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A Study of Spontaneous $\mathcal{CP}$ Violation in the Scale Invariant 2HDM

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Table 1: Abbreviations used throughout the paper with the given explanation.

Abbreviation	Explanation
2HDM	Two Higgs Doublet Model
$\mathcal{CP}$	Charge conjugation Parity
EW	electroweak
FCNC	Flavour Changing Neutral Currents
GW	Gildener-Weinberg
H.c	Hermitean conjugate
SM	Standard Model
vev	Vacuum Expectation Value

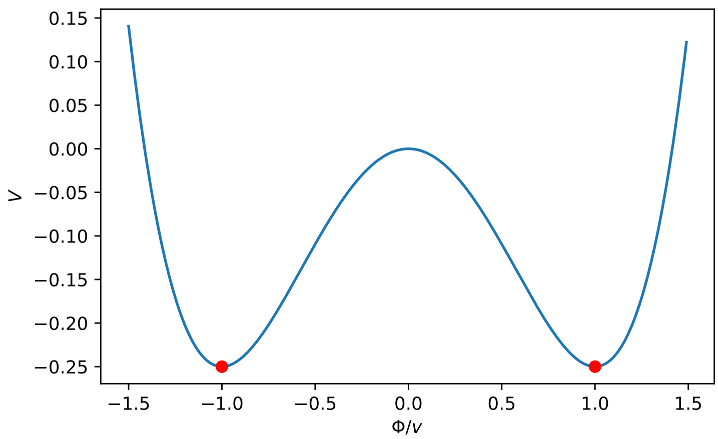
Abstract

In this thesis, we study the Higgs mass spectrum and spontaneous $\mathcal{CP}$ violation, where one of the Higgs vacuum expectation values acquires a non-trivial phase θ in a Type-I 2HDM. We begin by investigating $\mathcal{CP}$ violating effects at tree-level, and then expand our studies by taking into account one-loop radiative corrections. After imposing tadpole conditions we discover that this model gives the same mass spectrum as that found in the $\mathcal{CP}$ invariant theory. This occurs due to the cancellation of θ-dependency in the λ_5 coupling, and hence in the mass spectrum. We therefore conclude that the considered model does not feature spontaneous $\mathcal{CP}$ violation, at least, up to the one-loop approximation.

Populärvetenskaplig beskrivning

Inom partikelfysik försöker forskare hitta en så djupgående och allmän modell som möjligt. Den modell som experimentellt har verifierats kallas för Standardmodellen (SM). Det finns dock andra teorier, såsom Tvåhiggsdubbletmodellen (2HDM), som innehåller mer exotisk fysik. Dock är experimentella verktyg begränsade och som fysiker vill man gärna kunna ha en teori för att kunna förutspå egenskaper som man kan leta efter experimentellt. Detta leder till att teoretiker konstruerar teorier och generaliserar dem till en så allmän struktur som möjligt. Varför ska till exempel bara en Higgspartikel existera i SM? Varför bara ett vakuumvärde? Det är ingenting som teoretiskt begränsar att Higgspartikeln ska vara den enda av sin sort. Likaså behöver det nödvändigtvis inte finnas fler. Man har trots allt bara upptäckt att det är såhär det är med de experimentella verktyg som finns tillgängliga.

Vi ska använda en allmän modell där man tillåter mer än en Higgsdubblett som teoretisk grund, i vårt fall har vi två Higgsdubbleter. I modellen som skrivs om i denna uppsats används representationer som har två komponenter. En komponent som beskriver ett elektriskt laddat fält, och en annan som beskriver ett eletriskt oladdat komplext fält som ger upphov till två olika vakuumvärden v_1 och v_2 för vardera dubblet. Vi konstruerar det så att v_1 och v_2 är punkter i fältet där vi har ett så kallat symmetribrott. Dessa vakuumvärden förhåller sig till varandra via trigonometri. Trigonometrin mellan v_1 och v_2 är sådan att vardera är en katet för en triangel med hypotenusa v.



Figuren visar en förenklad Higgspotential: $V\left(\frac{\Phi}{v}\right) = \frac{1}{4}\left(\frac{\Phi}{v}\right)^4 - \frac{1}{2}\left(\frac{\Phi}{v}\right)^2$. Det finns två (markerat rött) extremvärden skilda från origo, dessa punkter är där vi har vakuumvärdet, och lägst energi för potentialen V. Att man ska kunna studera en Higgsmodell där fysik är likadan under en så kallad laddningskonjugering-paritet ($\mathcal{CP}$) är inte fallet om vakuumvärdet är komplext. Vi väljer då ut att vakuumvärdet inte ska se likadnt ut i en $\mathcal{CP}$-speglad värld. Detta kallas för $\mathcal{CP}$-brott.

Contents

1 Introduction

In 1964 Peter Higgs and others published a set of papers [1, 2] regarding spontaneous electroweak (EW) symmetry breaking. In these papers, an $\mathrm{SU}(2)_{\mathrm{EW}}$ doublet was theorised with an electrically neutral field which acquires a vacuum expectation value (vev) giving masses to the weak force carriers $W^{\pm}$ and Z^0 through gauge interactions, as well as fermions through Yukawa couplings. This is known as the Higgs mechanism [3], which is incorporated into the formulation of the Standard Model (SM) [4] with the Higgs potential being an important part of its Lagrangian. The existence of the Higgs mechanism was experimentally confirmed in July 2012 at CERN, with a 5σ signal for the discovery of an electrically neutral scalar Higgs boson at CMS [5] and ATLAS [6]. Its current experimentally measured mass is 125.10 ± 0.14 GeV [7].

There are models that consider an additional $\mathrm{SU}(2)_{\mathrm{EW}}$ Higgs doublet in the scalar potential, hence going beyond the SM theory: these theories usually include elements of Charge conjugation Parity ($\mathcal{CP}$) violation and neutrino oscillations [8]. $\mathcal{CP}$ violating phenomena exist in nature, as was first discovered in 1964 with the asymmetry between K^0 and $\overline{K}^0$ decays [9, 10]. T. D Lee, therefore, proposed in 1973 [11] that a Two Higgs Doublet Model (2HDM) could give rise to time reversal parity $\mathcal{T}$ violation in the Higgs sector. Because of the $\mathcal{CPT}$ theorem in quantum field theory, $\mathcal{T}$ violation is equivalent to $\mathcal{CP}$ violation. The motivation for studying $\mathcal{CP}$ violating phenomena has a basis in Sakharov conditions [12], wherein one explanation for the matter-antimatter imbalance in the universe could have an origin from $\mathcal{CP}$ violation.

In 1976 Gildener and Weinberg proposed [13] a multi-Higgs model that realises the EW symmetry breaking by means of 1-loop radiative corrections [14] in a massless theory. Such corrections modify the Higgs masses squared. In this paper, Gildener and Weinberg discuss proper renormalisation of the scalar potential at 1-loop level [15, 16].

In this thesis, we study the impact of a non-trivial phase θ in one of the vevs on the Higgs mass spectrum in the Type-I 2HDM. We also consider tree-level scale invariance, thus making it a massless theory, for which we can apply the Gildener-Weinberg (GW) approach in order to incorporate 1-loop radiative corrections. We also impose the discrete $\mathbb{Z}_2$ symmetry on the Higgs doublets (Φ_1, Φ_2) to remove Flavour Changing Neutral Currents (FCNC) [17]. We then investigate possible $\mathcal{CP}$ violating effects at tree-level. The corresponding analysis has then been generalised to the 1-loop case using GW theory [13].

The structure of this thesis is as follows: Section 2 covers an introductory discussion of the Higgs mechanism and the 2HDM, and the method to calculate the mass spectrum at tree-level. In Section 3 we calculate the tree-level mass spectrum. Section 4 contains the derivation of the mass spectrum at 1-loop level. In Section 5 we investigate phenomenological aspects regarding mixing between the Higgs fields and their coupling to the $W^{\pm}$ and Z^0 gauge bosons. Section 6 covers our conclusions from this study.

2 Introduction to the Two Higgs Doublet Model

2.1 The Higgs doublets

The SM $\mathrm{SU}(2)_{\mathrm{EW}}$ Higgs doublet spontaneously breaks the EW gauge group $\mathrm{SU}(2)_{\mathrm{EW}}\otimes \mathrm{U}(1)_{\mathrm{Y}}$ down to a $\mathrm{U}(1)_{\mathrm{EM}}$ symmetry [1]. The decomposition of the doublet into its components is

$$H_{\mathrm{SM}} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + H^0 + iG^0\right) \end{pmatrix}, \quad \langle H_{\mathrm{SM}} \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{2.1}$$

Here $v \approx 246$ GeV is the SM Higgs vev [7], and H^0 is the scalar Higgs field [1]. The Goldstone bosons $G^\pm$ and G^0 get 'eaten' by the weak gauge bosons $W^\pm$ and Z^0 to become their longitudinal degrees of freedom [14].

We extend the SM by adding another $\mathrm{SU}(2)_{\mathrm{EW}}$ Higgs doublet with hypercharge $Y = +1$ [11], such that both doublets have the same quantum numbers[1] [18, 19]. The structure of the Higgs doublets that we investigate has the following form:

$$\begin{aligned} \Phi_1 &= \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \varphi_1^0 + ia_1^0) \end{pmatrix}, & \langle \Phi_1 \rangle &= \frac{v_1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ \Phi_2 &= e^{i\theta} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \varphi_2^0 + ia_2^0) \end{pmatrix}, & \langle \Phi_2 \rangle &= \frac{v_2 e^{i\theta}}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \end{aligned} \tag{2.2}$$

Where $v_1^2 + v_2^2 = v^2$, $v_1, v_2 > 0$, and $\frac{v_2}{v_1} = t_\beta$ for $\beta \in (0, \frac{\pi}{2})$ [18, 19]. The fields in Eq.(2.2) are defined as: ϕ^+ is a $+1$ electrically charged complex field, while φ^0 and a^0 are electrically neutral real fields [18, 19]. We impose a $\mathbb{Z}_2$ symmetry on our doublets (Φ_1, Φ_2) to remove tree-level FCNC [17] in the Yukawa sector. The $\mathbb{Z}_2$ transformations are [18, 19, 20, 21]

$$\Phi_j \leftrightarrow \Phi_j, \quad \Phi_k \leftrightarrow -\Phi_k, \quad j, k \in \{1, 2\} : \ j \neq k. \tag{2.3}$$

In this thesis we choose that $j = 1$ and $k = 2$. Thereby, Φ_1 couples to all fermions as we have a Type-I 2HDM [18].

By picking a non-trivial phase θ where $e^{i\theta} \notin \{1, i, -1, -i\}$ we obtain:

$$\langle \Phi_1 \rangle^* = \langle \Phi_1 \rangle, \quad \langle \Phi_2 \rangle^* \neq \pm \langle \Phi_2 \rangle. \tag{2.4}$$

We have defined θ such that we have a $\mathcal{CP}$ breaking vacuum. Our theory is therefore a candidate [20] for spontaneous $\mathcal{CP}$ violation [11]. Furthermore, the $\mathcal{CP}$ properties of φ^0 and a^0 are

$$\mathcal{CP}\varphi_i^0(\mathcal{CP})^\dagger = (+1)\varphi_i^0, \quad \mathcal{CP}a_i^0(\mathcal{CP})^\dagger = (-1)a_i^0, \quad i \in \{1, 2\}. \tag{2.5}$$

[1] Hypercharge is also a quantum number, we mention it to distinguish the 2HDM from other beyond SM theories which extend the SM with an hypercharge $Y = -1$ Higgs doublet [22]. One such model is the Minimal Supersymmetric Standard Model.

We will therefore refer to φ^0 as $\mathcal{CP}$-even, and a^0 as $\mathcal{CP}$-odd [18].

We note that the value of t_β is a basis dependent quantity [19]. In the inert model, one uses the Higgs basis [19, 20, 21]. It has the same, except for the $\mathbb{Z}_2$ symmetry charge, quantum numbers as our basis (Φ_1, Φ_2) and is defined as [18]

$$\mathcal{H}_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H_1 + iG^0) \end{pmatrix}, \qquad \mathcal{H}_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H_2 + iA) \end{pmatrix}, \tag{2.6}$$

where the first doublet $\mathcal{H}_1$ obtains the SM Higgs vev v, and the second doublet $\mathcal{H}_2$ has a vanishing vev. We have 5 Higgs fields $H_1, H^\pm, H_2$, and A, and 3 Goldstone bosons $G^\pm$ and G^0 [18, 19, 21]. Thus it is easier to do calculations in the Higgs basis, as the Goldstone bosons are well defined.

2.2 Properties of the tree-level potential

To transform our basis defined in Eq.(2.2) into the Higgs basis in Eq.(2.6), by means of a Higgs basis transformation [18], we investigate the invariance of the Lagrangian $\mathcal{L}$ given by:

$$\mathcal{L} = T - V_{\text{tree}}, \quad T = (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2), \quad \mathcal{L}_{\text{int}} = -V_{\text{tree}}. \tag{2.7}$$

In Eq.(2.7) we have defined the kinetic term T with the covariant derivative D_μ. The interaction potential V_{tree} has the following general form [18]:

$$\begin{aligned} V_{\text{tree}} &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{H.c}] \\ &+ \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+ \left[\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \text{H.c}\right]. \end{aligned} \tag{2.8}$$

In Eq.(2.8) we have squared mass terms m_{ij}^2 and quartic couplings λ_k. We use the notation [... + H.c] to imply that we also add the hermitean conjugates of the terms within the bracket. Regarding the domains of the mass terms and quartic couplings in V_{tree} we have the following: $m_{11}^2, m_{22}^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{R}$, and $m_{12}^2, \lambda_5, \lambda_6, \lambda_7 \in \mathbb{C}$ [19, 23, 24].

We impose additional symmetries to decrease the number of free parameters in the potential V_{tree}: these are scale invariance and exact $\mathbb{Z}_2$ symmetry. From scale invariance it follows that $m_{11}^2, m_{22}^2, m_{12}^2 = 0$, thus making it a massless theory [13]. We defined the $\mathbb{Z}_2$ transformations in Eq.(2.3), and when chosen to be an exact symmetry we set $\lambda_6, \lambda_7 = 0$. Thus we obtain that $\lambda_5 \in \mathbb{C}$ is the only complex quartic coupling for V_{tree}, and thus there are six free parameters $(\lambda_1, \lambda_2, \lambda_3, \lambda_4, |\lambda_5|, \arg(\lambda_5))$.

The interaction Lagrangian $\mathcal{L}_{\text{int}}$ is, based on the structure of V_{tree}, invariant under the global SU(2)_{HF} Higgs flavour transformation (change of basis) [18, 21] for a reparametrisation of the quartic couplings [25]. Furthermore, a global $\mathrm{U}(1)_\mathrm{Y}$ transformation extends the group to be $\mathrm{SU}(2)_\mathrm{HF}\otimes \mathrm{U}(1)_\mathrm{Y} \cong \mathrm{U}(2)_\mathrm{HF}$ [18]. The kinetic term T in Eq.(2.7) is also invariant [25].

To transform from our basis (Φ_1, Φ_2) to the Higgs basis $(\mathcal{H}_1, \mathcal{H}_2)$ we pick $\Omega \in \mathrm{U}(2)_\mathrm{HF}$ to obtain

$$\Omega \equiv \begin{pmatrix} c_\beta & s_\beta e^{-i\theta} \\ -s_\beta & c_\beta e^{-i\theta} \end{pmatrix}: \qquad \begin{pmatrix} c_\beta & s_\beta e^{-i\theta} \\ -s_\beta & c_\beta e^{-i\theta} \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{pmatrix}. \tag{2.9}$$

We therefore re-write the potential V_tree with respect to the Higgs basis:

$$\begin{aligned} V_\mathrm{tree} = &\frac{1}{2}Z_1(\mathcal{H}_1^\dagger\mathcal{H}_1)^2 + \frac{1}{2}Z_2(\mathcal{H}_2^\dagger\mathcal{H}_2)^2 + Z_3(\mathcal{H}_1^\dagger\mathcal{H}_1)(\mathcal{H}_2^\dagger\mathcal{H}_2) + Z_4(\mathcal{H}_1^\dagger\mathcal{H}_2)(\mathcal{H}_2^\dagger\mathcal{H}_1) \\ &+ \left[\frac{1}{2}Z_5(\mathcal{H}_1^\dagger\mathcal{H}_2))^2 + Z_6(\mathcal{H}_1^\dagger\mathcal{H}_1)(\mathcal{H}_1^\dagger\mathcal{H}_2) + Z_7(\mathcal{H}_2^\dagger\mathcal{H}_2)(\mathcal{H}_1^\dagger\mathcal{H}_2) + \mathrm{H.c}\right]. \end{aligned} \tag{2.10}$$

In Eq.(2.10) we have introduced new quartic couplings Z which are written as linear combinations of $\{\lambda_k\}_{k=1}^5$:

$$\begin{aligned} Z_1 &= c_\beta^4\lambda_1 + s_\beta^4\lambda_2 + 2\left(s_\beta c_\beta\right)^2\lambda_{345}, \\ Z_2 &= s_\beta^4\lambda_1 + c_\beta^4\lambda_2 + 2\left(s_\beta c_\beta\right)^2\lambda_{345}, \\ Z_3 &= \lambda_3 + \left(s_\beta c_\beta\right)^2\left(\lambda_1 + \lambda_2 - 2\lambda_{345}\right), \\ Z_4 &= \lambda_4 + \left(s_\beta c_\beta\right)^2\left(\lambda_1 + \lambda_2 - 2\lambda_{345}\right), \\ Z_5 &= \left(s_\beta c_\beta\right)\left(\lambda_1 + \lambda_2 - 2\lambda_{345}\right) + \Re(e^{i2\theta}) + c_{2\beta} i\Im(\lambda_5 e^{i2\theta}), \\ Z_6 &= -s_\beta c_\beta\left(c_\beta^2\lambda_1 - s_\beta^2\lambda_2 - c_{2\beta}\lambda_{345} - i\Im(e^{i2\theta})\right), \\ Z_7 &= -s_\beta c_\beta\left(s_\beta^2\lambda_1 - c_\beta^2\lambda_2 + c_{2\beta}\lambda_{345} + i\Im(\lambda_5 e^{i2\theta})\right), \end{aligned} \tag{2.11}$$

where we use $\lambda_{345} = \lambda_3 + \lambda_4 + \Re(\lambda_5 e^{i2\theta})$. We note how θ-dependence only emerges as $\lambda_5 e^{i2\theta}$. Furthermore, we do not necessarily have that $Z_6, Z_7 = 0$, as the exact $\mathbb{Z}_2$ symmetry affects the quartic couplings in regards to our basis (Φ_1, Φ_2).

In Eq.(2.11) we have quartic couplings $Z_5, Z_6, Z_7 \in \mathbb{C}$ with imaginary parts

$$\Im(Z_5) = c_{2\beta}\Im(\lambda_5 e^{i2\theta}), \quad \Im(Z_6) = s_\beta c_\beta\Im(\lambda_5 e^{i2\theta}), \quad \Im(Z_7) = -s_\beta c_\beta\Im(\lambda_5 e^{i2\theta}). \tag{2.12}$$

Eq.(2.12) is used to investigate $\mathcal{CP}$ violation due to the following reason: there exists a formalism called Jarlskog invariants [10] for identifying $\mathcal{CP}$ violating effects in the CKM matrix. Such invariants have been found for the 2HDM [26] in the form:

$$J_1 \propto \Im(Z_5^* Z_6^2), \quad J_2 \propto \Im(Z_5^* Z_7^2), \quad J_3 \propto \Im(Z_6^* Z_7). \tag{2.13}$$

If $J_1 = J_2 = J_3 = 0$, then V_tree is a $\mathcal{CP}$ conserving potential [10, 18, 26], and we therefore do not have $\mathcal{CP}$ violation, which is satisfied when $\Im(\lambda_5 e^{i2\theta}) = 0$ given Eq.(2.12).

To investigate the Higgs mass spectrum we introduce the squared mass matrix $\mathfrak{M}$ with elements given by:

$$[\mathfrak{M}]_{jk} = \left\langle \frac{\partial^2 V_\mathrm{tree}}{\partial\varphi_j\partial\varphi_k} \right\rangle. \tag{2.14}$$

In Eq.(2.14) φ_j, φ_k are defined as any of the Higgs fields, and the notation $[\mathfrak{M}]_{jk}$ is used to indicate the element of the squared mass matrix $\mathfrak{M}$. Due to electric charge conservation at an interaction vertex, $\mathfrak{M}$ can be reduced into sub-matricies. A 4×4 submatrix would contain derivatives with respect to electrically neutral fields, and a 2×2 submatrix would contain derivatives with respect to electrically charged fields.

Due to Goldstone's theorem [1, 18, 19], when the Goldstone bosons have been identified, we then only have to consider a 3×3 squared mass matrix $\mathcal{M}$ for the electrically neutral Higgs fields, and a squared mass term $m_{H^\pm}^2$ for the electrically charged Higgs field $H^\pm$. Such is achieved in the Higgs basis $(\mathcal{H}_1, \mathcal{H}_2)$, as the Goldstone bosons G^0 and $G^\pm$ are already defined.

Since we want to calculate the mass spectrum at the point of a non-trivial minimum of the potential V_{tree}, we thereby impose tadpole conditions. These are conditions for an extremum [27], and can be understood as the removal of 1-loop external legs in Feynman diagrams [28]. The algebraic representation of tadpole conditions is given by:

$$\left\langle \frac{\partial V_{\text{tree}}}{\partial \varphi} \right\rangle \equiv T_\varphi = 0. \tag{2.15}$$

In Eq.(2.15) we note that φ is any of the eight fields defined in any Higgs doublet, however, we write out the ones containing any numerical significance. We can then write some of the quartic couplings in terms of other parameters.

3 Mass spectrum at tree-level

We calculate the tadpole conditions at tree-level using Eq.(2.15):

$$T_{H_1}^{\text{tree}} = \frac{v^3}{2} Z_1, \qquad T_{H_2}^{\text{tree}} = \frac{v^3}{2} \Re(Z_6), \qquad T_A^{\text{tree}} = -\frac{v^3}{2} \Im(Z_6). \tag{3.1}$$

We re-write Eq.(3.1) in terms of λ to reduce the amount of degrees of freedom for the quartic couplings by combining it with Eq.(2.11):

$$c_\beta^2 \lambda_1 = -s_\beta^2 \lambda_{345}, \quad s_\beta^2 \lambda_2 = -c_\beta^2 \lambda_{345}, \quad \Im(\lambda_5 e^{i2\theta}) = 0. \tag{3.2}$$

In Eq.(3.2) especially note the implication that $\Im(\lambda_5 e^{i2\theta}) = 0$. A consequence is that we have a $\mathcal{CP}$ conserving [26] tree-level potential V_{tree} because the terms in Eq.(2.13) become implicitly 0. Furthermore we can find a constraint for λ_5 and θ:

$$\Im(\lambda_5 e^{i2\theta}) = |\lambda_5| \sin(\arg(\lambda_5) + 2\theta) \quad \Longrightarrow \quad |\lambda_5| = 0 \vee \sin(\arg(\lambda_5) + 2\theta) = 0. \tag{3.3}$$

The implication in Eq.(3.3) can only be realised based on the mass spectrum written out in terms of λ. We will firstly find the masses squared in terms of Z.

The squared mass term for $H^{\pm}$ is given by:

$$m_{H^{\pm}}^2 = \frac{v^2 Z_3}{2}. \tag{3.4}$$

We have the following 3×3 squared mass matrix $\mathcal{M}$ for neutral Higgs fields using Eq.(2.14) and ordering them as $(H_1 - H_2 - A)$:

$$\mathcal{M}_{\text{tree}} \equiv \frac{v^2}{2}\begin{pmatrix} 3Z_1 & 3\Re(Z_6) & -3\Im(Z_6) \\ 3\Re(Z_6) & Z_3 + Z_4 + \Re(Z_5) & -\Im(Z_5) \\ -3\Im(Z_6) & -\Im(Z_5) & Z_3 + Z_4 - \Re(Z_5) \end{pmatrix}. \tag{3.5}$$

We combine Eqs. (3.1) and (3.5):

$$\mathcal{M}_{\text{tree}} = \frac{v^2}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & Z_3 + Z_4 + \Re(Z_5) & 0 \\ 0 & 0 & Z_3 + Z_4 - \Re(Z_5) \end{pmatrix}. \tag{3.6}$$

From Eq.(3.6) we note that $\mathcal{M}_{\text{tree}}$ is diagonal after imposing tadpole conditions, and because $\Im(Z_5) \propto T_A^{\text{tree}}$ by Eq.(2.12). We therefore have that H_1, H_2, and A are eigenstates of $\mathcal{M}_{\text{tree}}$. We obtain that H_1 corresponds to a pseudo-Goldstone state, associated with the spontaneous breaking of the scale symmetry [13] along a flat direction of H_1 in the potential V_{tree}.

The Higgs masses squared written in terms of λ using Eqs.(2.11),(3.4), and (3.6) yield that

$$\begin{aligned} m_{H^{\pm}}^2 &= -\frac{v^2(\lambda_4 + \Re(\lambda_5 e^{i2\theta}))}{2}, & m_{H_1}^2 &= 0, \\ m_{H_2}^2 &= -v^2(\lambda_3 + \lambda_4 + \Re(\lambda_5 e^{i2\theta})), & m_A^2 &= -v^2\Re(\lambda_5 e^{i2\theta}). \end{aligned} \tag{3.7}$$

In Eq.(3.7) we obtain implicit θ-dependence of the squared mass terms. However, if the mass spectrum is non-negative[2] up-to pseudo-Goldstone bosons, then we obtain that $|\lambda_5|$ is non-zero. Furthermore, $\arg(\lambda_5)$ is then given by:

$$\arg(\lambda_5) = \pi - 2\theta, \tag{3.8}$$

which is a direct consequence of Eq.(3.3) and A being massive. The mass spectrum given in Eq.(3.7) is thus simplified using Eq.(3.8):

$$\begin{aligned} m_{H^{\pm}}^2 &= \frac{v^2(|\lambda_5| - \lambda_4)}{2}, & m_{H_1}^2 &= 0, \\ m_{H_2}^2 &= -v^2(\lambda_3 + \lambda_4 - |\lambda_5|), & m_A^2 &= v^2|\lambda_5|. \end{aligned} \tag{3.9}$$

We have then proven that we do not have any θ-dependence at tree-level. We also note that t_β is an unphysical quantity at tree-level, as it does not contribute to any of the masses

[2]If A was a pseudo-Goldstone boson, we would have a $U(1)_{PQ}$ instead of a $\mathbb{Z}_2$ symmetry [29].

squared. Such effects are stressed by Haber and O'Neil in [19]. Furthermore, we find that only λ_3, λ_4, and $|\lambda_5|$ have physical significance in the tree-level Higgs mass spectrum. We therein obtain 3 free parameter choices.

The quartic couplings $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ can be written in terms of the tree-level masses squared, t_β, and θ due to Eq.(3.8):

$$\lambda_1 = \frac{m_{H_2}^2 t_\beta^2}{v^2}, \quad \lambda_2 = \frac{m_{H_2}^2}{v^2 t_\beta^2}, \quad \lambda_3 = \frac{2m_{H^\pm}^2 - m_{H_2}^2}{v^2}, \quad \lambda_4 = \frac{m_A^2 - 2m_{H^\pm}^2}{v^2}, \quad \lambda_5 = -\frac{m_A^2}{v^2} e^{-i2\theta}. \tag{3.10}$$

The quartic couplings λ_1 and λ_2 are derived using Eqs.(2.11) and (3.2). Eq.(3.10) is similar to Lee and Pilafsis' [30] results up to a phase in λ_5. However the implicit θ-dependence cancelled out with the phase of Φ_2.

The minimum of V_{tree} is obtained as:

$$\lambda_1, \lambda_2 > 0 \wedge \lambda_3 > \lambda_{345} \wedge \Re(\lambda_5 e^{i2\theta}) < 0 \wedge \Im(\lambda_5 e^{i2\theta}) = 0. \tag{3.11}$$

Thus we have found that we are at the minimum due to convexity of the potential. We also obtain a flat direction of H_1:

$$\sqrt{\lambda_1 \lambda_2} + \lambda_{345} = 0. \tag{3.12}$$

Therein we can add a 1-loop radiative correction term to the potential V_{tree}.

4 1-loop radiative corrections

4.1 Properties of the 1-loop effective potential

The motivation to investigate 1-loop radiative corrections [14] in our model is a generalisation of GW theory [13]. Such corrections lift the flat direction for H_1, and provide radiative masses. For this purpose, we expand around the minimum of V_{tree} obtained in the previous section. The 1-loop effective potential $V_{\text{1-loop}}$ is expressed as [13, 14, 30]:

$$\begin{aligned} 64\pi^2 V_{\text{1-loop}} = {} & \mu_{H_2}^4 \left(-\frac{3}{2} + \log\left(\frac{\mu_{H_2}^2}{Q^2}\right)\right) + \mu_A^4 \left(-\frac{3}{2} + \log\left(\frac{\mu_A^2}{Q^2}\right)\right) \\ & + 2\mu_{H^\pm}^4 \left(-\frac{3}{2} + \log\left(\frac{\mu_{H^\pm}^2}{Q^2}\right)\right) + 6\mu_W^4 \left(-\frac{5}{6} + \log\left(\frac{\mu_W^2}{Q^2}\right)\right) \\ & + 3\mu_Z^4 \left(-\frac{5}{6} + \log\left(\frac{\mu_Z^2}{Q^2}\right)\right) - 12\mu_t^4 \left(-1 + \log\left(\frac{\mu_t^2}{Q^2}\right)\right). \end{aligned} \tag{4.1}$$

The effective potential is written using the $\overline{MS}$ renormalisation scheme, Q is a renormalisation scale [13], and μ_j^2 are background field-dependent masses squared [13, 30].

A compact expression of Eq.(4.1) is given by:

$$V_{\text{1-loop}} = \frac{1}{64\pi^2}\sum_{j\in\Pi}\beta_j\mu_j^4\left(\alpha_j + \log\left(\frac{\mu_j^2}{Q^2}\right)\right), \quad \Pi = \{H_2, A, H^{\pm}, W^{\pm}, Z, t\}. \tag{4.2}$$

The background field-dependent masses squared are defined as [30]:

$$\begin{aligned}
\mu_{H_2}^2 &= -2v^2\lambda_{345}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), & \mu_{W^\pm}^2 &= \frac{g^2}{2}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2),\\
\mu_A^2 &= 2v^2|\lambda_5|(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), & \mu_Z^2 &= \frac{g^2}{2c_w^2}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2),\\
\mu_{H^\pm}^2 &= v^2\left(|\lambda_5| - \lambda_4\right)(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2), & \mu_t^2 &= 2\frac{m_t^2}{v^2c_\beta^2}\Phi_1^\dagger\Phi_1.
\end{aligned} \tag{4.3}$$

In Eq.(4.3) we introduce SM values: m_t is the top quark mass, g is the weak coupling constant, and c_w is the cosine of the Weinberg angle θ_w [30]. We emphasize that the fermion sector, which in our analysis only contains the top quark, only couples to Φ_1 due to the $\mathbb{Z}_2$ symmetry in a Type-I 2HDM [20]. The motivation to not consider the other fermions is that we only consider the heaviest particles. Such approximations have been imposed in other studies [31, 32, 33] on GW theory based on the work by Lee and Pilafsis [30].

The renormalisation scale Q for the effective 1-loop correction potential $V_{\text{1-loop}}$ is defined by the GW renormalization scale Λ_{GW} [13], which satisfies the following:

$$\log\left(\frac{v^2}{\Lambda_{GW}^2}\right) + \frac{\mathcal{A}}{\mathcal{B}} + \frac{1}{2} = 0. \tag{4.4}$$

In Eq. (4.4) the values for $\mathcal{A}$ and $\mathcal{B}$ are defined as [13]:

$$\mathcal{A} = \frac{1}{64\pi^2v^4}\sum_{j\in\Pi}\beta_j\left\langle\mu_j^4\right\rangle\left(\alpha_j + \log\left(\frac{\langle\mu_j^2\rangle}{v^2}\right)\right), \tag{4.5}$$

$$\mathcal{B} = \frac{1}{64\pi^2v^4}\left\langle\mu_A^4 + \mu_{H_2}^4 + 2\mu_{H^\pm}^4 + 6\mu_W^4 + 3\mu_Z^4 - 12\mu_t^4\right\rangle. \tag{4.6}$$

These quantities are derived in [13] and re-written for our notation choice of Eqs.(4.2) and (4.3).

We note that GW theory predicts a manifestly massive particle called the Scalon. The Scalon replaces the pseudo-Goldstone boson H_1, and it has squared mass [13]:

$$m_S^2 = 8v^2\mathcal{B}. \tag{4.7}$$

The Scalon can be understood as a massive particle that emerges due to radiative corrections at 1-loop level by lifting the flat direction of H_1 [13].

We define new functions to write down compact expressions:

$$\delta(S) = \left\langle \frac{1}{64\pi^2} \sum_{j\in S} \kappa_j \beta_j \mu_j^2 \left(2\alpha_j + 2\log\left(\frac{\mu_j^2}{\Lambda_{GW}^2}\right) + 1 \right) \right\rangle,$$

$$\Delta(S) = \left\langle \frac{1}{64\pi^2} \sum_{j\in S} v^2 \kappa_j^2 \beta_j \left(2\alpha_j + 2\log\left(\frac{\mu_j^2}{\Lambda_{GW}^2}\right) + 3 \right) \right\rangle, \tag{4.8}$$

where we perform sums over some set $S \subseteq \Pi$ of fields, and we note that $\delta(S), \Delta(S) \in \mathbb{R}$. We derive Eq.(4.8) from taking field derivatives of Eq.(4.2) in terms of the Higgs fields.

The variable κ_j is used to express the background field-dependent masses squared in Eq.(4.3):

$$\kappa_j \equiv \frac{2\langle \mu_j^2 \rangle}{v^2\left(1 - \delta_j^t s_\beta^2\right)}. \tag{4.9}$$

We introduce the Kronecker delta δ_j^t because of the top quark coupling. The terms of interest using Eq.(4.8) are

$$M = \delta(\Pi \setminus \{t\}), \quad \delta_t = \delta(\{t\}), \quad \Delta_\mu = \Delta(\Pi \setminus \{t\}), \quad \Delta_t = \Delta(\{t\}). \tag{4.10}$$

In Eq.(4.10) we have used set theory notation. The representation of $\Pi \setminus \{t\}$ is that we take the set Π and exclude t. We have therein isolated top quark contributions.

Eq. (4.10) is used to obtain log-free relations:

$$\Delta_\mu - 2M = \sum_{j\in\Pi\setminus\{t\}} \frac{\beta_j \langle \mu_j^4 \rangle}{8\pi^2 v^2}, \qquad \Delta_t c_\beta^2 - 2\delta_t = \frac{\beta_t \langle \mu_t^4 \rangle}{8\pi^2 v^2 c_\beta^2}. \tag{4.11}$$

Furthermore, Eq. (4.10) is used to relate M and δ_t as a consequence of Eq.(4.4):

$$\begin{aligned} M &= \frac{1}{16\pi^2 v^2} \left\langle \sum_{j\in\Pi\setminus\{t\}} \beta_j \mu_j^4 \left(\alpha_j + \log\left(\frac{\mu_j^2}{\Lambda_{GW}^2}\right) + \frac{1}{2} \right) \right\rangle + c_\beta^2(\delta_t - \delta_t) \\ &= \underbrace{4v^2 \left(\mathcal{A} + \mathcal{B}\left(\frac{1}{2} + \log\left(\frac{v^2}{\Lambda_{GW}^2}\right)\right)\right)}_{=0} - \delta_t c_\beta^2 \\ &= -\delta_t c_\beta^2, \end{aligned} \tag{4.12}$$

where we have also used Eqs.(4.5), (4.6), and (4.8).

4.2 Mass spectrum with 1-loop radiative corrections

The tadpole conditions using Eq.(2.15) with the added effective potential $V_{\text{1-loop}}$ are

$$T_{H_1}^{\text{1-loop}} = v\left(\frac{v^2 Z_1}{2} + M + \delta_t c_\beta^2\right), T_{H_2}^{\text{1-loop}} = v\left(\frac{v^2 \Re(Z_6)}{2} - \delta_t c_\beta s_\beta\right), T_A^{\text{1-loop}} = -\frac{v^3 \Im(Z_6)}{2}. \tag{4.13}$$

To investigate θ-dependence of the mass spectrum we investigate the constraints for the quartic couplings λ similarly to what we did in Eq.(3.3). We combine Eqs.(2.11) and (4.13):

$$\lambda_1 c_\beta^2 = -s_\beta^2 \lambda_{345} - \frac{2}{v^2}(M + \delta_t), \quad \lambda_2 s_\beta^2 = -c_\beta^2 \lambda_{345} - \frac{2}{v^2} M, \quad \Im(\lambda_5 e^{i2\theta}) = 0. \tag{4.14}$$

In this subsection $\lambda_{345} = \lambda_3 + \lambda_4 + \Re(\lambda_5 e^{i2\theta})$ until $\Re(\lambda_5 e^{i2\theta})$ can be simplified. With 1-loop radiative corrections we also have a $\mathcal{CP}$ conserving potential as a consequence of Eqs.(2.12) and (2.13).

The 1-loop squared mass term for $H^\pm$ is given by:

$$m_{H^\pm}^2 = \frac{v^2 Z_3}{2} + M + \delta_t s_\beta^2. \tag{4.15}$$

We note that it appears as if $H^\pm$ obtains a radiative mass. Expressing it in terms of the quartic couplings λ, we obtain:

$$\begin{aligned} m_{H^\pm}^2 &= \frac{v^2 Z_3}{2} + M + \delta_t s_\beta^2 \\ &= \frac{v^2}{2}\left(\lambda_3 + (s_\beta c_\beta)^2(\lambda_1 + \lambda_2 - 2\lambda_{345})\right) + M + \delta_t s_\beta^2 \\ &= \frac{v^2}{2}\left(\lambda_3 - \lambda_{345} - \frac{2}{v^2}\left(M + \delta_t s_\beta^2\right)\right) + M + \delta_t s_\beta^2 \\ &= -\frac{v^2(\lambda_4 + \Re(\lambda_5 e^{i2\theta}))}{2}, \end{aligned} \tag{4.16}$$

where we have used Eqs.(2.11) and (4.14). We find that the squared mass term for $H^\pm$ is the same as at tree-level.

We get the following squared mass matrix $\mathcal{M}_{\text{1-loop}}$ by means of the same procedure performed for Eq.(3.5):

$$\mathcal{M}_{\text{1-loop}} \equiv \begin{pmatrix} \rho_1 + \frac{3}{v} T_{H_1}^{\text{1-loop}} & \omega_1 + \frac{1}{v} T_{H_2}^{\text{1-loop}} & \frac{3}{v} T_A^{\text{1-loop}} \\ \omega_1 + \frac{1}{v} T_{H_2}^{\text{1-loop}} & \rho_2 + \omega_2 & -\frac{v^2}{2}\Im(Z_5) \\ \frac{3}{v} T_A^{\text{1-loop}} & -\frac{v^2}{2}\Im(Z_5) & \rho_2 - \omega_2 \end{pmatrix}, \tag{4.17}$$

where we use the following definitions for the elements of $\mathcal{M}_{\text{1-loop}}$

$$\begin{aligned} \omega_1 &= v^2 \Re(Z_6) - \Delta_t s_\beta c_\beta^3, & \omega_2 &= \frac{1}{2}\left(v^2 \Re(Z_5) + \Delta_t s_\beta^2 c_\beta^2\right), \\ \rho_1 &= 8\mathcal{B} v^2, & \rho_2 &= \frac{1}{2}\left(v^2(Z_3 + Z_4) + 2M + (\Delta_t c_\beta^2 + 2\delta_\tau) s_\beta^2\right). \end{aligned} \tag{4.18}$$

We also note that $\Im(Z_5) \propto T_A^{\text{1-loop}}$, as is given in Eq.(2.12).

Combining Eq.(4.13) and (4.17) yield:

$$\mathcal{M}_{\text{1-loop}} = \begin{pmatrix} \rho_1 & \omega_1 & 0 \\ \omega_1 & \rho_2 + \omega_2 & 0 \\ 0 & 0 & \rho_2 - \omega_2 \end{pmatrix}. \tag{4.19}$$

We have thus obtained the mass spectrum with 1-loop corrections, and we investigate if it is θ-dependent in the same way as we did at tree-level.

Combing Eqs.(4.13) and (4.18) allows us to find the off-diagonal element ω_1 in Eq.(4.19);

$$\begin{aligned}\omega_1 &= v^2\Re(Z_6) - \Delta_t s_\beta c_\beta^3 \\ &= s_\beta c_\beta \left(2\delta_t - \Delta_t c_\beta^2\right) \\ &= \frac{3m_t^4 t_\beta}{2\pi^2 v^2}.\end{aligned} \tag{4.20}$$

The last equality follows from Eq.(4.11). Thus H_1 and H_2 mix depending on the top quark mass m_t and t_β defined in Eq.(2.2), and we therefore do not obtain a trivial expression for their masses squared. Furthermore, t_β is a physically significant parameter with 1-loop radiative corrections.

We use Eqs.(4.8), (4.12), and (4.14) to calculate the squared mass of A:

$$\begin{aligned}m_A^2 &= \rho_2 - \omega_2 \\ &= \frac{v^2}{2}\left(Z_3 + Z_4 - \Re(Z_5)\right) + M - \delta_t s_\beta^2 \\ &= -v^2\Re(\lambda_5 e^{i2\theta}) \\ &= v^2|\lambda_5|.\end{aligned} \tag{4.21}$$

The last equality follows as a consequence of Eq.(3.8). We notice that the mass of A is unaffected by radiative corrections. A general result is that $\Re(\lambda_5 e^{i2\theta}) = -|\lambda_5|$. We conclude that our mass spectrum does not have θ-dependence, even including 1-loop radiative corrections.

Furthermore, we investigate Eq.(4.19) in the special case when we do not have top quarks. We let $m_t \to 0$ to obtain that

$$\begin{aligned}m_{H^\pm}^2 &= \frac{v^2(|\lambda_5| - \lambda_4)}{2}, & m_{H_1}^2 &= 8v^2\mathcal{B}, \\ m_{H_2}^2 &= -v^2(\lambda_3 + \lambda_4 - |\lambda_5|), & m_A^2 &= v^2|\lambda_5|.\end{aligned} \tag{4.22}$$

The calculation for the mass term of H_2 is identical to that at tree-level. We would therefore, in the absence of the top quark mass m_t, obtain that H_1 is the only Higgs boson that obtains mass from radiate corrections.

We now consider t_β-dependence. In Eq.(4.20) we have that $\omega_1 \propto t_\beta$. If we had switched order for $\mathbb{Z}_2$ symmetry assignment in Eq.(2.3), such that $j = 2, k = 1$, then we would get that $\omega_1 \propto \frac{1}{t_\beta}$. This is obtained because the top quark would then couple to Φ_2 instead of Φ_1 in this case. However, this choice would not significantly change our results, and would only lead to a difference in changing s_β and c_β, and exchanging $\pm$ signs, whenever necessary. We thereby conclude that t_β is dependent on the choice of $\mathbb{Z}_2$ symmetry assignments.

The result that we have obtained for the mass spectrum is similar to other studies [30, 31, 32, 33] performed in the scale invariant 2HDM with 1-loop corrections.

5 Phenomenological aspects

5.1 Mixing angles

We begin our phenomenological study by investigating the mixing between H_1 and H_2. Firstly, we write out the elements of $\mathcal{M}_{\text{1-loop}}$ which are involved in the mixing:

$$[\mathcal{M}_{\text{1-loop}}]_{11} = m_S^2, \tag{5.1}$$

$$[\mathcal{M}_{\text{1-loop}}]_{22} = -v^2\lambda_{345} + \frac{3m_t^4}{4\pi^2 v^2}\left(1 - 2t_\beta^2 - 2\log\left(\frac{m_t^2}{\Lambda_{GW}^2}\right)\right), \tag{5.2}$$

$$[\mathcal{M}_{\text{1-loop}}]_{12} = \frac{3m_t^4 t_\beta}{2\pi^2 v^2}. \tag{5.3}$$

The squared mass matrix $\mathcal{M}_{\text{1-loop}}$ can be diagonalised as $D = R\mathcal{M}_{\text{1-loop}}R^T$ where:

$$R \equiv \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \in \text{SO}(3). \tag{5.4}$$

The mixing angle θ_{12} can be found using Eqs.(5.1), (5.2), and (5.3):

$$\tan(2\theta_{12}) = \frac{6m_t^4 t_\beta}{3m_t^4\left(1 - 2t_\beta^2 - 2\log\left(\frac{m_t^2}{\Lambda_{GW}^2}\right)\right) - 4\pi^2 v^2\left(v^2\lambda_{345} + m_S^2\right)}. \tag{5.5}$$

We impose mass ordering by denoting h as the lightest, and H as the heaviest Higgs boson obtained from the mixing of H_1 and H_2.

From Haber and O'Neil's paper [19] we adopt a method of finding the physical Higgs bosons, and the electrically neutral Goldstone bosons G^0, in terms of the fields in (Φ_1, Φ_2) from Eq.(2.2). This is given by:

Table 2: Table with the couplings as defined in Haber and O'Neil's paper [19].

k	q_{k1}	q_{k2}
1	$c_{12}c_{13}$	$-s_{12} - ic_{12}s_{13}$
2	$s_{12}c_{13}$	$c_{12} - ic_{12}s_{13}$
3	s_{13}	ic_{13}
4	i	0

$$h_k = \frac{1}{\sqrt{2}}\sum_{j=1}^{2}\left[(q_{k1}^*[\Omega]_{1j} + q_{j2}^*[\Omega]_{2j})\overline{\Phi}_j + \text{H.c}\right]. \tag{5.6}$$

The convention in [19] is used here, with a modification. In Eq.(5.6) we use $\overline{\Phi}_j = \frac{1}{\sqrt{2}}e^{i\theta_j}\left(\varphi_j^0 + ia_j^0\right)$, where $\theta_1 = 0$ and $\theta_2 = \theta$.

For the angles in Tab.2 we have $\theta_{13} = 0$, and θ_{12} is defined in Eq.(5.5), while h_k for $k \in \{1, 2, 3, 4\}$ is defined as:

$$h_1 \equiv h, \quad h_2 \equiv H, \quad h_3 \equiv A, \quad h_4 \equiv G^0. \tag{5.7}$$

We then obtain:

$$\begin{aligned} h &= c_\psi \varphi_1^0 + s_\psi \varphi_2^0, & H &= -s_\psi \varphi_1^0 + c_\psi \varphi_2^0, \\ A &= -s_\beta a_1^0 + c_\beta a_2^0, & G^0 &= c_\beta a_1^0 + s_\beta a_2^0. \end{aligned} \tag{5.8}$$

Here $\psi = \beta - \theta_{12}$ for the case of 1-loop radiative corrections, and for the tree-level study we achieve that $\psi = \beta$. The Higgs bosons h, H, and A have their expected $\mathcal{CP}$ properties for Eq.(2.5).

The coupling[3] g_{hVV} with vector bosons $W^\pm$ and Z^0 is given by [19, 25, 30]:

$$g_{h_j VV} \equiv q_{j1} g_{\mathrm{SM,hVV}}, \quad j \in \{1, 2\}, \tag{5.9}$$

where the identification from Eq.(5.7) is assumed, and $g_{\mathrm{SM,hVV}}$ is given by the SM Higgs coupling to gauge bosons $W^\pm$ and Z^0 [5, 19]. We set $g_{\mathrm{SM,hVV}} \equiv 1$ for this thesis without loss of generality [25]. For analysis of couplings we will consider $1 \geq |g_{h_j VV}| \geq 0.99$ as a limit to be SM-like. Furthermore, the Higgs couplings to other fields can be found using a more extensive formalism in [30, 33, 32, 34].

5.2 Experimental analysis

We have alignment with the SM Higgs boson when $\theta_{12} = 0$ [34], as can be seen in Eq.(5.9), we will refer to this as the alignment limit [18, 34]. Such can be achieved by means of having ω_1 very small compared to the difference of the elements defined in Eqs.(5.1) and (5.2).

In the alignment limit we find that the Scalon mass squared is

$$m_S^2 = (125 \text{ GeV})^2, \tag{5.10}$$

thus equal to the SM Higgs boson mass squared [7]. We can then combine Eqs.(4.6), (4.7), and (5.10) for a bound of the 4th power sum of the masses for the tree-level Higgs bosons:

$$\sum_{j \in \mathcal{H}} m_j^4 = (540 \text{ GeV})^4, \quad \mathcal{H} = \{H_t, A, H^+, H^-\}. \tag{5.11}$$

We write H_t as it is the massive $\mathcal{CP}$-even tree-level Higgs boson, and to generalise Eq.(5.8) for tree-level. While for the 1-loop $\mathcal{CP}$-even Higgs boson we will write h and H as given previously.

[3] Since $\theta_{13} = 0$, the $\mathcal{CP}$-odd Higgs boson A is not considered as $g_{AVV} = 0$.

The value 540 GeV comes from summing up the SM masses for $W^{\pm}$, Z^0 and the top quark [7]. We recognise that Eq.(5.11) is equal to other evaluations of this quantity [31, 32, 35], and comes as a general consequence of GW theory when identifying the SM Higgs boson with the Scalon. The overall implication of Eq.(5.11) is profound, as it therefore sets an upper bound of tree-level mass spectrum in Eq.(3.9). We use Eq.(5.11) to find bounds on the tree-level masses to study phenomena with a 1-loop correction by finding suitable parameters λ. We then find constraints on the tree-level masses through Eq.(3.10) and (5.11). Since our theory is θ-independent, we therefore have the same degrees of freedom for parameter choice as in other studies [30, 31, 32] without the assumption of a phase θ for Φ_2. The free parameters that we have are $(m_H^{\pm}, m_A, t_\beta, \lambda_3)$. This is motivated by Eq.(3.10) where we can re-write λ_3, λ_4, and $|\lambda_5|$ in terms of the tree-level masses squared. The renormalisation scale Λ_{GW} will be updated based on the tree-level masses in Eq.(4.4), and is thus not a free parameter.

In this analysis, we do not restrict our study to experimental data. B-meson decays ($b \to s\gamma$) restrict which choice of the mass for $H^{\pm}$ we can pick in combination with t_β in the Type-II 2HDM [36, 37]. This result is generalised for the Type-I 2HDM and an interpolation of data has been performed in [31]. Furthermore, we do not consider EW precision constraints in this thesis. However it is worthwhile to mention that authors usually pick $m_{H^{\pm}} \approx m_A$ [30, 31, 32, 38] as a consequence of these constraints.

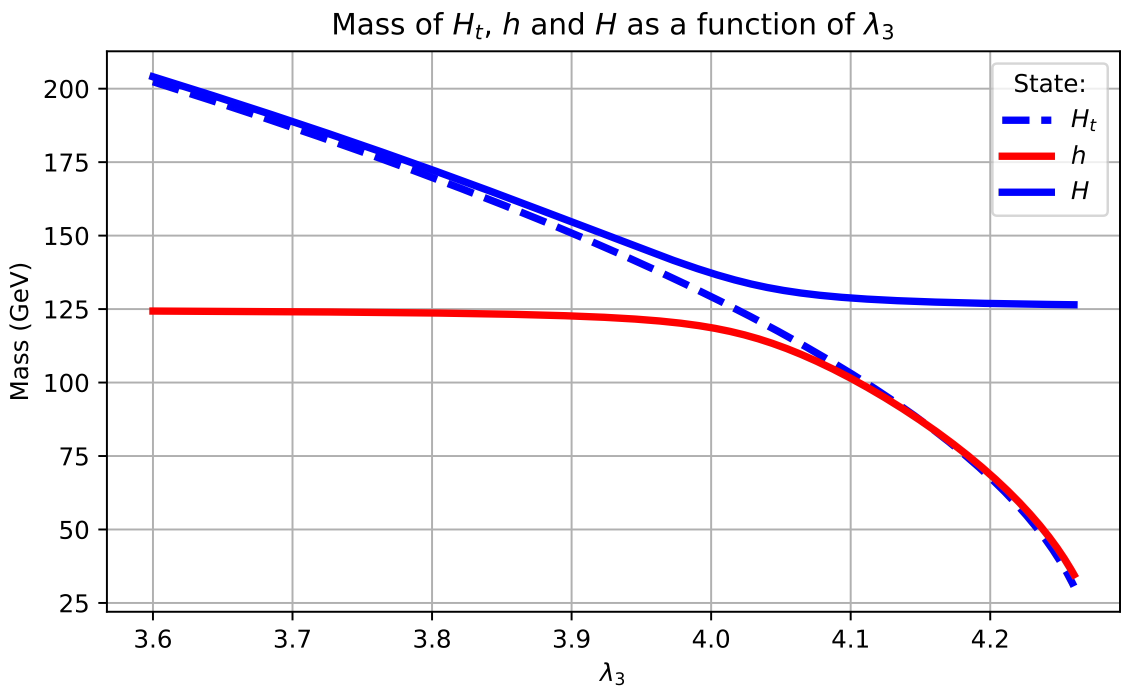


Figure 1: Figure showcasing the masses of h, H and H_t as a function of λ_3. We find crossing of the tree-level heavy $\mathcal{CP}$-even H_t between the 1-loop $\mathcal{CP}$-even Higgs bosons h and H at $\lambda_3^c \approx 4.02$.

In Fig. 1 we have fixed $m_A = 400$ GeV, $m_{H^\pm} = 360$ GeV, and $t_\beta = 1$. With respect to Eq.(3.9), this corresponds to $\lambda_4 = -1.6$, $|\lambda_5| = 2.6$ while we let $\lambda_3 \in [3.6, 4.26]$. Furthermore, the initial value for $\Lambda_{GW} = 250$ GeV.

In particular we note that we always have one of the states going asymptotically along the mass of the SM Higgs boson H^0, which we associated with the Scalon. We also notice some differences compared to the Lee and Pilafsis model used in [30].

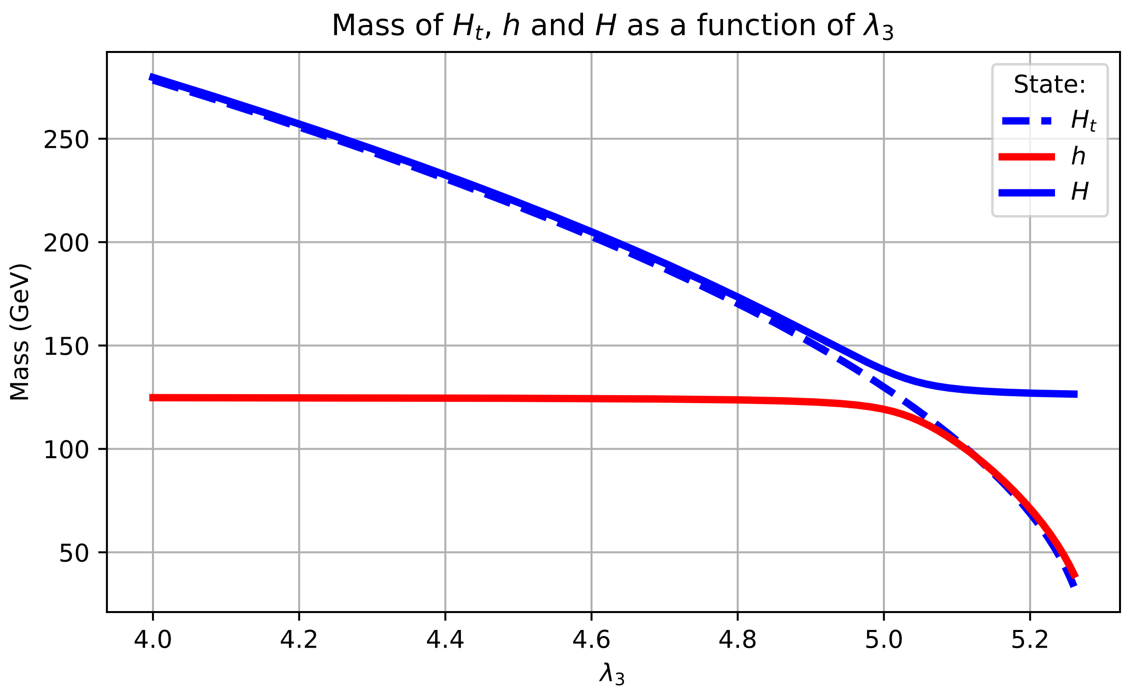


Figure 2: Figure showcasing the masses of h, H and H_t as a function of λ_3. This is using the Lee and Pilafsis model [30] where crossing occurs at $\lambda_3^c \approx 5.06$.

In Fig. 2 we use the same values as Lee and Pilafsis in [30]. They fixed $m_A = m_{H^\pm} = 400$ GeV, and $t_\beta = 1$. With respect to Eq.(3.9), this corresponds to $\lambda_4 = -|\lambda_5| = -2.6$ while we let $\lambda_3 \in [4, 5.26]$. Furthermore, the initial value for $\Lambda_{GW} = 254$ GeV.

However, all the differences are because $\lambda_4, |\lambda_5|$, and Λ_{GW} have different values due to different choice of masses for A and $H^\pm$. We note that the corresponding numerical significance of Λ_{GW} arises in Eq.(5.2). We therefore have identified the differences in values for λ_3^c and the shape of the curves.

Now we investigate the couplings for hVV defined in Eq.(5.9) and Tab. 2. Specifically the coupling g_{hVV} squared for the Higgs states h and H to the vector bosons $W^\pm$ and Z^0. These couplings are given by q_{k1}, where $k \in \{1, 2\}$ for the cases that we investigate.

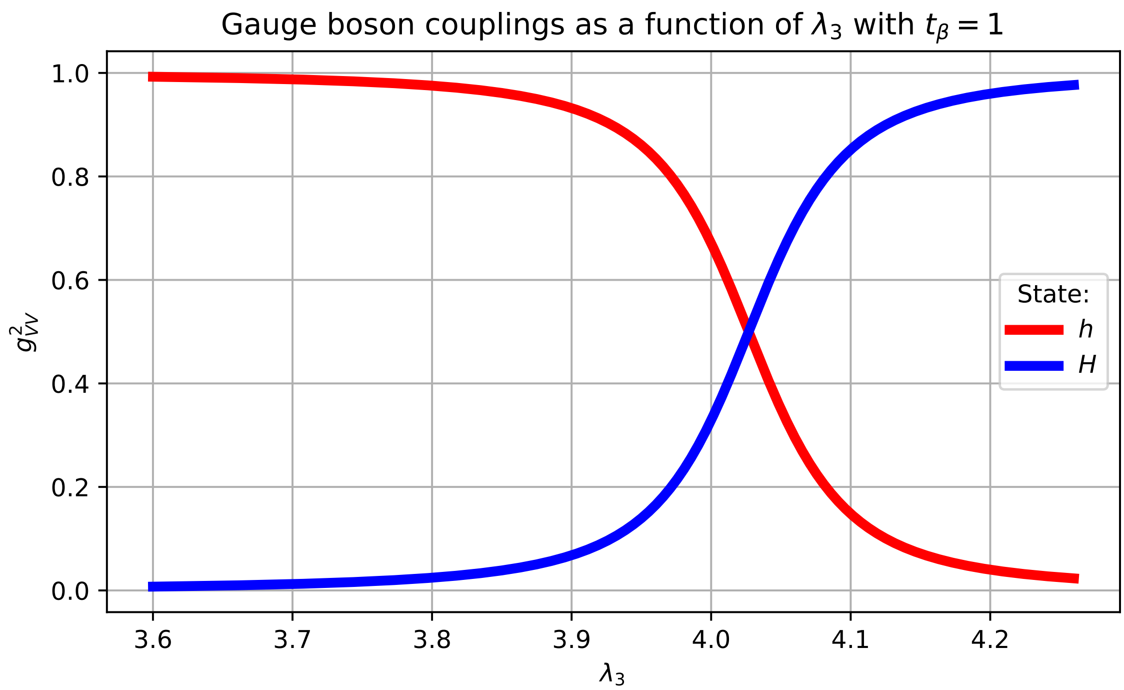


Figure 3: Figure showcasing the coupling g^2_{hVV} and g^2_{HVV} as a function of λ_3. The y-label g^2_{VV} is used while the legend shows which Higgs boson the coupling is to.

In Fig. 3 we observe that the couplings to vector bosons are favoured for the SM-like mass as seen in Fig.1 when we compare the parameter space points for different λ_3.

As is seen when comparing Figs. 1 and 3, the couplings to vector bosons appear to strongly correlate with the Higgs state with mass almost equivalent to the Higgs boson H^0 in SM. However, the coupling of H to gauge bosons is not fully aligned. Due to numerical limitations, when the $\mathcal{CP}$-even tree-level Higgs boson H_t obtains 0 mass at $\lambda_3 = \frac{2}{v^2} m^2_{H^\pm}$, we do not observe exact alignment. With $t_\beta = 1$ we obtain that $\max(|g_{HVV}|) \approx 0.98$, which is not within a conservative limit of $1 \geq |g_{HVV}| \geq 0.99$ to account for theoretical limitations.

We do notice that there exist crossing points where the Higgs states H and h have almost equivalent squared coupling strengths. Therefore there could exist some deviations that could come about experimentally to verify the mixing angle if the 2HDM proves to be correct for these regions where neither h nor H are aligned. However, this assertion has a limitation, as SM-like alignment is expected and assumed [20, 33].

Given that there is a dependence on t_β for the masses of h and H it is therefore important to see how this influences the coupling strength g_{HVV} and g_{hVV}. We will therefore investigate this for $t_\beta \in \{\frac{1}{3}, 3\}$, using the same masses for A and $H^\pm$ as in Figs.1 and 3.

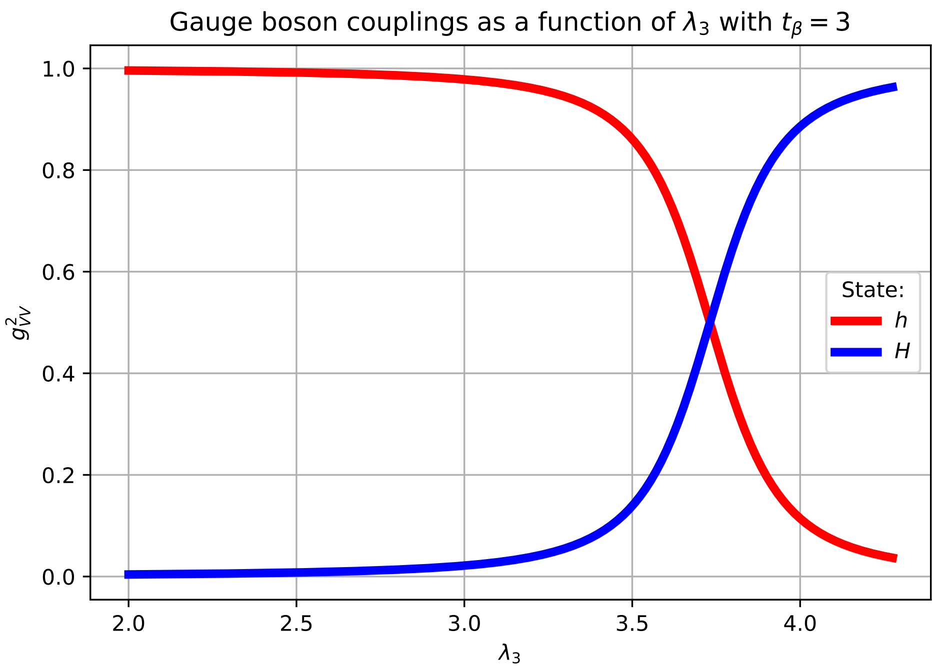


Figure 4: Similar to Figure 3, $t_\beta = 3$.

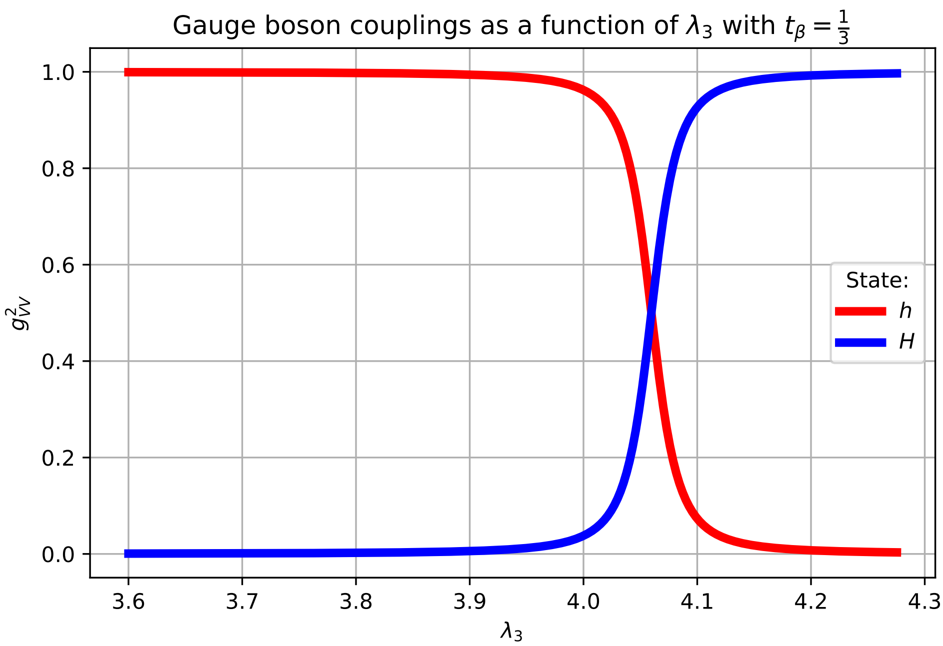


Figure 5: Similar to Figure 3, $t_\beta = \frac{1}{3}$.

The other values for t_β are shown in Figs. 4 and 5, and the graphs can be compared to Fig. 3 to showcase that t_β is physically significant for Higgs couplings with Z^0 and $W^\pm$.

In Fig. 4 the critical value for λ_3 when the coupling is favouring the H and h is shifted. Furthermore, to even see that h is SM-like we shifted the graph to be $\lambda_3 \in [2, 4.26]$.

The shift of λ_3 can be understood through the formulation of the diagonalisation of h and H, where we note that the contribution from the top quark is proportional to t_β^2 in Eq.(5.2). Therefore the mass contribution from λ_3 needs to be smaller for a crossing. However, this result has experimentally been ruled out due to B-meson decay [36, 31]. Furthermore, the coupling of H is not SM-like in the numerical limit.

The opposite conclusion can be observed in Fig. 5, where the smaller contribution from the top quark leads to a sharper switch about the same critical point λ_3^c as compared to the case in Fig. 3. This effect is observed by the concavity of the graph. Furthermore, h and H are SM-like in the alignment limit, and the domain where the coupling is SM-like is realised. Such a result is critical, since this choice of t_β is allowed by the $b \to s\gamma$ decay [36, 37], and choices of $m_{H^\pm}$ have been interpolated in [31] and agree with our finding. Finally we showcase its alignment with the mass with the same parameters as used for Fig.1 but with $t_\beta = \frac{1}{3}$.

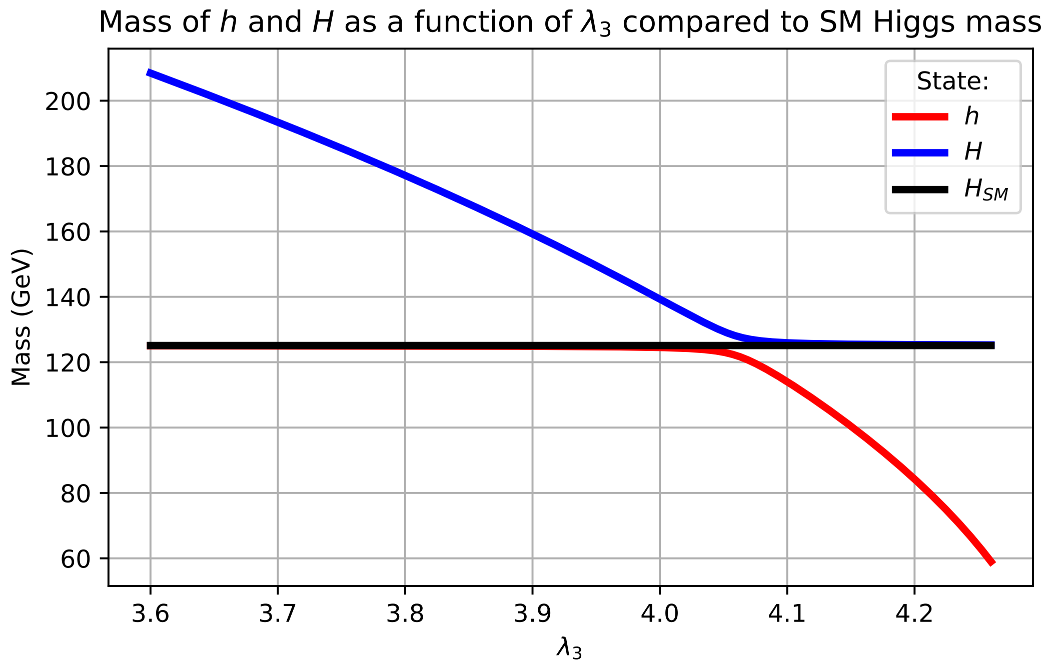


Figure 6: Figure showcasing the masses of h and H compared to the SM Higgs mass as a function of λ_3.

In Fig.6 it appears as if we have alignment, but in the limits we have an error of ± 0.3 GeV, which is currently outside the experimental error [7]. This could, however, occur due to our simplified model and us not being able to achieve exact alignment due to numerical cut-offs and other assumptions made in the theory. The latter would include not taking into account other fermions for Eq.(4.1), and because the alignment cannot be exact due to the mixing terms in Eq. (5.3) being non-zero. However, likewise to what we studied in regards to Figs.3, 4, and 5 we have only considered this error for the heavy $\mathcal{CP}$-even Higgs boson H. For h, we get alignment for all studied cases in the limit where we start our graphs.

There are other parameter space and coupling studies that have be performed with the Type-I 2HDM. However, these cases have already been covered in [30, 31, 32, 33] given that our mass spectrum is equivalent to theirs. Such a conclusion is given because the θ-dependence that occurred in our model does not have any significance on the masses squared.

We finalise this section by considering how θ-dependence can appear in the mass spectrum. If exact $\mathbb{Z}_2$ symmetry is applied but the theory is not scale invariant, we set $m_{12}^2 = \lambda_6 = \lambda_7 = 0$. Suppose further that no symmetry is applied which sets $\lambda_5 = 0$; we then obtain that λ_5 is the only complex parameter. But with a reparametrisation, we can generalise the expression for λ_5 to incorporate θ-dependency through tadpole conditions. It could therein become a situation where it is unclear if θ-dependence exists, or if it is just an unphysical phase for $\arg(\lambda_5)$. In our model, the tadpole conditions fixed the phase of λ_5 to eliminate θ-dependence. This is a general result when there is only a single complex parameter in the theory.

Other classes of $\mathbb{Z}_2$ symmetry need be also considered. Such can be softly applied where we let λ_6 and λ_7 be small. In that model, tadpole conditions do not cancel θ-dependence exactly. However one could be free to pick dependencies, and let θ be dependent on the phases of $\lambda_5, \lambda_6, \lambda_7$, thus eliminating explicit θ-dependence, as we can make θ a combination of the other phases without loss of generality.

Furthermore, there exist more papers which investigate the effect of $\mathcal{CP}$ violation with symmetries applied to allow the potential V_{tree} to not obtain $\mathcal{CP}$-conservation; which is achieved using the criteria written about above. Some examples of this can be found in [18, 19, 39].

6 Conclusions

In this thesis we have investigated how spontaneous $\mathcal{CP}$ violation and spontaneous EW symmetry breaking affects the mass spectrum for the 2HDM at tree-level given scale invariance and exact $\mathbb{Z}_2$ symmetry. We then generalised our study including 1-loop radiative corrections. We conclude that there does not exist any observable $\mathcal{CP}$ violating effect as the masses squared are θ-independent. In particular, we discovered that one could perform a set-up where one uses tadpole conditions to fully include the effects of the phase of the $\mathcal{CP}$ breaking vacuum $\langle\Phi_2\rangle$ into the phase of λ_5. Such a step is conducted by finding an expression for $\arg(\lambda_5)$ from the tadpole conditions.

By doing a direct transformation to the Higgs basis we still obtain evidence of $\mathcal{CP}$ violating quantities in Eq.(2.13) being a priori non-zero. However such $\mathcal{CP}$ violating quantities can be removed through tadpole conditions. Therefore we then have manifestly real quartic couplings, and thus obtain $\mathcal{CP}$ conservation.

A valid hypothesis would be that such manipulations could not be conducted when including the 1-loop corrections to eliminate $\mathcal{CP}$ violating effects. However, the θ-dependence gets eliminated by the tadpole conditions, which similarly was achieved on tree-level. Therefore we also obtained a mass spectrum which does not contain θ-dependence. We also make a careful assessment that the conclusion regarding $\mathcal{CP}$ conservation would only hold true for scale invariance and an exact $\mathbb{Z}_2$ symmetry, as there could manifest other phases for m_{12}^2, λ_6, and λ_7 which would make this conclusion invalid for other models. Further careful analysis can be found in [20].

Since we do not observe θ-dependence of the mass spectra nor for couplings, and that the potential is always $\mathcal{CP}$ conserving, we could therein conclude that we do not obtain spontaneous $\mathcal{CP}$ violation. We could still derive some properties in our study, such as an upper bound of the tree-level masses. We found that the tree-level masses cannot have a mass greater than 540 GeV in this model. This is, however, a general result of GW theory.

We also studied how t_β affects the couplings of $\mathcal{CP}$-even Higgs bosons to gauge bosons $W^\pm$ and Z^0 with 1-loop corrections. Such a task was conducted by fixing the mass of A and $H^\pm$. We noticed a strong dependence on the SM alignment for t_β.

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