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There is nothing certain but the uncertain

Evaluating the price dynamics of vol-of-vol in the cross-section of stock returns

by

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Abstract

Risk and risk aversion are crucial concepts in finance. Models in finance typically assume a known probability distribution of returns, which does often not hold in reality. This papers aims to measure the uncertainty surrounding the probability distribution in equity markets and to evaluate if such uncertainty is priced. Our study defines uncertainty as the volatility of the option implied volatility (vol-of-vol). By employing a Factor Mimicking Portfolio approach, we observe a significant underperformance of the high vol-of-vol portfolio compared the low vol-of-vol portfolio. Over the sample period from January 2005 to March 2023, the average annualized return difference between the two portfolios is 11.9%. Interestingly, the vol-of-vol effect cannot be explained by the Carhart Four Factor Model, as the High-Minus-Low Portfolio exhibit a significant annual Carhart 4-Factor alpha of 13.9%.

Key words: Uncertainty, Vol-of-vol, Ambiguity, Asset Pricing, Factor Mimicking Portfolio, Investor Sentiment, Efficient Market Hypothesis

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Introduction

A wide range of financial models are built upon the ideas of risk and risk aversion. Contrarily, uncertainty, which may be an even more important aspect of portfolio selection, receives relatively little attention (Epstein, 1999). Risk and uncertainty are distinguished by the availability of probabilities, and within a financial context they are therefore arguably important to keep apart. While risk refers to the known probability distribution of returns, uncertainty refers to ambiguity of the probability distribution (Knight et al., 2001; Park and Shapira, 2017; SEC, 2011), and an everyday challenge for investors relates to *ambiguity aversion.* Ambiguity aversion is defined as the preference of known risk over unknown risk, also known as the *Ellsberg paradox* (Ellsberg, 1961), and thus directly applicable to portfolio selection. Classic factor models such as Sharpe's (1964) and Lintner's (1965) *Capital Asset Pricing Model* (CAPM), Fama and French's (1992; 2015; 2018) factor models, and *Carhart*'s (1997) *Four Factor Model* (C4FM) try to reduce the obscurity in the probability of the return distribution by including company-specific factors. Although there exists evidence of the factors' significance, the models are criticized for not being able to capture the ambiguity itself or the market sentiment (Campbell, 1996). Consequently, this research aims at decoding ambiguity in stock returns by introducing a factor for uncertainty and examining its significance.

While uncertainty refers to incomplete knowledge or predictability of future events (Knight et al., 2001; Park and Shapira, 2017; SEC, 2011), volatility is a risk measurement based on historical values. On the contrary, expected volatility refers to the probability of future distribution and can be measured by *implied volatility* (IV), which represents the market's opinion on a stock's future volatility, and is thus forward-looking by nature. IV is calculated using an option pricing model where a higher IV corresponds to a perception in the market that the riskiness of the future return distribution is higher. Uncertainty is characterized by an unknown probability of future returns, whereas IV reflects the market's perception of the likelihood of future returns. The volatility of IV can be viewed as a mathematical metric for uncertainty (Baltussen et al., 2012) and we define *vol-of-vol* as uncertainty in the return distribution accordingly. Based on the concept of vol-of-vol, this study aims to clarify if stocks with higher uncertainty, i.e. vol-of-vol, perform better than stocks with lower uncertainty in the probability distribution of returns. We do this by addressing the following question;

How does the market price the uncertainty surrounding the return distribution?

Investors have been able to trade the market's ambiguity since the introduction of derivatives of the *Chicago Board Options Exchange Volatility of Volatility Index* (VVIX), an index based on the IV methodology with S&P 500 as the underlying asset. Consequently, solid research on VVIX exists and empirical evidence implies that the index is able to measure tail risk in the return distribution of S&P 500 (Park, 2013; Branger et al., 2018). Beside existing evidence on VVIX as an estimation of market uncertainty, Baltussen et al. (2012) investigates the amplitude of vol-of-vol further by examining its relation with individual stock return. Further on, the paper defines stocks' uncertainty by its vol-of-vol, and when testing its influence on a cross-section of equity returns during 1996-2009 the findings suggest that stocks with a higher vol-of-vol yield significantly lower returns. Baltussen et al. (2012) propose that the vol-of-vol effect may be attributed to overconfidence bias in the market which aligns with Bossaerts et al.'s (2010) idea, suggesting that investors are willing to pay a premium for stocks with greater uncertainty regarding the company's future fundamentals.

To enhance existing evidence on vol-of-vol further we will assess the influence of vol-of-vol during different sentiments in the market. The readiness of investors to accept financial risk is commonly defined by the term "risk appetite", which is frequently used in media (ECB, 2007). With countless derivatives in the market numerous indicators can be used to gauge the risk sentiment and a few examples of indicators are market momentum, stock price strength, stock price breadth, options, market volatility, bond demand, and consumers' purchasing power. However, since our research aims to shed light on investors potentially being overconfident, we require an index that accurately reflects investor confidence in the current state. To do so we use the *Consumer Confidence Index* (CCI) and the *American Association of Individual Investors* (AAII) which are indices based on surveys measuring consumer and investor sentiment (OECD, 2023; AAII, 2023). Due to the indices' composition, they are able to enclose investors' current attitude towards risk which is favorable compared to other technical indicators which tend to capture trends rather than actual sentiment (Oriani and Coelho, 2016). By using CCI and AAII separately we will investigate the interaction between investor confidence and vol-of-vol, and attack the following question;

Does investor confidence affect the significance of the uncertainty factor and its pricing dynamics?

In this study, we analyze the impact of vol-of-vol on individual stock returns within S&P 500 over the sample period January 2005 to March 2023. Vol-of-vol is calculated as the monthly volatility of IV derived from *30-day to maturity at-the-money* options. Our methodology involves adopting the *Factor Mimicking Portfolio* (FMP) framework and utilizing a double-sorting approach inspired by Fama and French's (1992; 2015; 2018) factor models to construct the vol-of-vol factor. To quantify the vol-of-vol effect, we conduct a four factor regression analysis similar to C4FM, incorporating factors for market risk premium, firm size, book-to-market ratio, and momentum on the vol-of-vol portfolios obtained from the FMP approach.

Our findings indicate that stocks with low vol-of-vol exhibit higher average excess returns with a highly significant *Carhart 4-Factor* (4F) alpha, highlighting the existence of a vol-of-vol factor in the market. Additionally, we identify that investor confidence, as measured by AAII, has a significant influence on the vol-of-vol effect. Lower investor confidence is associated with worse performance by high vol-of-vol stocks. Finally, we can not find evidence supporting that the vol-of-vol is a priced factor within the market.

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Literature Review

2.1 Uncertainty

Within the context of asset pricing, investors are often confronted with uncertainty about future stock returns. Uncertainty is characterized by incomplete knowledge or predictability of future events (Knight et al., 2001; Park and Shapira, 2017; SEC, 2011) and is a fundamental aspect of asset pricing and risk management. In order to address how uncertainty affects individuals' decision-making, a common assumption in asset pricing models is that investors are ambiguity averse. This implies that investors want to maximize their expected wealth, while minimizing the unknown risk. Based on this assumption, Ellsberg (1961) argues that individuals are willing to pay a premium to reduce ambiguity and provides a framework for understanding how market participants make choices when faced with uncertain outcomes.

Uncertainty and risk have distinct definitions. Risk refers to the known probabilities associated with future events, while uncertainty arises when these probabilities are unknown to the decision-maker. From a foundational standpoint in mathematical finance, researchers argue that the uncertainty in expected return nests in the ambiguity surrounding its probability distribution (Knight et al., 2001; Park and Shapira, 2017; SEC, 2011). This implies that the probability of the return distribution reflects the riskiness of the expected return and that the market volatility of this probability reflects the market's perception of the uncertainty. As this probability is forward-looking, the ability to measure uncertainty depends on the capacity of the measurement to capture investor sentiment. Hence, only measurements with dynamic attributes should be considered reliable instruments for uncertainty.

Cremers and Yan's (2012) evaluate forecast dispersion regarding quarterly earnings among analysts as a proxy for uncertainty, however, the paper does not find evidence of a positive correlation between the analyst forecast error variables and equity valuation. Although analyst dispersion does not show any significance as a proxy for uncertainty, the paper supports the idea of firm age as a proxy for uncertainty, and that younger firms have a higher valuation in terms of the price-to-book ratio (Pastor and Veronesi, 2003). Due to its forward-looking character, Park (2013) and Branger et al. (2018) argue that uncertainty can be measured by using the volatility of expected volatility. Expected volatility is typically related to option IV which is derived from the *Black-Scholes Model* (Black and Scholes, 1973). As options are forward-looking and directly linked to the price of the underlying asset, IV reflects all available information regarding the stock and gauges the market's view. One can say that IV measures the risk-neutral prediction of a stock's realized return volatility and an alternative approach to measure the uncertainty in the probability of the return distribution is to analyze variance swaps, i.e. the difference between IV and realized volatility (Carr and Wu, 2009). Other research also suggests that the relationship between IV and asset pricing is negatively correlated, i.e. that negative (positive) shocks in return relate to higher (lower) IV, the so-called *asymmetric volatility phenomenon* (Bekaert and Wu, 2000; Dennis et al., 2006).

Baltussen et al.'s (2012) investigate the pricing dynamics of uncertainty, defined as vol-of-vol, and finds that stocks with low vol-of-vol outperform high vol-of-vol stocks by 8% per year on average during the sample period 1996-2009. The negative vol-of-vol effect is considered robust after distinguishing the effect of vol-of-vol from other explanatory factors within the Fama and French's (1992; 2015; 2018) factor model framework, hence the authors infer that the market fails to discount uncertainty in stock prices. Further on, Baltussen et al. (2012) explain the negative vol-of-vol premium by stating that if the ambiguity aversion among agents varies sufficiently, the risky assets are held onto by agents with low ambiguity aversion. Bossaerts et al. (2010) use this reasoning to analyze low return in growth stocks and argue that investors are prepared to pay a premium to make investments in stocks with high levels of uncertainty. The authors address that this explanation can be assigned to the overconfidence bias, where investors think they are more knowledgeable than others.

2.2 A factor model approach

Factor models are financial instruments that investors can use to identify and manage investments by examining the influence that a certain objective may have on the return of stocks and portfolios (Lhabitant, 2017). CAPM is familiar factor model, and the model weighs the covariance of a stock's excess return with that of the market portfolio and is considered to measure a stock's historical performance in relation to the market by its beta value. Empirical evidence accepts CAPM as a simplistic approximation of the risk premium across stocks (Jensen et al., 1972; Fama and French, 1973). However, CAPM is criticized for having omitted-variable bias, and several models trying to remedy this shortcoming exist. The limitation of CAPM has fueled researchers to develop the model further and reduce still outstanding idiosyncratic risk. One such model is the Fama and French's (1992) *Three Factor Model* (FF3FM) which expands CAPM by introducing factors for firm size (SMB) and book-to-market ratio (HML). By including additional elements for company-specific characteristics the model is considered to capture more of the idiosyncratic risk (Xu and Malkiel, 2004) and empirical evidence suggests that FF3FM successfully increase the explanation of stock returns (Fama and French, 1993).

FF3FM has served as a foundation in researchers' search for new factors that can explain stock returns. One such model is C4FM which expands FF3FM by adding a factor loading for momentum (UMD) (Carhart, 1997). The idea of UMD is that stocks that have performed well over the previous months tend to do so in the near future, while equities that have performed poorly tend to continue doing so. To account for the criticism regarding the omitted-variable bias, Fama and French (2015; 2018) have introduced UMD into FF3FM along with additional factors for profitability (RMW) and the effect of investments (CMA), giving rise to the *Fama and French Five-* (2015) and *Six-* (2018) *Factor Model* (FF5FM and FF6FM) respectively. Hou et al.'s (2015) *Q-Factor Model* (QFM) is yet another model which considers sensitivities to the market risk premium, firm size, investments, and profitability in order to obtain superior results, similarly to FF5FM and

FF6FM. Another approach to explain the variation in stock returns is *Arbitrage Pricing Theory* (APT) (Ross, 1976), which instead of creating factor loadings for company-specific factors, argues that macroeconomic factors are preferable when untangling stock returns. A benefit of the model is that the researcher has the opportunity to adjust which factors to include according to sample characteristics, e.g. geography and size. However, Ross (1976) argues that APT lacks specific theoretical background and empirical evidence.

Following the variety of research proposing different factors to explain stock returns, researchers warn practitioners to apply too many factors as such models might get problems with endogeneity, overfitting, and data mining (Harvey et al., 2016; Blitz et al., 2016). Avoiding these biases is essential when designing the model and we do only want to include necessary factors in order to isolate the impact of vol-of-vol without causing problems with the interpretation of the results.

2.3 Investor confidence

Fundamental financial theory of investor behavior relies on the assumption that individuals have different attitudes toward the uncertainty of outcomes. While risk-averse individuals prefer investments with low volatility, where the payoff might be lower but to a higher probability, risk-loving individuals prefer high volatility in outcomes with a probability of giving a higher payoff. Moreover, investors' relationship to uncertainty is highly individual and perceived uncertainty is by definition depending on the level of confidence in the probabilities of the return distribution. Ambiguity aversion relates to the preference for known risk over unknown risk (Ellsberg, 1961), and empirical evidence suggests that it has a negative relationship with market participation and stock ownership (Dimmock et al., 2013). De Long et al. (1990) propose a model that views changes in investor sentiment as a market risk. The model distinguishes between rational investors and noise traders, where noise traders are influenced by investor sentiment, while rational investors are not.

Investor confidence is regularly measured by indicators based on underlying observations such as the performance of stocks and market derivatives, sentiment analysis, and macroeconomic variables. Among these indicators, the *CNN Fear & Greed Index* stands out as a widely recognized measure of market sentiment. The indicators building up the index aim to capture the psychological aspect of investor behavior, revealing whether investors are driven by fear or greed in their decision-making. The CNN Fear & Greed Index combines several factors, including stock price momentum, market volatility, options activity to generate a composite sentiment reading (CNN, 2023). Although this kind of index might be a good approximation of the momentum in the market, it lacks the ability to seize current investor confidence as technical indicators are lagging and considered indirect (Beer et al., 2013; Oriani and Coelho, 2016).

Direct approaches of measuring investor confidence are indicators based on surveys, and such indices are frequently used in financial research as proxies for investor sentiment (Corredor et al., 2013). One of the most prominent indices within this dimension is the *Consumer Confidence Index* (CCI), which is based on a monthly survey that reflects the degree of optimism among the population in different countries (Daskalopoulou, 2014). CCI and other indicators of consumer confidence have the advantage of being based on actual and current sentiment within an entire population, however their dependence on the selection of individuals and their economic incompetence can be seen as limitations (Dominitz and Manski, 2004). To make up for the drawbacks of broad survey-based indices, surveys targeting individuals that are active in the financial market are consequently superior at capturing confidence among market participants. One index that does this is the *American Association of Individual Investors Index* (AAII) whose survey goes out to the organization's members, consisting of approximately 150,000 investors (AAII, 2023). Since the organization aims at helping investors globally, the members' answers can be considered to reflect individual investors' views on the market, implying that AAII is able to seize current investor confidence (Fisher and Statman, 2000; Beer et al., 2013). Although AAII is considered a good proxy for investor confidence, research suggests that the sentiment of individual- and professional investors do not move in lockstep (Fisher and Statman, 2000), and that investors are prone to the sentiment itself (Bank and Brustbauer, 2014).

3

Data

3.1 Sample

Our analysis focuses on vol-of-vol and its relationship with the stock market. To ensure reliable measurements, we select stocks with a liquid option market. VIX is a volatility index derived from S&P 500 options, and it is widely recognized for capturing overall risk sentiment in the stock market by reflecting the collective IV of companies operating in different sectors. Given the liquidity of the US options market and the S&P 500's prominence, we follow the stock selection criteria of VIX and retrieve data from stocks within the index S&P 500 (CBOE, 2023). This approach aligns with Baltussen et al.'s (2012) finding that stocks with larger market capitalization tend to have more liquid options markets. We obtain data from the *Bloomberg Terminal* during the time period January 2005 to March 2023. In addition to IV and stock returns, we also gather data on market capitalization to facilitate an appropriate factor model methodology.

3.2 IV

In this study, we use IV data on 30-day-to-maturity call- and put options with a delta close to 50, downloaded from Bloomberg's historical database using *Listed Implied Volatility Engine* (LIVE) calculator. An option with a delta of 50 is considered to be at-the-money, with equal probabilities of expiring profitable, i.e. *in-the-money*, and non-profitable, i.e. *out-the-money*. All IV data in the sample have the equal delta which makes the volatility comparable between different stocks. We consider that delta 50 options with 30-day to maturity are reasonable to use since these options are the most liquid and close to being at-the-money, making IV from the options more reliable and suited for the purpose of this paper. An alternative approach could involve averaging options with different deltas, where lower (higher) delta corresponds to higher (lower) IV, as the options' prices are less (more) affected by underlying asset movements.

3.3 Additional data

In order to test if vol-of-vol is a significant factor for stock returns using a factor model approach we need data on the remaining factors which we will use in the model. We download historical daily data for SMB, HML, and MOM from the *Data Library of Kenneth R. French* which is a database consisting of all factor loadings used in the Fama and French (1992; 2015; 2018) framework. Data on the investor sentiment indices CCI and AAII are downloaded for the sample period from *OECD* and *AAII*'s own website, respectively.

4

Methodology

4.1 Vol-of-vol

We apply vol-of-vol as a measurement for uncertainty about stocks' return distribution and compute it using the average IV from call- and put option prices. Since IV is forward-looking by nature it successfully captures overall market expectations on the future volatility of a stock's return. Furthermore, the options market is traded on a daily basis which makes changes in market expectation of future volatility easy to measure, unlike analysts earning dispersion which is suggested as a measure of uncertainty in previous literature. Additionally, IV is derived from market prices which evades optimism bias in analyst forecasts (Thorstensson and Blazevic, 2022).

We address potential disparity in IV changes between high- and low-volatile stocks by scaling the standard deviation of IV for each stock i on day j $\sigma_{i,t}^{IV}$ by the average IV for the preceding month $\bar{\sigma}_{i,t}^{IV}$ in our calculations of monthly vol-of-vol. This adjustment helps filter out the impact of the markets already priced risk level in the calculations. Finally, we determine the stock's vol-of-vol on day j by evaluating the volatility of its IV fluctuations using Equation (4.1).

$$VOV_{i,t}^{Monthly} = \frac{\sqrt{\frac{1}{Days\ in\ month\ t}\sum_{j=1}^{t}(\sigma_{i,j}^{IV} - \bar{\sigma}_{i,j}^{IV})^2}}{\bar{\sigma}_{i,j}^{IV}} \tag{4.1}$$

Where $VOV_{i,t}^{Monthly}$ is the monthly vol-of-vol for stock i at month t. $\sigma_{i,t}^{IV}$ is IV for stock

i at day j, $\bar{\sigma}_{i,t}^{IV}$ is the average implied volatility for stock i during month t. The formula therefore yields an estimation of the uncertainty on a monthly basis, scaled by its already priced risk of the expected return.

4.2 Factor Mimicking Portfolio

Fama and French (1992; 2015; 2018) factor model extensions of CAPM uses a FMP methodology, and the approach is prevalent in asset pricing research. An advantage of FMP is that it allows for the construction of a portfolio that represents a specific background factor, even when the realization of that factor is not returns. Moreover, this methodology enables the isolation of the target factor's impact and to attain an accurate analysis (Asgharian, 2004). We adopt the portfolio approach within the FMP framework to construct a factor representing vol-of-vol and assess its significance. Employing this strategy involves dividing the sample into quintile portfolios ranked according to the chosen factor, followed by the construction of the High-Minus-Low Portfolio that takes long positions in assets with the highest sensitivities to the background factor, while simultaneously shorting assets with the lowest sensitivities.

To construct the vol-of-vol factor, we begin by dividing the sample into two groups according to their market capitalization, i.e. small or big, on a monthly basis. Within each group, the stocks are further divided into five quintile portfolios based on their vol-of-vol rankings. With a sample size of 500 companies, we believe five quintiles to be sufficient to capture the variation in vol-of-vol between high and low vol-of-vol stocks, while ensuring diversification and mitigating concentration risk and potential biases associated with the individual stocks. Thus, we obtain a total of ten portfolios each month which are consolidated into five quintile portfolios (see Table 4.1). The double-sorting approach is implemented in order to remove potential biases in the results because of the size factor.

Table 4.1: Quintile Portfolios

Quintile Portfolio	Firm size	Vol-of-vol	Firm size	Vol-of-vol
1	Small	1st quintile	Big	1st quintile
2	Small	2nd quintile	Big	2nd quintile
3	Small	3rd quintile	Big	3rd quintile
4	Small	4th quintile	Big	4th quintile
5	Small	5th quintile	Big	5th quintile

To compute the daily returns for the quintile portfolios we employ both a value-weighted approach (Fama and French, 1992) and an equally-weighted approach (Chan et al., 1998), separately. By utilizing these approaches separately, we aim to gain a more comprehensive understanding of the vol-of-vol factor and its potential significance. The benefit of an equally-weighted approach when constructing the portfolios relates to its ability to reduce the impact of individual stocks. However, one limitation of equally-weighted portfolios is that larger firms may have a disproportionate impact on the overall portfolio performance due to their size. The calculation of equally-weighted daily log excess returns is displayed in Equation (4.2).

$$R_{p_{t-1},j}^{P,Equally} = \sum_{i=1}^{n} \frac{r_{i,p_{t-1},j}}{n_{p_{t-1}}} \tag{4.2}$$

Where R is the excess return of portfolio p, r is the excess return of stock i within portfolio p at time t, and n is the number of stocks in portfolio j at time *t-1*.

The advantages of a value-weighted approach for constructing portfolios is that it enables a more accurate representation of the market portfolio compared to an equally-weighted approach. That is because the value-weighted approach put more weight in larger stocks according to market capitalization, similarly to Fama and French's (1992; 2015; 2018) framework and the construction of market indices, e.g. S&P 500. Additionally, asset pricing models such as CAPM assume that investors hold portfolios that are proportional to the market value of all outstanding stocks. In such models, value-weighted portfolios may be used to represent the market portfolio more accurately. The calculation of daily log returns for the value-weighted portfolios can be found in Equation (4.3).

$$R^{P,Value}_{p_{t-1},j} = \sum_{i=1}^{n} r_{i,p_{t-1},j}\left(\frac{Mcap_{i,p_{t-1},j-1}}{\sum_{i=1}^{n} Mcap_{i,p_{t-1},j-1}}\right) \tag{4.3}$$

Where R is excess return for portfolio j, r is excess return for stock i within portfolio p at time t, n is the number of stocks in portfolio j at time *t-1*, and *Mcap* is the market cap for stock i in portfolio j at time *t-1*.

The quintile portfolios containing stocks with individual excess return and monthly vol-of-vol are rebalanced on the last trading day of the month according to the stocks' vol-of-vol ranking, which by definition is renewed each month. Since the portfolios contain daily returns we create the vol-of-vol factor on a daily basis by taking the return of the highest quintile portfolio minus the lowest quintile portfolio each day throughout the sample period, see Equation (4.4). Consequently, we refer to the vol-of-vol factor as the High-Minus-Low Portfolio in accordance with the FMP methodology (Asgharian, 2004). Additionally, the portfolios are rebalanced on a fixed schedule during market close and we also assume that the market is frictionless, i.e. no costs associated with the rebalancing of the portfolios. However, it is important to note that these limitations do not directly impact the paper's primary objective of measuring the price dynamics of the factor being analyzed.

$$VOV = \frac{SH + BH}{2} - \frac{SL + BL}{2} \tag{4.4}$$

4.3 Vol-of-vol factor model

In order to evaluate the significance and performance of the vol-of-vol factor, we compute a multivariate linear regression model and assess the 4F alphas of the vol-of-vol portfolios. The dependent variable in the regression models is the excess return of each portfolio and the High-Minus-Low Portfolio on a daily basis, calculated as the portfolio returns minus the risk-free rate. The independent variables in the regression are based on C4FM, which includes market risk premium, SMB, HML, and UMD. Computing separate regression models for each portfolio enables us to evaluate the intercepts, which represents the parts of the abnormal return unexplained by these factors. A statistically significant 4F alpha would indicate that the portfolio has exhibited abnormal performance in addition to what can be accounted for by the factors alone. The equation for the regression model is displayed in Equations (4.5) and (4.6).

$$R^{P,Equal}_{p_{t-1},j} = \beta_0 + \beta_1(r^m_j - r^f_j) + \beta_2 r_{SMB,j} + \beta_3 r_{HML,j} + \beta_4 r_{MOM,j} + \epsilon \tag{4.5}$$

$$R^{P,Value}_{p_{t-1},j} = \beta_0 + \beta_1(r^m_j - r^f_j) + \beta_2 r_{SMB,j} + \beta_3 r_{HML,j} + \beta_4 r_{MOM,j} + \epsilon \tag{4.6}$$

4.4 Factor loadings

To empirically test the price dynamics of the vol-of-vol factor, we construct a vol-of-vol factor-mimicking portfolio derived from the stocks' factor loading with respect to vol-of-vol. This is done by a time-series regression with the daily excess returns for each stock, our created vol-of-vol factor, and the factors in C4FM as independent variables. The regressions are computed using a rolling window of 500 days (approximately 2 years), producing daily vol-of-vol factor loadings for each stock, see Equation (4.7). These factor loadings are then used to create portfolios according to the stock's ranking in terms of factor loading using a FMP approach.

$$r_{i,j} - r^f_j = \beta_0 + \beta_{1,j}(r^m_j - r^f_j) + \beta_{2,j} r_{SMB,j} + \beta_{3,j} r_{HML,j} + \beta_{4,j} r_{MOM,j} + \beta_{5,j} r_{VOV,j} + \epsilon \tag{4.7}$$

Where r is return for stock i at day j, r_f is the daily risk free rate at day j, $r^m_j - r^f_j$ is market risk premium at day j, r_{SMB} , r_{HML}, and r_{MOM} is the return for the factors in C4FM at day j. r_{VOV} is the return for vol-of-vol factor (see Equation 4.1) at day j. The regression is on a rolling window of 500 days. Hence, the output from the regression is daily factor loadings for each stock from December 2006 to March 2023.

We mitigate the problem with a potential correlation between a stock's vol-of-vol factor loading and vol-of-vol characteristic by using a three-step sorting approach. The vol-of-vol quintile portfolios have already been sorted according to size, however in order to evaluate the pricing dynamics of the vol-of-vol factor loading we must cancel out the vol-of-vol effect. This is done by sorting each vol-of-vol quintile portfolio into five new quintile portfolios based on their vol-of-vol factor loading, resulting in 25 portfolios. Subsequently, these portfolios are aggregated into five quintile portfolios, where the quintile portfolio consisting the stocks with the highest factor loadings has consolidated the top quintile portfolio within each original vol-of-vol quintile portfolio in Table 4.1.

The triple-sorting approach generates five portfolios with similar vol-of-vol characteristics and size but distinct vol-of-vol factor loadings, enabling the assessment of the market's pricing dynamics associated with uncertainty. The portfolios are then constructed using an equally- and value-weighted approach separately. To form the factor mimicking portfolio, the sum of the highest quintile values from all ten portfolios is subtracted from the sum of the lowest ten quintile values.

$$VOV^{FMP} = \frac{SHH + SNH + SLH + ... + BNH}{10} - \frac{SLL + SNL + SHL + ... + BHL}{10} \quad (4.8)$$

Where the first letter stands for the size, i.e. S is for small and B is for big. The second letter is for the vol-of-vol characteristics, i.e. H for High, N for neutral, and L for low. The third letter is for the vol-of-vol factor loading, i.e. H for quintile with the highest vol-of-vol factor loading, and S for quintile with the smallest vol-of-vol factor loading.

4.5 Market sentiment

To answer the question if investor confidence changes the pricing of uncertainty we use the market sentiment indicators CCI and AAII. CCI is an index that measures the overall sentiment and optimism of consumers regarding the current and future state of the economy, indicating their willingness to spend and make significant financial decisions (Daskalopoulou, 2014). The sentiment is defined as positive when the index is over 100 and negative if it is under 100. AAII on the other hand is a survey conducted among individual investors to gauge their collective sentiment and outlook on the stock market and classifies investors as either bullish, bearish, or neutral (AAII, 2023). To incorporate the findings from the surveys, we include the average bull-bear spread each month from AAII and the monthly CCI in the US from our dataset.

The impact of investor confidence on the vol-of-vol factor is examined by performing two regression models with dummy variables representing the bull state of the respective indices, AAII and CCI. By nature, the reference group in the regression is when the indices define the sentiment as bearish. If the dummy variable is significant, the regression suggests that the vol-of-vol factors' influence on stock returns differs depending on the market sentiment.

$$R^{P,Equal}_{p_{t-1},j} = \beta_0 + \gamma_0^{Bull\ AAII} + \beta_1(r^m_j - r^m_j) + \beta_2 r_{SMB,j} + \beta_3 r_{HML,j} + \beta_4 r_{MOM,j} + \epsilon \tag{4.9}$$

$$R^{P,Value}_{p_{t-1},j} = \beta_0 + \gamma_0^{Bull\ CCI} + \beta_1(r^m_j - r^m_j) + \beta_2 r_{SMB,j} + \beta_3 r_{HML,j} + \beta_4 r_{MOM,j} + \epsilon \tag{4.10}$$

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Results

5.1 Vol-of-vol effect

5.1.1 Portfolio characteristics

Table 5.1 shows that there is no substantial difference in terms of volatility in daily returns across the vol-of-vol portfolios. This stands in direct contrast to the perception that stocks exhibiting more uncertainty should display higher volatility and can be attributed to our definition of vol-of-vol (see Equation 4.1), which effectively isolates the uncertainty of the future return distribution. Thus, the vol-of-vol formula successfully quantifies the uncertainty of a stock's return despite the presence of already incorporated volatility in its price. When comparing Portfolio 5, i.e. the highest vol-of-vol stocks, with Portfolio 1, i.e. the lowest vol-of-vol stocks, one can observe that the average vol-of-vol over the specified time period is 15.6% for Portfolio 5, whereas Portfolio 1 exhibits an average vol-of-vol of 4.8%.

Table 5.1: Stock's vol-of-vol and volatility in vol-of-vol portfolios

	Portfolio					
	1	2	3	4	5	H-L
Vol-of-vol						
Average	0.048	0.065	0.080	0.099	0.156	0.109
75th percentile	0.052	0.071	0.087	0.107	0.169	0.124
25th percentile	0.035	0.050	0.063	0.080	0.134	0.092
St. deviation	0.022	0.027	0.030	0.032	0.038	0.024
Volatility						
Average	0.321	0.313	0.314	0.314	0.314	0.000
75th percentile	0.350	0.345	0.340	0.327	0.324	0.021
25th percentile	0.248	0.241	0.243	0.244	0.246	-0.030
St. deviation	0.107	0.109	0.113	0.120	0.130	0.056
Skewness	1.543	1.739	1.850	2.015	2.292	1.489
Kurtosis	1.699	2.487	2.964	3.778	5.176	5.140

Note: Table 5.1 reports values derived from individual stocks within each vol-of-vol portfolio, where Portfolio 1 (5) consists of the stocks with the lowest (highest) vol-of-vol stocks, during the sample period January 2005 to March 2023. Portfolio H-L is the High-Minus-Low Portfolio and calculated by taking Portfolio 5 minus Portfolio 1 (see Equation 4.4). Vol-of-vol is last month's volatility of Bloomberg's LIVE calculated IV, scaled by the last month's average IV (see Equation 4.1). Volatility is reported on an annual basis and is calculated using a 252-day rolling window. The average vol-of-vol each month for Portfolio 1 and 5 can be viewed in Appendix A.0.

Table 5.2 provides an overview of the disparities in market capitalization and CAPM beta between high and low vol-of-vol stocks, structured in quintile portfolios. The results show discernible differences in average size across the portfolios where bigger companies tend to pull towards the high vol-of-vol portfolio, albeit using a double sorting approach where the stocks have been categorized as big and small initially. This trend persists across all quintile portfolios, suggesting that the market possesses greater uncertainty in the return distribution of larger companies. On the other hand, the CAPM beta values of the stocks remain consistent across the quintile portfolios when the firm size has been taken into account. This indicates that high vol-of-vol does not equal a high CAPM beta and demonstrates that our definition of vol-of-vol distinguishes itself from other risk measures, such as CAPM beta and volatility.

Table 5.2: Stock's market cap and CAPM beta in vol-of-vol portfolios

	Portfolio				
	1	2	3	4	5
Market cap					
Average	31.0	35.8	39.5	44.7	49.5
Median	26.0	30.0	34.0	38.4	39.6
Max	127.2	108.9	150.0	147.1	164.1
Min	10.8	11.3	14.2	14.1	16.3
CAPM beta					
Average	0.987	1.002	1.007	0.998	0.970
Median	0.998	1.001	1.009	0.999	0.970
Max	1.135	1.108	1.116	1.110	1.100
Min	0.790	0.784	0.877	0.771	0.769
Avg. # of stocks	86	85	85	85	86

Note: Table 5.2 reports values derived from individual stocks within each vol-of-vol quintile portfolio, i.e. Portfolio 1 (5) consists of the stocks with the lowest (highest) vol-of-vol stocksduring the sample period January 2005 to March 2023. Vol-of-vol is last month's volatility of Bloomberg's LIVE calculated IV scaled by the last month's average IV (see Equation 4.1). Market cap is reported in billion USD. CAPM beta for each stock is calculated by regressing the daily excess return of stock i using C4FM with a rolling window of 500 days.

5.1.2 Frequency changes

Table 5.3 presents the frequency of monthly transitions from one quintile portfolio to another. The findings suggest that stocks tend to exhibit a balanced distribution of changes in their classification regarding the uncertainty of their returns in the following month. However, the results indicate that the lowest and highest vol-of-vol stocks have the highest likelihood of remaining in the same portfolios, with probabilities of 26% and 31%, respectively. These results indicate that stocks with the lowest and highest uncertainty tend to maintain their respective levels of uncertainty more frequently compared to other transitions. Additionally, the frequent rebalancing of the factor mimicking portfolio implies higher transaction costs associated with the strategy, however, we do not consider any of these.

Table 5.3: Frequency changes of stocks between vol-of-vol portfolios

	Portfolio					
From/to	1	2	3	4	5	**Total**
1	4,794	3,831	3,534	3,466	3,011	**18,636**
2	3,945	3,891	3,822	3,695	3,046	**18,399**
3	3,544	3,875	3,947	3,823	3,197	**18,386**
4	3,361	3,722	3,854	3,814	3,648	**18,399**
5	2,953	3,092	3,246	3,615	5,670	**18,576**
1	26%	21%	19%	19%	16%	**100%**
2	21%	21%	21%	20%	17%	**100%**
3	19%	21%	21%	21%	17%	**100%**
4	18%	20%	21%	21%	20%	**100%**
5	16%	17%	17%	19%	31%	**100%**

Note: Table 5.3 reports how frequently the stocks in our sample change vol-of-vol quintile portfolios from one month to another during the sample period January 2005 to March 2023. The first column represents the starting portfolio for a stock and the first row represents the next month's portfolio for the same stock. Total is the total stocks in each portfolio during the sample period January 2005 to March 2023.

5.1.3 Vol-of-vol factor

The relative performance of the vol-of-vol quintile portfolios based on the equally- and value-weighted approaches are presented in Table 5.4. The results show that low vol-of-vol stocks have a higher average excess return than high vol-of-vol stocks. By combining the portfolios of the highest- and lowest vol-of-vol stocks, the High-Minus-Low Portfolio assumes a long position in the former and a short position in the latter. When utilizing the equally weighted approach, the High-Minus-Low Portfolio achieves an average annual return of -9.4%, while the value-weighted approach yields -11.9%. The return differences cannot be attributed to higher risk, as the stocks within the portfolios have been standardized in terms of volatility and CAPM beta. Furthermore, the trend that low vol-of-vol stocks have a higher average excess return in average return is consistent across the vol-of-vol quintile portfolios using both portfolio weighting approaches.

The standard deviation of the vol-of-vol quintile portfolios in Table 5.4 show that the portfolios with higher vol-of-vol stocks exhibit wider return distributions than portfolios containing low vol-of-vol stocks. Besides higher standard deviation, the portfolios with higher vol-of-vol stocks also display greater extreme values. These results suggest that higher vol-of-vol stocks are more exposed to tail events than low vol-of-vol stocks, and hold for both equally and value-weighted portfolios.

Table 5.4: Summary statistics of vol-of-vol portfolios

	Portfolio					
Panel A: Equally-weighted	1	2	3	4	5	H-L
Excess return						
Average*	0.132	0.086	0.054	0.056	0.039	-0.094
St. error	0.000	0.000	0.000	0.000	0.000	0.000
Median*	0.247	0.185	0.188	0.207	0.229	-0.008
St. deviation	0.198	0.205	0.213	0.217	0.243	0.109
Sample variance	0.002	0.003	0.003	0.003	0.004	0.001
Kurtosis	10.088	9.474	10.963	12.415	21.684	33.040
Skewness	0.106	0.105	0.112	0.116	0.134	0.079
Minimum**	-0.116	-0.107	-0.130	-0.148	-0.186	-0.092
Maximum**	0.106	0.105	0.112	0.116	0.134	0.079
	Portfolio					
Panel B: Value-weighted	1	2	3	4	5	H-L
Excess return						
Average*	0.143	0.100	0.039	0.043	0.023	-0.119
St. error	0.000	0.000	0.000	0.000	0.000	0.000
Median*	0.298	0.189	0.128	0.170	0.209	-0.042
St. deviation	0.184	0.193	0.195	0.200	0.239	0.131
Sample variance	0.002	0.002	0.002	0.003	0.004	0.001
Kurtosis	12.774	11.204	10.269	11.275	23.264	26.762
Skewness	-0.254	-0.248	-0.397	-0.544	-0.958	-0.323
Minimum**	-0.122	-0.112	-0.115	-0.131	-0.181	-0.094
Maximum**	0.109	0.111	0.100	0.109	0.153	0.109
Count	4 592	4 592	4 592	4 592	4 592	4 592
Avg. # of stocks	86	85	85	85	86	86

*Note: Tables 5.4 reports values derived from individual stocks within each vol-of-vol portfolio, where Portfolio 1 (5) consists of the stocks with the lowest (highest) vol-of-vol stocks, during the sample period January 2005 to March 2023. Portfolio H-L is the High-Minus-Low Portfolio and calculated by taking Portfolio 5 minus Portfolio 1 (see Equation 4.4). * (**) means that the value is expressed on an annual (daily) basis. Calculations are made on daily excess log return data, meaning that the data is on daily basis if nothing else is noted. ** is calculated manually by taking the result times 252 (approximately 252 trading days in a year). Further, Panel A presents values when the portfolios are computed using an equally weighted approach while Panel presents values when the portfolios are computed using a value-weighted approach. In the equally-weighted portfolios every stock is weighted equal (see methodology in Equation 4.2) while the value-weighted approach weighs every stock according to size (see methodology used in Equation 4.3). Count is the total excess returns used in the calculations.*

5.2 C4FM regression

To assess the significance of the vol-of-vol factor independent of other return factors, we conduct a regression analysis using excess returns as the dependent variable and the factors within C4FM as independent variables, i.e. market risk premium, SMB, HML, and MOM. If the vol-of-vol factor has no explanatory power for stock returns in advance to C4FM, we would expect the intercept to be close to zero and no significant differences between the portfolios. The results in Table 5.5 show that the portfolio comprising the lowest vol-of-vol stocks generates a positive 4F alpha of 4.5% annually, while the portfolio consisting of the highest vol-of-vol stocks exhibits a negative 4F alpha of 6.3% on a yearly basis. Consequently, the High-Minus-Low Portfolio shows a negative 4F alpha of 10.7%. This indicates that, the High-Minus-Low Portfolio underperforms the market by 10.7% annually, after accounting for C4FM. The results are even more pronounced when considering the portfolios constructed using a value-weighted approach, with the High-Minus-Low Portfolio exhibiting a negative 4F alpha of 13.9%. These results hold significant statistical relevance throughout various time periods, including 2005-2009, 2010-2014, 2015-2019, and 2019-2023 (see Appendices B.3, B.4, B.5, and B.6).

Table 5.5: Regression results of vol-of-vol portfolios

	Portfolio					
Panel A: Equally-weighted	1	2	3	4	5	H-L
Vol-of-vol	0.048	0.065	0.080	0.099	0.156	0.109
Excess returns	0.131	0.085	0.053	0.056	0.039	-0.093
Intercept	0.045***	-0.006	-0.041***	-0.040***	-0.063***	-0.107***
	(3.74)	*(-0.56)*	*(-4.06)*	*(-3.58)*	*(-3.56)*	*(-4.67)*
MKT-RF	0.921***	0.965***	0.998***	1.017***	1.086***	0.164***
	(230.29)	*(279.87)*	*(293.74)*	*(271.57)*	*(183.77)*	*(21.31)*
SMB	0.161***	0.126***	0.122***	0.100***	0.036***	-0.124***
	(20.37)	*(18.45)*	*(18.19)*	*(13.50)*	*(3.12)*	*(-8.18)*
HML	0.111***	0.149***	0.190***	0.175***	0.279***	0.168***
	(16.97)	*(26.45)*	*(34.22)*	*(34.22)*	*(28.71)*	*(13.38)*
MOM	-0.027***	-0.024***	-0.020***	-0.031***	-0.082***	-0.055***
	(-5.35)	*(-5.44)*	*(-4.69)*	*(-6.64)*	*(-11.01)*	*(-5.66)*
Avg. # of stocks	86	85	85	85	86	86
Adj. R^2	0.934	0.954	0.959	0.952	0.905	0.190

	Portfolio					
Panel B: Value-weighted	1	2	3	4	5	H-L
Vol-of-vol	0.048	0.065	0.080	0.099	0.156	0.109
Excess returns	0.142	0.099	0.039	0.042	0.023	-0.118
Intercept	0.057***	0.010	-0.052***	-0.052***	-0.082***	-0.139***
	(4.24)	*(0.85)*	*(-4.76)*	*(-4.45)*	*(-4.29)*	*(-5.05)*
MKT-RF	0.886***	0.935***	0.958***	0.987***	1.116***	0.230***
	(194.76)	*(228.64)*	*(258.88)*	*(252.42)*	*(174.26)*	*(24.84)*
SMB	-0.050***	0.014***	0.032***	0.032***	0.094***	0.118***
	(-5.61)	*(-10.42)*	*(-11.81)*	*(-17.86)*	*(-20.06)*	*(-11.14)*
HML	-0.024***	0.014***	0.032***	0.032***	0.094***	0.118***
	(-3.21)	*(2.11)*	*(5.28)*	*(5.00)*	*(9.00)*	*(7.81)*
MOM	0.010	-0.005	0.022***	0.028***	-0.062***	-0.071***
	(1.73)	*(-0.88)*	*(4.84)*	*(5.62)*	*(-7.66)*	*(-6.15)*
Avg. # of stocks	86	85	85	85	86	86
Adj. R^2	0.901	0.927	0.942	0.939	0.884	0.191

*Note: Table 5.5 reports a summary of the regression results of Equations 4.5 and 4.6 during the sample period January 2005 to March 2023 with the vol-of-vol portfolio, where Portfolio 1 (5) consists of the stocks with the lowest (highest) vol-of-vol stocks. Portfolio H-L is the High-Minus-Low Portfolio and calculated by taking Portfolio 5 minus Portfolio 1 (see Equation 4.4). The intercept values are reported on an annual basis by taking the result multiplied with 252. The t-statistics are reported in parentheses and ** (***) indicate significance on the 5% (1%) level. Further, Panel A presents values when the portfolios are computed using an equally weighted approach while Panel B presents values when the portfolios are computed using a value-weighted approach (see Equations 4.2 and 4.3). Full regression results for the High-Minus-Low Portfolio are presented in Appendices C.10 and C.10.*

5.3 Price dynamics of vol-of-vol

In order to assess the pricing of the vol-of-vol factor we employ a FMP approach and construct triple-sorted portfolios based on stocks' vol-of-vol factor loadings (see Methodology 4.4). If the market prices the vol-of-vol factor appropriately, we would expect a portfolio with higher factor loadings to exhibit lower average returns compared to a portfolio with lower factor loadings. Table 5.7 presents the summary characteristics of the vol-of-vol factor loading quintile portfolios, and the regression results of Equations 4.5 and 4.6 using these portfolios. To mitigate the influence of vol-of-vol characteristics and hence multicollinearity, we divide the vol-of-vol factor loading into quintile portfolios within each portfolio. This approach effectively neutralizes the impact of vol-of-vol characteristics on returns, enabling a more accurate analysis of the portfolios. The results indicate no significant difference in average vol-of-vol characteristics across the portfolios. Therefore, we would expect a difference in excess return between the high and low portfolios, that is not explained by the vol-of-vol characteristics, if vol-of-vol would be a priced factor in the market. With the exception of Portfolio 4, the regression analysis does not reveal any significant 4F alphas for the portfolios. Consequently, the results do not provide evidence of a factor-based explanation for the vol-of-vol effect. This indicates that we cannot find evidence that investors are adequately adjusting their required returns to compensate for the additional risk these stocks exhibit.

Table 5.6: Frequency changes of stocks between vol-of-vol factor loading portfolios

	Portfolio					
From/to	1	2	3	4	5	**Total**
1	79%	18%	2%	1%	0%	**100%**
2	20%	53%	23%	4%	1%	**100%**
3	2%	24%	49%	23%	2%	**100%**
4	1%	3%	24%	53%	19%	**100%**
5	0%	0%	2%	18%	79%	**100%**

Note: Table 5.8 reports how frequently the stocks in our sample change factor loading portfolios from one month to another during the sample period December 2006 to March 2023. The first column represents the starting portfolio for a stock and the first row represents the next month's portfolio. The portfolios are the vol-of-vol factor loading portfolios with a triple-sorting (see Methodology 4.4).

Table 5.7: Regression results of vol-of-vol factor loading portfolios

	Portfolio					
Panel A: Equally-weighted	1	2	3	4	5	H-L
Factor loading	-0.300	-0.125	-0.029	0.073	0.295	0.596
Vol-of-vol	0.092	0.091	0.093	0.092	0.092	0.000
Excess returns	0.073	0.070	0.062	0.066	0.082	0.009
Intercept	-0.018	-0.020*	-0.031***	-0.029***	-0.022	-0.004
	(-0.88)	*(-1.71)*	*(-2.79)*	*(-2.26)*	*(-1.15)*	*(-0.127)*
Mkt-RF	0.957***	0.941***	0.970***	1.004***	1.124***	0.168***
	(148.18)	*(254.81)*	*(273.68)*	*(241.25)*	*(185.10)*	*(16.67)*
SMB	0.062***	0.041***	0.074***	0.117***	0.248***	0.186***
	(4.81)	*(5.56)*	*(10.42)*	*(14.03)*	*(20.42)*	*(9.24)*
HML	0.156***	0.140***	0.127***	0.166***	0.339***	0.183***
	(14.96)	*(23.42)*	*(22.11)*	*(24.54)*	*(34.44)*	*(11.19)*
MOM	0.019***	-0.005	-0.018***	-0.060***	-0.141***	-0.159***
	(2.31)	*(-1.06)*	*(-4.12)*	*(-11.33)*	*(-18.33)*	*(-12.55)*
Avg. # of stocks	82	79	79	79	81	82
Adj. R^2	0.865	0.950	0.957	0.947	0.923	0.232

	Portfolio					
Panel B: Value-weighted	1	2	3	4	5	H-L
Factor loading	-0.300	-0.125	-0.029	0.073	0.295	0.596
Vol-of-vol	0.092	0.091	0.093	0.092	0.092	0.000
Excess returns	0.065	0.083	0.110	0.098	0.155	0.090
Intercept	-0.032	-0.028	-0.019	-0.027	-0.027	0.005
	(-1.33)	*(-0.97)*	*(-0.55)*	*(-0.79)*	*(-0.47)*	*(0.08)*
Mkt-RF	0.991***	1.121***	1.285***	1.284***	1.876***	0.885***
	(126.17)	*(122.48)*	*(117.64)*	*(118.73)*	*(100.11)*	*(42.10)*
SMB	-0.090***	-0.174***	-0.125***	-0.139***	-0.068***	0.022
	(-5.72)	*(-9.52)*	*(-5.73)*	*(-6.40)*	*(-1.81)*	*(0.52)*
HML	-0.004	-0.048***	-0.112***	0.070***	0.026***	0.029
	(-0.28)	*(-3.23)*	*(-6.34)*	*(3.99)*	*(0.85)*	*(0.87)*
MOM	0.150***	0.059***	0.156***	0.078***	0.044***	-0.105***
	(15.10)	*(5.08)*	*(11.35)*	*(5.68)*	*(1.88)*	*(-3.96)*
Avg. # of stocks	82	79	79	79	81	82
Adj. R^2	0.803	0.797	0.779	0.791	0.732	0.347

*Note: Table 5.7 reports a summary of the regression results of Equations 4.5 and 4.6 during the sample period January 2005 to March 2023 with the factor loading portfolios, where Portfolio 1 (5) consists of the stocks with the stocks with the lowest (highest) factor loadings (see Methodology 4.4). Portfolio H-L is the factor mimicking portfolio and calculated by taking Portfolio 5 minus Portfolio 1 (see Equation 4.8). Factor loading is the average factor loading of the stocks in portfolios. Vol-of-vol is the average vol-of-vol characteristics of the stocks in portfolios. Intercept is reported on an annual basis by taking the result multiplied with 252. The t-statistic for each factor loading is reported in parentheses. Companies' factor loading of the vol-of-vol factor is calculated using a time-series regression with a rolling window of 500 days with daily excess log returns as dependent variables and the factors in C4FM and the vol-of-vol factor as independent variables (see Methodology 4.4). Further, Panel A presents values when the portfolios are computed using an equally-weighted approach while Panel B presents values when the portfolios are computed using a value-weighted approach (see Equations 4.2 and 4.3). * (**) (***) indicate significance at the 10% (5%) (1%) level. Full regression results for the High-Minus-Low Portfolio are presented in Appendices C.12 and C.13.*

5.4 Investor confidence

Table 5.8 presents the comparison of vol-of-vol under different market sentiments, as measured by CCI and AAII separately. One notable finding is that the indices differ in terms of the number of bullish days, with CCI being less bullish compared to AAII. To further analyze the impact of investor confidence, a dummy variable is included in the regression analysis (Equation 4.10). The results indicate an even more negative 4F alpha when investor sentiment is bearish. When AAII's bull-bear spread is positive, the intercept for the vol-of-vol factor is -6.6%, suggesting that high vol-of-vol stocks perform better when market sentiment is positive. Although the dummy variable is statistically significant, the standard error is high, and the corresponding 95% confidence interval is 7.0-29.0%. Consequently, the evidence suggests that market sentiment has a significant effect on the vol-of-vol factor, but its exact effect is vague.

Table 5.8: Regression results with indices for investor confidence

	AAII	CCI
Excess return		
Bull	-0.024	-0.072
Bear	-0.257	-0.152
Intercept	-0.246***	-0.181***
	(-5.71)	*(-5.06)*
Intercept bull	0.180***	0.103*
	(3.22)	*(1.83)*
Mkt-RF	0.228***	0.230***
	(24.71)	*(24.85)*
SMB	-0.204***	-0.203***
	(-11.20)	*(-11.13)*
HML	0.118***	0.118***
	(7.82)	*(7.84)*
MOM	-0.072***	-0.072***
	(-6.2)	*(-6.18)*
# of bullish days	2,720	1,862
# of bearish days	1,872	2,730

*Note: Table 5.8 reports the regression summary of Equation 4.10 during the sample period January 2005 to March 2023 with excess returns from the vol-of-vol High-Minus-Low Portfolio using a value-weighted approach. Intercept is reported on an annual basis by taking the result multiplied with 252. The t-statistics for the factor loadings are reported in parentheses under the factor. Intercept bull is a dummy variable taking the value 1 (0) when market sentiment is defined as bull (bear) according to AAII and CCI, respectively. * (**) (***) indicate significance at the 10% (5%) (1%) level. Full regression results for the High-Minus-Low Portfolio using a value-weighted approach with dummy variables for AAII and CCI are presented in Appendices C.14 and C.15*

6

Discussion

This chapter focuses on interpreting the results concerning the purpose of the report which is to evaluate the price dynamics of uncertainty in the cross-section of stock returns. The analysis will discuss the potential explanations of the vol-of-vol effect, starting with a discussion of what drives vol-of-vol as it can be seen as a risk measure explaining stock returns, followed by an analysis of the pricing dynamics of vol-of-vol and its relationship with investor sentiment. Furthermore, we will discuss and critically analyze potential methodological concerns that may influence the reliability and validity of the results obtained.

6.1 Drivers of the vol-of-vol effect

In the first part of the results, we illustrate the characteristics of a stock with high and low vol-of-vol respectively. Surprisingly, we find that vol-of-vol is not related to risk measures such as volatility and CAPM beta. Initially, one might expect vol-of-vol, being a risk measure itself, to have a positive relationship with other risk measures such as volatility and CAPM beta. However, our findings suggest that market uncertainty regarding future volatility is independent of whether a stock is classified as "high-risk" or "low-risk" according to traditional finance literature.

Part of the explanation is found in the definition of vol-of-vol, in which we scale the previous vol-of-vol with the average IV of the previous month. This allows us to measure the market's perception of stock-specific uncertainty in its rawest form, free from the influence of IV of the stock itself, as the IV is already reflected in the stock price. In addition to risk characteristics, we find a positive relationship between high vol-of-vol and the size of companies. This finding is also surprising since Zhang (2006) shows that larger companies tend to have more analyst coverage and lower dispersion in analyst forecasts, which they use as a proxy for lower uncertainty about the future. Overall, our results indicate that the factors driving vol-of-vol are distinct from traditional risk measures and are not solely dependent on the company's risk profile.

6.2 Vol-of-vol

The results show that stocks with higher vol-of-vol underperform stocks with low vol-of-vol. Specifically, the value-weighted High-Minus-Low Portfolio has a negative average annual return of 11.9%. The results are also significant with a 4F alpha of -13.9% on an annual basis, and is consequent when dividing the sample period into multiple time periods. This aligns with Baltussen et al.'s (2012) findings, suggesting that stocks with higher vol-of-vol might be overpriced relative to their fair fundamental value, potentially due to an optimism bias in the market. Miller (1977) and Chen et al. (2001) propose that stocks tend to have a more optimistic valuation when short-sale constraints are in place, as pessimistic investors' views are not fully reflected in the price. When there is a disagreement among market participants regarding the profitability of a company, the price tends to increase relative to its intrinsic value, resulting in a negative expected return. The significant disparity in returns between high-low vol-of-vol stocks may be partially attributed to the direct effect of short-sale constraints, as vol-of-vol is a measure of disagreement about risk. In contrast, short-sale constraints are usually more applicable to smaller stocks (Boehme et al., 2006), and the results show a positive correlation between higher vol-of-vol and larger companies. This contradicts the explanation that optimism bias in the presence of short-sale constraints can explain the vol-of-vol effect.

Overoptimism is closely linked to the overconfidence bias, which can contribute to the vol-of-vol effect. When there is variation in agents' aversion to uncertainty, those with lower ambiguity aversion, often associated with overconfidence bias, tend to hold onto risky assets (Bossaerts et al., 2010). This suggests that investors may be willing to pay a premium for stocks with high uncertainty levels because they perceive themselves as more knowledgeable than other market participants or because overconfident investors have a preference for positive skewness. This explanation aligns with Mitton and Vorkink's (2007) findings that idiosyncratic skewness can impact equilibrium prices, and stocks with high skewness tend to underperform on average.

Despite the high vol-of-vol portfolio having a significantly lower average return during the observed time period, it exhibits higher skewness (2.29) and a higher maximum daily return (15.3%) compared to the low vol-of-vol stock portfolio (with skewness of 1.54 and maximum daily return of 10.9%). However, the presence of asymmetric return distributions may not be visual in the portfolios and limited to individual stocks. Therefore, additional percentiles of stock returns are provided in Appendix A.2. Although high vol-of-vol stocks show higher return percentiles (0.75, 0.85, 0.95), indicating a willingness by investors to pay a premium for the upside tail risk, we do not consider this sufficient justification for the vol-of-vol effect.

In light of this, we investigated the overconfidence bias further by looking at the time variation of the vol-of-vol effect conditional on the market sentiment. By adding a dummy variable for when the market sentiment, according to both CCI and AAI, is defined as bullish or bearish, we observe that the vol-of-vol effect is even stronger during bearish sentiment. In this case, the High-Minus-Low Portfolio has a negative 4F alpha of 24.6% when AAI's Bull-Bear spread was negative. This indicates that the phenomenon known as "flight to safety", where investors shift their capital from riskier asset classes in potential downturns, applies to stocks as well. Potential further research in this area would be to investigate whether this behavior is driven by increased activity from noise traders in uncertain stocks, as suggested by De Long et al. (1990), that these traders are influenced by market sentiment. However, the impact of sentiment was weaker when defining it using CCI. In CCI, the dummy variable for the bullish sentiment was positive, indicating that high vol-of-vol stocks underperform less when the sentiment is bullish. The t-statistic of 1.83 suggests the need for further investigation to determine if consumer confidence truly affects the vol-of-vol effect. One approach could be to analyze changes in consumer confidence instead of focusing solely on the level of confidence, as these changes may have a more significant impact on decision-making shifts.

6.3 Criticism of the factor loading methodology

We are not able to show that the underperformance related to the vol-of-vol effect is a priced factor in the market. A possible reason might be that the market is inefficient when it comes to pricing uncertainty or that the vol-of-vol effect is priced through another factor model such as APT, which allows for various systematic risk factors. However, it can also be explained by our definition of uncertainty. Vol-of-vol measures the market's uncertainty regarding the one-month forward-looking volatility. In other words, if the market is uncertain about the volatility for the next month, it results in a high vol-of-vol. This measure is distinct from IV, which is a measure of the priced "risk" in the stock. If the market prices in higher volatility for the next month but then subsequently agrees on the higher volatility, it results in a lower vol-of-vol. This definition leads to companies frequently switching between being high and low vol-of-vol stocks, see Table 5.3, as vol-of-vol measures the market's uncertainty about the pricing of future volatility. This is significantly different from constructing portfolios with the FMP approach based on factor loadings against the vol-of-vol factor, as an individual stock's factor loading does not change as frequently, see Table 5.8.

The use of linear regression with a 500 day lookback horizon to examine the price dynamics of the vol-of-vol factor may be called into question based on the findings presented. Despite the evident impact of vol-of-vol characteristics on returns, the construction of portfolios using the FMP approach with companies' factor loadings does not yield a significant 4F alpha. This suggests that the relationship between the vol-of-vol factor and companies' returns may not be linear, or only exist in the short-term. Consequently, utilizing portfolios with a non-linear regression model or a shorter lookback horizon can potentially lead to more reliable insights into how companies' sensitivity to the vol-of-vol factor is priced within a factor model. However, since factor loadings from non-linear regression models are hard to interpret and a short lookback horizon could yield biased results, another appropriate methodology would be the *variance risk premium beta* proposed by Carr and Wu (2009). On the other hand, Carr and Wu (2009) does not find evidence of how the market prices in uncertainty. Instead, the study concludes that the market may exhibit high inefficiency in pricing uncertainty or that investors willingly accept lower returns as a hedge against spikes in overall market volatility.

7

Conclusion

This paper aims to evaluate the pricing dynamics of uncertainty in the stock market. Traditional finance literature focuses extensively on risk and risk preferences as the primary drivers of investors' choices and, ultimately, asset pricing. However, risk in the form of volatility assumes a known probability distribution of expected returns, which does not fully capture the complexity of market dynamics. Uncertainty surrounding the probability distribution has been argued to be a crucial factor in asset pricing, yet it has received relatively little attention in research. Hence, this study explores the pricing dynamics of uncertainty in the cross-section of stock returns.

Our primary findings demonstrate the existence of a significant vol-of-vol effect in the market. Specifically, the quintile portfolio of high vol-of-vol stocks exhibits an average annual underperformance of 11.9% compared to the quintile portfolio comprising low vol-of-vol stocks. This finding is noteworthy as it surpasses the impact of the factors within C4FM, i.e. SMB, HML, and MOM, over the sample period January 2005 to March 2023. Furthermore, we do not find evidence that the vol-of-vol effect is driven by different stock characteristics, as there is no significant disparity in volatility or CAPM beta between high and low vol-of-vol stocks. Moreover, the result remains significant when controlling for other factors, with the High-Minus-Low Portfolio exhibiting a negative annual 4F alpha of 13.9%. The vol-of-vol factor is robust during multiple time period within the sample period, and when considering other factor models (see Appendix B). A possible explanation of the vol-of-vol effect is overconfidence bias, where investors exhibit a preference for upside tail exposure, a characteristic commonly found in high uncertainty stocks. We examine the influence of overconfidence bias in the vol-of-vol effect by adding a dummy variable for a bullish investor sentiment, finding that the High-Minus-Low Portfolio's 4F alpha is even more negative during bearish sentiment. This implies that our perception of overconfidence bias being an explanation to the vol-of-vol is plausible, although the significance of consumer confidence impact is vague. Consequently, we propose further research to investigate noise traders' impact on the sentiment varying effect of vol-of-vol.

Our findings do not provide conclusive evidence that the vol-of-vol effect is a priced factor in the market. This could be due to the possibility that vol-of-vol is already priced through another factor, or the existence of a non-linear relationship between vol-of-vol and its factor loadings. Another explanation is that the market is inefficienct in terms of pricing uncertainty, as indicated by Carr and Wu's (2009) examination of the market's variance risk premium. However, it is important to acknowledge the limitation of this study that portfolios are rebalanced on a monthly basis, which incorporates transaction costs that have not been considered in this analysis. Hence, to further explore the efficiency in the market with regard to uncertainty, a promising research direction would involve a closer examination of investment strategies constructed based on the vol-of-vol effect. As for now, we draw the conclusion that *there is nothing certain but the uncertain.*

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Appendix

A Descriptive statistics

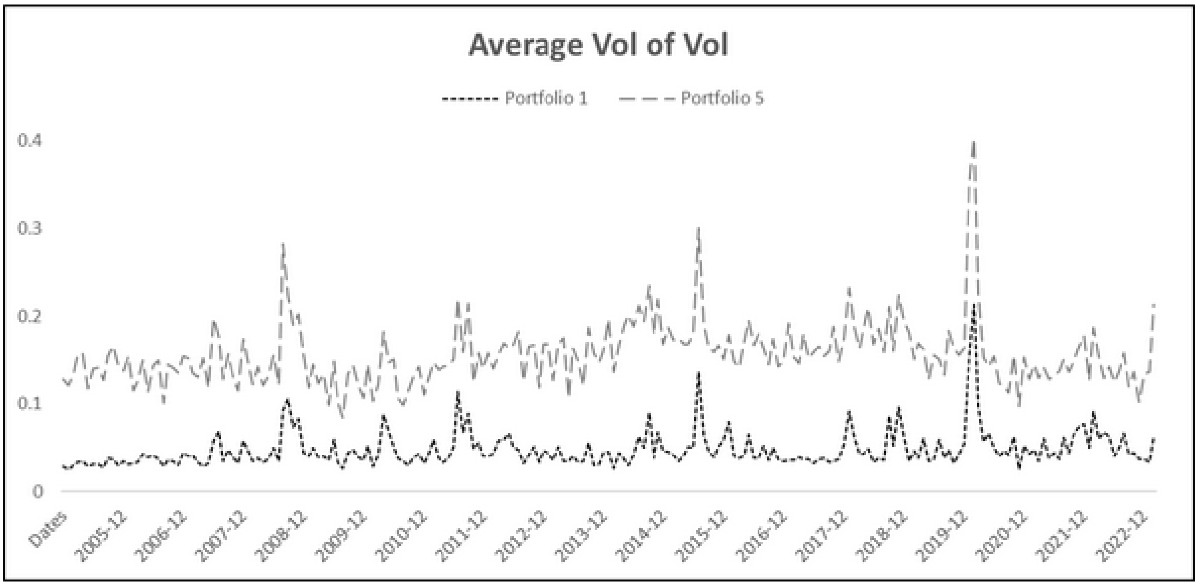

Figure A.0: Average vol-of-vol during January 2005 to March 2023

Table A.1: Correlation matrix

	MKT-RF	SMB	HML	MOM	VOV
MKT-RF	1.000				
SMB	0.186	1.000			
HML	0.182	-0.071	1.000		
MOM	-0.279	-0.057	-0.435	1.000	
VOV	0.370	-0.089	0.231	-0.233	1.000

Table A.2: Average return statistics of individual stocks within the quintile portfolios

	Portfolio				
	1	2	3	4	5
Skewness	-0.071	-0.157	-0.215	-0.298	-0.529
Kurtosis	8.364	8.406	8.647	10.393	14.668
Max	0.111	0.113	0.116	0.122	0.142
95th Percentile	0.030	0.030	0.030	0.031	0.033
85th Percentile	0.017	0.016	0.017	0.017	0.018
75th Percentile	0.010	0.010	0.010	0.011	0.011
65th Percentile	0.006	0.006	0.006	0.006	0.006
55th Percentile	0.002	0.002	0.002	0.002	0.002
50th Percentile	0.001	0.001	0.001	0.001	0.001
45th Percentile	-0.001	-0.001	-0.001	-0.001	-0.001
35th Percentile	-0.005	-0.005	-0.005	-0.005	-0.005
25th Percentile	-0.009	-0.009	-0.010	-0.010	-0.010
15th Percentile	-0.015	-0.016	-0.016	-0.016	-0.017
5th Percentile	-0.029	-0.030	-0.031	-0.032	-0.035
Min	-0.112	-0.117	-0.122	-0.133	-0.173

*Note: * stands for daily excess return.*

B Robustness checks

Table B.3: Regression vol-of-vol factor 2005-2009

Regression statistics						
Adj R^2	0.486					
St. error	0.008					
Observations	1258					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.083	0.021	297.972	0.000	
Residual	1253	0.087	0.000			
Total	1257	0.171				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.000	-3.080	0.002	-0.001	0.000
MKT-RF	0.189	0.019	9.848	0.000	0.151	0.226
SMB	-0.310	0.037	-8.368	0.000	-0.383	-0.237
HML	0.519	0.039	13.430	0.000	0.443	0.595
MOM	-0.128	0.027	-4.697	0.000	-0.182	-0.075
Intercept annual	-0.183	0.000	-3.080	0.002	-0.299	-0.066

Table B.4: Regression vol-of-vol factor 2010-2014

Regression statistics						
Adj R^2	0.052					
St. error	0.005					
Observations	1258					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.002	0.000	18.139	0.000	
Residual	1253	0.026	0.000			
Total	1257	0.028				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	-3.167	0.002	-0.001	0.000
MKT-RF	0.122	0.015	8.022	0.000	0.092	0.152
SMB	-0.114	0.028	-4.153	0.000	-0.169	-0.060
HML	0.067	0.032	-2.086	0.037	-0.129	-0.004
MOM	0.030	0.022	1.350	0.177	-0.014	0.074
Intercept annual	-0.103	0.000	-3.167	0.002	-0.167	-0.039

Table B.5: Regression vol-of-vol factor 2015-2019

Regression statistics						
Adj R^2	0.154					
St. error	0.005					
Observations	1258					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.006	0.001	58.407	0.000	
Residual	1253	0.031	0.000			
Total	1257	0.037				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	-2.039	0.042	-0.001	0.000
MKT-RF	0.194	0.017	11.674	0.000	0.161	0.226
SMB	-0.004	0.029	-0.124	0.901	-0.061	0.054
HML	-0.179	0.029	-6.101	0.000	-0.236	-0.121
MOM	0.066	0.021	3.133	0.002	0.025	0.108
Intercept annual	-0.072	0.000	-2.039	0.042	-0.142	-0.003

Table B.6: Regression vol-of-vol factor 2019-2023

Regression statistics						
Adj R^2	0.073					
St. error	0.008					
Observations	1070					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.006	0.002	22.112	0.000	
Residual	1065	0.076	0.000			
Total	1069	0.082				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.000	-2.740	0.006	-0.001	0.000
MKT-RF	0.174	0.019	9.386	0.000	0.138	0.210
SMB	-0.049	0.037	-1.338	0.181	-0.121	0.023
HML	0.014	0.023	0.606	0.545	-0.031	0.059
MOM	0.039	0.02	1.988	0.047	0.001	0.078
Intercept annual	-0.178	0.000	-2.740	0.006	-0.306	-0.051

Table B.7: Regression vol-of-vol factor CAPM

Regression statistics						
Adj R^2	0.136					
St. error	0.008					
Observations	4592					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	1	0.042	0.042	725.930	0.000	
Residual	4590	0.269	0.000			
Total	4591	0.311				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.000	-5.009	0.000	-0.001	0.000
MKT-RF	0.242	0.009	26.943	0.000	0.225	0.260
Intercept annual	-0.143	0.000	-5.009	0.000	-0.198	-0.087

Table B.8: Regression vol-of-vol factor FF3FM

Regression statistics						
Adj R^2	0.181					
St. error	0.007					
Observations	4592					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	3	0.057	0.019	339.722	0.000	
Residual	4588	0.254	0.000			
Total	4591	0.311				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.000	-5.067	0.000	-0.001	0.000
MKT-RF	0.242	0.009	26.489	0.000	0.224	0.260
SMB	-0.182	0.018	-9.866	0.000	-0.218	-0.146
HML	0.192	0.014	13.816	0.000	0.165	0.220
Intercept annual	-0.140	0.000	-5.067	0.000	-0.195	-0.086

Table B.9: Regression vol-of-vol factor FF5FM

Regression statistics						
Adj R^2	0.247					
St. error	0.007					
Observations	4592					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	5	0.077	0.015	300.911	0.000	
Residual	4586	0.234	0.000			
Total	4591	0.311				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	-4.420	0.000	-0.001	0.000
MKT-RF	0.172	0.009	18.201	0.000	0.153	0.190
SMB	-0.227	0.018	-12.539	0.000	-0.263	-0.192
HML	0.342	0.016	21.855	0.000	0.311	0.373
RMW	-0.228	0.025	-8.944	0.000	-0.277	-0.178
HML	-0.563	0.034	-16.517	0.000	-0.630	-0.496
Intercept annual	-0.118	0.000	-4.420	0.000	-0.170	-0.065

C Extended regression results

Table C.10: Regression vol-of-vol factor value-weighted

Regression statistics						
Adj. R^2	0.191					
St. error	0.007					
Observations	4,592					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.060	0.015	271.961	5e-210	
Residual	4,587	0.251	0.000			
Total	4,591	0.311				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.000	-5.053	0.000	-0.001	0.000
MKT-RF	0.230	0.009	24.844	0.000	0.212	0.248
SMB	-0.203	0.018	-11.136	0.000	-0.239	-0.168
HML	0.118	0.015	7.814	0.000	0.088	0.147
MOM	-0.071	0.012	-6.151	0.000	-0.094	-0.049
Intercept annual	-0.139	0.000	-5.053	0.000	-0.193	-0.085

Table C.11: Regression vol-of-vol factor equally-weighted

Regression statistics						
Adj. R^2	0.190					
St. error	0.007					
Observations	4,592					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.041	0.010	270.266	7e-209	
Residual	4,587	0.175	0.000			
Total	4,591	0.216				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	-4.669	0.000	-0.001	0.000
MKT-RF	0.164	0.008	21.314	0.000	0.149	0.179
SMB	-0.124	0.015	-8.182	0.000	-0.154	-0.095
HML	0.168	0.013	13.380	0.000	0.144	0.193
MOM	-0.055	0.010	-5.660	0.000	-0.074	-0.036
Intercept annual	-0.107	0.000	-4.669	0.000	-0.152	-0.062

Table C.12: Regression vol-of-vol factor loading value-weighted

Regression statistics						
Adj. R^2	0.232					
St. error	0.008					
Observations	4,091					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.078	0.019	309.261	0.000	
Residual	4,086	0.256	0.000			
Total	4,090	0.334				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	-0.127	0.899	0.000	0.000
MKT-RF	0.168	0.010	16.688	0.000	0.148	0.188
SMB	0.186	0.020	9.239	0.000	0.147	0.226
HML	0.183	0.016	11.191	0.000	0.151	0.215
MOM	-0.159	0.013	-12.547	0.000	-0.184	-0.134
Intercept annual	-0.004	0.000	-0.127	0.899	-0.065	0.057

Table C.13: Regression vol-of-vol factor loading equally-weighted

Regression statistics						
Adj. R^2	0.347					
St. error	0.017					
Observations	4,091					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	4	0.597	0.149	545.519	0.000	
Residual	4,086	1.118	0.000			
Total	4,090	1.716				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	0.075	0.940	0.000	0.000
MKT-RF	0.885	0.021	42.100	0.000	0.844	0.927
SMB	0.022	0.042	0.522	0.602	-0.061	0.104
HML	0.029	0.034	0.865	0.387	-0.037	0.096
MOM	-0.105	0.027	-3.961	0.000	-0.157	-0.053
Intercept annual	0.005	0.000	0.075	0.940	-0.123	0.133

Table C.14: Regression vol-of-vol factor AAII

Regression statistics						
Adj R^2	0.193					
St. error	0.007					
Observations	4592					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	5	0.060	0.012	220.081	0.000	
Residual	4586	0.251	0.000			
Total	4591	0.311				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.000	-5.705	0.000	-0.001	-0.001
Sentiment Dummy	0.001	0.000	3.216	0.001	0.000	0.001
MKT-RF	0.228	0.009	24.706	0.000	0.210	0.247
SMB	-0.204	0.018	-11.196	0.000	-0.240	-0.169
HML	0.118	0.015	7.817	0.000	0.088	0.147
MOM	-0.072	0.012	-6.200	0.000	-0.095	-0.049
Intercept annual	-0.246	0.000	-5.705	0.000	-0.330	-0.161
Dummy annual	0.180	0.000	3.216	0.001	0.070	0.290

Table C.15: Regression vol-of-vol factor CCI

Regression statistics						
Adj R^2	0.191					
St. error	0.007					
Observations	4592					
ANOVA	*df*	*SS*	*MS*	*F*	*Significance*	
Regression	5	0.060	0.012	218.351	0.000	
Residual	4586	0.251	0.000			
Total	4591	0.311				
	Beta	*Std. error*	*t-stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.000	-5.063	0.000	-0.001	-0.000
Sentiment Dummy	0.000	0.000	1.831	0.067	-0.000	0.001
MKT-RF	0.230	0.009	24.845	0.000	0.212	0.248
SMB	-0.203	0.018	-11.125	0.000	-0.239	-0.167
HML	0.118	0.015	7.843	0.000	0.089	0.148
MOM	-0.072	0.012	-6.175	0.000	-0.094	-0.049
Intercept annual	-0.181	0.000	-5.063	0.000	-0.251	-0.111
Dummy annual	0.103	0.000	3.216	0.001	-0.007	0.213