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Planning safe paths for robots with Exact Risk Allocation

Imagine a robot that has been sent to explore Mars. The robot must drive around to different spots, all the while maintaining a safe distance to obstacles it may encounter on its way. In a real-world environment like this, everything is subject to some degree of uncertainty which must be considered when deciding how to navigate the robot. Since it is very expensive to send a robot to Mars, we want to plan paths for the robot where the risk of collision with any of the obstacles is very low. Such paths can be planned using existing algorithms but a problem with some existing algorithms is that they generate paths that are too conservative, meaning that the robot will take unnecessarily big detours around all obstacles. To tackle this problem, a new path planning algorithm called "DR-RRT-ERA" has been constructed. This algorithm can generate less conservative paths than its predecessor "DR-RRT" while enforcing the same strict risk guarantees against obstacle collision.

This is a popular science summary of a master thesis entitled "Distributionally Robust Risk-Bounded Path Planning Through Exact Spatio-temporal Risk Allocation" [1], written at the department of Automatic Control at LTH, Lund University.

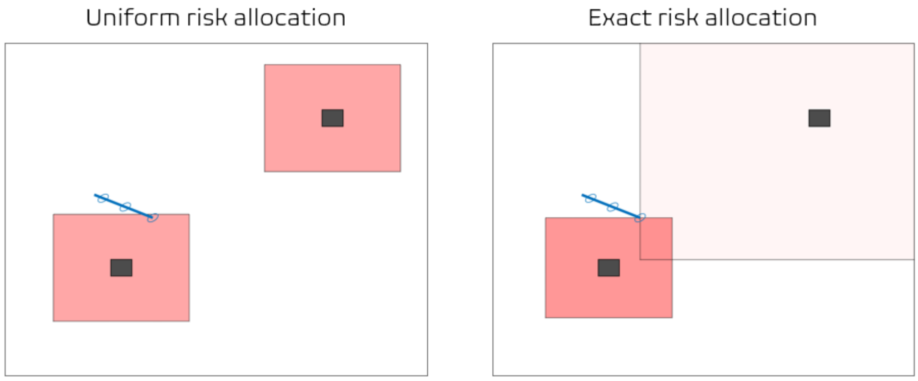
In a real-world environment, the exact distribution of the uncertainty is often unknown and assuming for instance a normal distribution can lead to the risk being seriously underestimated. In this case, the uncertainty was modeled as "distributionally robust", which means that only the mean and variance of the distribution is considered known and not the exact nature of the distribution. While this gives a more realistic representation of the uncertainty, it also increases the conservatism of path planning algorithms even further since we have to plan against to the worst possible uncertainty distribution. This means that reducing the conservatism when considering distributionally robust uncertainty becomes even more important.

One path planning algorithm that considers distributionally robust uncertainty is called "DR-RRT" which stands for "Distributionally Robust Rapidly Exploring Random Tree". This algorithm works by picking a random point in the environment and trying to steer to it using a steering method. The generated path is then checked against the risk constraints. If the path fulfills the risk constraints it is added to a "tree" of paths.

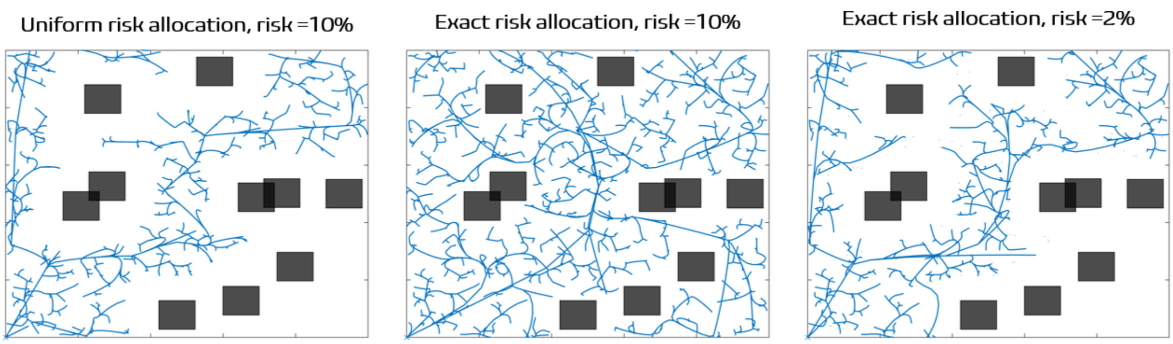
In the original DR-RRT-algorithm, the risk constraint for a generated path is checked by distributing the total risk budget equally over all timesteps and obstacles, so that all obstacles at all timesteps are assigned the same risk-parameter. The path fulfills the risk constraint if the risk of colliding with a specific obstacle at a specific timestep lies below the allocated risk-parameter. Instead of assigning the risk uniformly, the new Exact Risk Allocation (ERA) method assigns the exact amount of risk that is needed for each obstacle-constraint to be fulfilled.

To check if the risk of colliding with an obstacle is below the allocated risk parameter, the obstacle is enlarged in proportion to the risk-parameter. If the mean state of the robot is outside of all enlarged obstacles at all timesteps in the path, it means the path fulfills the risk constraint and can be added to the tree.

Here is a simple example to illustrate the difference between exact and uniform risk allocation. Imagine a path with 5 timesteps and 2 obstacles and the highest risk for collision that we can tolerate is 1%. With uniform risk allocation, all obstacles and timesteps are assigned the same risk parameter; 0.1%. This risk parameter is then used to enlarge all obstacles and check if the mean state of the robot lies outside. With exact risk allocation, we begin by assigning risk parameters to all obstacles such that the mean state of the robot is always just outside of the enlarged obstacle. All individual risk parameters are then summed up and compared to the risk budget, in this case 1%. The picture below shows an example of exact- and uniform risk allocation for one particular timestep, where the red areas represent the enlarged obstacles.



With uniform risk allocation, the mean state of the robot lies inside of the enlarged obstacle, which would mean this path is not feasible with uniform risk allocation. This particular path was however feasible when using ERA. When implementing the two different risk allocation methods into the previously mentioned path planning algorithm, we get the simulations below. As can be seen, when using the same upper bound of 10%, ERA leads to much less conservatism than using uniform risk allocation. We can also see that it is possible to give stronger risk guarantees, 2% as opposed to 10%, when using ERA and get similar results.



References

1. K. Ekenberg, “Distributionally Robust Risk-Bounded Path Planning Through Exact Spatio-temporal Risk Allocation”, Master thesis, TFRT-6174, Lund University, faculty of Engineering, department of Automatic Control, 2022. Available for download from https://lup.lub.lu.se/student-papers/.