New Ways to Describe Certain Mathematical Structures

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Mathematicians like to study different kinds of structures, and this thesis is concerned with one structure in particular, Polynomial Algebras. The main purpose of the thesis is to develop a new method for describing such algebras; by using of conditions.

Usually in mathematics when we speak of a certain "structure", we mean a set of elements that adhere to a specific set of rules. Such structures of different flavours arise in all different fields of mathematics. An algebra, as you might guess from the name, is an algebraic structure, meaning that an algebra is a set of elements which follow certain algebraic rules. Such sets can often contain infinitely many elements, for example, the set of real numbers form an algebra. This thesis is concerned with polynomial algebras. You might remember polynomials from school, for example $x^2 + 2x + 1$ is a polynomial, so is $x^{100} - 9999x^{44} + 0.4$ and $xy + x + y$ as well. A polynomial algebra is an algebra where the elements are polynomials.

This thesis develops a new way to describe polynomial algebras. Usually such algebras would be described by a set of generators. You may think of these generators as providing a recipe of sorts. In this text we learn how polynomial algebras can be described in a new way, namely by a set of conditions. This research was started initiated in the paper [1] only two years ago. There the authors developed the theory for polynomials in one variable. This thesis generalizes parts of their theory to multiple variables. Since this is such a new field of study, applications and consequences of this theory are not yet known, but in some ways, that makes it all the more exciting. Who knows what will come of this? It is not rare that new perspectives of viewing various objects leads to new results. We will just have to wait and see!

References

[1] Rode Grönkvist, Erik Leffler, Anna Torstensson, and Victor Ufnarovski. "Describing subalgebras of $K[x]$ using derivatives". In: (2021). arXiv: `2107.11916 [math.RA]`.