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Portfolio optimization using factor models

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Abstract

In this thesis model predictive control (MPC) is used to dynamically optimize a portfolio where data is sampled at the closing price. Previous research has shown that MPC optimization applied on financial data can yield a portfolio that exceeds the value of traditional portfolio strategies. MPC has also been observed having computational advantages when return forecasts are updated when a new observation are sampled. Factor models such as the Capital Asset Pricing Model (CAPM) and Fama and French factor models are used to forecast the financial return of stocks taken from the Standard & Poor's 500 index Global. Portfolio optimization are performed using single-period forecast where the portfolio contains one stock and a zero interest rate cash account and also a large portfolio with 10 stocks and a risk-free asset. Transactions cost are included to better reflect the real world and address prediction-error. The MPC portfolio are outperforming a buy and hold strategy in both risk and return. Between the factor models then difference is negligible in case of the small portfolio but both Fama and French models outperforms CAPM in the larger portfolio.

Acknowledgements

I would like to thank my fellow thesis writers in Gupta for interesting discussions and sense of comradery. Especially Wilhelm Ålander for our many lunches together. My friends and family for their support. Last, but not least, my supervisor, professor Erik Lindström. This thesis would not have been possible without your guidance and input.

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Chapter 1

Introduction

1.1 Background

In the financial sector, investors goal is to maximize returns on investments while at the same time reduce risks related to those investments. Many investors have different opinion on the relationship between risk and return. Modern finance have introduced a lot of different financial assets investors can invest in, all with different risk profile. This creates problems when collecting the optimal set of assets.

A portfolio is the collection of assets an investor hold. Through time, several different frameworks have been developed to optimally solve the choice of portfolio. In 1952, the Nobel laureate Markowitz introduced such a framework called Modern Portfolio Theory (also called mean-variance analysis), where expected returns for a portfolio are maximized for any given risk (1952).

Not only do investors have to choose the assets in their portfolios but also how often do they need to reevaluate those choices. It's popular among investors to apply a strategy called Strategic asset allocations (SAA) where asset-weights in the portfolio are determined by the investors risk-type according to some mean-variance framework and are re-balanced periodically. The lengths of the periods between re-balances are typically chosen based on investment horizon and strategy. A problem with SAA is that between periods, market behaviour can change. For some funds a period could be a month or even a quarter. Different periods in the market are called regimes and their causes can for example be changes in economic policy and regulations. This becomes apparent when there is a case of a large drawdown between re-balancing periods and the value of the portfolio are down the weights are not adjusted until the next re-balancing period.

An alternative to SAA is dynamic allocation strategy (DAA). A dynamic strategy allows weights to be changed continuously as new information arises in the markets. This allows investors to take advantage of beneficial regime shifts and reduce the impact of adverse regimes. It has been shown that when the market shifts between regimes, a DAA strategy are more beneficial than a SAA strategy (Sheikh et al. 2012). An important part of a DAA strategy is the model the investors use to predict new information on the markets. One of the most celebrated financial model of all time is the Capital Asset Pricing Model (CAPM). CAPM is a factor model and it became wildly used because of its simplicity of only relaying on one factor, the market risk of the security in relation to the market. Throughout time the assumptions of the model has come in to question, specifically that they don't hold up in reality. Several other factor models have been introduced since then. Two researchers, Eugene Fama and Kenneth French invented two of the more popular ones, Fama and French three-factor model (1993) and five-factor model (2014). In their research they found that value stocks tend to out-perform growth stock and small-cap companies tend to out-perform large-cap companies, therefore the three-factor model accounts for observed small-cap and value stock out-performance. Since 1993 other researchers have expanded on the three-factor model and Fama and French's answered by expanding the model with two new factor, profitability and investment. Profitability refers to that companies reporting higher future earnings have greater returns in the stock market. Investment refers to the concept of internal investment, suggesting that companies directing profit towards growth projects are likely to experience lower returns in the stock market.

When trading, investors needs to consider more than return and risk because executing trades cost money. Even large financial institutions have costs associated with trading. Therefore it is important to take this into account when optimizing holding and trading costs. not only is this more realistic in a real-world setting but can also be beneficiary by reducing errors when forecasting future quantities by restricting sub-optimal trades. Model predictive control (MPC) have been been suggested by Boyd et al. (2017) as an approach to solve a stochastic portfolio optimization that includes constrains for trading. In 2007 Herzog at al. (2007) came to the conclusion that MPC is a sub-optimal control strategy for stochastic system that uses new information as it is more efficient from a computational perspective than stochastic programming models. The idea of MPC is to control a portfolio based on forecasts of quantities such as asset returns.

In their 2017 paper, Boyd et al. describes a general framework for both single-period optimization (SPO) and multi-period optimization (MPO) of a portfolio. In case of the MPO, convex optimization are done over several periods and a series of optimal trades are formed. But only the first trade are executed. When including trading costs, the MPO case will differ from the SPO. This is because MPO takes in consideration not only how the trades affects the current period, but how that trade will affect futures trades over a time horizon. For example, the SPO suggests the portfolio should position itself long in some asset. The MPO can investigate several periods ahead and evaluate if it's still optimal to go long or if the trading cost over time are to severe. The most fundamental part of MPC is the model used for forecasting future values.

1.2 Thesis objective

In previous papers, Nystrup and coauthors have successfully demonstrated the use of HMM and portfolio optimization in using MPC (Nystrup et al. 2019). The aim of this thesis is to explore the potential of factor models such as CAPM, FF3FM and FF5FM as alternative models for the asset dynamics.

Chapter 2

Single-period portfolio selection

2.1 Problem formulation

In single-period portfolio optimization, the aim is to optimize the portfolio value over a planning horizon $K = 1$. Because future portfolio value are unknown the problem are formulated as a stochastic control problem. This type of formalization follows from Nystrup et al. (2019) inspired by Boyd et al. (2017). The goal is to maximize the expectation of the total portfolio υ_K over the horizon T^{invest} while subject to cost penalties $\gamma(h_t, u_t)$, based on portfolio holdings $h_t \in \mathbb{R}^n$ and value of trades $u_t \in \mathbb{R}^n$

$$\underset{h}{\text{maximize}} \quad \mathbb{E}\left[\upsilon_{T+K} - \sum_{t=0}^{T^{invest}-1} \gamma_\tau(h_\tau, u_\tau) \middle| \mathcal{F}_t\right]. \tag{2.1}$$

Then, the post-trade portfolio are defined as

$$h_t^+ = h_t + u_t, \quad t = 1, ..., T-1 \tag{2.2}$$

where $(h_t)_i < 0$ implies a short position on security i and $(u_t)_i > 0$ implies that an asset is bought. Trades are assumed to be executed at the end of each holding period. Assuming that the penalty function is convex formulation of the problem in (2.21) creates a convex objective function ensuring the existence of an unique solution. υ_K are assumed to be a stochastic variable subject to returns of the assets in the portfolio. To mimic an investors preferences constraints on the holdings can be included

$$h_t^{min} \leq h_t \leq h_t^{max}. \tag{2.3}$$

Where the constraint $0 \leq h_t$ represents a long-only portfolio. It's realistic to assume an investor does not have access to an infinite amount of cash to go long on assets, therefore it's natural approach to add a self-financing condition to the portfolio

$$\mathbf{1}^T u_t + \kappa^T |u_t| \leq 0 \quad t = 1, ..., T-1. \tag{2.4}$$

Meaning that the total proceeds from purchases and sales has to be less or equal to the total transactions cost of the particular trade. Were κ is a vector of transaction costs that can be accessed directly from market information or estimated.

Using MPC the stochastic optimization problem is reformulated as an deterministic problem by replacing the unknown expected total portfolio value in terms of weighted forecasted returns $\hat{r}_{\tau|t}, \tau = t+1, ..., T+H$.

$$\text{maximize} \quad \sum_{t=0}^{T^{invest}-1} \left(\hat{r}_{\tau|t}^T w_\tau - \gamma_\tau(w_\tau) \right). \tag{2.5}$$

Subject to $\mathbf{1}^T w_\tau = 1$, $\tau = t+1, ..., t+K$,

where once again $\gamma_\tau(w_\tau)$ is the penalty function for the cost of trading and holding. All the steps in the algorithm are summarized below.

Algorithm 1: Single-period portfolio selection via MPC.

1 Update model parameters based on most recent observation (optional step)
2 Forecast future unknown quantities one step in the future
3 Compute the optimal sequence of trades
4 Execute the trade and return to step 1

2.2 Trading aversions

The general penalty cost function from (2.5) can be partitioned into several functions in order to represent common trading aversion such as trading cost, transaction cost and risk aversion. In portfolio optimization there are a lot of different risk measures being used and essentially any of them could be implemented in the framework of Boyd et al. (2017).

Assuming $\sum_{t=\tau}$ is the estimated covariance matrix of the forecasted returns r_t, a common quadratic risk measure of the portfolio is

$$\gamma_\tau^{risk}(w_t) = \rho \cdot w_t^T \sum_t w_t \tag{2.6}$$

where ρ is a risk aversion parameter that can be tuned. If there are two assets and one of them is the risk free rate the covariance matrix will have zeros in the last row and column since the risk free rate is known. Combining this risk measure and objective function (2.5) corresponds to mean-variance preferences over the changes in portfolio value in each time periods. Also, if returns are independent random variables then the objective function is equivalent to the mean-variance criterion of Markowitz (1952).

Costs for trading are important when comparing dynamic and static strategies as frequent trading will incur transactions costs and can offset a dynamic strategy's excess return. Therefore a penalty for trading,

$$\gamma_t^{trade}(w_t) = \kappa \cdot |w_t - w_{t-1}| \tag{2.7}$$

are included in the objective function, where κ is the penalty factor. In this thesis κ is scalar but could replaced with a vector. A vector could have different values for each asset depending on uncertainty related to estimation (for example) and could better reflect real-world trading.

By holding the post-trade portfolio w_t over the tth a holding-based cost can incur. The basic holding-cost model used in this thesis imposes a charge for borrowing assets when going short,

$$\gamma_t^{hold} = s_t^T (w_t)_-, \tag{2.8}$$

where $(s_t)_i \geq 0$ is the borrowing fee for shorting asset i in period t and $(w)_-$ denotes the negative part of w. This is a fee for shorting the asset over one investment period.

2.3 Benchmark metrics

A set of metrics are used to evaluate performances of the different models. The portfolio return are defined as the average return over a period $1, .., T$,

$$R_p = \frac{1}{T} \sum_{t=1}^{T} R_t. \tag{2.9}$$

The portfolio risk are the standard deviation of the portfolio,

$$\sigma_p = \sqrt{\frac{1}{N} \sum_{t=0}^{N} (R_t - R_p)^2}. \tag{2.10}$$

In the case when the benchmark consists of cash, we define the excess return as

$$R_e = R_t - (r_t)_{n+1}, \tag{2.11}$$

where $(r_t)_{n+1}$ is the risk-free return and R_t the benchmark return in period t.

The Sharpe ratio, also called information ratio, are used to evaluate the realized returns given the risk of the portfolio,

$$\frac{R_p}{\sigma_p} = \frac{R_p - r}{\sigma_p}, \tag{2.12}$$

The maximum drawdown measures the largest drop from a peak to a valley. It's an indication of downside risk. The maximum is denoted by

$$M_t = \max_{t \in (0,T)} P_t. \tag{2.13}$$

Drawdown D_t are now defined as the difference in price compared to maximum,

$$D_t = \frac{M_t - V_t}{M_t}. \tag{2.14}$$

Finally, Maximum drawdown are then defined as

$$MDD_t = \max_{t \in (0,T)} D_t. \tag{2.15}$$

Chapter 3

Factor models

3.1 Capital asset pricing model

The Capital Asset Pricing Model (CAPM) tries to explain an asset's return based on the amount of risk it contains given the market as whole. The CAPM was developed during the 1960s by William Sharpe (1964), Jack Treynor (1962), Jan Lintner (1965) and Jan Mossin (1966). This is the first model of its kind and are still used by investors to price risk assets. The mathematical formula for the model are given by

$$R_{i,t} - R_{F,t} = b_i(R_{m,t} - R_{F,t}) + e_{i,t} \tag{3.1}$$

$R_{i,t}$ is the return of security or portfolio i for period t, $R_{F,t}$ the risk-free return, R_m return of the market portfolio, which makes $(R_m - R_f)$ the market risk premium (MRP). MRP provides investors with an excess return as compensation for increased volatility of returns over the risk-free return. b_i measures the amount of risk the asset adds to the portfolio. $b_i > 1$ implies the stock is riskier than the market itself and will increased the risk of the portfolio. Equivalently, $b_i \leq 1$ implies the risk of portfolio will be reduced by adding the stock. For instance, by investigating the underlying factors behind returns an investor could find out if some funds returns are caused by active investing or maybe it's because the fund is exposed to risk-factors that generated high returns.

3.2 Fama and French three-factor model

Over time, several assumptions behind the CAPM have been shown not to hold up in reality. Such as constant risk measure and normally distributed returns. Two researchers, Kenneth French and Eugene Fama published a series of articles questioning the practical uses of the CAPM. In 1993, they presented the Fama-French three factor model that expanded on the CAPM by adding two new factors to already existing one (1993). The first one being SMB (small minus big) which represent the typical out-performance of small-cap companies versus big-cap companies. Second one are HML (high minus low) which represents the out-performance of high book/market versus small book/market companies. However this may not hold throughout the business cycle (Barroso et al. 2013). Fama-French three-factor model expressed in mathematical terms,

$$R_{i,t} - R_{F,t} = \alpha_{i,t} + b_i(R_{m,t} - R_{F,t}) + s_i SMB_t + h_i HML_t + e_{i,t}. \tag{3.2}$$

If the factor-parameters b_i, s_i and h_i captures all variations in expected returns, α_i (intercept) is zero for all securities and portfolios i.

3.3 Fama and French five-factor model

Just as with the CAPM, the FF3FM were also being criticised for not being able to model variations in expected returns for securities. Therefore, in 2015, Fama and French introduced an expanded factor model (2014). They included two new factors, profitability (RMW, robust minus weak) and investment factors (CMA, conservative minus aggressive). RMW adds the idea that companies reporting higher future earnings will also have greater returns in the market. CMA relates the concept of internal investments which suggests that companies directing profits to growth projects are more likely to have losses in the stock market,

$$R_{i,t} - R_{F,t} =$$
$$\alpha_{i,t} + b_i(R_{m,t} - R_{F,t}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{i,t}.$$

Where RMW_t is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and CMA_t is the difference between the returns on diversified portfolio of the stocks of low and high investments firms. The reasoning behind this was evidence indicating FF3FM overlooked variations in average returns related to profitability and investment factors. As before, if factor-parameters capture all variations, α_i are zero. Fama and French also reported in their 2015 paper, when including the two new factors RMW and CMA, HML factor becomes redundant.

3.4 Recursive estimation in linear models

Typically, estimation of coefficients in models such as CAPM and Fama and French factor model are done by performing a regression. In this thesis, the estimation are done recursively.

In general, linear models can be written as,

$$Y = Z\theta + \epsilon, \tag{3.3}$$

where θ are parameters, Z factors and ϵ are independent and identical Gaussian random variables. Estimate of $\hat{\theta}$ is

$$\hat{\theta} = (Z^T Z)^{-1}(Z^T Y), \tag{3.4}$$

which can be done in a recursive manner.

We have a set of input samples $\{(Z_t)\}_{t=1}^N$ and a desired signal $\{(y_t)\}_{t=1}^N$. Computing their output,

$$y_t = \sum_{s=0}^{M} Z_t^T \theta. \tag{3.5}$$

The aim is to recursively find parameters $\{(\theta_t)\}_{t=1}^N$. $\hat{\theta}_t$ can be written as

$$\hat{\theta}_t = R_t^{-1} \xi_t, \tag{3.6}$$

where

$$R_t = \sum_{s=1}^{t} Z_t^T Z_t. \tag{3.7}$$

$$\xi_t = \sum_{s=1}^{t} Z_t^T Y_t. \tag{3.8}$$

Equations (3.6) - (3.8) implies we can now write

$$R_t = R_{t-1} + Z_t^T Z_t \tag{3.9}$$

and

$$\xi_t = \xi_{t-1} + Z_t^T Y_t. \tag{3.10}$$

Now, finally

$$\begin{aligned} \hat{\theta}_t = R_t^{-1} \xi_t = R_t^{-1}(\xi_{t-1} + Z_t^T Y_t) = \\ R_t^{-1}(R_{t-1}\hat{\theta}_{t-1} + Z_t^T Y_t) = \\ R_t^{-1}(R_t\hat{\theta}_{t-1} - Z_t^T Z_t \hat{\theta}_{t-1} + Z_t^T Y_t) = \\ \hat{\theta}_{t-1} + R_t^{-1} Z_t^T (Y_t - Z_t \hat{\theta}_{t-1}). \end{aligned}$$

We have in (3.9) an equation for R_t, however we are interested in R_t^{-1}. The matrix inversion lemma states that

$$[A + BCD]^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1}.$$

Yielding the expression we need for the inverse correlation matrix,

$$R_t^{-1} = R_{t-1}^{-1} - \frac{R_{t-1}^{-1}(Z_t^T Z_t) R_{t-1}^{-1}}{Z_t R_{t-1}^{-1} Z_t^T + 1}. \tag{3.11}$$

When estimating time-varying parameters using RLS one might reduce the impact of older data. A common way to handle this is using a forgetting factor $\lambda \in (0,\, 1]$ which gives exponentially less weight to older error samples. This approach begins with a modified loss function

$$L(\theta_t) = \sum_{s=1}^{t} \lambda^{t-s} (Y - Z\theta)^2. \tag{3.12}$$

From here it's straightforward to derive the RLS based on (3.12) and we end up with

$$R_t^{-1} = \lambda^{-1} R_{t-1}^{-1} - \frac{\lambda^{-1} R_{t-1}^{-1} (Z_t^T Z_t) \lambda^{-1} R_{t-1}^{-1}}{Z_t \lambda^{-1} R_{t-1}^{-1} Z_t^T + 1} \tag{3.13}$$

where λ are set to 0.99. All steps in the estimation are summarized in Algorithm 2 below.

Algorithm 2: The recursive least square algorithm.

Data: $\{(y_t, Z_t)\}_{t=1}^N$
Result: $\{(\theta_t)\}_{t=1}^N$

$\theta_0 = 0,\ R_0 = Z^T \mathrm{Z}$; `// Initialization`
for $t \leftarrow N$ **do**
  $\mathrm{y}_t \leftarrow Z_t^T \theta_t$; `// Filter output`
  $\mathrm{e}_t \leftarrow d_t - y_t$; `// Error signal`
  $R_t \leftarrow \lambda^{-1} R_{t-1}^{-1} - \frac{\lambda^{-1} R_{t-1}^{-1} (Z_t^T Z_t) \lambda^{-1} R_{t-1}^{-1}}{Z_t \lambda^{-1} R_{t-1}^{-1} Z_t^T + 1}$; `// Correlation matrix`
  $\Delta_t \leftarrow R_t Z_t^T e_t$
  $\theta_t \leftarrow \theta_{t-1} + \Delta_t$
end

Model	θ	Z
CAPM	$\begin{pmatrix} \mathrm{b} \\ \alpha \end{pmatrix}$	$\begin{pmatrix} R_m - R_F \\ 1 \end{pmatrix}$
FF3FM	$\begin{pmatrix} \mathrm{b} \\ \mathrm{s} \\ \mathrm{h} \\ \alpha \end{pmatrix}$	$\begin{pmatrix} R_m - R_F \\ SMB \\ HML \\ 1 \end{pmatrix}$
FF5FM	$\begin{pmatrix} \mathrm{b} \\ \mathrm{s} \\ \mathrm{h} \\ \mathrm{r} \\ \mathrm{c} \\ \alpha \end{pmatrix}$	$\begin{pmatrix} R_m - R_F \\ SMB \\ HML \\ RMW \\ CMA \\ 1 \end{pmatrix}$

Table 3.1: θ is the parameter vector and Z the factors.

In table 3.1, factor Z for the factor models are listed and the resulting parameters θ we get from the RLS algorithm.

3.5 Forecasting

In order for the MPC algorithm to work, we need to forecast future returns. The forecasting model is simple. To get a prediction for period $t+k$ given period t, we forecast parameters k periods ahead and plug into the model with factors for period t. Here, we are assuming that factors don't change to much day to day and that they affect each other in the long run. Predictions in the FF3FM and FF5FM for period $t+k$ are computed by,

$$R_{i,t+k} = R_{F,t} + b_{i,t+k} \cdot (R_{M,t} - R_{F,t}) + s_{i,t+k} \cdot SMB_t + h_{i,t+k} \cdot HML_t + \alpha_{i,t+k} + e_{i,t+k}. \tag{3.14}$$

In the case of CAPM, there is only have one factor-parameter, b and we use a standard AutoRegressive model to predict one period ahead. For FF3FM and FF5FM we use a Vector AutoRegressive model (VAR). Essentially, the VAR model is an extension of the standard autoregressive model. The VAR model is useful when one is interested in predicting multiple time series variables using a single model. In a VAR model we regress a vector of time series variables on lagged vectors of these variables. For the VAR(1) we have,

$$\theta_{t+1} = A_i \theta_t + \eta_t. \tag{3.15}$$

Where A is a $k \times k$ coefficient matrix, θ_t parameters and $\eta \in \mathbb{MVN}(0, \Sigma_\eta)$.

The best linear predictor of parameters in terms of minimum mean square error, of θ_{t+1} based on information available at time t:

$$\hat{A} = (\sum_{t=1}^{T-1} \theta_{t+1} \theta_{t+1}^T)(\sum_{t=1}^{T-1} \theta_t \theta_t^T)^{-1}. \tag{3.16}$$

Chapter 4

Method

4.1 Data

The data considered for this study are 10 stocks chosen from Standard & Poor's 500 index (S & P Global). It is important to choose assets for the algorithm that are liquid enough. An illiquid asset could cause problem for the forecasting process, e.g. time intervals where asset price doesn't change or even missing samples. The data consists of daily closing prices from the year 1996 to 2018. One could of course choose smaller sampling intervals such as one hour, 30 minutes or even five minutes. For the empirical study, the algorithm are tested with two different portfolios. One small with the Microsoft stock and a risk-free asset. The larger portfolio contains all 10 stocks and a risk-free asset. In figure 4.1 and 4.3 are two of the assets log-prices shown. They both display the typical behaviour of financial time-series data. Factor-data are collected through French's own data library. He updates the website on a monthly basis with new factors. One could also estimate them by yourself if you have access to market data. For investors operating of smaller time frames, say, on a daily basis you would have to create the factors yourself using some provider of market-data.

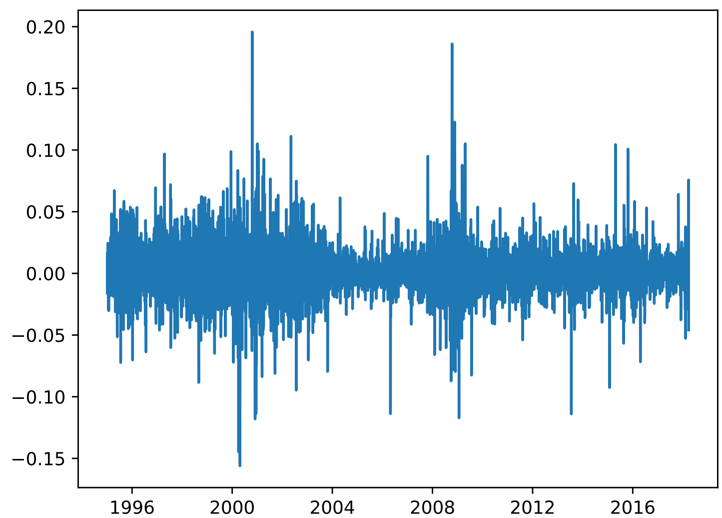

Figure 4.1: Log-returns of Microsoft.

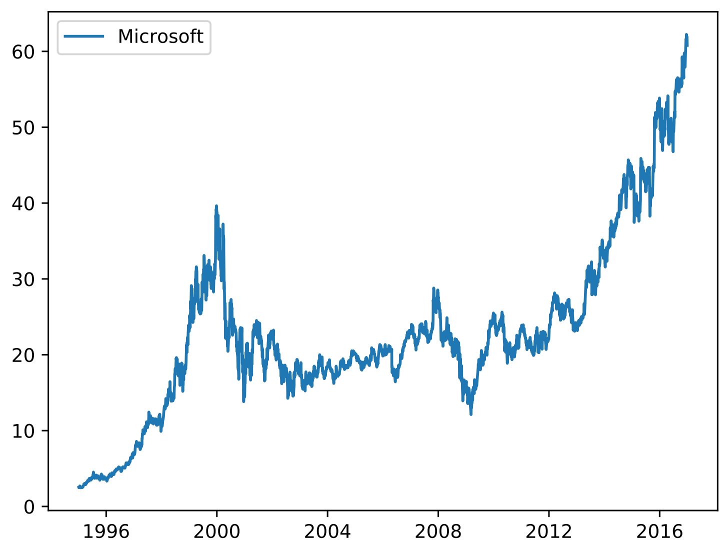

Figure 4.2: Price of Microsoft in dollars.

Dot com crisis hits Microsoft around year 2000.

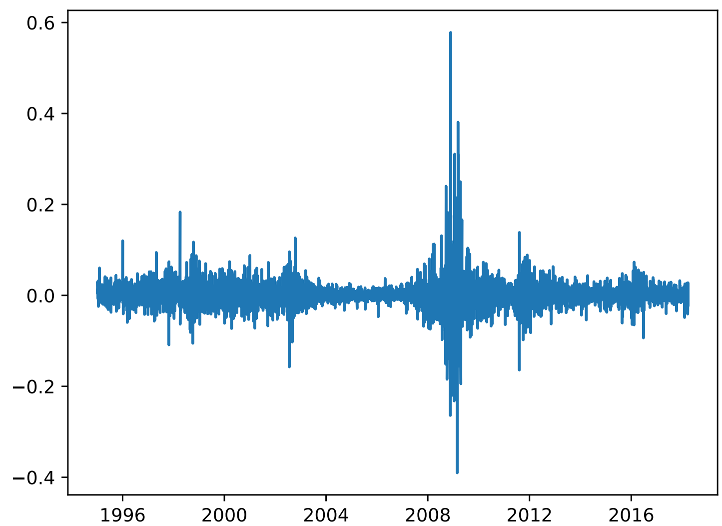

Figure 4.3: Log-returns of Citigroup.

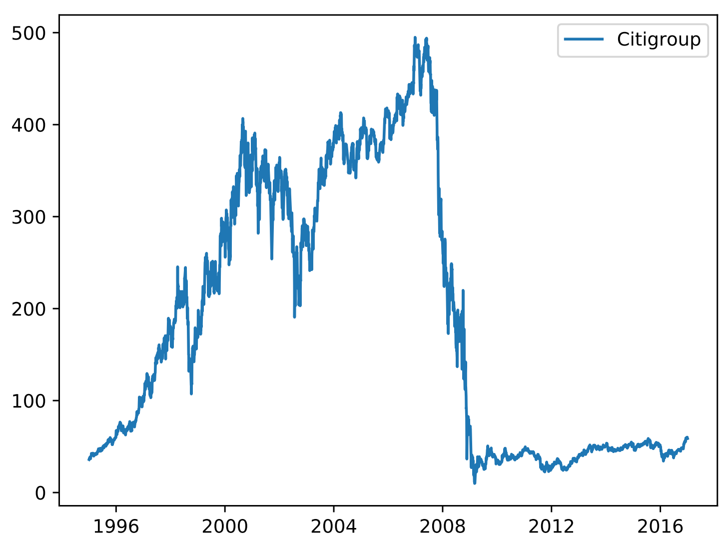

Figure 4.4: Price of Citgroup in dollars.

2008 financial crisis affects American investment bank Citigroup in a extreme manner. The stock has yet to rise ever since.

4.2 MPC parameters

We have three hyperparameters to consider when backtesting the models. The risk aversion parameter γ, the trading aversion parameter κ and the holding cost multiplier ϕ (all defined in section 2.2). A high value on ϕ makes the SPO algorithm to avoid short positions. However, testing different values yields that long only position performs worse. Therefore, ϕ are set to 0.0001, which somewhat reflects the price of the risk-free asset (price of risk-free asset changes over time, but relatively small). One could input a vector for time-varying holding cost, which probably would be more realistic in a real-world setting.

Transaction cost κ are set to 0.0001. This is believed to be a realistic transaction cost for a high frequency trading institution. Maximum amount of leverage are set to one. Meaning shorting is allowed. By testing different values for γ is becomes clear that a low value will yield high returns, but also high maximum drawdowns. By setting γ to 200 we get a reasonable balance between returns and drawdowns.

4.3 Implementation

The implementation of the portfolio optimization is done using the programming language Python. In 2017 Boyd et al. released a library called CVXPortfolio that are used for MPC. An advantage of the package is that forecasting is performed separately from the optimization, allowing any method of forecasting.

Chapter 5

Empirical results

5.1 Forecasting model

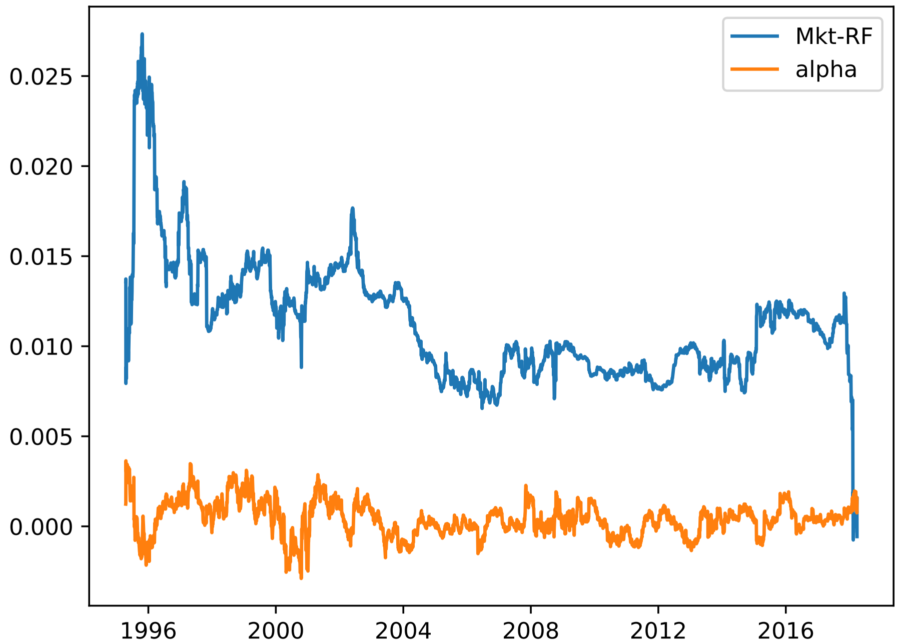

Figure 5.1: Forecasted factor coefficients for Microsoft using CAPM.

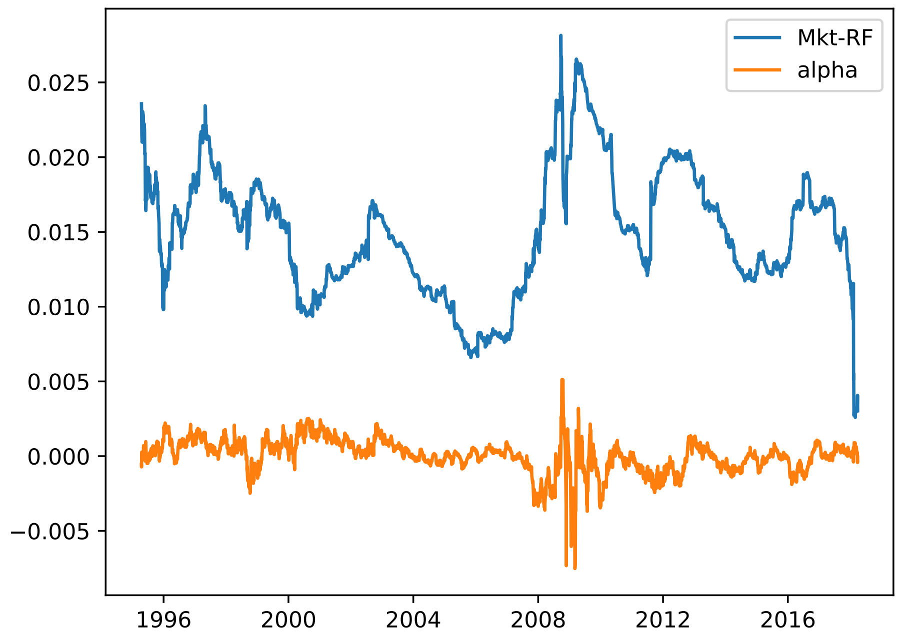

Figure 5.2: Forecasted factor coefficients for Citigroup using CAPM.

Factor b (Mkt-RF) displays are more volatile and almost cyclical manner in Citigroup with the greatest spike at 2008 financial crisis.

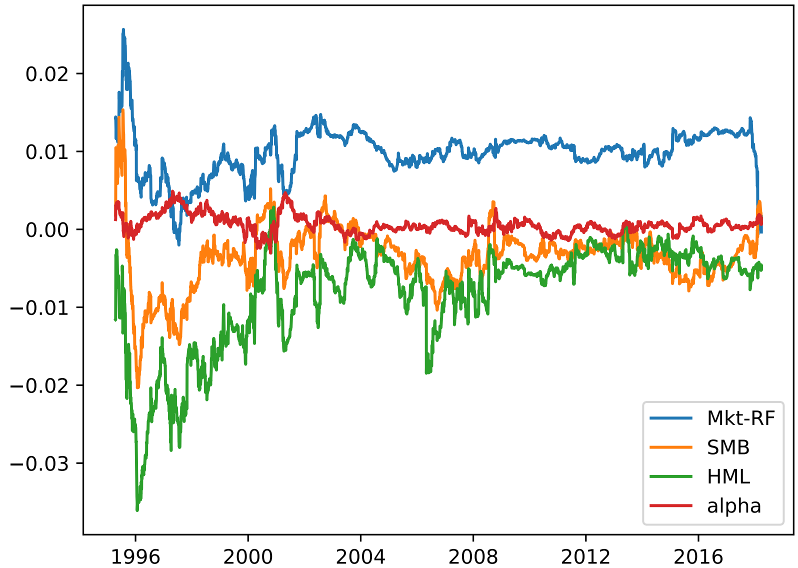

Figure 5.3: Forecasted factor coefficients for Microsoft using FF3FM.

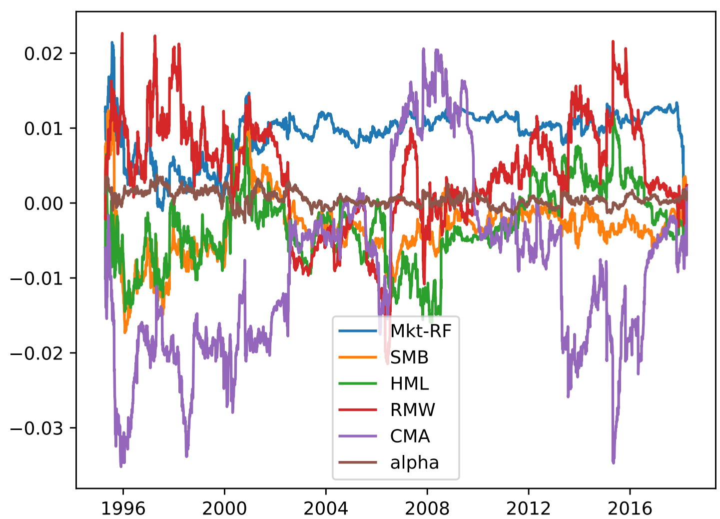

Figure 5.4: Forecasted factor coefficients for Microsoft using FF5FM.

Comparing plots of forecasted parameters in figure 5.4 and 5.3 we see that

α hovers around zero most of time for both model. An α around zero indicates that factors are capturing variations in the data. Around 1997-1997 HML for Microsoft goes deeply negative reflecting that future earnings were in danger for the tech company. RMW and CMA seems to be negatively correlated to each other. From 2008 RMW factor remains high which makes sense because of how good Microsoft stock performs.

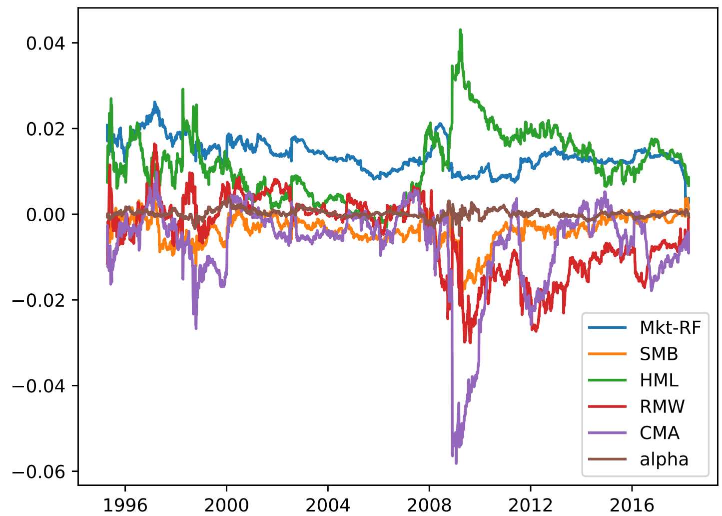

Figure 5.5: Forecasted factor coefficients for Citigroup in FF5FM.

During the 2008 crisis can also be seen in the forecasted parameters in figure 5.5 with massive spikes in CMA and HML. When the bank gets into the crisis profitability factor RMW dips, which makes sense since the banks future earnings are deemed to be affected. RMW continues to be negative until 2016 which does not surprise considering the price in figure 4.4. The investment factor CMA has an even greater spike. For Microsoft the two new factors were negatively correlated but this is not true for Citibank.

Table 5.1: Chosen hyper-parameters.

parameter	value
γ	200
κ	0.0001
ϕ	0.0001

5.2 SPO small portfolio

Table 5.2: Total performance of SPO portfolio containing one stock and one risk-free asset over the period between 2003-01-03 and 2018-01-03.

	Buy & hold	CAPM	FF3FM	FF5FM
Annual return	0.11	0.26	0.28	0.29
Excess risk	0.26	0.15	0.15	0.15
Sharpe ratio	0.48	1.5	1.6	1.7
Maximum Drawdown	58	23	20	23

The three factor models all outperform the buy and hold strategy. All the models have the same excess risk 0.16. This is probably due to the fact that all three models have the same parameter b that decides risk of the stock. In terms of annual returns FF3FM and FF5FM are superior to CAPM, leading to a higher Sharpe ratios. But the added complexity in the jump from FF3FM to FF5FM can be hard to justify since their is little difference in performance between the and FF5m comes with higher maximum drawdown.

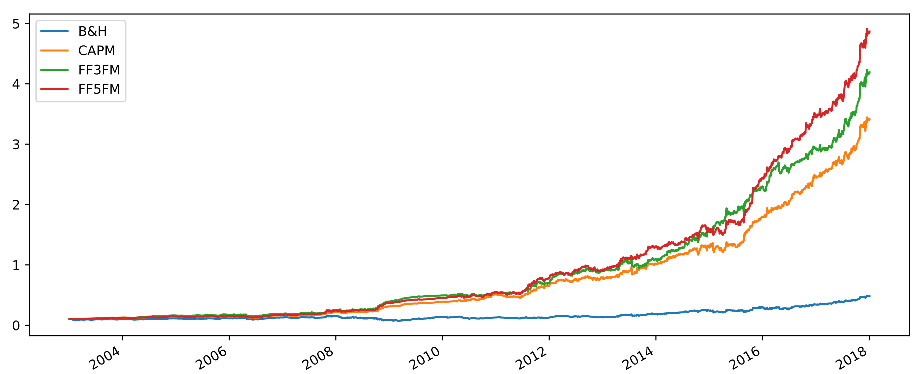

Figure 5.6: Value for portfolios containing Microsoft stock and a risk-free asset and starting value $V_0 = 0.1$.

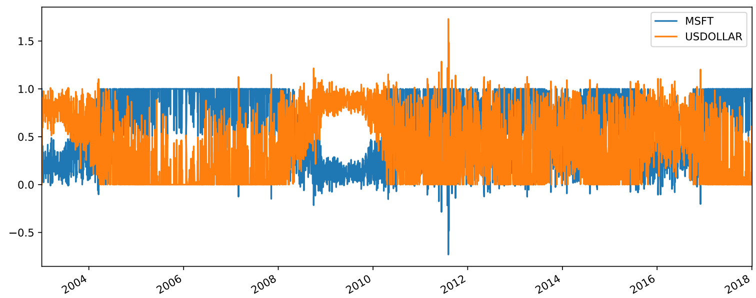

Figure 5.7: Weights in the FF5FM portfolio containing Microsoft stock.

5.3 SPO of portfolio of 10 stocks

Table 5.3: Total performance of SPO portfolio containing 10 stocks over the period between 2003-01-03 and 2018-01-03.

	Buy & hold	CAPM	FF3FM	FF5FM
Annual return	0.23	0.31	0.54	0.56
Excess risk	0.24	0.16	0.16	0.16
Sharpe ratio	0.9	1.7	2.7	2.8
Maximum drawdown	59	23	22	18

As in the small portfolio all three factor models all outperform the buy and hold strategy. All the models have the same excess risk 0.16. Remember . In terms of annual returns FF3FM and FF5FM are superior to CAPM, leading to a higher Sharpe ratios. But the added complexity in the jump from FF3FM to FF5FM can be hard to justify since their is little difference in performance between the and FF5FM has higher maximum drawdown.

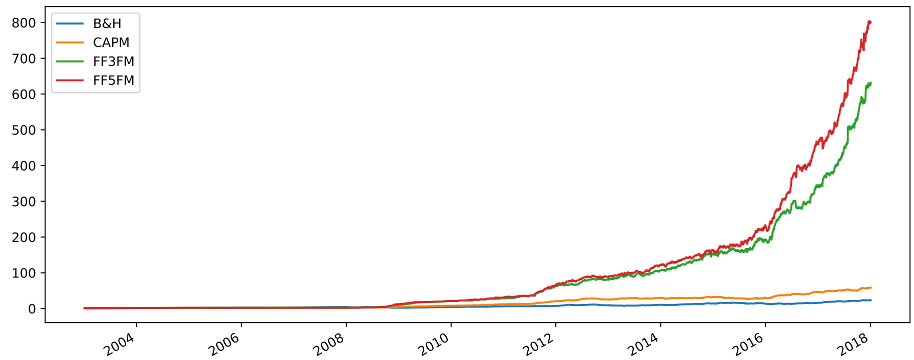

Figure 5.8: Comparing the value over time for different strategies and starting value $V_0 = 0.1$.

The result of the backtest shows that for both portfolios, all factor models outperforms the buy and hold strategy in the sampled period. As can be seen in figure 7.3 and 5.7 both portfolios goes in time of crisis. The large portfolio are shorting some stocks during the banking crisis, notably the investment bank Citigroup. This is a positive result because the models, especially FF3FM and FF5FM manages to make a profit in large downturns and uncertainties as seen in figure 5.8. Even though shorting is allowed, it is rarely used in the small portfolio. This is probably due to the good performance of the underlying stock.

Chapter 6

Conclusion

The SPO hyper-parameters were manually tuned. It resulted in a model that worked, but it is not the ideal way to proceed (Nystrup et al. 2020). The hyper-parameters are dependent and a small change in one of them means you might have to calibrate the other ones. This led to a manual approach where a set of parameters were chosen and the SPO algorithm was run. Obviously, this is not an ideal approach. You would like to run a coarse search and then a fine search of parameters to get the optimal ones. Instead this manual testing led to a set of parameters that where good enough. The risk-aversion parameter γ were scaled up relatively high at 200. The reasoning behind this choice was to reduce the excess risk and more importantly, reduce the maximum drawdown which was high for small values of γ.

Comparing the two Fama and French models, it's not obvious if the advantages of using the more complicated model outweighs the disadvantages. There are two more factors included but the FF5FM are only able to reduce the drawdown compared to FF3FM. Approaching SPO using factor models. As stated before, the excess risk for all Fama French models are the same. It would be interesting to include a model which includes factor for better model risk. Applying factor models for portfolio optimization has been success full and further research in this area could try to do a similar study but include multi-period forecasting. In theory it should outperform the SPO portfolio due to multi-period forecast which should predict longer periods of downturns and prevent costly and unnecessary trades. As stated earlier regarding downloading factors, French updates the data on a delayed monthly basis and there are factors available on a monthly basis and daily basis. This is problematic if one want's to use this model for trading in real-time. If one gets access to streaming market-data the forecasting and MPC could be combined in one program making the factor models viable in real-world trading.

The Python library CVXPortfolio used for this thesis are simple to setup in the case of single-period optimization but in the more complex case of multi-period optimization the library has some issues regarding dependence on other popular libraries such as pandas. This is probably due to the fact that CVXPortfolio has not been updates since April 2020. Therefore interested readers might consider waiting for the library to be updated or use another similar library called CVXPY (also created by Boyd) when utilizing multi-period optimization.

Chapter 7

Appendix

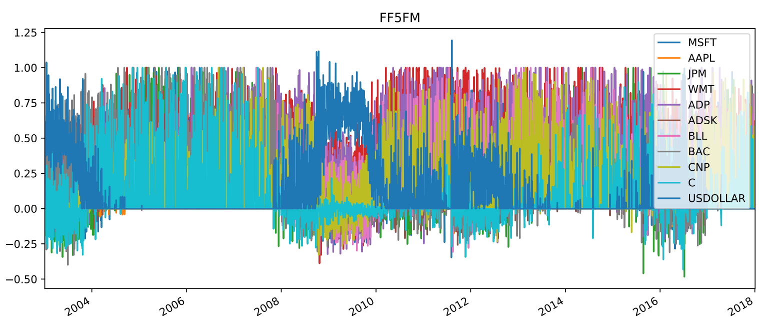

Figure 7.1: Weights in the FF5FM portfolio containing 10 stocks.

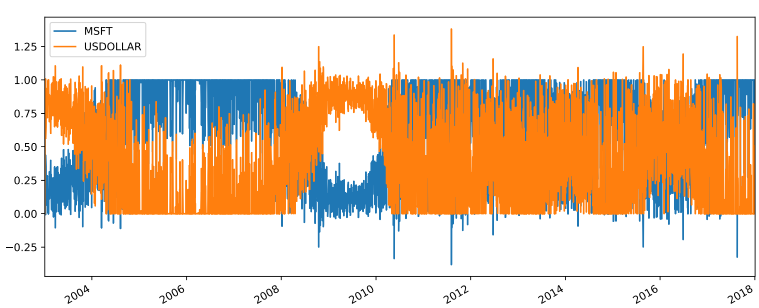

Figure 7.2: Weights in the CAPM portfolio containing Microsoft stock.

Figure 7.3: Weights in the FF3FM portfolio containing Microsoft stock.

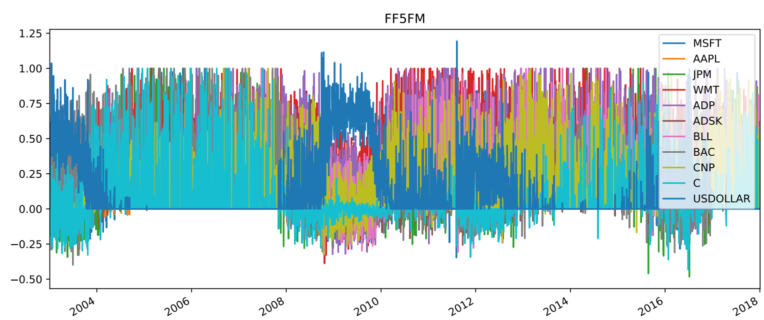

Figure 7.4: Weights in the FF5FM portfolio containing 10 stocks.

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