but in time the methods developed to include contracts with several dates of possible exercise as well. One of which is Bermudan options, a contract that allows the owner to exercise the option at a finite number of dates. However, the methods are not flawless. The aim of our thesis was to examine the accuracy and robustness of some of these pricing methods when pricing European and Bermudan options. In addition, attempts have been made to develop them further. Our research is important for several reasons, for instance for traders that want to set a proper price when issuing an option for which there is no current market. Also when option prices already are available, there is a need to check whether they are considered reasonable or not.

Through our research, we found that the methods' accuracy varied a lot depending on how the underlying asset price was simulated. The Monte Carlo-based European prices were more aligned with the real ones when using models with time-varying volatility. However, when including random jumps in the model to consider for news that affect the underlying asset price, the resulting prices got worse.

The pricing of Bermudan options is much more complex. The general idea is to decide which date the holder receives the optimal payoff in each price trajectory, and use these payoffs when estimating the option price. However, due to lack of data on the real option prices, the only way of checking whether our prices were reasonable, was to compare them to the corresponding European option price. From basic financial theory is it known that the Bermudan prices should be equally as large or larger. This was the case for almost all put options, but almost none of the call options. The reason is that it is difficult to approximate the correct size of the payoffs and when they should occur for each trajectory. The first method implemented was the LSM approach. In short, it is about regressing a function at each point in time the holder can exercise the option, and using it as a criterion to decide whether the holder should exercise the option or not. To improve the results, a modification of the standard LSM was made to form the so-called exercise boundary and use it as a decision criteria for whether the holder should exercise the option. Basically, the exercise boundary is a graph from which one can identify for which asset prices and times left to the options expiration date, it is worth exercising the option. However, the result got even worse with the method due to amplified errors that already existed with the standard LSM approach.

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Date: January 2022

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