‘Portfolio Optimization – Bitcoin & Downside Risk’

Vilgot Kraft

Lund University School of Economics and Management

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Supervisor: Thomas Fischer

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Abstract

The purpose of this paper is to analyze how the inclusion of cryptocurrency, specifically Bitcoin, affects downside risk in a diversified portfolio. The analysis utilizes a number of performance measures and combines Modern Portfolio Theory with a Post-Modern Portfolio Theory optimization in order to evaluate different portfolios. The portfolios are also benchmarked against a naive diversification portfolio. All of the included portfolios are evaluated on a time period starting from the 1st of January 2015 and ending by the 31st of March 2022.

The results of this study show that Bitcoin can be utilized to obtain higher expected returns. Depending on the risk appetite of the investor, different weight of Bitcoin can be included in a portfolio. A Sharpe Ratio optimized Bitcoin portfolio exhibits 27,77 percentage points higher annual expected return than a Sharpe Ratio optimized portfolio excluding Bitcoin. However, the annual downside risk of the portfolio also increases by 6,87 percentage points when including Bitcoin. The results also show that a smaller weight of 7,5% allocated to Bitcoin can be advantageous compared to a Sharpe Ratio optimized portfolio excluding Bitcoin. The minor weight of Bitcoin improves the performance measures but demonstrates a slight decrease in expected return. This paper therefor concludes that Bitcoin can positively affect performance in a diversified portfolio, as long as it is balanced with major weights in less risky allocations, such as bonds.

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1 Introduction

The technological development made it possible to create decentralized digital currency which challenged the fiat money issued by central banks. In 2008, Nakamoto published a white-paper stating the background and technological prerequisites for the now known cryptocurrency, Bitcoin. Bitcoin relies on a technology known as blockchain, which is an authentication system for transactions. To validate transactions, hashes are mined with processor power from computers (Nakamoto, 2008).

Since then, Bitcoin is known to be both a currency and a speculative asset with increasing interest throughout the years. On 9th of June 2021, El Salvador announced Bitcoin as official tender in the country (Solomon, 2021). It is widely discussed among academics whether Bitcoin is an asset, or a currency, and researchers have investigated both classifications. However, this thesis will focus on Bitcoin as an asset, with a portfolio management approach.

Portfolio management research focuses on finding the optimal portfolio by balancing risk with returns. Modern Portfolio Theory and its model mean-variance analysis was introduced by Harry Markowitz in 1952. The model provides investors the capabilities to weight risks and variance with the expected return of different portfolio combinations (Markowitz, 1952). The mean-variance approach is the underlying assumptions behind the now well-known Sharpe Ratio which was introduced by William Sharpe in 1966. The Sharpe Ratio is a ratio of expected return and risk. To find desirable weights of assets in a portfolio, Sharpe Ratio is a common optimization problem (Sharpe, 1966). Due to limitations in the assumptions of Modern Portfolio Theory, a Post-Modern Portfolio Theory was developed. Post-Modern Portfolio Theory focuses on the downside risk with performance measures such as downside standard deviation (downside risk), Sortino Ratio and Conditional Value-at-Risk.

Combining both Modern Portfolio Theory and Post-Modern Portfolio Theory this thesis aims to examine the existence of Bitcoin in a diversified portfolio and analyze the downside risks associated with a positive weight in Bitcoin. The research question is specified as:

'How does Bitcoin affect downside risk measures in a diversified portfolio?'

To answer this question, portfolios containing Bitcoin, ETF funds and the S&P500 were created. ETF funds were used for simplicity as they provide a well-diversified fund with a large range of exposure. The portfolios were then optimized for different performance or risk measures and benchmarked with each other.

The findings in this thesis demonstrate that Bitcoin is included in the different optimizations, Bitcoin provides a portfolio with extraordinarily returns which tend to outperform the risk taken, risk-return measures are thus improved when Bitcoin is included. The risk-return measures are also improved when Bitcoin is included in a naive diversification portfolio. In one of the Conditional Value-at-Risk optimized portfolios a positive weight of Bitcoin is balanced by a major weight in the least risky asset. This portfolio composition results in an improved risk-return performance with just a slight decrease in expected return compared to a Sharpe Ratio optimized portfolio excluding Bitcoin. The thesis therefor concludes that Bitcoin can improve performance of a diversified portfolio.

The thesis is organized as follows: Chapter 2 introduces the theoretical background behind portfolio theory and latter in this chapter performance and risk measures are introduced. Chapter 3 summarizes previous and adjacent research. Chapter 4 explains the methodology used, Chapter 5 presents the data and descriptive statistics. Chapter 6 summarizes the results followed by a discussion. The thesis ends with concluding remarks in Chapter 7.

2 Theoretical Background

2.1 Modern Portfolio Theory

1952 Harry Markowitz published his famous paper 'Portfolio Selection', introducing mean-variance analysis. Markowitz (1952) argues that an investor should see return as desirable and variance as undesirable. Therefor the model aims to create portfolios where the investor can determine between different expected returns given a level of risk, more specifically their variance. The portfolio returns and its variance are computed in the following way:

$$E(R_p) = \sum_{i=1}^{n} W_i U_i$$

Equation 2.1

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \sigma_{i,j}$$

Equation 2.2

In Equation 2.1, $E(R_p)$ is the expected return of the portfolio and W_i the fraction of the total portfolio invested in asset i with U_i being the return of the asset. In Equation 2.2, the portfolio variance is computed with the fraction of the total portfolio of some asset i and some asset j and the covariance between i and j, denoted as $\sigma_{i,j}$ Covariance is defined as:

$$\sigma_{i,j} = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

Equation 2.3

In Equation 2.3, $[R_i\text{-}E(R_i)]$ is the deviation from the mean for asset i and $[R_j\text{-}E(R_j)]$ is the deviation from the mean for asset j. Thus, this can also be calculated with the correlation coefficient (ρ) times the standard deviation for asset i and j, expressed as:

$$\sigma_{i,j} = \rho_{i,j} \sigma_i \sigma_j$$

Equation 2.4

The different combinations of variance and targeted expected return can be illustrated with a minimum-variance frontier. The frontier demonstrates the lowest obtainable standard deviation given a specific expected portfolio return. If short selling is allowed, all individual assets will lay to the right of the frontier. This means that a portfolio of only one risky asset is expected to have a higher standard deviation than a portfolio with more assets given the same expected return. Diversification allows an investor to choose a portfolio with lower standard deviation and the same or higher expected return. From the global minimum and upwards is the efficient frontier, the other part of the frontier contains combinations where an investor can choose a higher expected return at a lower given standard deviation and is thus non-efficient (Bodie, et al., 2014).

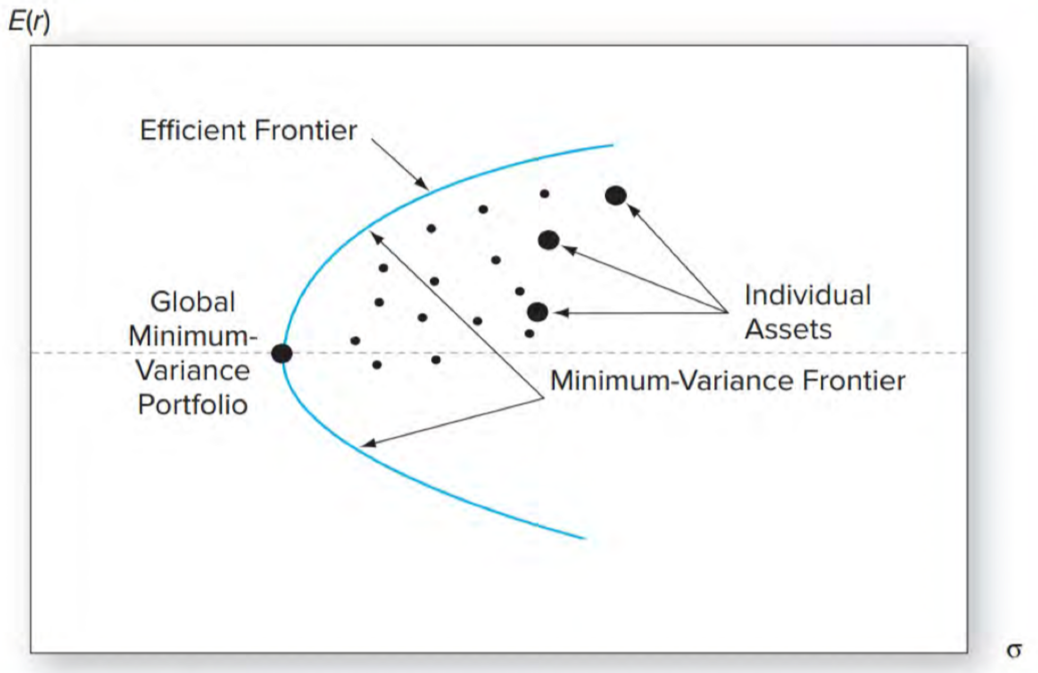

Figure 2.1: The minimum-variance frontier of risky assets (Bodie et al., 2014, p. 220).

2.2 Post-Modern Portfolio Theory

Markowitz' Modern Portfolio Theory (MPT) is limited by symmetrical standard deviation and normal distribution. The model implies that a portfolio with more upside returns than downside returns appears riskier than it is, as standard deviation covers both the upside and downside risk. This is a major shortcoming in Modern Portfolio Theory, and it is widely recognized that an investor does not view risk as positive returns above their investment targets. On the other hand, risk is viewed as the outcomes below the targets, i.e., bad outcomes (Rom and Ferguson, 2001).

Downside risk can provide insights when comparing performance results and give an understanding if an investor were compensated for the level of risk taken. However, critics of Post-Modern Portfolio Theory argue that the upside is ignored when solely focusing on the downside. Some would argue that high positive returns would imply equivalent negative returns. However, Rom and Ferguson (2001) emphasize that this criticism is not supported by published research and thus inaccurate.

2.3 Performance Measures & Risk Measures

To analyze portfolio and conduct portfolio optimization a set of performance metrics will be introduced in this chapter.

2.3.1 Sharpe Ratio

William F. Sharpe (1966) outlines that the performance of a given portfolio is explained with expected return and its standard deviation of the return. With the Sharpe Ratio one can measure the performance of a single asset or a portfolio of assets against its risk. The Sharpe Ratio is defined as:

$$S_p = \frac{E(R_p) - R_f}{\sigma_p}$$

Equation 2.5

In Equation 2.5, S_p is the Sharpe ratio for a given portfolio or asset, $E(R_p)$ the expected return for a given portfolio or asset and σ_p the standard deviation for a given portfolio or asset. R_f is the risk-free rate.

2.3.2 Sortino Ratio

Downside risk is measured with semi-standard deviation which measures the variance under a specific target rate. The target rate can either be a benchmark or the risk-free rate. Downside risk is defined as follows:

$$\sigma_d = \sqrt{\sum_{i=1}^{N} \frac{\min[(R_i - R_t), 0]^2}{N}}$$

Equation 2.6

In Equation 2.6, R_i is the return of asset i, R_t the target return and 0 are the positive returns. As the positive returns are not included in the equation, one must be careful with the data and assure there are enough data points to calculate the downside risk (Bacon, 2010).

Sortino is an extension of Sharpe ratio and uses the downside risk measure. The ratio is given by the following equation:

$$SR_p = \frac{R_p - R_t}{\sigma_d}$$

Equation 2.7

In Equation 2.7, R_t is the target return or the risk-free rate, R_p the portfolio return and σ_d the downside risk (Bacon, 2010).

2.3.3 Conditional Value-at-Risk

Conditional Value-at-Risk (CVaR) has numerous superior properties to its similar measure Value-at-Risk (VaR) even though they both are risk measures. While VaR does not work well with non-normal distributed losses, CVaR can handle different distribution of losses (Rockafellar and Uryasev, 2002). CVaR is also proven to be coherent and has properties such as: transition-equivariant, positively homogenous, convex, and monotonic. Optimization of a portfolio with respect to CVaR is a stochastic optimization problem.

Rockafellar and Uryasev (2000) define a cumulative distribution function as $\Psi(x, *)$ for a loss $z = f(x,y)$. $\Psi(x,\zeta)$ is the left limit of $\Psi(x, *)$ at ζ. y is a vector of uncertain market variables affecting the value of f(x,y). Elaborated to:

$$\Psi = (x, \zeta) = P\{y|f(x,y) < \zeta\}$$

Equation 2.8

Given a specific confidence level α the VaR is defined as:

$$\zeta_\alpha(x) = \min\{\zeta|\Psi(x,\zeta) \geq \alpha\} \; and \; \alpha \in [0,1]$$

Equation 2.9

Furthermore, the CVaR is expressed as:

$$CVaR(x) = (1-\alpha)^{-1} \int_{f(x,y)>\zeta_\alpha(x)}^{\cdot} f(x,y)p(y)dy$$

Equation 2.10

CVaR will measure the average loss when the loss exceeds or is equal to the VaR given the specific confidence level. I.e., the average loss in the tail end of the distribution (Rockafellar and Uryasev, 2002). Given a 5% α means that the loss of the 5% worst scenarios is the CVaR(95%).

2.3.4 Modigliani Risk-Adjusted Performance

Risk-adjusted measures were introduced to benchmark not only by total return but instead by the return relative to the risk of the portfolio. M^2 will measure the performance of a portfolio compared to an unmanaged market portfolio, a market index, or a world index (Modigliani and Modigliani, 1997). The equation is denoted as follows:

$$M^2 = R_p + S_p(\sigma_m - \sigma_p)$$

Equation 2.11

In Equation 2.11, R_p is the return of the portfolio, S_p the Sharpe ratio of the portfolio, σ_m is the market risk (standard deviation of the unmanaged market portfolio) and σ_p the standard deviation of the portfolio. However, M^2 can also be measured with the Sortino Ratio in the following way:

$$M_s^2 = R_p + SR_p(\sigma_{dm} - \sigma_d)$$

Equation 2.12

In Equation 2.12, σ_d is the downside risk for the portfolio calculated in equation 2.6 and with same equation σ_{dm} is the downside risk for the unmanaged market portfolio. SR_p is the Sortino Ratio for a portfolio derived in equation 2.7.

2.4 Cryptocurrency

Cryptocurrency emerged as an opposition to the centralized fiat currencies. Cryptocurrency aims to revolutionize the financial system by utilizing the technological advancements made in the recent years. The idea is that a cryptocurrency should rely on its framework which provides the currency with safety, supply, and verification in a decentralized manner. Cryptocurrencies are built on the mechanisms of cryptography combined with mathematical protocols (Narayanan, et al., 2016). However, neither cryptography nor mathematical protocol are in the scope of this thesis.

2.4.1 Bitcoin

The white paper of Bitcoin suggests a variant of electronic cash with immediate transactions without an intermediate financial institution. The underlying technological solution behind this is now known as blockchain using peer-to-peer networks to validate and secure transactions. The technology is set up to be decentralized and as the processor power is spread around the world and there are no restrictions to add nodes into the network (Nakamoto, 2008). However, Nakamoto (2008) emphasizes that the network has to have a sufficient amount of honest nodes to remain secure. If someone would control a majority of the processor power in the network, they can defraud and steal coins. Nevertheless, by convention the nodes are incentivized by transaction fees and issuing of new coins to stay honest and thus more profitable and as the network grows it is more difficult to assemble a majority of the processor power.

The characteristics and nature of Bitcoin are widely discussed. The matter of discussion is whether it is a currency as Nakamoto (2008) intended and therefore meant for payments or if it is an asset. As this thesis will include Bitcoin in a portfolio it will be deemed to be an asset. This corresponds to the conclusions Glaser et al (2014) made when analyzing trading data of Bitcoin. The study indicated that first time buyers bought Bitcoin for speculative purposes rather than using it as a currency. Although, the study might be inconclusive as the study covers a limited time period. Dyhrberg (2016) contributes to the discussion and concludes that Bitcoin is similar to gold and thus has hedging capabilities and is reactive to news. Dyhrberg (2016) suggests that Bitcoin is somewhere in between an asset and a currency because of its nature and that is a useful tool for portfolio management as well as risk analysis.

Cheah and Fry (2015) highlights that cryptocurrency is under-explored from an academic point of view. They stress that Bitcoin among other asset classes can be exposed to speculative bubbles and that the fundamental value of Bitcoin is zero. They reach the conclusion that the long-term viability of Bitcoin is scarce.

3 Previous research

Since the creation of Bitcoin in 2008, research has been conducted to analyze the nature of cryptocurrencies, the technology behind them and the application of them. Economists have analyzed if Bitcoin is an investable asset, and some have analyzed how Bitcoin acts in a currency point of view. However, the understanding of Bitcoin in a portfolio theory perspective is not as comprehensive.

Briére et al. (2013) found in an early study comprising portfolio theory and Bitcoin that Bitcoin did not correlate with other assets during the period investigated (2010-2012). Using a mean-variance approach in a portfolio with traditional assets, their main finding was that inclusion of Bitcoin in a portfolio enhances returns with a steep increase in volatility. Furthermore, Briére et al. (2013) mentions that Bitcoin might be contaminated by early-stage behaviour and thus deviate from more extensive analysis.

Platanakis and Urquhart (2020) analyzed Bitcoin in a portfolio theory perspective. By using a stock and bond portfolio the analysis concludes that in an out-of-sample analysis Bitcoin enhances the performance. However, this is in a setting where short-selling is permitted. The analysis is extended to a commodity, stock, and bond portfolio with consistent findings. The paper focuses mainly on the benefits of an out-of-sample analysis using different portfolio optimization models. In addition to Markowitz mean variance, models like Bayes-Stein and Black-Litterman are used.

Ahnhem and Lindberg (2017) also analyzed Bitcoin but in a Swedish portfolio using the Black-Litterman model. They include commodities, gold, Swedish bonds (OMRX), OMX Stockholm 30 (OMXS30) and Swedish Housing Index (HOX) in their analysis. Their analysis concludes that optimized risk-adjusted performance measures of a Swedish portfolio improve by including Bitcoin. Due to the high volatility of Bitcoin, they also conclude that Bitcoin is not included in a minimum variance portfolio. Ahnhem and Lindberg (2017) conclude that their result are characterised by high weights in real estate which was possible since Swedish real estate in their chosen time period had a very low variance and a notably high return.

In a more recent study, Ottosson (2021) explores the inclusion of Bitcoin in an All-Weather portfolio by optimizing for minimum variance and Sharpe Ratio. An All-Weather portfolio is a portfolio theory originated from Ray Dalio, the founder of Bridgewater Associates. The portfolio strategy is to perform during different possible scenarios such as inflation and growth and is thus a way to hedge for unforeseen scenarios in the future. Ottosson (2021) also found that Bitcoin is not included in a minimum variance portfolio due to the high level of volatility. With this said, Bitcoin did have a positive impact on Sharpe Ratio for the portfolio.

Eisl et al. (2015) analyzes the inclusion of Bitcoin under different portfolio optimizations. They suggest a Bitcoin of weight ranging from 1,65% to 7,69%. Their findings are in line with both Ottosson (2021) and Ahnhem and Lindberg (2017) who also found that Bitcoin does have a positive weight in similar portfolios. Furthermore, Eisl et al. (2015) note that Bitcoin does increase the expected return as well as the risk of the portfolios and can thus be used to balance the risk and return in optimal portfolios. It is notable that Bitcoins return performance seems to outweigh the risk.

In conclusion, Bitcoin is a relatively young asset. The research throughout the years has become more complex, starting from a mean-variance approach for a very short time-period to a more complex analysis with optimization under different recognized frameworks. The main findings are that Bitcoin has had a positive weight even though it has a high volatility. This thesis will enhance the analysis by looking at the downside risk in portfolios containing Bitcoin and utilizing a more long-term view.

4 Methodology

4.1 Portfolios

Some initial assumptions are made when constructing the portfolios and analyzing the assets. Short-sale are prohibited and the portfolios are fully invested at all times. Therefor the total value of the portfolio weights will always sum to one. Mathematically, this is expressed as:

$$\sum_{i=1}^{N} W_i = 1 \ and \ W_i > 0$$

Equation 4.1

In Equation 4.1, W_i is the weight in each asset.

Initial transaction costs are not accounted for. All assets are denoted in US dollar. The portfolios were optimized for minimum-variance, Sharpe Ratio and CVaR. Along with this, a non-optimized naive diversification portfolio is created. To further extend the analysis and compare the effect of Bitcoin, a set of portfolios excluding Bitcoin are optimized. The asset included in the analysis is presented in chapter 5.1.

A naive portfolio is a portfolio with equal weights in each asset and mathematically this is denoted as:

$$W_n = \frac{1}{N}$$

Equation 4.2

In Equation 4.2, N is the total number of assets in the portfolio and W_n the naive weight in each asset.

To conclude, the portfolios in the analysis are Sharpe Ratio (SRP), minimum-variance (MINV), naive diversification (ND) and Portfolio(CVaR(95%)) nr. 1-10. With their portfolio counterparts which exclude Bitcoin, they are referred to as Sharpe Ratio (SRP-NOBTC), minimum-variance (MINV-NOBTC), naive diversification (ND-NOBTC) and Portfolio (CVaR(95%)-NOBTC) nr. 1-10.

Standard deviation, downside risk and CVaR will act as risk measures while expected return, Sharpe Ratio, Sortino Ratio and M^2 will act as performance measures.

Due to the nature of Bitcoin being traded 24 hours a day, 7 days a week, all weeks of the year while most other assets is traded approximately 252 days per year all metrics will be reported in daily figures and not annualized.

4.2 Optimizations

Sharpe Ratio

Equation 2.5 is the expression for Sharpe Ratio that is subject for maximization. To calculate this, the excel tool solver is utilized.

Minimum-variance

Equation 2.2 is the expression for variance that is subject for minimization. The optimization was carried out using Microsoft Excel's solver tool.

Conditional Value-at-Risk

Equation 2.8-2.10 expresses CVaR and a confidence level, $\alpha = 5\%$ were chosen in this thesis. To optimize this MathWorks software MATLAB was used. The software has an optional financial toolbox and statistics package. When optimizing for CVaR the package can be utilized to create a matrix of ten portfolios which will be combined to plot an efficient frontier, illustrating how different weights in the assets affect the CVaR measure.

Sortino Ratio

The portfolios were not optimized for Sortino Ratio, but this measure will be used to compare and evaluate the performance of the portfolios.

Modigliani Risk-adjusted performance

The portfolios are not optimized for M^2 as it is used as a benchmark performance measure. To calculate M^2 as in Equation 2.11 the standard deviation of an unmanaged market portfolio is needed. In this case the asset iShares MSCI ACWI ETF is used, which is an ETF meant to replicate the global stock market in a single fund. Further details regarding the fund is given in Chapter 5.2.

5 Data

5.1 Data Collection

The time frame of the data is 2015-01-01 until 2022-03-31. The data has been collected from Yahoo! Finance as well as Investing.com. The data contains daily observations of the included assets. Since Bitcoin is traded 24 hours a day, 7 days a week and 365 days a year it has been normalized to the average of 252 trading days per year. This enables the opportunity to estimate the correlation between Bitcoin and the other assets included. Most of the assets included are exchange traded funds (ETFs). An ETF operates like a mutual fund and aim to track a specific asset class, exchange, or commodity. However, an ETF is traded on an exchange and can thus be sold or bought like a regular stock. Each ETF and what they replicate is specified in 5.2.

5.2 Assets

All assets are denoted in US dollars and the assets included in the analysis and optimization are shown in Table 5.1.

Table 5.1: Summary of assets.

Asset	*Ticker*
Bitcoin	BTC-USD
iShares MSCI Emerging Markets ETF	EEM
iShares S&P GSCI Commodity Indexed Trust	GSG
iShares 7-10 Year Treasury Bond ETF	IEF
iShares Global Energy ETF	IXC
iShares Global Tech ETF	IXN
Aberdeen Standard Gold ETF Trust	SGOL
S&P500	S&P500
iShares MSCI ACWI ETF	ACWI

Bitcoin

The nature of Bitcoin is explained in chapter 2.4.1.

iShares MSCI Emerging Market ETF

EEM is an exchange traded fund launched (ETF) in 2003 with exposure to large- and mid-capitalization equities in emerging markets. It is specified to be diversified and per 2021-03-

31, the number of holdings was 1247. The top 3 sectors in the ETF are financials (22,08%), information technology (21,64%) and consumer discretionary (12,34%) and by geographic the top 3 countries are China (30,03%), Taiwan (16,13%) and India (13,09%) (iShares, 2022a).

iShares S&P GSCI Commodity Indexed Trust

GSG is an indexed trust launched in 2006 that attempts to follow the performance of a fully collateralized investment in future contracts containing a group of commodities futures. The top 3 sectors in the trust are energy (60,76%), agriculture (18,25%) and industrial metals (11,26%) (iShares, 2022b).

iShares 7-10 Year Treasury Bond ETF

IEF is a treasury bond ETF tracking the index composed with US treasury bonds with maturities between seven to ten years. The ETF was launched 2002 and per 2022-03-31 the number of holdings was 11. The credit rating of the underlying securities is AAA rated (iShares, 2022c).

iShares Global Energy ETF

IXC is composed of global equities in the energy sector. The fund was created 2001 and the number of holdings is 47 per 2022-03-31. The fund gives an investor exposure to companies that produce and distribute oil and gas. The top 3 sectors are integrated oil & gas (55,59%), oil & gas exploration & production (20,98%) and oil & gas storage & transportation (11,04%). The top 3 countries are United States (58,81%), Canada (13,16%) and United Kingdom (12,28%) (iShares, 2022d).

iShares Global Tech ETF

IXN is a technology ETF composed of global equities. The fund was launched in 2001 and the number of holdings is 131 per 2022-03-31. The idea of the fund is to give exposure to electronics, software-, hardware- and information technology companies. The top 3 sectors are software & services (42,82%), tech hardware & equipment (31,88%) and semiconductors & semiconductor equipment (25,21%). The top 3 countries in the fund are United States (81,67%), Taiwan (4,04%) and Japan (3,42%) (iShares, 2022e).

Aberdeen Standard Gold ETF Trust

SGOL is a physical-backed gold ETF launched in 2009. As per 2021-12-31 the trust held approximately 40 585 kg of gold. Thus, the idea of the ETF is to follow the performance of the price of gold (Abrdn, 2022).

S&P500

S&P500 contains 500 of the leading companies in the US market. The index was launched in 1957 and is rebalanced quarterly. As per 2022-03-31 the top 3 sectors in the index are information technology (28%), health care (13,6%) and consumer discretionary (12%) (S&P Global, 2022).

iShares MSCI ACWI ETF

ACWI was launched in 2008 and aim to give exposure to both developed and emerging markets. The ETF give investors access to global equities in one single fund. The top 3 sectors are information technology (22,33%), financials (14,50%) and health care (11,86%) and is spread over the following top 3 countries United States (61,26%), Japan (5,41%) and United Kingdom (3,72%) (iShares, 2022f).

Risk-Free Rate

When applicable 3-month US treasury bond is used as risk-free rate.

5.3 Descriptive Statistics

Table 5.2: Descriptive statistics of assets.

Asset	*Mean Return (Annual) %*	*Standard Deviation (Annual) %*	*Skewness*	*Kurtosis*
EEM	7,06	21,75	-0,637	8,438
GSG	5,97	22,86	-0,799	7,669
BTC	305,08	73,75	-0,0530	6,840
IEF	0,38	5,75	0,0630	5,0948
IXC	5,94	28,89	-0,582	19,137
IXN	35,12	22,70	-0,455	12,126
SGOL	11,37	14,02	-0,103	3,604
SP500	20,32	1,81	-0,638	18,309
ACWI	14,21	17,30	-0,993	16,100

Table 5.2 describes the fundamental statistics for the assets included. All assets have a kurtosis exceeding 3 which is the kurtosis for a normal distribution. Bitcoin has a remarkable mean return as well as standard deviation. The standard deviation is well exceeding the other assets.

Figure 5.1 displays a histogram of the daily returns. All assets are centered around zero with different skewness. The assets with negative skewness have a slightly larger range in the left tail-end. EEM, GSG and ACWI exhibit the largest negative skewness. IEF and BTC both have a rather small skewness compared to the other assets. IEF are slightly positively skewed while BTC slightly negatively skewed. Regardless, the range of BTC is much wider with a higher kurtosis than IEF. This indicates a higher standard deviation and higher downside risk. SP500 are centered around zero with rather small tail-end variations, which indicates a lower standard deviation as well as lower downside risk.

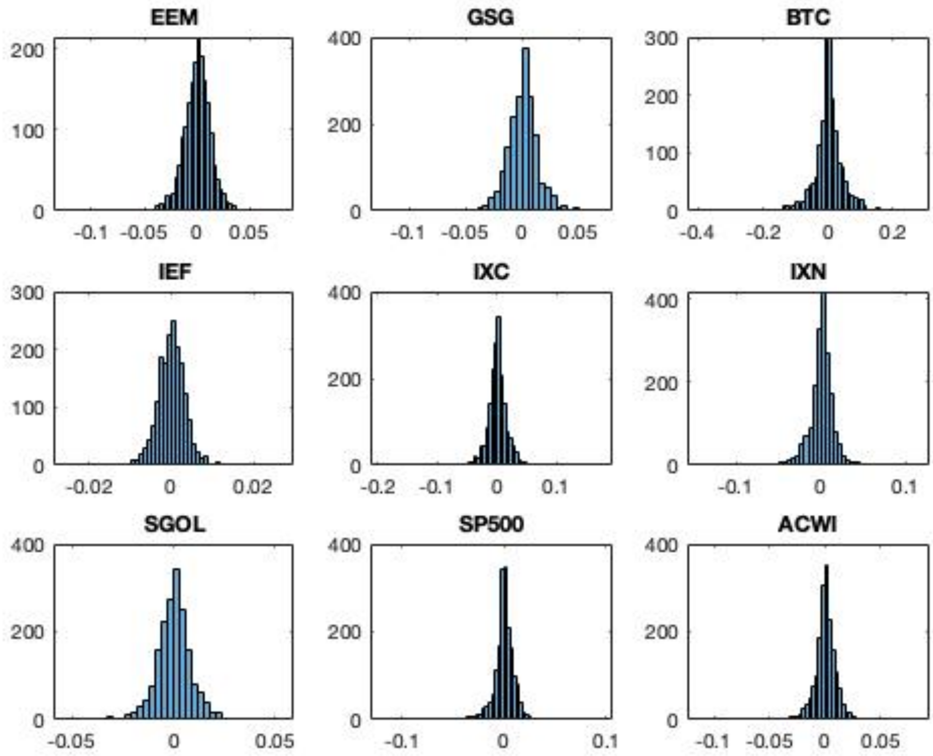

Figure 5.1: Histogram - daily return of assets.

To emphasize the price development in the scope of this thesis, the logarithmic price development of Bitcoin for the time period 2015-01-01 to 2022-03-31 is presented in Figure 5.2. Bitcoin has a positive trend in this period. During 2017-2018 there is a substantial increase. During 2018-2019 Bitcoin is experiencing a slightly negative trend. In general, there has been a continuous growth in Bitcoin, but it seems as if the growth has slowed down from 2021 and onwards.

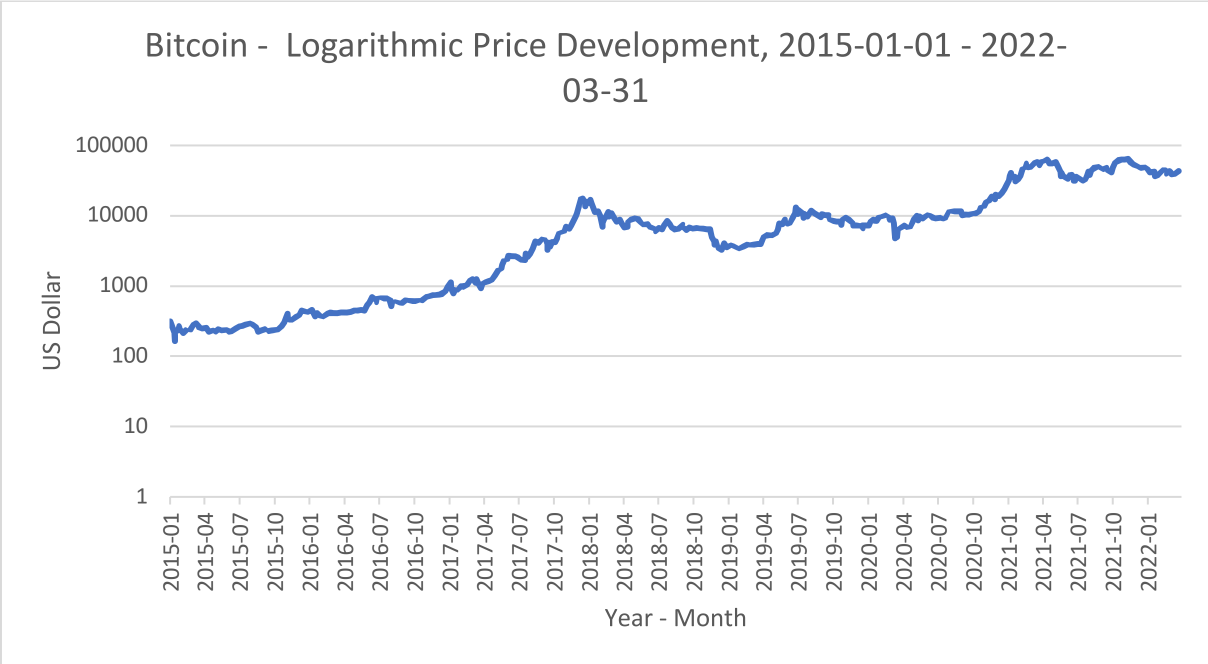

Figure 5.2: Bitcoin - logarithmic price development, 2015-01-01 – 2022-03-31. Data from investing.com.

Table 5.3 illustrates the correlation among the assets which ranges between 1 (blue) and -1 (red). The correlations between Bitcoin and the other assets are ranging from -0,040 to 0,173 which indicate that Bitcoin has a slight positive but low correlation with the other assets. Notable is SP500 and ACWI with a correlation of 0,968. Hence, the SP500 index co-moves almost identically with ACWI. The asset with the most frequent negative correlation is IEF. This indicates that the treasury bond is negative correlated to some extent with the other assets and can thus be used as a diversification tool in portfolios.

Table 5.3: Correlation matrix of assets.

Ticker	*EEM*	*GSG*	*BTC*	*IEF*	*IXC*	*IXN*	*SGOL*	*SP500*	*ACWI*
EEM	1								
GSG	0,391	1							
BTC	0,142	0,056	1						
IEF	-0,260	-0,210	-0,040	1					
IXC	0,655	0,657	0,133	-0,309	1				
IXN	0,786	0,305	0,173	-0,300	0,555	1			
SGOL	0,079	0,128	0,070	0,367	0,043	-0,001	1		
SP500	0,779	0,371	0,167	-0,362	0,688	0,931	-0,021	1	
ACWI	0,871	0,400	0,172	-0,357	0,729	0,916	0,014	0,968	1

6 Empirical Analysis

6.1 Portfolios including Bitcoin

In this section, portfolios containing Bitcoin will be presented. First, three portfolios are presented in Table 6.1; one portfolio optimized for Sharpe Ratio (SRP), a naive diversification (ND) portfolio and a portfolio optimized for minimum variance (MINV). Secondly, ten portfolios are presented in Table 6.2, illustrating an efficient frontier of CVaR optimization in Figure 6.1.

Table 6.1: Weights and annual performance metrics of portfolios including Bitcoin.

		Portfolios	
	SRP	*ND*	*MINV*
EEM	-	0,11	-
GSG	-	0,11	0,040
BTC	0,18	0,11	-
IEF	0,14	0,11	0,82
IXC	-	0,11	-
IXN	0,37	0,11	-
SGOL	0,32	0,11	-
SP500	-	0,11	0,079
ACWI	-	0,11	0,063
		Annual Performance Metrics	
Expected Return %	48,88	29,33	2,88
Standard Deviation %	17,62	16,03	4,54
Variance %	3,1	2,57	0,21
Sharpe Ratio	2,73	1,78	0,46
Downside Risk %	20,32	19,53	6,08
Sortino Ratio	2,36	1,46	0,34
CVaR %	3,94	3,69	1,19
M^2 %	48,35	31,85	8,80
Note: CVaR is calculated with a α of 5%.			

Due to the low standard deviation of IEF it is obvious that this asset takes part in the minimum variance portfolio. This is also the case for SP500 and ACWI which both also exhibit a low standard deviation. Surprisingly, SGOL is not included in this portfolio. From Figure 5.4 we see that SGOL has a positive and relatively high correlation with IEF and is thus excluded. Both the standard deviation and downside risk is lowest in this portfolio along with CVaR being 1,19%.

The SRP portfolio is optimized for Sharpe Ratio and will thus observe the highest Sharpe Ratio possible. Sharpe Ratio is calculated with respect to standard deviation and covers both the upside and downside risk. This portfolio exhibits just a slightly higher downside risk than the ND portfolio, but with a substantial increase in expected return. This leads to a considerable increase in both the Sortino Ratio as well as the M^2 measure. This indicates that an investor who is optimizing his or her portfolio for Sharpe Ratio will in this case have an increased downside risk, but with increased risk-return performance measures and thus perform better than the ND and MINV portfolios.

By construction, the ND portfolio has equal weights in each asset. This portfolio provides an indication of how well a diversified but not optimized portfolio will perform. Even though this portfolio exhibits a positive return, the risk taken is far from optimal. The portfolio has similar risk measures as the SRP portfolio, but the performance measures are extraordinarily lower. An investor who uses a naive diversification will thus take unnecessary risks and yet achieve a lower return than an investor that utilizes an optimized portfolio.

The following ten portfolios is optimized for CVaR, and their performance is specified in Table 6.2.

Table 6.2: Weights and annual performance metrics of CVaR portfolios including Bitcoin.

Portfolios CVaR nr. 1-10

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
EEM	-	-	-	-	-	-	-	-	-	-
GSG	0,040	-	-	-	-	-	-	-	-	-
BTC	-	0,075	0,15	0,23	0,34	0,45	0,58	0,72	0,86	1
IEF	0,83	0,56	0,19	-	-	-	-	-	-	-
IXC	-	-	-	-	-	-	-	-	-	-
IXN	-	0,18	0,30	0,41	0,51	0,55	0,42	0,28	0,14	-
SGOL	-	0,18	0,36	0,36	0,15	-	-	-	-	-
SP500	0,13	-	-	-	-	-	-	-	-	-
ACWI	-	-	-	-	-	-	-	-	-	-
Annual Performance Metrics										
Expected Return %	2,99	19,93	39,68	60,67	89,31	120,60	157,06	198,43	247,70	305,08
Standard Deviation %	4,54	8,43	14,83	21,75	29,21	37,15	45,72	54,77	64,13	73,82
Variance %	0,21	0,71	2,19	4,73	8,53	13,80	20,90	29,99	41,13	54,49
Sharpe Ratio	0,48	2,27	2,62	2,75	3,03	3,23	3,42	3,61	3,85	4,12
Downside Risk %	5,98	11,49	17,94	23,97	30,00	34,77	40,00	45,08	50,16	55,24
Sortino Ratio	0,37	1,66	2,17	2,49	2,95	3,45	3,91	4,38	4,92	5,51
CVaR %	0,69	1,16	2,05	3,04	4,15	5,32	6,58	7,89	9,24	10,60
M^2 %	9,21	40,39	46,55	48,83	53,66	57,06	60,42	63,75	67,96	72,71

Note: CVaR is calculated with a α of 5%.

The CVaR optimized portfolios provides insight in how exposed an investor are to tail-end risks. The CVaR ranges from 0,69% up to 10,60%. A risk averse investor should therefor choose portfolio nr. 1, with the least downside risk. Supposing an investor is not risk averse, the investor would then choose portfolio nr.10 as this portfolio provides the highest performance measures. It is noteworthy that both the M^2 and Sortino Ratio are increasing for every additional weight put in Bitcoin, indicating that the return of Bitcoin outperforms the additional risk taken.

These ten portfolios are plotted in an efficient frontier in Figure 6.1, the frontier illustrates the daily mean return on the y-axis and the CVaR on the x-axis. The frontier emphasize the CVaR when an investor is focusing only on the expected return. But the frontier does not take other risk-return measures into consideration. As noted above, the additional return appears to outperform the additional risk taken.

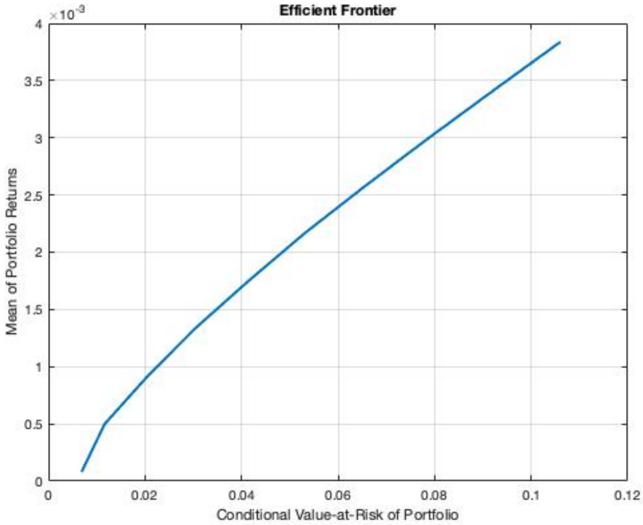

Figure 6.1: Efficient frontier – CVaR including Bitcoin.

Figure 6.2. illustrates how the portfolio weights develop throughout the efficient frontier. Bitcoin is ranging from 0% in portfolio nr. 1 up to 100% in portfolio nr. 10. Noteworthy is that EEM, IXC and ACWI has a weight of 0% which is illustrated both in Table 6.2 and Figure 6.2. The asset IEF which has lower volatility than the other assets is only included in portfolio nr. 1-3.

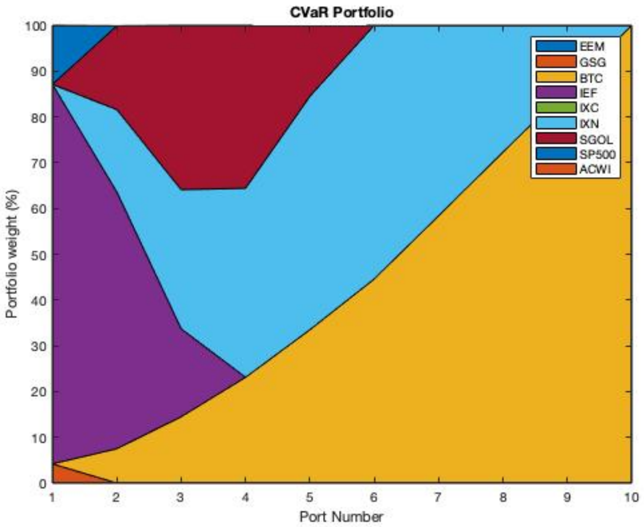

Figure 6.2: Portfolio weights – CVaR including Bitcoin.

6.2 Portfolios excluding Bitcoin

In this section portfolios excluding Bitcoin will be presented. The underlying assumptions denoted in chapter 6.1 are still valid. First, three portfolios will be presented in Table 6.3, one portfolio optimized for Sharpe Ratio (SRP-NOBTC), a naive diversification (ND-NOBTC) portfolio and a portfolio optimized for minimum variance (MINV-NOBTC). Secondly, ten portfolios will be presented in Table 6.4, illustrating an efficient frontier of CVaR optimization in Figure 6.3.

Table 6.3: Weights and annual performance metrics of portfolios excluding Bitcoin.

	Portfolios		
	SRP-NOBTC	*ND-NOBTC*	*MINV-NOBTC*
EEM	-	0,125	-
GSG	-	0,125	0,040
IEF	0,10	0,125	0,82
IXC	-	0,125	-
IXN	0,49	0,125	-
SGOL	0,41	0,125	-
SP500	-	0,125	0,079
ACWI	-	0,125	0,063
	Annual Performance Metrics		
Expected Return %	21,11	12,10	2,88
Standard Deviation %	12,41	13,87	4,54
Variance %	1,54	1,92	0,21
Sharpe Ratio	1,64	0,82	0,46
Downside Risk %	13,45	14,97	6,08
Sortino Ratio	1,51	0,76	0,34
CVaR %	2,59	2,82	1,19
M^2 %	29,35	15,01	8,80
Note: CVaR is calculated with a α of 5%.			

The MINV-NOBTC portfolio optimization is exactly the same as the one MINV portfolio including Bitcoin, Bitcoin is not included in either of these portfolios due to the high variance.

SRP-NOBTC portfolio: Due to the exclusion of Bitcoin this portfolio exhibits increased weights in IXN and SGOL and a decreasing weight in IEF compared to the portfolio including Bitcoin. When comparing the performance metrics, it is clear that the portfolio including Bitcoin outperforms the one excluding Bitcoin, the expected return is 27,77 percentage points higher with increased values in both Sharpe and Sortino Ratio. This implies that the portfolio including Bitcoin compensates for the increased risk with a substantial increase in return. However, a risk averse investor would still choose the portfolio excluding Bitcoin as this portfolio observes lower risk measures.

The naive diversification portfolio excluding Bitcoin has a worse performance than the SRP-NOBTC portfolio, the naive portfolio has increased risk measures as well as lower performance measures. This was not the case regarding the naive diversification portfolio including Bitcoin. Even though a non-optimized weight in Bitcoin adds a slight increase in risk, the risk-return measures are higher. This indicates that an investor who is not including Bitcoin in its naive diversification may miss out on potential return.

The following ten portfolios are optimized for CVaR. Their performance will be presented in an efficient frontier in Figure 6.3 and their weights will be illustrated in a weight plot in Figure 6.4.

Table 6.4: Weights and annual performance metrics of CVaR portfolios excluding Bitcoin.

Portfolios CVaR – NO BTC nr.1-10										
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
EEM	-	-	-	-	-	-	-	-	-	-
GSG	0,04	0,0089	0,0043	0,00039	-	-	-	-	-	-
IEF	0,83	0,71	0,58	0,44	0,29	0,13	-	-	-	-
IXC	-	-	-	-	-	-	-	-	-	-
IXN	-	0,11	0,22	0,30	0,37	0,44	0,53	0,69	0,84	1
SGOL	-	0,11	0,19	0,27	0,34	0,44	0,47	0,31	0,16	-
SP500	0,13	0,058	-	-	-	-	-	-	-	-
ACWI	-	-	-	-	-	-	-	-	-	-
Annual Performance Metrics										
Expected Return %	2,99	6,13	9,39	12,76	16,22	19,75	23,44	27,18	31,09	35,12
Standard Deviation %	4,54	5,21	6,33	7,89	9,69	11,68	13,69	16,19	19,21	22,70
Variance %	0,21	0,27	0,40	0,62	0,94	1,37	1,88	2,62	3,69	5,15
Sharpe Ratio	0,48	1,02	1,36	1,52	1,59	1,62	1,65	1,63	1,58	1,51
Downside Risk %	5,98	6,99	8,32	9,81	11,35	12,92	14,41	15,75	17,15	18,4
Sortino Ratio	0,37	0,76	1,03	1,22	1,36	1,47	1,57	1,68	1,77	1,86
CVaR %	0,69	0,77	0,93	1,15	1,41	1,69	1,98	2,36	2,86	3,41
M^2 %	9,21	18,66	24,47	27,24	28,54	29,09	29,62	29,22	28,31	27,18
Note: CVaR is calculated with a α of 5%.										

In opposition to the CVaR optimization including Bitcoin the optimization in Table 6.3 does not exhibit an increase in performance measures for every additional weight in IXN. Instead, the performance increases up to portfolio nr. 7. Portfolio nr. 7 exhibits both the highest Sharpe Ratio as well as M^2 this indicates that this is the optimal portfolio in a risk-return point of view. The Sortino Ratio is increasing throughout the portfolios, which contradicts the results of the other performance measures. Sortino Ratio is explained in Equation 2.7 and by construction this ratio depends on the increases in downside risk and expected return. Hence, an investor has to look at multiple measures to foresee misleading results.

These ten portfolios are plotted in an efficient frontier in Figure 6.3. The frontier illustrates the daily mean return on the y-axis and the CVaR on the x-axis. Compared to the efficient frontier including Bitcoin this frontier has consistent lower CVaR. The optimal risk-return portfolio has a CVaR of 1,98%.

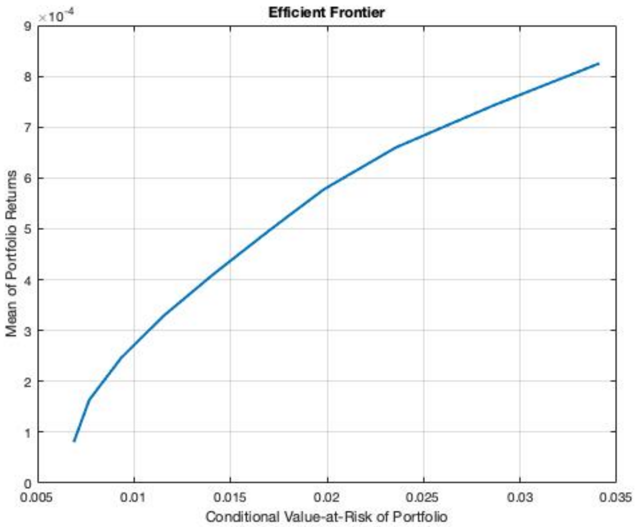

Figure 6.3:Efficient frontier – CVaR excluding Bitcoin.

Figure 6.4 provides a visual understanding for how the weights of the portfolios progress across the frontier. As noted earlier, IXN is the asset with highest mean return when Bitcoin is excluded and thus this is the asset that ends up having 100% allocation in CVaR portfolio nr. 10.

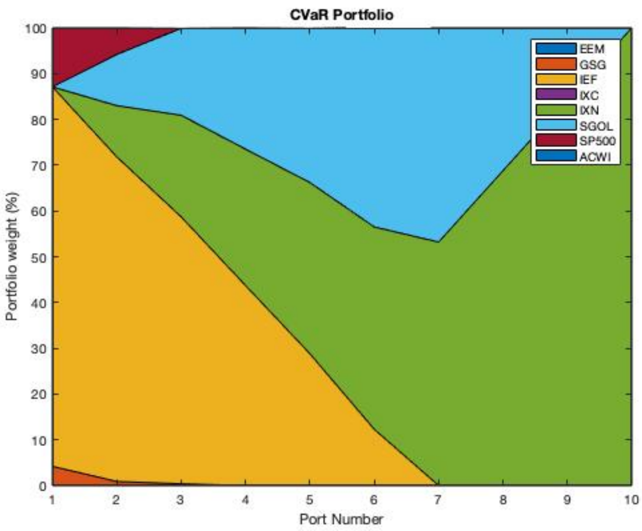

Figure 6.4:Portfolio weights – CVaR excluding Bitcoin.

6.3 Discussion

In this study, different performance metrics have been implemented to evaluate how the inclusion of Bitcoin can affect a portfolio. The analysis was achieved through optimization for specific performance measures. The portfolios are then benchmarked against each other as well as with a naive diversification portfolio.

The research question this thesis aims to answer is:

'How does Bitcoin affect downside risk measures in a diversified portfolio?'

The main findings in the result are that Bitcoin does exhibit a protruding expected return as well as standard deviation considering the assets. However, when included in a Sharpe Ratio optimized portfolio, Bitcoin enhances the expected return while also increasing the risk. When comparing the SRP portfolio with the SRP-NOBTC portfolio, it is apparent that the SRP portfolio has a notable amount allocated to Bitcoin and balances this with a higher allocation in the least risky asset, IEF. The SRP portfolio achieves more than a double increase in expected return while the downside risk increases by 6,57 percentage points. This leads to a considerable increase in Sortino Ratio, Sharpe Ratio and M^2.

According to a risk-averse investor, Bitcoin is not a sound investment due to the associated risks. An investor who is not risk-averse would gain on either having a naive diversification or a Sharpe Ratio optimized weight in Bitcoin. Even a smaller weight can provide a portfolio with increased performance. The CVaR portfolio nr. 2 has a minor weight of 7,5% allocated to Bitcoin and outperforms the SRP-NOBTC portfolio for all performance measures except for expected return, which is 1,18 percentage points lower. This indicates that Bitcoin can be used to increase risk-return performance, which is achieved by balancing a minor weight of Bitcoin with a larger weight in the least volatile and risky asset in the sample, IEF.

The CVaR portfolio nr.10 including Bitcoin is the portfolio with overall highest performance measures. This portfolio is 100% invested into Bitcoin and is no longer diversified meaning it will be overlooked as a potential investment strategy.

The general findings are that an investor should include Bitcoin in an optimal portfolio, which is aligned with previous research. In the time period evaluated, Bitcoin is experiencing a high volatility with significant returns. The downside risk is affected negatively by including Bitcoin and the optimized weight are not higher than 18.1% of the total portfolio. As previously noted, Briére et al. (2013) found that the inclusion of Bitcoin in a portfolio has a substantial effect on the volatility, complementary to this study this thesis found that a smaller weight of Bitcoin, 7,5% can be utilized to decrease the downside risk of a portfolio but with a considerably large allocation in IEF of 56%.

In contraction to Eisl et al. (2015)'s this study found that a larger weight of Bitcoin is optimal in the Sharpe Ratio optimized portfolio specifically, 18%. This could be explained by different time samples, sample assets or returns in the sample. The sample of Bitcoin used in this thesis exhibits exceptional returns in the years 2017-2018 and 2020-2021. Another explanation could be that the correlation among the chosen assets is different. As observed in Figure 5.4 the correlation of Bitcoin has a slight positively skew.

As emphasized earlier, Cheah and Fry (2015) states that the fundamental value of Bitcoin is zero and Bitcoin might experience bubble behaviour. This can be associated with significantly higher risks than the risk measures used in this thesis.

7 Conclusion

The purpose of this study was to examine the risks associated with investing in cryptocurrency, specifically Bitcoin. To assess the risks, Bitcoin was incorporated in portfolios containing eight other assets. These assets were selected ETF funds and indexes which created a well-diversified composition. Different optimizations were conducted and evaluated with performance measures as well as downside risk measures.

The study concludes that Bitcoin can be utilized to substantially increase the expected return of a portfolio with considerable increases in risk. Depending on the risk appetite of the investor, different weight of Bitcoin are suggested. In the CVaR nr. 2 portfolio, a weight of 7,5% allocated to Bitcoin is observed and this portfolio showed to have almost as high expected return as the portfolio optimized for Sharpe Ratio excluding Bitcoin. Nevertheless, the portfolio maintains lower risk measures.

To further investigate the risks of Bitcoin, future research could explore additional risk measures, extended testing period or including more assets. Further research could also study the correlation of Bitcoin with other assets and examine how this relationship develops through time.

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