# Comparison of Level Control Strategies for a Flotation Series in the Mining Industry

Frida Norlund

LUND UNIVERSITY

Department of Automatic Control

MSc Thesis
TFRT-6178
ISSN 0280-5316

Department of Automatic Control
Lund University
Box 118
SE-221 00 LUND
Sweden


Printed in Sweden by Tryckeriet i E-huset
Lund 2022

# Abstract

Separating valuable minerals from waste rock is an important step in the production of metals. This is for copper ore done through a process called flotation. A flotation series consists of tank cells in series where the minerals are collected in a froth on top of the cells. The level control of the flotation cells is important in order to be able to collect the froth. The first four flotation cells and the buffer tank before the series is the process considered in this thesis. This process is found in the concentrator connected to Boliden's copper mine Aitik located near Gällivare in the north of Sweden. A simulation model of the process was developed using both physical modeling and experimental data from the real process. When the simulation model of the process had been developed, different control structures were tested and evaluated. The control structures that were tested were coupled PI-controllers, an LQ-controller, an MPC-controller and a state feedback controller where the state feedback was determined using reinforcement learning. The reference-tracking properties of the different controllers were similar while a bigger difference could be seen when it came to disturbance rejection. The PI-controllers gave a stable performance but their disturbance rejection was not as good as for the other controllers. One advantage with the PI-structure is its simplicity. Unlike the LQ- and the MPC-controllers, it does not need a model of the process to control it. The MPC-controller outperformed the other controllers when it came to disturbance rejection, but it was a bit more sensitive to model errors than the LQ-controller which also performed well. The reinforcement-learning-based controller did not give a better performance than the LQ-controller and it had issues with robustness in the tuning process, making it less reliable than the other controllers. The tuning process for it also required experiments that are unreasonable to perform on the real process. There is potential in reinforcement learning approaches to deal with drifting operating conditions but this particular approach was not successful. Overall, the results indicate that model-based controllers have good potential to perform better than the PI-structure that controls the real plant today.

# Acknowledgements

This thesis has been conducted at Boliden and I am grateful for the opportunity to do it in cooperation with the company. Thank you to the MACE-section for the warm and inclusive environment and the generous help I have received whenever needed. A special thanks goes out to my supervisor at the company, Rasmus, for always supporting me throughout all different phases of this thesis project. Thank you for being someone I could turn to in all situations, no matter if it was for input and help when I was lost, discussions when I felt confused or celebrations when breakthroughs were made.

The next and equally important thank you goes out to my academic supervisor Tore, for generously sharing your rich experiences in the field and giving me tons of valuable input. But mostly thank you for always looking after me, guiding me back on the right path when I was lost and looking after my well-being during stressful periods. It has been a pleasure working with the two of you during my thesis project and I am very grateful for the interest and engagement you have shown in my work and the time you have dedicated to helping me.

At last I would like to say thank you to my family and friends for supporting and cheering me on during the course of the project.

Frida Norlund

# Contents

# 1

# Introduction

This thesis has been conducted at the mining company Boliden. When metals are produced, separating the valuable minerals from the rest of the ore is a very important process step. When copper ore is considered this can be done with a process called flotation. The flotation process consists of tank cells in series that are filled with a slurry of milled ore and water. The purpose of the process is to collect the minerals in a froth on top of the slurry, this froth is then collected and further processed. For this process to work properly and give good recovery of the minerals, the level control is important. The aim of the thesis is to study how different control strategies perform when it comes to level control of the flotation process and if there is potential for improvement compared to the control structure that controls the plant today. Apart from classical PI-controllers, model-based controllers such as LQ and MPC will be explored as well as a state feedback controller that uses reinforcement learning to determine the state feedback.

In the modeling section, a simulation model that describes the pulp levels in the process is developed. This is done through physical modeling combined with parameter estimations based on step response experiments performed on the plant. The tuning-chapter explores the tuning of the different controllers. The result section evaluates the different controllers performance when it comes to disturbance rejection and reference tracking. Real process data is used to design the tests. A feed-forward extension of the existing control structure that includes a measurable disturbance is also evaluated as well as robustness towards model errors for the model based controllers.

# 2

# Background

Boliden was founded in 1925 when gold was first found at the location that would become the town Boliden. The company has since grown and today Boliden has mines in Sweden, Finland and Ireland and the main products are zinc, copper, lead, nickel, gold and silver. One of the Swedish mines is the open-pit copper mine in Aitik, located just south of Gällivare in the north of Sweden. The mine has been active since 1965 and it is Sweden's largest open-pit copper mine. Apart from copper the mine also contains gold and silver [*Bolidens Historia* n.d.].

The process of producing metals can be divided into three main areas: mining, concentrating and smelting. The ore is first mined in the mine and then transported to the concentrator where the first step in extracting the valuable minerals is performed. What takes place in the concentrator can also be divided into three main steps: milling, flotation and dewatering. The final product from the concentrator is a concentrate with a higher percentage of minerals than the ore that entered it. This concentrate is transported to a smelter where the mineral concentrate is refined into pure metals [Stenlund, 2002].

The flotation process takes place in the concentrator. It makes use of the differences in surface properties of minerals and rock to separate the two. A flotation cell is a big tank filled with a mixture of milled ore and water. This mix is referred to as slurry or pulp. A chemical reagent is mixed with the pulp to make the minerals water repellent and air bubbles are generated at the bottom of the tank. As these bubbles rise to the top of the tank, the minerals attach to them. This results in that a mineral froth is formed on top of the pulp. The froth is collected by letting it flow over the edge of the tank. This requires good control of the pulp level in the cells. The level is to be kept high enough for the froth to escape the cell, while the pulp itself should remain in the tank. There are a number of factors that affect the pulp level in the cell: the inflow of pulp, the addition of reagents, the generation of bubbles, the outflow of pulp and the froth leaving the cell. For the cell, the addition of reagents, the amount of bubbles produced and the outflow of pulp can be controlled. How much froth that escapes the cell is a result of how much bubbles there are and how the total pulp volume in the cell changes [Kawatra, 2011] [Paulina Quintanilla, 2021]. In Figure 2.1 a schematic picture of a flotation cell is shown.

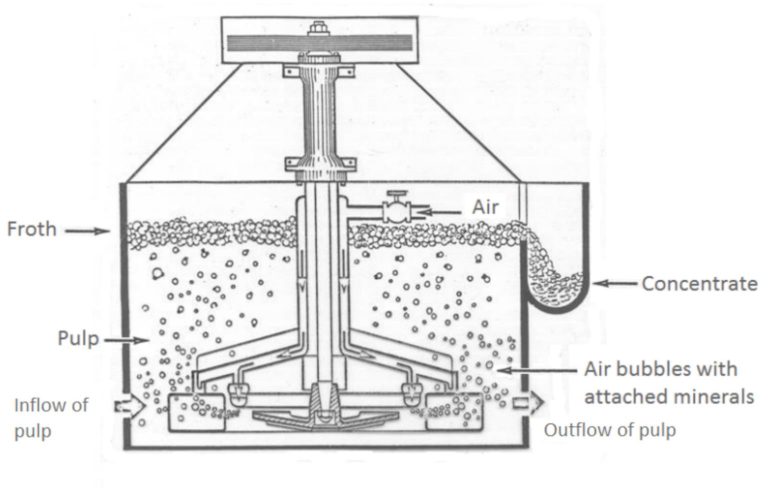


**Figure 2.1** The main principles of flotation are shown in the figure. The minerals are separated and concentrated in the froth.

The flotation cells are connected in series to form the flotation process. They are connected in such a way that the outflow of pulp from the first cell becomes the inflow to the second cell, the outflow from the second cell is the inflow to the third and so on. Between the cells there is a valve that can be controlled. The cells are mounted in such a way that there is a physical height difference between two successive cells. This difference in height, together with the level difference of pulp in the cells, is what drives the flow of pulp from the first to the second cell. How many cells the series consists of depends on which minerals are to be extracted [Kawatra, 2011].

Historically, the majority of the flotation plants have been controlled by PI-controllers. However, since there are a lot of cross-couplings within the process that the single-input-singe-output (SISO) control loops do not explicitly account for there may be better options. Therefore, the focus of research within the field has for quite some time been on multi-variable controllers. In [Stenlund, 2002] and [P. Kämpjärvi, 2003] decoupling strategies for the SISO loops are considered, as well as feed-forward and LQ-control. These papers test the different strategies in simulation and show that the performance of the level control can be improved with more advanced control structures. [Paulina Quintanilla, 2021] states that MPC is widely accepted to be able to deal with complex processes such as the flotation process. [Hodouin, 2011] explores MPC as a control structure for both the flotation and the milling process and [Kevin Brooks and Bauer, 2019] studies if improvement can be found by combining the MPC controller for the milling line with the flotation controller. Both these papers are based on simulation studies.

In order to perform simulation studies, a model of the process is needed. There are two main approaches to developing such a simulation model. [Stenlund, 2002] and [P. Kämpjärvi, 2003] take a starting point in mass balance and make use of Torricelli's law to model the dynamics of the pulp level in the cells while the models in [Kevin Brooks and Bauer, 2019] are developed through system identification from step response experiments.

# 3

# Modeling

The part of the flotation process that is to be considered in this project is called the raw series and in the Aitik concentrator it consists of a blending tank and four flotation cells. A pump pumps the pulp from the blending tank to the first flotation cell. Between the flotation cells there are valves that control the flow from the previous tank to the successive one. A schematic overview of the process is seen in Figure 3.1.

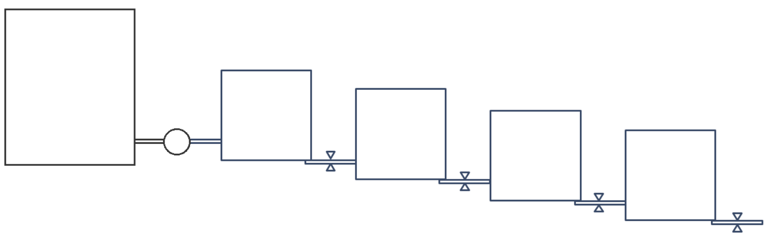

**Figure 3.1** A schematic overview of the process. A pump is connecting the blending tank to the left to the first flotation cell. Between the flotation cells there are valves that control the flow of pulp from the previous cell to the successive one.

## 3.1 Blending Tank

When the pulp enters the flotation process it first enters the blending tank. This tank does not add any air or reagents to the pulp and it functions purely as a buffer tank to make the inflow to the flotation cells more stable. Pulp is pumped from the blending tank to the first flotation cell. The relation between the outflow of pulp $q_{out_b}$ [m$^3$/s] and the control signal to the pump $u_b$ [%] is approximated from real process data to be

$$q_{out_b} = K_{flow_b} u_b, \tag{3.1}$$

where $K_{flow_b} = 1.11$ [m$^3$/s]. The level $h_b$ [%] of the blending tank is modeled as a simple integrator,

$$\frac{dh_b}{dt} = K_{blender}(q_{in} - q_{out_b}) \tag{3.2}$$

where $K_{blender} = 0.003$ [m$^{-3}$] is a constant tuning parameter adjusted such that the dynamics of the model matches the real dynamics of the blending tank. In Figure 3.2 the dynamic behavior of the fitted model is compared to the real dynamics. The step responses are detrended, meaning that the mean value and trends in the data are removed. This makes it easier to observe the dynamic behavior. During the experiment the inflow of pulp to the blender, $q_{in}$, is assumed to be constant.

## 3.2 Physical Modeling of Flotation Cells

The outflow from each flotation cell is positioned at the bottom of the tank. The next tank cell is mounted $h_{diff} = 1$ meter lower compared to the previous one. This height difference, together with differences in total pulp level in the cells, generate a hydrostatic pressure over the valve that causes pulp to flow through it. The change of pulp level in a tank can be described as

$$\frac{dh}{dt} = \frac{1}{A}(q_{in} - q_{out}), \tag{3.3}$$

where $A$ [m$^2$] is the cross section area of the tank, $q_{in}$ [m$^3$/s] is the inflow of pulp to the tank and $q_{out}$ [m$^3$/s] is the outflow. For the flotation cells, $h$ is measured in meters. Since the valve opening is small in relation to the cross section area of the cell, the speed of the outflow can be approximated using Torricelli's law. This means that it is proportional to the square root of the height difference of pulp between the tanks connected through the valve. However, since the valves are rarely fully open, the relation between the valves' control signal and the flow through it must also be considered to fully be able to describe the outflow.

The relation between the control signal, $u$ [%], and the outflow is tabulated from the manufacturer for a typical flotation slurry with a 1 meter hydraulic head and it typically has a quadratic relation. Therefore, the outflow is modeled as

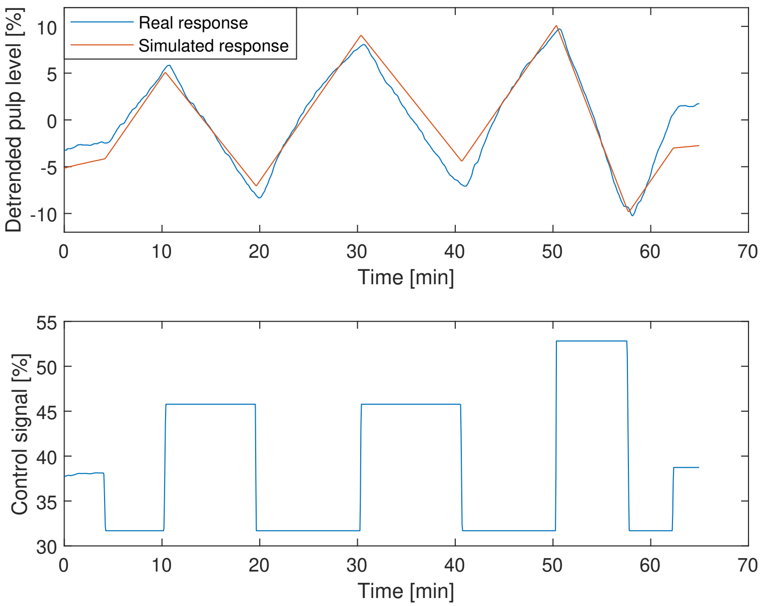


**Figure 3.2** The response of the level $h_b$ for the real blending tank compared to the response of the model when the control signal $u_b$ is changed.

$$q_{out_i}(u) = (k_2u^2 + k_1u + k_0)\sqrt{2g(h_i - h_{i+1} + h_{diff})} \tag{3.4}$$

where $g = 9.8$ [m/s$^2$] and $k_2$ [m$^2$], $k_1$ [m$^2$] and $k_0$ [m$^2$] are constants that are to be determined, $h_i$ [m] is the level in the flotation cell and $h_{i+1}$ is the level in the successive cell.

To be able to adjust the coefficients so that the model agrees with the real process, step response experiments performed on the real process were used. The model coefficients described above were adjusted so that the model's dynamic behavior agreed with the dynamic behaviour of the real process. This was done by performing a parametric sweep for $-0.023 \leq k_2 \leq -0.20$ and $0.11 \leq k_1 \leq 0.56$. To satisfy the steady state condition of the process before the step response experiment was started, $k_0$ was adjusted. The physical interpretation of allowing $k_0 \neq 0$ is that $q_{out} = 0$ will not necessary coincide with $u = 0$. This is not physically reasonable but it was a trade-off made to be able to more accurately model the dynamic behavior at normal operating conditions. The different parameter sets were evaluated by comparing the root-mean-square of the difference between the simulated step responses and the real response.

| Tank | $k_2$ [m$^2$] | $k_1$ [m$^2$] | $k_0$ [m$^2$] |
|---|---|---|---|
| 1 | -0.16 | 0.47 | -0.10 |
| 2 | -0.045 | 0.23 | 0 |
| 3 | -0.079 | 0.34 | -0.045 |
| 4 | -0.17 | 0.45 | -0.11 |

**Table 3.1** Values of constants in equation (3.4) adjusted at medium and high production rate.

In the experiments the control signal was shifted according to square waves and the resulting changes in level were recorded. There were two or three step responses available for each of the considered tanks, the difference between the responses were the production rate. The production rate is related to the inflow of pulp to the system. A high production rate corresponds to a higher inflow of pulp to the series. By inspecting each response separately it became clear that the optimal parameter set for medium and high production rates are quite similar, while the parameter set for low production rates differs more from the other two. A normal production rate can from data be seen to be in the range between medium and high production rates. Therefore, for the tanks with 3 responses available the parameters chosen were a combination between the medium and high parameter sets. For the tanks with only two steps available, one for high production and one for low, only the response for high production was used to determine the model since it will be more representative of normal operating conditions. The resulting tuned parameters for each individual cell can be seen in Table 3.1 and the models output along with the real step response for medium production in the first cell is seen in Figure 3.3.

## 3.3 System Identification

An alternative approach to the physical modeling is to use system identification to identify the dynamic properties of the process. This can be done by fitting a transfer function model to the available step response experiments. This approach also confirms that the operating conditions for high and medium production rates are much more similar than for the low production rate. In Figure 3.4 the step response for medium production on the first cell is seen for the model determined by system identification where the system was modeled as a simple integrating process,

$$H(s) = \frac{-0.036}{s} U(s). \tag{3.5}$$

Reflecting on the shape of the step responses in Figure 3.3 and Figure 3.4, it is clear that the more complex physical model reflects the dynamic behavior of the process better than the more simple model determined from system identification. It follows the shape of the step responses more smoothly and also reflect the shape of the step

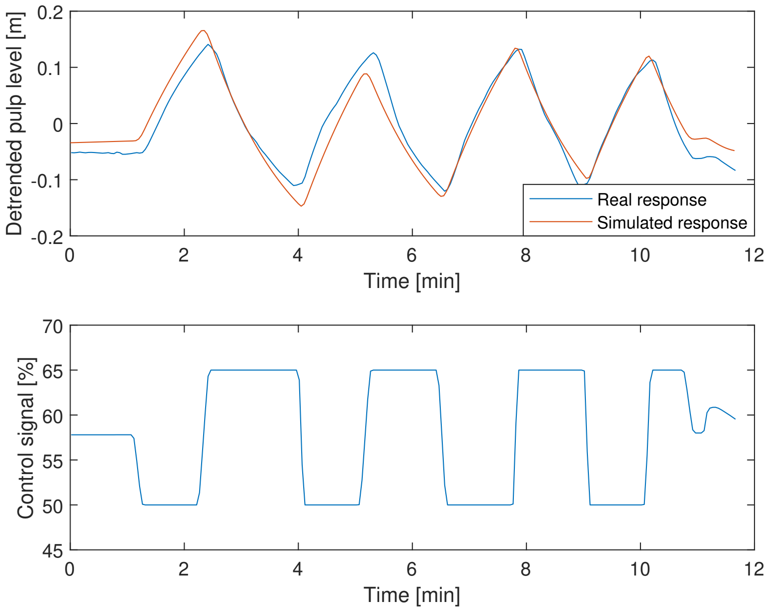


**Figure 3.3** Detrended step responses for the real process compared to the simulated model. The responses are from the first cell and the model is obtained from physical modeling.

responses more accurately. Therefore, the model developed in Section 3.2 will be the model used in this project.

## 3.4 Validation of Physical Model

To validate the model of the flotation cell developed in Section 3.2, the model's response to a reference change in pulp level is considered. The reference change is considered since when the reference is changed the control signal must react to make the pulp level follow the reference. This reaction is what was needed in the real process to adjust the pulp level and the same control signal can be applied to the model to see how the simulated level reacts to the same control signal. This makes the reference change suitable to validate the model. The inflow to the cell, and the corresponding control signal to the valve, are fed to the model and its response is compared to the real process' response. The resulting behavior is shown in Figure 3.5. The model manages to reflect the dynamic behavior of the real process in a satisfying way and the offset before removing the mean from the data is small.

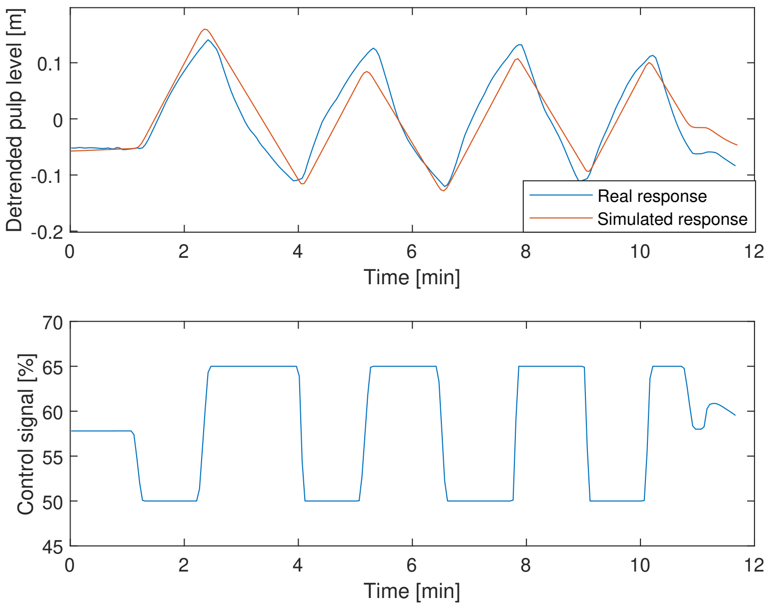


**Figure 3.4** Detrended step responses for the real process compared to the simulated model. The responses are from the first cell and the model is obtained from system identification.

## 3.5 Process Model of the Raw Series

When the model for each individual cell is decided it is time to put the cells together. They are connected as visualized in Figure 3.1 where the outflow of the first cell is the inflow to the second and so on. The complete model is described by equation (3.6) where $q_{out_b}$ is described by equation (3.1) and $q_{out_i}$ is described by equation (3.4).

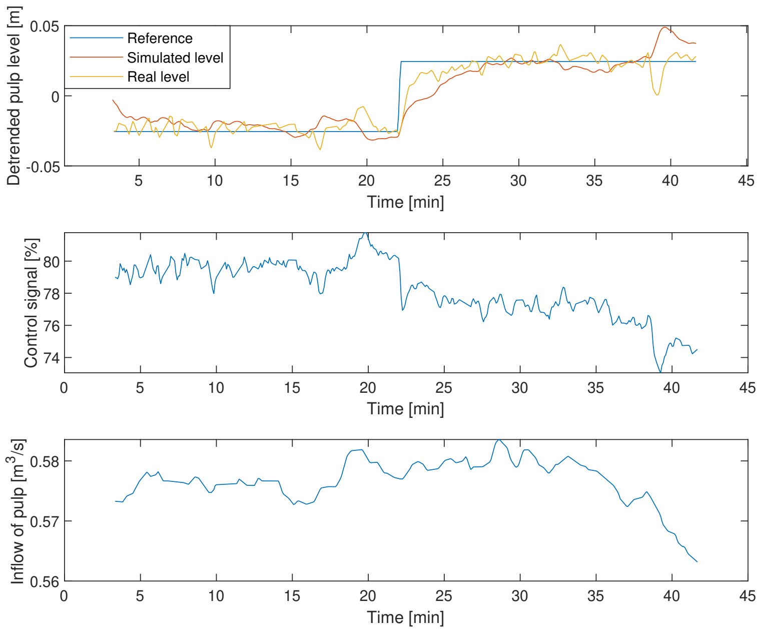


**Figure 3.5** The model response and the real process response to a reference change. The model is fed with the control signal and inflow from process data.

$$
\begin{aligned}
\frac{dh_b}{dt} &= K_{blender}(q_{in} - q_{out_b}) \\
\frac{dh_1}{dt} &= \frac{1}{A_1}(q_{out_b} - q_{out_1}) \\
\frac{dh_2}{dt} &= \frac{1}{A_2}(q_{out_1} - q_{out_2}) \\
\frac{dh_3}{dt} &= \frac{1}{A_3}(q_{out_2} - q_{out_3}) \\
\frac{dh_4}{dt} &= \frac{1}{A_4}(q_{out_3} - q_{out_4})
\end{aligned}
\tag{3.6}
$$

The LQ- and the MPC-controllers described in Chapter 4 need a linear process model to determine the control gain. This model is obtained by linearizing the model for the complete system around a linearization point that represents normal operating conditions. For this process the linearization point is given by $h_0 = [0.65\ 4.63\ 4.62\ 4.65\ 4.6]$ and $u_0 = [0.54\ 0.62\ 0.63\ 0.61\ 0.61]$. Since the controller operates in discrete time the process model is discretized and the state space representation takes the form shown in equation (3.7).

$$\begin{aligned} \Delta h(t+1) &= \Phi \Delta h(t) + \Gamma \Delta u(t) \\ \Delta y(t) &= I \Delta h(t) \end{aligned} \tag{3.7}$$

Here $\Delta h$ and $\Delta u$ represent deviations from the linearization point and the discrete time linearized dynamics are described by $\Phi$ and $\Gamma$ given below.

$$\Phi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.952 & 0.04707 & 0.0008829 & 1.494 \cdot 10^{-5} \\ 0 & 0.04707 & 0.9174 & 0.03461 & 0.0008838 \\ 0 & 0.0008829 & 0.03461 & 0.9174 & 0.04711 \\ 0 & 1.494 \cdot 10^{-5} & 0.0008838 & 0.04711 & 0.952 \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} -0.01666 & 0 & 0 & 0 & 0 \\ 0.1916 & -0.2083 & -0.003098 & -5.757 \cdot 10^{-5} & -7.446 \cdot 10^{-7} \\ 0.004728 & 0.2044 & -0.1225 & -0.003406 & -5.902 \cdot 10^{-5} \\ 5.889 \cdot 10^{-5} & 0.003835 & 0.1225 & -0.1815 & -0.004739 \\ 7.437 \cdot 10^{-7} & 6.49 \cdot 10^{-5} & 0.003101 & 0.185 & -0.1918 \end{pmatrix}$$

## 3.6 Airflow as a Measurable Disturbance

Each cell in the flotation series has an inflow of air at the bottom of the cell. If the amount of air injected is changed this also affects the level in the cell since the gas holdup in the pulp changes. To gain a better understanding of the effect the air has on the level, experiments were performed on the process. The resulting detrended step responses are shown in Figure 3.6 along with the fitted first order process model given by the transfer function

$$H_{air}(s) = \frac{1.2873}{1 + 26.66s} Q_{air}(s), \tag{3.8}$$

where $H_{air}$ represents the additional level due to the airflow $Q_{air}$. Since the experiments were only performed on one cell, the other cells are assumed to have the same dynamic response to changes in the airflow.

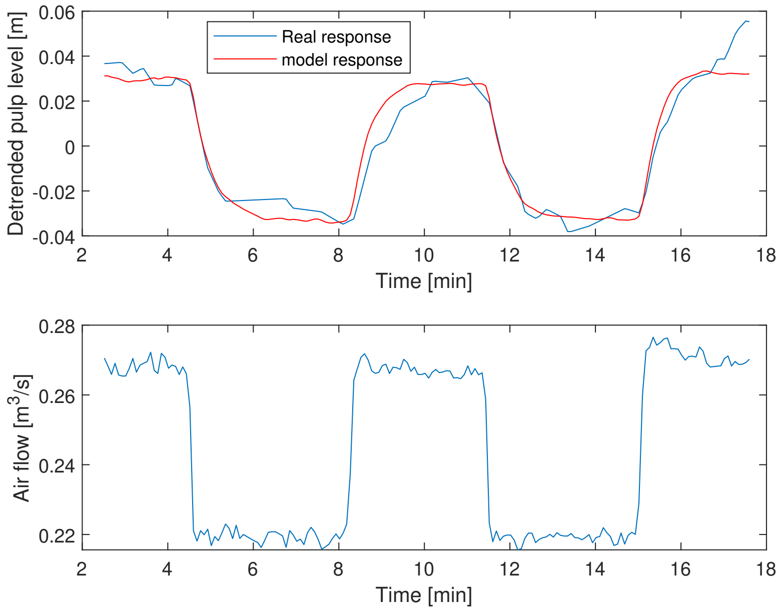


**Figure 3.6** The dynamic response for the pulp level when the airflow into the cell is changed.

The effects of the airflow can be included in the simulation model to make it more representative of the real process. But including it is not straightforward and has to be given some extra thought. The level changes caused by the air will not affect the flow rate out of the cell even though equation (3.4) suggests so. The levels associated with this equation assumes the same density in both cells. The changed airflow changes the density in the cell slightly, but this change is minor and assumed to be negligible. To separate the level changes due to the airflow and the level changes due to a changed pulp volume in the cell, the model developed keeps track of two separate levels. The first one is the measured and actual level of pulp in the cell and the second one is a virtual level that subtracts the influence of the air such that equation (3.4) holds.

## 3.7 Other Disturbances

There are of course more disturbances to the process than the airflow, this is easy to observe when looking at Figure 3.5 where a lot of noise is present. There are a

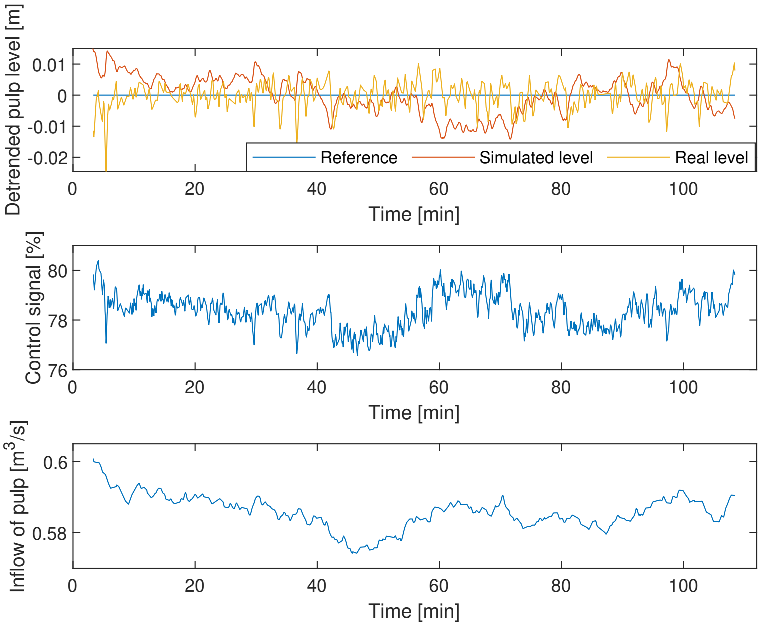


**Figure 3.7** The model output and process output used to model the disturbance.

lot of potential causes for noise in the process, some are measurement related and some may arise from phenomenon that the model does not account for.

To get a more representative model of these disturbances, a noise model was developed to catch the over all characteristics of the noise. Starting from a model on the form

$$y(t) = G(q)u(t) + H(q)e(t), \tag{3.9}$$

one can rearrange and find that

$$w(t) = H(q)e(t) = y(t) - G(q)u(t). \tag{3.10}$$

Here the noise component $w(t)$ can be seen as filtered white Gaussian noise, $y(t)$ is the process output, $u(t)$ the process input and $G(s)$ represents the process model developed in previous sections [Ljung and Glad, 2016]. In Figure 3.7 the process output and the model output is shown along with the control signal and the inflow of pulp.

The noise is to be modeled with an autoregressive model (AR-model) which means that the process can be described as

$$A(q)w(t) = w(t) + a_1 w(t-1) + \ldots + a_n w(t-n) = e(t), \tag{3.11}$$

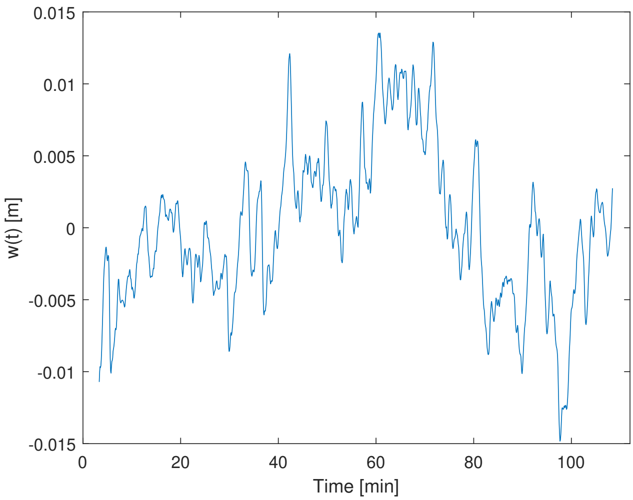


**Figure 3.8** The noise component $w(t)$.

where $n$ represents the order of the model. This can equivalently be expressed as

$$w(t) = H(q)e(t) = \frac{1}{A(q)}e(t). \tag{3.12}$$

To fit the model, $w(t)$ is calculated according to equation (3.10), the resulting noise component can be seen in Figure 3.8. The prediction error $\varepsilon$ for the model is given by

$$\varepsilon(t) = y(t) - \hat{y}(t|\hat{\theta}) \tag{3.13}$$

where $\hat{\theta}$ represents the estimations of $a_i$ in equation (3.11). Comparing the mean square of $\varepsilon$ for different model orders is a measure that indicates which model order to choose to describe the system. This is shown for ten different model orders in Figure 3.9.

The figure implies that a model order of at least two should be chosen, but a significantly higher one does not seem to be necessary.

The next step is to ensure that the prediction errors are free of additional structure, which means that the model captures the dynamics in a good way. One way of validating this is to check if the residuals are mutually independent. This can be checked by forming the estimate

$$\hat{R}_\varepsilon(\tau) = \frac{1}{N}\sum_{t=1}^{N} \varepsilon(t+\tau)\varepsilon(t) \tag{3.14}$$

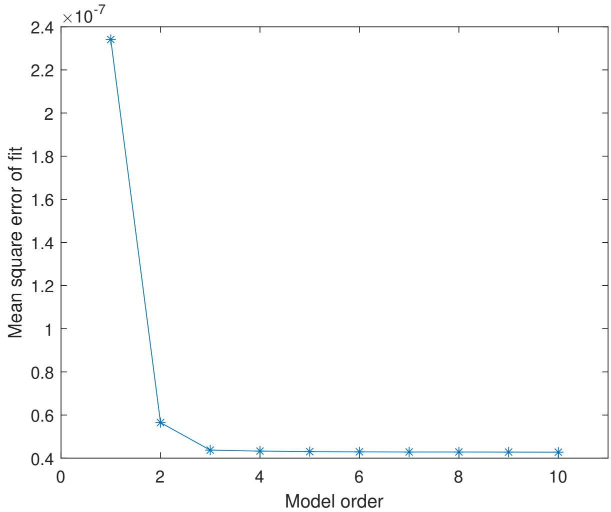


**Figure 3.9** Mean square error of fit for different model orders.

and checking if the values are close enough to zero. This is often referred to as residual auto-correlations and for the AR-model of order two this is shown in Figure 3.10.

The residuals for the second order model are correlated and hence it is preferable to choose a model of higher order. A model of order three is evaluated next and its residual correlations are shown in Figure 3.11. For this model the residual correlation is not significant.

To further validate the noise model a spectral analysis was performed on the noise component $w$ as well as on data generated from the model when it was fed with Gaussian white noise. The periodogram $\Phi_\omega(\omega)$ and the spectral estimate $\hat{\Phi}_N(\omega)$ of the signals are shown in Figure 3.12 and they are very similar.

The resulting model is an AR-model of order three and the modeled noise component $\hat{w}$ is described as

$$\hat{w}(t) = \frac{1}{1 - 2.275q^{-1} + 1.752q^{-2} - 0.473q^{-3}} e(t). \tag{3.15}$$

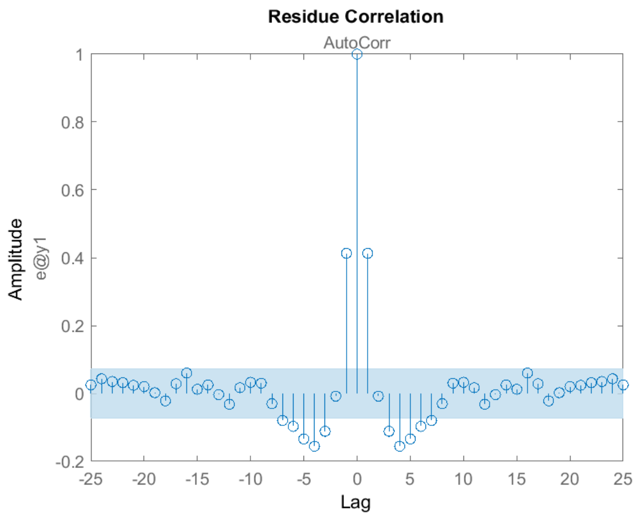


**Figure 3.10** Residual correlations for AR-model of order 2.

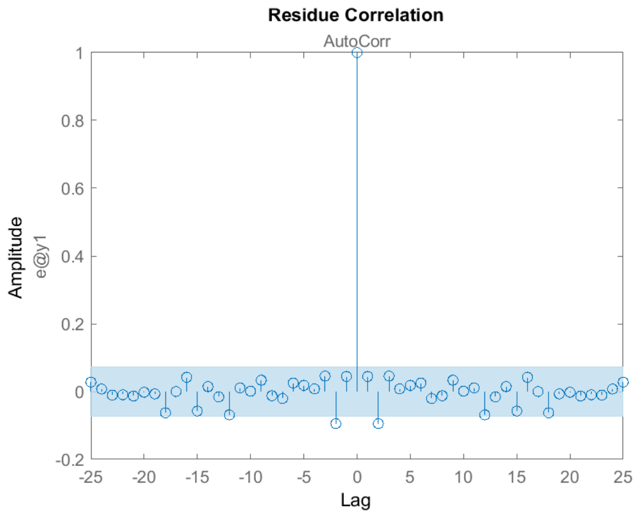


**Figure 3.11** Residual correlations for AR-model of order 3.

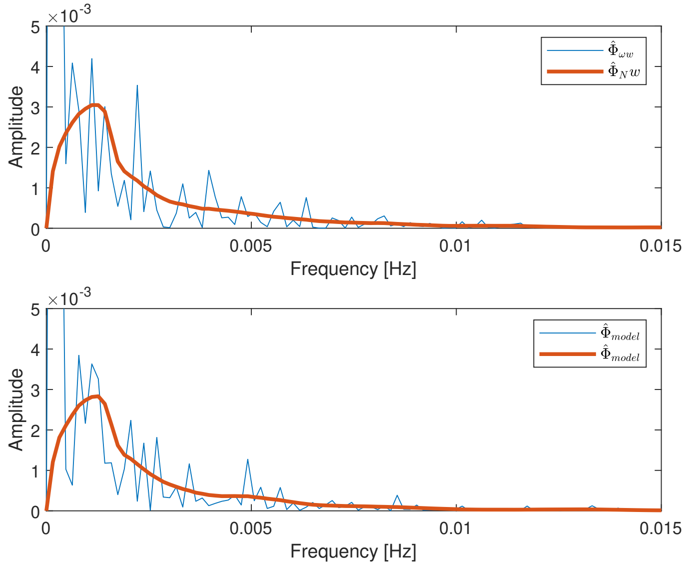


**Figure 3.12** Periodogram and spectral estimate for the noise signal, $w(t)$, and the model of the noise, $\hat{w}(t)$ fed with Gaussian white noise.

# 4

# Control Theory

There are different control strategies that can be used to control a process. The theory of the strategies that are to be considered in this project are summarized in this chapter.

## 4.1 PI- control

A proportional-integral controller, referred to as a PI-controller, is a controller that makes use of the difference between the reference $r$ and the measurement $y$ to determine the control signal. The difference between the reference an the actual value is the control error $e = r - y$. The control signal $u$ of the PI-controller is calculated as

$$u = K(e + \frac{1}{T_i} \int e \, dt), \tag{4.1}$$

where $K$ is the gain and $T_i$ is called the integral time (not to be confused with the time over which $e$ is integrated). These two parameters are the two parameters to be tuned in the controller. Since the controller bases its control action on the error, an error must occur before the controller can change the control signal to compensate for it. If the disturbance that causes the control error can be measured, feed forward can be used to compensate for the otherwise resulting error before it occurs.

In Figure 4.1 a control structure of a controller including feed forward is seen. The additional part of the control signal that enters the process, $u_{ff}$, is defined by a gain $K_{ff}$ times the disturbance $v$. This allows the controller to start compensating for the disturbance as soon as it appears, instead of waiting for it to cause an error before correcting it.

Some practical adjustments can be made to the PI-controller to avoid fast changes in the control signal due to changes in the reference value. One such adjustment is to place a weight on the reference value in the proportional part of the controller such that the control signal is calculated as

$$u = K(\beta r - y + \frac{1}{T_i} \int e \, dt), \tag{4.2}$$

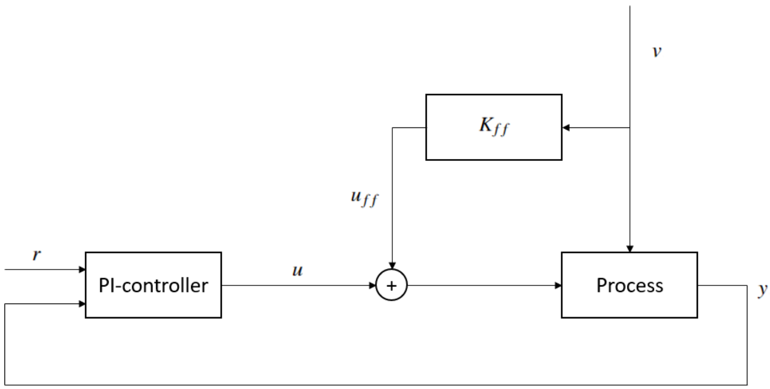


**Figure 4.1** A control loop with feed forward.

were $0 \leq \beta \leq 1$. If the reference changes as a step, the initial change of the control signal will become smaller if $\beta$ is smaller than 1. This setup also reduces the overshoot when a reference change is made. Choosing $\beta \neq 1$ only affects the behavior of the controller when it comes to reference changes, the behavior for disturbance rejection is unchanged [Hägglund, 2019].

## 4.2 LQ-control

LQ stands for linear quadratic. This type of controller makes use of a linear process model, $x_{k+1} = \Phi x_k + \Gamma u_k$, to minimize the value of the quadratic cost function

$$J = \sum_{k=0}^{\infty} x_k^T Q_1 x_k + 2x_k^T Q_{12} u_k + u_k^T Q_2 u_k. \tag{4.3}$$

Here $x_k$ are the states of the model at time step $k$ and $Q_1$, $Q_{12}$ and $Q_2$ are weight matrices which are chosen when designing the controller. The relative sizes of the elements in the weight matrices affect the resulting control law that is derived by solving the algebraic Riccati equation

$$S = \Phi^T S \Phi + Q_1 - (\Phi^T S \Gamma + Q_{12})(\Gamma^T S \Gamma + Q_2)^{-1}(\Phi^T S \Gamma + Q_{12})^T. \tag{4.4}$$

When the solution $S$ to the Ricatti equation is found, the optimal feedback gain can be determined according to

$$K = (\Gamma^T S \Gamma + Q_2)^{-1}(\Phi^T S \Gamma + Q_{12})^T. \tag{4.5}$$

The feedback gain will be static and can be solved for offline. The resulting control law becomes

$$u_k = -K x_k. \tag{4.6}$$

This control law is optimal if the model of the process matches the real process perfectly and if there are no disturbances to the process, this however is rarely the case. A way to compensate for differences between the model and the real process is to include the integral of the control error of the states as additional states in the model. This expansion of the model also allows the controller to follow references that are different from the linearization point [Murray, n.d.]. In this project the controller will include integral states of the tracking error to make reference tracking possible. The controller assumes that all states are measurable. If this is not the case or if the measurements are noisy, a state estimator (e.g. a Kalman filter) can be combined with the controller to fulfill this requirement.

## 4.3 MPC-control

The MPC-controller is closely related to the LQ-controller, but there are a couple of differences. While the LQ-controller has a static feedback gain that can be solved offline, the gain of the MPC-controller will change in every time step and it must be computed online while controlling the plant. The MPC-controller solves

$$\begin{aligned} \min_{u_0,\ldots,u_N} J &= \sum_{k=0}^{N} x_k^T Q_1 x_k + 2x_k^T Q_{12} u_k + u_k^T Q_2 u_k \\ \text{Subject to} \;\; x_{k+1} &= \Phi x_k + \Gamma u_k \end{aligned} \tag{4.7}$$

over a finite prediction horizon of $N$ time steps [Glad and Ljung, 2003]. This results in a sequence of control signals $u_1,\ldots,u_N$. Only the first control signal in the series, $u_1$, is used and it is applied to the system until the next sample. At the next sampling instance, the optimization problem is solved again with the updated states and the first control signal in the next solution is applied during this time step. In this optimization problem, other constraints such as limitations for states or rates of change for the manipulated variables ($u$) can be imposed. This way, some physical limitations of the process can be taken into account in the solution to the problem, as long as there is a feasible solution under these constraints. Similar to the LQ-controller, integral states of the control errors can be introduced to deal with stationary errors and reference tracking.

## 4.4 Reinforcement Learning for LQ-control

The model is significant for the performance of the model-based controllers, but it can be hard to find a representative model or the operating conditions in the real process can drift far from the conditions under which the model was developed. Based on these observations, it would be desirable if a controller could be developed without a model while still using the successful state feedback approach of the LQ-controller. Using reinforcement learning in the controller may be a way of achieving this. Reinforcement learning can be used to update control laws and in this context it is well described by [F. L. Lewis, 2012]: "In this scheme, reinforcement learning is a means of learning optimal behaviors by observing the real-time responses from the environment to nonoptimal control policies."

For now, this will be limited to linear state feedback. In short the strategy is to gather information of how one linear state feedback controls the system and from that information determine a new improved state feedback. Recall the cost function from equation (4.3),

$$J = \sum_{i=0}^{\infty} x_i^T Q_1 x_i + 2x_i^T Q_{12} u_i + u_i^T Q_2 u_i, \tag{4.8}$$

and assume that $Q_{12} = 0$. Introduce $J = Q(x_i, u_i)$ as the target function. At time $i$ it can be expanded as [Enqvist, 2020]

$$\begin{aligned} Q(x_i, u_i) = (x_i^T Q_1 x_i + u_i^T Q_2 u_i) + \sum_{k=i+1}^{\infty} x_k^T Q_1 x_k + u_k^T Q_2 u_k = \\ (x_i^T Q_1 x_i + u_i^T Q_2 u_i) + Q(x_{i+1}, u_{i+1}). \end{aligned} \tag{4.9}$$

Since state feedback is used, $u_i$ can be expressed as $u_i = -Kx_i$. Combining this with equation (4.9) we get

$$Q(x_i, u_i) = x_i^T Q_1 x_i + u_i^T Q_2 u_i + Q(x_{i+1}, -Kx_{i+1}) \tag{4.10}$$

In the case when $\Phi$ and $\Gamma$ are known along with the solution to the algebraic Riccati equation $S$, $Q$ can be expressed as

$$Q(x_i, u_i) = \begin{pmatrix} x_i \\ u_i \end{pmatrix}^T \begin{pmatrix} \Phi^T S \Phi + Q_1 & \Phi^T S \Gamma \\ \Gamma^T S \Phi & \Gamma^T S \Gamma + Q_2 \end{pmatrix} \begin{pmatrix} x_i \\ u_i \end{pmatrix}. \tag{4.11}$$

$Q$ is minimized by the control law in equation (4.6) with $K$ derived by equation (4.5). When there is no available model of the system, $Q$ can still be expressed similar to equation (4.11) as

$$Q(x_i, u_i) = \begin{pmatrix} x_i \\ u_i \end{pmatrix}^T \begin{pmatrix} S_{xx} & S_{xu} \\ S_{ux} & S_{uu} \end{pmatrix} \begin{pmatrix} x_i \\ u_i \end{pmatrix}. \tag{4.12}$$

The symmetry properties of the matrices in equations (4.11) and (4.12) are the same, that is $S_{xx}$ and $S_{uu}$ are symmetrical and $S_{xu}^T = S_{ux}$. Minimizing equation (4.12) with respect to $u$ is in the same way as for equation (4.11) done by the control law in equation (4.6), with $K$ derived by

$$K = S_{uu}^{-1} S_{ux}. \tag{4.13}$$

What now remains to conclude is how to gain information of $S_{xx}$, $S_{xu}$ and $S_{uu}$. Rearranging (4.10) to

$$Q(x_i,u_i) - Q(x_{i+1},-Kx_{i+1}) = (x_i^T Q_1 x_i + u_i^T Q_2 u_i) \tag{4.14}$$

leaves a right hand side where all the parameters are known and a left hand side that are linearly dependent on the parameters in $S_{xx}$, $S_{xu}$ and $S_{uu}$. With this observation $Q$ can be written as

$$Q(x,u) = \varphi(x,u)^T \theta, \tag{4.15}$$

where $\theta$ contains the parameters of $S_{xx}$, $S_{xu}$ and $S_{uu}$ and $\varphi(x,u)$ contain the corresponding quadratic combination of the elements of $x$ and $u$. With $Q$ expressed as in equation (4.15), equation (4.14) can be rewritten as

$$(\varphi(x_i,u_i) - \varphi(x_{i+1},-Kx_{i+1}))^T \theta = (x_i^T Q_1 x_i + u_i^T Q_2 u_i). \tag{4.16}$$

In this equation, the only unknowns remaining are the $\theta$ parameters. But since there are always more $\theta$ parameters than one, (even a system with one state and one control signal will have three parameters in its $\theta$ vector) a single equation is not enough to estimate them all. However, equation (4.15) can be calculated again for the next time step, and the next after that and so on, until there are enough equations to estimate all the unknown $\theta$ parameters.

Let $\Psi$ represent the matrix where each row is the left hand side of equation (4.16) except $\theta$ and let $Y$ be the vector where each row contains the corresponding right hand side of (4.16). This gives

$$\Psi\theta = Y \tag{4.17}$$

and using least squares the estimate $\hat{\theta}$ can be found according to

$$\hat{\theta} = (\Psi^T\Psi)^{-1}\Psi^T Y. \tag{4.18}$$

When the new parameter estimate is found the control law can be updated according to equation (4.13).

# 5

# Tuning of Controllers

The control structures can be tuned in different ways. Therefore, it is interesting to compare different tunings of the same control strategy.

## 5.1 PI-tunings

In the PI-case, one of the tunings is the one that is run in the plant today. This tuning is presented in Table 5.1. The blending thank also has an alarm-triggered addition that adds 10% control signal when the blender level exceeds 95% and it subtracts 5% of the control signal when the level goes below 40%.

An alternative tuning was also developed where the controllers were adjusted to be about two times more aggressive while keeping the same damping as before. This resulted in the parameters summarized in Table 5.2. Neither the parameters for the blending tank nor the feed forward factors were changed and they are therefore left out of the table. A rate limiter was also implemented in the controllers. It makes sure that the rate of change of the control signal cannot exceed $\pm 1\%/s$. This allows for the controller to be more aggressive without violating the limitation in how fast the control signal can be changed. The limitation is approximated from the available open loop step response experiments used to model the process in Chapter 3. From those experiments, the rate of change during the steps can be observed.

| Tank | $K$ | $T_i$ [s] | $K_{ff}$ |
|---|---|---|---|
| blender | -0.651 | 1203 | * |
| 1 | -1.183 | 87 | 0.8 |
| 2 | -1.279 | 85 | 0.8 |
| 3 | -1.236 | 83 | 0.8 |
| 4 | -0.990 | 81 | 0.8 |

**Table 5.1** Controller parameters for the PI-structure that operates in the plant today.

| Tank | $K$ | $T_i$ [s] |
|---|---|---|
| 1 | -2.1 | 74 |
| 2 | -3.4 | 73 |
| 3 | -2.4 | 74 |
| 4 | -2.3 | 71 |

**Table 5.2** A more aggressive setting for the controller parameters for the PI-structure than the one that operates in the plant today.

A comparison between the two tunings can be seen in Figure 5.1 where the reference for the second tank is changed. The effects this has on the other flotation cells are also shown. The level adjusted a bit faster for the more aggressive tuning but at the cost of a bigger dip in the control signal before it finds its new steady state. The level in the blending tank is not visualised in this section since it is not affected by the reference change when the system is controlled by the PI-controllers.

## 5.2 LQ-tunings

When tuning of the LQ- and MPC- controllers are considered, the tuning parameters are the weights in the weight matrices. A suitable start can be to specify acceptable deviations from the linearizarion point and use them to construct the weight matrices in such a way that the diagonal elements are decided as $1/dx^2$. When a suitable balance has been found for the LQ-controller without integral states, a weight can be added to the integral states to allow the controller to follow the reference. The weight matrices $Q_1$ and $Q_2$ are shown below for one alternative tuning.

$$Q_1 = \begin{pmatrix} 0.44 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.7\cdot 10^{-5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.016 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.016 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.016 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.016 \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 51 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

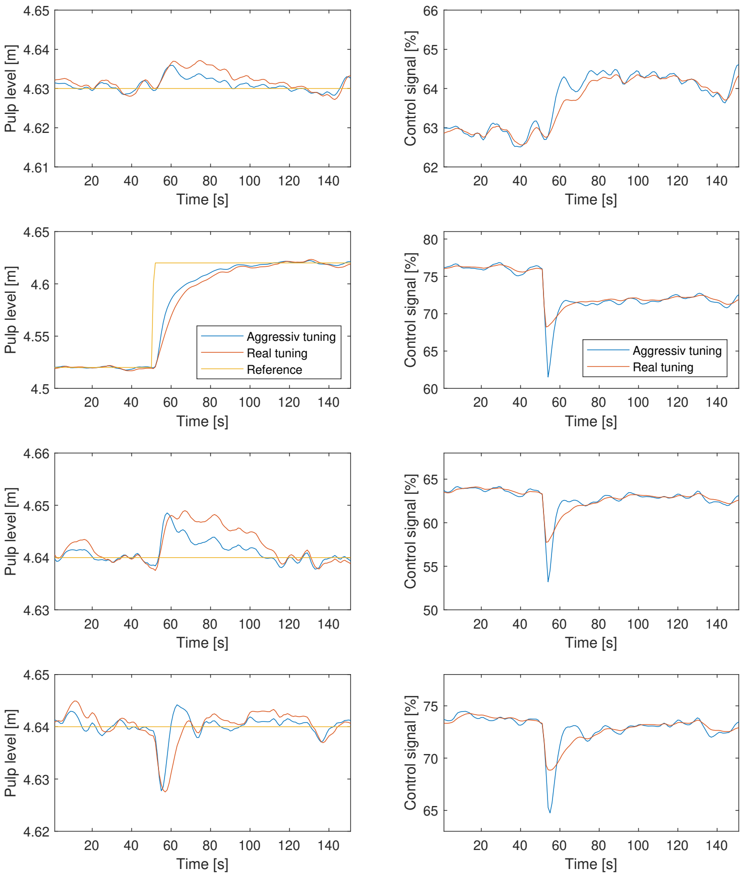


**Figure 5.1** The pulp levels for the different flotation cells and their respective control signals when the system is controlled by PI-controllers with two different tunings. The reference change in the second cell at $t = 50$ causes the changes.

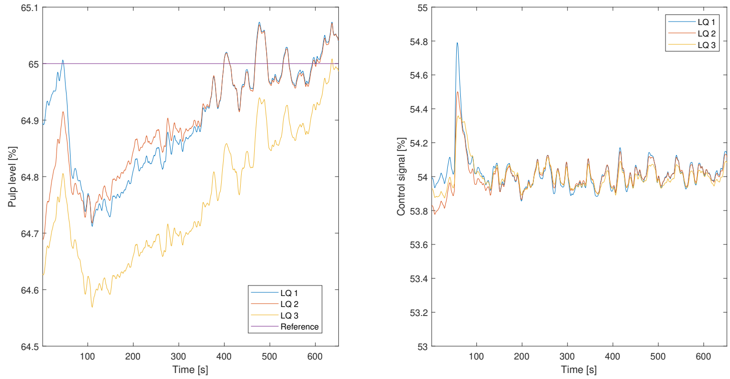


**Figure 5.2** The response in the blender tank when a reference change happens in the second cell. The system is controlled by three different LQ-controllers.

The control law designed with these weight matrices will be referred to as LQ 2. To make the reference tracking more or less aggressive, the weights for the integral states, which are the diagonal elements in columns six to ten in the matrix $Q_1$, can be changed. Two alternatives are $[4.4 \cdot 10^{-5}, 0.04, 0.04, 0.04, 0.04]$ (LQ 1) or $[4.9 \cdot 10^{-6}, 0.0044, 0.0044, 0.0044, 0.0044]$ (LQ 3) depending on how aggressive the tracking is supposed to be. The rate limiter was also implemented for the LQ controller to allow more aggressive tuning without violating the system limitations. The resulting reaction in the flotation cells to a reference change in the second cell are shown in Figure 5.3. The corresponding response in the blender is shown in Figure 5.2 (note the different time scales in the figures). The bigger the weight on the integral states the faster the response but this causes the control signal deviations to become bigger as well. Since the blender has less weight on its integral state it takes the blender longer to return to its reference compared to the flotation cells. This is desired since the blender is meant to function as a buffer tank that uses its volume to buffer when the inflow to the series change.

## 5.3 MPC-tunings

A good starting point when tuning the MPC controller is to start from the weights chosen for the LQ controller. For this system the matrices tuned for the LQ controller gives an MPC controller with too little effect from the states and the integral states. Therefore, the changes that were made in the weight matrices, compared to the LQ case, were that the weights for the states and the integral states were increased. The weights for the control signals were also decreased slightly. For the controller that will be referred to as MPC 2 the $Q_1$ and $Q_2$ matrices are

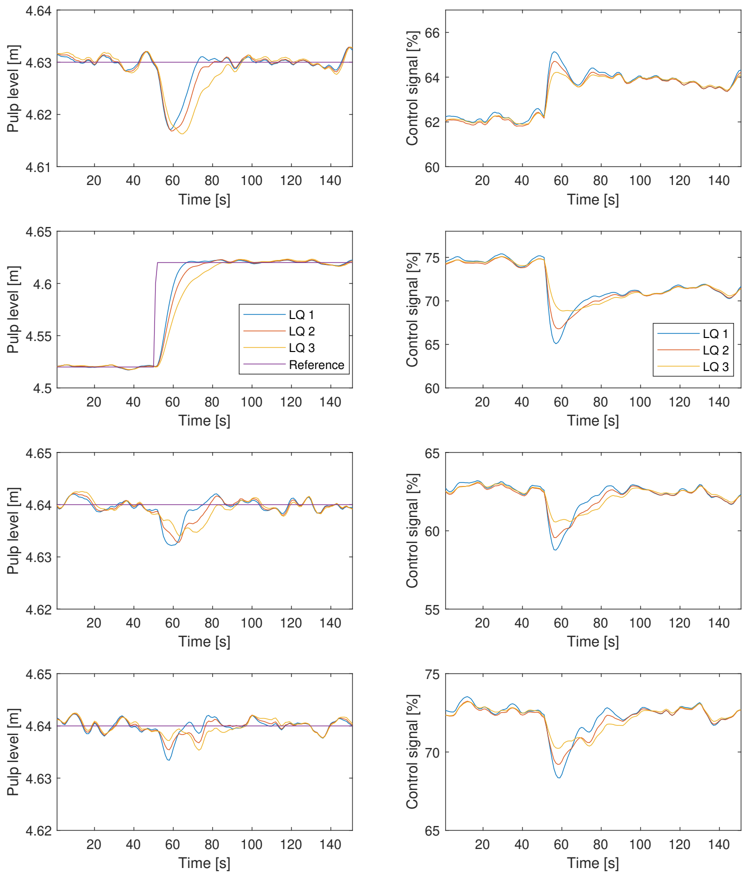


**Figure 5.3** The pulp levels for the different flotation cells and their respective control signals when the system is controlled by three different LQ-controllers. The reference change in the second cell at $t = 50$ causes the changes.

$$Q_1 = \begin{pmatrix} 11.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0044 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} 25 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Other alternative tunings where the reference tracking is made more or less aggressive can be MPC 1 with integral state weights (diagonal elements six to ten in $Q_1$) [0.028, 25, 25, 25, 25] or MPC 3 with [0.0011, 1, 1, 1, 1]. For the MPC controller, there are other parameters to be decided for the optimization problem in equation (4.7). The prediction horizon of $N$ samples needs to be chosen. A suitable choice is to choose it such that the dynamics of the system has time to settle, but not much longer to keep the optimization problem small. For the flotation cells this corresponds to the number of samples in 150 seconds. Limitations of the states, control signals and the rate of change of the control signal can be imposed on the solution. The limitation on the states is set to match the measured range for the levels in the cells and the control signal is to stay between 0% and 100%. The rate of change of the control signal is set to stay within $\pm 1\%/s$. Since the rate of change of the control signal is regulated by the optimisation problem, no rate limiter needs to be implemented in the MPC controller. The response from the different controllers to a reference change in cell two is shown in Figure 5.4 for the blending tank and in Figure 5.5 for the flotation cells.

## 5.4 Reinforcement Learning Based State Feedback Controller

Finding the control law with the reinforcement learning algorithm described in Section 4.4 is an iterative process that goes on until the state feedback gains have converged to a solution. When this has happened the gain can be kept constant until there are major changes in the system, caused by either reference changes or other disturbances. When new conditions apply the state feedback must once again be

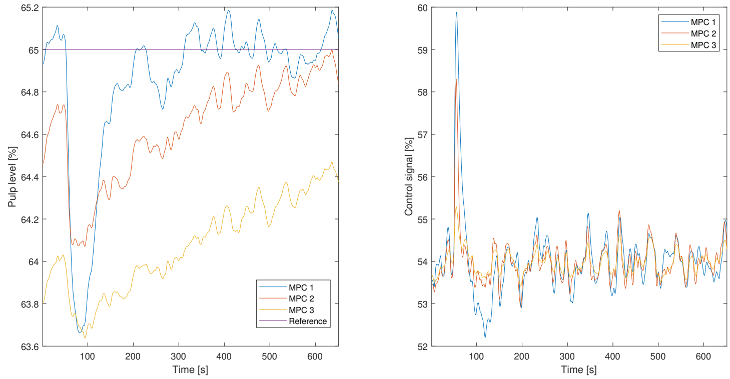


**Figure 5.4** The response in the blender tank when a reference change happens in the second cell. The system is controlled by three different MPC-controllers.

recalculated. A condition for the tuning to work in a proper way is that the system is properly excited during the period when data is collected. Otherwise the parameter estimation may not be accurate. Poor excitation leads to poor parameter estimates and the resulting control law is hence not guaranteed to be an improvement from the previous one. It could also in some cases lead to a destabilizing state feedback when the parameter estimation is poor. To assure that the excitation was sufficient a disturbance signal was added to the control signal. This disturbance consisted of a low frequent square wave and white Gaussian noise. During the time the algorithm was run, the inflow to the system was assumed to be constant. During each iteration 2000 samples were gathered before the control law was updated. There are 25 parameters in the matrix of feedback gains. In Figure 5.6 the 5 elements on the main diagonal are shown as they approach their stationary values. In this case the references are set to the initial values of the references in Figure 6.2.

The weight matrices used in the tuning algorithm were

$$Q_1 = \begin{pmatrix} 11 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 100 \end{pmatrix},$$

$$Q_2 = \begin{pmatrix} 25 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

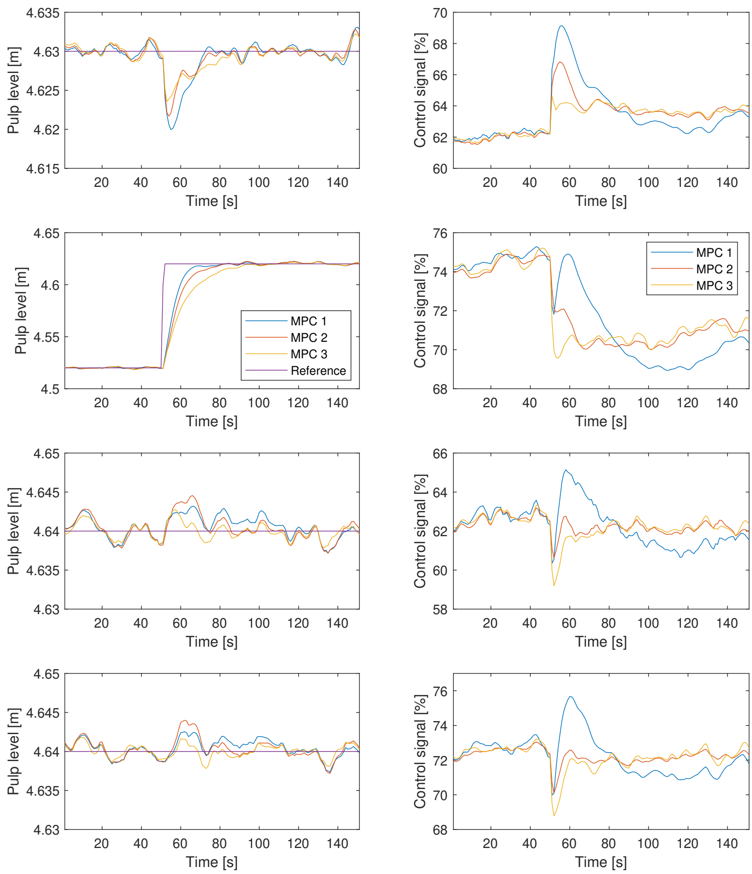


**Figure 5.5** The pulp levels for the different flotation cells and their respective control signals when the system is controlled by three different MPC-controllers. The reference change in the second cell at $t = 50$ causes the changes.

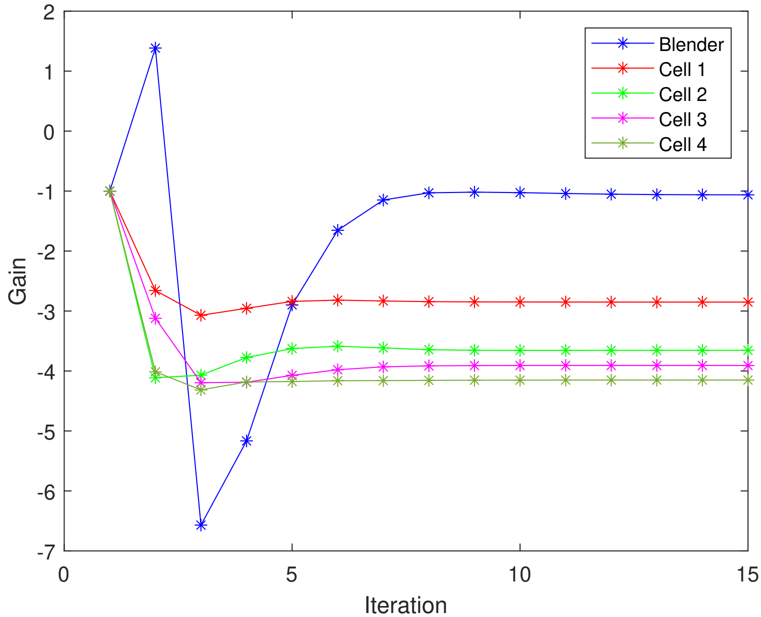


**Figure 5.6** The updates for the state feedback gains on the main diagonal of the gain matrix.

The state feedback found by the RL-algorithm is similar to the state feedback of a traditional LQ-controller which is designed with the same weight matrices and the system model developed in Chapter 3. A graphical representation of the state feedbacks are shown in Figure 5.7. The main difference between the two, in this case, is that the gains in the first column have more negative values for the RL-algorithm than for the traditional LQ. These gains correspond to the level in the blending tank and it means that the level in the blending tank will influence the control signal to all cells in the series more in the RL-based feedback gain.

The controller developed so far does not have integral action which means that it wont be able to follow references if the reference values change or if disturbances change the operating conditions in other ways. Introducing integral action can be done by expanding the system with integral states before the RL-algorithm is used. But this means that there will be more parameters to estimate which will make the problem harder to solve in an accurate way if the excitation of the system is problematic. An other approach that will give integral action is to choose the state feedback for the integral states separately and use knowledge of the system to set the parameters to appropriate values. This feedback law will not be optimal in the sense that the integral feedback for the LQ-controller is but if the only purpose

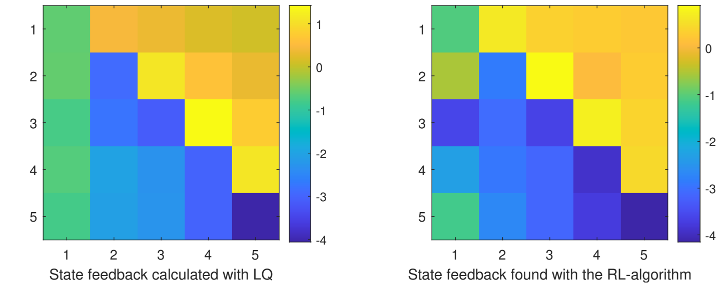


**Figure 5.7** Gain matrix for the state feedback when it is designed with a traditional LQ-controller and when it is found with the RL-algorithm.

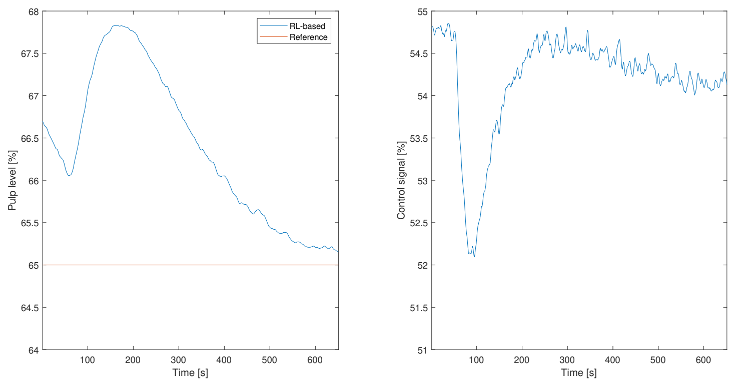


**Figure 5.8** The response in the blender tank when a reference change happens in the second cell. The system is controlled by the state feedback controller designed with the RL-algorithm.

is to ensure reference tracking this is enough. For this system the diagonal matrix with diagonal elements [0.001 0.1 0.1 0.1 0.1] is one such choice. Using the state feedback found with the RL-algorithm and adding the integral action the controller can be tested. In Figure 5.8 the level in the blending tank is shown and in Figure 5.9 the cell levels are shown. One thing that is obvious is that the integral feedback is not as developed and therefore the cells after the one where the reference change takes place are more affected by it.

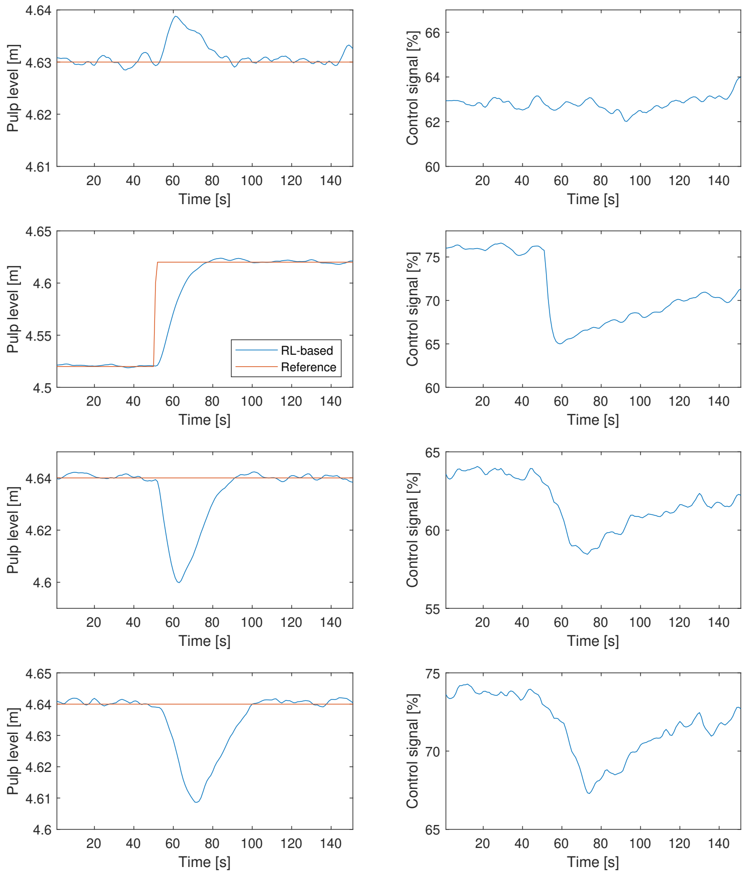


**Figure 5.9** The pulp levels for the different flotation cells and their respective control signals when the system is controlled by the state feedback controller designed with the RL-algorithm. The reference change in the second cell at $t = 50$ causes the changes.

# 6
# Results

The different controllers that are to be compared are the PI-structure with the parameter set represented in Table 5.1, the LQ-controller referred to as LQ 2 in Section 5.2, the MPC controller referred to as MPC 2 in Section 5.3 and the state feedback controller designed with reinforcement learning.

## 6.1 Inflow and References from Real Process Data

A good way to get an idea of the overall performance of the controllers is to run a simulation where the inflow and the reference values are extracted from real process data. Such a test touches upon a lot of things that are likely to happen in production and therefore it is important for the controllers to be able to handle them. The inflow to the flotation series includes various periodic disturbances as well as three stops on one milling line, which can be seen as the big drops in the inflow in Figure 6.1. The references for the flotation cells during this time period are shown in Figure 6.2. The reference for the level in the blending tank is 65% during the entire time period.

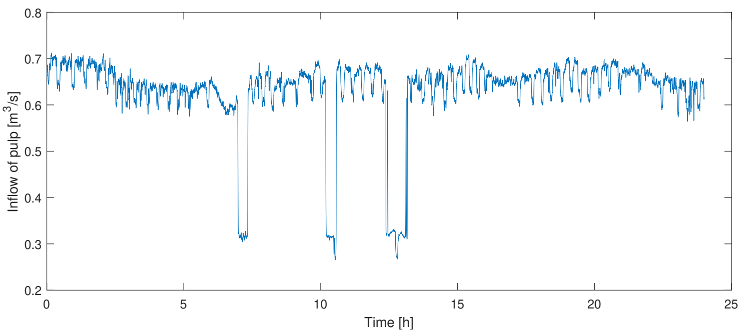


**Figure 6.1** The inflow of pulp to the blending tank during 24 hours.

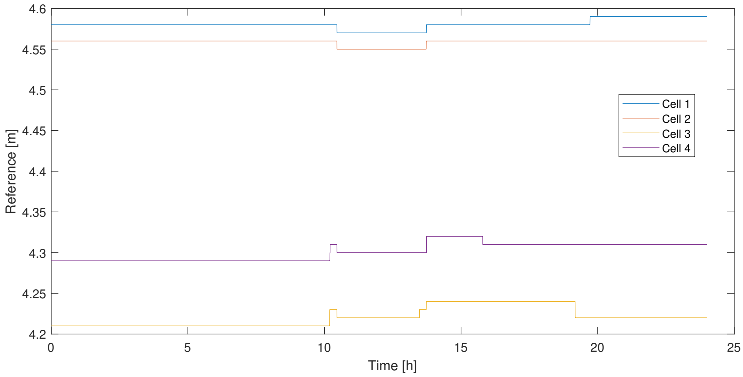


**Figure 6.2** The references for the levels in the flotation cells during 24 hours.

The inflow and the references are used as inputs to the system and the simulation is run with the four different controllers. In Figures 6.3, 6.4, 6.5 and 6.6 the levels in the cells are shown along with the level of the blending tank and the control signal for the different controllers.

It is easy to observe that both the the LQ-controller and the MPC-controller are better than the PI-controllers at following the reference for the flotation cells, they reflect its shape in a more accurate way. The bumps caused by the abrupt changes in inflow due to issues with one of the milling lines are visible for the cells in all the figures, but to different extent. For the model based controllers the bumps are barely visible, while the disturbances have a bigger impact for the PI-controllers and the RL-based state feedback controller. It is also notable that the model based controllers and the RL-based controller suppress the noise in the system more efficiently than the PI-controllers.

The behavior of the level in the blending tank differs more between the control structures. For the PI-controllers it can be noted that the level at some points exceeds the alarm limits and therefore the extra addition to the control signal that helps it to stay within the acceptable interval is activated. This is the main reason for the sharp peaks in the tracking error for the cell levels. For the LQ-controller, the blending tank successfully acts as a buffer tank. It uses its volume to handle the majority of the disturbance and then in a slow manner returns to its reference. This is the main reason why the levels for the cells are a lot less affected by the disturbance than for the PI-controllers. For the MPC-case the blender tank also buffers when disturbances enter. The flexibility of being able to recalculate the control law for each sample makes it possible for the blender level not to drift as far away from its reference and to return to it faster than for the other control structures. The behavior of the blending tank for the RL-based controller is similar to the LQ-controllers but it does not allow the level in the blender to drift as far as for the LQ-controller.

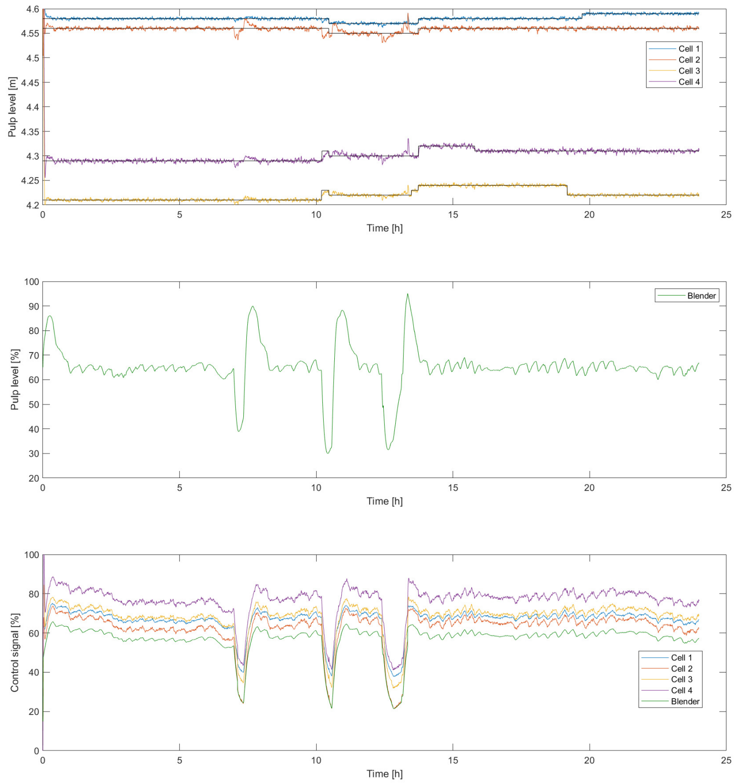


**Figure 6.3** The levels for the different flotation cells, the blending tank and the control signal when the system is controlled by the PI-controllers.

To compare the performance of the controllers the root mean square of the tracking error is calculated for each simulation. The results are presented in Table 6.1 where the values are normalized with the biggest value found among the flotation cells. Since the blending tank is intended to be buffering its deviations are not of interest as long as the level stays within its limitations.

Since the simulation period is quite long and the effects of the major disturbances are relatively small the noise suppressing qualities of the controllers are reflected in the values presented in Table 6.1. The PI-controllers noise canceling properties are not as good as the other controllers. The difference in performance

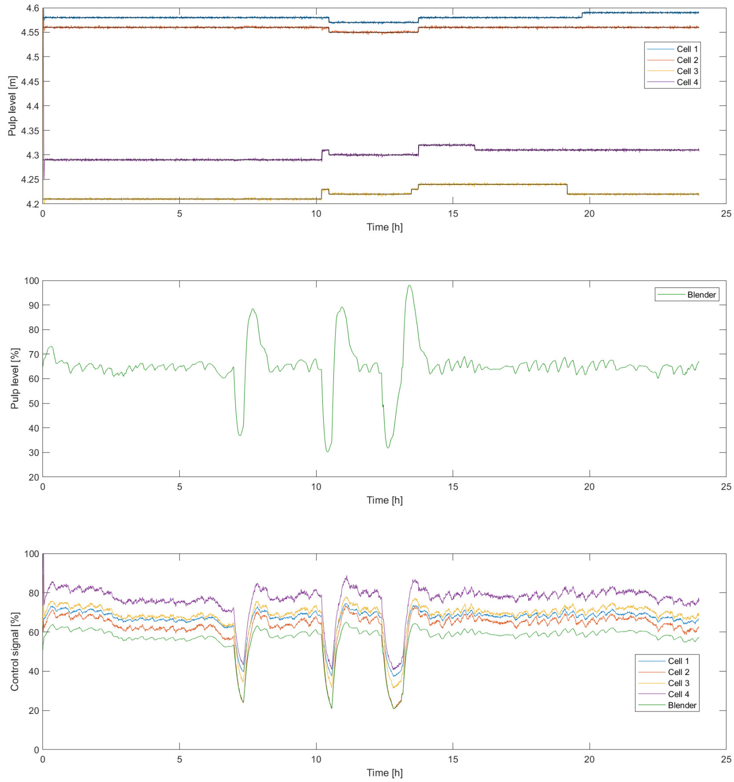


**Figure 6.4** The levels for the different flotation cells, the blending tank and the control signal when the system is controlled by the LQ-controller.

| Tank | PI | LQ | MPC | RL-based |
|---|---|---|---|---|
| blender | 94 | 70 | 13 | 55 |
| 1 | 0.43 | 0.26 | 0.15 | 0.31 |
| 2 | 1 | 0.30 | 0.13 | 0.54 |
| 3 | 0.54 | 0.31 | 0.15 | 0.35 |
| 4 | 0.80 | 0.32 | 0.15 | 0.40 |

**Table 6.1** Root mean square of the tracking error, The table is normalized with the biggest value among the flotation cells.

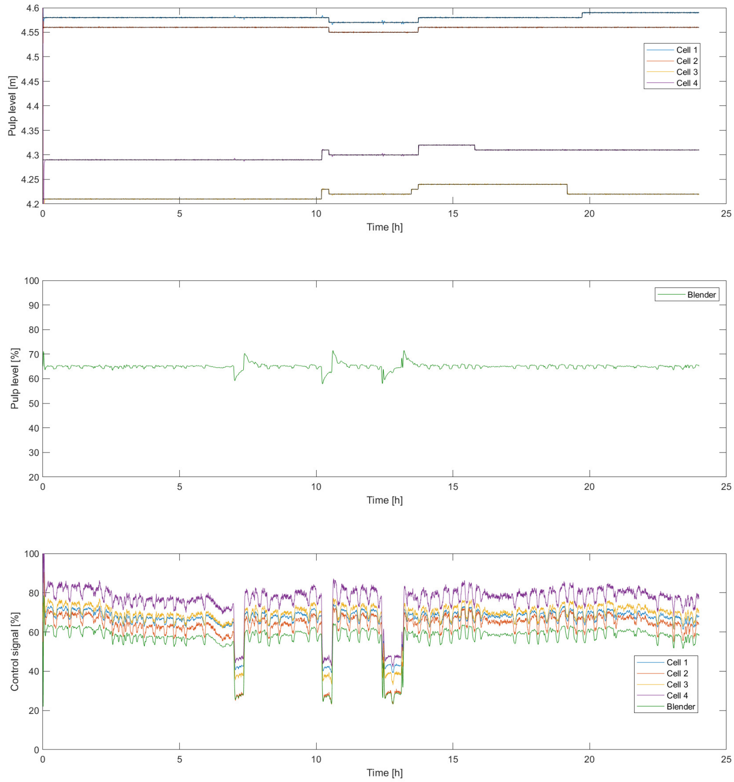


**Figure 6.5** The levels for the different flotation cells, the blending tank and the control signal when the system is controlled by the MPC-controller.

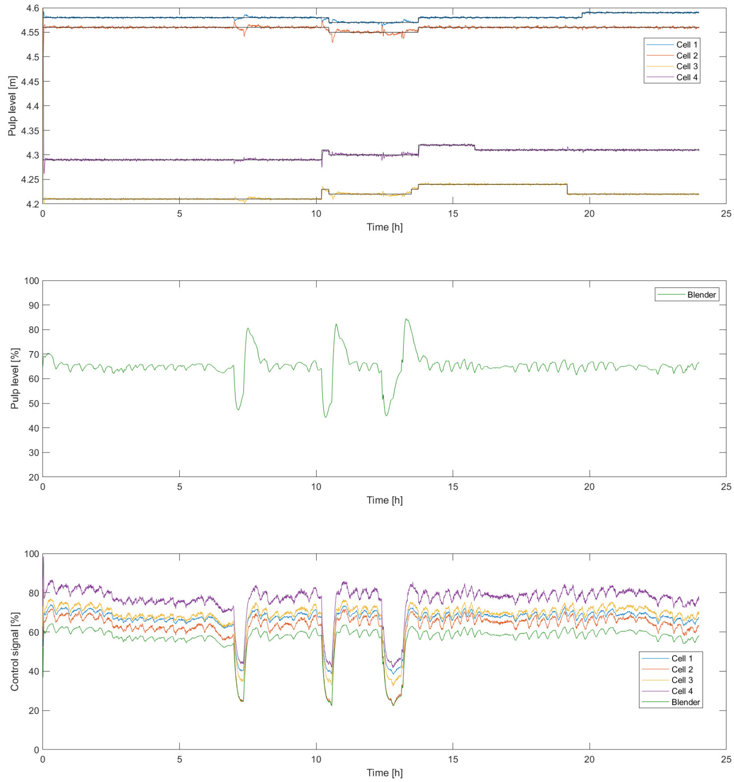


**Figure 6.6** The levels for the different flotation cells, the blending tank and the control signal when the system is controlled by the RL-designed state feedback controller.

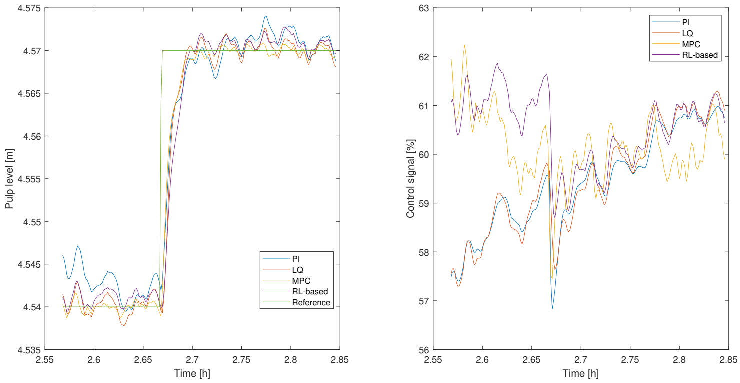


**Figure 6.7** Step responses for the second flotation cell with the different controllers.

between the LQ- and the MPC-controller is smaller but also mostly related to the noise suppressing qualities since the bumps caused by the major disturbances are small for both controllers. The RL-based controller has a couple of bumps that are bigger than for the model based controllers. This is part of the reason why it has worse over all performance than the LQ-controller even though their noise suppressing qualities are similar. The RL-based controller still however performs better than the PI-controllers.

## 6.2 Step Response

Step responses are an often used way to compare performance of different controllers. This is also a relevant case for the flotation series since references for levels are normally changed during production. A reference change from production data in cell 2 is chosen to evaluate the performance of the controllers. The references in production are often changed as ramps and this is the case for this reference change as well. In Figure 6.7 the resulting step responses are shown with the different controllers.

The noise suppressing qualities of the controllers play an important role in this case too and it is hard to tell from the figure how the controllers perform. To compare the performances the integral of the absolute value of the tracking error,

$$\int |e|dt, \tag{6.1}$$

was calculated. The integral was taken from $t = 2.64$ just before the step to $t = 2.72$ which is shortly after the levels have reached the new reference value. The values are summarized in Table 6.2.

| Tank | PI | LQ | MPC | RL-based |
|---|---|---|---|---|
| 1 | 0.46 | 0.27 | 0.09 | 0.27 |
| 2 | 0.85 | 0.83 | 0.68 | 1 |
| 3 | 0.42 | 0.20 | 0.07 | 0.66 |
| 4 | 0.30 | 0.20 | 0.06 | 0.65 |

**Table 6.2** Integral of the absolute value of the tracking error during a short time window covering the reference change, the table is normalized with the biggest value.

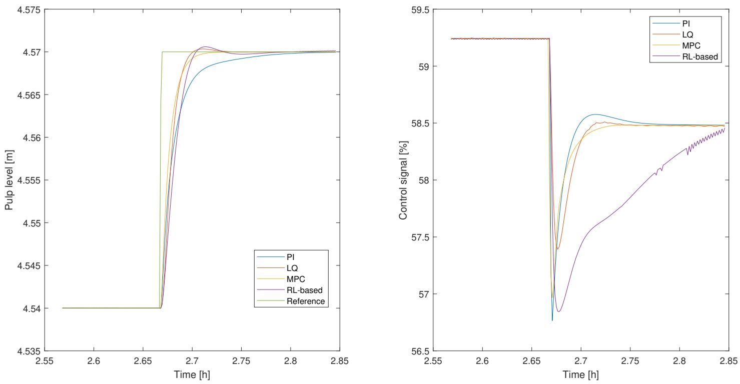


**Figure 6.8** Step responses for the second flotation cell with the different controllers. The noise in the system is removed from the simulation.

For the RL-based controller, the effects on the cells after the one that has the reference change that were observed earlier in Section 5.4 are also clearly seen in the table as cell three and four have quite big values. The RL-based controller also seems to have the slowest step response. From the table it seems that the PI controllers handle the step change almost as good as the LQ-controller but it is worth noting that for the surrounding cells the PI-controllers have the worst performance after the RL-based controller. Taking a closer look at Figure 6.7 it can be seen that the level for the PI-controller in fact also has a head start on the other controllers since its level actually was too high before the reference change. To exclude the noise canceling properties from the analysis the noise in the simulation was removed. The controllers response to the same reference change without the noise is shown in Figure 6.8 and the integral of the absolute value of the tracking error is summarized in Table 6.3.

| Tank | PI | LQ | MPC | RL-based |
|---|---|---|---|---|
| 1 | 0.34 | 0.15 | 0.03 | 0.22 |
| 2 | 1 | 0.70 | 0.58 | 0.99 |
| 3 | 0.27 | 0.13 | 0.02 | 0.74 |
| 4 | 0.22 | 0.07 | 0.01 | 0.70 |

**Table 6.3** Integral of the absolute value of the tracking error when the noise is removed from the simulation, the table is normalized with the biggest value.

In Figure 6.8 it becomes more clear what the reference tracking properties of the different controllers are. It can be observed that even though there are differences in reference tracking properties, the differences are relatively small. This is partly explained by the rate limiter on the control signal that prevents the controllers from acting arbitrary fast. When studying the step responses closely some differences between the controllers can still be observed. The model based controllers and the RL-based controller reach the new reference quicker than the PI controllers. The start of the LQ and RL-based controllers responses are slower than the MPC:s and these controllers also have a small overshoot when they reach the reference. The initial part of the PI-controllers response is as fast as the LQ:s but as it approaches the new reference it slows down and takes longer time to converge. One notable thing when studying Table 6.3 is that the surrounding cells are affected in different extent depending on the controller. When the MPC is used the surrounding cells are barely affected. The LQ-controller gives a small deviation in the cell before and after but the fourth cell gets a smaller impact than the third. For the PI-controllers the effect of the response to the reference change in cell 2 affects all the surrounding cells more than for the model based controllers. For the RL-based controller it is once again the cells after the one with the reference change that also gets an impact from the reference change. This is part of the explanation to why the control signal for the RL-based controller behaves differently from the other controllers in Figure 6.8.

## 6.3 Feed Forward of Air

In the raw series the main variable that is manipulated to change the quality of the froth, apart from the reagents, is the pulp level. The references for the levels change regularly, while changes in the airflow are more uncommon. However, in other parts of the flotation series, for example in the repetition step, the air is a more commonly used control variable. Since the airflow is a measured variable the PI-controller could also include feed forward from the airflow. To demonstrate the effects of extending the controller the raw series is studied when the airflow is changed as a step in the third cell according to Figure 6.9. The reactions in the levels are recorded both with feed forward from the airflow and without the addition in the

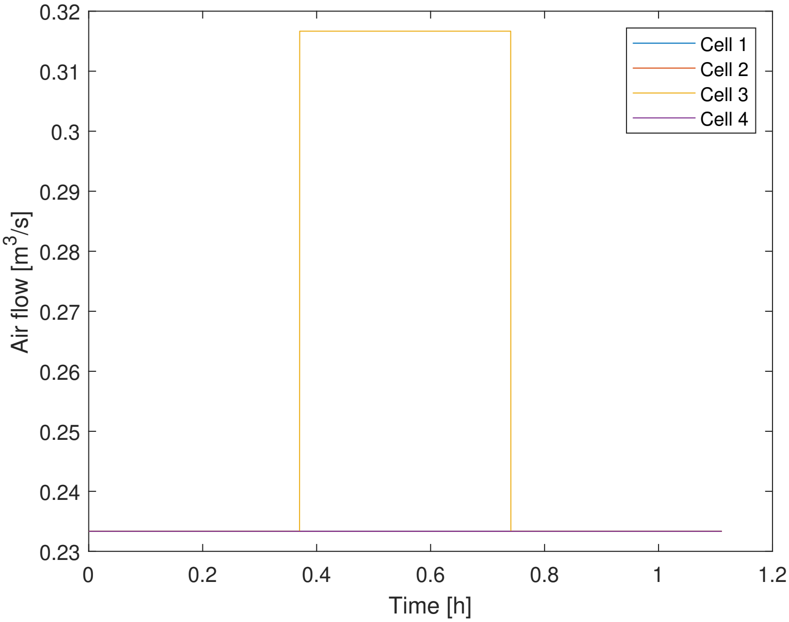


**Figure 6.9** The inflow of air to the flotation cells.

controller. The resulting levels are shown in Figure 6.10 and the root mean square of the tracking error is summarized in Table 6.4. To better demonstrate the effects of the feed forward the noise is removed from the signals and the deviations due to the change in airflow are shown in Figure 6.11.

The effects of the disturbance in the third cell is reduced with 45% by adding the feed forward from the airflow. In cell two and four the effects of the disturbance becomes a little bit bigger, but it only increases with 9.5% and 11% respectively. That the fourth cell is affected is intuitive since it is downstream from the cell where the changes take place but it may seem strange that also the the second cell is affected by the air change in the third cell. Recall from Chapter 3 that the volume of pulp in the cells affect the flow between them, changing the airflow changes the volume and hence a reaction in the cell before is also expected. The reason that the disturbance can not be completely eliminated even though it can be measured is the limitation in how fast the control signal can change. When the airflow changes the rate of change for the control signal reaches its max while reacting to the disturbance when the controller has feed forward. This means that the controller does not give the full addition that the feed forward factor adds instantly, it takes a while for the control signal to get to the desired level.

| Tank | PI | PI + ff |
|---|---|---|
| 1 | 0.39 | 0.39 |
| 2 | 0.66 | 0.70 |
| 3 | 1 | 0.55 |
| 4 | 0.67 | 0.75 |

**Table 6.4** Root mean square of the tracking error when the airflow to the third cell is changed. The table is normalized with the biggest value.

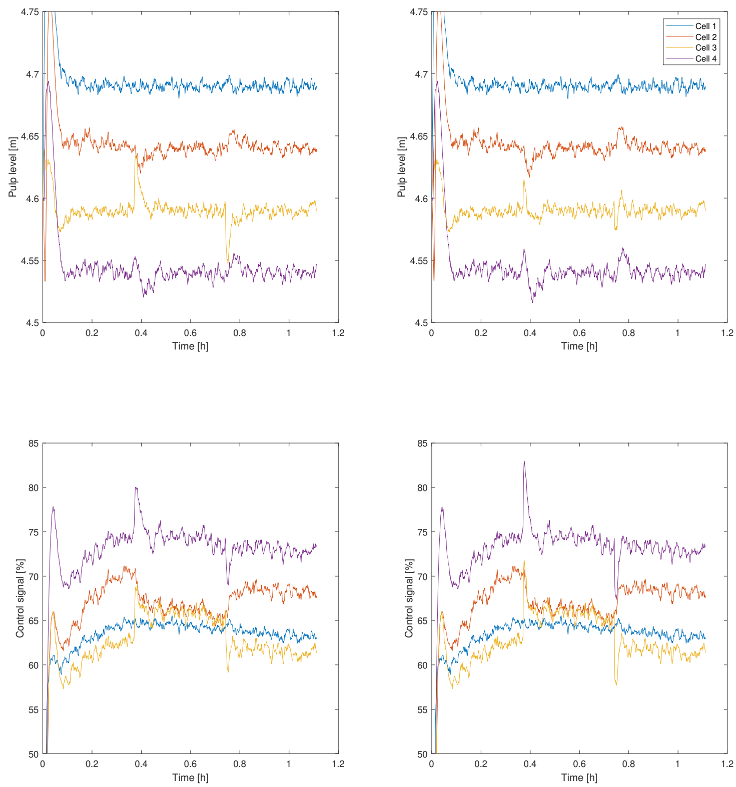


**Figure 6.10** The levels for the flotation cells when the airflow changes in the third cell. In the left figures the series is controlled with the original PI-controllers and in the right figures the controllers also include feed forward from the airflow.

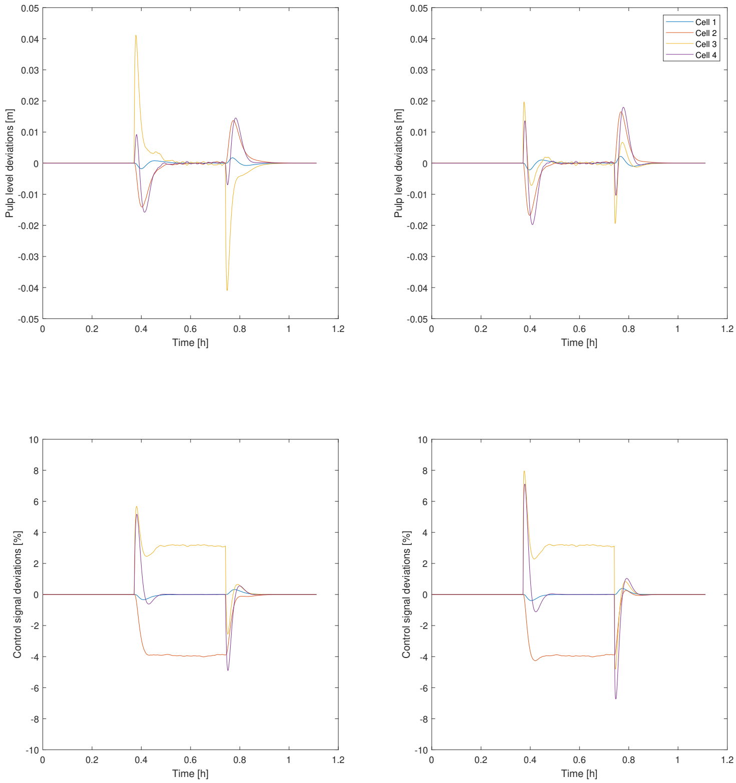


**Figure 6.11** The levels deviations in the flotation cells when the airflow changes in the third cell. In the left figures the series is controlled with the original PI-controllers and in the right figures the controllers also include feed forward from the airflow. The noise is removed to more clearly demonstrate the effects.

## 6.4 Model Errors and Robustness

For the model based controllers the accuracy of the model is important for the performance but to create a model that agrees perfectly with reality under all circumstances is unreasonable to say the least. Therefore it is interesting to investigate how much the model can deviate from the real process before the control performance becomes poor. This is interesting to study for the LQ- and tha MPC-controller since they are model based. The PI-controllers and the RL-based state feedback controller will not be considered in this section since these controllers do not make use of a model. The main uncertainties in the system model is the valve and pump dynamics and how they relate to different control signals. Both the noise canceling properties and the reference tracking properties can be affected by model errors and are therefore evaluated. For this purpose the same step change that was used in Section 6.2 is used and both the RMS of the tracking error and the integral of its absolute value is evaluated. One interesting thing about this sequence is that there is an issue with the milling line shortly before the reference change, so this kind of disturbance is also included in the test set. To alter the valve dynamics, equation (3.4) is multiplied by a constant term, $\Delta$, according to

$$q_{out_i}(u) = \Delta(k_2 u^2 + k_1 u + k_0)\sqrt{2g(h_i - h_{i+1} + h_{diff})}.$$

This leads to that the relation between the control signal and the pulp flow through the valve changes. The constant terms used were $\Delta = 0.5$ and $\Delta = 1.5$. These modified valve dynamics were used in the model when the controllers were designed, which lead to that the model for the controller and the process does not agree. In Figure 6.12 a part of the test sequence is shown for the LQ-controllers designed with the differing models and for the LQ-controller with the original model. The same is shown for the MPC-controllers in Figure 6.13.

The impact of the model errors is mostly visible when the issue with the milling line appears. And it is also a lot more apparent in the MPC case than in the LQ case since the MPC changes its control action based on the model between samples. In this case the model for the second cells valve was the only part of the model that was changed but if all the valves are altered in the same way the effects are similar but smaller.

In Figure 6.14 the root mean square of the tracking error for the test sequence, with controllers designed with different models are shown. The values were normalized with the biggest of the values. For the LQ case the cell with the model errors have good noise canceling properties when the valve is assumed to be slower ($\Delta = 0.5$) in the model than it is in the actual process, however in this case the surrounding cells are more affected. The opposite is true when the valve model has faster dynamics ($\Delta = 1.5$) than the actual process. For the MPC-case both a too fast and a too slow model gives bad performance for the second and third cell, as the RMS of the error increases the further away from the correct dynamics the model

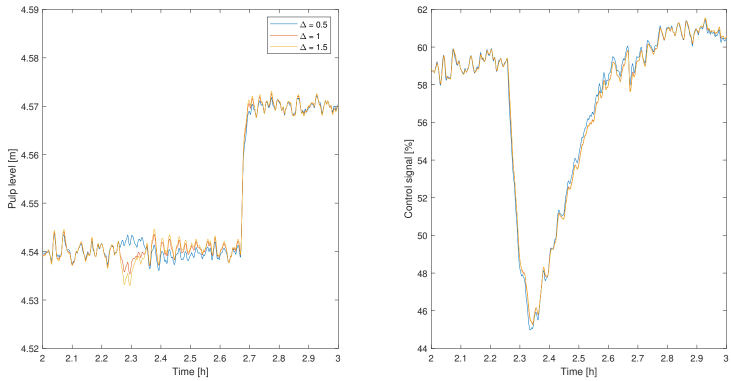


**Figure 6.12** The level in the second cell when the model for the LQ-controller is altered to differ from the process.

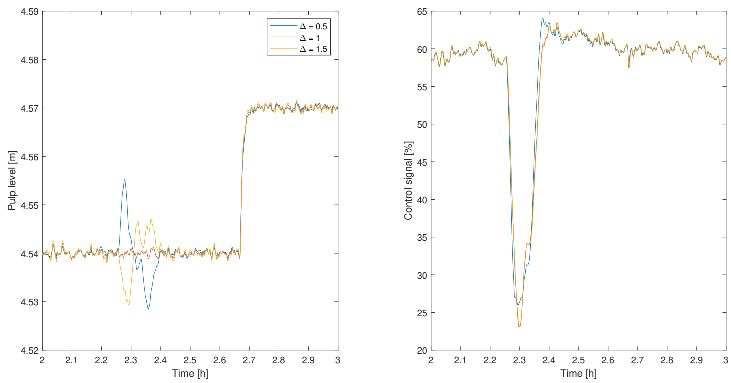


**Figure 6.13** The level in the second cell when the model for the MPC-controller is altered to differ from the process.

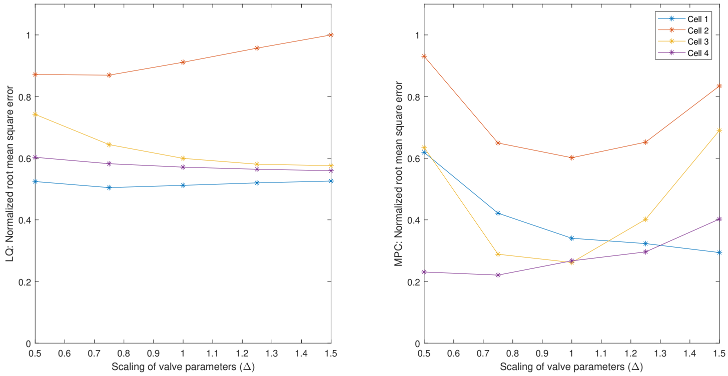


**Figure 6.14** The normalized root mean square of the tracking error for the different cells when the valve parameters in the model of cell 2 are scaled in the controller design.

drifts in any direction. This can also be observed in Figure 6.13 when the milling line issue happens, the level of the second cell deviates in different directions depending on if the valve dynamics in the model are too fast or too slow but the deviations are roughly of the same size.

Figures 6.15 and 6.16 are zoomed in on the step response part of the test set to more clearly see what happens to the reference tracking properties. The model errors make some difference when it comes to reference tracking, but the effects are small. The normalized integrals of the absolute error during the step response are shown in Figure 6.17. The effects of the model error on the LQ and MPC controller are similar, if the internal model is slower than the actual dynamics the reference tracking abilities are a bit worse than when the dynamics match but the difference is not big. Letting the valve dynamics be faster in then model than in the process does not seam to improve or worsen the performance when it comes to reference tracking.

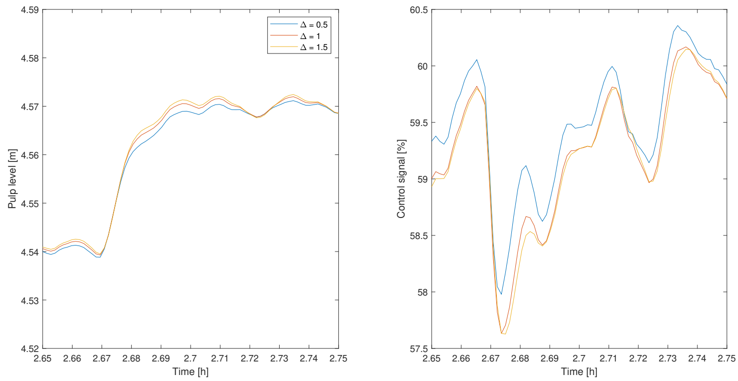


**Figure 6.15** The level in the second cell when the model for the LQ-controller is altered to differ from the process.

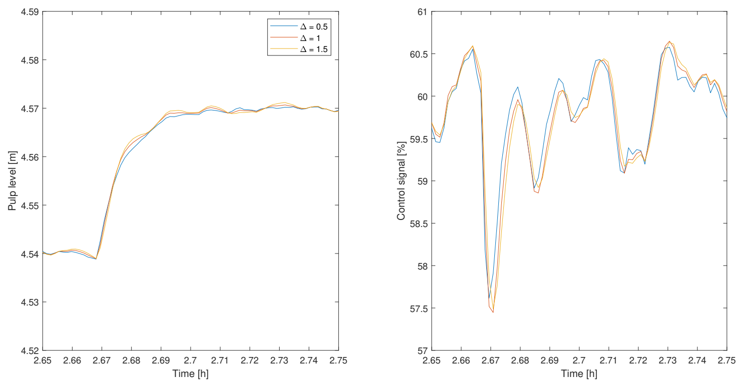


**Figure 6.16** The level in the second cell when the model for the MPC-controller is altered to differ from the process.

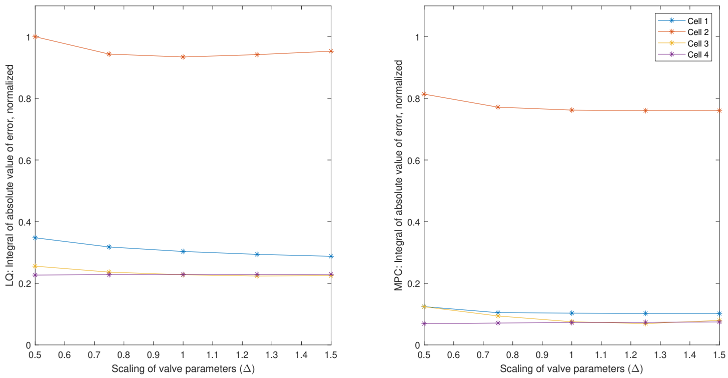


**Figure 6.17** The normalized integral of the absolute value of the tracking error during the step response for the different cells when the valve parameters in the model of cell 2 are scaled in the controller design.

# 7

# Discussion

### RL-based State Feedback Controller

One of the main challenges with the learning based LQ-controller is to gain data that contains enough information about the system dynamics to accurately being able to approximate the parameters. If the excitation of the system is not sufficient, the parameter estimate will be poor and it will result in a potentially even unstable control law. As it turns out, the natural noise in the system is not on its own sufficient to excite the system enough. Therefore a disturbance to the control signal had to be added to excite the system more but even with this addition the excitation of the system was problematic. With weight matrices that had bigger differences between the weights on the control signal and the states the algorithm got the gains to converge to reasonable values. However if the difference was too small in the weight matrices, the feedback gains often converged to unreasonably big values.

The parameters that the estimation had the biggest issues with was the ones related to the blending tank, probably due to the structure of the weight matrices that treats the blender tank very differently compared to the rest of the flotation cells. It could also be an effect of that the blending tank has a big influence on the rest of the flotation series. It could be easy for the algorithm to associate effects in a cell with the blender that may actually be related to interaction with another cell close by. These are issues that probably partly or fully could be solved by better excitation of the system, more qualitative data for the algorithm to base its decisions on would improve its robustness and the reliability of the control law. In theory this would also allow to expand the system with integral states and find the state feedback gain for the integral states with the algorithm simultaneously to the feedback gain for the states. The reason that this was not done in this thesis was that the current excitation is too poor to give accurate parameter estimates for such a big system since introducing the integral states approximately doubles the number of parameters that need to be estimated by the algorithm.

All these experiments were performed in simulation environment and in that setting it is easy to try different excitations and run many different simulations over long time periods to collect the data needed. In reality however, running the experiment on the real process is already very time consuming. It would probably have to run during even longer time periods to collect enough data since the real processes has more disturbances and non-linear phenomena than the simulation model reflects. Not to mention the fact that performing the experiment heavily will affect the production since the level changes made by the noisy control signals not only will affect the cells in the raw series but also the following process steps. To be able to do it in the real process it also requires that the inflow to the series is kept as close to constant as possible which can be hard to guarantee during longer time periods. The algorithm must also be able to run for a couple of iterations until it converges to a state feedback gain. Overall the experiment in this setting would not be realizable, and the need to add more excitation to the system while collecting the data makes it even harder to realize in reality. The consequences of a failed or poorly working control law in the iterations would also be a lot more severe if it happened in reality compared to in the simulation that could just be restarted.

The initial vision was that the internal noise in the system would be enough to excite it. This would have lead to that data could have been collected online while running the process under normal operating conditions. If this would have worked the algorithm could have been a good way to adjust the control law online as the operating conditions change. Changing operation conditions caused by degradation of the equipment is a common problem in processes, but the processes in the concentrator also deals with changes in operating conditions due to variations in the ore. The properties of the ore varies a lot over a deposit and this causes the operating conditions to drift in different directions as well. This is one of the main reasons why reinforcement learning approaches are interesting in the first place and even though there seems to be better options for level control of the flotation series there may be other areas where RL-approaches can bring things to the table. This particular approach seems to work better when the system is smaller and fever things influence the system. The main characteristics of the flotation process are quite easy to understand and model, but there are other processes where this may not be the case and where RL-approaches potentially can be helpful in understanding unknown dynamics and correlations.

When implementing and exploring this algorithm, the knowledge of the system has been really important to have a feeling for what might work and what may not in terms of weight matrices and the resulting state feedbacks. Without a prior knowledge of how the system is connected the process would have been a lot harder. One could argue that this algorithm may have the potential to be run offline in simulation environment and that the resulting control law then could be applied to the real system, however that would require a simulation model of the system and if this model can be developed, then so can models for model based controllers.

To summarize, the issues with exciting the system enough gives problems both with reliability of the resulting control law and it also makes the experiments that need to be performed to collect data hard to realize in the real process. This makes the algorithm unsuitable to use in this process where there are other more reliable methods to design a controller that also show better performance for the system. However this does not mean that RL-approaches are irrelevant for the flotation process, just that this particular use of it was not successful but there may be other areas where this approach or similar ones have more success.

## Model Based Controllers and their Sensitivity to Model Errors

There are many similarities and differences between the LQ-controller and the MPC-controller. One similarity is the role the weight matrices play when tuning the controllers. But even though the setup is similar, a weight matrix that works well for the LQ-controller is not guaranteed to work well for an MPC-controller. This is the case for this system where the weight matrices that worked well for the LQ-controller give a slow MPC-controller with very slow integral action. The weight matrices that work well for the MPC-controller on the other hand would give an LQ-controller that is too aggressive and gets oscillative when big disturbances enter the system. The tunings for the two different controller types however both have good noise canceling properties and follow references in a satisfying way. As seen in Section 6.4 the MPC-controllers performance is more affected by model errors. When the model is good the MPC-controller performs better than the LQ-controller but if the model drifts far from the real process this affects the MPC-controller more than the LQ-controller. In this sense the LQ-controller can be said to be more robust towards model errors. The disturbance rejecting properties are more affected by the model errors than the reference tracking properties for both controller types.

Another interesting question is whether the full system should be controlled by a model based controller or if for example the blending tank should be controlled by an independent controller and the model based controller only should control the flotation cells. This would make the tuning procedure easier in the model based controller since the blending tanks desired behavior is so different from the cells and the model complexity would be lower. However including the blending tank in the model gives the controller the possibility to coordinate the actions of the blender tank with the other flotation cells. This is an advantage since the flow from the blending tank to the flotation cells affect the levels in the flotation cells.

## PI, LQ and MPC, a Comparison

As the result section indicated the model based controllers generally give better performance than the PI-controllers, but the model based structures are more complex. The need for a good model is the biggest issue with the model based controllers since their performance are correlated with the accuracy of the model. However, if the model is good enough, the model based controllers will be superior to the PI-controllers. With an accurate model the MPC-controller gives the best performance, followed by the LQ-controller. When the model is good their performance is better than the performance of the PI-controllers. On the other hand an advantage of the PI-controllers is that they do not need a model of the system to control it. This is a big advantage if accurate models of the system are hard to find or if the operating conditions change a lot in such a way that models for different operation conditions differ a lot.

When comparing controllers it is also worth noting that the performance of the controllers depend on how they are tuned. Tuning the LQ- and MPC-controllers is not a straightforward process and there are many different tunings that may work properly depending on which characteristics that are desirable for the controllers. The PI-controllers could also be retuned to be more ore less aggressive which also would give different performances.

An advantage of the MPC-controller is that limitations of the system can be imposed on the controller, making the limitations of the process a natural part of the controller. These limitations must of course be respected by the other controllers too, but it requires good tuning and maybe some additions in the controllers to guarantee that no limitations will be violated. Implementing the rate limiter in the PI- an LQ-controller is one such example.

It is important to remember that the flotation process only is one part of the process line in the concentrator and that even though level control of it is important, the main goal of the process is to get the best possible recovery of minerals. With this perspective one could understand that the interaction between the different process steps is important for the overall performance of the plant. In this thesis the different control structures have been compared to each other for a specific part of the process, but there is nothing preventing us from combining the different control structures to achieve better overall control. A common approach is for example to use an MPC-controller as a high level controller for setting the references for PI-controllers on a lower level in the control structure.

## Feed Forward of Measurable Disturbances

For the cells in the later part of the flotation process where the air is a more commonly used control variable, introducing a feed forward addition to the controller that helps compensate for the level changes due to variations in the airflow could be considered. Today changes in the airflow are made as slow ramps in order for the level control to be able to compensate as the airflow changes. With the feed forward addition these changes in air could be done faster without reducing the performance of the level control. Since the airflow is already a measured variable no new instrumentation would be needed to make the addition possible. The only thing that would have to be done is include the feed forward of the signal in the controller.

# 8

# Conclusions and Future Work

This thesis has covered and compared different control structures for the first section of the flotation series, the raw series. Even though the control structure of PI-controllers that control the plant today works well, model based controllers show even greater potential. The biggest difference where there is room for improvement is when it comes to the disturbance rejecting properties of the controllers. Here the model based controllers outperform the traditional PI-controllers. The model based controllers, and especially the LQ-controller, show good robustness towards model errors in the model. Since the robustness towards model errors is good the reinforcement learning based state feedback controller that was developed only seems to over complicate matters. It shows poor robustness and the experiments needed are unreasonable to perform in the real process. Reinforcement learning approaches however may be more suited to use for other purposes related to the flotation process.

If a model based controller is to be implemented in the system a valid question that needs further investigation is whether the blending tank should be a part of the model or if it should be left out. Both approaches have advantages and disadvantages and this thesis does not cover this area. Another related question is how many flotation cells to include in the model based controller. Should only the raw series be considered or should the scavenger series or parts of it be included as well?

An addition that can be made to the existing controllers is to extend them with feed forward from the airflow into the cells. This extension would help the controllers to handle faster changes in the airflow without loosing performance when it comes to level control. All the necessary measurements already exist so no new instrumentation would be needed.

# Bibliography

*Bolidens Historia* (n.d.). URL: `https://www.boliden.com/sv/verksamhet/om-boliden/bolidens-historia`.

Enqvist, M. (2020). *En introduktion till lärande reglering*. Department of Electrical Engineering (ISY), Linköping University.

F. L. Lewis, D. V. o. K. G. V. (2012). "Reinforcement learning and feedback control – usingnatural decision methods to design optimal adaptive controllers." *IEEE Control Systems Magazine* 32(6), pp. 76–105.

Glad, T. and L. Ljung (2003). *Reglerteori : flervariabla och olinjära metoder*. 2. uppl. Studentlitteratur, Lund. ISBN: 9144030037.

Hägglund, T. (2019). *Praktisk processreglering*. Fjärde upplagan. Studentlitteratur, Lund. ISBN: 9789144130668.

Hodouin, D. (2011). "Methods for automatic control, observation, and optimization in mineral processing plants". *Journal of Process Control* 21, pp. 211–225.

Kawatra, S. (2011). *Fundamental principles of froth flotation*. Last accessed 26 January 2022. URL: `https://www.researchgate.net/publication/281944301_Fundamental_principles_of_froth_flotation`.

Kevin Brooks, M. W. and M. Bauer (2019). "Combined mpc for milling and flotation – a simulation study". *IFAC PapersOnLine* 52-14, pp. 24–29.

Ljung, L. and T. Glad (2016). *Modeling and identification of dynamic systems*. 1. ed. Studentlitteratur, Lund. ISBN: 9789144116884.

Murray, R. M. (n.d.). *Cds 110b lecture 2 – lqr control*. URL: `https://www.cds.caltech.edu/~murray/courses/cds110/wi06/lqr.pdf`.

P. Kämpjärvi, S.-L. J.-J. (2003). "Level control strategies for flotation cells". *Minerals Engineering* 16, pp. 1061–1068.

Paulina Quintanilla Stephen J. Neethling, P. R. B.-P. (2021). "Modelling for froth flotation control: a review". *Minerals Engineering* **162**:106718.

Stenlund, B. (2002). *Model-based Control and Monitoring of the Flotation Process*. PhD thesis. Luleå Univerisy of Technology.

**Lund University**
**Department of Automatic Control**
**Box 118**
**SE-221 00 Lund Sweden**

*Document name*
MASTER'S THESIS

*Date of issue*
June 2022

*Document Number*
TFRT-6178

*Author(s)*
Frida Norlund

*Supervisor*
Rasmus Tammia, Boliden
Tore Hägglund, Dept. of Automatic Control, Lund University, Sweden
Kristian Soltesz, Dept. of Automatic Control, Lund University, Sweden (examiner)

*Title and subtitle*
Comparison of Level Control Strategies for a Flotation Series in the Mining Industry

*Abstract*

Separating valuable minerals from waste rock is an important step in the production of metals. This is for copper ore done through a process called flotation. A flotation series consists of tank cells in series where the minerals are collected in a froth on top of the cells. The level control of the flotation cells is important in order to be able to collect the froth. The first four flotation cells and the buffer tank before the series is the process considered in this thesis. This process is found in the concentrator connected to Boliden's copper mine Aitik located near Gällivare in the north of Sweden. A simulation model of the process was developed using both physical modelling and experimental data from the real process. When the simulation model of the process had been developed, different control structures were tested and evaluated. The control structures that were tested were coupled PI-controllers, an LQ-controller, an MPC-controller and a state feedback controller where the state feedback was determined using reinforcement learning. The reference-tracking properties of the different controllers were similar while a bigger difference could be seen when it came to disturbance rejection. The PI-controllers gave a stable performance but their disturbance rejection was not as good as for the other controllers. One advantage with the PI-structure is its simplicity. Unlike the LQ- and the MPC-controllers, it does not need a model of the process to control it. The MPC-controller outperformed the other controllers when it came to disturbance rejection, but it was a bit more sensitive to model errors than the LQ-controller which also performed well. The reinforcement-learning-based controller did not give a better performance than the LQ-controller and it had issues with robustness in the tuning process, making it less reliable than the other controllers. The tuning process for it also required experiments that are unreasonable to perform on the real process. There is potential in reinforcement learning approaches to deal with drifting operating conditions but this particular approach was not successful. Overall, the results indicate that modelbased controllers have good potential to perform better than the PI-structure that controls the real plant today.

*Keywords*

*Classification system and/or index terms (if any)*

*Supplementary bibliographical information*

*ISSN and key title*
0280-5316

*ISBN*

*Language*
English

*Number of pages*
1-65

*Recipient's notes*

*Security classification*

http://www.control.lth.se/publications/