Heuristic and Exact Evaluation of Two-Echelon Inventory Control Systems

Bachelor's Degree in Mathematics Statistics
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The essence of inventory control is to decide when we should order new items and how much we should order. With an efficient inventory control model, companies can maximize their profits and avoid stocking up unnecessary amount of goods in warehouses.

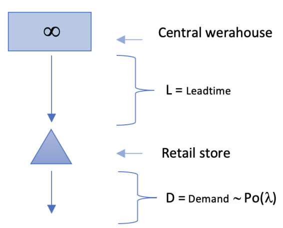

Figure 1: Single-echelon model

In order to determine the optimal base-stock level, i.e., the level S for a two-echelon model, we will first consider a single-echelon model. A single-echelon model consists of one central warehouse and one retail store, where the warehouse has infinite capacity, see Figure 1. We assume that the model follows a so-called order-up-to S policy and that there is a continuous review. That is, as soon as item has been purchased we immediately order another item. Customer demands at the retailers follow independent Poisson demand processes and each customer only demands one unit. Customer demands which are not satisfied directly from stock on hand are assumed to backordered, i.e., no lost sales exist. In order to do this optimization, we will consider two costs, the holding cost and the shortage cost, and balance them. The holding and shortage costs depends on the inventory level. In order to find the probability function of the inventory level we need to take the lead-time into account. In many inventories control models, the lead-time is assumed to be exponentially distributed since the mathematical analysis becomes considerably simpler, due to the Markov property. However, in the real world the lead-time is often constant. By analyzing the probability function of the inventory level when the lead-time is constant and the case when the lead-time is exponential distributed we find that the probability function of the inventory level is Poisson distributed in both cases. We conclude that whether the lead-time is constant, or exponential distributed, we will have that the probability function of the inventory level is Poisson distributed. In matter of fact, the choices of distribution for the lead-time does not matter, it will always be Poisson distributed according to Palm's theorem. After constructing the cost function and proving that the function is convex, we are able to find the optimal inventory position S.

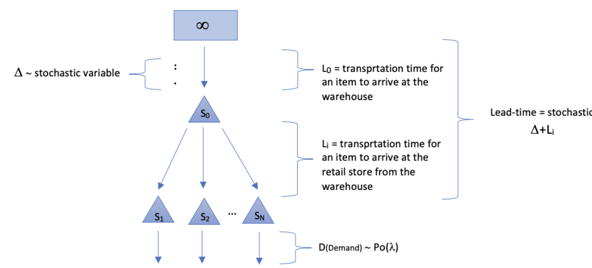

Figure 2: Two-echelon model

A two-echelon model consist of one central warehouse and N retailers, see Figure 2. As for the single-echelon model, customer demands at the retailers follow independent Poisson demand processes and each customer only demands one unit. Customer demands which are not satisfied directly from stock on hand are assumed to backordered. The first main goal is to derive an expected system cost function which consist of inventory holding costs and backorder costs. Secondly, we will optimize this cost function with respect to the base-stock levels S_i, $i = 0, \ldots, N$, where S_i represents the base-stock-level at retailer i (index 0 is for the warehouse).

We will first consider an exact method to optimize the expected system cost function. In this exact method the lead-times for the retailers are stochastic due to possible delays when replenishing from the central warehouse. However, in practice, it is common to use an approximate method where the stochastic lead-times for the retailers are replaced by the corresponding mean values. Here, we will investigate the robustness of this approximate method in terms of changes in system parameters.

When analyzing the optimal values obtained when using the approximated method respective the exact method, we find that the approximated method often finds the true optimal policy in most cases. However, in some cases the optima values for the base-stock level are underestimated when using the approximated method. We also note that the evaluated total cost is always lower when using the approximated methods. The reason for this is that, when using METRIC approximation we usually underestimate the values of the expected positive and negative inventory levels ($E(IL^+)$ and $E(IL^-)$).

In practice, it can be very hard to understand and use the exact method. Companies that want to use inventory control to maximize their profits and avoid stocking up unnecessary amount of goods in their warehouses, do not always have the understanding of how the exact method works due to the stochastic lead-time. However, if we know that the optimal inventory position at the retailers is sometimes underestimated when using the approximated method, keeping these factors in mind, companies can use that in their advantages, allowing them to use the approximated method. The METRIC approximation has been used successfully in practice, and in general, using this method works well as long as the demand at each retail store is low relative to the total demand.