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The Higgs Width in Higgs production in association with a W^+ boson

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Abstract

The width of the Higgs boson is a quantity that cannot be measured directly from the on-shell Higgs peak; it would have to be measured indirectly. In 2013, Caola and Melnikov developed a method to determine the Higgs width by using off-shell and on-shell cross-sections and applying the narrow-width approximation. In this thesis, we will use the indirect method of finding the Higgs width for the production of the Higgs boson in association with a W^+ boson. The specific decay mode of the Higgs boson will be its decay to two Z bosons. In theory, this specific decay mode of the scalar boson must result in an off-shell and on-shell region. We simulated proton-proton collisions that produce the Higgs and W^+ boson using the MadGraph5_MC@NLO software and, using the generated data, we were able to plot a histogram for when the Higgs width equals the Standard Model Higgs boson width calculated at LO accuracy. The same was done when the width is two times the Standard Model Higgs boson width and when it is five times the Standard Model Higgs boson width. From the analysis of the histograms and finding the total cross-section of the on-shell and off-shell region in each of the histograms, we found that the results agree with the relationship between the on-shell cross-section and the Higgs width when the narrow-width approximation is applied. The same can be said for the off-shell cross-section.

Popular Science Summary

The Higgs Width: a tool for the search for new particles

Mass is an intrinsic property of all matter; this is what we've been taught in school. However, particle physicists do not think this is the case. Not all particles have mass; an example of a massless particle is the photon, the particle of light. All particles get their mass from the Higgs field, whose particle manifestation is the Higgs boson discovered at CERN in 2012. The discovery of the Higgs boson provided evidence of the existence of the Higgs field and this confirmed the validity of the Higgs mechanism. This theory explains how particles get mass from the Higgs field. But this doesn't mean that our work with the Higgs boson is over. In fact, it was just the beginning of a new chapter in particle physics! There's more to investigate about the Higgs boson such as its width. A measurement of this property of the boson will provide clues to the existence of new particles.

How particles gain mass from the Higgs field and what the Higgs boson is can be explained easily by using an analogy created by physicist David Miller of University College London [1]. Imagine the Higgs field as a room full of physicists at a cocktail party. When the tax collector enters the room, no one interacts with him because who likes to pay taxes? The tax collector isn't held back by people so he moves freely. One could think of the tax collector as a massless particle such as the photon. When a famous person such as Peter Higgs, the physicist who theorized the existence of the Higgs boson, enters the room, the physicists are thrilled to see him and everyone tries to talk to him. Peter Higgs interacted with the crowd much more than the tax collector and he represents a heavy particle. When the tax collector and Peter Higgs are out of the room there is no difference between their masses but when they enter the room Peter Higgs has much more mass than the tax collector.

The Higgs boson is a particle just like the photon and electron. It has no electric charge and has a mass of about 125 GeV (which is 125 times heavier than the proton). It is the particle of the Higgs field just as the photon is the particle of the electromagnetic field. Unlike protons and electrons, the Higgs boson doesn't live very long, its lifetime is only 1.6×10^{-22}s [2]. Nevertheless, we've still detected it albeit indirectly. It is the short lifetime of the Higgs boson which physicists find very useful.

Short-lived particles decay into more stable particles and this is the case with the Higgs boson. Thus in the detector what we detect is not the Higgs boson but its final decay products which are photons. Due to mass-energy conservation, the total energy of the photons will be equal to the total mass of the Higgs boson. If we plot a graph of the total number of particles detected over the total energy of the detected particles we get a hump in the graph which has a considerable width. This is known as the width of the Higgs boson and the hump represents the Higgs particle. All short-lived particles have a width and this width is related to their lifetime. The longer the lifetime the narrower the width.

There are different ways of producing Higgs bosons. So far not all of the production modes have been investigated by experimental particle physicists. The production mode that we will be looking at is one of the modes that hasn't been investigated yet. By generating events in a particle collision simulator we can get to find a value for the Higgs width obtained by the production mode we are interested in. If the width is different from the value given by the Standard Model, then this indicates that there may be new physics such as the existence of dark matter particles.

There's still more to uncover about the Higgs boson. Even though it was discovered more than 10 years ago it still is relevant to the particle physics community. The Higgs boson may hold the answer to the existence of dark matter particles and we'll never know unless we work hard and do our research.

Contents

List of acronyms

1. SM : Standard Model
2. BSM : Beyond Standard Model
3. LHC : Large Hadron Collider
4. LHE : Les Houches Event
5. NLO : Next to Leading Order

6. LO : Leading Order
7. NWA : Narrow Width Approximation

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1 Introduction

The Higgs boson is the only elementary scalar boson in the Standard Model of particle physics. Theorized back in 1964, it was discovered about 50 years later at the LHC at CERN. The Higgs width is one important facet of the Higgs boson and further study of it will help us answer questions relating to dark matter and matter anti-matter asymmetry.

It is important to mention that the Higgs boson is the quanta of the Higgs field. The Higgs field is described by the Brout-Englert mechanism. This mechanism was named after the three physicists who formulated it in 1964; Robert Brout, François Englert (both worked on the paper describing the mechanism together), and Peter Higgs [3]. The purpose of the mechanism was to fix an error in the theory of electroweak interaction which predicted that the masses of the W and Z bosons are zero which is not in agreement with experiment. Particles which interact with the Higgs field acquire mass and the amount of mass they have is proportional to the strength of their interaction with the Higgs field. The Higgs boson interacts with the Higgs field as well and as a result, it has a mass $m_H = 125.38 \pm 0.14$ GeV [4].

Only 5% of the universe can be explained by the Standard Model, 26% of it should be dark matter and the rest dark energy. Astronomical observations lead to the suggestion of the existence of dark matter. A well-known example is Fritz Zwicky's study of the Coma cluster, a cluster of galaxies, and gravitational lensing observations which were carried out in the 1970s [5]. In Zwicky's study, he found that the mass of the cluster that was calculated using the virial theorem was much larger than the mass calculated using the mass-to-luminosity ratio of the cluster. He coined the term "dark matter" which refers to the extra mass that does not give off light. The Standard Model does not have a candidate for dark matter and this indicates that this theory must be extended. Such theories that can be considered extensions of the SM are known as "Beyond the Standard Model" or BSM. One such BSM theory which has a possible candidate for dark matter is supersymmetry. The dark matter candidate is called the neutralino. It is a neutrally charged weakly interacting particle also known as a WIMP. These types of particles interact via the weak force with SM particles, however, the interactions are very weak for them to be detected easily. Their effects can only be seen when they interact via the gravitational force (Since dark matter candidates are WIMPS we have already mentioned these effects above). As dark matter particles have mass there is a possibility of them interacting with the Higgs boson. The mass of the lightest neutralino lies in the range of about 10- 10,000 GeV [6]. Hence, if the mass of the dark matter particles is less than the mass of the Higgs boson then the boson can decay into these dark matter particles. One can deduce that by studying more about the Higgs boson, specifically its width, we can learn about the existence of dark matter particles.

The width of the Higgs boson is affected by the particles which interact with it. According to the Standard Model, the width of the Higgs boson is $\Gamma_H = 4.1$ MeV [2]. If the width of this boson is measured to be larger than the Higgs width this means that it has interacted with dark matter particles. Since the width of the Higgs boson is very small it can only be measured indirectly. One way of measuring it indirectly was proposed by Fabrizio Caola and Kirill Melnikov in 2013 and it is described in detail in the theory section of this paper. This method is possible because of the validity of the narrow-width approximation to the Higgs width and the independence of the off-shell cross-section from the Higgs width. One gets the width by taking the ratio of the total cross-section in the off-shell region and the total cross-section of the on-shell region. The width is proportional to this ratio.

It should be noted that only certain Higgs boson decays result in an off-shell and on-shell region. The decay of the boson to either two Z bosons or two W bosons are some examples. This is due to the fact that the difference between the mass of these two types of vector bosons and the Higgs boson is not very large compared to the difference in mass between other particles in the SM and the mass of the scalar boson. So when the Higgs boson is off-shell the vector bosons will be produced on- shell and this results in an enhanced cross-section around the mass of the vector bosons, hence an off-shell cross-section that can be used to find the width of the Higgs boson.

Besides the width of the Higgs boson the mass of this particle is important as well. This is because it hints at physics beyond the Standard Model, specifically supersymmetry. The Standard Model does not have a specific value for the mass of the Higgs boson, it is a free parameter. Although this is the case in the Standard Model, experiments have shown that the mass of the Higgs always lies close to 125 GeV. At larger energy scales, that is, scales beyond the weak scale, and also when the Higgs boson interacts with heavy particles such as the top quark the mass of the Higgs boson is affected by radiative corrections which comes from quantum loops. These quantum loops contain fermions and bosons. Additionally, these corrections cause the mass of the Higgs boson to be about 10^{15} GeV. This, of course, is much larger than the observed mass of the boson. To explain the lightness of the Higgs boson one needs to extend the Standard Model, and one of the candidates for that is to use the principle of supersymmetry, also known as SUSY. In SUSY, each fermion in the Standard Model has a partner boson, and vice versa [7]. Therefore, these partner bosons and fermions can be used to cancel out the fermions and bosons which contribute to the corrections of the mass of the Higgs boson, thus maintaining the light mass of the Higgs boson which, is about 125 GeV. Therefore, we can conclude that SUSY provides an explanation as to why the mass of the Higgs boson is smaller than expected, and some of the particles predicted by SUSY are dark matter particles.

The specific process we are going to study in this project is the production of the Higgs boson via Higgs Stralung and the decay of this boson to two Z bosons. To generate the events and the Feynman diagrams a software called MadGraph5_MC@NLO will be used. MadGraph5_MC@NLO is capable of generating particle collisions using Monte Carlo methods. Using the information generated by the software for each run a histogram with the cross-section per bin over the total invariant mass of the decay products will be plotted. The process will be repeated for the Higgs width which is twice the Standard Model Higgs width and five times the Standard Model width. We expect to see a pronounced peak at around the rest mass of the Higgs boson which is 125 GeV and also a peak around 220 GeV.

The thesis is structured as follows. The first section, namely the Theory section, contains the preliminary background that is required to understand the research and builds up with the required theory needed to find the width of the Higgs boson indirectly. Also, why we expect to see two prominent peaks in the off-shell and on-shell regions of the Higgs boson is also explained. The next section gives a brief summary of what was done in my research and presents the results obtained. In the final section, I will discuss whether the results agree with the theory presented in the Theory section and the assumptions made etc.

2 Theoretical Background

2.1 Preliminary Background

The Higgs boson is a resonance particle, it has a short lifetime. It has a lifetime of about 10^{-22}s. The particle only travels a distance of 10^{-15}m, about the diameter of the proton, before decaying [8]. Thus we can't measure it directly. Only the final state particles (the decay products) are detected. The total energy of the detected particles is measured and a graph of the total cross-section over the total invariant mass is plotted. There will be a peak at a particular energy value and this indicates the presence of a resonance particle. Fig. 1 illustrates what was said above. The peak indicates that the intermediate particle causes an increase in the chances of the collision happening. It must have a non-zero width due to the resolution of detectors and also due to Heisenberg's uncertainty principle. Heisenberg's uncertainty principle states that the uncertainty in energy times the uncertainty in time is bounded, to be more precise:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{2.1}$$

where ΔE is the uncertainty in the energy measurement and Δt is the uncertainty in the lifetime measurement.

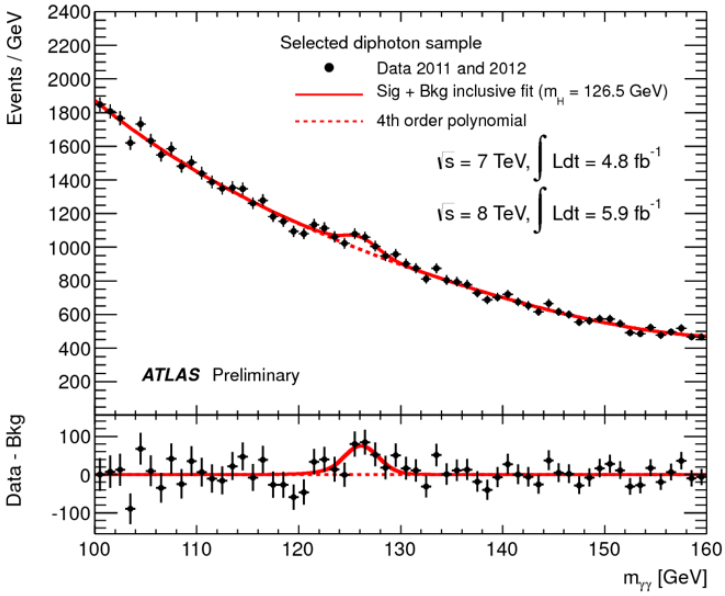

Figure 1: A graph of the events detected over the invariant mass of the diphotons from the H $\to \gamma\gamma$. There is a peak around 125 GeV and this was identified as the Higgs boson Image taken from `https://atlas.cern/updates/feature/higgs-boson`

An important measurement of unstable particles is their lifetime. The lifetime of a particle is a statistical value therefore we measure the mean lifetime of the particle. The mean lifetime of a particle is also known as its proper lifetime and it is related to the rate of decay in the following way:

$$\tau = \frac{1}{\Gamma} \tag{2.2}$$

where τ is the mean lifetime and Γ is the rate of decay . The rate of decay Γ is also known as the width of the resonance particle, referring to the full width at half maximum of the graph shown in Fig.1 when detector resolution can be ignored. The reason for why this holds can be found if we consider Heisenberg's Uncertainty principle as shown in Eq. 2.1. The uncertainty in the lifetime Δt and τ are in the same order of magnitude so the equality $\Delta t = \tau$ holds. Consequently, we replace τ in Eq. 2.1 by Δt. Δt can be expressed in terms of ΔE (see Eq. 2.1) and so ΔE is related to the rate of decay. In fact ΔE is directly proportional to Γ thus Γ is also known as the decay width or the width of the unstable particle.
The general form for the rate of decay of an unstable particle into two particles is given in Eq.2.3 [9]

$$\Gamma(\overrightarrow{p}) = \frac{(2\pi)^4}{2E(\overrightarrow{p})} \int \frac{d^3k_1}{(2\pi)^3 2E_1(\overrightarrow{k_1})} \frac{d^3k_2}{(2\pi)^3 2E_2(\overrightarrow{k_2})} \delta^4(p - k_1 - k_2) g^2 , \tag{2.3}$$

where $\overrightarrow{p}$ is the total momentum of the system of particles, $\overrightarrow{k_i}$ is the momentum of the i^{th} decay product and $E(\overrightarrow{p})$ is equals to $\sqrt{\overrightarrow{p}^2 + m^2}$, i.e. the total energy of the system. The total energy of the i^{th} decay product is given by $E_i(\overrightarrow{k_i})$ and g^2 is the coupling constant. Finally, the delta function represents the conservation of mass-energy in the system.

The number of events in a particular process is directly proportional to the cross-section of the process. Thus, it tells us the likelihood of a particular process happening. The cross-section is mostly used in scattering processes and it depends on the size of the target particles, the forces involved when the incident particles and the target particles interact and the outgoing and incoming velocities of the final and initial state particles (in other words, their momentum). For a scattering event which involves two particles in the initial state and one final product particle the cross section is given as follows:

$$d\sigma = |\mathcal{M}|^2 \frac{S}{\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2)^2}} \left[\left(\frac{d^3 \overrightarrow{p_3}}{(2\pi)^3 2E_3} \right) \right] \times (2\pi)^4 \delta^4 (p_1 + p_2 - p_3)\,, \qquad (2.4)$$

where $\mathcal{M}$ is the amplitude of the process, p_i is then four vector of the i^{th} particle, $\overrightarrow{p_3}$ is the three vector of the final particle, E_i is the total energy of the i^{th} particle and m_i is the rest mass of the i^{th} particle [10]. The Dirac-Delta function at the end of the equation, $\delta^4(p_1 + p_2 - p_3)$ makes sure that the total momentum of the system is conserved and S is a statistical quantity that is used to make sure that particles of the same kind are only considered once.

For the process we consider in this project the two protons collide to give a W^+ boson and a Higgs boson. The Higgs boson is unstable so it decays into two Z bosons. The differential cross-section for such a process which involves an unstable intermediate particle is given in Eq. 2.5. Suppose two particles A and B collide to produce two particles C and D. Particle C is stable while particle D is unstable and continues to decay into particles denoted by Y. The total process is denoted by $A + B \rightarrow C + Y$ where $A + B$ represent the particles in the initial state and $C + Y$ represent the particles in the final state. The differential cross-section

$$\begin{aligned} d\sigma(A + B \rightarrow C + Y) &= \frac{d\Gamma(D \rightarrow Y)}{\Gamma(D \rightarrow \text{all})} d\sigma(A + B \rightarrow C + D) \\ &\times \left[\frac{1}{\pi} \frac{m_D \Gamma}{(m^2 - m_D^2)^2 + m_D^2 \Gamma^2} \right] \sqrt{\frac{\overrightarrow{p}^2 + m_D^2}{\overrightarrow{p}^2 + m^2}} d^3 p \, dm^2\,, \end{aligned} \qquad (2.5)$$

shows the likelihood of A and B colliding to give C and D from happening at a particular energy [9]. The variable m is the invariant mass of the unstable particle D which can be a virtual particle, the mass of the unstable particle is denoted by m_D and $\overrightarrow{p}$ is the momentum of this unstable particle D. $d\Gamma(D \rightarrow Y)$ is the partial decay width of D to one of the possible final states with particles Y and $d\sigma(A + B \rightarrow C + D)$.

There is also another special factor that affects the cross-section of the scattering process which will be relevant here; it is the presence of resonance particles. These particles increase the total cross-section of the scattering process and how the total cross-section is affected by the resonance particle is described mathematically by the relativistic Breit-Wigner formula. The Breit-Wigner formula is present within brackets in Eq. 2.5, indicating that this formula does indeed affect the cross-section of the scattering process. The formula for the Breit-Wigner distribution for an unstable particle is as follows:

$$W(m^2) = \frac{1}{\pi} \frac{m_D \Gamma}{(m^2 - m_D^2)^2 + m_D^2 \Gamma^2}, \tag{2.6}$$

where m^2 is the possible mass values that the resonance particle can take, m_D is the on-shell mass of the particle and Γ is the decay width of the resonant particle [9].

2.2 Off-shell and On-shell Higgs boson production

In a scattering process, when the Higgs boson is produced as an intermediate state (a resonance particle), it can be produced **on-shell** or **off-shell**. If a particle is produced on-shell this means that its mass is close to the nominal mass value. The nominal mass of the Higgs boson is m_H and the value, along with its uncertainty, is given in the introduction section. Off-shell Higgs bosons have a mass value that is far away from the nominal value. Fig. 2 shows the off-shell and on-shell regions in the graph of cross-section per mass interval over invariant mass for a process in which the produced Higgs boson decays into two Z bosons.The concept of off-shell and on-shell production of the Higgs boson will be important in the following subsection.

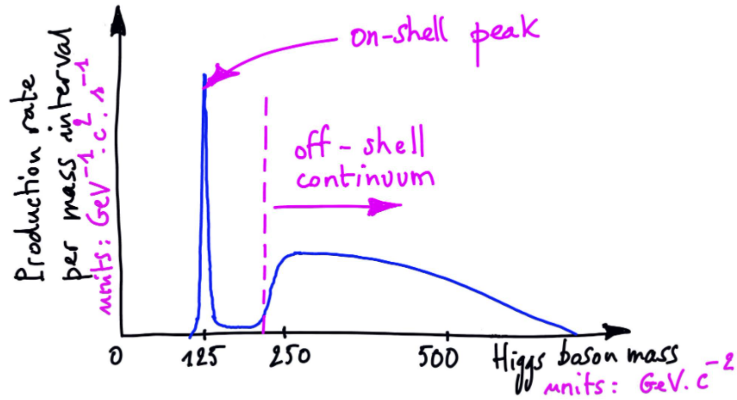

Figure 2: This graph refers to the Higgs boson decay into two Z bosons. There are two main regions in the graph; the on-shell peak and the off-shell continuum. The off-shell continuum has a peak, albeit not so well defined compared to the on-shell peak. Image taken from `https://cms.cern/news/life-higgs-boson`.

2.3 Finding the width of the Higgs boson indirectly.

The Standard Model width of the Higgs boson is $\Gamma_H = 4.1$ MeV [2]. Compared to the width of the Z boson, which is $\Gamma_Z = 2.5$ GeV [11], the width of the Higgs boson is much smaller than the width of the Z boson and Γ_Z can be measured directly from the line shape (which is described by Eq.2.6), unlike Γ_H. This drawback can be attributed to the resolution of present-day LHC detectors; the resolution of these detectors is up to the order of 1 GeV and it cannot be made smaller as of now. Ergo, the Higgs width must be measured indirectly and this is done by making use of the on-shell and off-shell Higgs boson production cross-sections. The method which I will describe briefly is given in detail in [12]. Before looking into this method it is essential to know about the narrow width approximation and a description of what it is and how it can be used will be given.

The narrow-width approximation is applied when the width of the resonance particle Γ is much smaller than its on-shell mass m_D (i.e. $\Gamma << m_D$). It makes use of the Dirac-delta function and this approximation is apt for much smaller widths. As you may already know, the Dirac-delta function makes calculations much easier. Suppose $m_D\Gamma$ is equals to ϵ and $(m^2 - m_D^2)^2$ is equals to η^2, then Breit-Wigner's formula becomes $W(m) = \frac{1}{\pi}\frac{\epsilon}{\eta^2+\epsilon^2}$ [9]. When the condition above is fulfilled the following limit will hold:

$$\lim_{\epsilon \to 0} \left\{ \frac{1}{\pi} \frac{\epsilon}{\eta^2 + \epsilon^2} \right\} = \delta(\eta)\,. \tag{2.7}$$

As a result, the narrow width approximation tells us that when ϵ approaches zero the probability distribution function $W(m)$ is approximately equal to $\delta(m^2 - m_D^2)$.

As was mentioned earlier, the narrow-width approximation makes calculations of total cross-section easier due to the presence of the Dirac-delta function. As a result, the total cross-section can be factorized into the production cross-section and decay cross-section. If we take the integral of Eq. 2.5 over all possible momenta values that the final states particles can take we can obtain the equation for the total cross-section for the process which is given in Eq. 2.8. Eq. 2.5 has three important parts in it, one part which represents the decay of the unstable particle $\left(\frac{d\Gamma(D\to Y)}{\Gamma(D\to all)}\right)$ the other is the Breit-Wigner distribution (Eq. 2.6) and the final one is the production of the Higgs boson along with the W^+ boson ($d\sigma(A + B \to C + D)$). The total cross-section of the scattering and decay process is given by σ and the variable p^2 is the magnitude of the total momentum of the virtual particle involved in the process (in this thesis we consider the Higgs virtual particle). The equation for the total cross-section of the production and decay of the resonant particle is

$$\sigma = \frac{1}{2s}\left[\int_{\vec{p}^{\,2}_{max}}^{\vec{p}^{\,2}_{min}} \frac{dp^2}{2\pi}\left(\int d\phi_p |\mathcal{M}_p(\vec{p}^{\,2})|^2 W(\vec{p}^{\,2}) \int d\phi_d |\mathcal{M}_d(\vec{p}^{\,2})|^2\right)\right] . \tag{2.8}$$

Note that $\vec{p}^{\,2}$ is the total three-momentum of the Higgs, s is the total four-momentum squared [13], $d\phi_p$ is the scattering angle and $d\phi_d$ is the decay angle.

If we were to apply the narrow width approximation to this process then the cross section is given by:

$$\sigma_{\text{NWA}} = \frac{1}{2s}\left(\int d\phi_p |\mathcal{M}_p(m_D)|^2\right)\left(\int_{-\infty}^{\infty} dp^2 Y(\vec{p}^{\,2})\right)\left(\int d\phi_d |\mathcal{M}_d(m_D)|^2\right) , \tag{2.9}$$

where $Y(p^2) = KW(p^2)$ and $K = \frac{\pi}{m_D\Gamma}$. Since at the narrow width approximation $W(\vec{p}^{\,2}) = \delta(\vec{p}^{\,2} - m_D^2)$ the second integral in Eq. 2.9 is equals to K. We finally get the following expression for the total cross-section in the narrow width approximation:

$$\sigma_{\text{NWA}} = \frac{1}{2s}\left(\int d\phi_p |\mathcal{M}_p(m_D)|^2\right)\frac{1}{2m_D\Gamma}\left(\int d\phi_d |\mathcal{M}_d(m_D)|^2\right) . \tag{2.10}$$

From the equation above one can infer that the total width of the resonance particle is inversely proportional to the total cross-section of the process which involves the production and decay of this particle. The two expressions in the parenthesis are proportional to the coupling constants squared, more precisely

$$\left(\int d\phi_p |\mathcal{M}_p(m_D)|^2\right) \propto g_i^2 , \tag{2.11}$$

where g_i is the coupling of the resonance particle to the initial states and

$$\left(\int d\phi_d |\mathcal{M}_d(m_D)|^2\right) \propto g_f^2, \tag{2.12}$$

where g_f is the coupling of the resonant particle to the final states. Overall, we obtain the expression for the total cross section and it depends on both the coupling factors and the total width of the resonant particle;

$$\sigma_{\text{on-shell}} \propto \frac{g_i^2 g_f^2}{m_D \Gamma}. \tag{2.13}$$

2.4 Caola and Melnikov's method of measuring the width of the Higgs boson indirectly

The method for finding the width of the Higgs boson and bounding it was described in Caola's and Melnikov's 2013 paper titled "Constraining the Higgs boson width with ZZ production at the LHC" (See [12]). The particular process that was investigated in this paper was $gg \to H \to ZZ$. Whether this method is applicable to the process we are studying will be one of the questions we will try to answer in the Discussion section. The method described here will be how we will obtain a bound to the Higgs boson width Γ_H but for the production of the Higgs boson in association with a W^+ boson.

The Caola-Melnikov method is applied to processes in which the Higgs boson decays to a pair of ZZ bosons because of the enhanced cross-section in the off-shell region of the Higgs boson. This enhancement can be attributed to the relatively large mass of the Z boson, whose mass is 35 GeV less than the mass of the Higgs boson. Another reason would be the instability of the Z boson. The Z boson, like the Higgs particle, is also a resonance particle so it can be described by the Breit-Wigner distribution given in Eq. 2.6. From Eq. 2.6 one can see that the cross-section is larger when the invariant mass of the decay products is equal to the mass of the resonance particle. Therefore, we could say that the cross-section is enhanced when the propagator is on-shell.

For processes such as $H \to bb$ there is no off-shell region of the Higgs boson with an enhanced cross-section, it can only be seen in processes such as $H \to ZZ$ and $H \to WW$. This is due to the mass of the bottom quark being much smaller than the mass of the Higgs. The mass of the bottom quark is around 4.18 GeV and, like the Higgs, it is also unstable. For the bottom quarks to be on-shell the invariant mass of the final decay products must be greater than or equal to 8.36 GeV. The mass of the Higgs boson lies in this range so the bottom quarks and the Higgs boson can be on-shell at the same time unlike the pair of Z bosons or the pair of W bosons. For this reason, there is no enhanced off-shell cross section in the case when the Higgs decays to a pair of bottom quarks and so the Caola-Melnikov method can not be used.

The Higgs boson decay to a pair of Z bosons is much more desirable to investigate than the decay of the Higgs to a pair of W bosons because of the final decay products when the W boson decays. The W boson either decays into a quark pair or into a lepton and a neutrino. The problem arises when the W boson decays into a lepton and neutrino. Neutrinos interact with matter only via the weak force, hence they can't be detected easily. This means we don't have information about the momentum of the neutrino.

One would notice that the on-shell cross-section for the $H \to ZZ$ decay mode is much larger than the off-shell cross-section (see Fig. 2). This is so due to the Higgs width Γ_H being much smaller than the width of the Z boson Γ_Z (the values for the widths can be found in section 2.3). For mass values less than the on-shell mass of the Z boson, the Breit-Wigner distribution doesn't fall down so quickly compared to the distribution of the Higgs boson. So there is a chance of producing Z bosons with masses less than the mass of the vector boson. This contributes to the cross-section when the Higgs boson is produced on-shell and this is the main reason why the on-shell cross-section of the Higgs boson is larger than that of the off-shell region.

A short description of Caola and Melnikov's method will now be given. The equation for the cross-section for the production of the Higgs boson via two protons and its decay to ZZ bosons which in turn decay to four leptons ($4l$) is given by:

$$\sigma_{pp\to H+W^+\to ZZ+W^+} \sim \frac{g_i^2 g_f^2}{(M_{4l}^2 - m_H^2)^2 + m_H^2\Gamma_H^2}, \tag{2.14}$$

where M_{4l} is the mass of the final state lepton particles, m_H is the invariant mass of the Higgs boson, g_i is the coupling of the Higgs boson to the W^+ and g_f is the coupling of the Higgs boson to the ZZ bosons which decay into the leptons which are then detected.

When on-shell masses of the Higgs boson are considered, the narrow width approximation can be applied and the cross-section is given by equation 2.13 while the off-shell cross-section is given by the following:

$$\sigma_{\text{off-shell}} \propto \frac{g_i^2 g_f^2}{(M_{4l})^2}. \tag{2.15}$$

The off-shell cross section doesn't depend on the Higgs boson width while the on-shell one does. Such a result is obtained due to the applicability of the narrow-width approximation in the case of the Higgs boson width. This makes it possible to find the width of the Higgs boson directly since if we take the ratio of the on-shell and off-shell cross-sections we get Eq. 2.16 given below:

$$\frac{\sigma_{\text{off-shell}}}{\sigma_{\text{on-shell}}} \propto \frac{m_H \Gamma_H}{(M_{4l})^2} . \tag{2.16}$$

It is important to note that this method makes it possible to find the width of the Higgs boson independent from model constraints. This again is due to the property of the off-shell cross-section. If the couplings and the width were each to increase or decrease by factors which are more or less the same, the off-shell cross-section is changed due to the factors but the on-shell cross-section remains unchanged due to the presence of Γ_H. Thus, only investigating the on-shell region makes it difficult to know whether couplings which are not in the Standard Model are present since whether they are there or not we notice no change in the cross-section.

2.5 Higgs width and beyond the Standard Model

The Caola-Melnikov method is used to investigate BSM (Beyond Standard Model) models due to the off-shell cross-section not depending on the Higgs boson width. In such models, the Higgs width can be greater than the Standard Model Higgs width. The BSM models deal with the scalar boson decaying into invisible states that are candidates for dark matter particles.

To know whether a particular measurement of the Higgs width hints at the existence of dark matter, one must find a limitation on the branching fraction of the Higgs particle decaying into invisible states. To do this, one must find the relationship between the branching fraction of the scalar boson to invisible final states (Br_{inv}), the full BSM Higgs width (Γ_H^{BSM}) and the SM Higgs width (Γ_H^{SM}). The full BSM Higgs width is equal to the following:

$$\Gamma_H^{BSM} = \Gamma_{inv} + \sum_{i \in vis} \Gamma_i . \tag{2.17}$$

To find the relationship between the variables stated above we must make the assumption that the coupling of the Higgs to the visible states in the BSM is equal to a factor λ times the coupling of the visible states to the Higgs boson in the Standard Model. It is also assumed that the cross-section of the Higgs production and decaying process in BSM is equal to the cross-section of that in the Standard Model. This and the fact that the decay rate (Γ_i) is proportional to the coupling constant g_i^2 implies that $\Gamma_H^{BSM} = \lambda^4 \Gamma_H^{SM}$. The information above can be used to derive equation 2.18 which is given below:

$$\Gamma_H^{BSM} (1 - \text{Br}_{inv})^2 = \Gamma_H^{SM} , \tag{2.18}$$

Using 2.17 and 2.18 we get an upper bound on the branching fraction of Higgs decay to invisible final states, precisely:

$$\mathrm{Br}_{inv} = 1 - \sqrt{\frac{\Gamma_H^{SM}}{\Gamma_H^{BSM}}}\,. \tag{2.19}$$

3 Method and Results

The particular case we will be studying is the production of the Higgs boson along with a W^+ boson. A proton-proton collision produces a W^+ boson which emits a Higgs boson. The Higgs boson then decays into a pair of Z bosons which in turn decay into leptons. Fig. 3 shows a Feynman diagram for this process.

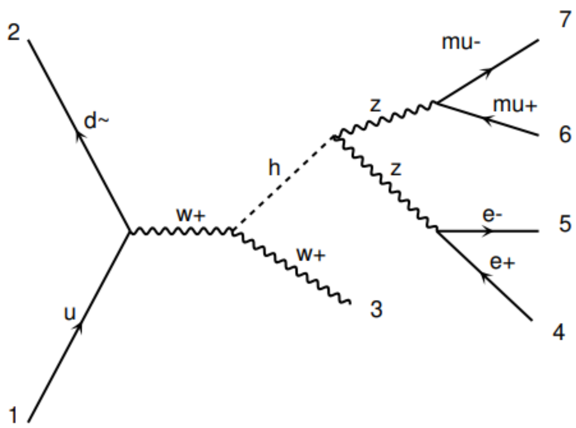

Figure 3: The Feynman diagram for $pp \to H + W^+ \to ZZ + W^+$. The Z bosons decay into leptons.

The MadGraph5_MC@NLO computer program was used to generate events for $pp \to H + W^+ \to ZZ + W^+$. Precisely, the following command was used to generate the process:

```
p p > h > w+ e+ e- mu+ mu-,
```

where the `p` is a proton, `h` is the Higgs boson, `w+` is the W boson, `e` and `mu` are the electron and muon respectively. The process of collecting events was repeated three times; in the second time, the width of the Higgs boson was changed to be two times that of the Standard Model Higgs width and in the final time the width was set to be five times that of the SM width. The data containing information about the events was obtained as an

LHE file which was then converted to a text file in Python. After the text file was read the relevant data was collected and histograms were plotted. The Appendix contains all of the code that was used to read the text file, plot the histograms and find the total off-shell cross-section and total on-shell cross-section. The on-shell region starts at 120 GeV and stops at 130 GeV while the off-shell region starts at 190 GeV and stops at 715 GeV.

When the Higgs width is equal to the SM Higgs width (set as 6.38 MeV in MadGraph) the number of events generated is 10000 and the integrated weight is $5.712 \times 10^{-6} \pm 3.088 \times 10^{-8}$ pb. Likewise, when the Higgs width is two times that of the SM Higgs width the number of events generated is 10000 and the integrated weight is $3.0578 \times 10^{-6} \pm 1.441 \times 10^{-8}$ pb. The number of events generated, when the Higgs width is five times the Standard Model width, is the same as for the previous cases but the integrated weight is lower, $1.4691 \times 10^{-6} \pm 6.885 \times 10^{-9}$ pb. For each simulation that was carried out the total energy of each of the two beams of protons was 6500 GeV. The bwcutoff was kept at its default value which is 15 GeV. All of the simulations were carried out in the laboratory frame, which is the default setting in MadGraph.

Presented below are the final results obtained by carrying out the method which was described earlier.Fig. 4 is the distribution of cross-section for each Higgs width and Tab. 1 shows the total cross-section for each region for each Higgs width.

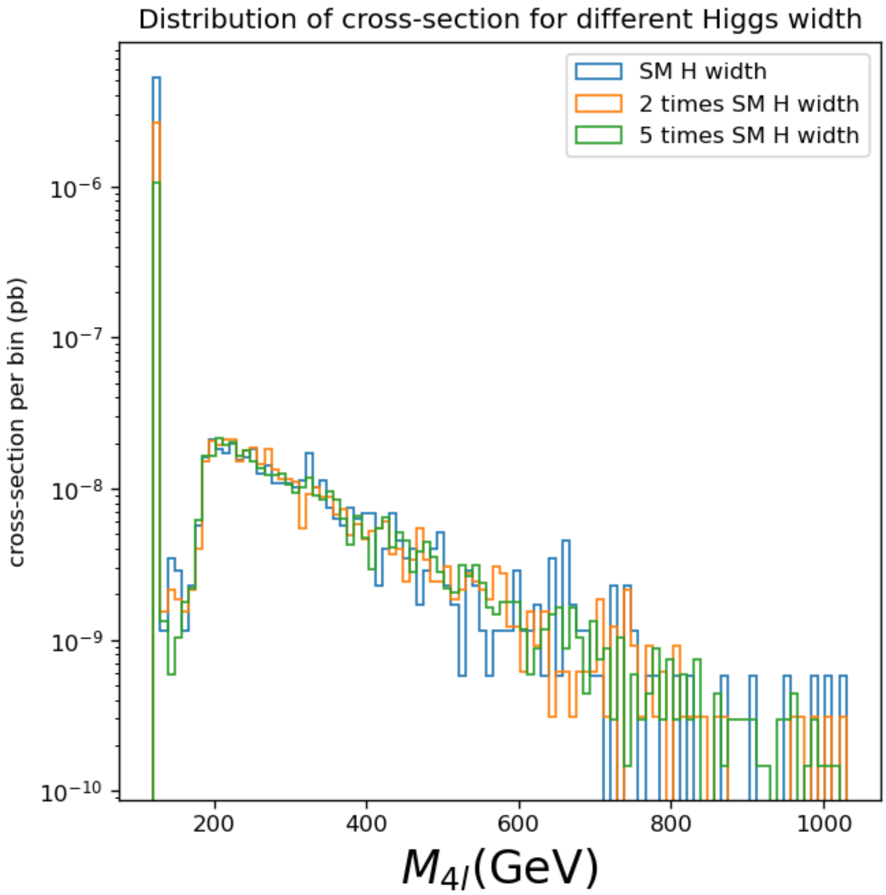

Figure 4: Histograms of cross-section per bin given in picobarns (pb) over total invariant mass given in GeV.

Higgs Width	**On-shell region total cs (pb)**	**Off-shell region total cs (pb)**	$\frac{\sigma_{\text{off-shell}}}{\sigma_{\text{on-shell}}}$
Γ_H^{SM}	$5.3 \times 10^{-6} \pm 2.9 \times 10^{-8}$	$3.7 \times 10^{-7} \pm 10 \times 10^{-13}$	$0.070 \pm 3.8 \pm 10^{-4}$
$2\Gamma_H^{SM}$	$2.7 \times 10^{-6} \pm 1.3 \times 10^{-8}$	$3.7 \times 10^{-7} \pm 10 \times 10^{-13}$	$0.14 \pm 6.9 \times 10^{-4}$
$5\Gamma_H^{SM}$	$1.1 \times 10^{-6} \pm 10 \times 10^{-13}$	$3.7 \times 10^{-7} \pm 10 \times 10^{-13}$	$0.34 \pm 1.2 \times 10^{-6}$

Table 1: Table of the total cross-section (cs) in the on-shell and off-shell regions for each Higgs width and the ratio of total off-shell region cross-section and total on-shell region cross-section for each value of Higgs width.

The results presented in both Tab. 1 and Fig. 4 agree with the theory that was presented before. In Fig 4 all of the three histograms have two distinct regions, a narrow peak which is centred at about 125 GeV and a short peak centred at about 200 GeV. This short peak falls off at a small rate and looks similar to the peak in Fig. 2, hence the results obtained agree with the theory. Upon reading the table we can notice that the total cross-section in the off-shell region doesn't change with Higgs width. On the other hand, the total cross-section of the on-shell region decreases as Higgs width increases.

4 Discussion

We must first make note of the approximations we have made and factors we haven't taken into account. Firstly, there will be a continuum background which interferes destructively with the scattering amplitude of the decay of the Higgs boson to ZZ particles.A Feynman diagram of a process that contributes to this interference is given in Fig.5 As a result of this interference, the cross-section of the off-shell region will be increased. The interference effects are important for large invariant mass values i.e. values that are greater than 200 GeV [2]. The interference effects are redundant in the on-shell region. Including the continuum, the background would affect the total cross-section of the off-shell region thereby giving an overestimate to the width of the Higgs boson. With that in mind, all of the processes which contributed to a continuum background were omitted while obtaining the results for this thesis.

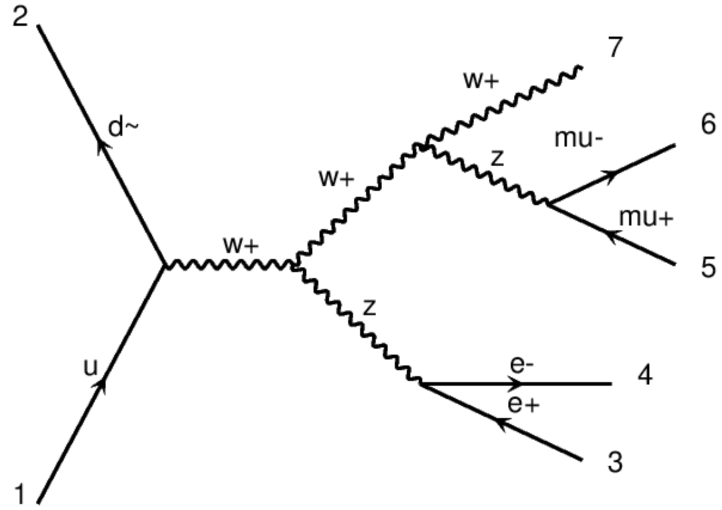

Figure 5: A Feynman diagram of a $pp \rightarrow W^+ \rightarrow e^+e^-\mu^+\mu^-$ process which does not produce an intermediate H boson.

In our analysis the W^+ boson was assumed to be stable, however, this does not reflect reality. The W boson must decay into either a positron or a positively charged muon and a neutrino. Neutrinos have no electric charge which makes them hard to detect although this is not the issue in our case. The leptons produced from the decay cause an issue because they are indistinguishable from the leptons produced from the decay of the Z bosons. This means that we could use the wrong momentum and energy values (instead of using the ones from the Z bosons we would instead use the ones from the W^+ boson) to calculate the invariant mass of the Higgs boson. Consequently, this makes our analysis more complicated.

Tab. 1 shows the same total off-shell cross-section for all of the Higgs widths whereas the total on-shell cross-section decreases as the value of the width increases. This is also evident in the histograms shown in Fig 4. In all cases of the Higgs width, the total on-shell cross-section is always much larger than the total off-shell cross-section. Hence, we can affirm that the results agree with the theory; the on-shell cross-section is inversely proportional to Higgs width (see Eq. 2.13) while the off-shell cross-section is independent of it (see Eq. 2.15). Due to the fact that the off-shell and on-shell cross-sections obtained from the simulation agree with the theory we expect the ratios of these cross-sections to be directly proportional to Higgs width. This is indeed the case, the value of the ratio for Higgs width equals Γ_H^{SM} is the smallest and the value of the ratio for $5\Gamma_H^{SM}$ is the largest.

In MadGraph5_MC@NLO the Standard Model Higgs boson width is equal to 6.38 MeV.

This is higher than the Higgs width of 4.1 MeV because this value is the width computed at LO accuracy. Thus, the Higgs width in the software is acceptable. When 6.38 MeV is multiplied by 2 and 5 the Higgs width is beyond the maximum value for the SM Higgs width and this indicates what the Higgs width would be if there was a BSM physics phenomenon present. If we multiply the ratios in Tab.1 by a factor of 10 the values are then approximately equal to the magnitude of their corresponding Higgs widths given in MeV.

As was said previously in the Introduction section, we expect to find an off-shell region peak about 200 GeV. This is stated in the results of the experiments carried out by the CMS collaboration [2]. In the Theory section, it was stated that when the Higgs boson is off-shell the Z bosons are on-shell. If both of the Z bosons are on-shell then the invariant mass of their decay products must be around 180 GeV. There is a considerable cross-section value for masses around this value (see Fig. 1). However, there is a peak for invariant mass values close to 200 GeV. A somewhat crude explanation to this observation would be that at invariant mass values greater than $2m_Z$ the coupling of the Higgs boson to the Z bosons is very large (i.e. g_f is large). From Eq. 2.15 we infer that the total of-shell cross-section is directly proportional to the coupling of the Higgs boson to the Z bosons.

5 Conclusion

The production of the Higgs boson in association with a W^+ boson is one of the production modes that can be used to measure the width of the Higgs boson directly. It must be specified that the decay channel $H \to ZZ$, the one that we studied in this thesis, is desirable because of the difference between the mass of the Z boson and the mass of the Higgs boson not being very large and two times the mass of the Z boson,m_Z, being larger than the mass of the Higgs boson. An off-shell region will be present because of this.

The cross-section of the off-shell region not being dependent on the width of the Higgs boson while the on-shell region cross-section does is another advantage of this decay mode ($H \to ZZ$). Taking the ratio of the off-shell cross-section to the on-shell cross-section gives us a quantity that is proportional to the width of the Higgs. We found the ratio for when the Higgs width was equal to its SM value, when it was two times the SM value and when it was five times the SM value and the value of the ratios did increase with width.

The fact that the on-shell cross-section is inversely proportional to the Higgs width and the off-shell cross-section not depending on the width was derived by applying the narrow-width approximation. Since the results presented in Tab. 1 agree with Eq. 2.13 and Eq. 2.15, we can conclude that the narrow width approximation is applicable to the values of Higgs width studied in this thesis.

6 Appendices

6.1 Code for converting an LHE file to a text file

```
### Converting the LHE file into a text file ###
f = open('unweighted_events.lhe/unweighted_events.lhe','r')
content_list = f.readlines()
f.close()

x = open('unweighted_events.lhe/textfile.txt','a')
for i in range(len(content_list)):
    x.write(content_list[i])
x.close()

print("Done")
#### THE CODE ABOVE MUST BE RUN ONLY ONCE! ####
```

6.2 Reading the text file and getting relevant information

```

import xml.etree.ElementTree as ET
from math import sqrt

## Function for getting lepton information from each event ##
def getEventsLeptons():
    tree = ET.parse('unweighted_events.lhe/sense.txt')
    root = tree.getroot()

    events = []
    for child in root:
        if (child.tag == 'event'):
            lines = child.text.strip().split('\n')
            lines.pop(
                0)  # Remove the first line i.e. header information in
    event

            leptons = []
            for particle in lines:
                particle = particle.strip().split()
                particle = list(map(float, particle))

                desired = {-11, 11, -13, 13}
                if (particle[0] in desired):
                    leptons.append(particle)

            events.append(leptons)
    return events

```

```
## Function for getting momenta and energy values for each lepton in each
    event ##
def getMomentaAndEnergy(event):
    for i in range(len(event)):
        event[i] = event[i][6:10]
    return event






def calculate_total_invariant_mass(m_and_e):
    p_x1, p_y1, p_z1, E_1 = 0, 0, 0, 0
    p_x2, p_y2, p_z2, E_2 = 0, 0, 0, 0
    p_x3, p_y3, p_z3, E_3 = 0, 0, 0, 0
    p_x4, p_y4, p_z4, E_4 = 0, 0, 0, 0

    for lepton in m_and_e:
        if lepton == m_and_e[0]:
            p_x1, p_y1, p_z1, E_1 = lepton
        elif lepton == m_and_e[1]:
            p_x2, p_y2, p_z2, E_2 = lepton
        elif lepton == m_and_e[2]:
            p_x3, p_y3, p_z3, E_3 = lepton
        elif lepton == m_and_e[3]:
            p_x4, p_y4, p_z4, E_4 = lepton

    Total_inv_mass = sqrt((E_1+E_2+E_3+E_4)**2-((p_x1+p_x2+p_x3+p_x4)
   **2+(p_y1+p_y2+p_y3+p_y4)**2+(p_z1+p_z2+p_z3+p_z4)**2))
    return Total_inv_mass
```

Listing 1: The code above is for reading the text file and getting the relavant information from it. Moreover

6.3 Getting a list of total invariant mass for each event

```
    # -------------- Main Code ----------------------
events = getEventsLeptons()

momenta_and_energy = []
for event in events:
    values = getMomentaAndEnergy(event)
    momenta_and_energy.append(values)


total_invariant_mass = []
for m_and_e in momenta_and_energy:
    values = calculate_total_invariant_mass(m_and_e)
```

```
    total_invariant_mass.append(values)



num = 1
for i in total_invariant_mass[50:55]:
    print("Event {}".format(num))
    print(i)
    print("\n")
    num += 1

print(events[0])
```

6.4 Plotting a histogram for a given Higgs width

```
    ####### HISTOGRAM PLOTTING #######
import matplotlib.pyplot as plt
import numpy as np
from matplotlib.axis import Axis

%matplotlib notebook


res1 = np.array(total_invariant_mass)

# Create a histogram with 100 bins and cumulative=True


number1 = (5.712 * 10**-6)/10000
weight1 = [number1 for i in range(10000)]
plt.figure(figsize=(6,6))
plt.hist(res1, bins=100, histtype='step',weights=weight1)
plt.yscale('log')
plt.xlabel('$M_{4l}$GeV', fontsize=20)
plt.ylabel('cross-section per bin')
plt.show()
```

6.5 Finding Off-shell and On-shell total cross-section

```
    ####### CUMULATIVE HISTOGRAM PLOTTING: On-shell region #######
import matplotlib.pyplot as plt
import numpy as np
from matplotlib.axis import Axis

%matplotlib notebook

#plt.style.use(['science', 'notebook', 'grid'])
```

```
res1 = np.array(total_invariant_mass)

# Create a histogram with 100 bins and cumulative=True


number1 = (5.712 * 10**-6)/10000        #  Insert integrated weight / total absolute uncertainty of integrated weight
weight1 = [number1 for i in range(10000)]
plt.figure(figsize=(6,6))
plt.hist(res1, bins=100,range=[120.804,129.707],weights=weight1, histtype='step', cumulative=True)
plt.xlabel('$M_{4l}$GeV', fontsize=20)
plt.ylabel('cross-section per bin(pb)')
plt.title('Cumulative histogram')
#plt.show()
```

```
   ####### CUMULATIVE HISTOGRAM PLOTTING: Off-shell region #######
import matplotlib.pyplot as plt
import numpy as np
from matplotlib.axis import Axis

%matplotlib notebook

#plt.style.use(['science', 'notebook', 'grid'])
res1 = np.array(total_invariant_mass)

# Create a histogram with 100 bins and cumulative=True


number1 = (5.712 * 10**-6)/10000      #  Insert integrated weight / total absolute uncertainty of integrated weight
weight1 = [number1 for i in range(10000)]
plt.figure(figsize=(6,6))
plt.hist(res1, bins=100,range=[191.592,712.041],weights=weight1, histtype='step', cumulative=True)
plt.xlabel('$M_{4l}$GeV', fontsize=20)
plt.ylabel('cross-section per bin(pb)')
plt.title('Cumulative histogram')
#plt.show()
```

Listing 2: The y-axis value for the very last bin of the cumulative histogram equals to the total cross-section.

6.6 Plotting the histograms for different Higgs widths in graph

```
    #-------Main Histogram -------


# Define bin edges
```

```
bin_edges = np.linspace(min(res1), max(res1), 101)  # This is done so
    that each of the histograms will have the same bin width.

# Plot histogram
plt.figure(figsize=(6,6))
plt.hist(res1, bins=bin_edges, histtype='step',weights=weight1,label = '
    SM H width')
plt.hist(res2, bins=bin_edges, histtype='step',weights=weight2, label = '
    2 times SM H width')
plt.hist(res3, bins=bin_edges, histtype='step',weights=weight3,label = '5
     times SM H width')
plt.yscale('log')
plt.xlabel('$M_{4l}$(GeV)', fontsize=20)
plt.ylabel('cross-section per bin (pb)')
plt.title('Distribution of cross-section for different Higgs width')
plt.legend()
plt.show()
```

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