House Transactions in the Macroeconomy as Needs Change over the Life Cycle

Abstract: A long-term overlapping generations macroeconomic framework is presented, in which a downsizing old generation bargains with an upsizing middle-aged generation to settle on house prices. In this model, both expectations, which are coordinated by the relative bargaining power of each generation, and credit conditions, which influence the range of prices each side will accept, are important for the resulting house price. High house prices bind more wealth to an asset without dividend, so decreasing the house price is found to generally be welfare-improving in the long run. A decreased price can be accomplished by strengthening the relative bargaining position of the middle-aged buyer generation or by tightening credit conditions, although the latter can have negative side effects.

Keywords: House Prices, Bargaining, Multiple Equilibria

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1 Introduction

People choose to move for many reasons. Perhaps they need a larger or smaller home. Perhaps they need to live closer to some particular location. Or perhaps they just want something different from their old home. But when they do decide to move and start looking for a new home, they determine an upper limit to how much they are willing to pay for that new home, and, if applicable, a lower limit to what they are willing to sell their old home for.

Once matched with another household for a transaction, their respective limits will not generally be equal. If the minimum acceptable price for the seller is higher than the maximum acceptable price for the buyer, they will not be able to reach an agreement, as all prices are too high for one side and/or too low for the other. But if there is room above the minimum that is also below the maximum, then any price in between could satisfy both sides. Thus, once they have found someone they can deal with, then many deals will generally be possible.

These phenomena also apply on the aggregate level. In particular, as a generation grows older, its typical household will have a growing need for a large house as the size of the family grows, and then a falling need as children become independent and move out. Then, as those who downsize from a large house sell to those who upsize, the average price level can and does fluctuate. In fact, the degree to which prices can vary would be even greater than on the individual level, as future financial outlooks, which especially influence maximum acceptable buying prices, are given on an individual level but endogenous on an aggregate level.

The purpose of this paper is to investigate these aggregate price levels. Specifically, the main question is as follows: What are the limits on house prices, and how is the final price determined within those limits? On an aggregate level, “prices” refers to the sequence of prices, which means the relevant limits are both absolute limits on these prices as well as constraints on how these prices can move over time. This is far from the first time the determination of house prices has been investigated in macroeconomic analysis. However, in the interest of including more macroeconomic features or arriving at a unique equilibrium, previous studies on this matter have sacrificed some key feature of the housing market, such that the bargaining room for house prices is reduced to a single point. This paper makes no such compromises, and instead embraces the multiplicity of equilibria and steady states.

The resulting flexibility of house prices raises another important question for this paper: What are the long-run macroeconomic consequences of the realized house price level? In the framework presented here, there are both direct effects on steady-state outcomes from the selected equilibrium, as well as new indirect channels through which classic variables can take effect.

Figure 1 illustrates why house prices are an interesting line of economic inquiry. The house price level has increased massively over time in Sweden, while the general price level barely moved at all, until very recently. The simplest possible economic framework for explaining growing house prices would be to suppose that house supply is constant while the supply of other goods grows. Then, as the economy experiences general growth, houses become relatively scarce, and house prices should grow at the same rate as the rest of the economy. However, consider a recent period of relative economic stability, 2012 to 2022. House prices nearly doubled, which is beyond what the corresponding GDP growth of 58% can explain; disposable income does even worse, only having grown by up to about 30%. A key task of the macroeconomic literature on housing is to create models capable of explaining the remaining price growth in scenarios like this.

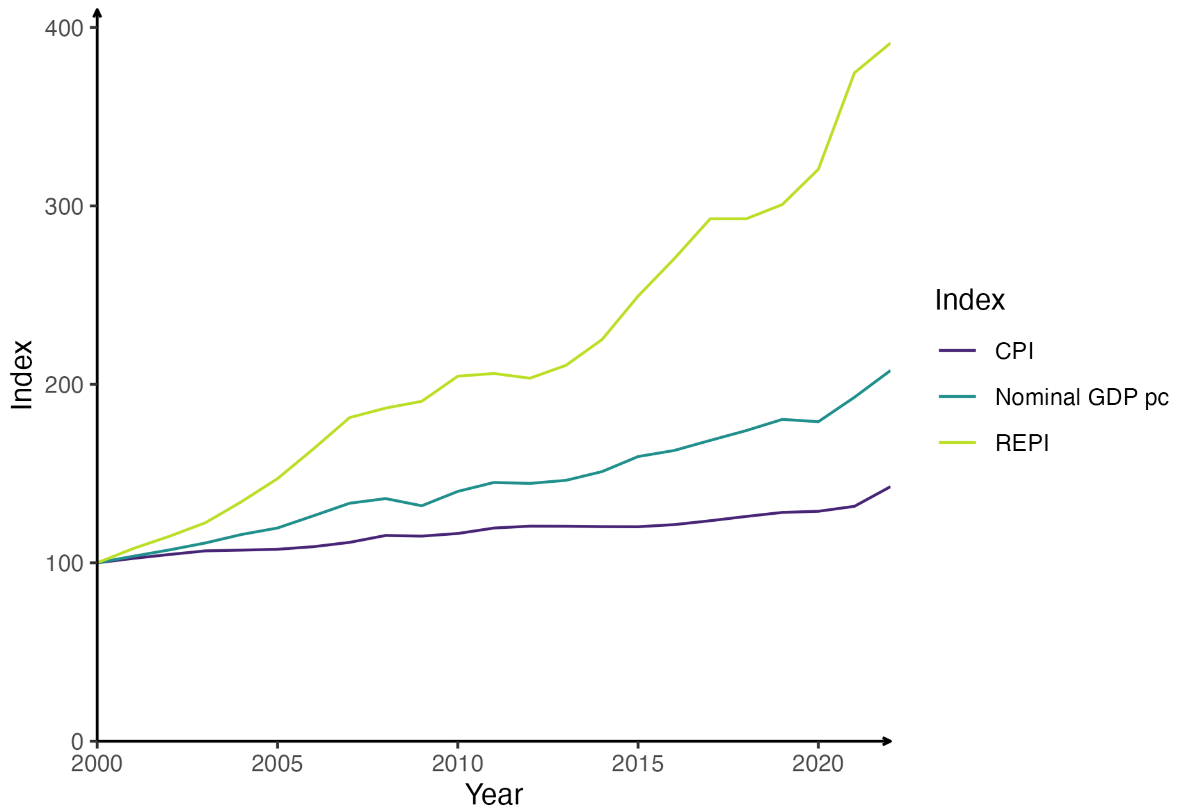

Figure 1: Real estate price index, indexed nominal GDP per capita, and consumer price index, Sweden. Data source: Statistics Sweden

The simple calibration of the model presented here is found to have more than enough room for variation in the steady-state house price to be able to serve as a possible explanation for the remaining price growth. More generally, this overlapping generations framework, with two generations that trade in houses, finds that a high level of house prices has a negative effect on long-run consumption and welfare, while rising house prices could have stimulating short-run effects. Improving the bargaining position of the buying middle-aged generation reduces the long-run house price and increases steady-state welfare, but the opposite holds for improving the selling old generation's position. It is also possible to lower house prices within the model by tightening credit, but this also restricts the set of available consumption-savings alternatives, which may have a negative welfare effect that can dominate the positive effect of lowering the steady-state house price. Thus, both credit conditions and expectations are found to be important for the aggregate house price, with the former affecting price limits and the latter being a channel through which aggregate relative bargaining power takes effect.

This paper makes three main contributions: First, it sets up a framework that captures the repeated house transactions that arise from changing housing needs over the life cycle. Second, within that framework, it derives limits on house prices and proposes a bargaining mechanism for selecting a price level within those limits. Third, it investigates the steady-state effects of changes to the underlying model parameters on the house price level, and incorporates the house price as a channel through which those same parameters affect the remaining outcome variables.

The rest of this paper is structured as follows: Relevant literature is reviewed in the second section. The third section introduces the model and solves for equilibrium conditions, steady state conditions, and proposes a rule for selecting an equilibrium. In the fourth section, the insights offered by the model are presented, including a calibration for the Swedish economy. The fifth section concludes.

2 Literature Review

This paper is connected to three interrelated strands of macroeconomic literature: The study of housing's role in the general economy, in particular its role as collateral for endogenous borrowing constraints. The investigation into the movement of house prices, particularly their booms and busts. And the analysis of rational bubbles, where asset prices can move because of coordinated, rational expectations, not just changes in the assets' underlying dividends. These different branches all investigate the consequences and determination of house prices or asset prices in general.

In their review of the literature on macroeconomics and housing, Piazzesi and Schneider (2016) cover the first two strands. They preface the economics of housing by noting many of housing's unusual properties, including its indivisibility, its illiquidity, and its potential for nonlinear pricing, many of which are very difficult to model simultaneously. House prices are much more flexible than the prices of other assets, particularly because of the limited possibility of shorting it along with its indivisibility property. They note that this price variation is primarily driven by variations in land value, not in structure prices, as construction costs are fairly stable. Furthermore, they show that mortgage debt makes up the majority of household debt, which means the housing market has major implications for aggregate economies.

On the general macroeconomics of housing, Iacoviello (2005) nicely illustrates key empirical regularities of the relationship between housing and the macroeconomy. There is positive feedback between house prices and output, as positive house price shocks increase spending, and positive output shocks bolster house prices. Raised interest or inflation, meanwhile, both prompt a drop in house prices. The model presented in the paper lets houses serve both as factors in production and as providers of housing services to households, and emphasizes their effect as collateral. However, the theoretical approach is ultimately quite distant from the one here, as housing is modeled as infinitely divisible – a continuous good. In another paper, Iacoviello and Pavan (2013) present a model that shares some traits with the framework of this paper, featuring both overlapping generations and indivisible houses; however, they impose an exogenous house price by making housing services proportional to the housing stock. They propose that high risk and low downpayments contribute to stabilizing the economy during normal times, when the economy is hit by occasional small shocks, but that this makes the economy especially vulnerable to large shocks.

Martin and Ventura (2018) provide an overview of the literature on rational bubbles as well as its underlying ideas and approaches. This literature is dedicated to investigating the conditions under which multiple macroeconomic equilibria are possible. Furthermore, it studies how rational expectations are coordinated to realize one specific equilibrium using the concept of *market psychologies*. They explain that the price equaling the value of the underlying assets, a default assumption for models incorporating the stock market, is just one of many such psychologies. Bubbles in productive assets such as stocks can have positive effects on economic activity by encouraging investment in these assets. Housing bubbles, explicitly, as well as other unproductive asset bubbles are mentioned as often having a negative effect on the economy, as they can divert investment from productive assets.

The observation that high asset prices redistribute between generations with potentially welfare-reducing effects is not entirely unique to this paper. Bonchi (2023) presents another model in which this can be the case, within the framework of rational bubbles. However, the channel through which increased asset prices increase redistribution from the young to the old in his model is indirect via an increased natural interest rate, as opposed to the direct effect because of indivisibility in this paper. With this mechanism, bubbles have the potential of letting an economy escape a zero lower bound episode, which increases the welfare of the overall economy, reversing the general effect, and can even boost overall output enough to dominate the welfare loss from redistribution for the young under some conditions.

Basco (2014) develops a rational bubbles model where bubbles can either apply to houses or a pure bubble asset and where these bubbles are driven by globalization. He observes that house prices are inversely related to the current account, tying them to global financial markets. In his model, the economy can be bubbleless or have a bubble in housing or in some other asset. He finds that increased globalization drives rising house prices in financially developed economies if that economy is bubbleless or if it has a house price bubble, but has no effect on house prices if the economy has a non-housing bubble. However, like Iacoviello (2005), this model does not feature the indivisibility aspect of housing.

As for models on housing booms and busts in general, Piazzesi and Schneider (2016) note that there are two main competing explanations: Credit conditions and expectations. However, credit conditions struggle to explain house price volatility and the two most notable booms and busts in recent history – the 1970s period and the 2000s leading up to the financial crisis – on its own, while expectations serve as a much more promising potential mechanism. Kaplan, Mitman, and Violante (2020) provide an expectation-driven model of house price booms and busts with overlapping generations and indivisible houses. Like in other models, house price booms can lead to more general economic booms, and vice versa for busts. Their paper shares many features with rational bubbles setups, but they coordinate expectations by imposing a strict law of motion on house prices;[1] this serves as a market psychology in practice, but their assumption that house prices are proportional to house size also majorly limits house price flexibility. Furthermore, their booms and busts are based on shifts in probability of future episodes of high housing demand, which resembles an approach common in the rational bubbles literature. Even so, they note that while price movements in their model can coherently be interpreted as bubbles, the primary interpretation should be as results of news shocks. A similar situation is the case for the model in this paper, which should primarily be interpreted as agents influencing the price to claim a larger share of the surplus out of self-interest, although it would be possible to reinterpret this framework as a one of rational bubbles by replacing the bargaining mechanism with a bubbly mechanism.

3 Model

The focal transaction of this model resembles one of the fundamental, but rarely discussed, economic games – that of bargaining or surplus sharing. In this game, there is a surplus to be shared between two agents. The agents simultaneously make a numeric claim. If these claims sum to the surplus, or less, both agents receive their claim, if not, no agent receives anything. There is also a bargaining framing, where seller's/buyer's claim is the difference between their minimum/maximum acceptable price and their suggested price. In addition to a no-deal equilibrium that is not particularly interesting, there is a continuum of equilibria from one agent receiving the entire surplus to the other agent receiving the entire surplus. Real bargaining for discrete goods is more complex, but it nevertheless illustrates the point that there can be a continuum of prices associated with rational, Pareto-improving deals.

Such a game would be intriguing from a macroeconomic perspective when repeated. All kinds of patterns of price variation could emerge: Stable prices, which could be interpreted as a steady state. Increasing and decreasing price sequences, which might be seen as inflation and deflation. Or perhaps there will not be a clear pattern, which is much alike a noisy time series. While this paper is focused on comparative statics, this type of setup potentially enables interesting foundations for dynamics or even a channel through which prices can be volatile.

In game theory, the mechanism that selects an equilibrium among many tends to be called a focal point, whereas it would be called a market psychology in the literature of rational

[1] Which they do with their equation 14.

bubbles. Regardless of the name, it coordinates expectations and thus realizes the corresponding equilibrium, as there is then no room for improving one's outcome through unilateral deviation. This paper only considers a deterministic focal point, but there is theoretical room for variation both on the aggregate and individual transaction levels.

There is one market in particular that exhibits this type of repeated surplus sharing on a macroeconomic scale – the housing market. As generations grow older, needs for housing predictably shift, creating room for regular Pareto-improving reallocations of housing, wherein different generations can receive a larger or smaller share of these improvements. The actual process of house transactions is complex, but on a fundamental level, there is a surplus sharing component and outcomes are influenced by cultural or structural expectations.

3.1 Model Setup

In this model, households live for two periods. During their first period, they are middle-aged, indexed by m. The second, old period is similarly indexed by o. Let there be no population growth, so the number of households per generation is constant across generations and across time. The households within a generation are identical, so the number of households has no bearing on per-capita outcomes and generational representative households can be used. With this setup, periods are long-term and should be interpreted as around twenty years, possibly even up to thirty.

These households maximize their lifetime utility, which is given by:

$$U = u_c(c_{m,t}) + u_{m,h}(h_{m,t}) + \beta\left(u_c(c_{o,t+1}) + u_{o,h}(h_{o,t+1})\right)$$

Here, $c_{i,t}$ represents consumption of the non-durable consumption good by generation i at time t. The felicity function for the consumption good is assumed to be the natural logarithm, $u_c(c_{i,t}) = \log(c_{i,t})$. Future utility is discounted by $\beta \in (0,1)$, the patience parameter.

$h_{i,t} \in \{0,1\}$ is an indicator for whether generation i owns a house or not at time t. Note that a value of zero corresponds to an abstract outside alternative. It need not necessarily represent homelessness, but could rather correspond to something like rented apartments or elderly homes. For simplicity, the outside alternative is assumed to be costless, but relaxing that assumption would not change the mechanics of the model, it would only make the mathematics more convoluted. As households have different housing needs depending on their age, the housing utility functions $u_{i,h}$ need to be indexed by age. While all generations prefer owning their house, $u_{i,h}(1) > u_{i,h}(0)$, the central assumption on housing is that this premium is greater for the middle-aged generation, $u_{m,h}(1) - u_{m,h}(0) > u_{o,h}(1) - u_{o,h}(0)$. With logarithmic consumption good utility, the shorthand $\nu_i = \exp\left(u_{i,h}(1) - u_{i,h}(0)\right)$ is useful, in which case $\nu_m > \nu_o > 1$ follows from the main assumptions. Note that ν_i has an interpretation: Owning a house increases utility as much as multiplying consumption in the same period by ν_i does, holding consumption in other periods constant.[2]

The households' maximization problem is subject to three flow budget constraints:

$$y_m = c_{m,t} + p_{h,t}h_{m,t} + b_{m,t}$$

$$y_o + (1+r)b_{m,t} + p_{h,t+1}h_{m,t} = c_{o,t+1} + p_{h,t+1}h_{o,t+1} + b_{o,t+1}$$

$$(1+r)b_{o,t+1} + p_{h,t+2}h_{o,t+1} \geq 0$$

The first two budget constraints correspond to the first and second periods' budgets, while the third signifies that the household must be able to pay off remaining debts at the end of its life. Prices are real and the consumption good is the numeraire. Meanwhile, $p_{h,t}$ is the price of a house at time t.

There is no economic growth, so the income endowment y_i does not depend on time, but it does depend on age. For reasons such as retirement, the endowment is assumed to be larger for the middle-aged generation, $y_m > y_o$.

$b_{i,t}$ is the amount generation i saves at time t for period $t+1$ in one-period bonds. r is the exogenous interest rate, which could be seen as the global interest rate if the economy is interpreted as a small open economy. Debt is represented by negative savings.

Household debt is collateralized and constrained. The main borrowing constraint is assumed to follow from regulation and/or an implicit incentive compatibility problem in the banking sector. This first borrowing constraint is:

$$b_{i,t} \geq -\omega p_{h,t}h_{i,t}$$

where ω is a parameter capturing how loose this constraint is, with it being the maximum fraction of their owned house's value they are allowed to borrow. ω is assumed to be greater than $\beta/(1+\beta)$; note that this minimum is at most 0.5, so many real-life values (like 0.85)

[2] Mathematically put, $\log(c_{i,t}) + u_{i,h}(1) = \log(\nu_i c_{i,t}) + u_{i,h}(0)$ as $\log(\nu_i) = u_{i,h}(1) - u_{i,h}(0)$.

satisfy this condition regardless of how patient or impatient people are. Furthermore, ν_o is assumed to be low enough that $\omega\nu_o < 1$.[3]

Beyond the first borrowing constraint, households are not allowed to borrow more than they would be able to repay by selling the house:

$$b_{i,t} \geq -\frac{p_{h,t+1}h_{i,t}}{1+r}$$

While the first borrowing constraint represents regulation and keeping household incentives in check, the second constraint corresponds to borrowing limits imposed because of the risk of falling prices and the associated inability to pay back. In a general-equilibrium model with risk and default, there would only be the main borrowing-constraint, and the considerations for the second constraint would enter as part the determination of the borrowing limit ω.

Finally, the number of (large, privately owned) houses the economy is endowed with equals the number of households in a single generation. Much like rental costs for the outside option aren't modeled, neither are depreciation and maintenance of houses.[4] This endowment can be seen as following from there being incentives in the implicit construction sector to only build large houses for households that actually need large houses.

3.2 Market Clearing

With the number of houses equaling the number of households in one generation, market clearing in the housing market requires that only one generation owns houses. In terms of model variables, that means $h_{m,t} + h_{o,t} = 1$, the number of owned houses equals the number of houses available for ownership.

Fundamentally, there are many possible sequences of homeownership that fulfill this condition, but only equilibria where the middle-aged generation owns the houses (that is: $h_{m,t} = 1$ and $h_{o,t} = 0$) in every period will be considered here. This is the most realistic sequence of ownership in the long run. The most interesting departures from this sequence would be dynamics following certain types of shocks, especially ones that increase the relative size of the old endowment. However, such shocks are not covered here. To satisfy middle-aged

[3] This assumption is necessary because credit conditions changing for very large loans isn't explicitly modeled here. This point is elaborated on where it becomes relevant (appendix A1).

[4] An equivalent assumption would be that all rental costs are exactly equal to the sum of maintenance and depreciation costs, in which case these costs can just be baked into the wage endowment.

ownership, equilibrium needs to not only satisfy household optimality conditions, but also the related homeownership incentive compatibility constraints where the old prefer selling to keeping in every period, and the middle-aged prefer buying every period.

This incentive compatibility approach resembles subgame perfect equilibria for sequential games, but without detailed modeling of strategies off the equilibrium path, and is therefore suitably approached with backward induction.

3.3 Old Household Decisions

Once a household has become old, it has reached its final opportunity to consume, and thus no longer has any savings motive. Then, they will want to consume as much as they are able, thereby reducing their decision problem to one of whether to own a house or not.

If an old household owns a house at the beginning of the period due to buying one last period, as they will along the equilibrium path, choosing to sell their house will yield the following utility:

$$\log\left(y_o + (1+r)b_{m,t} + p_{h,t+1}\right) + u_{o,h}(0)$$

Meanwhile, if they choose to keep their house, they will want to borrow up to their limit, as any money left after selling the house at the end of their lives that is not spent on repaying their loan is wasted. Thus, they will choose a debt level $b_{o,t+1} = -\omega p_{h,t+1}$,[5] yielding utility of:

$$\log\left(y_o + (1+r)b_{m,t} + \omega p_{h,t+1}\right) + u_{o,h}(1)$$

Incentive compatibility with the desired type of equilibrium requires that the former be greater than the latter. This condition simplifies to:

$$p_{h,t+1} \geq \frac{\left(y_o + (1+r)b_{m,t}\right)(v_o - 1)}{1 - \omega v_o}$$

For a more detailed derivation of this, see appendix A1. Note that a negative right-hand side, which is possible if the households are sufficiently deep into debt, does not imply that the old will accept any price – buyer incentive compatibility will still require a positive house price for the house purchase decision to have made financial sense in the first place.

[5] This imposes a minor condition on the off-equilibrium path: That the main borrowing constraint is tighter there. Even if this is not the case, the conditions here remain sufficient, as a violation would make old homeownership less attractive.

3.4 Middle-aged Household Decisions

Given a sequence of homeownership values $(h_{m,t}, h_{o,t+1})$ over time, the middle-aged problem behaves like a typical macroeconomic household problem. The household will have a standard Euler equation:

$$c_{o,t+1} = (1+r)\beta c_{m,t}$$

This Euler equation will be followed as long as none of the borrowing constraints prevent them from doing so. However, the different sequences of homeownership will have different implications for how much of the budget is left over for the consumption-saving decision, which means actual consumption values will differ across these sequences.

For the middle-aged generation, whether they are borrowing-constrained or not and, if so, by which constraint will depend on both parameter and variable values. First off, which borrowing constraint is tighter determines which constraint to check for. The first borrowing constraint is tighter if the following holds:

$$\omega(1+r)p_{h,t} \leq p_{h,t+1} \tag{1}$$

This is relevant if they choose to buy a house, as in the desired equilibrium. It is, however, not relevant for the comparison case where they do not buy a house, where both borrowing constraints just require non-negative savings, which will be fulfilled if:

$$(1+r)\beta y_m \geq y_o$$

This is assumed to be the case.[6] If the first borrowing constraint is tighter, the buyer will be constrained if the following is true:

$$p_{h,t} \leq \frac{p_{h,t+1}}{(1+r)\big(\omega - \beta(1-\omega)\big)} - \frac{y_m - \dfrac{y_o}{\beta(1+r)}}{\left(\dfrac{1+\beta}{\beta}\omega - 1\right)} \tag{2}$$

Meanwhile, if the second constraint is the tighter one, they will be constrained if:

$$p_{h,t} \geq y_m - \frac{y_o}{\beta(1+r)} + \frac{p_{h,t+1}}{1+r} \tag{3}$$

The derivations for these conditions can be found in appendix B.

[6] In fact, under the common simplifying macroeconomic assumption that $\beta(1+r) = 1$, the condition is identical to one of the assumptions in the model setup. Even without that simplifying assumption, a violation would require an implausibly large discrepancy between the interest and discount rates.

Then, for the comparison case, never buying a house ($h_{m,t} = 0$), the middle-aged household will plan to consume and save as follows:

$$c_{m,t} = \frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}$$

$$c_{o,t+1} = \beta(1+r)\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}$$

$$b_{m,t} = \frac{\beta y_m - y_o(1+r)^{-1}}{(1+\beta)}$$

These directly follow from the generic unconstrained solutions, also found in appendix B, along with the Euler equation for the old-age consumption quantity. The same goes for a buyer household ($h_{m,t} = 1$) if unconstrained:

$$c_{m,t} = \frac{y_m + y_o(1+r)^{-1} + p_{h,t+1}(1+r)^{-1} - p_{h,t}}{(1+\beta)}$$

$$c_{m,t} = \beta(1+r)\frac{y_m + y_o(1+r)^{-1} + p_{h,t+1}(1+r)^{-1} - p_{h,t}}{(1+\beta)}$$

$$b_{m,t} = \frac{\beta y_m - \beta p_{h,t} - y_o(1+r)^{-1} - p_{h,t+1}(1+r)^{-1}}{(1+\beta)}$$

For constrained buyer households, savings follow from the corresponding borrowing constraints being binding with equality, while consumption quantities follow from inserting those savings into the budget constraints. If constrained by the first borrowing constraint:

$$c_{m,t} = y_m - (1-\omega)p_{h,t}$$

$$c_{o,t+1} = y_o - (1+r)\omega p_{h,t} + p_{h,t+1}$$

$$b_{m,t} = -\omega p_{h,t}$$

And finally, the solutions for buyer households bound by the second constraint:

$$c_{m,t} = y_m - p_{h,t} + \frac{p_{h,t+1}}{1+r}$$

$$c_{o,t+1} = y_o$$

$$b_{m,t} = -\frac{p_{h,t+1}}{1+r}$$

As was the case for the old household decisions, incentive compatibility with the focal class of equilibria requires the relevant buyer case to yield higher utility than the comparison non-owner case. The full derivations for these incentive compatibility conditions are found in appendix A2. For price sequences for which the middle-aged households would not be constrained by any of the borrowing constraints, middle-aged households will prefer buying houses if:

$$p_{h,t} \leq p_{h,t+1}(1+r)^{-1} + \frac{\left(v_m^{\frac{1}{1+\beta}} - 1\right)(y_m + y_o(1+r)^{-1})}{v_m^{\frac{1}{1+\beta}}} \tag{4}$$

For sequences where the first constraint is tighter and binds, the buyer incentive compatibility condition is:[7]

$$p_{h,t+1} \geq \frac{(1+r)\beta(y_m + y_o(1+r)^{-1})^{\frac{1+\beta}{\beta}}}{(1+\beta)^{\frac{1+\beta}{\beta}}\left(y_m - (1-\omega)p_{h,t}\right)^{\frac{1}{\beta}} v_m^{\frac{1}{\beta}}} - y_o + (1+r)\omega p_{h,t} \tag{5}$$

And for price sequences where the second constraint is the tighter, binding constraint, incentive compatibility with the middle-aged buying houses is requires:

$$p_{h,t} \leq y_m + \frac{p_{h,t+1}}{1+r} - \frac{(1+r)^{\beta}\beta^{\beta}(y_m + y_o(1+r)^{-1})^{1+\beta}}{(1+\beta)^{1+\beta} v_m y_o^{\beta}} \tag{6}$$

3.5 Equilibrium

With the restriction on the sequence of homeownership, equilibrium reduces to an infinite sequence of the following variables: House prices, $p_{h,t}$. Consumption quantities for each of the generations, $c_{m,t}$ and $c_{o,t}$. Savings for the middle-aged generation, $b_{m,t}$. An equilibrium is then defined as an infinite sequence of these variables such that the middle-aged generation always prefers buying houses, the old generation always prefers selling their houses, and the optimal consumption and savings rules are followed.

The remaining variables must be the same in every period in this type of equilibrium. The old will never save, as they are in the last period of their lives, $b_{o,t} = 0\ \forall t$. The middle-aged always own houses, and the old never do, by design, $h_{m,t} = 1, h_{o,t} = 0\ \forall t$. The equilibrium values for consumption and savings are reported in the previous subsection, while house prices are the subjects of equilibrium selection.

Buyer incentive compatibility constraints were presented in the previous subsection, but to complete the notion of equilibrium, the seller constraint in section 3.3 has to be updated

[7] This one is expressed in terms of $p_{h,t+1}$, the minimum future price, as it has a closed-form solution while $p_{h,t}$, the maximum current price, does not.

with equilibrium savings values.[8] The savings choice in the last period depends on the price sequence, and they may or may not have been borrowing-constrained at the time. If they were not constrained, prices have to satisfy the following:

$$p_{h,t+1} \geq \frac{(\nu_o - 1)\left((1+r)y_m - (1+r)p_{h,t} + y_o\right)}{1 - \nu_o \frac{\omega + \omega\beta - 1}{\beta}} \tag{7}$$

Meanwhile, if they were constrained by the first borrowing constraint:

$$p_{h,t+1} \geq \frac{(\nu_o - 1)\left(y_o - (1+r)\omega p_{h,t}\right)}{1 - \nu_o \omega} \tag{8}$$

The intuition for why these two depend negatively on the past price is that the higher the past price was, the poorer the old house-sellers will be, thus having higher marginal utility of consumption and accepting smaller increases to that consumption. Finally, if the price sequence constrained them by the second borrowing constraint, the price has to fulfill:

$$p_{h,t+1} \geq \frac{y_o(\nu_o - 1)}{\nu_o(1 - \omega)} \tag{9}$$

The derivations of these constraints are in appendix A3.

With mathematical expressions for all the incentive compatibility conditions out of the way, these conditions can be illustrated graphically. Figure 2 illustrates a basic case. The orange line graphs equation 4, the condition for unconstrained buyers preferring to buy a house, which is satisfied by any point below or to the right of the line. Similarly, the green curve shows equation 5, the same condition under the first borrowing constraint, again satisfied toward the bottom-right. The dashed, dark purple line illustrates equation 2, the threshold for being unconstrained by the first borrowing constraint, to the left, or constrained by it, to the right. These thresholds also govern the transitions between different constraints within the buyer and seller roles; the relevant constraint for each region of the figure is outlined in black. The dashed red line shows equation 3, the threshold for the second borrowing constraint (with constrained points being above the line), but the unconstrained incentive constraint is already tight enough in this example for the second borrowing constraint to never come into play. The yellow, turquoise, and blue lines display seller-side incentive compatibility constraints when unconstrained (equation 7), constrained by the first borrowing constraint (equation 8) as well

[8] Even so, the version with last period's middle-aged savings still will hold, and would be relevant for the first period as well as right after unexpected shocks.

as the second (equation 9), respectively, which are satisfied to the right of the corresponding line. Relevance among these is determined by the same thresholds, and they are outlined in black where relevant, like the buyer constraints. In short, any point to the right of both black-outlined curves satisfies all incentive compatibility constraints.

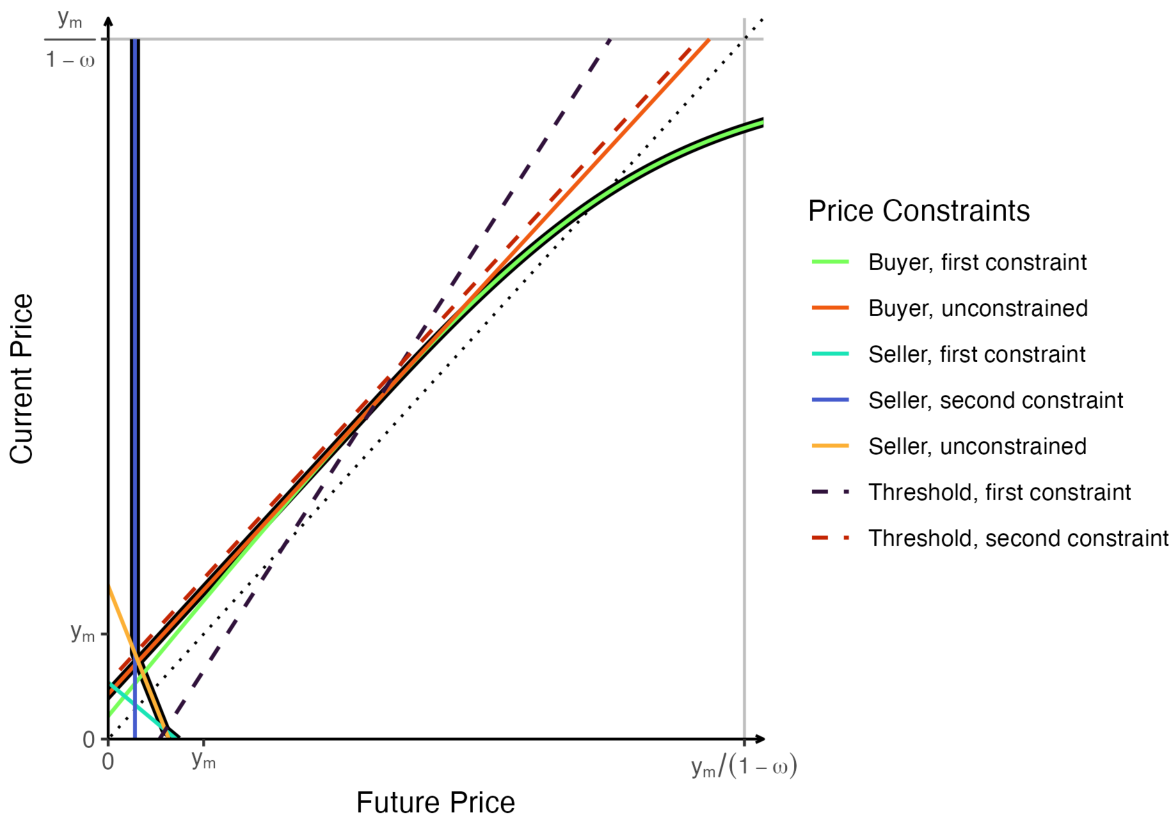

Figure 2: Intertemporal house price conditions, second borrowing constraint never binding

Furthermore, note that the dotted line shows where prices are equal across the two periods (which is also the steady state condition; see figure 6 in the next subsection for an illustration of the steady states and the effective incentive compatibility limits) – to its left, the house price is falling, to its right, the price is rising. The gray lines represent the absolute upper bound on possible prices, as a house price of $y_m/(1-\omega)$ or higher implies that the minimum downpayment (from the first borrowing constraint) becomes unaffordable. Finally, note that these figures only illustrate the incentive compatibility constraints for one generation – that generation also deals with both the previous generation when buying their houses and the next generation when selling their houses. In extension, all generations in the equilibrium sequence are connected, and these conditions apply between any two adjacent periods of time.

Figure 3 covers a more advanced case. Here, the second constraint is tight enough to start applying before buyers run into their unconstrained limit on acceptable price sequences. The light blue line corresponds to equation 6,[9] with points to the right or below fulfilling the condition that the middle-aged households prefer to buy when constrained by the second borrowing constraint. When the second borrowing constraint is tight enough that it can bind, the dashed lime-green line, graphing equation 1, shows which of the borrowing constraints is tighter in the region where both constraints are tighter than what an unconstrained agent would want to borrow. The main difference for the set of valid equilibrium price sequences compared to the case in figure 2 is that, when the second borrowing constraint is tight, there is a kink in the buyer-side maximum price when it transitions to the first borrowing constraint.

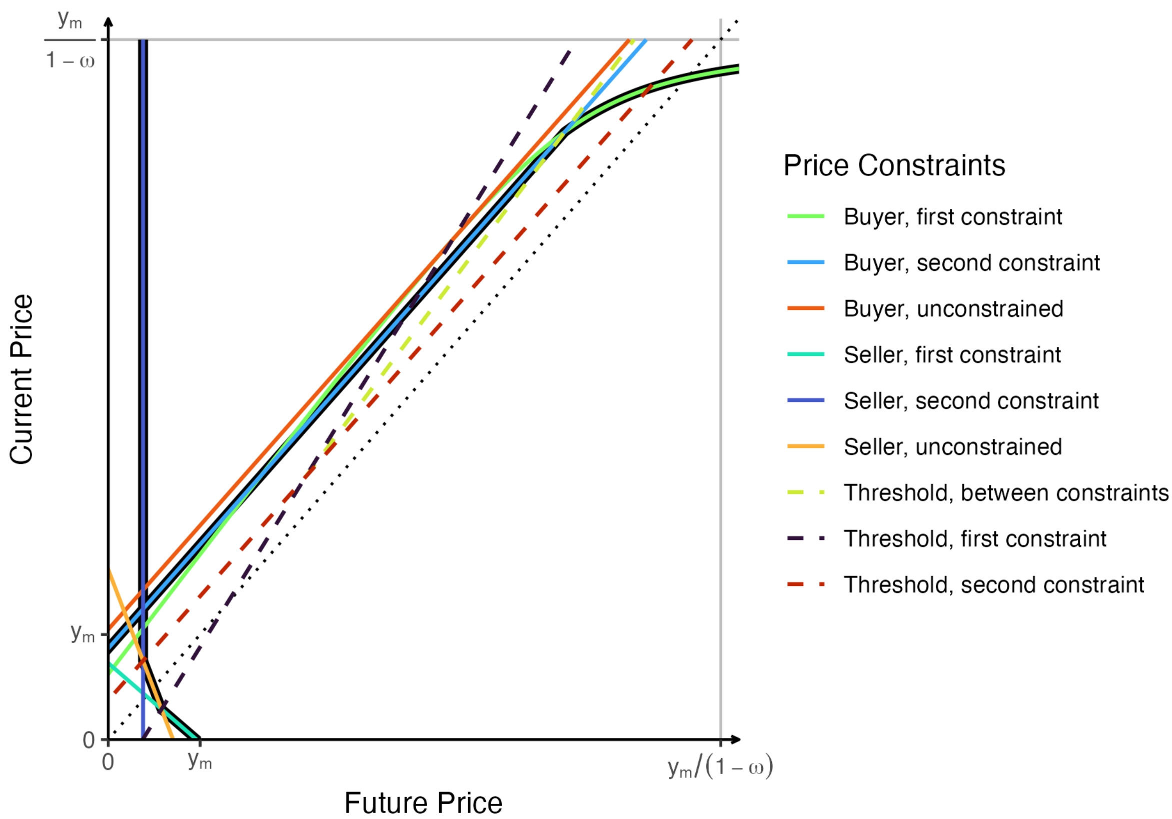

Figure 3: Intertemporal house price conditions, second borrowing constraint relevant

[9] Note that the unconstrained maximum price, the maximum price under the second borrowing constraint, and the threshold for the second constraint will always be parallel, so there are no transitions between these two maximum price lines. Which of the two is relevant for decreasing price sequences depends only on parameters.

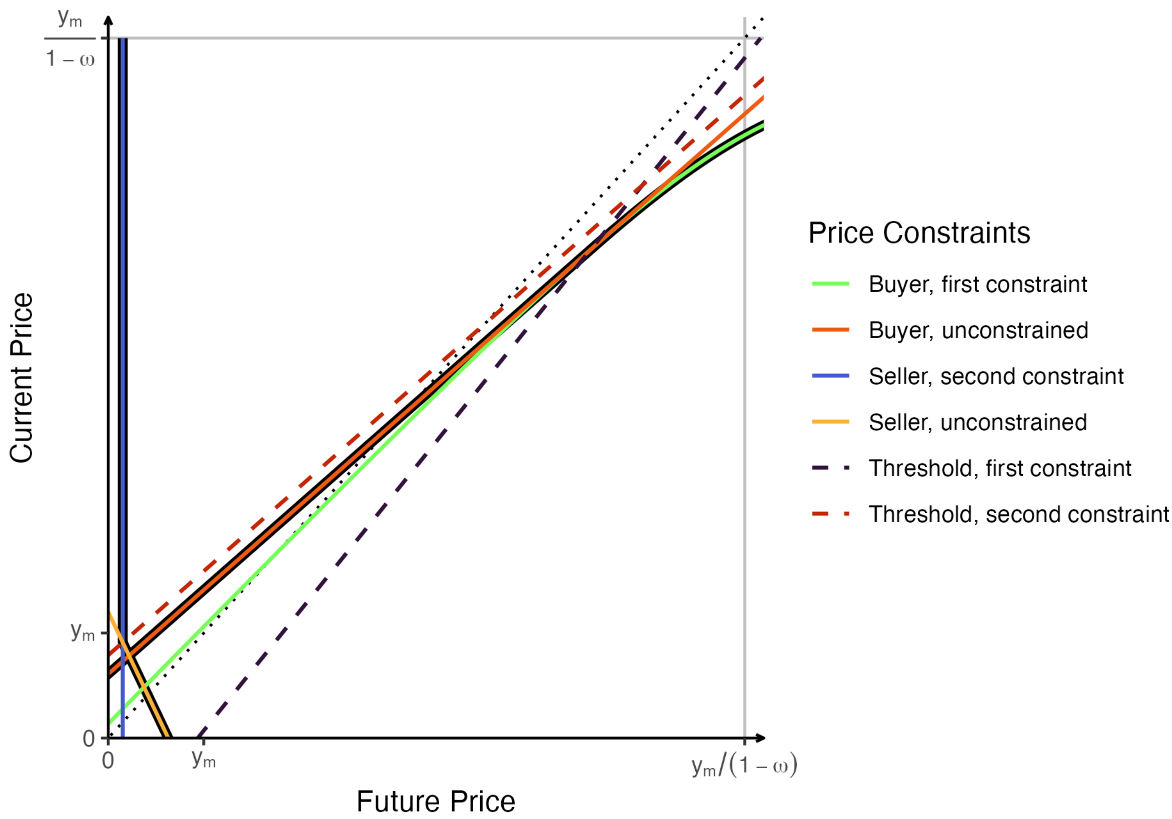

Figure 4: Intertemporal house price conditions, unconstrained steady state maximum

Figures 4 and 5 illustrate two additional scenarios where the first borrowing constraint is less likely to bind, but otherwise correspond to the cases in figures 2 and 3, respectively. Their main differences, compared to the last two, is that the steady state maximum price is not determined by the buyers' maximum price under the first borrowing constraint, but rather the unconstrained incentive compatibility condition and the one for the second borrowing constraint, respectively.

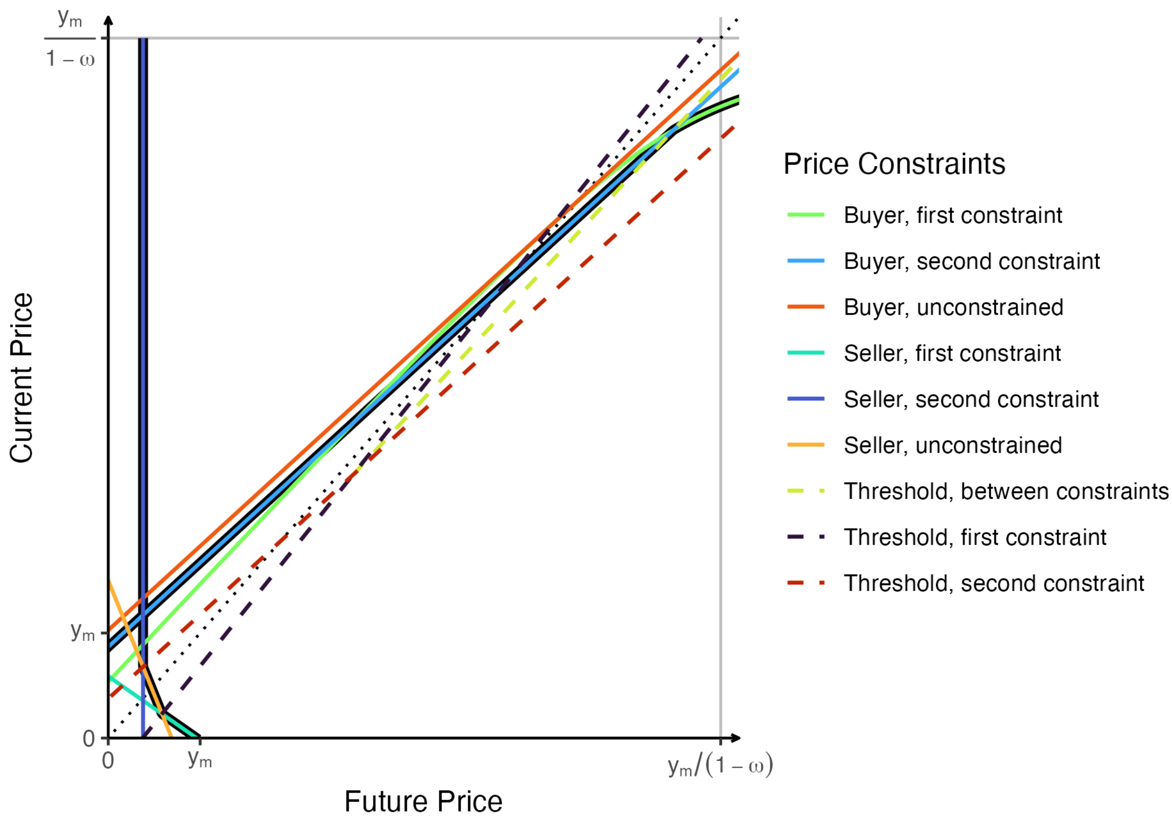

Figure 5: Intertemporal house price conditions, second borrowing constraint in steady state

3.6 Steady States

For the economy to be in steady state, the equilibrium variables (house price, consumption for both generations, middle-aged savings) must repeat the same values each period, thus dropping the time index. The most important of these conditions is that the house price is constant over time, as the other variables can be derived from the house price level, once selected, using the equations in section 3.4. The steady-state range of house values can be seen in the figures above (specifically, figures 2 through 5) as the points on the dotted line (which illustrates the steady-state condition) between the sellers' minimum price curve and the buyers' maximum price curve, both of which are outlined in black. See figure 6 below for a demonstration, which shows the range of steady states in yellow and the effective incentive compatibility constraints for the same case as in figure 2 above. This range can also be calculated without any graphs, using the equations and process outlined below.

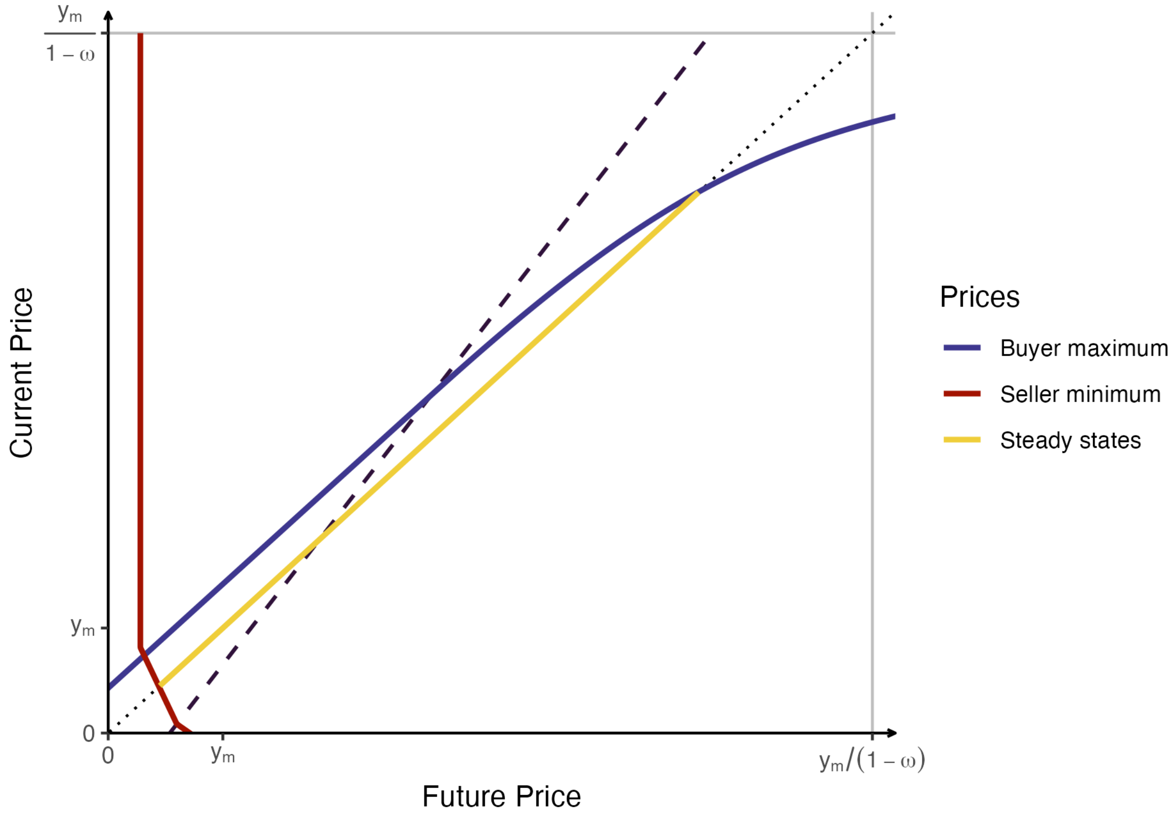

Figure 6: Steady states and effective price constraints

In steady state, the representative household may or may not be borrowing-constrained while middle-aged. Note that only one borrowing constraint will need to be checked in steady state, as the threshold between the two constraints, equation 1, simplifies to a condition that only depends on parameters:

$$\omega(1+r) \leq 1 \tag{10}$$

which, if true, implies that the first borrowing constraint is tighter. This will likely be the case under real parameter values, and if it is, the price threshold for the middle-aged households being borrowing-constrained simplifies from equation 2 to:

$$p_h \geq \frac{(1+r)\beta y_m - y_o}{1-(1+r)\big(\omega - \beta(1-\omega)\big)} \tag{11}$$

Meanwhile, if that is not the case, and the second borrowing constraint is tighter instead, the condition for them being borrowing-constrained goes from equation 3 to:

$$p_h \geq \left(y_m - \frac{y_o}{\beta(1+r)}\right)\frac{1+r}{r} \tag{12}$$

The maximum price captures the fact that incentives must be such that buying a house is the optimal decision for middle-aged households. The steady-state form of the unconstrained household maximum price – $p_{h;max,u}$, which follows from equation 4 – is:

$$p_{h;max,u} = \frac{\left(v_m^{\frac{1}{1+\beta}} - 1\right)(y_m(1+r) + y_o)}{v_m^{\frac{1}{1+\beta}} r} \qquad [13]$$

If the maximum price is constrained by the first borrowing constraint, it is the solution to

$$p_{h;max,f} = \frac{(1+r)\beta(y_m + y_o(1+r)^{-1})^{\frac{1+\beta}{\beta}}}{(1+\beta)^{\frac{1+\beta}{\beta}}\left(y_m - (1-\omega)p_{h;max,f}\right)^{\frac{1}{\beta}} v_m^{\frac{1}{\beta}}} - y_o + (1+r)\omega p_{h;max,f} \qquad [14]$$

such that[10]

$$p_{h;max,f} \in \left[\frac{(1+r)\beta y_m - y_o}{1 - (1+r)\left(\omega - \beta(1-\omega)\right)}, \frac{y_m}{1-\omega}\right)$$

if that solution exists. Equation 14 here adapts equation 5 to steady state. This solution price exists and is unique if it is needed as the maximum price, which is proven in appendix C. It does, however, lack a closed-form solution, but is at least easy to perform a numeric search for. If households are constrained by the second borrowing constraint instead, the maximum steady-state price $p_{h;max,s}$ follows from equation 6, leading to:

$$p_{h;max,s} = \frac{1+r}{r} y_m - \frac{\beta^\beta\left((1+r)y_m + y_o\right)^{1+\beta}}{r(1+\beta)^{1+\beta} v_m y_o^\beta} \qquad [15]$$

Meanwhile, the minimum price reflects how it must be optimal for the household to sell the house again in the next period for the desired pattern of ownership to be maintained. The unconstrained-household minimum price, $p_{h;min,u}$, is given by:

$$p_{h;min,u} = \frac{(v_o - 1)\left((1+r)y_m + y_o\right)}{1 - v_o \frac{\omega + \omega\beta - 1}{\beta} + (v_o - 1)(1+r)} \qquad [16]$$

which follows from equation 7. Under the first borrowing constraint, the minimum price $p_{h;min,f}$ follows from equation 8, being:

$$p_{h;min,f} = \frac{(v_o - 1)y_o}{1 - v_o\omega + (v_o - 1)(1+r)\omega} \qquad [17]$$

[10] This range captures prices for which the first constraint would be (weakly) binding under optimal behavior (the minimum, which restates the right-hand side of equation 11), such that houses are still affordable (the supremum).

And finally, the minimum price for the second borrowing constraint, $p_{h;min,s}$, is just a change of variable of equation 9:

$$p_{h;min,s} = \frac{y_o(\nu_o - 1)}{\nu_o(1-\omega)} \quad [18]$$

The maximum and minimum prices in steady state, $p_{h;max}$ and $p_{h;min}$, can then be determined using a fairly simple algorithm: First, check which of the two borrowing constraints is tighter using equation 10. Second, calculate the unconstrained price limit (equation 13 for the buyer side and equation 16 for the seller side). Then, check if the relevant constraint (equation 11 if equation 10 held, equation 12 if not) would be binding at this price. If it would not be binding, then the unconstrained price limit is the maximum or minimum price. If it would be binding, the price limit under the relevant constraint (equations 14, 15, 17, and 18) is the maximum or minimum price. This process can be used to verify that the maximum price is higher than the minimum one, which enables the type of middle-aged ownership equilibria that are of interest here.[11]

3.7 Steady State Equilibrium Selection

As this paper is focused on comparative statics, the focal point or market psychology only needs to be defined for the long run – that is, for the steady state.[12] A natural approach to selecting the equilibrium in this setting is to define and compare the bargaining power of the sellers (the old generation) and the buyers (the middle-aged generation). The more relative bargaining power one side has, the more of the surplus – the difference between the sellers' minimum

[11] Unfortunately, the assumption $\nu_m > \nu_o$ turns out to not quite be a sufficient condition to guarantee that this is the case. However, I have only been able to generate scenarios where they are not possible – where the minimum price exceeds the maximum price, thus not enabling any deals – with extreme parameter values, such as the combination of very close ν-values and a very high interest rate. Furthermore, there would presumably not be incentives in the implicit construction sector to put the housing market in a position where the stock of housing is not sufficiently attractive to the generation with the highest income, so no-deal parameters should not occur in the first place.

[12] However, note that not all equilibria need to converge to a steady state, neither within the restricted notion of equilibrium used here nor beyond it. A fairly simple example of a valid equilibrium sequence of house prices that never converges to a steady state would be one that swaps back and forth between two values; such equilibria can actually also be seen in figures 2 through 5: If the axes are reinterpreted to odd and even periods, they are (roughly) the points to the right of the black-outlined curves but to the left of the dotted, steady-state line.

acceptable price and the buyers' maximum acceptable price – they can claim. Let the non-negative number θ_i represent bargaining power of generation i, then the steady-state price can easily be pinned down with a weighted average, as in the following equation:

$$p_h = \frac{\theta_m}{\theta_m + \theta_o} p_{h;min} + \frac{\theta_o}{\theta_m + \theta_o} p_{h;max}$$

Defining a dynamic focal point is left for future research.[13] Given how houses are usually sold through auctions, sellers would most likely have most of the relative bargaining power, meaning that the steady-state price should be close to the maximum price the middle-aged will accept, $p_{h;max}$.

4 Analysis

In the introduction, the two questions this paper aims to answer were presented. The first of these concerns the limits on house prices, which were derived on a theoretical level in the previous section, and will be briefly discussed on a practical level with the calibrated model. The second question is about the macroeconomic consequences of economic shifts and policy changes, which are discussed in detail below.

The first result of note is that the simplest explanation for rising house prices – that when the rest of the economy grows, but the supply of houses remains constant, house prices should grow at the same rate in the long run – holds in this framework, as expected. The formal condition for this is that the steady-state maximum and minimum prices and borrowing constraint thresholds are homogeneous of degree one as functions of the endowments. For the minimum prices (equations 16, 17, and 18), the thresholds (equations 11 and 12), and the unconstrained and second borrowing-constrained maximum prices (equations 13 and 15, respectively), this is trivially the case, as all of their terms are proportional to linear[14] sums of the endowments. It is also the case for the borrowing-constrained maximum price for the first

[13] A candidate starting point could be to take inspiration from autoregressive processes, with an equation that looks something like:

$$p_{h,t} = (1-\rho)p_{h,t-1} + \rho\left(\frac{\theta_m}{\theta_m + \theta_o} p_{h,t;min} + \frac{\theta_o}{\theta_m + \theta_o} p_{h,t;max}\right)$$

Where $p_{h,t;min}$ is connected to $p_{h,t-1}$ and $p_{h,t;max}$ is connected to $p_{h,t+1}$, which might be possible to solve recursively.

[14] In the strict sense. That is, not affine.

borrowing constraint, shown in appendix D. As all the price conditions are scaled up by the same factor, the realized steady-state house price, as a weighted average of the relevant price limits, also scales by that factor. The same is true for the consumption quantities in extension, which can be seen by repeating the proportionality argument for those equations.

A more novel result is that an increase in the selected steady-state house price has a similar effect to a transfer from the middle-aged to the old; specifically, an increased house price decreases total long-run consumption in the economy as well as long-run welfare, defined as steady-state lifetime utility. This is because of the nature of repeated house transactions in the model, which effectively sets up such a transfer every period. There are two intuitive reasons for these negative consequences: First off, high house prices and transfers have the effect of delaying availability of funds in steady state, which means households are worse off because those funds get locked behind discounting or interest. Second, a high house price or high transfer has the effect of reducing savings or increasing debt, which means the economy loses out on some interest payments or has to pay more interest itself.[15] This is shown in detail in appendix E. There could also be additional negative effects due to the raised debt level leading to increased risk, but that risk is not modeled in this framework. The requisite increase in house prices can be caused by increasing the raw bargaining power of the old sellers, increasing the size of their relative endowment,[16] or improving their outside option; conversely, implementing changes such that buyers have a more favorable bargaining position would lower house prices and have positive steady-state welfare effects.

The model has two explicit parameters that relate to credit – the interest rate and the tightness of the borrowing constraint. The former has very complex effects, so its effects are simulated in the calibrated model. For the latter, tightening the borrowing constraint will lower the steady-state house price. It will also improve long-run welfare if middle-aged households will be unconstrained after the change, but if they will be constrained, the welfare effect is ambiguous. The proofs for these effects are in appendix F. These effects mark an important difference between this model and many other macroeconomic models with housing – even if

[15] Recall that an exogenous interest rate was assumed. The most natural interpretation, then, is that the economy is a small open economy and interest is paid to foreigners. If the financial market is at least partially domestic, this second intuition holds to a lesser degree, but there may still be consequences from the ensuing redistribution.

[16] This aspect further implies that the effect of redistribution to the old is amplified by house prices.

the households are not borrowing-constrained, changing credit conditions can change economic outcomes, as that change impacts the bargaining positions of the market agents.

While the model here does not provide a full dynamic specification, some preliminary dynamic results can be discussed. Specifically, suppose one of the changes in the previous paragraphs were to occur, as a form of permanent shock, and the economy immediately jumps to the new steady state. Then, most effects reverse for the current old generation. For instance, if an old generation suddenly receives more bargaining power, then that generation will be able to enjoy greater consumption[17] and utility at the expense of the long run.

Another worthwhile note on dynamics is the great degree of asymmetry in the figures (see figures 2 through 5, for instance). Buyer incentive compatibility prevents rapidly falling price sequences, while there are almost no limits on rising sequences. In fact, immediately going all the way from the lowest possible price to the highest possible steady-state price and then staying there indefinitely is a perfectly valid equilibrium price sequence. This would likely only hold to a limited degree in a model with risk, however, as then households will have to take the possibility of a house price crash into account.

4.1 Calibration

This is not a quantitative model, chiefly because it only considers two generations, thus lacking key features like saving up for a house purchase, as well as because it does not specify dynamics. Beyond that, it also lacks other important features like general equilibrium effects, risk, and heterogeneity. Nonetheless, a basic calibration yields a handful of insights.

Two calibrations for Sweden are presented here. The first is a naïve calibration using annual values for observables. This underestimates the funds available to the households, as it does not let them save up for a big house purchase. It also uses parameters like the interest rate for a much shorter term than what is appropriate for housing decisions, and matches generations for house transactions that are too close. The second strategy is to use long-term values. This matches generations more appropriately and uses more accurate parameters, however, it also overestimates the funds available to households at the start of the period,

[17] Furthermore, as they have a higher marginal propensity to consume than the middle-aged, total consumption would also increase, further resembling a form of stimulus.

which actually become available over time. Extending the model to become more appropriate for quantitative analysis would be preferable, but is beyond the scope of this paper.

Parameter values are calibrated using three different methods. Values that are observable overall, like relative endowments and interest rates, are taken from official sources. The discount factor, which is not directly observable, is matched with estimates used in the literature. Remaining parameters are specified on grounds that are ultimately arbitrary.

Statistics Sweden reports that mean annual disposable income grew from 504.1, 513, and 352.6 (in thousand SEK) to 611.9, 666.5, and 463.9 for households in the 30-49, 50-64, and 65-79 age ranges, respectively, between 2011 and 2021. That is, disposable incomes grew by about 21% for younger households, 30% for older households, and 31% for retired households. The ratio between the disposable income of retired households and the older households was stable at around 0.7 over the time period, while it grew to about 0.75 by the end of the period when comparing with the younger households instead. The endowments are therefore calibrated to 100 and 70 for the middle-aged and old households, respectively.

From the end of 2014 to mid-2022, the flexible mortgage lending rate mostly stayed in the 0.015-0.02 range, according to Statistics Sweden's numbers. However, between 2009 and 2014, it was less stable, peaking at about 0.04 in early 2012. The model interest rate is calibrated to 0.02, the high end of the stable range, as a compromise. For the long-run calibration, the corresponding 20-year interest rate is about 0.49.[18]

Since 2010, the ceiling on housing-securitized loans (that is, ω) has been set to 85% of the house's value in Sweden by Finansinspektionen. In 2016 and 2018, they introduced further regulation on amortization for households with large such loans (see FFFS 2016:16 for the current rules). Somewhat simplified, a household whose loan-to-value ratio is between 70% and 85% has to at least amortize 2% of the loan's value per year, while one whose ratio is between 50% and 70% has to amortize at least 1% per year. For a long-term period of 20 years, this implies that a household that borrowed up to the limit will have to amortize down to about 70% over 10 years, and then down to about 63% over the following 10 years. Therefore, ω is calibrated to 0.85 for the naïve calibration and 0.63 for the long-term calibration.

[18] Note that this interest rate implies that the long-run endowment, which is the discounted sum of the annual endowments over the period, is about 16.7 times as high as the short-run endowment.

As for discounting, De Lipsis (2021) uses time series data to estimate the discount rates by income quintile in various European countries. For Sweden, the average estimated discount rate is 0.0224, which implies a discount factor of about 0.978, which is used for the calibration here. For this value, the product $\beta(1+r)$ is close to one, which is a widely used simplifying assumption. This value is also close to the value of 0.964 used in Kaplan, Mitman, and Violante (2020). Furthermore, it is between the values used in Iacoviello and Pavan (2013), which are 0.999 for patient households and 0.941 for impatient ones. Both of these studies calibrated their values to match another economy, however. The long-run discount factor corresponding to 0.978 is about 0.64.

There are unfortunately no easy ways of estimating the ν-values, as they are novel, and their direct effects are on boundaries that cannot be directly observed. ν_o is chosen to equal 1.1, which is roughly in the middle of the smallest and greatest possible numbers consistent with the basic assumptions of the model, for the given value of ω. ν_m is calibrated to 2, implying that middle-aged households would be willing to swap to a home that is too small if offered doubled consumption in this period in return, while keeping future consumption constant. I consider this estimate to be on the low end of plausible values, but larger values expand the range of possible steady-state prices, so a low estimate is prudent.

The resulting intertemporal constraints on the prices from these specifications are illustrated in figure 7 for the short-term calibration, and in figure 8 for the long-term one. Note that the normalization of the middle-aged endowment to 100 specifically lets the axis values in the figures below be interpreted as percentages of the middle-aged endowment. See the data appendix for a summary of the calibration values and detailed data sources.

In both cases, it turns out that the first borrowing constraint is tighter in steady state, but the second borrowing constraint is still tight enough to determine maximum prices for some descending price sequences. For seller-side minimum price incentive compatibility, neither borrowing constraint is binding in steady state.

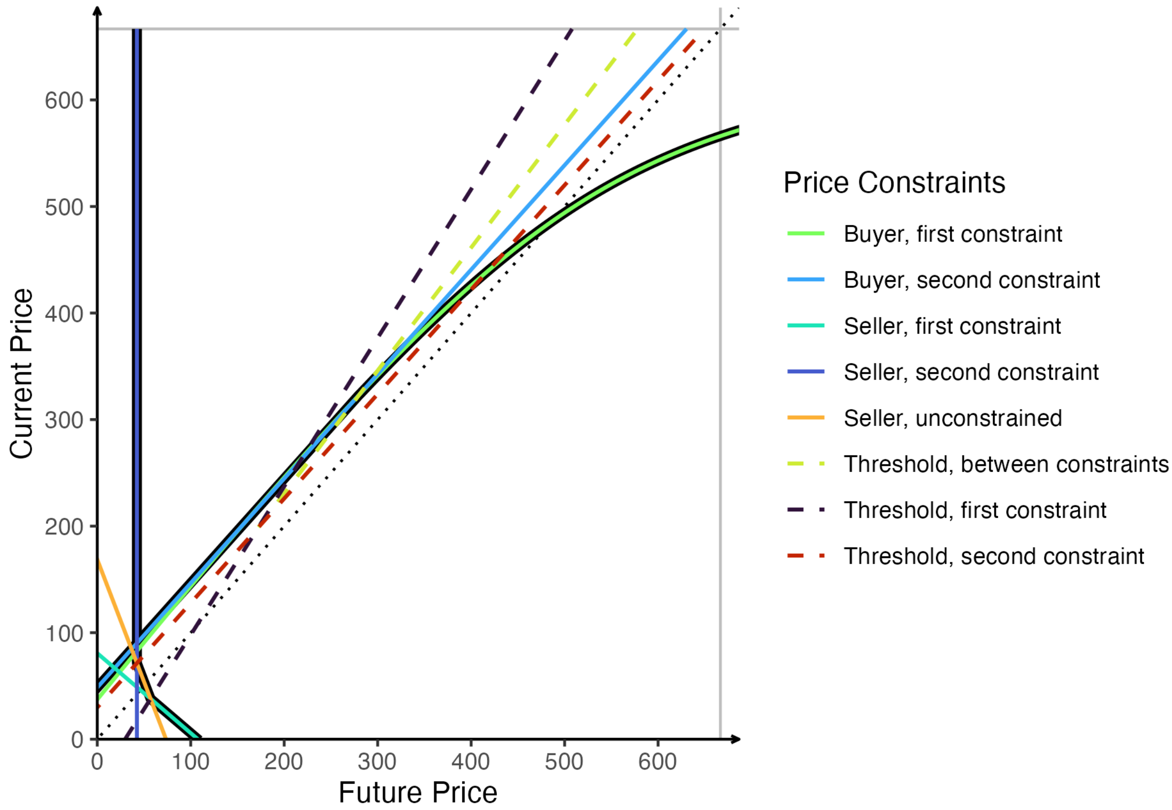

Figure 7: Intertemporal house price conditions in Sweden, naïve calibration

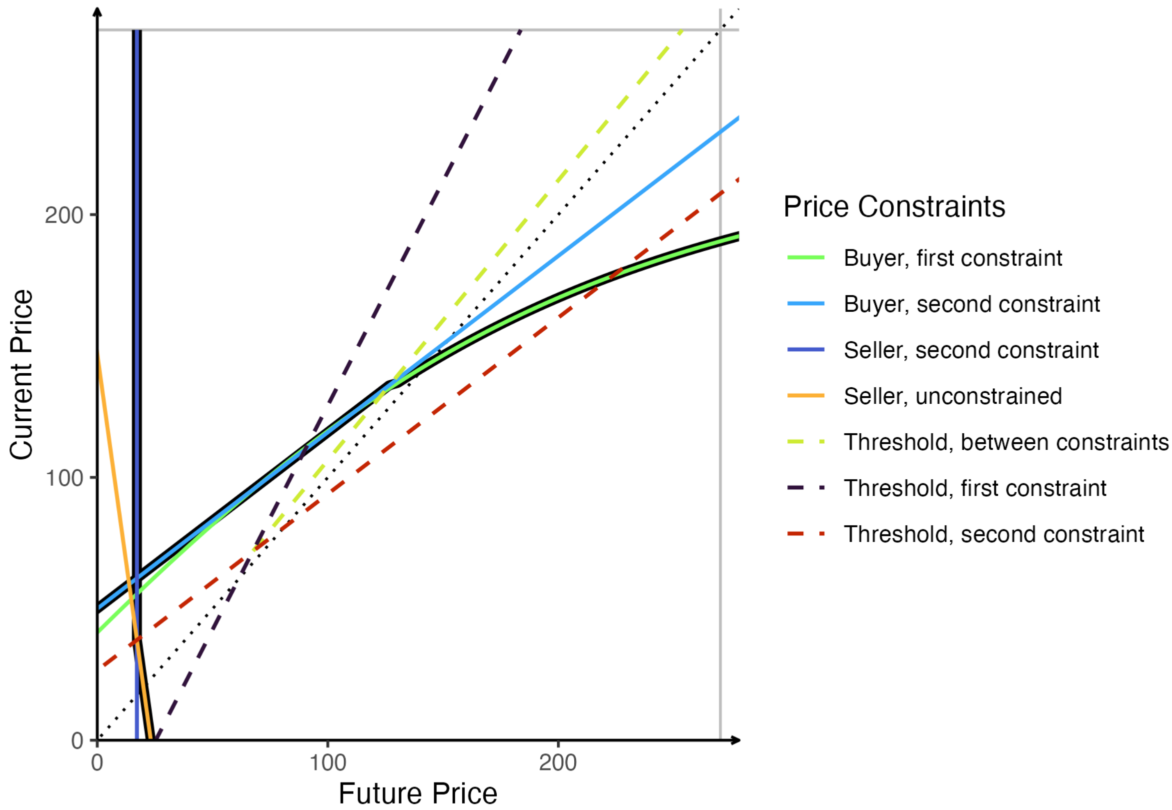

Figure 8: Intertemporal house price conditions in Sweden, long-run calibration

Using the process outlined in section 3.6, the minimum steady-state price in the naïve specification is about 51.2% of the middle-aged endowment, while the maximum price is approximately 486.5% of that endowment. The ratio between the maximum and minimum possible steady-state prices, then, is about 9.5 – a massive range. The long-run specification is somewhat more subdued, with a range from 20% to 143.3% of the endowment for the middle-aged, a ratio of about 7.2.

To account for the near-doubling of house prices during the period discussed in the introduction, income growth may be able to explain an increase by up to 30%, as shown in the beginning of this section. On top of that, another increase by 50% needs to be explained. These ranges are more than capable of providing such an increase through a movement from one steady-state price to another, which shows that this theoretical framework has the potential to explain large movements in house prices. A plausible narrative would be that the bargaining position of sellers recovered during the stable 2012-2022 period, after deteriorating under the preceding period of economic instability. With that said, however, it is too early to say how good that explanation is – that would require a thorough empirical investigation of a more fully featured dynamic model.

As the effects of modifying the interest rate are too complicated to be fruitfully discussed generally, simulations of raising the relevant interest rate by two percentage points to 0.04 are illustrated below. Figure 9 shows the new intertemporal price conditions for the naïve calibration. For the long-run calibration, the corresponding new interest rate is 1.19,[19] and the price conditions are graphed in figure 10. Do note, however, that this simulates an increase in the real interest rate, not the nominal one; without further model extensions, which might alter these results, there is no channel through which policy can affect the real interest rate.

[19] Before adding the one. That is, the interest factor is 2.19.

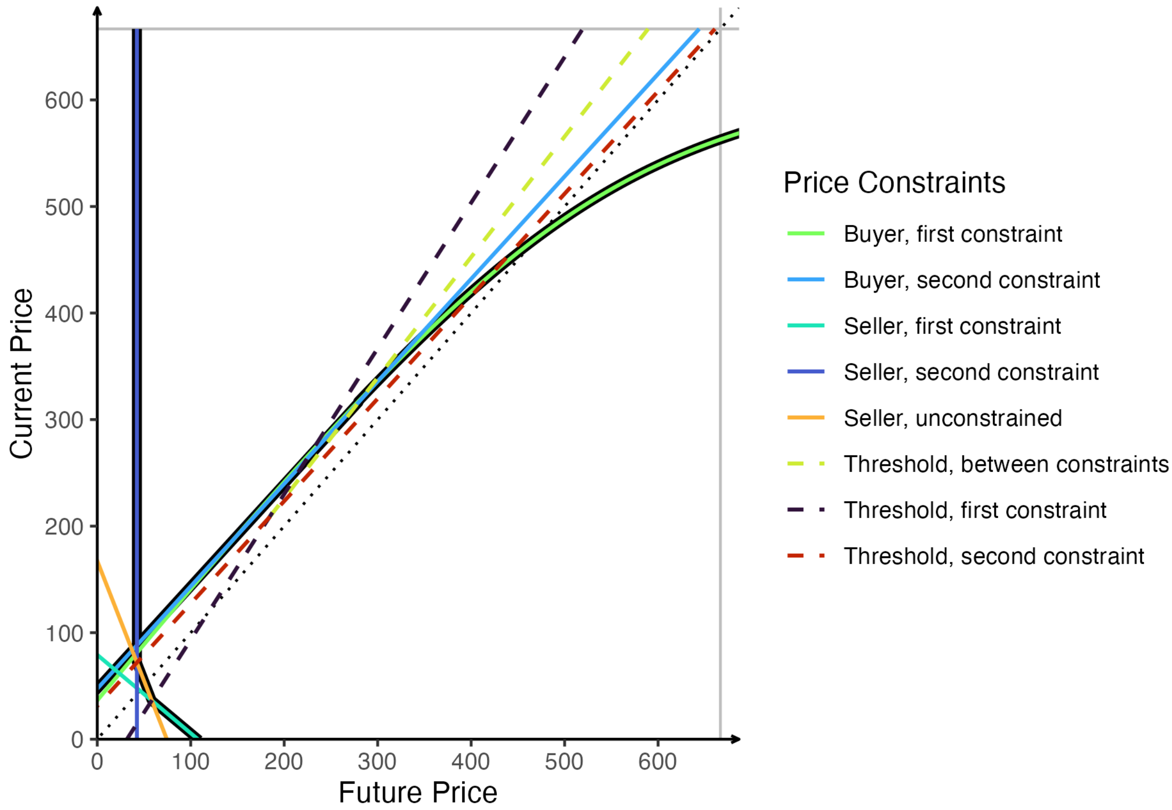

Figure 9: Intertemporal house price conditions with raised interest, naïve calibration

For the naïve calibration, raising the annual interest rate has relatively minor effects, but both the maximum and realized house prices fall a little bit. Meanwhile, for the long-run calibration, the maximum price drops drastically while the minimum price rises slightly. Furthermore, it alters incentives such that the borrowing constraints will never bind in any steady state. While the final price falling has a positive effect on consumption and utility, these effects are dominated by the negative effects from imposing higher interest on accessing future income. Still, the former effect implies that the usual reasons for low interest (corresponding to the latter effect) should be somewhat tempered. As the consequences of this simulated change vary greatly between the two simulations, this shows that the framework still needs to be extended further before yielding good quantitative predictions.

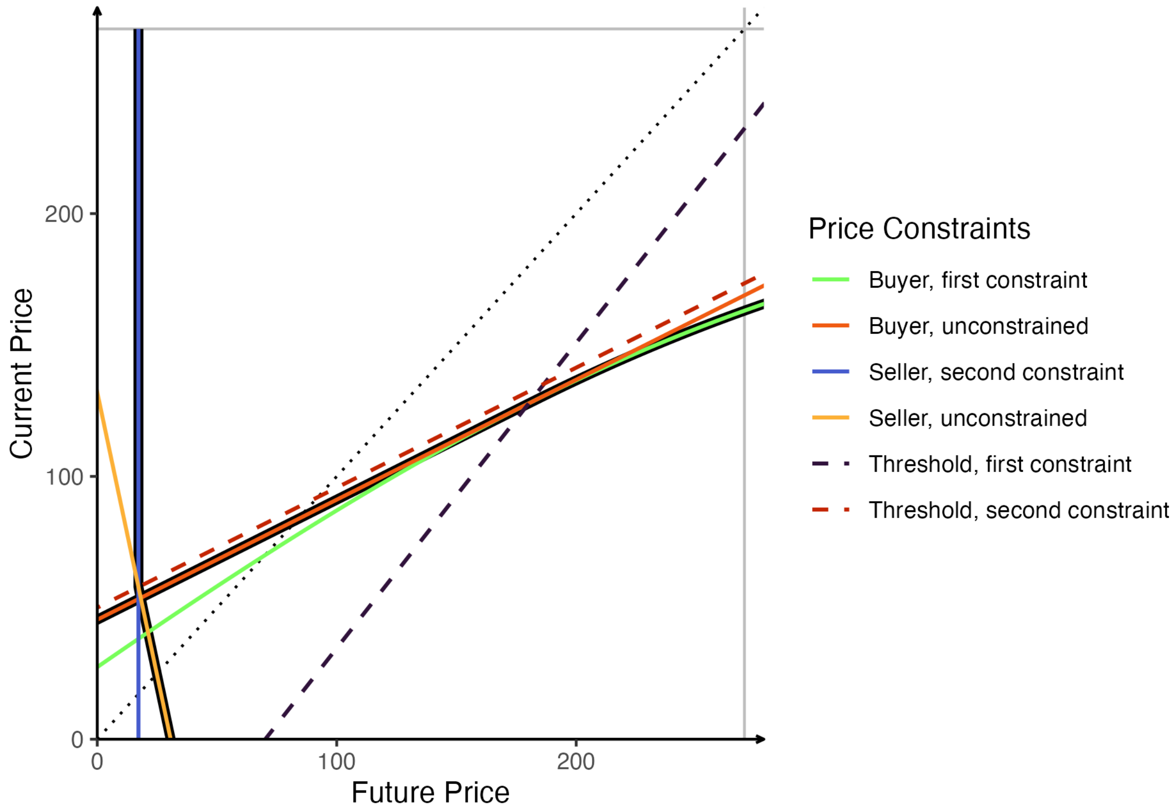

Figure 10: Intertemporal house price conditions with raised interest, long-run calibration

5 Conclusion

The framework presented here is one where both expectations and credit conditions are important for the determination of house prices. Expectations are coordinated by the relative bargaining power of the middle-aged, who need and therefore buy larger houses, and the old, who want to downsize and therefore sell such houses, which determines the realized house price within the price limits. The borrowing limit, meanwhile, plays a role in determining these limits, particularly if pushing the buyers to their maximum acceptable price would involve them hitting that limit. Improving the position of the middle-aged buyer generation would be associated with a lower house price, and the reverse holds for the bargaining position of the old seller generation. Both the proposed mechanism for expectation coordination and the result that credit tightness can have major effects even when households are not borrowing-constrained in the final outcome are novel in the macroeconomic literature on housing.

The long-run negative lifetime utility consequences of high house prices are a notable contrast against the short-run stimulating consequences of rising house prices in other macroeconomic housing models. This is not a contradiction, however, as the preliminary results on the short-run dynamics of this model appear to be in line with the stimulating effect found in other models. Rather, it is an observation that the short-run benefits of increasing house prices are likely to come at long-run costs, much like fiscal stimuli. Lowering expectations on house prices or otherwise changing the transaction structure such that buyers are more favored would have positive implications for the long run, whereas doing so through tightening credit is less clear, as that may have adverse side effects to take into account.

Worthwhile extensions for future research to explore include: Shortening the period length and increasing the number of generations would allow dynamics to operate over more reasonable time frames and may allow features like saving up for house purchases. Risk is the most natural feature to include to enable house price crashes within the framework. Nominal rigidities, which could not only be applied to a production sector, but also to house price expectations. Introducing a construction sector would add an additional seller agent and enables incorporation of demographic changes into the model. These extensions could further contextualize the intuitive features of the model while also introducing interesting twists.

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Appendices

Appendix A1: Old Generation Incentive Compatibility

First, note that there is a possibility that the old cannot afford keeping their house. If $y_o + (1+r)b_{m,t} \leq -\omega p_{h,t+1}$, then old homeowners would need to consume non-positive quantities of the consumption good, which is not allowed. They will always be able to consume a positive quantity of the consumption good by selling their house along the equilibrium path, though, as they would otherwise not have bought the house in the first place.

If they can afford keeping their house, incentive compatibility for the old selling their houses requires that:

$$\log\left(y_o + (1+r)b_{m,t} + p_{h,t+1}\right) + u_{o,h}(0) \geq \log\left(y_o + (1+r)b_{m,t} + \omega p_{h,t+1}\right) + u_{o,h}(1)$$

$$\log\left(\frac{y_o + (1+r)b_{m,t} + p_{h,t+1}}{y_o + (1+r)b_{m,t} + \omega p_{h,t+1}}\right) \geq u_{o,h}(1) - u_{o,h}(0)$$

$$\frac{y_o + (1+r)b_{m,t} + p_{h,t+1}}{y_o + (1+r)b_{m,t} + \omega p_{h,t+1}} \geq \exp\left(u_{o,h}(1) - u_{o,h}(0)\right) = \nu_o$$

$$p_{h,t+1}(1 - \omega\nu_o) \geq \left(y_o + (1+r)b_{m,t}\right)\nu_o - y_o - (1+r)b_{m,t}$$

Going from the last expression to the final one is why the $\omega\nu_o < 1$ assumption is necessary. If it does not hold, the old can take any price offer, no matter how exorbitant, to their bank and expand their loan to the limit according to that price, and they will prefer that to selling their house. In reality, banks (or the financial market in general) would impose tighter credit conditions for such very large loans, thus ensuring that there would be a limit to this, but that is not modeled here, so this assumption is needed as a technical assumption.

$$p_{h,t+1} \geq \frac{\left(y_o + (1+r)b_{m,t}\right)(\nu_o - 1)}{1 - \omega\nu_o}$$

This final condition actually ends up capturing both the case where they can and cannot afford keeping their house. It was derived from the former case. For the latter, the first factor of the right-hand numerator is a negative number, which means the entire right-hand side is negative, which captures the fact that the old households will accept prefer selling at any price.

Appendix A2: Middle-aged Generation Incentive Compatibility

With no binding constraint, the condition for the middle-aged to prefer owning their home is that owner utility exceeds non-owner utility (with solution quantities):

$$\log\left(\frac{y_m + y_o(1+r)^{-1} + p_{h,t+1}(1+r)^{-1} - p_{h,t}}{(1+\beta)}\right)$$
$$+ \beta\log\left(\beta(1+r)\frac{y_m + y_o(1+r)^{-1} + p_{h,t+1}(1+r)^{-1} - p_{h,t}}{(1+\beta)}\right) + u_{m,h}(1)$$
$$+ \beta u_{o,h}(0)$$
$$\geq \log\left(\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}\right) + \beta\log\left(\beta(1+r)\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}\right) + u_{m,h}(0)$$
$$+ \beta u_{o,h}(0)$$

This can be simplified through a couple of steps:

$$u_{m,h}(1) - u_{m,h}(0) \geq (1+\beta)\log\left(\frac{y_m + y_o(1+r)^{-1}}{y_m + y_o(1+r)^{-1} + p_{h,t+1}(1+r)^{-1} - p_{h,t}}\right)$$

$$\left(\exp\left(u_{m,h}(1) - u_{m,h}(0)\right)\right)^{\frac{1}{1+\beta}} = \nu_m^{\frac{1}{1+\beta}} \geq \frac{y_m + y_o(1+r)^{-1}}{y_m + y_o(1+r)^{-1} + p_{h,t+1}(1+r)^{-1} - p_{h,t}}$$

$$y_m + y_o(1+r)^{-1} + p_{h,t+1}(1+r)^{-1} \geq \frac{y_m + y_o(1+r)^{-1}}{\nu_m^{\frac{1}{1+\beta}}} + p_{h,t}$$

$$p_{h,t} \leq p_{h,t+1}(1+r)^{-1} + \frac{\left(\nu_m^{\frac{1}{1+\beta}} - 1\right)(y_m + y_o(1+r)^{-1})}{\nu_m^{\frac{1}{1+\beta}}}$$

The same utility comparison for the first borrowing constraint:

$$\log\left(y_m - (1-\omega)p_{h,t}\right) + \beta\log\left(y_o - (1+r)\omega p_{h,t} + p_{h,t+1}\right) + u_{m,h}(1) + \beta u_{o,h}(0)$$
$$\geq \log\left(\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}\right) + \beta\log\left(\beta(1+r)\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}\right) + u_{m,h}(0)$$
$$+ \beta u_{o,h}(0)$$

This can be rearranged to:

$$\log\left(\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)\left(y_m - (1-\omega)p_{h,t}\right)}\right) + \beta\log\left(\frac{\beta}{1+\beta}(1+r)\frac{y_m + y_o(1+r)^{-1}}{y_o - (1+r)\omega p_{h,t} + p_{h,t+1}}\right)$$
$$\leq u_{m,h}(1) - u_{m,h}(0)$$

Which, in turn, is equivalent to:

$$\frac{(1+r)^{\beta}\beta^{\beta}(y_m + y_o(1+r)^{-1})^{1+\beta}}{(1+\beta)^{1+\beta}\left(y_m - (1-\omega)p_{h,t}\right)\left(y_o - (1+r)\omega p_{h,t} + p_{h,t+1}\right)^{\beta}} \leq \nu_m$$

$$p_{h,t+1} \geq \frac{(1+r)\beta(y_m + y_o(1+r)^{-1})^{\frac{1+\beta}{\beta}}}{(1+\beta)^{\frac{1+\beta}{\beta}}\left(y_m - (1-\omega)p_{h,t}\right)^{\frac{1}{\beta}}\nu_m^{\frac{1}{\beta}}} - y_o + (1+r)\omega p_{h,t}$$

And the comparison for the second borrowing constraint:

$$\log\left(y_m - p_{h,t} + \frac{p_{h,t+1}}{1+r}\right) + \beta\log(y_o) + u_{m,h}(1) + \beta u_{o,h}(0)$$
$$\geq \log\left(\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}\right) + \beta\log\left(\beta(1+r)\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)}\right) + u_{m,h}(0)$$
$$+ \beta u_{o,h}(0)$$

Which rearranges to:

$$\log\left(\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)\left(y_m - p_{h,t} + \frac{p_{h,t+1}}{1+r}\right)}\right) + \beta\log\left(\frac{\beta}{1+\beta}(1+r)\frac{y_m + y_o(1+r)^{-1}}{y_o}\right)$$
$$\leq u_{m,h}(1) - u_{m,h}(0)$$

And then simplifies to:

$$\frac{y_m + y_o(1+r)^{-1}}{(1+\beta)\left(y_m - p_{h,t} + \frac{p_{h,t+1}}{1+r}\right)}\left(\frac{\beta}{1+\beta}(1+r)\frac{y_m + y_o(1+r)^{-1}}{y_o}\right)^{\beta} \leq v_m$$

$$p_{h,t} \leq y_m + \frac{p_{h,t+1}}{1+r} - \frac{(1+r)^{\beta}\beta^{\beta}(y_m + y_o(1+r)^{-1})^{1+\beta}}{(1+\beta)^{1+\beta}v_m y_o^{\beta}}$$

Appendix A3: Old Generation Equilibrium Incentive Compatibility

If not borrowing-constrained in the last period, the old will prefer selling their houses to keeping them as long as the utility is higher for doing so:

$$\log\left(y_o + (1+r)\frac{\beta y_m - \beta p_{h,t} - y_o(1+r)^{-1} - p_{h,t+1}(1+r)^{-1}}{(1+\beta)} + p_{h,t+1}\omega\right) + u_{o,h}(1)$$
$$\leq \log\left(y_o + (1+r)\frac{\beta y_m - \beta p_{h,t} - y_o(1+r)^{-1} - p_{h,t+1}(1+r)^{-1}}{(1+\beta)} + p_{h,t+1}\right)$$
$$+ u_{o,h}(0)$$

$$\log\left(\frac{(1+r)\beta y_m - (1+r)\beta p_{h,t} + \beta y_o + \beta p_{h,t+1}}{(1+\beta)}\right)$$
$$- \log\left(\frac{(1+r)\beta y_m - (1+r)\beta p_{h,t} + \beta y_o + (\omega + \omega\beta - 1)p_{h,t+1}}{(1+\beta)}\right)$$
$$\geq u_{o,h}(1) - u_{o,h}(0)$$

$$\log\left(\frac{(1+r)y_m - (1+r)p_{h,t} + y_o + p_{h,t+1}}{(1+r)y_m - (1+r)p_{h,t} + y_o + \frac{\omega + \omega\beta - 1}{\beta}p_{h,t+1}}\right) \geq u_{o,h}(1) - u_{o,h}(0)$$

$$\frac{(1+r)y_m - (1+r)p_{h,t} + y_o + p_{h,t+1}}{(1+r)y_m - (1+r)p_{h,t} + y_o + \frac{\omega + \omega\beta - 1}{\beta} p_{h,t+1}} \geq \nu_o$$

Note that there is no mechanism that guarantees that the denominator here is positive. However, if it is not, then an old household keeping their house would consume negative quantities, which is not allowed. The numerator, meanwhile, must be positive because of the second borrowing constraint. So, if $p_{h,t+1} \leq \frac{\beta\left((1+r)p_{h,t} - (1+r)y_m - y_o\right)}{\omega + \omega\beta - 1}$, the old generation will want to sell their houses (however, for there to be any risk of this, the right-hand side here must be a positive number, but then the final condition's right-hand side must be a negative number, which means the final condition already captures these prices, rendering this one redundant). Otherwise, this condition can be simplified further:

$$p_{h,t+1}\left(1 - \nu_o \frac{\omega + \omega\beta - 1}{\beta}\right) \geq (\nu_o - 1)\left((1+r)y_m - (1+r)p_{h,t} + y_o\right)$$

Note that $\frac{\omega+\omega\beta-1}{\beta} = \omega - \frac{1-\omega}{\beta} < \omega$, so, under $\nu_o\omega < 1$, then $\nu_o \frac{\omega+\omega\beta-1}{\beta} < 1$ must also hold. With that, this can be simplified to the final condition:

$$p_{h,t+1} \geq \frac{(\nu_o - 1)\left((1+r)y_m - (1+r)p_{h,t} + y_o\right)}{1 - \nu_o \frac{\omega + \omega\beta - 1}{\beta}}$$

For the first borrowing constraint:

$$\log\left(y_o - (1+r)\omega p_{h,t} + p_{h,t+1}\right) + u_{o,h}(0) \geq \log\left(y_o - (1+r)\omega p_{h,t} + p_{h,t+1}\omega\right) + u_{o,h}(1)$$

$$\frac{y_o - (1+r)\omega p_{h,t} + p_{h,t+1}}{y_o - (1+r)\omega p_{h,t} + p_{h,t+1}\omega} \geq \nu_o$$

Again, the denominator here will be negative, and the second logarithm invalid in the first place, if $p_{h,t+1} \leq (1+r)p_{h,t} - y_o\omega^{-1}$ (but again, this implies a negative final minimum, so this only yields a redundant condition). If the denominator is positive:

$$p_{h,t+1} \geq \frac{(\nu_o - 1)\left(y_o - (1+r)\omega p_{h,t}\right)}{1 - \nu_o\omega}$$

And if they were under the second borrowing constraint:

$$\log(y_o) + u_{o,h}(0) \geq \log\left(y_o - (1-\omega)p_{h,t+1}\right) + u_{o,h}(1)$$

$$\frac{y_o}{y_o - (1-\omega)p_{h,t+1}} \geq \nu_o$$

This time, if $p_{h,t+1} \geq \frac{y_o}{1-\omega}$, old households are always willing to sell their house (although the condition below yet again already implies this on its own). With a positive denominator:

$$p_{h,t+1} \geq \frac{y_o(\nu_o - 1)}{\nu_o(1-\omega)}$$

Appendix B: Borrowing Constraint Thresholds and Base Middle-age Decisions

For the middle-age decisions, the per-period budget constraints can be collected into a generic lifetime budget constraint:

$$y_m + \frac{y_o + p_{h,t+1}h_{m,t}}{1+r} = c_{m,t} + p_{h,t}h_{m,t} + \frac{c_{o,t+1}}{1+r}$$

To find generic unconstrained optimal consumption, $c_{o,t+1}$ is solved for:

$$c_{o,t+1} = (y_m - c_{m,t})(1+r) + y_o + \left(p_{h,t+1} - p_{h,t}(1+r)\right)h_{m,t}$$

This is substituted into the lifetime utility expression, leading to the first-order condition:

$$U' = \frac{1}{c_{m,t}} - (1+r)\beta\frac{1}{(y_m - c_{m,t})(1+r) + y_o + \left(p_{h,t+1} - p_{h,t}(1+r)\right)h_{m,t}} = 0$$

Which leads to the generic unconstrained solution:

$$c_{m,t} = \frac{y_m + y_o(1+r)^{-1} + \left(p_{h,t+1}(1+r)^{-1} - p_{h,t}\right)h_{m,t}}{(1+\beta)}$$

Savings then follow from the per-period budget constraint, leading to:

$$b_{m,t} = y_m - p_{h,t}h_{m,t} - \frac{y_m + y_o(1+r)^{-1} + \left(p_{h,t+1}(1+r)^{-1} - p_{h,t}\right)h_{m,t}}{(1+\beta)}$$

$$b_{m,t} = \frac{\beta y_m - \beta p_{h,t}h_{m,t} - y_o(1+r)^{-1} - p_{h,t+1}(1+r)^{-1}h_{m,t}}{(1+\beta)}$$

These generic expressions can be used for directly deriving the unconstrained consumption decisions. Further, the expression for base savings is the one to compare with the borrowing constraints to see if the unconstrained optimum is available for a specific set of parameters and prices or if one of the borrowing-constrained versions need to be used.

The condition for the first borrowing constraint being tighter than the second is:

$$-\omega p_{h,t} \geq -\frac{p_{h,t+1}}{1+r} \Rightarrow \omega(1+r)p_{h,t} \leq p_{h,t+1}$$

For the simplest case, where the household never owns a house, however, both borrowing constraints state that savings must be non-negative. This will be fulfilled if:

$$b_{m,t} = \frac{\beta y_m - y_o(1+r)^{-1}}{(1+\beta)} \geq 0 \Rightarrow (1+r)\beta y_m \geq y_o$$

The first borrowing constraint will bind, if tighter, if desired borrowing exceeds the limit:

$$\frac{\beta y_m - \beta p_{h,t} - y_o(1+r)^{-1} - p_{h,t+1}(1+r)^{-1}}{(1+\beta)} \leq -\omega p_{h,t}$$

$$y_m - \frac{y_o}{\beta(1+r)} - \frac{p_{h,t+1}}{\beta(1+r)} \leq \left(1 - \frac{1+\beta}{\beta}\omega\right) p_{h,t}$$

Note that the first factor of the right-hand side is negative, because of the $\omega > \frac{\beta}{1+\beta}$ assumption.

$$p_{h,t} \leq \frac{p_{h,t+1}}{(1+r)\beta\left(\frac{1+\beta}{\beta}\omega - 1\right)} - \frac{y_m - \frac{y_o}{\beta(1+r)}}{\left(\frac{1+\beta}{\beta}\omega - 1\right)}$$

$$p_{h,t} \leq \frac{p_{h,t+1}}{(1+r)\big(\omega - \beta(1-\omega)\big)} - \frac{y_m - \frac{y_o}{\beta(1+r)}}{\left(\frac{1+\beta}{\beta}\omega - 1\right)}$$

On the other hand, if the second constraint is tighter, it will bind if a similar comparison holds for that constraint:

$$\frac{\beta y_m - \beta p_{h,t} - y_o(1+r)^{-1} - p_{h,t+1}(1+r)^{-1}}{(1+\beta)} \leq -\frac{p_{h,t+1}}{1+r}$$

$$\frac{\beta p_{h,t}}{1+\beta} \geq \frac{\beta y_m - y_o(1+r)^{-1} + \beta p_{h,t+1}(1+r)^{-1}}{(1+\beta)}$$

$$p_{h,t} \geq y_m - \frac{y_o}{\beta(1+r)} + \frac{p_{h,t+1}}{1+r}$$

Appendix C: Proof for Existence and Uniqueness of First Borrowing-Constraint Maximum Steady-State Price

At the minimum price in the range of possible $p_{h;max,f}$ values, the first borrowing constraint's threshold holds with equality, which means both the unconstrained equilibrium consumption values and the same values for the first borrowing constraint are equal. As consumption and homeownership are equal for both, utility is also equal across the two. Then, the right-hand side of equation 14 is equal to the right-hand side of this rearrangement of equation 4

$$p_h = p_h(1+r) - \frac{\left(v_m^{\frac{1}{1+\beta}} - 1\right)(y_m + y_o(1+r)^{-1})}{v_m^{\frac{1}{1+\beta}}}(1+r) \qquad [C]$$

from which equation 13 was derived. Equation C is therefore also equivalent to equation 13.

If these right-hand sides are greater than their identical left-hand sides at this minimum, it follows that equation 13 holds for a lower price than that minimum (as the right-hand side of equation C shrinks faster as the price decreases). That minimum is also the minimum price for which equation 11 is satisfied, thus, that condition is not satisfied, and the constrained maximum price (and its potential nonexistence) is irrelevant.

If the right-hand side is less than or equal to the left-hand side, this logic reverses, and the constrained maximum price is the relevant maximum price. Then there must be a solution to equation 14 somewhere in the $p_{h;max,f}$ range, as, within this range, the left-hand side grows linearly while the right-hand side grows without bound (the first term approaches division by zero), which means the right-hand side must catch up at some point.

This solution is unique, as the derivative of the right-hand side expression must be greater than one at the point where it catches up with the left-hand side (whose derivative is exactly one), after which it cannot fall below one (as the right-hand side's second derivative is strictly positive), which would be necessary for the left-hand side to catch up again and generate additional solutions.

If the right-hand sides are equal to the left-hand sides, then equation 11 holds with equality, in which case equation 13 still yields the correct solution.

Appendix D: Proof for Endowment Homogeneity of First Borrowing-Constraint Maximum Steady-State Price

If the endowments are multiplied by some positive constant θ, then multiplying the price by that same constant is a solution to equation 14 (and the only solution, as was proven in appendix C), which is thus the maximum price. The left-hand side is cleanly multiplied by this factor. So are the second and third terms of the right-hand side. As for the first term, the denominator is multiplied by $\theta^{\frac{1}{\beta}}$, whereas the numerator is multiplied by $\theta^{\frac{1+\beta}{\beta}}$, whose ratio also simplifies to $\theta^{\frac{1+\beta}{\beta}-\frac{1}{\beta}} = \theta^{\frac{\beta}{\beta}} = \theta$. Thus, if the price is multiplied by the same factor as the endowments, the right-hand side is multiplied by the same factor as the left-hand side, which means the equation still holds. Therefore, the maximum steady-state price is positively homogeneous of degree one in the endowments.

Appendix E: House Prices' Effects on Other Steady-State Variables

If the middle-aged household is unconstrained in steady state, an increase in house prices would decrease consumption in both periods, as the negative effect of the current house price has a greater multiplier than the positive effect of the future house price in the relevant equations (see section 3.4). If constrained by the second borrowing constraint, consumption in their middle-aged period decreases for the same reason, while consumption in the future period stays constant. The case when constrained by the first borrowing constraint is a bit more complicated, as an increase in the house price decreases current consumption while increasing it in the future. However, the increase in future consumption by $(1-(1+r)\omega)p_h$ is smaller than the decrease in current consumption by $(1-\omega)p_h$, so total consumption is decreasing for all three cases.

As for lifetime utility, this is easy to see for the unconstrained case and the one for the second borrowing constraint, as neither period's consumption increases, and at least one drops. For the first borrowing constraint, the derivative of lifetime utility is negative:

$$-(1-\omega)\frac{1}{y_m-(1-\omega)p_h}+(1-(1+r)\omega)\beta\frac{1}{y_o+(1-(1+r)\omega)p_h}<0$$

$$\frac{1-(1+r)\omega}{1-\omega}\beta(y_m-(1-\omega)p_h)<y_o+(1-(1+r)\omega)p_h$$

$$\frac{1-(1+r)\omega}{1-\omega}\beta y_m-y_o<(1-(1+r)\omega)(1+\beta)p_h$$

$$\frac{(1+r)\beta y_m}{(1+r)(1-\omega)(1+\beta)}-\frac{y_o}{(1-(1+r)\omega)(1+\beta)}<p_h$$

Which can be shown to hold by replacing p_h with its minimum possible value in equation 11:

$$\frac{(1+r)\beta y_m}{(1+r)(1-\omega)(1+\beta)}-\frac{y_o}{(1-(1+r)\omega)(1+\beta)}$$
$$<\frac{(1+r)\beta y_m}{1-(1+r)\omega+(1+r)\beta-(1+r)\beta\omega}$$
$$-\frac{y_o}{1-(1+r)\omega+(1+r)\beta-(1+r)\beta\omega}$$

Here, the positive term on the left-hand side has a larger denominator than the corresponding term on the right-hand side, thus being smaller. Also, the left-hand negative term has a smaller denominator than its right-hand one, thus being larger. With a smaller positive term and a larger negative term, the left-hand overall expression must be smaller, thus proving that the inequalities hold.

All three expressions for savings depend negatively on both current and future prices, which means they naturally also negatively depend on prices in steady-state form.

These patterns also hold if a price change changes whether the relevant borrowing constraint is binding or not, as the derivatives have the same sign on both sides of the threshold, and there are no discontinuities at the thresholds.

Appendix F: Effects of Changes to Credit Tightness

Given a price sequence, tightening the main borrowing constraint makes those constrained by it (weakly) worse off. This is because them becoming better off leads to a contradiction, as their new forced choice was a consumption-savings choice accessible before the tightening, so if it is an improvement, then their original decision was not optimal. While utilities become (weakly) lower for all constraining price sequences, the reference utility is unconstrained and therefore constant. Thus, buying a house becomes less attractive, lowering the constrained maximum acceptable price. Note that all seller minimum prices are also lowered, as their reference utility is constrained by the first constraint, while the old are not constrained in equilibrium.

The price sequence will, however, change when the limit prices change. Specifically, as they all decrease or stay constant, with at least the minimum prices decreasing, the steady-state house price drops.

If households are not ever constrained even after the tightening, they are better off because of the lower price. But if they always were or become constrained, the price change makes them better off, while the tightened constraint pushes them further away from their underlying desired decision, so the effect is ambiguous.

Data Appendix

Calibration values:

Parameter	Short run	Long run	Source
y_o/y_m	0.7	0.7	Statistics Sweden
r	0.02	0.49	Statistics Sweden
omega	0.85	0.63	Finansinspektionen
β	0.978	0.64	De Lipsis (2021)
ν_m	2	2	Arbitrary
ν_o	1.1	1.1	Arbitrary

Statistics Sweden tables used:

Consumer Price Index (CPI), Fixed Index numbers, total annual average, 1980=100. Year 1980 - 2022
Real estate price index for one- or two-dwelling buildings for permanent living (1981=100) by year and region
GDP: expenditure approach (ESA2010) by type of use. Quarter 1980K1 - 2022K4
Disposable income for households. Mean value, SEK thousands by type of household, age, year and region
Lending rates to households for housing loans, percent by counterparty sector, agreement, original rate fixation, month and reference sector