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Bachelor's Thesis

Cryptocurrencies and Market Indices: A Markowitz Portfolio Optimization Problem

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Abstract

This thesis will explore the role of cryptocurrencies in a market index portfolio. The portfolios of a mix of Bitcoin, Ether and the market indices S&P 500, OMXS30 and VTI will be examined and optimized to maximize the Sharpe ratio. This process is done using the Markowitz Portfolio Theory for portfolios containing one market index and either one or both cryptocurrencies, numerically optimizing the portfolios in regard to the Sharpe ratio and comparing them to the descriptive statistics of the market indices. This process is then repeated for three subsamples, looking at the performance of the portfolios before the Covid-19 lockdown, during the lockdown period, and from the end of 2021 onward. Due to Bitcoin and Ethereum enjoying astronomical returns over the time period examined in the thesis, the optimal portfolios were heavily allocated into the cryptocurrencies. Similar results were shown for the pre- and mid-pandemic portfolios. The post-pandemic portfolios experienced negative returns, therefore allocating fully into the less volatile market indices. These results suggest that the inclusion of cryptocurrencies into market index portfolios is very beneficial for the Sharpe ratio, but should be approached with caution by the more risk-averse investor.

Contents

1. Introduction

This thesis will explore the role of cryptocurrencies in a market index portfolio. The portfolios of a mix of Bitcoin, Ether and the market indices S&P 500, OMXS30 and VTI will be examined and optimized to maximize the Sharpe ratio. Cryptocurrencies have become increasingly disruptive in the financial market, and this thesis aims to see if there is any validity to this rise in popularity. First of all, the concept of cryptocurrencies and their rise will be described.

1.1. Background

Cryptocurrencies were first presented to the public in a whitepaper published under the pseudonym Satoshi Nakamoto, explaining a new peer-to-peer cash system that would eliminate the need of a centralized financial institution. This would be achieved by using blockchain technology. (Nakamoto, 2008)

Blockchain is a distributed database or ledger that is shared across a network of computers. This database is structured in blocks that hold a certain amount of information. Once the block has reached its capacity, another block is created and linked to the previous block. Once a block is filled, it cannot be altered or removed. This way the data is structured in an irreversible timeline, where each block has a timestamp of when it is added to the chain. By storing the blockchain on multiple different nodes, one actor cannot alter the chain, therefore creating a safe and decentralized system. (Hayes, 2022)

Through the application of block-chain technology suggested in Nakamoto's paper, Bitcoin became the first viable option for true digital value. To this day, Bitcoin is still the largest cryptocurrency on the market, with a market cap of $441 billion, however it is far from the only one (Tretina, 2022). Alternative cryptocurrencies, each supported by different kinds of blockchain technologies, emerged soon after Bitcoin gained in popularity. The most prominent cryptocurrency following Bitcoin is Ether, created by Vitalik Buterin and launched in 2015. (Buterin, 2014)

The goal of cryptocurrencies is to offer a replacement for traditional fiat currency. However, due to volatility being very high and transaction speeds still being quite slow, cryptocurrency is currently used more commonly as a speculative financial asset, rather than a traditional currency (Baur et al., 2018).

During the last decade, cryptocurrencies have been growing rapidly and have become more accessible to buy and trade for the retail investor. During the Covid-19 pandemic, this growth was accelerated even further, with a market cap of the top 5 cryptocurrencies now exceeding $800 billion (Tretina, 2022) and cryptocurrencies getting more recognition by mainstream media and institutions.

Many private investors are now staking at least 1% of their wealth in cryptocurrencies, in order to benefit from the astronomical returns seen in recent years (Stephan, 2021). However, this rise has come with the cost of very high volatility, where the value of cryptocurrencies can shift dramatically during a single day. Therefore, investors suggest adding a small amount of cryptocurrency to your portfolio in order to participate in the growth in safe manner. This raises the question that will be explored in this thesis: will the average retail investor see a benefit of adding cryptocurrencies in their portfolio? How much has cryptocurrencies benefitted retail investors in the past, and does this indicate similar benefits in the future? The goal of this thesis is to answer these questions by adding Bitcoin and Ether to the stock indices S&P 500, OMXS30 and VTI.

1.2. Research question

- *How is the risk-adjusted return of a market index portfolio influenced, given the inclusion of Bitcoin and/or Ether?*
- *What effects have the cryptocurrencies had on said market index portfolios prior to, during and after the Covid-19 pandemic?* [1]

1.3. Definitions

1.3.1. Bitcoin

Bitcoin (BTC) is the first implementation of the concept cryptocurrency, first created in 2009. They are created through the process of mining, where computers solve complex computational problems in order to create the next block. One block awarded you 50 BTC in 2009, however the rewards are halved periodically and currently stand at 6.25 BTC per block (Hong, 2022). Due to

[1] After the pandemic and post-pandemic are shorthand terms for the selected time period, further explanation can be found in section 3.5.

the way Bitcoin is designed, only 21 million BTC will ever be in existence. Currently around 19 million Bitcoin have been mined.

Bitcoin's price is determined by the market's supply and demand (Bitcoin (2022), Ciaian et al., (2016)). In other words, there is no underlying factor determining the price of Bitcoin, only people's perception of its value. However, the limitation on the amount of Bitcoin that can exist stops the currency supply from becoming increasingly inflated, thus losing its value over time.

1.3.2 Ethereum

Ethereum, similarly to Bitcoin, is a platform powered by blockchain technology. However, Ethereum has the added benefit of being programmable, allowing for applications to be created that use the Ethereum network to store data. Ethereum has its own native token, or cryptocurrency, called Ether (ETH). This token is used similarly to BTC, with the added benefit of paying programmers for their services on the Ethereum network. ETH has no practical limit like BTC does, but is limited by the amount of time it takes to mine a block of ETH. (Frankenfield, 2022)

ETH is valued largely in the same way as BTC, through supply and demand. Although this could change in the future, due to ETH being required to pay transaction fees on the Ethereum network. (Ethereum, 2022)

1.4. Structure

The thesis is structured in six different sections. Section 1 includes the introduction, background, definitions and research question. Section 2 follows with a literature review. Section 3 contains the methodology of the thesis, prefaced by assumptions made in the calculations. Section 4 contains empirics, divided into data collection and descriptive statistics. Section 5 contains the empiric results and analysis, and section 6 is the conclusion.

2. Literature review

There is no shortage of research exploring the viability of cryptocurrencies in different ways: as a replacement for Fiat currency, as a speculative investment tool, or as a hedge or safe haven for your portfolio.

2.1. Crypto hedging versus crypto diversifying

Due to the pandemic and increased unrest on a geopolitical scale, the world has been experiencing an increasingly volatile market the past couple of years. Investors have been looking for many ways to lower the risk of losing money during a period of both high uncertainty and, for the US mainly, high inflation (Cox, 2022). Since treasury bonds have not been providing positive yields since the decline of interest rates in 2009 (Fleischmann, Fritz and Sebastian, 2019), investors are looking elsewhere to minimize their losses. In the search of such a safe haven, Bitcoin has been titled the "Digital gold", as is has many traits that coincide with the precious commodity; its value is derived partly due to scarcity of supply, the supply is not controlled by a centralized institution, both assets' prices are volatile and total supply is finite (Dyhrberg, 2016). Even if this hypothesis sounds like it should have the capabilities to hedge a stock portfolio, many researchers have found that these hedging capabilities are not significantly strong (Conlon & McGee (2020), Baur et. al. (2022), Löveråsen & Tyrsing (2021)).

Despite concluding that the hedging capabilities of Bitcoin are lacking, many of the same articles still recognize the diversification possibilities of cryptocurrencies in different portfolios (Baur. et al. (2022), Löveråsen & Tyrsing (2021)). Diversification is possible even with assets with high volatility, as long as they have a low correlation with the rest of the portfolio (Segal, 2022). This diversification property holds true for cryptocurrencies as well. When creating portfolios of 500 cryptocurrencies, Brauneis & Mestel (2019) found that naïvely, or equally, diversified mean-variance optimized portfolios outperformed single cryptocurrencies in terms of Sharpe ratio. Similarly, Borri (2019) found that creating cryptocurrency portfolios reduced the idiosyncratic risk of the individual cryptocurrency. Following this reasoning, it would be rational to diversify a non-crypto portfolio with crypto, in order to further reduce the individual risks of all assets, and potentially increasing returns in the process.

2.2. Using crypto to increase portfolio return

Many studies have found that the inclusion of cryptocurrency in a portfolio could lead to positive results for that portfolio. Andrianto & Diputra (2018) found that including Bitcoin in a portfolio consisting of ETF's and forex would help increase the Sharpe ratio. Ma et al. (2020) found that a portfolio consisting of the ten largest US stocks would increase in return and decrease in volatility after introducing a mix of five different cryptocurrencies, with Ethereum offering better diversification than Bitcoin. Kajtazi and Moro (2019) found that adding Bitcoin to a well-diversified portfolio improves its performance by increasing the returns more than the reduction in the volatility. The effectiveness of increasing your risk-adjusted returns using Bitcoin has been further supported by Bakry et al (2021), albeit for the risk-seeking investor mostly.

2.3. The articles contribution

Given the fact that cryptocurrencies are still a new financial instrument, with barely over a decade on the market, many findings about risk and return for portfolios including cryptocurrencies no longer hold the same weight as they once did. As time goes by, more data becomes available to analyze, giving future researchers more tools to work with. For example, when Dyhrberg (2016) found that Bitcoin serves as a proper hedge, it was still two years before the price spike of 2017, and even before Ethereum was properly introduced. This holds true even for more recent research such as Bakry et. al. (2021), who concluded their research just a few months before the post-pandemic market crash which begun at the end of 2021. Investigating the portfolios in times of market turbulence will show how the financial assets behave during that time. This gives us knowledge that can be used by the short-term risk-seeking investor in order to make profits off the trends, or for the more risk-averse long-term investor to hedge away that risk using alternative assets.

3. Methodology

The methodology section describes the data, theoretical framework and methodology used for the thesis. The purpose is to inform the reader about what method is used to explore the research question, what assumptions have been made to limit the scope, and a reflection of the method.

3.1. Assumptions

In order to make answer the thesis' research question, the following assumptions are made when calculating the empirical data.

First, short-sale constraints are imposed on the portfolios. This is partly because of avoiding potential portfolios with extreme weights, which is a possibility using the methodology explained in section 3.2.

Secondly, the transaction costs of trading assets are assumed to be zero. Because of different brokerages having different policies on transaction fees, as well as the burden of the fee differing depending on the amounts that are invested, this is an attempt to keep the findings as generalized as possible (Francis, 2022).

Thirdly, the risk-free interest rate is assumed to be zero. Because of the monetary policies and interest rates of different countries differing, this assumption attempts to keep the findings generalized geographically. It is however close to reality, due to the effective risk-free interest rates of many Western countries being extremely low during the observed time period (Fleischmann, Fritz and Sebastian, 2019).

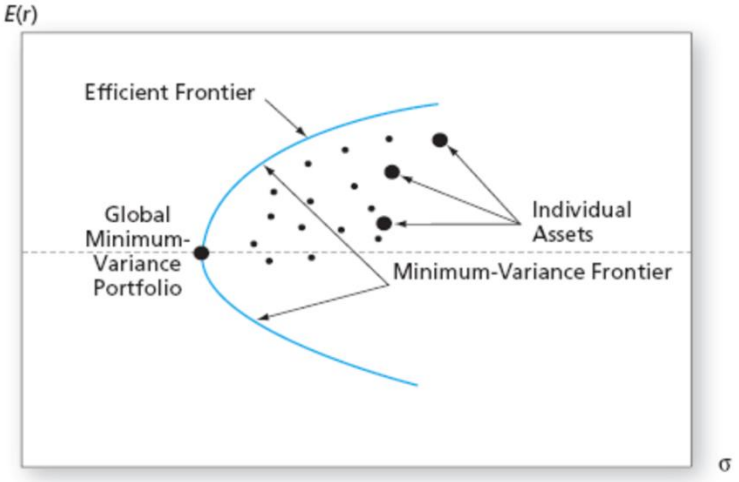
Lastly, it is assumed that the covariance between the index funds and the cryptocurrencies stay consistent, even on days when traditional assets are not tradeable. Because of BTC and ETH not relying on a centralized institution, they are tradeable at all times. In order to carry out the necessary calculations to compare the assets and create the portfolios, we have to assume that the data points removed from the time series of the cryptocurrencies are consistent with the rest of the time series, thus not differing in covariance with the index funds.

3.2. Markowitz portfolio for two risky assets

In order to find the portfolio that will maximize the risk-adjusted return, the Modern Portfolio Theory is utilized. Modern Portfolio Theory, or mean-variance analysis, is a portfolio selection theory developed by Harry Markowitz (1952). Since its inception it has become the most widely used theory to optimize portfolios of risky assets.

The problem that Markowitz was trying to solve in his paper was the investors desire to receive a high expected return, while also desiring a low variance. In order to solve this problem, Markowitz suggested looking at the expected return and variance of the financial assets simultaneously, creating the mean variance framework. By estimating the expected return of the assets in the portfolio, as well as the volatility, the different asset allocation combinations can be plotted, creating the mean variance frontier, displayed in Figure 3.1.

Figure 3.1 Mean Variance Frontier



Source: Bodie et al., 2014, p. 220

The portfolios that follow the line going through the global minimum variance portfolio are the ones with the best risk-return combinations. Any portfolio to the right of this line is considered inefficient, due to its combination having the same return but a higher risk than an efficient portfolio. The individual assets will be located to the right of this portfolio, proving the effectiveness of diversification in portfolio selection.

In order to find the portfolio in the mean-variance frontier that returns the highest Sharpe ratio, we find the point on the efficient frontier that tangents with the Capital Market Line

(CML). The CML is the line that present all portfolios that optimally combine the risk-free return and the risky asset portfolio (Bodie et al., 2014).

But in order to find the optimal portfolios, we first need to be able to estimate the necessary statistics. This process is explained in the following portion.

3.3. Estimating the expected return and volatility

In order to compare the different assets against each other, the rates of return need to be calculated. This allows us to see the percentage change of return of the asset between two time periods, instead of the nominal price point at a certain time. We calculate the rates of return using the following formula:

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

Where S_t is the spot price of the asset at time t, and S_{t-1} is the spot price at the previous point in time.

Now that the returns are standardized into rates of return, the expected rate of return of the asset can be estimated using the following formula:

$$E(r_i) = \frac{1}{n}\sum_{t=1}^{n} r_{i,t}$$

This formula uses the historical average returns of the asset in order to predict what growth we can expect in the future.

With the help of the rates of return and the mean return, or expected return rate, the variance, or volatility of the asset can be calculated using the formula presented below:

$$\sigma_i^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_{i,t} - \bar{r})^2$$

Where σ_i^2 presents the variance of the asset, and $\bar{r}$ presents the mean return rate of the asset, or the expected return. Because the data collected is just a sample of the entire population of data, we lose one degree of freedom in this calculation, which is reflected in the denominator being n-1.

In order to measure and compare the risk-adjusted returns for the assets and portfolios, the Sharpe ratio is utilized. The Sharpe ratio was introduced by William Sharpe in 1966 and

measures the reward, return, of a certain asset, seen to its risk, or volatility. This is done using the following formula:

$$SR_i = \frac{E(r_i - r_f)}{\sigma_i}$$

Where σ_i stands for the standard deviation of the asset, and r_f is the risk-free interest rate. Because we assume the risk-free interest rate to be zero in this thesis, the variable can be removed from the equation, giving us the following adjusted formula for our thesis:

$$SR_i = \frac{E(r_i)}{\sqrt{\sigma_i^2}}$$

Given these formulas it is now to compare and determine the different assets' roles in our portfolios. This gives us the opportunity to identify the optimal portfolios, with the help of the descriptive statistics of the portfolios. For that we need to use some adjusted formulas, where the two assets can be weighted in different ways. We define the expected return and the portfolio variance with the following formulas:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i)$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} w_i w_j$$

Where $E(r_p)$ is the expected return of the formula, w is the weight, or share, of the asset i in the portfolio and $E(r_i)$ is the average return of asset i. The portfolio variance, σ_p^2, consists of the weights (w) of two assets, i and j, and their covariance, $\sigma_{i,j}$. This covariance is calculated through the assets inherit variances, and their correlation to each other:

$$\sigma_{i,j} = \rho_{i,j} \sigma_i \sigma_j$$

Instead of using the CML to find the optimal portfolio to maximize the Sharpe ratio, we can use the portfolio variance and expected portfolio return to solve the optimal portfolio problem numerically. The problem we are solving is the following:

$$Max \frac{E(r_p)}{\sqrt{\sigma_p^2}}$$

Subject to

$$\sum_{i=1}^{m} w_i = 1$$

Using the Solver software in Microsoft Excel, it is possible to optimize equations given certain constraints. Using the equations given above and the weights of the assets as variables, setting a constraint of the sum of weights as one and the weights as positive to enforce the short-sale constraint, the maximum Sharpe ratio is given for each portfolio.

3.4. Markowitz portfolio for multiple risky assets

When calculating the optimal Markowitz portfolio with more than two assets, the same principle stands, however with some new equations involved. First of all, the covariance matrix is calculated. This matrix will identify the covariance between all the assets, and will be calculated using the following formula:

$$\Omega = \frac{1}{n-1} O^T O$$

Where O is the variance vector of the assets. Using the covariance matrix, the expected portfolio return, and portfolio volatility can be estimated. These formulas follow the same principle as with the two-asset portfolio, however using linear algebra instead. This makes it possible to portfolios of any sizes. The linear algebra formulas of the expected portfolio return and portfolio volatility are presented below:

$$E(r_p) = W^T ER$$

$$\sigma_p^2 = W^T \Omega W$$

Where W is the weight vector, ER is the expected return vector and Ω is the covariance matrix. Using the same Solver method in Microsoft Excel as in the two-asset portfolios, the optimal portfolios can now be found.

3.5. Comparisons between different time periods

In order to examine the performance of cryptocurrencies across different time periods, the procedures above need to be repeated on subseries of the time series collected. The different

subseries are chosen by identifying the starting points of each significant event observed. By looking at the price development of the S&P 500 for the full period observed, two points right before significant crashes are identified. The first being the crash at the start of the pandemic, identified to be at 2020-02-10, and the second being at the end of 2021, namely 2021-10-11. These three periods will be classified as pre-, mid-, and post-pandemic throughout this thesis. The goal is to compare these different time periods and see what effect the cryptocurrencies have when included into the index fund portfolios.

The names are chosen to differentiate between the bull market of the pandemic, and the bear market that followed the relaxation of lockdown policies and the inflation period that followed. This thesis recognizes the WHO classification of the Covid-19 as a pandemic at the time of writing.

3.6. Methodology Reflection

The methods used in this thesis are based on the standard method of Markowitz portfolio theory. Arithmetic means are used to estimate the expected returns, in order to keep the data as consistent as possible over the full time period. This allows for easier division of the time series into its subseries used to analyze the portfolios during irregular market conditions. Geometric means are an alternative that would be useful to account for compounding, leading to a more cohesive expected return. However, this makes comparing the subseries more difficult.

Both monthly and daily data have been used to solve the thesis' problem. This is in order to mimic two different trading frequencies. Daily data is aimed at a day trading situation, where the trader is highly responsive to changes in the market. This allows the trader to benefit from the high volatility assets more than somebody who trades less frequently. For the latter situation, the monthly data has been chosen. Monthly data is infrequent enough to normalize any outliers in the returns through its aggregation, but frequent enough to still see the way the market evolves over the years, as well as allowing us to examine the smaller time periods of mid- and post-pandemic.

4. Empirics

This section provides the data collection and descriptive portions of the thesis. First, the data collection process is explained. Secondly the data is transformed into descriptive statistics, that are explained in the descriptive statistics portion.

4.1. Data collection

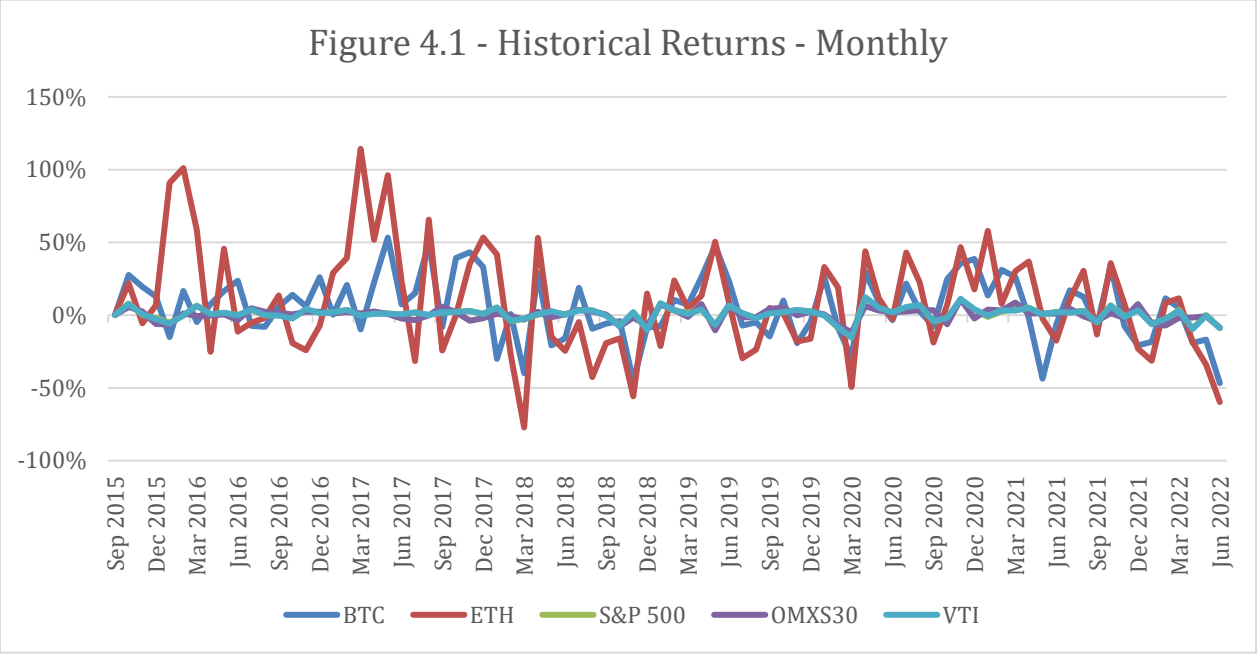
The data collected for this thesis consists of ten different time series, five different variables in two different time intervals: daily and monthly. The variables consist of two cryptocurrencies and three stock indices. The cryptocurrencies are Bitcoin (USD per BTC) and Ethereum (USD per ETH), where the price points are collected. The three stock indices used in this thesis are the Standard & Poor's Composite 500 (S&P 500), the OMX Stockholm 30 (OMXS30) and the Vanguard Total Stock Market Index Fund (VTI). These indices are so called market-value-weighted indices. Market-value-weighted indices are stock indices that are weighted according to the stocks' market cap. This means that larger stocks (e.g. Apple, Meta, Alphabet) have a higher share in the index than smaller stocks.

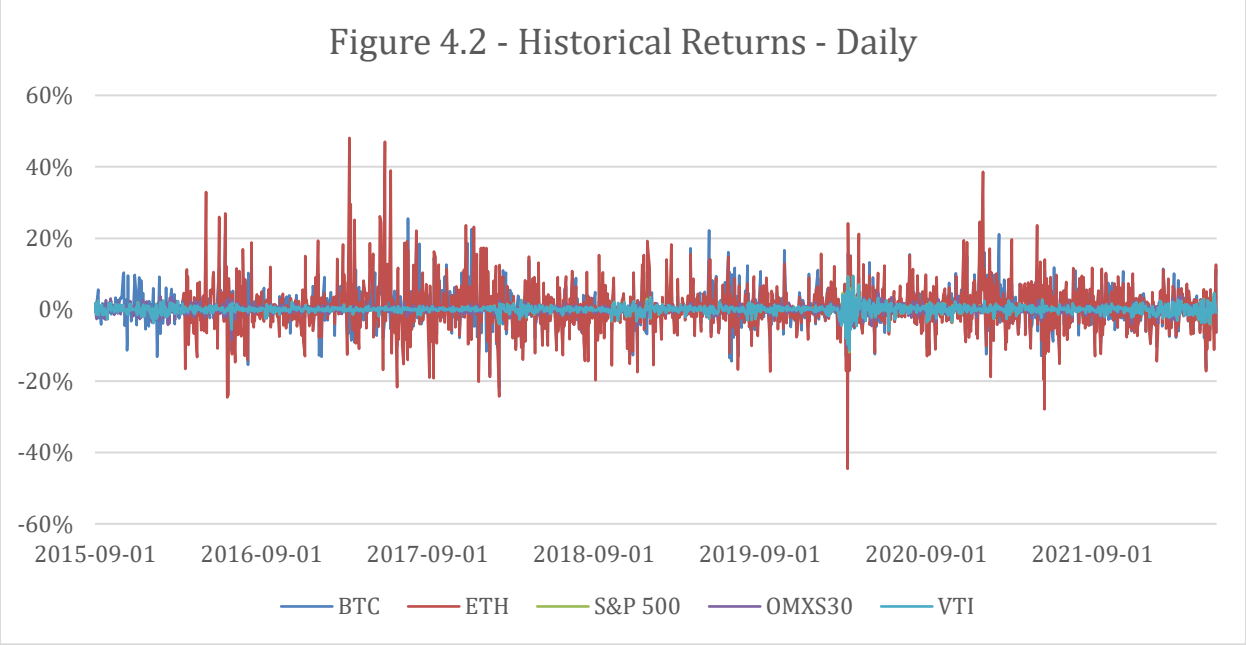
The data is collected from Investing.com. The monthly data samples consist of 82 observations, during the time period from September 2015 to June 2022. The data for Ethereum is supplemented with data from Yahoo! Finance for the period September 2015 to March 2016, due to the data not being available on Investing.com. The daily data samples consist of 1527 observations, during the time period from 2016-03-10 to 2022-06-01. These observations are already adjusted before processing, due to the difference in tradable dates for the different assets. All data points that are not shared between all assets are removed. This decision in data manipulation is what led to the thesis' assumption of covariance being constant between trading days over weekends and holidays.

4.2. Descriptive Statistics

Using the returns formula mentioned above, the collected data is transformed from price points at certain dates to rates of return between the time periods, presented in figures 4.1 - 4.2. There is a clear difference between both the returns and the volatility of the market indices funds and the

cryptocurrencies. BTC and ETH are showing much higher returns throughout the time period, however followed with higher volatility, presented by the higher peaks and troughs in the graphs.





This observation is supported by the descriptive statistics presented in tables 4.1 and 4.2. These tables were created by using the formulas for expected return, standard deviation and Sharpe ratio explained above, as well as finding the maximum and minimum returns in the historical returns series. The expected returns of the cryptocurrencies are significantly higher than those of the index funds, with Ethereum showing the highest return across all time intervals, and OMXS30 showing the lowest. The standard deviation follows the same pattern, however showing a very slightly higher volatility for OMXS30 in the weekly data than the other index funds. This is further supported by the much more extreme highs and lows shown by the cryptocurrencies than the index funds. To see how much the expected returns and standard deviations increased, we can look at the Sharpe ratio.

The Sharpe ratios for the cryptocurrencies are yet again significantly higher than their index funds counterparts. This implies that the expected returns outperform to a higher degree than the volatility does.

When looking at the skewness of the different asset classes, we can see that the index funds tend to have more negative skewness, indicating that more returns are positive, while having more negative outliers. For the cryptocurrencies we see the opposite, with weak positive skewness for BTC and strong positive skewness for ETH. This indicates that we have more frequent positive outliers in the return.

Considering the kurtosis, daily data shows excess kurtosis, indicating fat and extreme tails. S&P 500 and VTI have very high kurtosis, indicating that the vast majority of returns stay very close to the mean, with few but extreme outliers. The same principle applies to the cryptocurrencies, however less extreme. The monthly kurtosis is however very low, showing a low density of returns around the mean but with less extreme outliers. This contrast can be explained by the aggregation of returns for the monthly period. By collecting multiple returns and summing them up, the outliers get toned down by the more normal returns, while it also shows more trends due to time allowing to pass.

Table 4.1 - Descriptive Statistics - Monthly

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	0,05477	0,08982	0,00838	0,00344	0,00799
Variance	0,04920	0,13522	0,00199	0,00193	0,00216
Standard Deviation	0,22181	0,36773	0,04460	0,04394	0,04647
Sharpe Ratio	0,24693	0,24427	0,18789	0,07837	0,17199
Highest Return	0,53283	1,14261	0,11942	0,11010	0,12340
Lowest Return	-0,46720	-0,77191	-0,13367	-0,11845	-0,15445
Skewness	-0,14663	0,49581	-0,66698	-0,39253	-0,75343
Kurtosis	-0,22624	0,45209	1,22904	0,37524	1,76014

Table 4.2 - Descriptive Statistics - Daily

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	0,00396	0,00566	0,00055	0,00032	0,00053
Variance	0,00229	0,00479	0,00014	0,00013	0,00014
Standard Deviation	0,04784	0,06922	0,01193	0,01159	0,01201
Sharpe Ratio	0,08269	0,08178	0,04576	0,02776	0,04443
Highest Return	0,25559	0,48071	0,09383	0,07089	0,09490
Lowest Return	-0,39182	-0,44547	-0,11984	-0,10571	-0,11381
Skewness	-0,01386	0,71785	-0,73623	-0,83158	-0,78445
Kurtosis	6,03679	6,69878	18,30087	8,74394	16,70578

Table 4.3 and 4.4 illustrates the correlation between the assets. A higher number indicates a stronger correlation, where 1 is a perfect correlation. In all time periods we are seeing a weak positive correlation between the cryptocurrencies and the index funds. Within the asset classes we see very high correlations indicating that they move at around the same pace over time.

Table 4.3 - Correlation Matrix - Monthly

	BTC	ETH	S&P 500	OMXS30	VTI
BTC	1	0,54809	0,33111	0,22266	0,34265
ETH	0,54809	1	0,23519	0,14842	0,23733
S&P 500	0,33111	0,23519	1	0,76791	0,99457
OMXS30	0,22266	0,14842	0,76791	1	0,76591
VTI	0,34265	0,23733	0,99457	0,76591	1

Table 4.4 - Correlation Matrix - Daily

	BTC	ETH	S&P 500	OMXS30	VTI
BTC	1	0,61254	0,20508	0,14639	0,21016
ETH	0,61254	1	0,21719	0,14865	0,21717
S&P 500	0,20508	0,21719	1	0,57922	0,99439
OMXS30	0,14639	0,14865	0,57922	1	0,58860
VTI	0,21016	0,21717	0,99439	0,58860	1

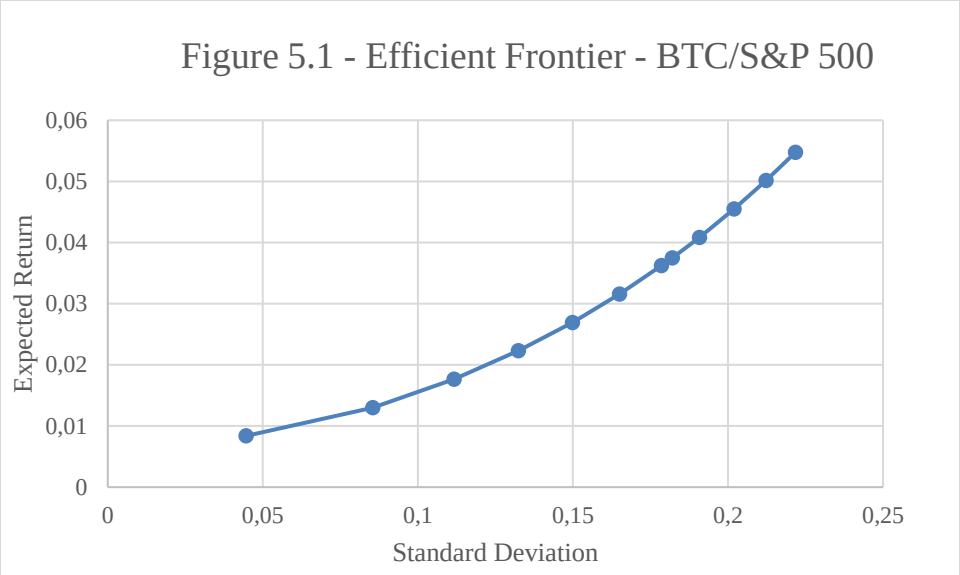
5. Empirical Results and Analysis

This section contains the empirical results found from the empirics section along with an analysis of the results. The section is structured as follows: first a presentation and analysis of all 2-asset portfolios. Secondly the 3-asset portfolio for the full time period is presented and analyzed, followed by the separated time periods in chronological order.

5.1. 2-asset portfolios

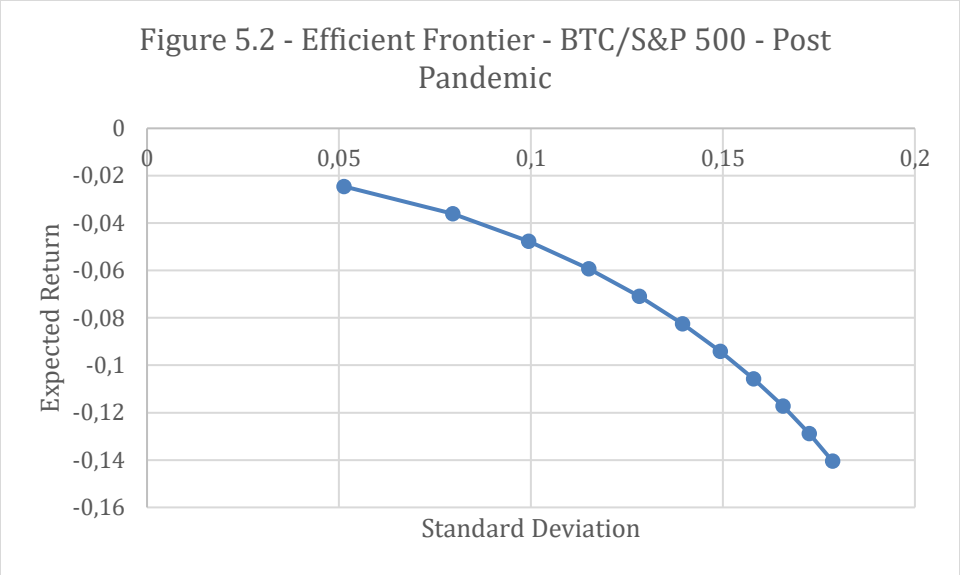
In this section the results for the 2-asset portfolios will be presented and analyzed. Because of the results being identical in all tests, the tables are presented in the appendix. The tables display the optimal portfolios of two assets, one cryptocurrency (CC) and one index fund (IF), when maximizing the Sharpe ratio.

The 2-asset portfolios are all optimized by investing fully into the cryptocurrencies across all time periods, with the exception of the post pandemic portfolios, showing full allocation into the index funds instead. This goes against the theory of the Efficient Frontier and diversification. In order to understand why these portfolios don't follow the theory I created the efficient frontiers for the 2-asset portfolios. All of them followed the same trend, displayed in the efficient frontier of the BTC/S&P 500 portfolio for the full time period with monthly frequency in Figure 5.1. As seen in the figure, this efficient frontier is not following the "C" shape explained in theory, following the initial decrease in risk while increasing the expected return, and later showing diminishing returns. Instead, the curve is following an exponential trend, the opposite from what is considered normal. This supports the allocation being solely in cryptocurrencies, due to more exposure into cryptocurrencies results in higher returns while only slightly increasing risk.



Index funds have had a period of abnormally high volatility during the last couple of years, making them less of a "safe investment" as historically considered. The pandemic created an unusual market, where all financial assets crashed very suddenly, followed by a period of extremely high returns. Because of this trend following the entire financial market, we also experienced a period of very high correlation between all assets. This makes diversification less of a possibility, since all assets are following more or less the same trends. So, if diversification is not an option anymore, it is only rational to invest in the asset that gives you the highest return.

In Figure 5.2, the efficient frontier of the post pandemic monthly portfolio of BTC and S&P 500 is displayed. This frontier follows a negative exponential trend, caused by the majority of the returns during this time period being negative. This explains the optimal portfolio weights being completely focused on the index funds, since increased exposure into the cryptocurrencies results in more risk exposure and higher losses.



5.2. Full time period

Using the descriptive statistics displayed above and the process described in the methodology, the following portfolios were the results of the optimizations of the data for the full time period. Tables 4.8 – 4.10 display the optimal portfolios of three assets, BTC, ETH and one index fund (IF), when maximizing the Sharpe ratio. The optimal weights for these portfolios are more diversely weighted than those of the 2-asset portfolios. For the monthly data set, the OMXS30 portfolio shows the most weight in the cryptocurrencies, at over 64%, while the S&P 500 portfolio only has a combined weight of around 34%.

Table 5.1 - Optimal Portfolios - 3 assets - Monthly

	S&P 500	OMXS30	VTI
Weight BTC	21,37%	41,67%	24,99%
Weight ETH	13,21%	22,76%	15,14%
Weight IF	65,42%	35,57%	59,88%
Er(p)	0,04670	0,07541	0,05232
St Dev	0,10965	0,18232	0,12410
Sharpe Ratio	0,42587	0,41359	0,42161

The daily portfolios show a complete investment in the cryptocurrencies for the S&P 500 and VTI portfolios, while weighting only 6,20% into the OMXS30 in its portfolio. All the portfolios are heavily allocated into ETH.

Table 5.2 - Optimal Portfolios - 3 assets - Daily

	S&P 500	OMXS30	VTI
Weight BTC	16,46%	15,34%	16,46%
Weight ETH	83,54%	78,45%	83,54%
Weight IF	0,00%	6,20%	0,00%
Er(p)	0,00438	0,00412	0,00438
St Dev	0,06296	0,05920	0,06296
Sharpe Ratio	0,06956	0,06957	0,06956

Comparing the Sharpe ratios to those of the indices in the descriptive statistics, it is clear that the inclusion of BTC and ETH in all portfolios significantly increases the Sharpe ratio of said portfolios. This finding supports the findings of the research presented in the literature review, showing that the recent turbulent years have not changed the characteristics of BTC and ETH as return-increasing assets.

In order to examine the changes that these portfolios have gone through over the years, the separated time periods are presented and analyzed below, followed by a comparison of the time periods.

5.3. Pre-Pandemic

The following result describe the optimal portfolios for the time period before the pandemic effects on the market. This time period stretches from the start of the full time period, shown in section 4.1, until 2020-02-07.

Displayed in Tables 5.3 – 5.4 are the optimal portfolios when utilizing both cryptocurrencies and one index fund in the time period before the Covid-19 pandemic. We can observe very high weights in the index funds for all portfolios, except the daily frequency portfolios, showing yet again almost exclusively crypto-focused portfolios, weighted heavily into ETH.

Table 5.3 - Optimal Portfolios - 3 assets Pre-Pandemic - Monthly

	S&P 500	OMXS30	VTI
Weight BTC	10,48%	21,09%	11,16%
Weight ETH	4,55%	8,15%	4,88%
Weight IF	84,97%	70,76%	83,96%
Er(p)	0,02896	0,04168	0,03003
St Dev	0,05006	0,08323	0,05256
Sharpe Ratio	0,57846	0,50080	0,57124

Table 5.4 - Optimal Portfolios - 3 assets Pre-Pandemic - Daily

	S&P 500	OMXS30	VTI
Weight BTC	17,54%	15,85%	17,54%
Weight ETH	82,46%	75,21%	82,46%
Weight IF	0,00%	8,94%	0,00%
Er(p)	0,00459	0,00420	0,00459
St Dev	0,06416	0,05862	0,06416
Sharpe Ratio	0,07161	0,07162	0,07161

The pre-pandemic time period chosen for this portfolio spans from the first available price points for ETH until the right before the crash following the start of the pandemic. This was a period of relatively low volatility and high returns for the market indices, making them and attractive asset for the investor. The cryptocurrencies however were in a period of higher volatility. Both BTC and ETH experienced a massive price spike in late 2017, which quickly returned to low levels over the next year. This period of high volatility and relatively low returns would explain the lower allocation in cryptocurrencies during this time period for the monthly portfolios, compared to the full time period.

For the daily data, the amount of extremely high returns for the cryptocurrencies gave them a very high daily mean, ten times as large as the S&P 500. Even though the volatility increased as well, it was not nearly as much, resulting in the daily Sharpe ratio for ETH being six times as high as the one for the S&P 500. Interestingly, the Sharpe ratio for the OMXS30 is the lowest of them all, but that is the only one included in its portfolio. The explanation comes from the low correlation between the OMXS30 and the cryptocurrencies.

5.4. Mid-Pandemic

The following portfolios display the optimal weights for the maximum Sharpe ratio during the pandemic, accounting for the time period between 2020-02-10 and 2021-10-08.

For the monthly 3-asset portfolios, we can see that they are all fully allocated into the cryptocurrencies, fully ignoring the index funds. This makes all the portfolios identical, due to only accounting for the cryptocurrencies.

Table 5.5 - Optimal Portfolios - 3 assets Mid-Pandemic - Monthly

	S&P 500	OMXS30	VTI
Weight BTC	28,27%	28,27%	28,27%
Weight ETH	71,73%	71,73%	71,73%
Weight IF	0,00%	0,00%	0,00%
Er(p)	0,17670	0,17670	0,17670
St Dev	0,25674	0,25674	0,25674
Sharpe Ratio	0,68824	0,68824	0,68824

The daily frequency portfolios are during this time period fully allocated into ETH.

Table 5.6 - Optimal Portfolios - 3 assets Mid-Pandemic - Daily

	S&P 500	OMXS30	VTI
Weight BTC	0,00%	0,00%	0,00%
Weight ETH	100,00%	100,00%	100,00%
Weight IF	0,00%	0,00%	0,00%
Er(p)	0,00485	0,00485	0,00485
St Dev	0,06962	0,06962	0,06962
Sharpe Ratio	0,06962	0,06962	0,06962

These portfolios are less predictable than expected. During the pandemic, the stock markets returned record-breaking profits, including having the highest expected return of the time periods in this thesis. However, the cryptocurrencies increased massively in price during the pandemic, with BTC going from $10,000 at the start to about $65,000 in April 2021 and ETH going from $300 at the start to around $4,200 at its peak in May 2021. This enormous increase in price

makes the following volatility quite trivial, showing that such an increase in price makes all six portfolios favor the cryptocurrencies.

5.5. Post-Pandemic

Lastly, these portfolios display the optimal weights for the maximum Sharpe ratio after the pandemic, accounting for the time period between 2021-10-11 and 2022-06-01.

For the post pandemic portfolios, we are yet again seeing very extreme portfolios, only consisting of either all index funds or all ETH.

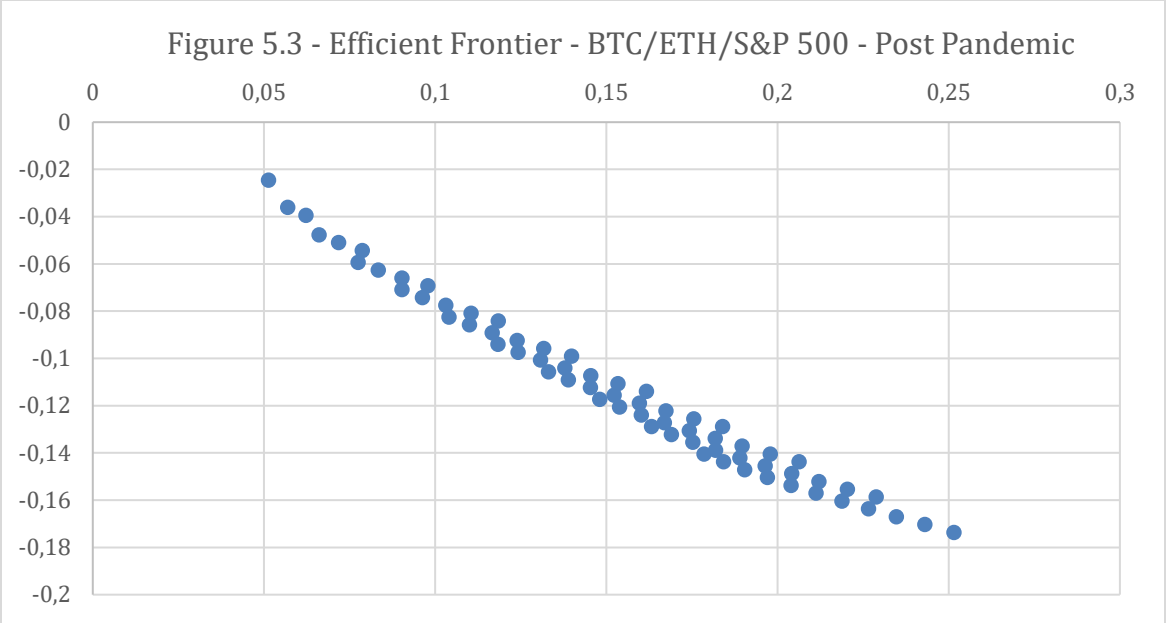
Table 5.7 - Optimal Portfolios - 3 assets Post-Pandemic - Monthly

	S&P 500	OMXS30	VTI
Weight BTC	0,00%	0,00%	0,00%
Weight ETH	0,00%	100,00%	0,00%
Weight IF	100,00%	0,00%	100,00%
Er(p)	-0,02451	-0,17362	-0,02849
St Dev	0,05130	0,25147	0,04999
Sharpe Ratio	-0,47781	-0,69041	-0,57002

Table 5.8 - Optimal Portfolios - 3 assets Post-Pandemic - Daily

	S&P 500	OMXS30	VTI
Weight BTC	0,00%	0,00%	0,00%
Weight ETH	0,00%	0,00%	0,00%
Weight IF	100,00%	100,00%	100,00%
Er(p)	-0,00034	-0,00050	-0,00053
St Dev	0,01367	0,01333	0,01422
Sharpe Ratio	-0,02483	-0,03751	-0,03707

These portfolios follow the same characteristics as the 2-asset post pandemic portfolios explained above. Because of the strong negative returns during this time period for all assets, the problem to solve becomes not just to maximize the Sharpe ratio for the portfolios, but to minimize the negative returns and volatility. This problem is visualized in Figure 5.3. displaying the mean-variance frontier of the S&P 500 portfolio using monthly data.



As shown in the figure, the frontier is rotated vertically from the theoretic frontier shown in the methodology. This gives us an indication of what we are achieving with the optimal portfolios. Due to the lack of a curve on the upper left side of the trend, there is no possibility to reduce the volatility of the portfolio by diversifying. This makes the only rational decision for the investor to choose the asset in the portfolio that has the least negative returns, as well as the least volatility. This portfolio highlights the risks involved in investing in highly volatile assets such as cryptocurrencies. When investing in a market index, the low volatility rewards less returns, but that accounts for both positive and negative returns. Choosing to invest in cryptocurrencies will give you the chance to take part in very high returns, however it also exposes you to huge losses.

If the short-sale constraint would be lifted, the mean-variance frontier would be mirrored vertically, creating a sideways parabola into the positive quadrant of the cartesian plane. This shows us that it is possible to still enjoy positive returns in times of negative market evolution, if short selling is available to you. This however comes with the added risk of losing more than the asset is worth. This shows that if you are willing to expose yourself to more risk, there is always more opportunity for you to profit off the exposure.

6. Conclusion

The objective of this study was to evaluate the change in Sharpe ratio for market index portfolios, given the inclusion of Bitcoin and/or Ethereum, as well as the past influence during pre-, mid- and post-pandemic time periods. The market indices evaluated are the S&P 500, OMXS30 and VTI. Using the Markowitz Portfolio Theory to maximize the Sharpe ratio, the optimal portfolios were identified.

The study concludes that the inclusion of ETH and/or BTC into market index portfolios leads to a positive change in the Sharpe ratio of these portfolios, with multiple portfolios optimizing by allocating all resources into the cryptocurrencies. This finding holds true for the pre- and mid-pandemic portfolios as well and is consistent across the 2-asset and 3-asset portfolios alike. For the post-pandemic period we see a risk minimizing strategy being executed due to the negative returns of this period, resulting in the portfolios allocating all resources into the market indices. These results conclude that the increased returns enjoyed by the cryptocurrencies outweigh the increased volatility that comes with it, making them a positive influence for the market indices in terms of Sharpe ratio, whenever the market is trending positively.

This conclusion does however not take into consideration the risk aversity of the individual investor. The inclusion of the cryptocurrencies increased the volatility of the portfolios considerably, which can be seen as undesirable by the more risk-averse investor. Due to the quantitative nature of the thesis, no fundamental analysis has been considered either in the portfolio selection. This means that the portfolios presented in this thesis are not the objectively best way to allocate your assets when investing. Every investor should look at the data provided and make their own subjective conclusions as to how to best allocate their own resources.

Future research on this topic is encouraged, due to the volatility of the financial market at the time of writing this thesis. The more data becomes available about cryptocurrencies, the better we can assess their role within the financial and economic world.

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Appendix

A.1 Descriptive Statistics for the separated time periods

Table A1.1 - Descriptive Statistics - Pre-Pandemic - Monthly

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	0,07076	0,10565	0,00998	0,00442	0,00970
Variance	0,04742	0,16368	0,00115	0,00145	0,00124
Standard Deviation	0,21775	0,40458	0,03398	0,03809	0,03525
Sharpe Ratio	0,32497	0,26112	0,29361	0,11612	0,27533
Highest Return	0,53283	1,14261	0,07972	0,07603	0,08195
Lowest Return	-0,45479	-0,77191	-0,09627	-0,10408	-0,10210
Skewness	-0,01996	0,61144	-0,79606	-0,56415	-0,85235
Kurtosis	-0,08498	0,23521	1,82526	0,65824	1,84800

Table A1.2 - Descriptive Statistics - Pre-Pandemic - Daily

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	0,00475	0,00668	0,00065	0,00042	0,00066
Variance	0,00236	0,00507	0,00014	0,00013	0,00014
Standard Deviation	0,04859	0,07120	0,01171	0,01138	0,01173
Sharpe Ratio	0,09780	0,09380	0,05548	0,03715	0,05610
Highest Return	0,25559	0,48071	0,09383	0,07089	0,09490
Lowest Return	-0,39182	-0,44547	-0,11984	-0,10571	-0,11381
Skewness	-0,02914	0,72214	-0,83606	-0,97738	-0,90848
Kurtosis	6,21403	6,53536	21,81705	10,34870	20,34124

Table A1.3 - Descriptive Statistics - Mid-Pandemic - Monthly

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	0,08956	0,15100	0,01696	0,01193	0,01765
Variance	0,04754	0,06902	0,00358	0,00274	0,00396
Standard Deviation	0,21803	0,26271	0,05980	0,05235	0,06292
Sharpe Ratio	0,41074	0,57478	0,28358	0,22783	0,28060
Highest Return	0,38503	0,57872	0,11942	0,11010	0,12340
Lowest Return	-0,43665	-0,49299	-0,13367	-0,11845	-0,15445
Skewness	-0,69076	-0,59225	-0,79314	-0,65625	-0,94996
Kurtosis	0,15557	0,27054	0,97713	0,99469	1,80945

Table A1.4 - Descriptive Statistics - Mid-Pandemic - Daily

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	0,00552	0,00925	0,00083	0,00058	0,00087
Variance	0,00262	0,00485	0,00031	0,00023	0,00031
Standard Deviation	0,05118	0,06962	0,01775	0,01514	0,01770
Sharpe Ratio	0,10779	0,13283	0,04697	0,03840	0,04936
Highest Return	0,21145	0,38654	0,09383	0,07089	0,09490
Lowest Return	-0,39182	-0,44547	-0,11984	-0,10571	-0,11381
Skewness	-0,85757	-0,23879	-0,62803	-0,90442	-0,69692
Kurtosis	9,83690	7,47418	12,07162	7,77236	11,37917

Table A1.5 - Descriptive Statistics – Post-Pandemic - Monthly

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	-0,14049	-0,17362	-0,02451	-0,02519	-0,02849
Variance	0,03188	0,06324	0,00263	0,00250	0,00250
Standard Deviation	0,17855	0,25147	0,05130	0,05000	0,04999
Sharpe Ratio	-0,78682	-0,69041	-0,47781	-0,50386	-0,57002
Highest Return	0,11495	0,11612	0,04269	0,07644	0,03365
Lowest Return	-0,46720	-0,59684	-0,09207	-0,08687	-0,09576
Skewness	-0,37617	-0,31413	-0,07199	1,06809	-0,19096
Kurtosis	0,86622	-0,73171	-1,38378	2,02770	-1,47716

Table A1.6 - Descriptive Statistics – Post-Pandemic - Daily

	BTC	ETH	S&P 500	OMXS30	VTI
Expected Return	-0,00293	-0,00306	-0,00034	-0,00050	-0,00053
Variance	0,00163	0,00234	0,00019	0,00018	0,00020
Standard Deviation	0,04037	0,04841	0,01367	0,01333	0,01422
Sharpe Ratio	-0,07259	-0,06325	-0,02483	-0,03751	-0,03707
Highest Return	0,11478	0,12613	0,04512	0,03936	0,04631
Lowest Return	-0,16424	-0,17222	-0,04040	-0,04037	-0,03913
Skewness	-0,01228	-0,24938	-0,12595	0,01800	-0,08506
Kurtosis	1,76579	0,62773	0,67861	0,83549	0,51940

A.2 2-asset optimal portfolios for all time periods

Table A2.1 - Optimal Portfolios - 2 assets Full Time Period - Monthly

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	0%	0%	0%	0%	0%	0%
Weight CC	100%	100%	100%	100%	100%	100%
Er(p)	0,0820	0,4897	0,0820	0,4897	0,0820	0,4897
St Dev	0,2380	0,4897	0,2380	0,4897	0,2380	0,4897
Sharpe Ratio	0,3445	0,3561	0,3445	0,3561	0,3445	0,3561

Table A2.2 - Optimal Portfolios - 2 assets Full Time Period- Daily

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	0%	0%	0%	0%	0%	0%
Weight CC	100%	100%	100%	100%	100%	100%
Er(p)	0,0040	0,0692	0,0040	0,0692	0,0040	0,0692
St Dev	0,0478	0,0692	0,0478	0,0692	0,0478	0,0692
Sharpe Ratio	0,0827	0,0818	0,0827	0,0818	0,0827	0,0818

Table A2.3 - Optimal Portfolios - 2 assets Pre-Pandemic - Monthly

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	0%	0%	0%	0%	0%	0%
Weight CC	100%	100%	100%	100%	100%	100%
Er(p)	0,0985	0,5646	0,0985	0,5646	0,0985	0,5646
St Dev	0,2407	0,5646	0,2407	0,5646	0,2407	0,5646
Sharpe Ratio	0,4093	0,3753	0,4093	0,3753	0,4093	0,3753

Table A2.4 - Optimal Portfolios - 2 assets Pre-Pandemic - Daily

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	0%	0%	0%	0%	0%	0%
Weight CC	100%	100%	100%	100%	100%	100%
Er(p)	0,0048	0,0712	0,0048	0,0712	0,0048	0,0712
St Dev	0,0486	0,0712	0,0486	0,0712	0,0486	0,0712
Sharpe Ratio	0,0978	0,0938	0,0978	0,0938	0,0978	0,0938

Table A.5 - Optimal Portfolios - 2 assets Mid-Pandemic - Monthly

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	0%	0%	0%	0%	0%	0%
Weight CC	100%	100%	100%	100%	100%	100%
Er(p)	0,1176	0,2967	0,1176	0,2967	0,1176	0,2967
St Dev	0,2286	0,2967	0,2286	0,2967	0,2286	0,2967
Sharpe Ratio	0,5142	0,6741	0,5142	0,6741	0,5142	0,6741

Table A.6 - Optimal Portfolios - 2 assets Mid-Pandemic - Daily

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	0%	0%	0%	0%	0%	0%
Weight CC	100%	100%	100%	100%	100%	100%
Er(p)	0,0055	0,0696	0,0055	0,0696	0,0055	0,0696
St Dev	0,0512	0,0696	0,0512	0,0696	0,0512	0,0696
Sharpe Ratio	0,1078	0,1328	0,1078	0,1328	0,1078	0,1328

Table A.7 - Optimal Portfolios - 2 assets Post-Pandemic - Monthly

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	100%	100%	100%	100%	100%	100%
Weight CC	0%	0%	0%	0%	0%	0%
Er(p)	-0,0231	0,0500	-0,0238	0,0498	-0,0270	0,0484
St Dev	0,0500	0,0500	0,0498	0,0498	0,0484	0,0484
Sharpe Ratio	-0,4615	-0,4615	-0,4777	-0,4777	-0,5580	-0,5580

Table A.8 - Optimal Portfolios - 2 assets Post-Pandemic - Daily

	S&P 500		OMXS30		VTI	
	BTC	**ETH**	**BTC**	**ETH**	**BTC**	**ETH**
Weight IF	100%	100%	100%	100%	100%	100%
Weight CC	0%	0%	0%	0%	0%	0%
Er(p)	-0,0003	0,0137	-0,0005	0,0133	-0,0005	0,0142
St Dev	0,0137	0,0137	0,0133	0,0133	0,0142	0,0142
Sharpe Ratio	-0,0248	-0,0248	-0,0375	-0,0375	-0,0371	-0,0371