Exchange Rate and Equity Market Dependence under Shifts in Volatility Expectations

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Abstract

Exchange rate movements have important implications for both policy makers and investors, as they can have large effects on the real economy and the return on investments. Lately, their relation to capital flows have attracted growing interest due to the failure of macroeconomic fundamentals to explain them. This paper uses a copula framework to investigate the short-term dependence between the SEK and OMX in reference to the three developed markets of USA, EU, and Japan, and for both the absolute and relative performance of OMX. By considering shifts in volatility expectations as a third variable, the dynamic dependence between the SEK and OMX is assessed in light of existing theories such as portfolio rebalancing, the hedging channel, return chasing, and the notion of safe havens. For the absolute performance of OMX, we document a negative dependence with the exchange rate which is more pronounced when volatility expectation shifts are large, which is in accordance with the theories of safe havens and the hedging channel of exchange rate determination. The dependence is strongest in reference to Japan, and weakest in reference to USA. For the relative performance of OMX, a positive but weak dependence with the exchange rate is identified, which is in agreement with the predicted effects of portfolio rebalancing and hedging adjustments but contrary to that of return chasing. The results have implications for investors in terms of hedging, as the results indicate that the referenced currencies provide a natural hedge in times of market distress. Furthermore, the results indicate that relative outperformance is linked to a depreciation of the currency, creating an opportunity for foreign investors to benefit from both.

Key words: Exchange rates, Equity markets, Volatility, ARMA-GARCH, Copula, Exchange rate determination, Safe-haven, Portfolio rebalancing, Return chasing

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Chapter 1

Introduction

Currency movements are notoriously hard to predict and have so far seemed to be largely disconnected with macroeconomic fundamentals. For example, in an often-quoted article by Meese and Rogoff [1], it was found that predictive models of exchange rate movements performed no better than simple random walk models. This is unfortunate since they have important implications for both policy makers and investors. Assumptions about future exchange rates are important inputs for financial predictions and decision making as they can have large effects on the real economy. For example, it has recently been argued that the weak SEK has added to the already high inflation currently being witnessed in Sweden [2]. Similarly, large currency appreciations could have adverse effects on exports and trade balances, why central banks such as Switzerland's has historically taken measures to fight them [3]. Furthermore, in an increasingly global financial system exchange rates affect investor returns both directly through foreign investments, and indirectly through the foreign exposure of domestic companies.

The impact of international capital flows in this context have attracted growing interest in the academic world [4][5][6]. This report narrows that scope down to investigate the dependence between the equity and foreign exchange markets, specifically the SEK and OMX, in reference to a selection of developed economies. Exploring a potential linkage is especially interesting from a Swedish perspective, since the weak SEK has garnered significant public attention as of late [2]. Furthermore, the dynamic property of this potential linkage is examined by taking shifts in volatility expectations into account.

The difficulty of understanding exchange rate movements is apparent from the contradictory theories of dependence even when limited to its relation to equity markets. This stems from a variety of reasons. First, the theories recognize different driving forces that work in opposite directions. Second, proposed explanations can be expected to take effect over different time spans, i.e., be lagged or (relatively) instantaneous. Third, due to the financial system being a complex network of dynamic relationships, the power of the explanatory forces would reasonably be expected to change under different financial and economic conditions. For example, a frequently used line of reasoning suggests that a weaker SEK is beneficial for the Swedish equity market since it would make many of its export-oriented companies more competitive internationally [7]. If true, it is reasonable to expect this dependence to be somewhat lagged in time since the increased sales of the domestic companies would be so.

This report is concerned with the short-term dependence between the equity and foreign exchange markets. In a general setting, a few theories have been suggested to this end. They acknowledge different driving forces and thus sign of dependence, including portfolio rebalancing, hedging adjustments, return chasing, and the concept of safe havens. The first three are related in that they concern the relative performance of markets. Portfolio rebalancing refers to adjustments in the portfolio composition in response to market movements, with the aim to keep a constant level of diversification (see e.g. [8]). In this way, the relative performance of markets is central. This is true for return chasing as well, in which investors turn to markets that have recently outperformed. The hedging channel of exchange rate determination, which emphasizes currency risk by stating that hedging adjustments rather than reallocations of the underlying assets move exchange rates, does not directly depend on the relative performance of markets (since adjustments are made in reference only to the specific foreign position). However, if one assumes that this capital flow is bi-directional for two markets, the respective sizes of the flows in either direction and thus the net effect will depend on the relative performance of the underlying positions (in addition to factors such as amount of foreign investments and hedging practices).

The theory of safe-haven currencies is somewhat different, stating that investors take to currencies considered safe in times of financial uncertainty. If true that investors exhibit a flight to quality during such periods, it is reasonable to assume that relative market performance will be less important. Put another way: in periods of high financial stress that affects several markets, the relative performance of those markets is of lower importance; investors seek out safe havens. Since this effect would be most noticeable during periods of high financial uncertainty it could potentially obscure the effects of the previous theories mentioned, which might emerge during more calm periods.

With this in mind, the aim of this report is to study the short-term dependence between equity and currency markets for both the absolute and relative performance of the former, and under shifts in financial uncertainty. Taking the SEK and OMX as our point of reference, we first investigate how their co-movement changes for different levels of financial uncertainty. Then, we do this but with the *relative* performance OMX. Three economies are used as reference: EU, USA, and Japan. They share some common characteristics, such as being developed markets and having been described as safe-haven currencies [9]. As measures of financial uncertainty, volatility indexes for each of the three markets are used: VIX, VSTOXX, and VXJ, respectively. Those are based on the implied volatility of options of different maturities, and are considered to be indicative of the expected future volatility [10]. Importantly, throughout the report exchange rates are expressed as SEK per unit of referenced currency. Accordingly, the strength of the SEK and its exchange rate are inversely dependent, and unless otherwise specified, the dependence will refer to the latter. Thus, a negative dependence between the SEK and OMX does in fact imply that the currency strengthens as the equity market appreciates, and vice versa.

This is achieved using the copula framework. A copula is a mathematical tool to flexibly construct and model multivariate dependence structures, and they have been extensively used in finance with applications such as risk modelling, pricing of derivatives, and portfolio optimization [11][12]. In contrast to ordinary multivariate distributions that aim to capture the whole dependence structure simultaneously, copulas allow them to be divided into univariate marginal distributions and the dependence between them. By modelling them separately it is possible to combine marginal distributions of fundamentally different kinds that in turn do not constrain the choice of joint dependence structure, which makes the set of possible distributions considerably greater than when limited to multivariate extensions of existing univariate distributions. Furthermore, by modelling the complete dependence structure, copulas can capture asymmetric and nonlinear dependencies.

By doing this, our study adds to the existing body of literature in two ways. First, while previous studies have examined the dependence between currency and equity markets, to our knowledge this has not previously been done for the Swedish market using the copula framework. Second, while previous studies have taken volatility into account by either discretizing data sets of observations based on crisis periods [13], or by letting the copula parameter vary with volatility [14], this is the first study that treats volatility expectations as a third variable in a three-dimensional dependence structure together with exchange rates and equity markets.

The results suggest that the three-dimensional dependence structure between SEK, OMX, and volatility indexes is best described by the Student's t-copula for both the absolute and relative performance of the OMX. Furthermore, in the case of absolute performance, a negative dependence between OMX and the exchange rates is identified which is greater when shifts in volatility expectations are large, and decreasing towards zero as the latter becomes smaller. This indicates a safe-haven effect. For the relative performance of OMX, a weak positive dependence is identified, which for USA and Japan are independent of the shifts in volatility expectations. This is in agreement with the theories of portfolio rebalancing or the hedging channel of exchange rate determination, rather than a return-chasing effect.

The structure of the report is as follows. First, some previous work that relates to ours is discussed. In Chapter 2, the necessary mathematical theory is introduced. In Chapter 3, that theory is applied and the results provided. Finally, Chapter 4 summarizes the findings and discusses them in light of existing theories.

1.1 Related work

While early models of exchange rate determination focused on macroeconomic variables such as money supplies, relative price levels, and current account balances [15][16], later studies have begun placing more emphasis on cross-border capital flows. This is likely due to the failure of macroeconomic fundamentals to explain exchange rate variations, especially in the short term [1][17], as well as the growth of cross-border order flows. For example, gross stocks of cross-border assets and liabilities have increased from 60% of the world's GDP in the mid-nineties to 200% in 2015 [18].

Still, existing theories acknowledge different sources of dependence and hence the sign of it. [8] develops a framework which posits a negative dependence between equity and exchange rates. They ascribe this result to portfolio rebalancing effects, where foreign investors will rebalance out of markets that outperform in order to remain diversified across markets, which puts downwards pressure on the local currency. In contrast, [19] identifies a return-chasing effect among US investors, i.e., positive net purchases in markets that perform well. Similarly, [20] finds evidence of trend chasing in that increases in returns lead to increased future inflows of capital. Reasonably, this would imply a positive dependence as it would increase the demand for currencies of markets that outperform.

[21] and [22] put emphasis on the hedging channel rather than adjustments in the underlying asset position and show that net foreign investments and country-specific hedging practices influence exchange rates. If foreign investors aim to keep a stable hedge ratio, hedging positions will be adjusted in accordance with movements in the underlying assets. This implies that when foreign assets depreciate, investors must decrease their hedging position, which effectively means buying the foreign currency and selling the home currency, putting downward pressure on the latter. In this way, strengthening and weakening of a country's currency and equity market should co-move inversely. By extension, the relative performance of equity markets will have an influence on the net effect since it will affect the sizes of hedging adjustments flowing in either direction.

Another strand of literature concerns the safe-haven theory of currencies by taking volatility and financial uncertainty into account. Using a factor model to capture linkages between currencies, stock, and bond markets, and with the VIX and a corresponding measure of foreign exchange volatility as proxies for market uncertainty, [9] documents that the JPY, CHF, and EUR exhibit safe-haven characteristics in reference to the USD in that they appreciate during times of increased uncertainty. In [23], they find that currencies of emerging markets appreciate and depreciate with their equity market, while the opposite is true for developed markets. They attribute this to a flight-to-quality mechanism in which capital flows out of emerging markets during times of financial stress as investors seek out assets considered more secure, and vice versa. [14] uses a copula-based method to identify safe-haven currencies. In addition to static copulas, they employ time-varying models where the copula parameters depend on the estimated market volatility, and find that the JPY, CHF, and USD have exhibited safe-haven properties during the last twenty years.

Considering all the above, it is reasonable to assume that results depend on what markets are studied and at what point in time. Unsurprisingly, therefore, empirical studies of the equity and exchange rate dependence have yielded mixed results [24]. Specifically, a few studies have used copulas to model the equity and currency dependence. [25], [26], [27], [28] looked at developed markets using static copulas, and found positive dependencies for US and Japan, but negative or insignificant dependencies for the rest. However, [29] employs a time-varying copula and shows that the dependence for US changes over time, and becomes more positive during crisis periods. Similar results are provided in [30], where they apply a dependence-switching copula for six major developed markets. Emerging markets have been studies with copulas but to a lesser extent. [31] finds negative and asymmetric dependencies for ten emerging countries against the USD. Similarly, in [32] they study six African markets and find negative dependencies when quoted against the USD and EUR.

Chapter 2

Theory

2.1 Time series Modelling

Accurate dependence modelling with copulas requires observations that are independent and identically distributed (i.i.d.), which is typically not true for financial time series. They exhibit various stylized facts, including autocorrelation (new observations are not independent of previous observations), trends in mean (the mean of the process varies with time, e.g., equity markets tend to show a positive trend in the long-term despite short-term fluctuations), volatility clustering (certain periods exhibit higher volatility than others), and skewed and leptokurtic distributions (extreme movements occur more frequently than expected under the normal distribution and primarily in one tail) [33].

To account for this, the marginals are modelled prior to fitting a copula. Due to the stylized facts above wherein new observations are dependent on previous ones in both mean and variance, the ARMA(p,q)-GARCH(r,s) is a common choice for financial time series modelling. Informally, by regressing observations and their variance onto past values, the idea is to remove their influence in terms of interdependence and trends, so that one ends up with residuals consisting of that which cannot be explained by the model. If so, those residuals should be i.i.d. The Autoregressive Moving Average (ARMA) model specifies the conditional mean of the process according to

$$\begin{aligned} y_t &= \mu + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{j=1}^{q} c_j \epsilon_{t-j} + \epsilon_t \\ \epsilon_t &= \sigma_t z_t \end{aligned} \tag{2.1}$$

where p and q are the model orders, a_i and c_i are the autoregressive and moving average model parameters, and ϵ the stochastic innovation (see for example [34]). Because of volatility clustering, σ_t is often assumed to follow a time-varying stochastic process. The conditional variance is therefore modelled according to

$$\sigma_t^2 = \omega + \sum_{i=1}^{r} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$

which is the frequently used Generalized autoregressive conditional heteroscedasticity (GARCH) model, first described in [35]. Here, w is a constant, r and s the model orders, and α_i and β_j the respective model parameters. In equation (2.1) above, z_t is assumed to be an i.i.d. random variable. For financial data, a common choice for this distribution is the Student's t-distribution, as it allows for fatter tails than the gaussian. It is defined as

$$f(x;\nu,\mu,\sigma) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\sigma\,\Gamma\left(\frac{\nu}{2}\right)} \left[1 + \frac{(x-\mu)^2}{\nu\,\sigma^2}\right]^{-\frac{\nu+1}{2}} . \tag{2.2}$$

Here, Γ is the Gamma function, ν describes the degrees of freedom, μ and σ are the location and scale parameters.

This study also uses an extension of (2.2) as presented in [36], where they incorporate a shape parameter α to allow for an asymmetric distribution,

$$f_s(x;\alpha,v,\mu,\sigma) = 2 \cdot f(x;v,\mu,\sigma) \cdot F\left(\alpha(\frac{x-\mu}{\sigma})\sqrt{\frac{v+1}{v+\left(\frac{x-\mu}{\sigma}\right)^2}}; v+1\right).$$

2.1.1 Model validation

A successfully fitted time series model captures all predictable information available in the data. It follows that the residuals should be random and independent and therefore autocorrelation and heteroscedasticity absent. The last two can be tested for with the autocovariance function (ACF),

$$r_x(k) = \mathrm{Cov}(x_t, x_{t-k}) = E[(x_t - \mu_x)(x_{t-k} - \mu_x)],$$

$$\hat{r_x}(k) = \frac{1}{N}\sum_{t=k+1}^{N}(x_t - \mu_x)(x_{t-k} - \mu_x)$$

with N the number of observations and μ_x the mean of the process.

ACF is calculated for the residuals to detect autocovariance, and for the squared residuals to detect heteroscedasticity. They can also tested for using the Ljung-Box statistic, which tests for autocorrelation at multiple lags jointly under the null hypothesis of $r_1 = r_2 = \ldots = r_m = 0$. Here, we use a weighted version as presented in [37],

$$\tilde{Q}_w = n(n+2)\sum_{k=1}^{m}\frac{(m-k-1)}{m}\frac{\hat{r_x}(k)}{n-k},$$

where m denotes the number of lags being tested for.

Apart from the stationarity condition, misspecified margins will lead to biased estimators of the copula [38]. It is therefore necessary to assess the fit of the residual distributions. Most goodness-of-fit methods do this by comparing the fitted distribution to the empirical one. This report uses the Cramer-von-Mises test, which quantifies the differences according to

$$T = \frac{1}{12n} + \sum_{i=1}^{n}\left[\frac{2i-1}{2n} - F(x_i)\right]^2 \tag{2.3}$$

where the set of observations $x_1, x_2, \ldots, x_n$ have been ordered in increasing order. If T is larger than the corresponding tabulated value, the null hypothesis of the two distributions being equal can be rejected [39].

A visual presentation of the goodness-of-fit can be achieved with the Quantile-Quantile plot. If the data comes from the fitted distribution, the plotted pairs should align at the theoretical quantile line. Any large and systematic deviation from it indicates a misspecification.

2.2 Copulas

We begin this chapter by defining the notion of a copula and stating Sklar's theorem. The latter, which states that a multivariate distribution can be decomposed into its univariate margins and a function (the copula) linking them together, is central to the theory of copulas and the foundation for its applications in statistical modelling.

Definition 1 (Copula)**.** *A d-dimensional copula is a joint cumulative distribution function with uniform margins, i.e.,* $C : [0,1]^d \to [0,1]$

Theorem 1 (Sklar's theorem)**.** *Let* F *be a d-dimensional cumulative distribution function with continuous margins* $F_1(x_1), F_2(x_2), ..., F_d(x_d)$*. Then there exists a (unique) d-dimensional copula* C *such that*

$$F(x_1, x_2, ..., x_d) = C(F_1(x_1), F_2(x_2), ..., F_d(x_d)),$$

for all $(x_1, x_2, ..., x_d) \in R^d$

Proof. See for example [40].

In this way, a copula "couples" a joint distribution function to its univariate margins [40].

The use of copulas comes with multiple advantages. First, it allows for greater flexibility when modeling since the marginals and the copula can be fitted separately. In other words, the marginal distributions can be of fundamentally different kinds, and they do not in turn constrain the choice of copula [11]. Second, one can easily show that copulas are invariant for strictly increasing transformations of the underlying variables. Intuitively, this follows from the fact that the ranks of the quantile functions of the original and transformed variables remain unchanged. Third, the copula approach allows for inference about not just global dependence measures, such as the conditional mean in the case of linear regression, but also for local ones such as tail dependencies.

2.2.1 Dependence measures

By far the most commonly used measure of association is Pearson's correlation coefficient. It is defined as

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y},$$

where $\text{cov}(X,Y)$ denotes the covariance between X and Y, σ_X and σ_Y are the standard deviations of X and Y, respectively.

While easy to implement, it has certain drawbacks. Most noticeably, it measures the linear relationship between two variables. That is, if two variables exhibit nonlinear dependence, p might fail to capture it. It follows that any nonlinear transformation of X and Y, e.g., a log-transform, will generally affect ρ. Furthermore, it is clear from the definition that it requires the existence of the first two moments.

Alternative measures of dependence are those based on rank correlation which measure ordinal association. As such, they are non-parametric, can detect nonlinear dependence, and are invariant to monotone transformations. This report uses Kendall's τ, which measures the degree of concordance between two variables. Given a pair of observations (x_i, y_i) and (x_j, y_j), we say that two pairs (x_i, y_i) and (x_j, y_j) are concordant if either ($x_i < x_j$ and $y_i < y_j$) or ($x_i > x_j$ and $y_i > y_j$). A pair of observations for which this does not hold true is said to be discordant. For a given data set, let c denote the number of concordant pairs, and d the number of discordant pairs. Kendall's τ is then defined as

$$\tau_K = \frac{c-d}{c+d}.$$

Both $\rho_{X,Y}$ and τ_K are measures of global association. Oftentimes in applications, the tails of a distribution are of particular interest. To this end, for a bivariate distribution $F(X,Y)$, one can define the upper and lower tail dependence, respectively as

$$\lambda_U = \lim_{u \to 1} \Pr(X > F_X^{-1}(u) \mid Y > F_Y^{-1}(u)),$$
$$\lambda_L = \lim_{u \to 0} \Pr(X \leq F_X^{-1}(u) \mid Y \leq F_Y^{-1}(u)),$$

Informally, it is the probability for a variable to take on an extreme value, conditional on the other variable doing so.

2.2.2 Copula models

In the following section, the copulas used in the report are introduced. Apart from the Student's t-copula they all belong to the Archimedean family of copulas, why we begin with a definition.

Definition 2 (Archimedean Copula)**.** *A copula C is called an* Archimedean copula *if there exists a* generator function $\varphi : [0, \infty) \to [0, 1]$ *such that*

$$C(u_1, \dots, u_d) = \varphi(\varphi(u_1)^{-1} + \cdots + \varphi(u_d)^{-1}),$$

where φ fulfills the following properties:

- $\varphi(0) = 1$
- $\lim_{x\to\infty} \varphi(x) = 0$
- $\varphi(x)$ *is completely monotonic, i.e., it has derivatives of all orders on* $(0, \infty)$ *and*

  $$(-1)^j \varphi^{(j)}(x) \geq 0,$$

  where $\varphi^{(j)}(x)$ denotes the jth order derivate of $\varphi(x)$.

$\varphi_A(x)$ *is called the* ***generator function*** *for the Archimedean copula A.*

It is obvious from the definition that copulas in this family are symmetric functions in their arguments, i.e., they are exchangeable. This can be a limitation in dimensions $d > 2$, since it implies identical marginal copulas. Still, Archimedean copulas allow for many different types of dependence structures, as well as asymmetric tail dependencies. Importantly, Definition 2 puts certain restrictions on the copula parameters and by extension Kendall's τ. For all Archimedean copulas used in this paper, the range of admissible Kendall's τ is [0,1), meaning that they do not allow for negative dependence. However, this is simply overcome by "rotating" the copula, i.e., replacing marginals u with their survival functions $1 - u$.

Student's t-copula
The t-copula belongs to the elliptical class of copulas. It is constructed by applying the multivariate t-distribution to t-distributed marginals,

$$C_T(\mathbf{u}; \mathbf{\Sigma}, v) = t_d\left(t_\nu^{-1}(u_1); \ldots; t_\nu^{-1}(u_d); \mathbf{\Sigma}, v\right)$$

where $t_d(\cdot; \mathbf{\Sigma}, v)$ is the multivariate extension of (2.2). As part of the elliptical family of copulas, the t-copula has elliptically contoured density level surfaces. This fact implies radially symmetry, which in turn implies identical upper and lower tail dependence. In contrast to the often-used Gaussian copula, however, the t-copula does allow for non-zero tail dependence.

Gumbel Copula
The generator function of the Gumbel copula is

$$\varphi_{\mathrm{G}}(x) = e^{-x^{1/\theta}},$$
$$\varphi_{\mathrm{G}}^{-1}(x) = (-\ln x)^{\theta}$$

with $\theta \in [1, \infty)$. This family has no lower tail dependence, but allows for upper tail dependence. It is the only Archimedean copula that is also an extreme value copula, as it satisfies the property

$$C(u_1^t, \ldots, u_d^t) = C(u_1, \ldots, u_d)^t$$

for all $t > 0$.

Clayton Copula
The Clayton copula allows only for lower tail dependence. For $\theta \in (0, \infty)$, it is given by

$$\varphi_{\mathrm{C}}(x) = (1 + x)^{-1/\theta},$$
$$\varphi_{\mathrm{C}}^{-1}(x) = -x^{1/\theta}.$$

Joe Copula
Like the Gumbel copula, the Joe copula allows for upper but not lower tail dependence. For $\theta \in [1, \infty)$, it is formulated as

$$\varphi_{\mathrm{J}}(x) = 1 - (1 - e^{-x})^{1/\theta},$$
$$\varphi_{\mathrm{J}}^{-1}(x) = -\ln(1 - (1 - x)^{\theta}).$$

Frank Copula

The Frank copula has zero tail dependence in both tails, and in the bivariate case, is the only Archimedean copula which is radially symmetric [41]. With $\theta \in (0, \infty)$, the generator function of the Frank copula is

$$\varphi_{\mathrm{F}}(x) = -\frac{1}{\theta} \ln\left(e^{-x}(e^{-\theta} - 1) + 1\right),$$
$$\varphi_{\mathrm{F}}^{-1}(x) = -\ln\left(\frac{e^{-\theta x} - 1}{e^{-\theta} - 1}\right).$$

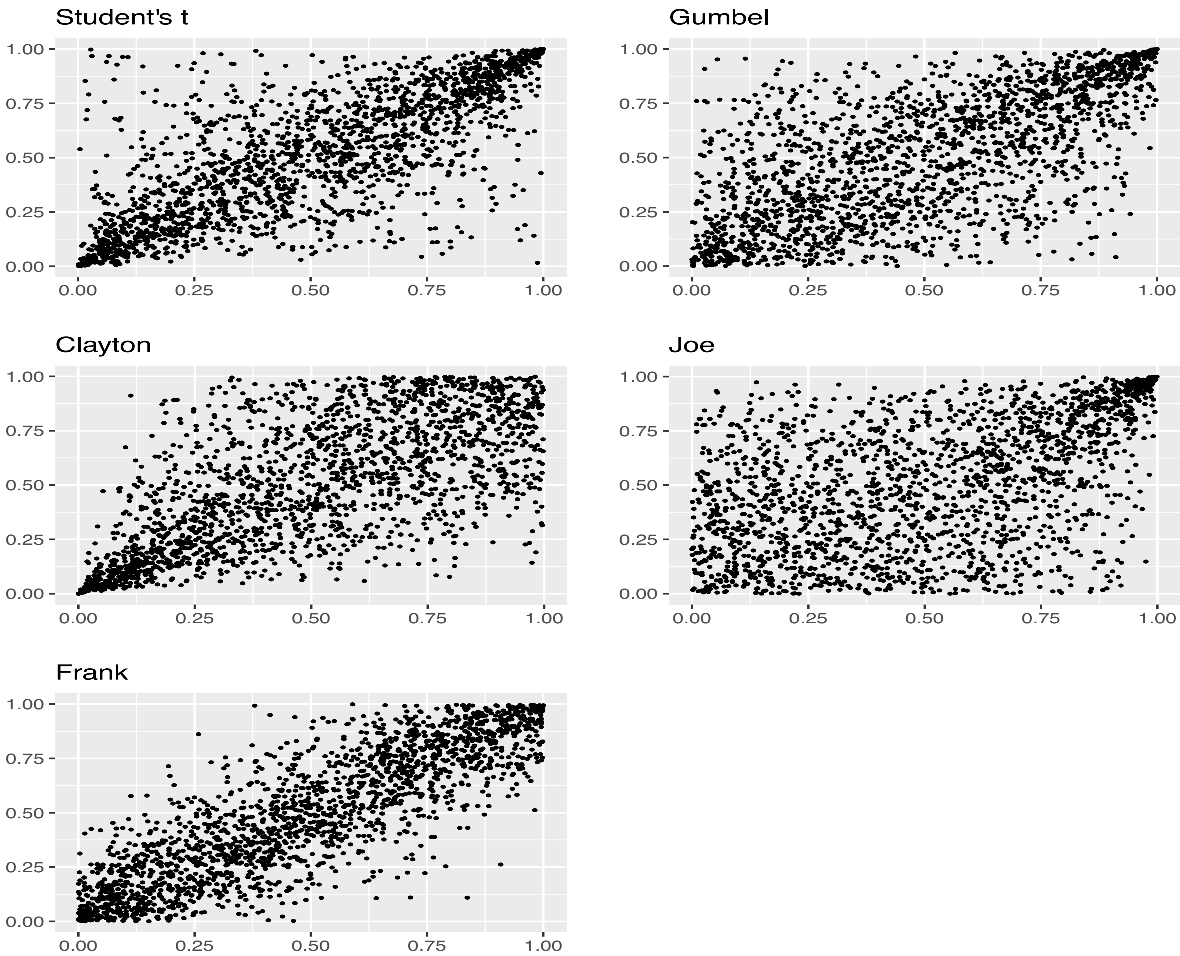

Figure 2.1: 2000 simulated copula samples from Student's t with $p = 0.8$, df $= 2$; Gumbel, $\theta = 2$; Clayton, $\theta = 2$; Joe, $\theta = 2$; Frank, $\theta = 10$

2.2.3 Model validation

To assess the goodness-of-fit of the copulas, an extension of the Cramer-von-Mises statistic (equation 2.3) is used. Informally, it compares the fitted (parametric) copula with the empirical copula C_E. Let r_{ij} be the rank of x_{ij} among all x_{kj}, $k \in 1, \ldots, n$. Then

$$\hat{U}_{ij} = \frac{r_{ij}}{n+1},$$

$$C_E(u_1, \ldots, u_d) = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{\{\hat{U}_{i,1} \le u_1, \ldots, \hat{U}_{i,d} \le u_d\}}$$

are the empirical cumulative distribution of margin j and the empirical copula C_E, respectively.

The test statistic is then defined as

$$S_n = \int\limits_{u=[0,1]^d} n(C_E - C_\theta)^2 dC_E(u_1, \ldots, u_d) \tag{2.6}$$

with the null hypothesis $H_0 : C_\theta \in C_E$. For more details, see [42] [43].

In addition, this report employs a rank-based analogue to equation (2.6) as described in [43]:

$$S_n^K = \int\limits_0^1 n(K_n - K_{\theta n})^2 dK_\theta(v)$$

with

$$K_{\theta n}(v) = \int\limits_{u=[0,1]^d} \mathbf{1}_{C_\theta(u) \le v} dC_\theta(u)$$

and

$$K_n = \sum_{i=1}^{n} \mathbf{1}(V_i \le v),$$

$$V_i = C_E(\hat{U}_{i,1}, \ldots, \hat{U}_{i,d}).$$

That is, the parametric and empirical rank distribution functions of the pseudo-observations are compared, and the null hypothesis of them being the same is rejected for large values of S_n^K. This is a weaker estimator than (2.6), since it is possible for different copulas to share the same rank distributions.

2.2.4 Conditional dependence

A bivariate copula conditional on a third can be acquired from their joint trivariate copula,

$$C_{12|3}(u_1, u_2|u_3 = v_3) = \int_0^{u_1}\int_0^{u_2} c_{123}(v_1, v_2, v_3)\, dv_1\, dv_2 =$$

$$= \int_0^{u_1}\int_0^{u_2} \frac{\partial^3}{\partial v_1 \partial v_2 \partial v_3} C_{123}(v_1, v_2, v_3)\, dv_1\, dv_2 = \frac{\partial}{\partial u_3} C_{123}(u_1, u_2, u_3)|_{u_3=v_3}$$

A couple of things to note. First, as this results in a copula for each value of the conditional variable, and as the conditional variable is assumed to be continuous, one ends up with an infinite number of copulas. Second, analytical differentiation of the trivariate copula can prove to be difficult. Therefore, this report employs a simulation-based procedure to numerically estimate the effect of shifts in volatilty expectations on the dependence between the SEK and OMX. This is done by sampling from the trivariate copula and partitioning the observation pairs of the dependent variables into discrete bins based on the value of the conditioning variable. One then ends up with a finite number of copulas of the kind

$$C_{12|3}(u_1, u_2|i < v_3 < (i + \epsilon))$$

which tends to the above when $\epsilon \to 0$. For interpretational purposes, summary statistics such as τ_K or $\lambda_{U,L}$ can be used to describe how the dependence changes depending on v_3.

2.3 Parameter estimation and model selection

The two tasks of (1) finding optimal parameters for a given data set and (2) choosing the best model from a set of candidates are separate, but because the selection criteria used in this report are based on the estimation procedure, they are presented together. Both marginals and copulas are modelled using Maximum Likelihood Estimation (MLE). Intuitively, this means that for a specified model with a set of parameters, the parameters are chosen so that the likelihood of the observed data is optimized.

In the case of a bivariate copula, for example, we have (from equation (2.2.1)),

$$L(\mathbf{x};\theta) = \prod_{i=1}^{n} f(x_{1i}, x_{2i}) = \prod_{i=1}^{n} c(F_1(x_{1i}), F_2(x_{2i}); \theta) * f_1(x_{1i}) * f_2(x_{2i}),$$

$$\ell(\mathbf{x};\theta) = \sum_{i=1}^{n} \ln f(x_{1i}, x_{2i}),$$

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}}\, \ell(\mathbf{x};\theta)$$

where the likelihood function is logarithmized for computational efficiency. Although $\ell(\mathbf{x};\hat{\theta})$ cannot be interpreted in absolute terms, it allows for comparison between different models fitted to the same data set, where higher likelihood the better. Due to the risk of overfitting, however, it is important to account for the complexity of the model. The Akaike Information criterion (AIC) is formulated as,

$$\mathrm{AIC} = 2k - 2\ell(\mathbf{x};\hat{\theta})$$

with k = the number of estimated parameters and a lower AIC indicates a better fit. Simply put, the first term penalizes the model's complexity, while the second rewards it fit. Closely related to AIC is the Bayesian information criterion (BIC),

$$\mathrm{BIC} = k\ln(n) - 2\ell(\mathbf{x};\hat{\theta}).$$

As such it penalizes complexity more than AIC. For a further discussion of AIC and BIC, see for example [44].

Chapter 3

Data and Results

3.1 Data

The data set consists of three types of data: equity indexes, exchange rates, and volatility indexes. The Swedish equity market is described by the OMSX30, which tracks the 30 largest companies listed on Nasdaq Stockholm and is a commonly used benchmark for Swedish equities. To derive the relative performance of OMX (which here is defined as the difference in weekly log-return between markets) S&P500, EUROSTOXX50, and NIKKEI225 have been used as references for USA, the Eurozone, and Japan respectively. Like the OMSX30 for Sweden, they are considered core indexes for each market. Volatility indexes for the three referenced markets have been used as proxies for financial uncertainty: VIX for USA, VSTOXX for EU, and VXJ for Japan. They measure the expected future volatility by calculating the implied volatility of index options. As such, they are considered to be forward-looking, and have often been referred to as "fear indexes" [45]. Finally, the exchange rates of the SEK against the USD, EUR, and JPY have been used, expressed as SEK per unit of the referenced currency. Importantly, this means that the strength of the SEK is inversely correlated with the exchange rates used. A negative dependence between the SEK and OMX, then, means that they do in fact appreciate and depreciate together.

The data points are of weekly frequency spanning the period from January 2003 to December 2022, resulting in 1042 observations. All data series come from Macrobond [46]. Finally, all series have been transformed to log-returns, i.e.,

$$r_t = \ln \frac{P_t}{P_{t-1}}$$

This is perhaps most important to keep in mind when we refer to the volatility indexes. Just like with the equity indexes and exchange rates, we are concerned with the changes in expected future volatility, not the absolute level of it. By plotting the raw volatility time series and colour by the log-change (Figure 3.1), we can see that large changes are spread out over time. So are the positive and negative observations of OMX's relative performance against the referenced markets, as illustrated in Figure 3.2.

Table 3.1: Data set

	Sweden	EU	USA	Japan
Equity Index	OMXS30	EUROSTOXX50	S&P500	NIKKEI225
Currency	SEK	EUR	USD	JPY
Volatility Index	-	VSTOXX	VIX	VXJ

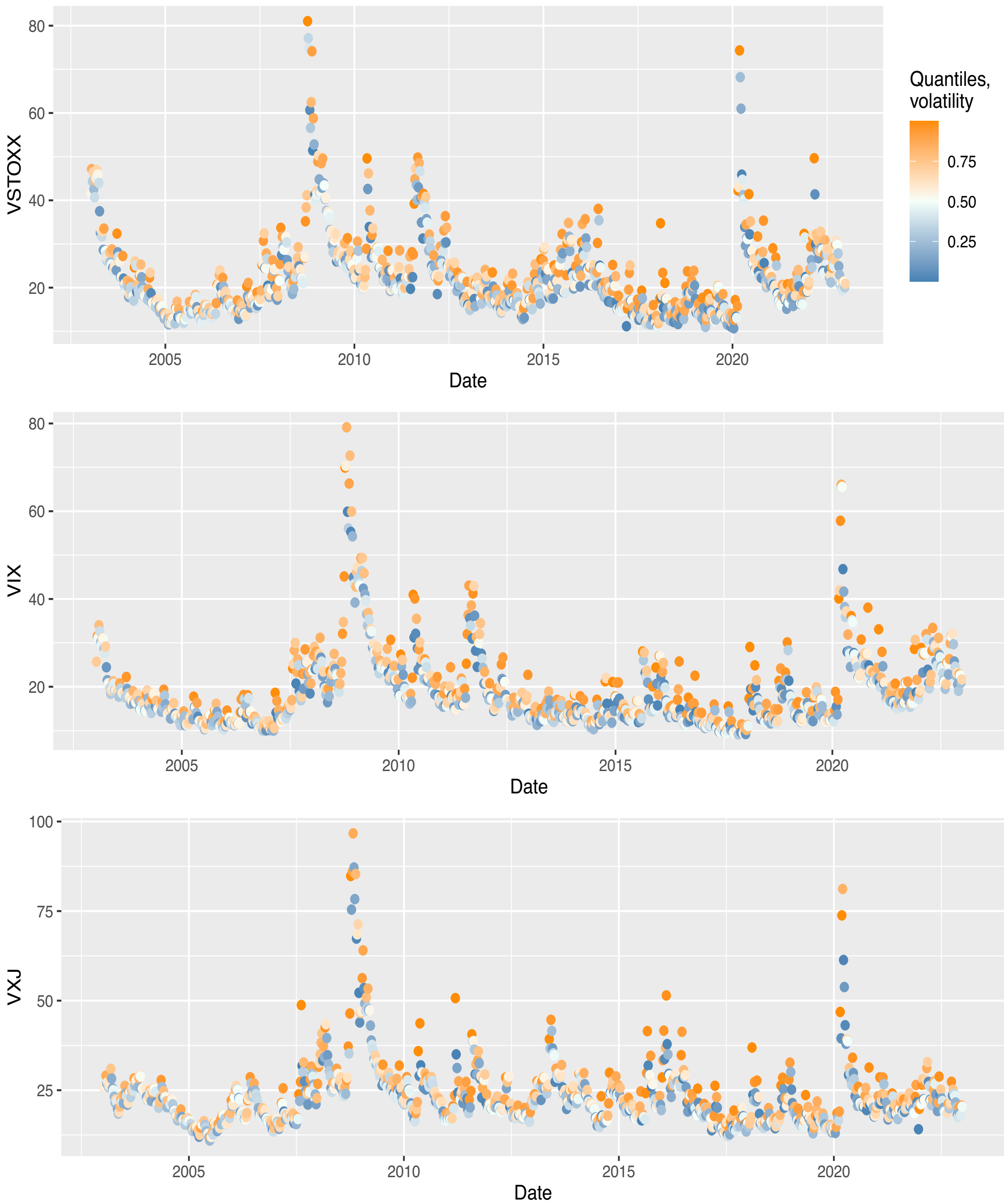

Figure 3.1: Volatility indexes, coloured by empirical quantiles of their weekly log-change.

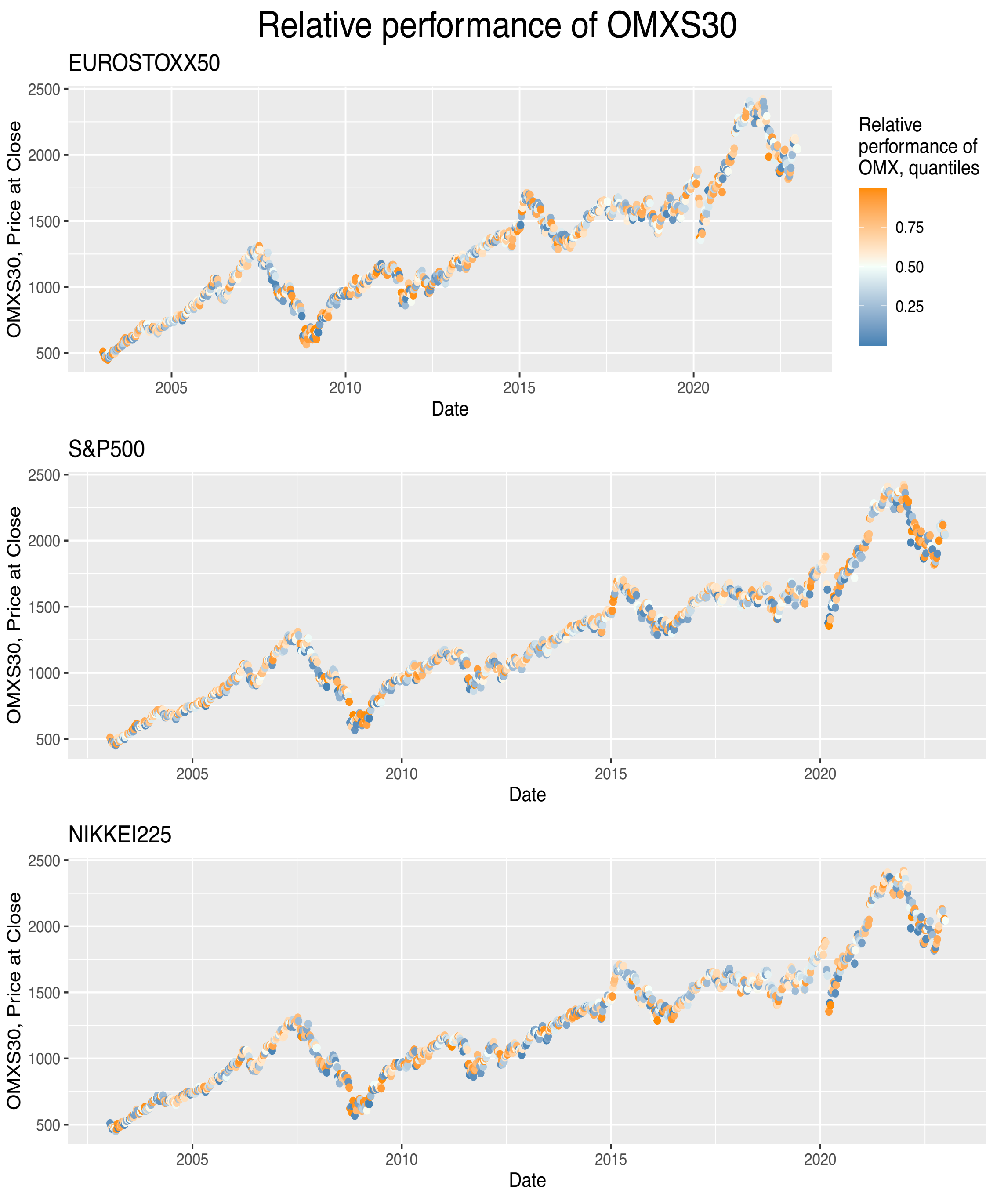

Figure 3.2: OMXS30, coloured by empirical quantiles of relative performance against the referenced markets.

3.2 Marginals

First, the presence of autocorrelation and heteroscedasticity in the data were confirmed with ACF plots. See Figure 3.3 for the case of OMX, and Appendix A for all series. The heteroscedasticity was considerably more noticeable, which suggested that the required model orders of the conditional variances might be higher than that of the conditional means. Differences were also visible between the different types of series. For example, the heteroscedasticity was most pronounced for the exchange rates.

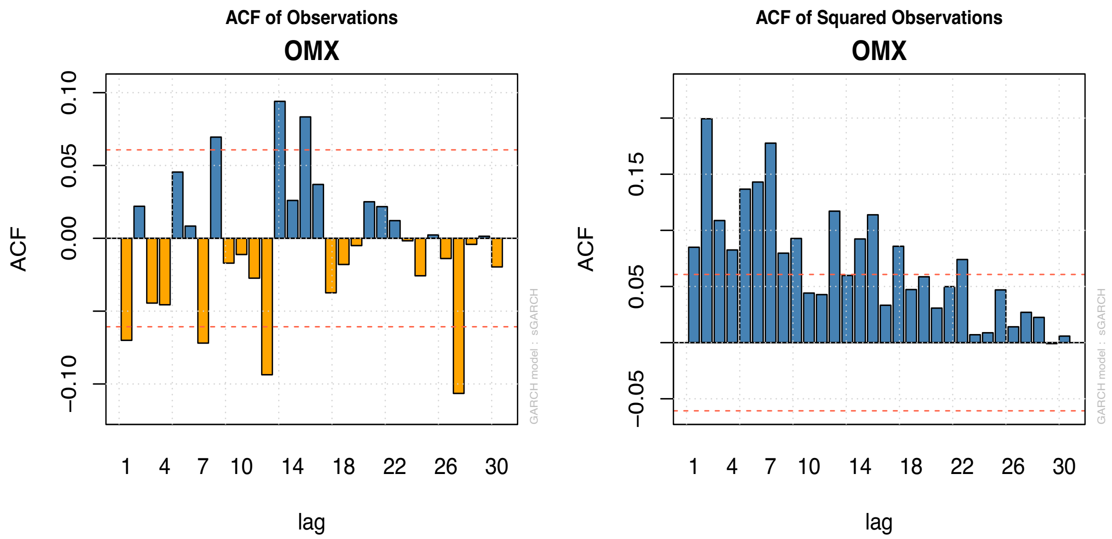

Figure 3.3: ACF for observations and squared observations of original OMX series

The time series were modelled using the *rugarch* package available in R [47]. Because the series described different types of data with varying characteristics, combinations in the parameter space up to a model order of ARMA(2,1)-GARCH(5,1) were tested. Model selection was then done in two steps, where the order combinations that passed the weighted Ljung-box tests for the residuals and squared residuals were retrieved, after which the model of choice was decided using AIC. For each of them, ACF plots were analysed to confirm the goodness-of-fit. See Figure 3.3 for the case of OMX, and Appendix A for all series. In Table 3.2, the final models for each time series are presented.

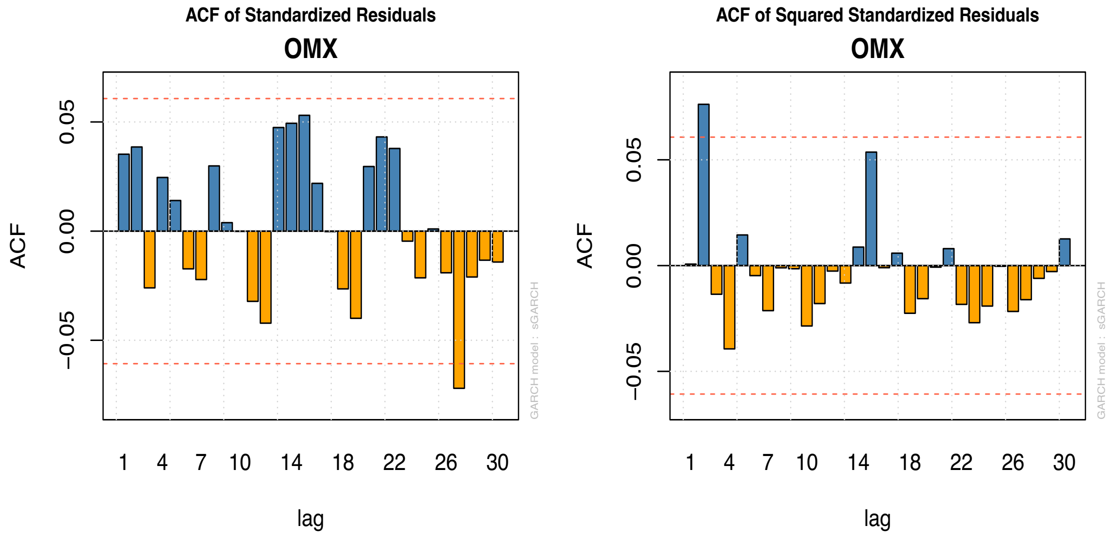

Figure 3.4: ACF for observations and squared observations of OMX residuals after fitting

Model fits were better for t-distributed residuals than skewed t. However, QQ-plots show that the former fails to accurately model the tails of the distributions. Therefore, skewed t-distributions were fitted to the standardized residuals using the package *sn* [48]. For all residual series this led to significantly better AIC. In contrast to the implementation in *rugarch*, *sn* allows for non-zero means and non-unit variance, which could explain the difference in performance. This is allowed since it can be shown that the parameter estimations of ARMA-GARCH models remain consistent under distribution changes of the innovations (see for example [49]). See Figure 3.5 for a comparison of fitted t- and skewed t-distributions for OMX. Resulting QQ-plots for all sets of data are provided in Appendix A.

Finally, probability transforms were applied to the residuals to obtain pseudo-observations. Unless misspecified, they should be uniformly distributed. Using the Cramer-von-Mises test the null hypothesis of the observations coming from a uniform distribution could not be rejected for any case. See Table 3.3 for distribution specifications and CvM test statistics.

Table 3.2: Specifications of fitted time series models

	μ	a_1	a_2	c_1	ω	α_1	α_1	α_2	α_4	α_5	β_1
OMX	2,98E-03	6,27E-01		-7,01E-01	3,06E-05	5,30E-02	6,06E-02				8,44E-01
EUR	1,43E-04	-1,01E-01			2,22E-06	7,08E-02	7,62E-07	3,79E-07	3,92E-07	3,07E-02	8,71E-01
USD	1,66E-04				6,31E-06	5,87E-02					9,17E-01
JPY	-5,37E-04				6,98E-06	5,52E-02					9,24E-01
VSTOXX	-5,38E-03	6,66E-01	5,38E-02	-9,06E-01	4,56E-03	1,61E-01					5,82E-01
VIX	-5,72E-03	6,62E-01	1,13E-01	-9,36E-01	8,95E-03	1,68E-01	1,46E-01				2,65E-01
VXJ	-8,25E-03	-1,76E-01			2,31E-03	1,47E-01					7,22E-01
(OMX-STOXX)	6,51E-04	8,50E-01		-9,06E-01	4,95E-06	5,77E-02					9,19E-01
(OMX-S&P)	5,41E-05	2,75E-01		-4,17E-01	1,79E-04	1,72E-01	9,55E-02	1,01E-01	9,11E-02		
(OMX-N225)	-5,37E-05	-1,50E-01			3,78E-04	1,96E-01	4,58E-02	1,27E-01			

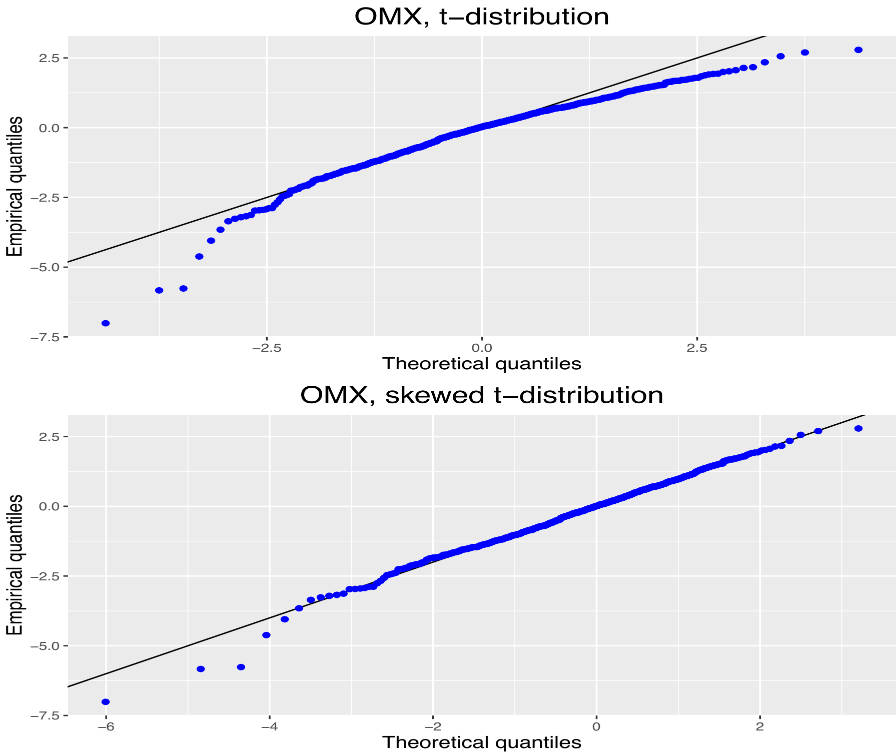

Figure 3.5: QQ-plots, OMX residuals.

Table 3.3: Statistical summary of marginal distributions

	Mean	Std	Skew	Df	CvM (p-value)
OMX	0.73	1.09	-1.48	6.89	0.95
EUR	-0.49	1.01	0.72	11.30	0.93
USD	-0.70	1.17	1.08	24.85	0.97
JPY	-0.46	0.93	0.73	6.50	0.96
VSTOXX	-0.77	1.12	1.71	7.07	0.83
VIX	-0.81	1.13	2.35	6.15	1.00
VXJ	-0.64	0.95	1.39	4.77	0.93
(OMX-STOXX)	-0.17	0.88	0.25	7.40	0.75
(OMX-S&P)	0.21	0.83	-0.32	5.67	0.99
(OMX-N225)	-0.15	0.87	0.22	7.68	0.99

3.3 Copulas

Next, copulas were fitted using the *copula* package available in R [42]. This was initially done for the absolute performance of OMX. Below, two scatter plots of the exchange rates and OMX for each of the three markets are presented, where the observations are coloured by the corresponding change in volatility expectations. In the first plots, we let the colour range from (large) increases in volatility to (large) decreases. In the second versions, we let the colour describe large shifts in either direction, contrasted to smaller changes. In this way, the colours in the second versions can be considered describing the level of extreme change in volatility expectations.

All three regions exhibited similar patterns with negative dependence between OMX and the exchange rates, although it was strongest for Japan. It is obvious from the first three plots that observations in the upper-left quartile are linked to large increases in volatility expectations, and observations in the lower-right quartile with large decreases. This indicates that in times of increased (decreased) uncertainty, both the SEK and OMX depreciate (appreciate). This agrees with the notion that the SEK depreciates against the three currencies of interest during periods of heightened uncertainty. This is true for OMX as well, as for most equity indexes, as volatility is negatively correlated with stock returns [50].

The three last plots provide a more interesting finding. From a quick visual inspection, there seems to be an increased dependence between the two variables when shifts in volatility expectations are greater in magnitude. During small movements in volatility expectations, the dependence between OMX and exchange rates is small if not non-existing. This effect is particularly pronounced for USA and the EU. We return to this issue after having fitted copulas to each dataset.

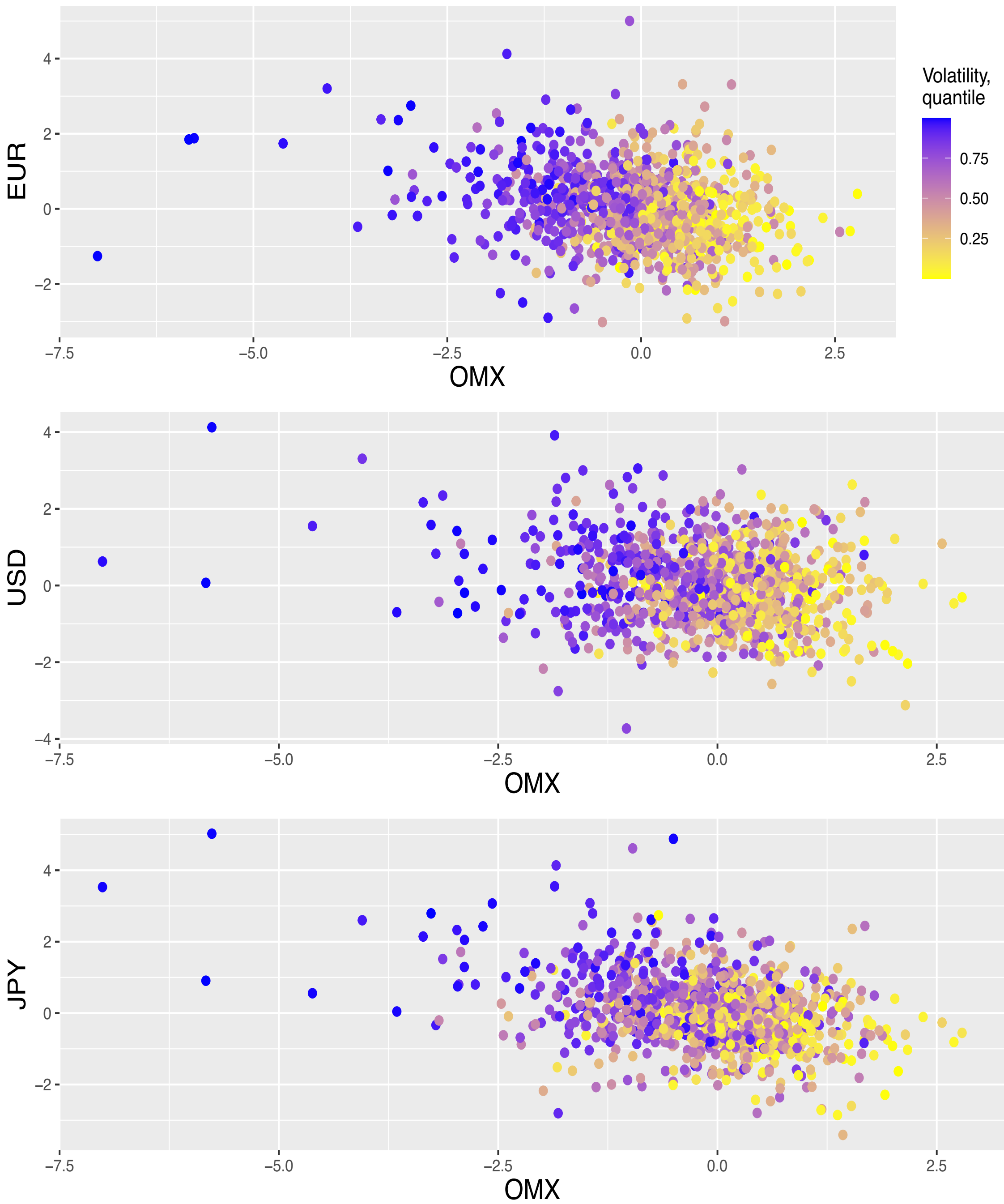

Figure 3.6: Scatter plots of observations (absolute performance of OMX), coloured by shift in volatility expectations.

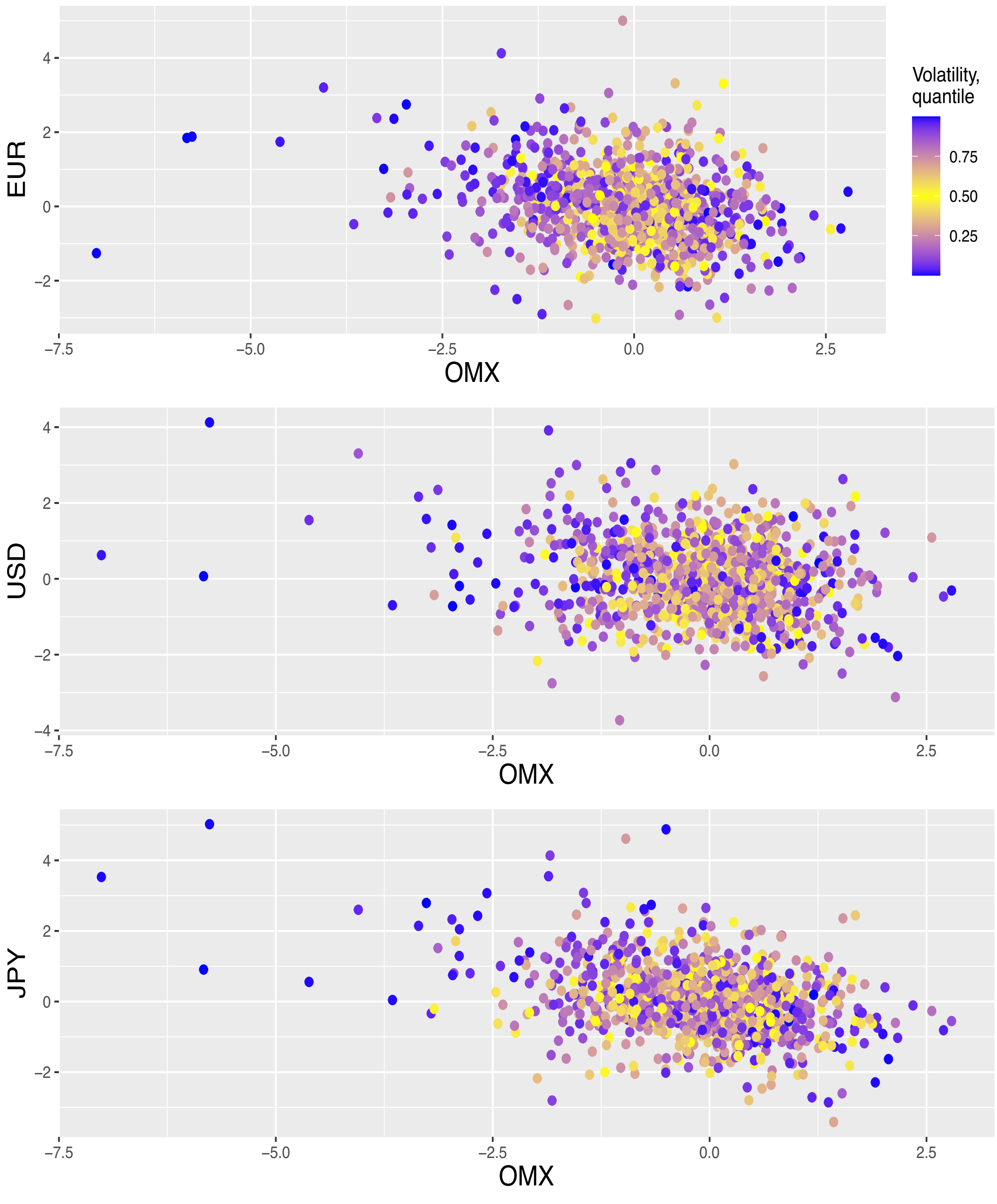

Figure 3.7: Scatter plots of observations (absolute performance of OMX), coloured by level of shift in volatility expectations.

Below, the same plots for the relative performance of OMX are presented. That is, the weekly log-returns of OMX are replaced with the difference in weekly log-returns when compared to the referenced equity indexes. Three differences to the plots above stand out. First, the dependence is weaker than above. Second, rather than negative it is slightly positive. The dependence seems to be strongest for Japan, and weakest for EU. Third, the impact of shifts in volatility expectations is less obvious, both when considering the sign of the shift (increase/decrease) and just the magnitude of it.

Copulas were then fitted to each data set. To reiterate, there were six sets of data: the absolute and relative performance of OMX against the three regions USA, EU, and Japan. For each data set, five three-dimensional copulas were fitted: the t-copula and the four Archimedean copulas Gumbel, Clayton, Joe, and Frank. The best fit was chosen according to the AIC criterion, see Table 3.4. As mentioned in Chapter 2, Archimedean copulas put restrictions on the values of Kendall's tau. For the absolute performance, OMX is negatively dependent with the exchange rates and volatility indexes (Figure 3.6), and rotated versions have therefore been used by replacing the probability functions U with their survival functions 1-U. In three dimensions, this can be done in two ways (i.e., by either rotating OMX or both the SEK and volatility index). While both combinations were tested, only the versions with the best AIC are presented in Table 3.4.

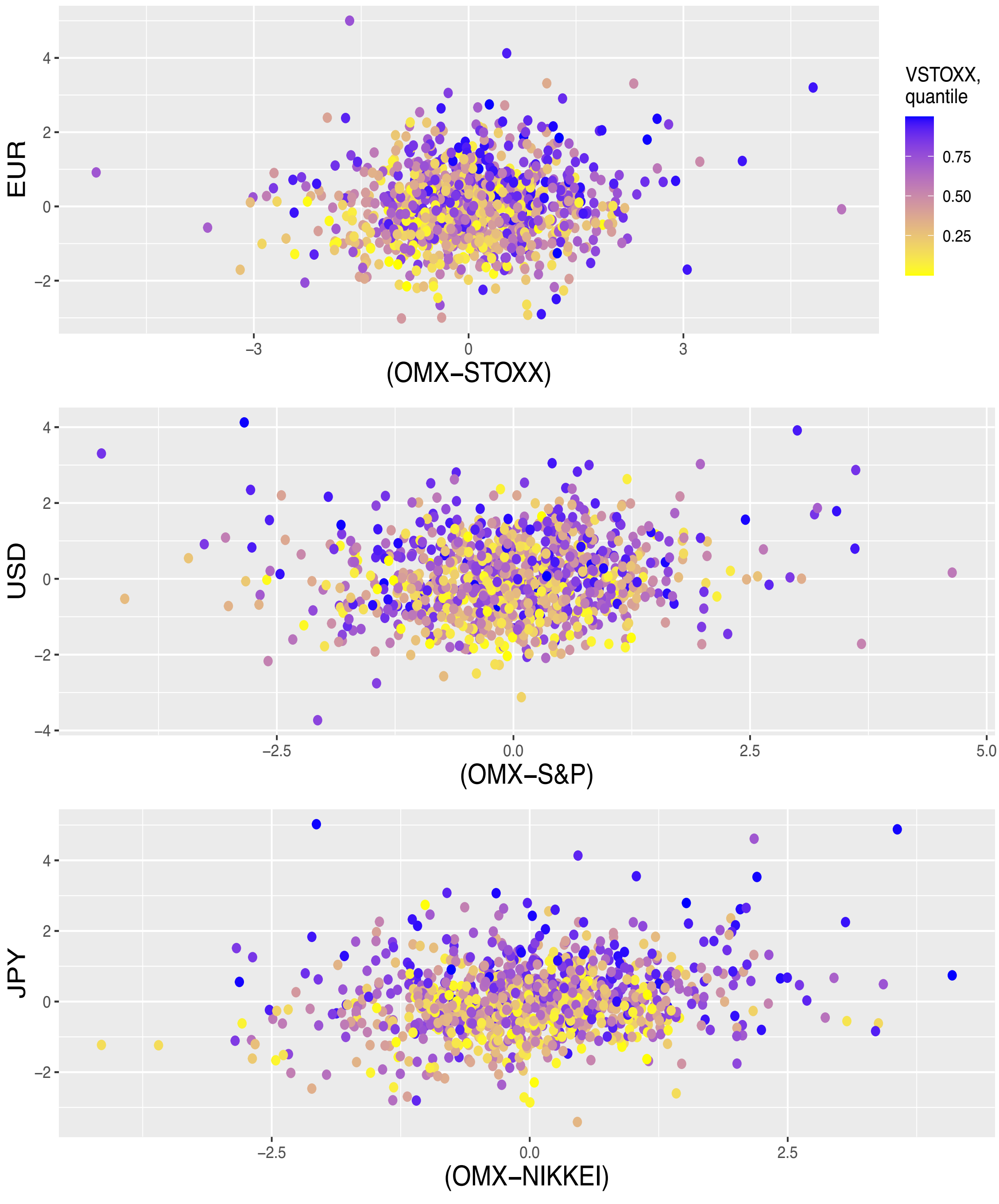

Figure 3.8: Scatter plots of observations (relative performance of OMX), coloured by shift in volatility expectations.

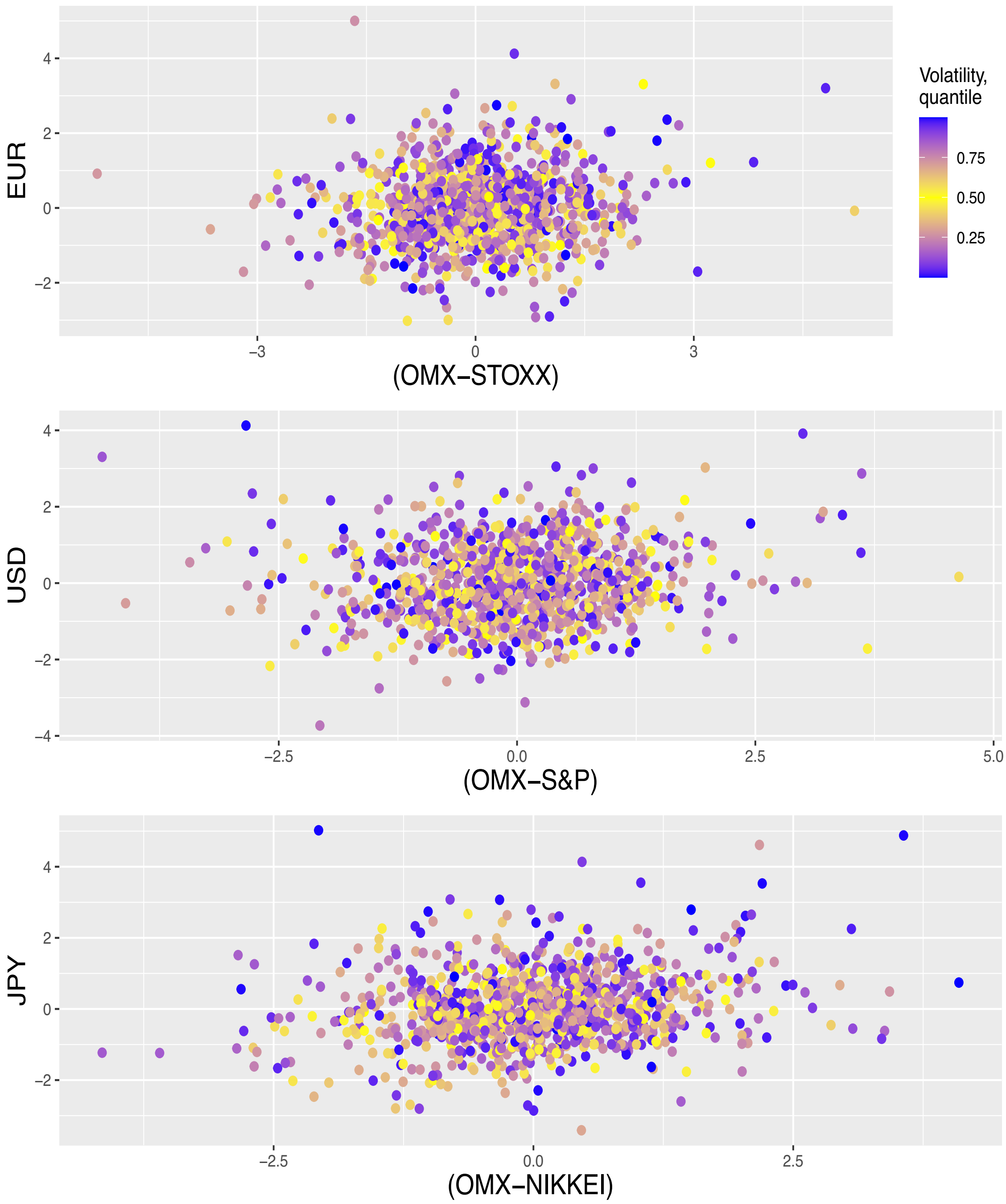

Figure 3.9: Scatter plots of observations (relative performance of OMX), coloured by level of shift in volatility expectation.

Table 3.4: AIC values of fitted copulas

	Student t	**Gumbel**	**Clayton**	**Joe**	**Frank**
OMX-SEK/EUR-VSTOXX	**-769.8**	-424.1	-439.0	-345.2	-430.5
OMX-SEK/USD-VIX	**-564.8**	-268.7	-312.0	-215.3	-263.7
OMX-SEK/JPY-VXJ	**-423.6**	-388.9	-388.5	-341.7	-322.5
(OMX-STOXX) -SEK/EUR-VSTOXX	**-116.4**	-84.7	-44.7	-69.0	-76.3
(OMX-S&P) -SEK/USD-VIX	**-62.9**	-33.4	-21.4	-24.7	-33.2
(OMX-NIKKEI) -SEK/JPY-VXJ	**-162.1**	-118.1	-50.8	-105.3	-81.7

In all six cases, the t-copula provided the best fit. Most likely, the exchangeability of the Archimedean copulas was in this context too strong of an assumption, since they only provide a reasonable fit when all pairwise dependencies of the variables are of a similar structure. In this case, it would imply that the dependence structure of equity-exchange rate, equity-volatility, and exchange rate-volatility are alike. Specifications of the resulting t-copulas, as well as p-values for the goodness-of-fit tests, are provided in Table 3.5. Two copulas did not pass the CvM test at significance level 0.05: EU in the absolute case and Japan in the relative case. For the former, the null hypothesis could however not be rejected for the rank-based test. While this is worth keeping in mind, it is important to remember that we are modelling three-dimensional empirical processes using a reasonably large data set. It is therefore not surprising that the models diverge from the true processes to some extent.

For the t-copula, there is a direct inverse linkage between degrees of freedom and tail dependencies in that, as degrees of freedom tend to infinity, the t-copula tends to the Gaussian copula which has zero tail dependence. Degrees of freedom were higher and tail dependencies lower for EU than the other two markets for both the absolute and relative performance of OMX. This is likely due to Sweden's stronger interconnectedness to the EU. For example, the relative performance of OMX against EUROSTOXX50 will exhibit fewer extremes than against the other two indexes. Looking at the correlation Θ between OMX and SEK, it decreased significantly more for EU than the other two markets when switching from absolute to relative performance of OMX. Again, this is likely due to the large co-movement between OMX and EUROSTOXX50. Disregarding the impact of the volatility expectations for a moment, this indicates that the link between the SEK and the relative performance of OMX is less important when using EU as reference, than USA or Japan.

Table 3.5: Specifications fitted copulas

Copula	Pair	Df	Θ	τ	$\lambda_{U,D}$	CvM, p-val.	CvM$_R$, p-val.
OMX-SEK/EUR-VSTOXX		20.8				0.005	0.450
	OMX-SEK/EUR		-0.267	-0.172	3.64E-06		
	OMX-VSTOXX		-0.699	-0.493	1.95E-10		
	SEK/EUR-VSTOXX		0.268	0.173	1.82E-03		
OMX-SEK/USD-VIX		16.2				0.064	0.628
	OMX-SEK/USD		-0.166	-0.106	1.33E-04		
	OMX-VIX		-0.628	-0.433	1.11E-07		
	SEK/USD-VIX		0.207	0.133	3.71E-03		
OMX-SEK/JPY-VXJ		18.5				0.311	0.946
	OMX-SEK/JPY		-0.336	-0.218	4.56E-06		
	OMX-VXJ		-0.469	-0.311	4.94E-07		
	SEK/JPY-VXJ		0.321	0.208	4.98E-03		
(OMX-STOXX)-SEK/EUR-VSTOXX		61.9				0.767	0.378
	(OMX-STOXX)-SEK/EUR		0.086	0.055	6.58E-10		
	(OMX-STOXX)-VSTOXX		0.204	0.131	1.83E-08		
	SEK/EUR-VSTOXX		0.267	0.172	9.28E-08		
(OMX-S&P)-SEK/USD-VIX		18.3				0.074	0.568
	(OMX-S&P)-SEK/USD		0.119	0.076	9.32E-04		
	(OMX-S&P)-VIX		0.054	0.035	5.06E-04		
	SEK/USD-VIX		0.204	0.131	1.97E-03		
(OMX-NIKKEI)-SEK/JPY-VXJ		12.1				0.005	0.02
	(OMX-NIKKEI)-SEK/JPY		0.179	0.115	9.66E-03		
	(OMX-NIKKEI)-VXJ		0.093	0.059	5.64E-03		
	SEK/JPY-VXJ		0.311	0.201	2.06E-02		

As mentioned in Chapter 2, the t-copula exhibits symmetric tail dependence. This agrees with the findings in for example [26], and differs from the asymmetric tail dependencies across international foreign exchange markets [51] and across equity-bond markets [52].

Of particular interest was how shifts in volatility expectations affect the dependence between the SEK and OMX. However, it is difficult to directly get a good understanding of a three-dimensional dependence structure. One possibility is to assess how the dependence changes for different values of the volatility variable. In this context, it is important to be aware of the t-copulas properties as a simplified copula [53]. Put simply, for a three-dimensional copula, this means that the third variable only affects the other two variables via their marginals and not the dependence between them. In [54], it is argued that this simplification holds surprisingly well in many applications. One consequence of this is that the conditional Kendall's τ, i.e.

$$\tau(v_3) = 4 \int \int C_{v_3}(u_1, u_2) dC_{v_3}(u_1, u_2) - 1,$$

where

$$C_{v_3}(u_1, u_2) = P(Y_1 \leq y_1, Y_2 \leq y_2 \mid F(y_3) = v_3)$$

will be unaffected by the value of the third variable and hence constant. Looking at the Figures 3.5 and 3.7, it is not obvious that this assumption is violated.

With this in mind, and encouraged by the findings in Figure 3.6 where the dependence seems to increase with larger shifts in volatility expectations, we employ the following strategy to calculate a slightly modified version of conditional Kendall's τ:

1. Simulate samples from the fitted three-dimensional copula
2. Transform the volatility samples to one-sided probabilities by replacing all observations $u_i > 0.5$ with $1 - u_i$. For example, $u_i = 0.9$ becomes 0.1. In this way, the probabilistic level of "extremeness" is considered but without its sign. If the marginal distribution of volatility shifts was symmetrical, this would be equivalent to fitting a folded distribution to the absolute values of the residuals.
3. Split the equity and exchange rate sample pairs into bins based on the values of the corresponding volatility observation. Here, we choose a step size of 0.01 and window size of 0.05.
4. Calculate Kendall's τ for each bin

We do this with 100,000 samples. See Figure 3.10 for a visualization of the conditioning based on the level of volatility changes. It is worth highlighting point two above again. By construction, we move from the tails of the volatility expectation distribution inwards to its mean. Had we let the volatility residuals follow a t-distribution, this would have been equivalent to conditioning on a folded distribution fitted to the absolute residuals. Because they are modelled by skewed t-distributions, this procedure was not possible since the process of taking the absolute values of the residuals would have obscured the true probabilities.

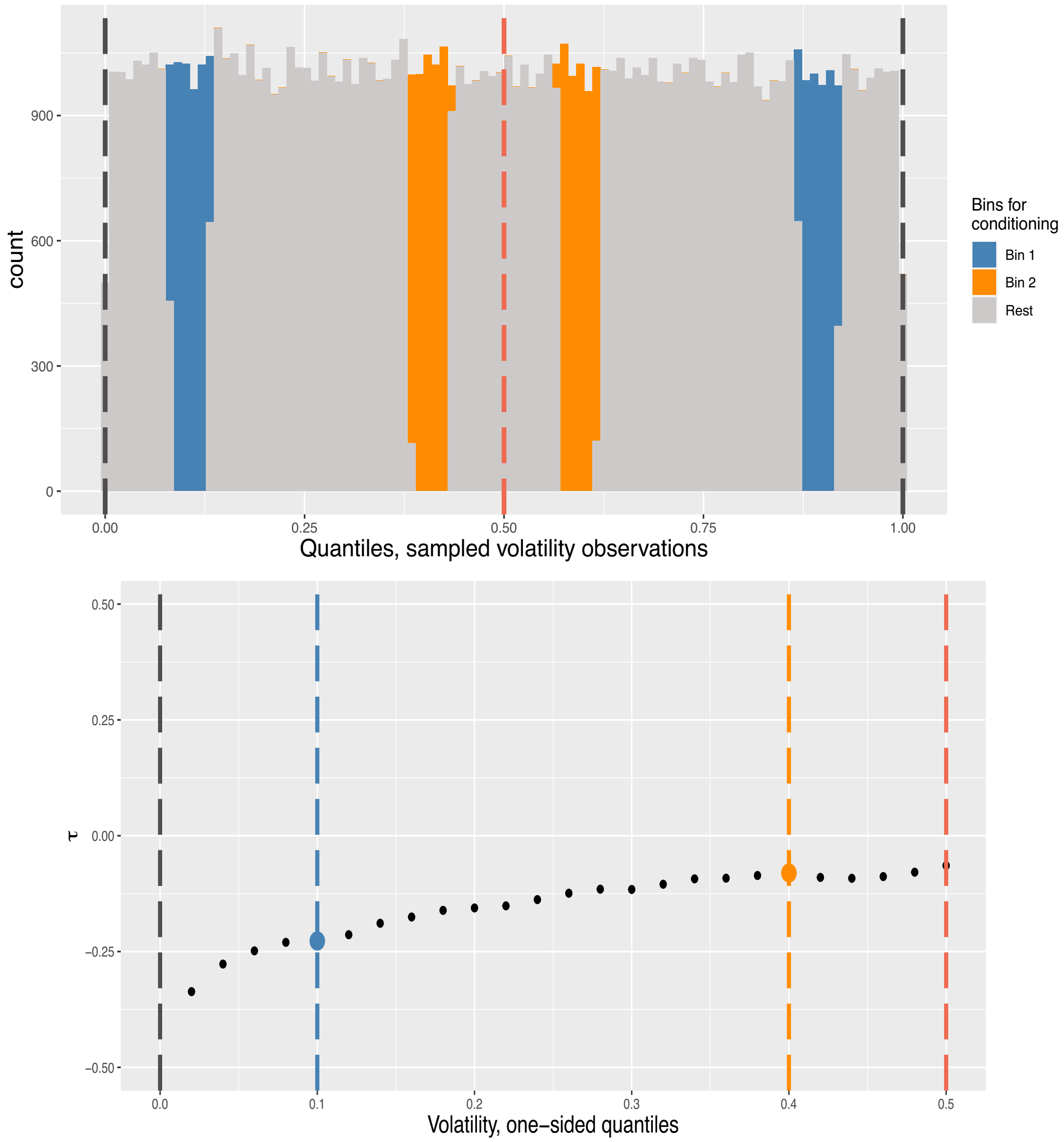

Figure 3.10: The upper plot illustrates the histogram of simulated pseudo-observations of volatility samples, and the bins used for conditioning based on transformed one-sided quantiles. The second plot visualizes an example of how the conditional Kendall's τ between the two other variables changes based on these bins (black dots), with the corresponding bins of the upper plot highlighted.

This procedure was initially performed for the copulas of the absolute performance of OMX. The evolution of Kendall's τ between OMX and the exchange rates as shifts in volatility expectations become smaller are plotted for the three countries in Figure 3.10. Clearly, they all exhibit a similar pattern of negative dependence when changes in volatility expectations are large, which tends towards zero as they decrease. To be more precise: when volatility expectations either increase or decrease greatly, OMX and the exchange rates tend to exhibit more (negative) dependence. This dependence becomes weaker when shifts in volatility expectations become smaller, although it remains negative. The dependence is the strongest for Japan, and weakest for USA. This result confirms the visible effect previously described in the scatter plots of Figure 3.7.

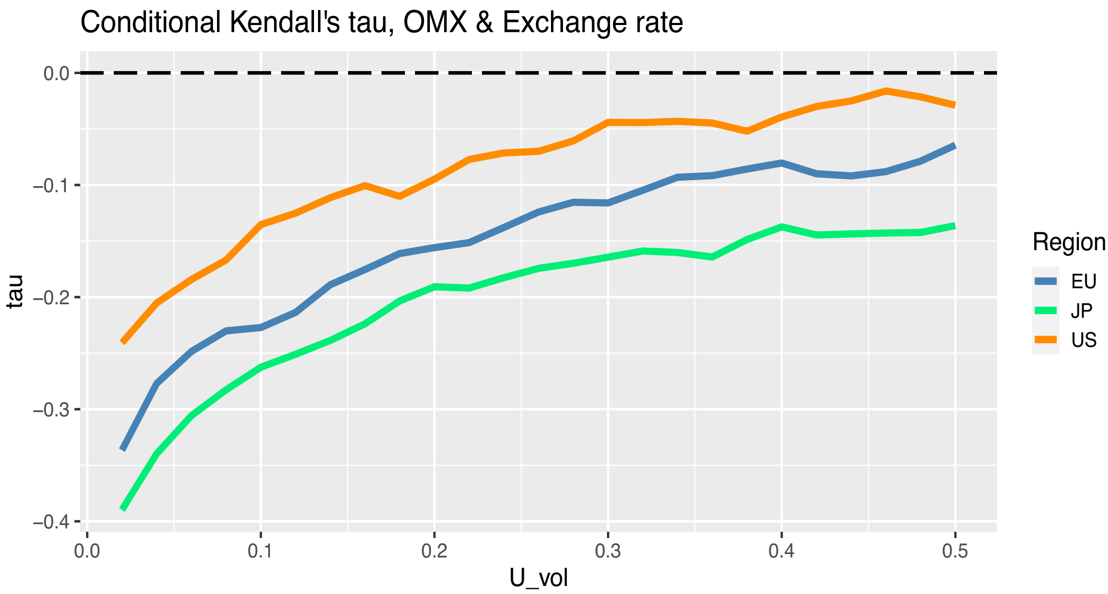

Figure 3.11: Conditional Kendall's τ between absolute performance of OMX and SEK given level of shift in volatililty expectations

One can also study the dependence structure in greater detail by plotting the observation pairs in different bins. We illustrate this in the case of EUR and VSTOXX. In Figure 3.12, the observation pairs corresponding to the volatility levels of the fields marked blue and orange in Figure 3.9 above are visualized, respectively. Here too, is the stronger dependence with larger volatility shifts visible. They can be compared to Figure 3.7, where a similar change of dependence is seen when volatility shifts increase.

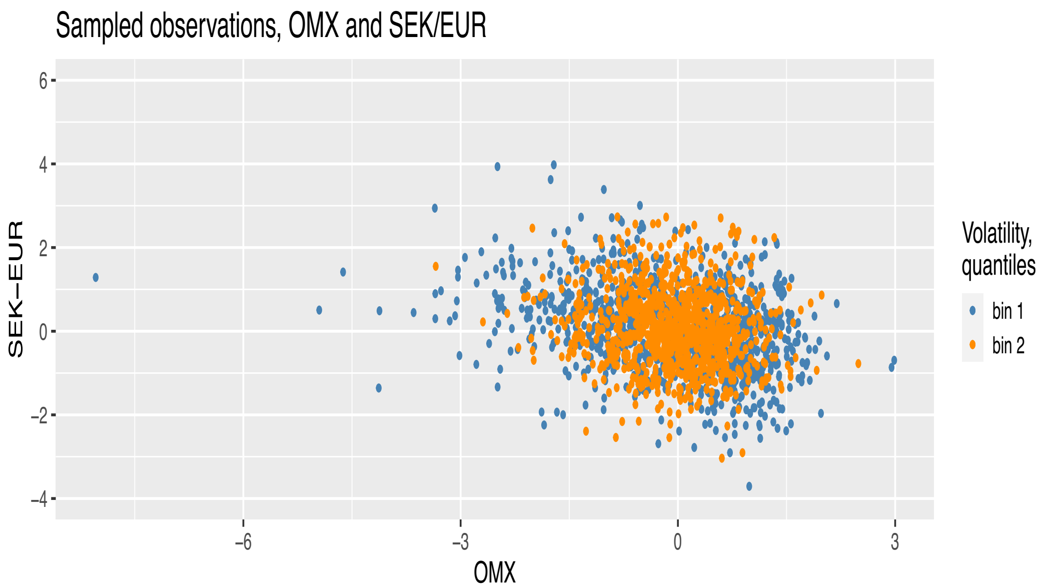

Figure 3.12: Samples from two bins of the fitted copula OMX-SEK/EUR-VSTOXX. The blue observations, corresponding to larger shifts in volatility expectations, exhibit stronger dependence.

Finally, the same procedure was performed for the relative performance of OMX. Corresponding plots of the change in Kendall's τ are visible in Figure 3.13. Again, all regions exhibit similar patterns. This time, however, the dependence is positive. It remains fairly constant for Japan and USA under different volatility shifts. For EU, it is stronger when shifts in volatility expectations are larger, and tends close to zero as they become smaller. While all markets exhibit similar results, the dependence is weak (below 0.125). One should therefore be careful about conclusions of non-zero dependence.

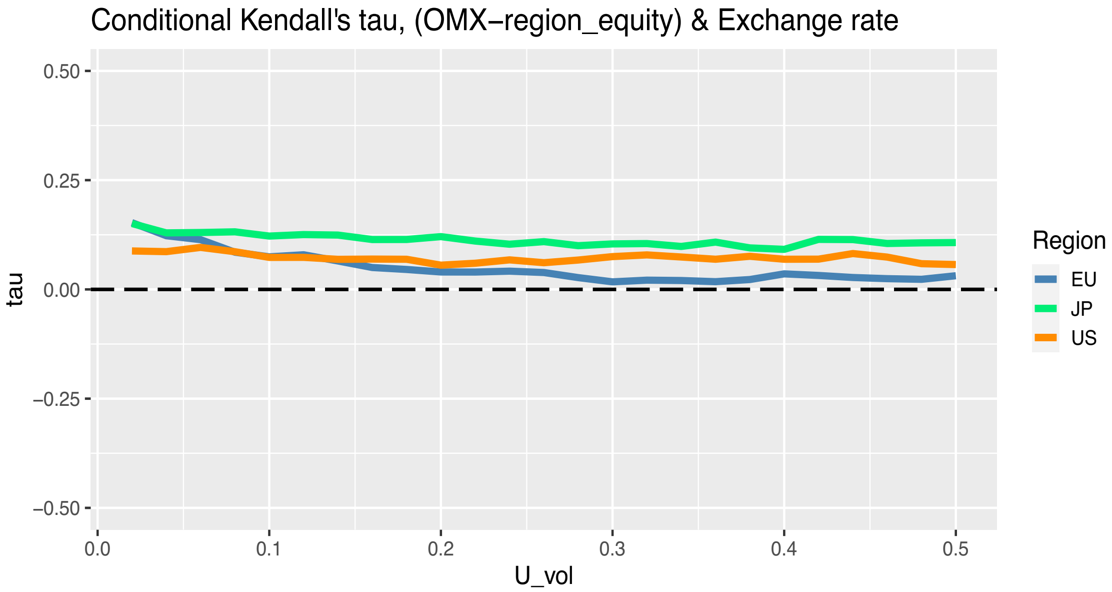

Figure 3.13: Conditional Kendall's τ between absolute performance of OMX and SEK given level of shift in volatililty expectations

Chapter 4

Discussion and Conclusions

In this report, we modelled the three-dimensional joint distribution of the SEK, OMX, and expected future volatility. This was done for both the absolute and relative performance of the OMX, and in reference to three developed markets. Of the copulas, the Student's t-copula provided the best fit for both absolute and relative performance. Likely, the exchangeability of the Archimedean copulas was too large of a restriction. Then, from the three-dimensional distribution, the impact of shifts in expected future volatility on the dependence between OMX and the SEK was derived. We found a negative dependence between the exchange rates and the absolute performance of OMX that was most pronounced when shifts in volatility expectations were large, and tended to zero as they became smaller. On the contrary, the relative performance of OMX was positively dependent with the exchange rates, but weak for all markets.

For the absolute performance of OMX, the increased negative dependence when shifts were large in either direction (meaning, that the SEK appreciated and depreciated with OMX) is in agreement with the safe-haven theory, which predicts that investors will move to currencies perceived to be safe when uncertainty is high. If the referenced currencies are considered safer than the SEK, then, the SEK and OMX should both depreciate when volatility expectations sharply increase. When they sharply decrease, indicating more positive outlooks, the effects will be reversed. We observed that as volatility shifts became smaller the OMX-SEK dependence approached zero, indicating a disconnect in calmer periods. Again, this is to be expected from the safe-haven theory, as the specific linkage it identifies will in calmer periods lose power relative to the many other factors affecting currency and equity markets.

Increased dependence during volatile periods could also align with the hedging channel theory of exchange rate dependence. As previously mentioned, its predicted effect depends not only on the relative performance of markets but also on country-specific factors such as the amount of foreign capital invested and the willingness to hedge it. It could therefore be visible in the data on absolute performance as well. Since uncertain periods often involve large market movements, it is reasonable that this effect is most pronounced during such times, since the capital flows of hedging adjustments will be so. However, the negative sign of the dependence is conditional on those adjustments being greater for Swedish investors in foreign markets than vice versa. Otherwise, positive dependence could have been observed. Whether this assumption is reasonable is left for future research to establish.

The results were similar for the three markets, indicating that whatever the underlying cause, there is little market level idiosyncrasy when comparing the SEK to large, developed markets. Still, Japan exhibited the strongest dependence, and USA the weakest. This is not surprising since both the JPY and EUR have been documented to appreciate against the USD in volatile periods. In [9] this is shown using the notion of safe-haven currencies, and in [21] using the hedging channel of exchange rate determination.

A question sparked by the results pertains to why the JPY would be considered a safer currency than the USD, which by all metrics must be considered "the world currency". While it would not be surprising to find a flight from smaller currencies, one would expect the USD to appreciate the most if the main driver of change was a psychological flight-to-safety effect. The stronger dependence in the case of JPY might thus constitute evidence in favour of the hedging channel theory. However, one would need to investigate the issue more closely to confidently say so. To compare the respective theories, data on capital flows could be incorporated in the model. For example, flows in derivative markets commonly used for hedging, such as exchange rate futures and options, could highlight the impact of hedging. Another option would be to inquire investment professionals who might have a recognition of the incitements and driving forces behind market movements.

The positive dependence (i.e., currency and equity market appreciate and depreciate inversely) when considering relative equity performance could be explained by portfolio rebalancing. That is, when OMX outperforms the referenced markets, investors in those countries sell off assets in order to maintain the desired diversification, leading to a weaker currency and hence an increased exchange rate. Here too, is the hedging channel a possible explanation. In contrast, the positive dependence disagrees with the predicted effects of return chasing which posits that currency and equity markets appreciate and depreciate together. However, one should be careful drawing conclusions given the low dependence ($\tau \leq 0.15$). For Japan and USA the dependence remained fairly constant under different levels of volatility shifts. This is interesting given our initial reasoning for testing both the absolute and relative market performance: the disconnect during volatile periods that might arise when investors prefer currencies with safe-haven properties is not visible. For EU, however, it decreased from 0.15 to zero as volatility shifts decreased. One could speculate that this result is reasonable if portfolio rebalancing is indeed a driving force of the dependence. Of the three equity indexes referenced, OMX exhibits the strongest dependence with EUROSTOXX50. This means that in terms of diversification across geographies, balancing between the Swedish and Eurozone markets could be considered less important than for the US and Japanese markets. This is also reflected in the change of dependence between OMX and the SEK when going from absolute to relative performance, which sees the greatest decrease in absolute terms for EU (see Table 3.5). On the other hand, because OMX and EUROSTOXX50 are closely correlated, it could also be a purely mechanical effect whereby the (on average) smaller differences in performance lead to smaller portfolio reallocations, that in turn get obscured by noise or other factors.

For professional investors active in both foreign and domestic markets, these findings have implications for diversification and hedging. First, when financial uncertainty increases, the SEK weakens against the referenced currencies. If the goal is to avoid large losses, then, Swedish investors should consider not hedging their investments in these countries as the local currencies provide a natural hedge. Of course, on the flip side, this can potentially result in smaller returns during positive market conditions when assets appreciate. Second, while simple return chasing must be considered a naïve investment strategy, portfolio reallocation effects could provide opportunities worth exploring in that outperforming markets seem to bring cheaper currencies, allowing foreign investors to benefit from both.

It is important to highlight some limitations and possible sources of error. First, the choice of copula. In this report, five copulas were fitted to the data. While the Student's t-copula provided the best fit, it is possible that other models could capture the true dependence better. Another possible source of uncertainty is that our model does not account for time-varying effects. Our dataset spans a time period of twenty years, and it is possible that the dynamics between currency and equity markets have shifted during this time. For future reference, the analysis could therefore be expanded to incorporate dynamic copulas, such as time-varying or regime-shifting models. Furthermore, alternative copula models that might provide good results include hierarchical Archimedean copulas, which allow for non-exchangeability.

Finally, as previously noted, it would be interesting to incorporate data of capital flows in the model to further analyse potential causal relationships. One could look at both hedging instruments as well as equities to assess the impact of hedging and portfolio reallocations, respectively. It would then also be possible to assess if the relationship between capital flows and relative equity performance appeared to be constant, or if one could detect a disconnect during periods of shifting financial uncertainty. The latter would then further corroborate the safe-haven effect suggested by this work.

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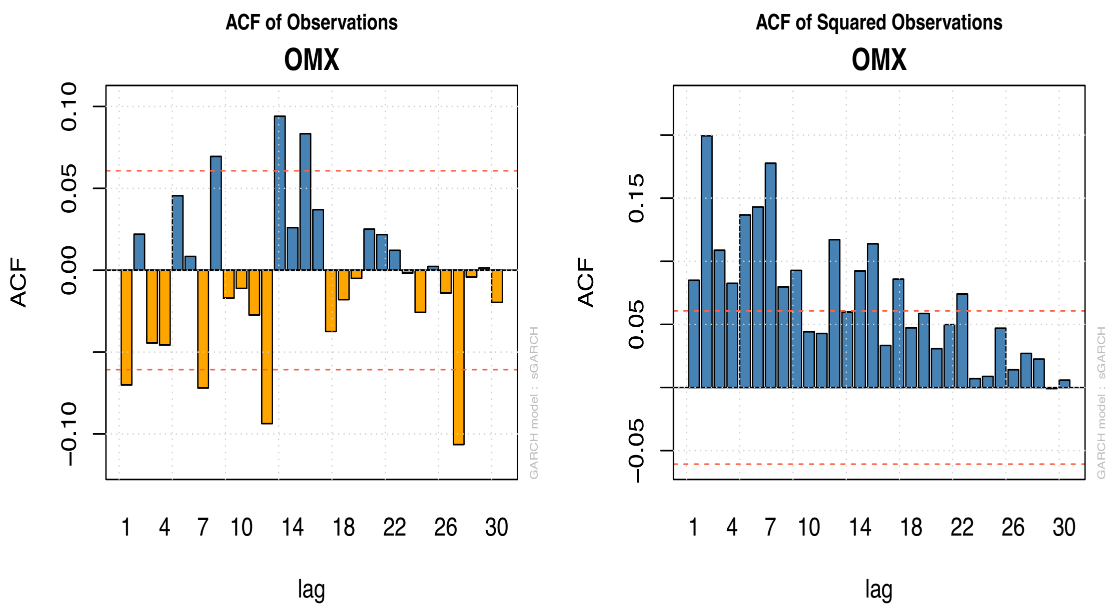
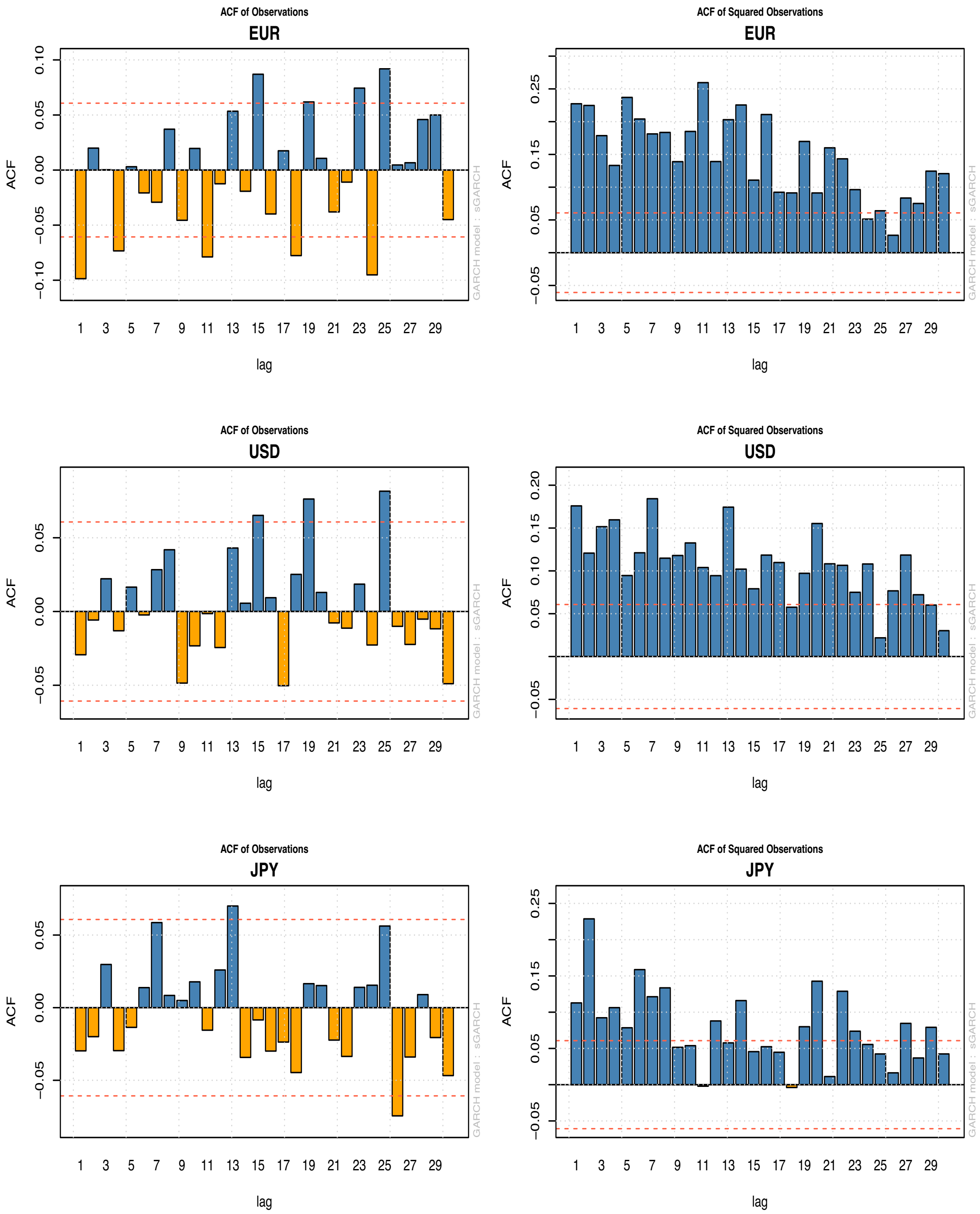
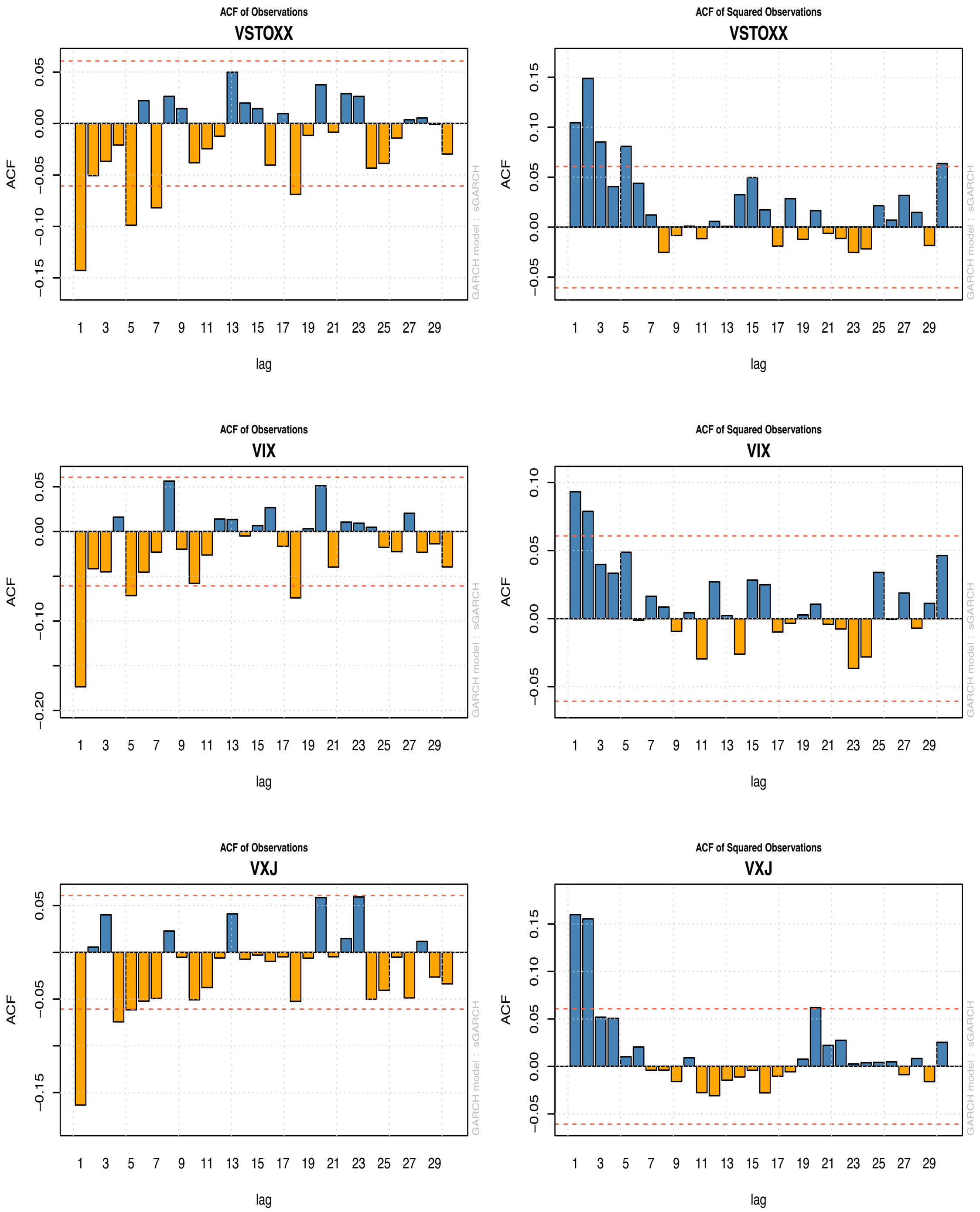
Appendix A

Figures

A.1 ACF plots, original series




ACF of Observations
EUR
ACF of Squared Observations
EUR
ACF of Observations
USD
ACF of Squared Observations
USD
ACF of Observations
JPY
ACF of Squared Observations
JPY
ACF
lag
GARCH model : sGARCH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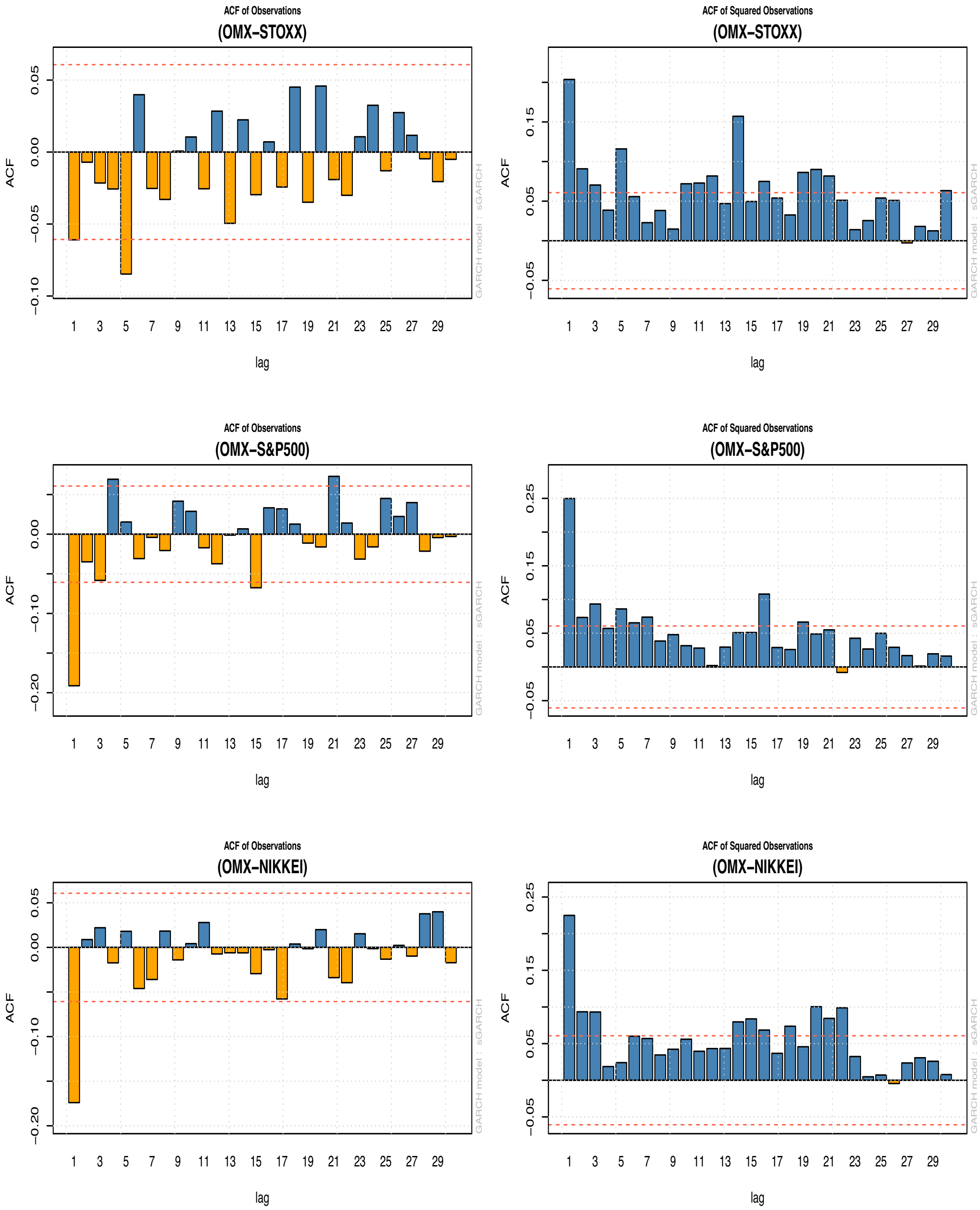
VSTOXX
ACF of Squared Observations
VSTOXX
ACF of Observations
VIX
ACF of Squared Observations
VIX
ACF of Observations
VXJ
ACF of Squared Observations
VXJ
ACF
lag
GARCH model : sGARCH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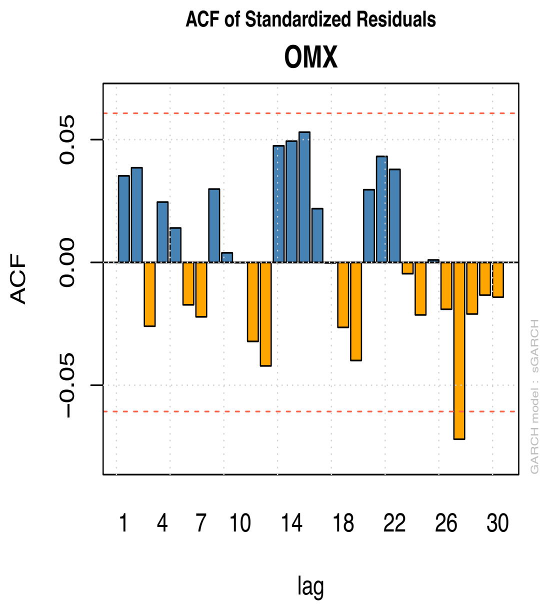
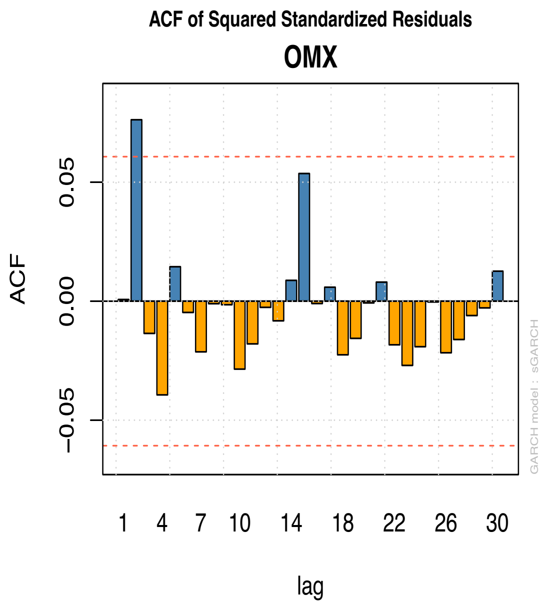
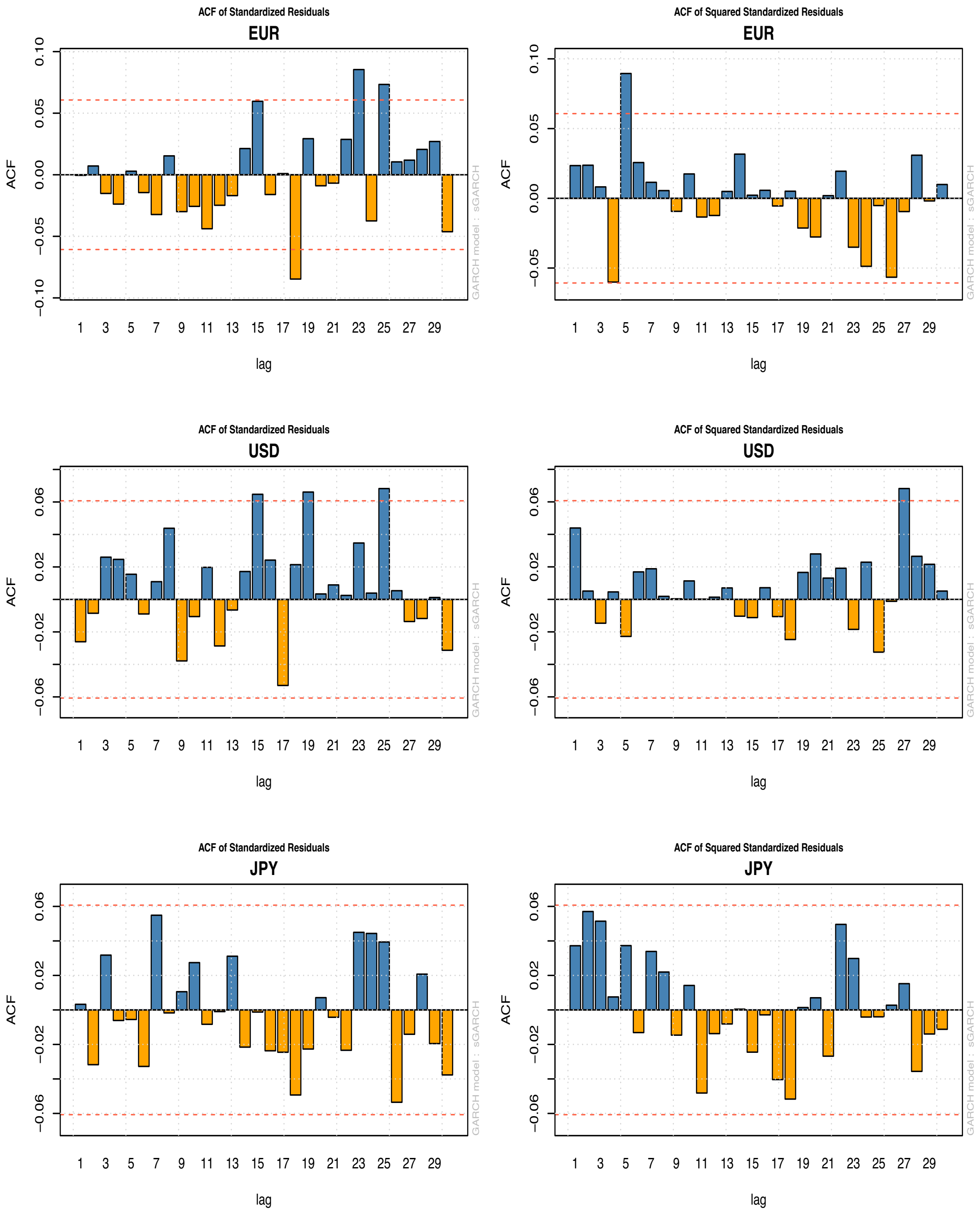
(OMX–STOXX)
ACF of Squared Observations
(OMX–STOXX)
ACF of Observations
(OMX–S&P500)
ACF of Squared Observations
(OMX–S&P500)
ACF of Observations
(OMX–NIKKEI)
ACF of Squared Observations
(OMX–NIKKEI)
ACF
lag
GARCH model : sGARCH

A.2 ACF plots, modelled series



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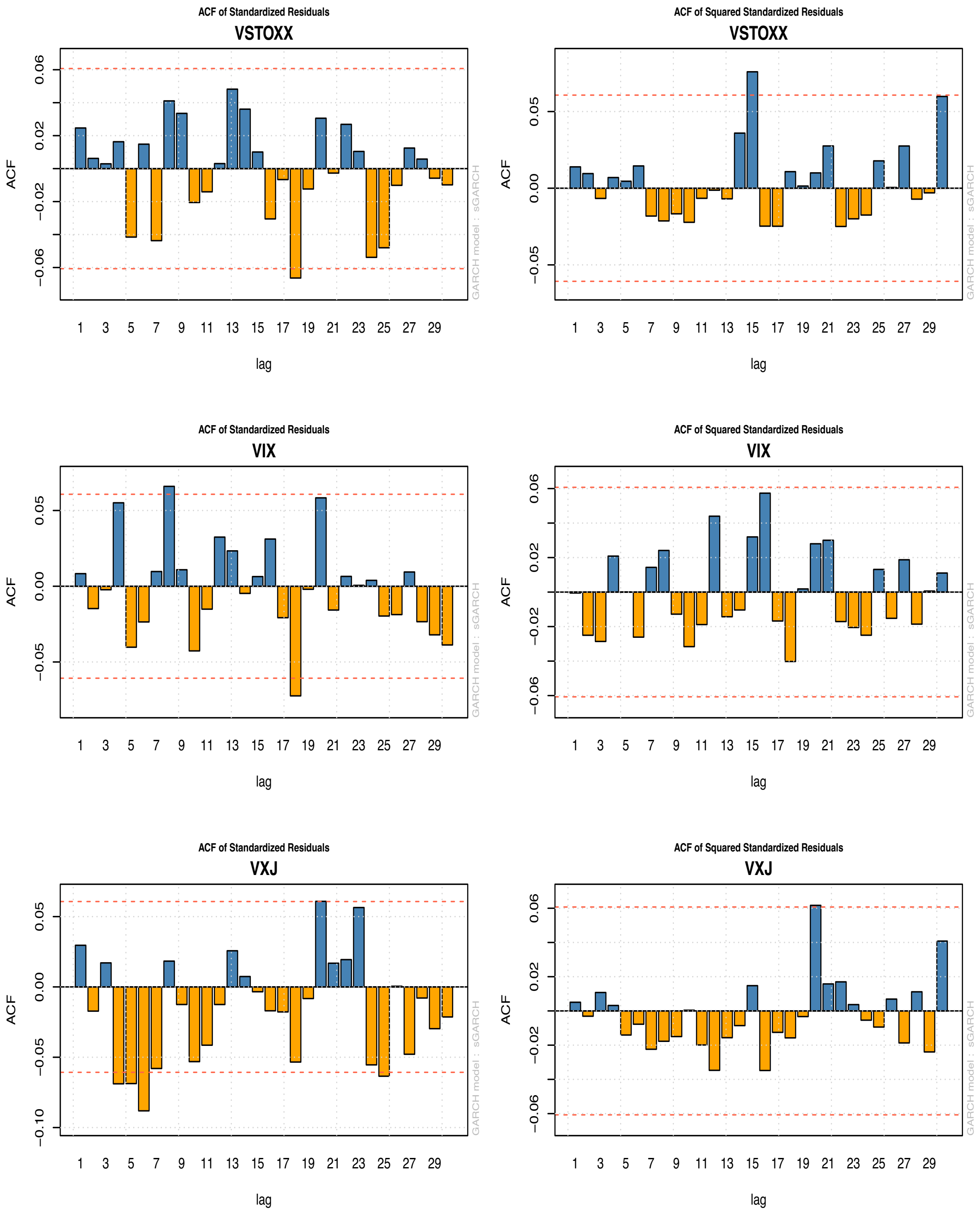
EUR
ACF of Squared Standardized Residuals
EUR
ACF of Standardized Residuals
USD
ACF of Squared Standardized Residuals
USD
ACF of Standardized Residuals
JPY
ACF of Squared Standardized Residuals
JPY
ACF
lag
GARCH model : sGARCH


ACF of Standardized Residuals
VSTOXX
ACF of Squared Standardized Residuals
VSTOXX
ACF of Standardized Residuals
VIX
ACF of Squared Standardized Residuals
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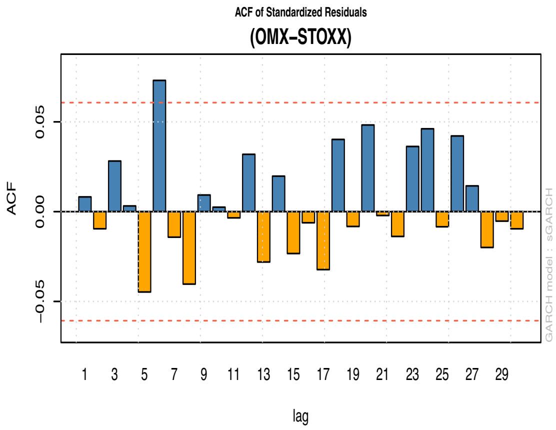
VXJ
ACF
lag
GARCH model : sGARCH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(OMX–STOXX)
ACF
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0.00
0.05
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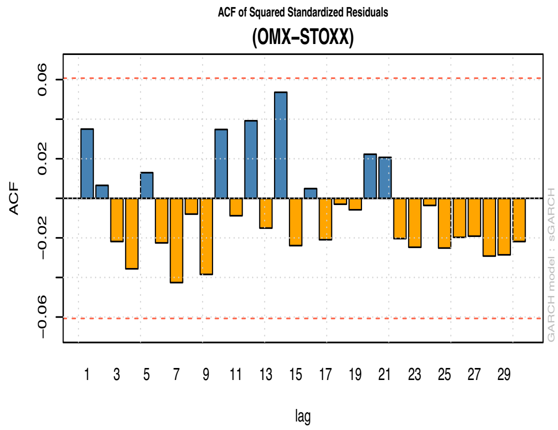
lag
GARCH model : sGARCH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(OMX–STOXX)
ACF
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-0.02
0.02
0.06
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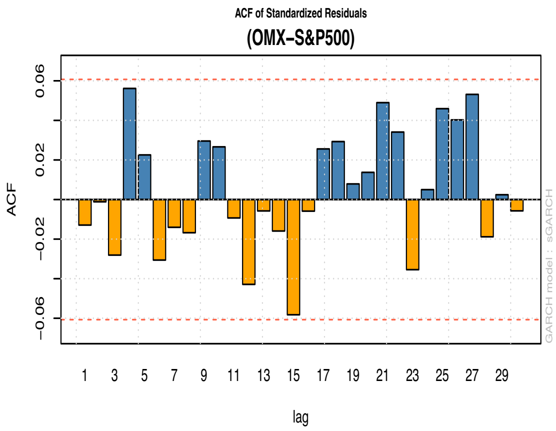
lag
GARCH model : sGARCH


ACF of Standardized Residuals
(OMX–S&P500)
ACF
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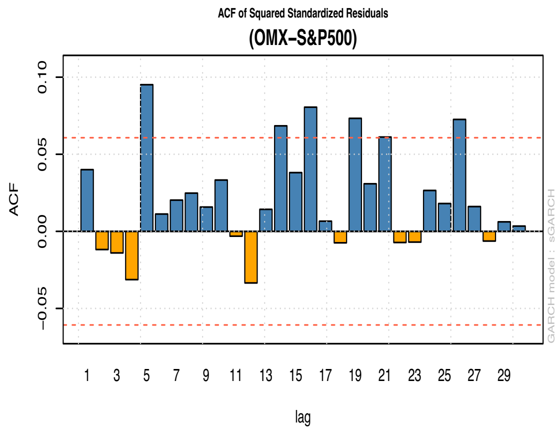
-0.02
0.02
0.06
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29
lag
GARCH model : sGARCH


ACF of Squared Standardized Residuals
(OMX–S&P500)
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0.00
0.05
0.10
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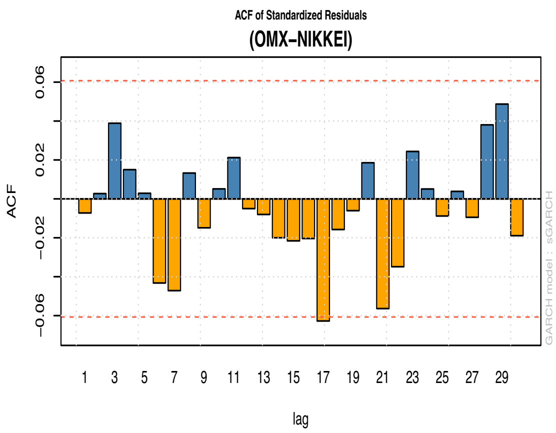
lag
GARCH model : sGARCH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(OMX–NIKKEI)
ACF
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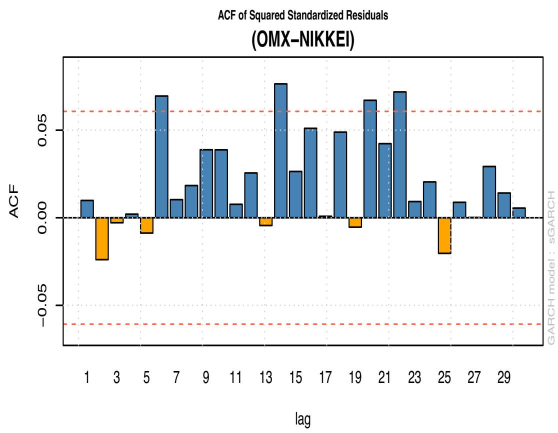
-0.02
0.02
0.06
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29
lag
GARCH model : sGARCH


ACF of Squared Standardized Residuals
(OMX–NIKKEI)
ACF
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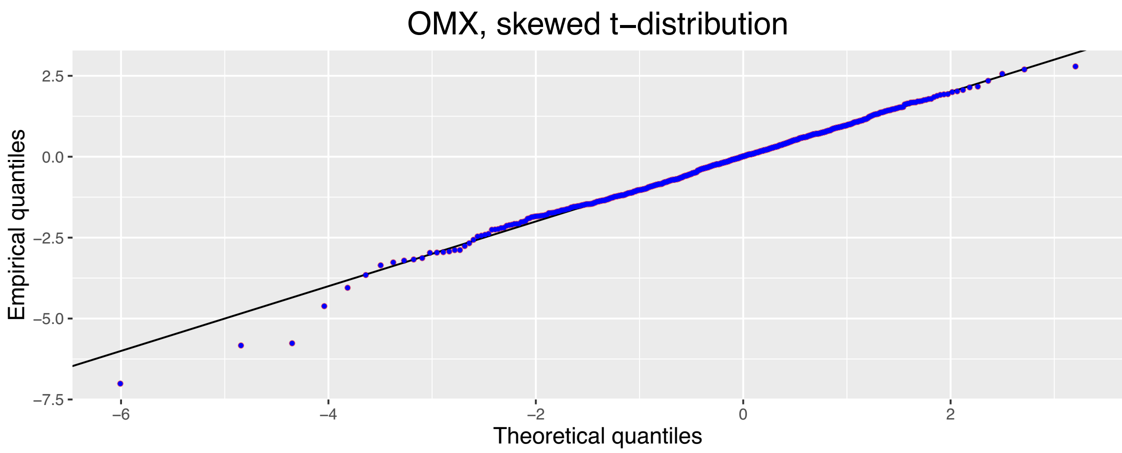
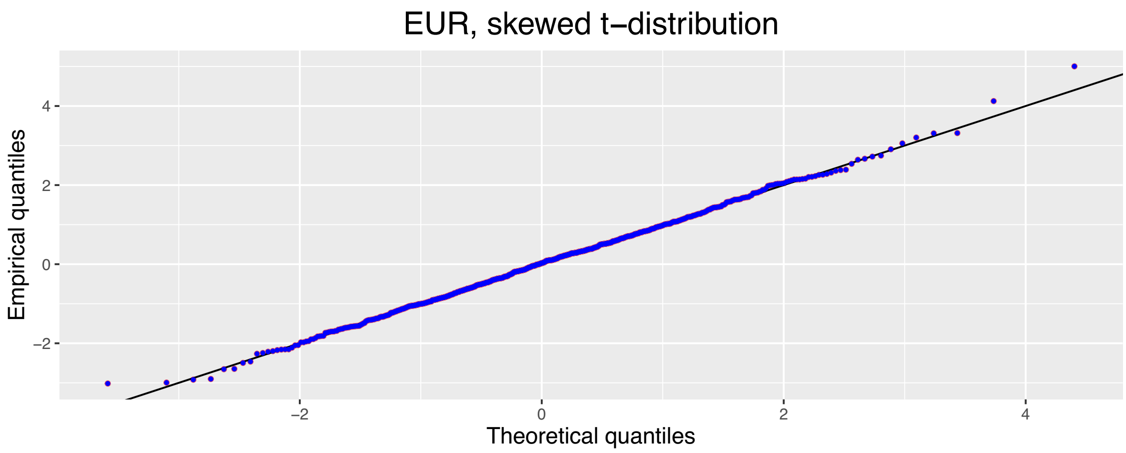
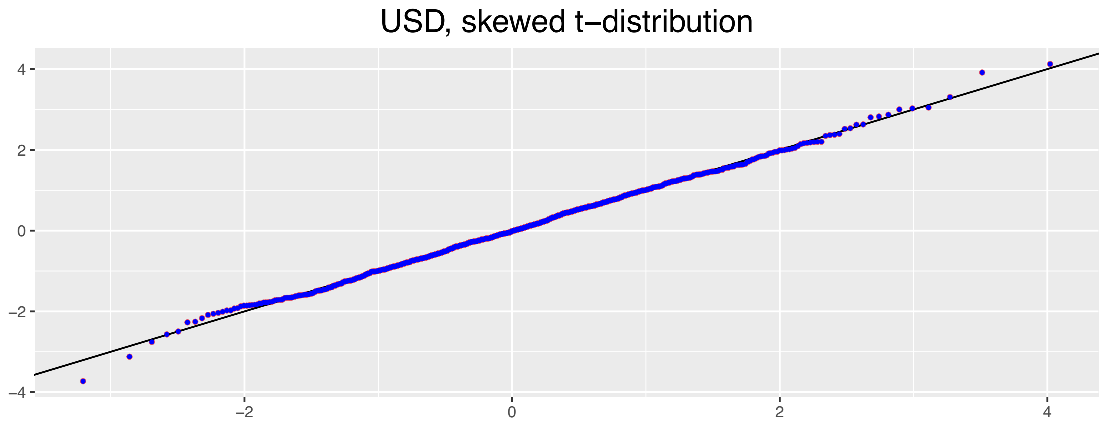
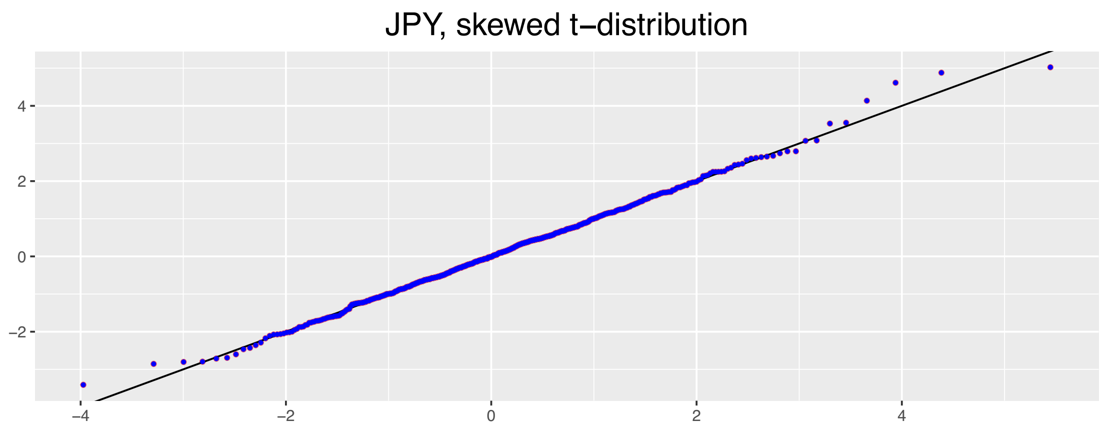
0.00
0.05
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29
lag
GARCH model : sGARCH

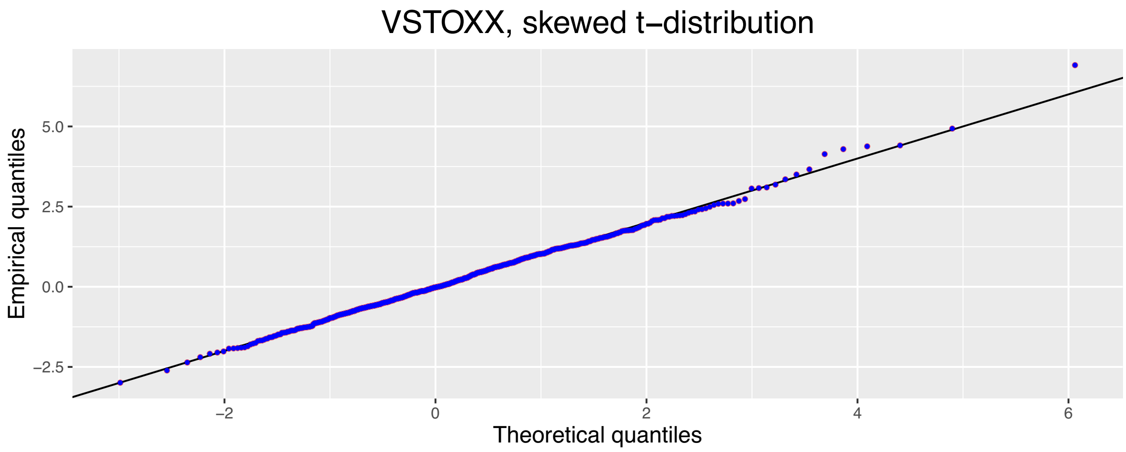
A.3 QQ-plots, residuals

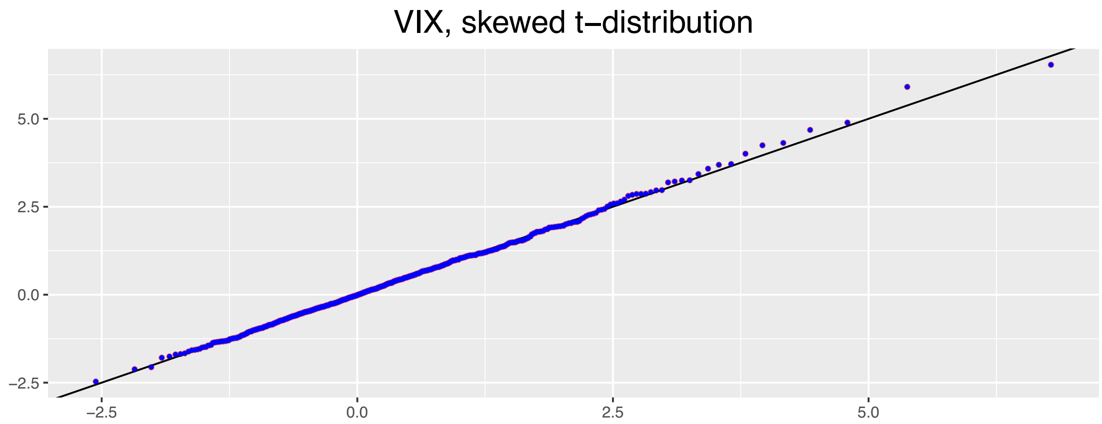


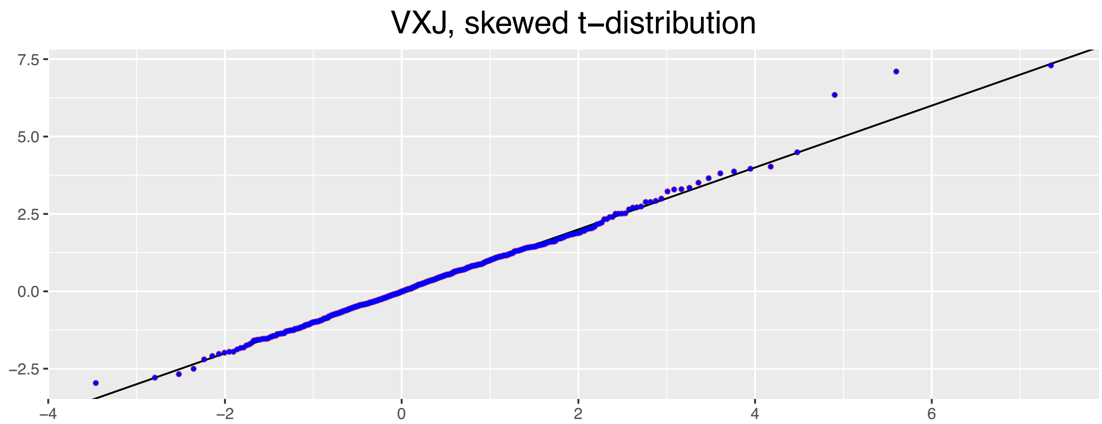


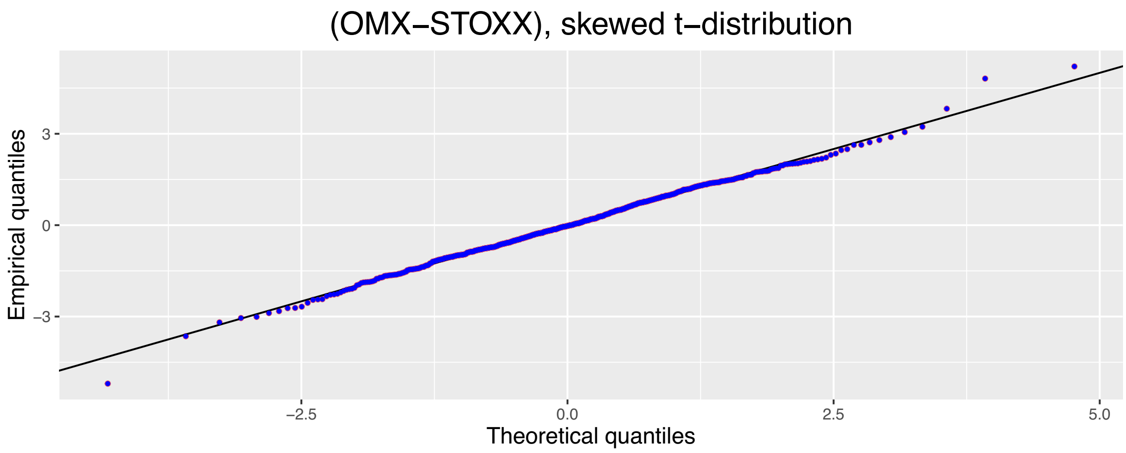

USD, skewed t−distribution
4
2
0
−2
−4
−2
0
2
4

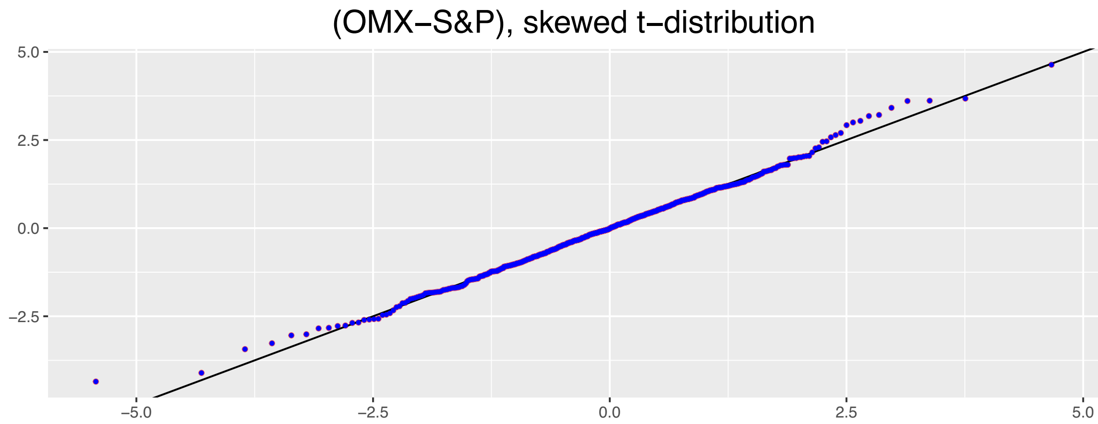

JPY, skewed t−distribution
4
2
0
−2
−4
−2
0
2
4

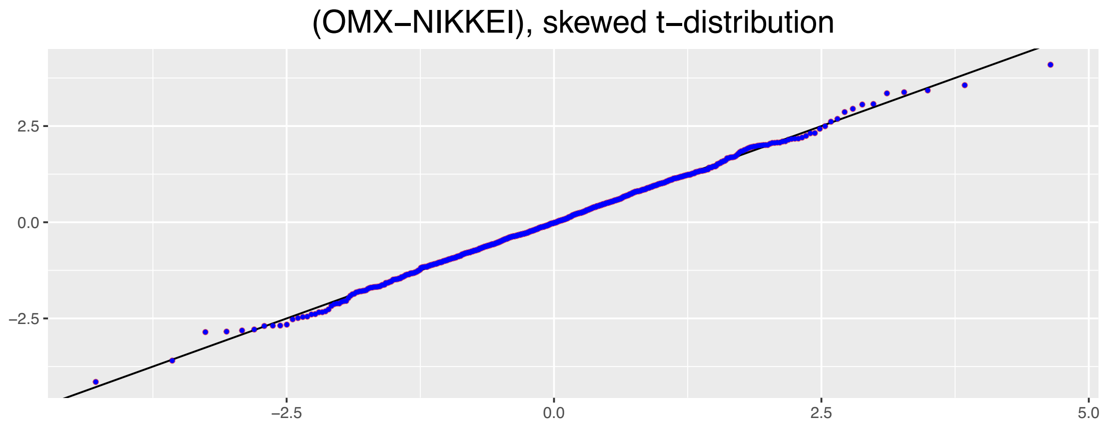

VSTOXX, skewed t−distribution
Empirical quantiles
5.0
2.5
0.0
−2.5
−2
0
2
4
6
Theoretical quantiles


VIX, skewed t−distribution
5.0
2.5
0.0
−2.5
−2.5
0.0
2.5
5.0


VXJ, skewed t−distribution
7.5
5.0
2.5
0.0
−2.5
−4
−2
0
2
4
6


(OMX−STOXX), skewed t−distribution
Empirical quantiles
3
0
−3
−2.5
0.0
2.5
5.0
Theoretical quantiles


(OMX−S&P), skewed t−distribution
5.0
2.5
0.0
−2.5
−5.0
−2.5
0.0
2.5
5.0


(OMX−NIKKEI), skewed t−distribution
2.5
0.0
−2.5
−2.5
0.0
2.5
5.0

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