LUND UNIVERSITY
School of Economics and Management

Macro-based Adjustment Factors for Valuations in Venture Capital

Tomás Pinto
to1363pi-s@student.lu.se

Abstract

This paper focuses on creating macro-based adjustment factors which can be applied to the valuation of start-up companies. Since start-up companies are private, market valuations are limited to events such as funding rounds or M&A transactions, which happen only at discrete points in time. This is a drawback when compared to the continuous valuations produced in public markets, with current literature focusing on accounting for the timing differences in the valuation of private firms through the application of market indexes. Alternatively, research into the interactions between performance in the venture capital sector and economic conditions suggests that macroeconomic indicators might also be useful in estimating timing adjustments for start-up valuations. The two proposed approaches are compared through a pooled ordinary least squares (OLS) estimation of a panel data sample of relative change in valuations in the United States' venture capital sector between 2007 and 2021 using both models based on market indexes and macroeconomic indicators. The adequacy of the adjustment factors is evaluated based on the comparison of prediction errors in an out of sample forecast between the market-based and macro-based models. Overall, a parsimonious macro-based model using per capita GDP as a sole explanatory variable is found to outperform both the complete macro-based model and the remaining market-based models. However, improvement relative to the market approach defined in Damodaran (2009) is slight. Between the market-based approaches studied, a small cap market index is found to be, in most scenarios, a more accurate predictor of timing adjustments than a synthetic index based on sectorial market performance. Furthermore, the introduction of time fixed effects in the regression generally worsens results, only presenting general improvement relative to the naive prediction when using either the complete macro-based model and a small cap market index as explanatory variables.

Keywords: Valuation, Start-ups, VCM, Multiples Approach

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To my family.

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1 Introduction

The venture capital sector is an important part of the financial sector, as it finances new companies with both great potential and high levels of uncertainty regarding future outcomes, that is, high levels of risk. Venture Capital or Private Equity firms finance these companies, typically called start-ups, through the issuance of equity while they are still private. (Gompers & Lerner 2001, p.145)

Since start-up companies are private, there are no continuous market valuations as in the case of public firms. Instead, market valuations are produced at specific points such as funding rounds, M&A transactions or even IPO events, constituting a limitation when compared to the valuations produced in public markets, which are by nature continuous.

The current methodology on the adjustment of private company valuations by timing differences can be found in Damodaran (2009, p.57), who suggests adjusting valuations by the performance of a small cap market index. This approach is still valid today, and can be seen in practice in the Refintiv VC Research Index, which focuses on the application of sectorial market indexes to adjust valuations. (Refinitiv 2021)

Past studies propose that macroeconomic and financial indicators can also be helpful in understanding the dynamics behind venture capital investments, since there are qualitative relations between the performance of the economy and investment decisions. (Gompers et al. 1998) To this end, factors such as risk-free interest rates, economic activity, exchange rates and public market quality are introduced as having influence over performance in the VC sector across time. (Füss & Schweizer 2012)

The purpose of this work is, therefore, to expand on the current methodology for the adjustment of valuations in start-up companies by creating adjustment factors reliant on macroeconomic indicators. Additionally, I will focus on assessing the performance of the macro-based adjustment factors relative to the current industry benchmarks.

The rest of this paper is structured as follows: Section 2 defines start-up companies, reviews common techniques regarding their valuation, establishes practical applications for the proposed adjustment factors and explores the literature on the relation between macroeconomic indicators and VC performance. Section 3 explains the model used in the creation of the adjustment factors, section 4 introduces the data utilized and section 5 explains the estimation techniques applied and presents results. Sections 6 and 7 discuss the results found and present the main conclusions. There is also an appendix with further details and results.

2 Literature Review

This section is divided into three parts. The first part introduces start-up companies and addresses the background on the application of public market indexes to estimate the timing adjustments of start-up valuations. The second part presents an overview of valuation techniques in start-up companies, explains the concept of market valuations in private firms and provides a practical application of the proposed adjustment factors: the multiples approach. Lastly, the third part explains the intuition behind the relation between macroeconomic and financial indicators and valuations in the VC sector.

2.1 Start-up Companies: Definition and Characterization

Start-up companies can be defined as independent private corporates, usually with low or negative profitability, high revenue growth and external investments from venture capital or private equity firms. They usually operate in highly technological sectors, such as information, communications and biotech. (Granlund & Taipaleenmäki 2005) Practical methods, such as the "rule of 40", could be used to identify these firms. Additionally, they allow for the comparison of the ideal profile of a start-up with the average results obtained by public companies. According to the rule of 40, the sum of a start-up's profitability and yearly revenue growth should total at least 40% (Depeyrot & Heap 2018), meaning there should be at least enough revenue growth (close to or above 40%) to compensate for the negative or low profitability. Looking at the S&P 500[1], average revenue growth rate in 2022 was 10,7% and the 5 year (2017-2022) average net profit was 11.4%. (Butters 2023) These values highlight that, on average, start-ups and public companies will differ in terms of revenue growth and focus on profitability, with start-ups focusing on maximizing revenue growth and public companies focusing mostly on maximizing profitability. The risk profile of start-ups and public companies also differs. For example, Korteweg & Sorensen (2010) introduce that start-up investments have an average Beta (β)[2] between 1.3 to 2.4 on the S&P 500. This means that, on-average, start-up investments are 30% to 140% more volatile than the S&P 500. The finance literature further expands on this by introducing size as a proxy for risk, with smaller firms being considered riskier than larger firms. (Fama & French 1992)

Gompers & Lerner (2001, pp.162-163) elaborate on the difference between the venture capital sector and public markets, pointing out that "many institutions, like public and private pension funds, have increased their allocation to venture capital and private equity in the belief that the returns of these funds are largely uncorrelated with the public markets." This, however, might be an incorrect assumption. According to these authors, since venture capital firms often "maintain the investments at book value until the company

[1]Stock market index of the 500 largest companies listed in stock exchanges in the United States.

[2]Beta is a measure of an asset's volatilty in relation to a market index.

goes public, reported returns during years with many initial public offerings are biased upwards: had the portfolio been marked to market, many of the gains would have been realized in the years before the initial public offering." For example, an assessment of data from a venture group where the portfolio was "marked-to-market" conducted by Gompers & Lerner (1997) shows singnificantly higher levels of correlation between venture capital and public market prices.

2.2 Valuation Methods in Start-up Companies

It has long been understood that valuing start-up companies requires adjustments to the traditional techniques. (Montani et al. 2020) First of all, it becomes important to consider the concept of valuing pre-revenue companies, where the focus is more on qualitative rather than on quantitative characteristics. These methods work by attributing a monetary value to key elements of a firm, such as product idea, the stage of completion of a prototype, the quality of the management team, among others.[3] At the same time, strategies to approach the lack of historical data and high degree of risk in more established start-ups are required. For example, option pricing models, such as those created by Black & Scholes (1973), have been used to value private companies. (Trigeorgis 1993) Alternatively, Scherlis & Sahlman (1987) introduce the venture capital method, which modifies the DCF valuation by using peer firms to estimate future cash-flows. Lastly, the multiples approach can also be applied to the valuation of start-up firms, where market valuations of similar firms are used to value a target firm. (Damodaran 2009)

These final strategies, the venture capital method and the multiples approach, will commonly focus on external elements to help craft valuations, instead of relying solely on internal elements, such as a company's financial statements. The external elements, usually valuations produced in the market for companies similar to the firm being valued, are constrained by the discreteness in the publication of these valuations for private companies, which justifies the need to apply timing adjustments. These valuation techniques which rely on external elements can therefore be seen as the key application of the proposed adjustment factors.

Since these valuations are built using the prices determined in deals (funding rounds, M&A transactions, IPOs) of firms which are similar to the target firm being valued, in line with the concept of price in Knight (1921), they can be understood as market valuations. They therefore represent the external perception of value of a firm as determined by the market, and can be used as elements in the valuation of other firms. It is also important to distinguish between pre-money and post-money valuations: pre-money valuations correspond to the value of a company before the inclusion of external funding provided in the corresponding event.

[3]See Payne (2011), Berkus (2016).

Based on the work by Damodaran (2009), where the suggested best practice is to adjust the valuations by the performance of a market index for similarly sized companies[4], current valuation-based indexes of VC performance, such as the Refintiv VC Research Index, adjust valuations of start-ups by a sectorial market index[5]. These methods therefore follow the conclusions in Gompers & Lerner (1997), highlighting the correlation between "market-to-market" venture capital portfolios and public market prices.

2.2.1 The Multiples Approach

The multiples approach is a valuation technique in which the value of a company can be estimated as a multiple of a certain indicator, usually revenue or earnings. This multiple is chosen by looking at peer firms (comparables) in the market and assessing similarities with the target company, usually related to the companies' growth trajectory, risk profile and the industry in which it operates. The multiples approach to valuation is ubiquitous in finance, with over 90% of equity analysts' reports including these indicators. (Asquith et al. 2005) This is further corroborated by Pinto et al. (2019), where 92.8% of respondents include multiples in equity reports. It is also commonplace when looking at private companies, appearing as the most common method of valuation. (Schmidt 2022)

A summary on the application of the multiples approach can be found in Plenborg & Pimentel (2016), who focus on aspects such as the choice of peer firms, the selection of adequate value drivers, methods of averaging multiples, differences in use of reported or forecasted earnings, concerns with accounting differences, the possibility of normalization of earnings, the impact of size in the accuracy of multiples and the existence of differences in the average multiples between public and private companies, caused by the control premium and the illiquidity discount. In addition to this, Barg et al. (2021) introduce that there are differences in the multiples of individual companies when comparing valuations at financing rounds and exit events, such as M&A transactions or IPO events. Regarding the application of the multiples approach, I find important to highlight two aspects: (i) the criteria for the selection of peer firms and (ii) the applicability of the multiples approach relative to firm size.

Historically, two different methodologies have been explored when choosing comparable firms for relative valuation: industry and fundamental properties. One of the first studies on this matter, by Alford (1992), analyses the selection of comparable firms based on both methods. His results find that "the widespread procedure of selecting comparable firms by industry is relatively effective, where industry is defined by the first three SIC digits".[6] (Alford 1992, p.106) The alternative approach for choice of comparables focuses

[4]The example used for small cap market index is the Russell 2000.

[5]See appendix A.10.

[6]Standard Industrial Classification (SIC) is a system for classifying industries by a four-digit code (although previous versions used three digits).

on firms with similar valuation fundamentals, such as profitability, growth and risk. According to Alford (1992, p.106), "similar accuracy occurs when risk and earnings growth are used together to construct portfolios of comparables firms". Further work, however, seems to focus on this second approach as preferred, with Herrmann & Richter (2003) reporting significantly lower predictions errors when using forecasts for earnings growth and rate of return versus SIC classifications. The current industry standard, therefore, follows from a combination of both approaches, with innovations in industry selection and fundamental characteristics analysis allowing for improvements in choice of comparables. This is, for example, the approach in Liu et al. (2002, p.163) who, when using firm fundamentals to choose comparable firms, mention that "selecting firms from the same industry improves performance for all value drivers". Lastly, when considering start-up companies, Bancel et al. (2021) have observed that choosing peers solely based on industry might lead to erroneous results, due to the high degree of variability which can occur within each industry. Because of this, focusing on firms' fundamental properties is considered adequate.

The importance of the size of firms in the accuracy of multiples application is also introduced in Alford (1992), who reports that absolute prediction errors can be cut by half when comparing large to small firms selected on the basis of industry.

2.3 Interactions between Venture Capital and the Economy

The last subsection of this literature review relates to the interactions between venture capital and the economy. Initial work by Poterba (1989) studies the idea that changes in the capital gains tax rate negatively affect the amount of venture capital investment, explained by the decrease in appeal of these projects for both investors and entrepreneurs. Gompers et al. (1998) expand this analysis, by considering an array of macroeconomic, regulatory and performance factors. Their results support the notion that higher GDP growth and increases in R&D spending lead to increased VC activity, with lower capital gains tax rates leading to increases in equilibrium investment amounts.

Füss & Schweizer (2012) introduce the quality of a country's financial markets as a determinant of venture capital performance, based on work by Black & Gilson (1998). They suggest that the existence of a public market to which start-up companies can converge is a determinant of VC performance, since it functions as an exit channel which allows investors to obtain liquidity. They also explain that public market performance (exit channel) is an adequate proxy for the quality of public markets, using this as an explanatory variable in their analysis. Füss & Schweizer (2012) additionally find that, in short-term dynamics, both industrial production, the proxy used for economic activity, and the exit channel Granger-cause venture capital performance.

Furthermore, short and long term interest rates are introduced as independent variables in their regression. An analysis over a period of 15 years (1991-2006) reveals that the value of VC investment is positively related to the long term interest rate and negatively related to the short term interest rate. In the case of long term interest rates, the result is introduced as a consequence of a demand side effect, where higher interest rates mean an increased cost of capital from the traditional lending sector, leading entrepreneurs to demand higher levels of venture capital and therefore increasing VC performance. In the case of short term interest rates, the effect is introduced through the impact on economic activity: higher interest rates lead to a decrease in aggregate demand in the economy, decreasing equilibrium output in the economy[7] and reducing both the amount of demanded and supplied capital, therefore depressing VC performance.

An additional factor that could be considered as a possible determinant of venture capital performance is the exchange rate between the currency in the country of analysis and key international currencies, such as the U.S. Dollar or the Euro. In general, exchange rates have been shown to affect total investment amounts, through their effect on the ability to capture external investment. Looking at the U.S. as a target country, Alba et al. (2010) have shown a significant positive relation between exchange rate (that is, a more valuable USD) and higher levels of foreign direct investment (FDI), possibly leading to higher valuations. This is justified by the improvement of "home-currency revenues and thus profitability of foreign firms entering the US market". (Alba et al. 2010, p.1)

In general, these results point to a strong relation between macroeconomic and financial indicators and venture capital performance, with Füss & Schweizer (2012) introducing the quality of public markets, interest rates, inflation and economic activity as explanatory variables. Additional works, such as Alba et al. (2010), lead to the idea that exchange rates could be meaningful additions to the analysis. Based on this, this paper intends to extend the research by theorizing that these variables could also be useful in explaining relative changes in the valuation of start-up firms.

[7]A description of this reasoning can be found, for example, in chapter 7 of Romer (2012).

3 Model

The objective in this paper is to estimate an adjustment factor for the valuations in start-up companies so that the estimated present value of a past valuation event can be obtained. This adjustment factor corresponds to the relative change in valuations in a given year.

$$Adj_t = Val_t/Val_{t-1} \tag{1}$$

Since Val_t is not known, to estimate the adjustment factor I apply a linear regression in which the dependent variable corresponds to the natural log of the returns on valuations for the years in the dataset, in line with the methodology followed in Refinitiv (2021, pp.9-10). As seen in equation (2), this can generally be described as:

$$\ln(Adj)_t = \beta_1 + \beta_2 x_{t2} + ... + \beta_K x_{tK} + u_t \tag{2}$$

This value can then be applied in order to compute the adjusted valuation of a firm, such that:

$$\begin{aligned} \widehat{Val_t} = & \quad Val_{t-1} * e^{\ln(\widehat{Adj})_t} \\ \Leftrightarrow \quad \widehat{Val_t} = & \quad Val_{t-1} * \widehat{Adj_t} \end{aligned} \tag{3}$$

The expression above represents the relative change in valuations which happens in any given year t. However, this result can be expanded to cover a period of several (n) years, such that:

$$\widehat{Val_t} = Val_1 * \prod_{n=1}^{t} \widehat{Adj_n} \tag{4}$$

Considering the approaches which have already been described in previous literature, as well as the qualitative relation between macroeconomic and financial variables and performance in the VC sector, three different models will be estimated:

1. The first model utilizes a small cap market index as an explanatory variable, in line with the proposal in Damodaran (2009). In this paper, the Russell 2000 index is chosen. The explanatory variable will be defined as the natural log of returns on the index, as observed in Refinitiv (2021, pp.9-10).

$$x_t = \ln(RUSS_t/RUSS_{t-1}) = \ln(russ)_t \tag{5}$$

2. The second model utilizes the values from the Refinitiv VC index as an explanatory variable. Here too, the explanatory variable will be defined as the natural log of returns on the index.

$$x_t = \ln(REFVC_t/REFVC_{t-1}) = \ln(refvc)_t \tag{6}$$

3. The third model utilizes macroeconomic and financial variables, in line with those proposed in section 2.3. In this work, the macroeconomic and financial variables used for model 3 are: long term interest rates, short term interest rates, a measure of economic activity (per capita GDP), exchange rates between the USD and EUR and a measure of the quality of public markets (exit channel). The interest rates are transformed using first differences, in order to be interpretable as the absolute change in interest rates in any given year:

$$x_t = LTIR_t - LTIR_{t-1} = ltir_t \tag{7}$$

$$x_t = STIR_t - STIR_{t-1} = stir_t \tag{8}$$

The remaining variables are defined as the natural log of the returns on the underlying variable in any given year:

$$x_t = \ln(GDPpc_t/GDPpc_{t-1}) = \ln(gdppc)_t \tag{9}$$

$$x_t = \ln(EXCH_t/EXCH_{t-1}) = \ln(exch)_t \tag{10}$$

$$x_t = \ln(EXIT_t/EXIT_{t-1}) = \ln(exit)_t \tag{11}$$

To measure the performance of the models used to estimate the adjustment factors, they are re-estimated for a subsample of the available dataset and the predicted values for the remaining years available are compared to the actual values obtained during that period. From here, a comparison is made between the root mean square errors (RMSE) obtained from the predictions of the estimated models and those of a naive prediction, allowing to understand whether the adjustment models improve on the naive prediction. Furthermore, this allows for a comparison between the different adjustment models. At the same time, Theil's U is calculated for the adjustment models and is also used to assess comparative model performance.

4 Data

4.1 U.S. Venture Capital Valuations (2006-2021)

4.1.1 Context and Descriptive Statistics

I start by choosing a series of valuations in start-up firms across time. It corresponds to the pre-money valuations produced in the VC sector between 2006 and 2021 in the U.S., as captured in the "Pitchbook Q3 2022 US VC Valuations Report". The U.S. was chosen as the country of study due to its highly representative nature, having captured around 60% of global VC investment in 2021. (KPMG 2022) The use of pre-money valuations, as described in section 2.2, avoids the effect of the amount of external funding on the valuation. For example, two similar companies with the same pre-money valuation could have different post-money valuations if the external amounts invested during a funding round are different, despite there being no difference in their fundamental properties.

Additionally, the valuations are categorized by industry and stage of financing to which companies correspond. The four industries considered are: "Biotech and Pharma", "Consumer Tech", "Enterprise Tech" and "Fintech". The four stages of financing considered are: "1 - Angel", "2 - Seed", "3 - Early-Stage" and "4 - Late-Stage". This means that this dataset can be considered as a panel dataset, since there are 16 individual panels (which combine industry and stage of financing) with data between 2006 and 2021. In total, 3 out of 256 (1.17%) observations for the series of valuations are missing. Although the dataset could be considered unbalanced, since this value is low, Stata still reports it as strongly balanced. Usual thresholds for considering data as unbalanced are set at 5% or even 10% of missing observations.

The values chosen correspond to the median of the valuations for each subcategory in each year. The choice of the median from the sample of valuations captured in the Pitchbook database is based on the work of Herrmann & Richter (2003), who suggest that averaging a sample of valuations using a simple mean will return values affected by the impact of outliers, therefore preferring the median or the harmonic mean. It is also important to note that only having summary statistics of the Pitchbook sample available results in a loss of information and a reduction of within-category variance, therefore resulting in lower predictive power in the models. (Midway 2022) Summary statistics for the available dataset are represented in Table 1. (Pitchbook 2022) Appendix A.1 introduces yearly values averaged by stage of financing, allowing for a practical illustration of the fact that valuations oscilate significantly over time.

As seen in section 3, the transformation required to measure the adjustment factor, which corresponds to the YoY relative change in valuations, is to take the natural logarithm of the quotient between the present and the past year's valuation. In addition to being the approach used in Refinitiv (2021), it can also be seen, for example, in Psaradakis et al. (2005), when considering industrial production as a proxy for economic activity.

Table 1: Summary Statistics - Valuation

	Mean	Max	Min	Std Dev	Count
Biotech and Pharma	20,00	85,00	1,48	19,45	63
1 - Angel	7,94	20,00	1,48	5,73	16
2 - Seed	5,66	11,77	2,36	2,73	15
3 - Early-Stage	17,34	45,00	8,73	9,73	16
4 - Late-Stage	48,17	85,00	30,00	14,66	16
Consumer Tech	18,87	112,00	1,29	22,74	64
1 - Angel	2,92	5,06	1,29	1,06	16
2 - Seed	4,69	8,50	2,40	1,66	16
3 - Early-Stage	16,01	40,00	6,55	9,70	16
4 - Late-Stage	51,84	112,00	30,84	20,49	16
Enterprise Tech	18,51	120,00	2,43	21,78	64
1 - Angel	3,73	6,00	2,43	0,90	16
2 - Seed	4,91	9,50	2,53	1,91	16
3 - Early-Stage	17,08	47,25	7,57	10,42	16
4 - Late-Stage	48,31	120,00	26,16	22,34	16
Fintech	28,42	245,00	1,24	41,74	62
1 - Angel	9,26	74,85	1,24	17,71	16
2 - Seed	6,07	10,00	3,53	1,93	14
3 - Early-Stage	19,25	60,00	5,66	13,38	16
4 - Late-Stage	76,30	245,00	19,47	56,53	16
Grand Total	21,40	245,00	1,24	27,91	253

4.1.2 Diagnostic Tests

Since in time series data non-stationarity can lead to spurious results, an analysis on stationarity and normality of the series is conducted. To test stationarity, a Fisher-type Dickey-Fuller unit root test is applied, where the null hypothesis is that all panels contain a unit root. (Choi 2001) The original data is considered to be non-stationary (p-value = 0.999) and the transformed data to be stationary (p-value = 0.000).[8] This can also be

[8]The corresponding Stata command is: xtunitroot fisher ln_adj, dfuller lag(1).

seen in figure 1.[9] The normality test found in D'Agostino & Belanger (1990) is applied for both the original and transformed data. It indicates the original data is skewed (p-value = 0.000), something which is singnificantly reduced after the transformation (p-value = 0.307).[10] A bar graph of the data in $ln(Adj)$ can be found in figure 2.

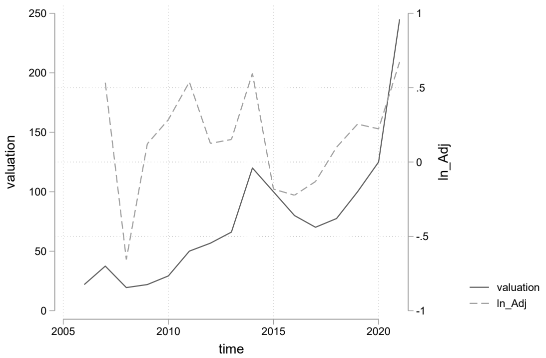

Figure 1: U.S. VC Valuations in Late-Stage Fintech Firms (2006-2021)

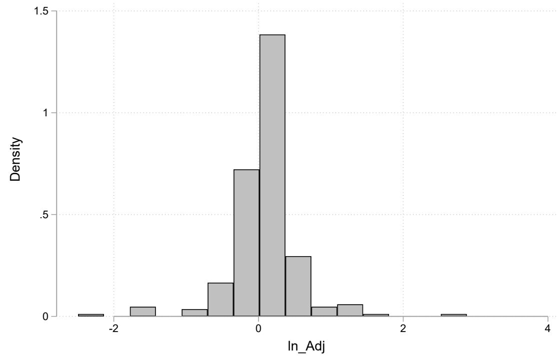

Figure 2: Distribution of ln(Adj)

As it can be seen, the transformation applied introduces desirable properties, since the original series is non-stationary and skewed, properties which can be verified after the transformation.

4.2 Macroeconomic and Financial Variables

4.2.1 Context and Descriptive Statistics

Regarding model 1, the Russell 2000 index is utilized (Capital IQ 2023*a*). In model 2, the Refinitiv Venture Capital index is applied. (Refinitiv Eikon 2023) Both in the case of the Russell 2000 Index, which has daily values, and in the case of the Refinitiv Venture Capital Index, which has monthly values, the yearly average is considered in the dataset. In the case of model 3, the choice of macroeconomic and financial variables was informed by previous research on the determinants of performance in the VC sector. In this work, long and short term interest rates, a measure of economic growth (per capita GDP), the EUR-USD exchange rate and a measure of quality of public markets are chosen. In line with Füss & Schweizer (2012), the S&P 500 index is used as a proxy for quality of public markets.[11] Summary statistics for the macroeconomic and financial variables can

[9] The figure corresponds to the data found in the panel corresponding to the "Fintech" industry and "Late-Stage" funding.

[10] The corresponding Stata command is: sktest ln_adj.

[11] In Füss & Schweizer (2012), the Nasdaq Composite index is used instead of the S&P 500. The Nasdaq Composite is seen as a representative measure of performance in the tech space, as many of the leading tech companies are listed here. The S&P 500, on the contrary, is defined based on market capitalization, and does not have a particular industry bias.

be found in Table 2.

The long term interest rate is represented by the Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity (FRED 2023*a*), the short term interest rate is represented by the 3-Month Treasury Bill Secondary Market Rate (FRED 2023*b*) and the economic activity variable is represented by U.S. per capita GDP (FRED 2023*c*). The EUR-USD Spot Exchange Rate (FRED 2023*d*) and the S&P 500 index (Capital IQ 2023*b*) are also included. The macroeconomic variables correspond to the average of the values for each year. Since both the market indexes and some of the macroeconomic indicators are expected to be trending (which is later verified in table 3), using the yearly average of daily or monthly values may also introduce bias since some of the within-year trending effects may be lost. A complete version of the data on macroeconomic and financial variables can be found in appendix A.2.

Table 2: Summary Statistics - Macroeconomic and Financial Variables

			Mean	Max	Min	Std Dev	Count
Model 1	RUSSELL_2000	*USD*	1114,45	2242,91	521,68	460,74	16
Model 2	REF_VC_INDEX	*USD*	5015,61	21304,28	934,08	5720,96	16
	LTIR	*(%)*	2,67%	4,79%	0,89%	1,06%	16
	STIR	*(%)*	1,04%	4,73%	0,03%	1,53%	16
Model 3	GDPpc	*USD*	55155,31	70150,25	46233,00	7269,16	16
	EXCH	*EUR*	0,802	0,903	0,679	0,076	16
	EXIT	*USD*	2004,59	4273,41	948,05	913,35	16

4.2.2 Diagnostic Tests

The considered macroeconomic and financial variables are also transformed as described in section 3. The Russell 2000 index, the Refintiv VC Index, the per capita GDP, the EUR-USD exchange rate and the S&P 500 index follow the same transformation applied to the series of valuations ($ln(Var_t/Var_{t-1})$), allowing for the interpretation of relative change. In the case of long and short term interest rates, the absolute change between the valuations in a given year is calculated through first differencing. The reason for the difference in approach is that long and short term rates suffer large changes in magnitude relative to the overall value, meaning symmetry may not apply when measuring a relative increase or decrease. An analysis of stationarity and normality can be found in table 3.

Table 3: Stationarity and Normality Tests - Macroeconomic and Financial Variables

Original Variable	Stationarity *(p-value)*	Normality *(p-value)*	Transformed Variable	Stationarity *(p-value)*	Normality *(p-value)*
RUSSELL_2000	0,995	0,150	ln(russ)	0,053	0,859
REF_VC_INDEX	1,000	0,004	ln(refvc)	0,182	0,492
LTIR	0,325	0,473	ltir	0,009	0,424
STIR	0,043	0,013	stir	0,136	0,010
GDPpc	0,997	0,435	ln(gdppc)	0,002	0,959
EXCH	0,632	0,085	ln(exch)	0,001	0,061
EXIT	1,000	0,087	ln(exit)	0,130	0,030

The log transformation introduces stationarity in the variables, although sometimes at significance levels higher than 5%. The long term interest rate variable is found to be highly stationary (p-value = 0.009), with the short term interest rate variable less so (p-value = 0.136). The transformed variables are also normally distributed, with some evidence of skewness appearing in $stir$, $ln(exch)$ and $ln(exit)$. Here, stationarity is calculated with recourse to a Dickey-Fuller unit root test and normality based on the test found in D'Agostino & Belanger (1990).

Table 4: Correlation Matrix - Macroeconomic and Financial Variables

	ln(russ)	ln(refvc)	ltir	stir	ln(gdppc)	ln(exch)	ln(exit)
ln(russ)	1.0000						
ln(refvc)	0.7677	1.0000					
ltir	0.6916	0.4416	1.0000				
stir	0.5777	0.4902	0.6020	1.0000			
ln(gdppc)	0.7723	0.5227	0.5831	0.4889	1.0000		
ln(exch)	-0.1516	-0.1123	-0.2564	0.2111	-0.1537	1.0000	
ln(exit)	0.9407	0.8669	0.5301	0.6048	0.7177	-0.1200	1.0000

An analysis of correlation between the independent variables referring to model 3 reveals no evidence of excess multicollinearity, when considering the 0.8 limit set by Kennedy (1998, p.187). It should also be mentioned that, considering the effect of multicollinearity in forecasting, Neter et al. (2005, p.283) note that "the fact that some or all of predictor variables are correlated among themselves does not, in general, inhibit our ability to obtain a good fit nor does it tend to affect inferences about mean responses or predictions of new observations". High levels of correlation can be observed between the transformed exit channel, Russell 2000 index and Refinitiv VC index.

5 Methodology and Results

5.1 Methodology

As introduced before, a panel data approach is used in this work. Panel data can be described as data in which there is a "pooling of observations" for different categories of a same entity across time. (Baltagi 2008, p.1) The description consequently fits the type of data present in the return series, where there are 16 different panels of valuations between 2007 and 2021, each corresponding to a combination of firm industry and stage of financing.

When considering panel data, the error term in the model ($u_{i,t}$) is usually split into two parts: an unobservable individual specific effect (μ_i) relating to each of the panels and a remainder disturbance ($v_{i,t}$). When disturbances are modelled this way, the models are said to have one-way error components and can be represented as:

$$\ln(Adj)_{i,t} = \beta_1 + \beta_2 x_{i,t2} + ... + \beta_K x_{i,tK} + \mu_i + v_{i,t} \tag{12}$$

Depending on the characteristics of the individual specific effect, panel data models can have three different categorizations: fixed effects, random effects or no panel effects. In fixed effects models, μ_i are assumed as fixed parameters to be estimated and the remainder disturbances stochastic with $v_{i,t} \sim IID(0, \sigma_v^2)$ for all i and t. The independent variables are assumed independent of $v_{i,t}$ for all i and t. It is appropriate if the focus is on a specific set of N categories, which are separate and independent, and the inference is restricted to the behavior of these sets of categories. Inference in this case is conditional on the particular N categories that are observed. (Baltagi 2008, pp. 16-17)

In random effects models, μ_i are assumed random, with $\mu_{i,t} \sim IID(0, \sigma_\mu^2)$ and $v_{i,t} \sim IID(0, \sigma_v^2)$, for all i and t. It is therefore an appropriate specification when drawing N individuals randomly from a large population. The individual effect is characterized as random, and inference pertains to the population from which this sample was randomly drawn. (Baltagi 2008, p. 24) It can also be found that $\mu_i = 0$, meaning no panel effects are detected. In this case, the error component is composed solely of $v_{i,t} \sim IID(0, \sigma_v^2)$.

Selection between random and fixed effects models can be made with recourse to the Durbin-Hu-Hausman specification test, which tests the null hypothesis that a random effects model is efficient when compared to a fixed effects model. (Hausman 1978) In practice, it is testing whether there is correlation between the independent variables and the unobservable individual specific effect (μ_i). If these are correlated, then μ_i is named a fixed effect. If not, μ_i is referred to as a random effect. (Roberts & Whited 2013) If no correlation is found between the specific effect (μ_i) and the independent variables, it might also be that no panel effects are detected. This can be assessed through the Breusch-Pagan Lagrange Multiplier test, which tests whether random effects are significant in panel data models. (Breusch & Pagan 1980)

Other error term components, such as time specific effects, can be included in models. In some cases, this means the error term in the model can also be split into three parts, where in addition to an unobservable individual specific effect (μ_i) and a remainder disturbance ($v_{i,t}$), there is a time specific effect (λ_t). When disturbances are modelled this way, the models are said to have two-way error components. (Baltagi 2008, p. 47) Time specific effects capture factors that affect the dependent variable and are common to all entities within a given time period. Considering the data available has time series, it could therefore be useful to control for time-specific effects through the introduction of time fixed effects.

In the models presented below, a one-way error component configuration is tested to compare between the applicability of fixed or random effects to model the unobservable individual specific effect and the possibility of no observable panel effects. In section 5.4, the possibility of time fixed effects is introduced as a robustness check to compare with the results from the initial configuration and understand the impact of time-specific effects in estimation results.

5.1.1 Model 1

In line with section 3, model 1 can be defined as:

$$\ln(Adj)_{i,t} = \beta_1 + \beta_2 \ln(russ)_{i,t} + u_{i,t} \tag{13}$$

1. $\ln(Adj)_{i,t}$ corresponds to the natural logarithm of the quotient between valuations in two consecutive years;

2. $\ln(russ)_{i,t}$ corresponds to the natural logarithm of the quotient between the Russell 2000 index in two consecutive years;

3. $u_{i,t} = \mu_i + v_{i,t}$ where μ_i corresponds to the unobservable individual specific effect and $v_{i,t}$ denotes the remainder disturbance;

4. $i = 1, ..., 16$ is an ID factor which corresponds to each industry and stage of financing considered.

The results from the Durbin-Hu-Hausman specification test comparing a one-way fixed effects model with a one-way random effects model indicate that using a random effects model is more appropriate, since it is efficient under the null hypothesis (p-value = 0.9436).[12] This also means no significant correlation is found between the independent

[12] Details on the Hausman specification test are reported in Appendix A.3.

variable and the individual specific effect. Running the Breusch-Pagan Lagrange Multiplier test, the null hypothesis ($\mu_i = 0$) cannot be rejected (p-value = 1.000), meaning that the reason no correlation is found in the Hausman test is that no signficant panel effects are detected. Consequently, the model is estimated using pooled ordinary least squares (OLS), meaning a simple OLS regression is applied to each of the panels.

Looking at some of the common problems in the data, the Breusch-Pagan test[13] gives evidence of heteroskedasticity, meaning clustered robust standard errors are applied throughout the estimation. Additionally, no correlation is found between the explanatory variable and the predicted remainder error terms ($v_{i,t}$), suggesting no endogeneity problems should arise.

5.1.2 Model 2

In line with section 3, model 2 can be defined as:

$$\ln(Adj)_{i,t} = \beta_1 + \beta_2 \ln(refvc)_{i,t} + u_{i,t} \tag{14}$$

1. $\ln(Adj)_{i,t}$ corresponds to the natural logarithm of the quotient between valuations in two consecutive years;
2. $\ln(refvc)_{i,t}$ corresponds to the natural logarithm of the quotient between the Refinitiv VC index in two consecutive years;
3. $u_{i,t} = \mu_i + v_{i,t}$ where μ_i corresponds to the unobservable individual specific effect and $v_{i,t}$ denotes the remainder disturbance;
4. $i = 1, ..., 16$ is an id factor which corresponds to each industry and stage of financing considered.

Here too, the results from the Durbin-Hu-Hausman specification test indicate that the one-way random effects model is efficient under the null hypothesis (p-value = 0.9303)[14], indicating no significant correlation between the independent variable and the individual specific effects. The Breusch-Pagan Lagrange Multiplier test also indicates that no significant panel effects are found (p-value = 1.000), leading the model to be estimated using pooled OLS.

The Breusch-Pagan test also gives evidence of heteroskedasticity, meaning clustered robust standard errors are applied. Additionally, no correlation is found between the independent variable and the predicted remainder error terms, also indicating no endogeneity issues.

[13]This corresponds to the Breusch-Pagan test for heteroskedasticity (Breusch & Pagan 1979) adapted to panel data specifications, different from the Breusch-Pagan Lagrange Multiplier test mentioned previously.

[14]Details on the Hausman Specification Test are reported in Appendix A.3.

5.1.3 Model 3

In line with section 3, model 3 can be defined as:

$$\begin{aligned} \ln(Adj)_{i,t} = \beta_1 + \beta_2 ltir_{i,t} + \beta_3 stir_{i,t} + \beta_4 \ln(gdppc)_{i,t} \\ +\beta_5 \ln(exch)_{i,t} + \beta_6 \ln(exit)_{i,t} + u_{i,t} \end{aligned} \tag{15}$$

1. $\ln(Adj)_{i,t}$ corresponds to the natural logarithm of the quotient between valuations in two consecutive years;
2. $ltir_{i,t}$ corresponds to the first differences of the long term interest rates;
3. $stir_{i,t}$ corresponds to the first differences of the short term interest rates;
4. $\ln(gdppc)_{i,t}$ corresponds to the natural logarithm of the quotient between per capita gross domestic product in two consecutive years;
5. $\ln(exch)_{i,t}$ corresponds to the natural logarithm of the quotient between the EUR-USD exchange rate in two consecutive years;
6. $\ln(exit)_{i,t}$ corresponds to the natural logarithm of the quotient between the S&P 500 Index in two consecutive years;
7. $u_{i,t} = \mu_i + v_{i,t}$ where μ_i corresponds to the unobservable individual specific effect and $v_{i,t}$ denotes the remainder disturbance;
8. $i = 1, ..., 16$ is an id factor which corresponds to each industry and stage of financing considered.

In model 3, the results from the Durbin-Hu-Hausman specification test also indicate that the one-way random effects model is efficient under the null hypothesis (p-value = 1.000)[15], indicating a lack of correlation between covariates and the individual specific effects. The Breusch-Pagan Lagrange Multiplier test does not allow for the rejection of the null hypothesis (p-value = 1.000), meaning no significant panel effects are found. The model is also estimated using pooled OLS.

There is evidence of heteroskedasticity, leading to the application of clustered robust standard errors. Here too, no correlation is found between the covariates and the predicted remainder error terms, leading to no presumption of endogeneity issues.

[15]Details on the Hausman Specification Test are reported in Appendix A.3.

5.2 Results

At this point, I will estimate the three models defined in 5.1 through pooled OLS regressions.[16] These are calculated for both the complete sample period (2007-2021) and subsample periods of five years. Even though only the complete sample period is used in the creation of the adjustment factors (see equations (16) - (19)), conclusions from the subsample periods in the estimation will be used in the robustness check section below. An additional model is created from the analysis of significance and robustness for model 3, with a reduced number of covariates, whose results are also estimated. Afterwards, I analyse the results from the estimation of each model, which can be found in tables 5, 6, 7 and 8 respectively.

5.2.1 Model 1

Table 5: Estimation - Model 1 - Pooled OLS

ln_adj	(1) Total	(2) 2007-2011	(3) 2012-2016	(4) 2017-2021
ln_russ	0.588**	0.200	0.296	1.120***
	(0.234)	(0.426)	(0.378)	(0.375)
Constant	0.0420**	0.0387	0.00697	0.0405
	(0.0154)	(0.0274)	(0.0285)	(0.0271)
Observations	236	76	80	80

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 5 presents the results from the estimation of model 1. When considering the entire period, it can be seen that both the constant term and the independent variable are significant at a 5% level. The independent variable is also found to be significant in the 2017-2021 subsample period at a 1% level.

The natural log of the returns on the Russell 2000 index, which corresponds to the relative change in the Russell 2000 index, is found to positively affect the natural log of the adjustment factor, meaning that, in any given year, an increase in the index of 1% leads to an expected additional increase in valuations of approximately 0,588%.

The 2007-2011 and 2012-2016 subsample periods report non-significant coefficients. In the case of the 2007-2011 subsample period, this could potentially be related to bias introduced from the unbalanced panel, since there missing observations corresponding to

[16] The corresponding Stata command is: reg (...), vce(cluster id).

5% of the total number of observations in the subsample. This observation can also be generalized to the remaining models studied.

The model is found to be statistically significant, with the p-value based on the obtained F-statistic (6.33) indicating model significance at a 5% level (0.0238). The remainder disturbance verifies the residuals are not skewed, with a p-value of 0.1073.[17] The distribution of the error terms for model 1 can be found in figure 11 (see appendix A.11.)

5.2.2 Model 2

Table 6: Estimation - Model 2 - Pooled OLS

	(1)	(2)	(3)	(4)
ln_adj	Total	2007-2011	2012-2016	2017-2021
ln_refvc	0.554**	0.322	1.343***	0.0132
	(0.247)	(0.607)	(0.302)	(0.495)
Constant	-0.0297	0.00783	-0.223**	0.182
	(0.0426)	(0.0479)	(0.0769)	(0.156)
Observations	236	76	80	80

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 6 presents the results from the estimation of model 2. When considering the entire sample, it can be seen that only the independent variable is found to be significant at a 5% level. Looking instead at the 2012-2016 subsample period, the constant term is significant at a 5% level and the independent variable is significant at a 1% level.

The natural log of the returns on the Refinitiv VC index, which corresponds to the relative change in the Refinitiv VC index, is also found to positively affect the natural log of the adjustment factor, meaning that, in any given year, an increase in the index of 1% leads to an expected additional increase in valuations of approximately 0,554%.

Model 2 is also found to be statistically significant, with the p-value obtained from the F-statistic (5.02) indicating significance at a 5% level (0.0406). The remainder disturbance also verifies that the residuals are not skewed, with a p-value of 0.278. The distribution of the error terms for model 2 can be found in figure 12 (see appendix A.11).

[17]This is calculated with the normality test from D'Agostino & Belanger (1990). The corresponding Stata command is: sktest residual.

5.2.3 Model 3

Table 7: Estimation - Model 3 - Pooled OLS

ln_adj	(1) Total	(2) 2007-2011	(3) 2012-2016	(4) 2017-2021
ltir	0.0314	-0.563	0.570***	-0.451
	(0.0405)	(0.785)	(0.176)	(0.488)
stir	-0.0392	0.282*	3.068*	0.238
	(0.0341)	(0.135)	(1.720)	(0.338)
ln_gdppc	5.663**		64.73*	5.100*
	(2.176)		(34.35)	(2.515)
ln_exch	0.536	-1.050	2.912**	
	(0.595)	(1.842)	(1.127)	
ln_exit	-0.128	-0.487		3.105
	(0.442)	(0.982)		(3.852)
Constant	-0.0678	0.0520	-2.000*	-0.481
	(0.0480)	(0.273)	(1.083)	(0.522)
Observations	236	76	80	80

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 7 presents the results from the estimation of model 3. When considering the entire sample, it can be seen that only one variable is found to be significant: the natural log of the quotient between per capita GDP in two consecutive years, which is found to be significant at a 5% level. The remaining variables: the absolute change in long and short term interest rates, the relative change in the EUR-USD exchange rate, the relative change in the S&P 500 index and the constant term are found to be non-significant.

The absolute change in long term interest rates is found to be significant at a 1% level in the 2012-2016 subsample period, with the exchange rate variable being significant at a 5% level and the short term interest rate, per capita GDP and constant term being significant at a 10% level. Lastly, the per capita GDP variable is also found to be significant at a 10% level in the 2017-2021 subsample period. The omission of some results is due to the presence of multicollinearity between the independent variables in the subsample periods.

Here too, the model is found to be statistically significant, with the p-value from the obtained F-statistic (8.12) indicating significance at a 1% level (0.0007). The remainder disturbance indicates some skewness may be present, with a p-value of 0.0114. Considering the distribution of the residuals present in figure 13 (see appendix A.11), this could possibly be related to the presence of outliers in the data.

5.2.4 Model 4

Table 8: Estimation - Model 4 - Pooled OLS

ln_adj	(1) Total	(2) 2007-2011	(3) 2012-2016	(4) 2017-2021
ln_gdppc	4.769***	1.414	13.68***	5.687**
	(1.257)	(3.018)	(4.601)	(1.956)
Constant	-0.0458	0.0201	-0.358**	-0.0384
	(0.0306)	(0.0456)	(0.147)	(0.0614)
Observations	236	76	80	80

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Model 4 is an extension of model 3, following the principle of parsimony, that is, removing the non-significant and non-robust variables. Results are presented in table 8. When considering the entire sample, the per capita GDP variable is still found to be significant, but now at a 1% level. The constant term is found to be non-significant, in line with the results for model 3.

The model is statistically significant, with the p-value relative to the obtained F-statistic (14.39) indicating significance at a 1% level (0.0018). Here too, the remainder disturbance finds some evidence of skewness, with a p-value of 0.0252. The distribution of the residuals is presented in figure 14 (see appendix A.11).

5.3 Adjustment Factors

From the results above, we can establish the following adjustment factors:

Adjustment Factor 1

$$\ln(\widehat{Adj})_t = 0.042 + 0.588 * \ln(russ)_t \tag{16}$$

Adjustment Factor 2

$$\ln(\widehat{Adj})_t = -0.030 + 0.554 * \ln(refvc)_t \tag{17}$$

Adjustment Factor 3

$$\ln(\widehat{Adj})_t = -0.068 + 0.031 * ltir_t - 0.039 * stir_t + 5.663 * \ln(gdppc)_t \\ +0.536 * \ln(exch)_t - 0.128 * \ln(exit)_t \tag{18}$$

Adjustment Factor 4

$$\ln(\widehat{Adj})_t = -0.046 + 4.769 * \ln(gdppc)_t \tag{19}$$

To exemplify, adjustment factor 1 can be interpreted such that in a year in which market returns on the Russell 2000 index are 5% (that is, $russ_t = 1.05$), valuations increase by 7.3%.

$$\ln(\widehat{Adj})_t = 0.042 + 0.588 * \ln(1.05) = 0.071 \tag{20}$$

$$\begin{aligned} \widehat{Val}_t = &\quad Val_{t-1} * e^{0.071} \\ \Leftrightarrow \widehat{Val}_t = &\quad Val_{t-1} * 1.073 \end{aligned} \tag{21}$$

5.4 Robustness Check

In this section, I will assess the robustness of the results estimated for the four models, which are presented in tables 5, 6, 7 and 8 respectively. This means understanding whether the estimated coefficients are comparable to results obtained in similar scenarios, both in terms of the sign and magnitude of the coefficient and its significance level.

The robustness check will follow two alternative approaches. The first approach will use the different subsample periods to make a preliminary analysis of robustness, that is, understand whether the coefficients estimated in each subsample period are similar to the coefficient estimated for the overall sample. The second approach will use the results from a regression which introduces time fixed effects to the original models estimated using pooled OLS. The coefficients will then be compared to those in the original pooled OLS estimation.

5.4.1 Subsample Test

Starting with model 1, the independent variable is consistent in its sign across all subsample periods considered, with similar magnitude. Significance is only found for the overall period (at a 5% level) and for the 2017-2021 period (at a 1% level). Overall, the fact that the coefficients are similar in all scenarios points to a highly robust result. The constant term is significant (at a 5% level) only in the overall period, with the sign being equal across all subsamples considered and magnitude similar to that of the overall sample in two of the three subsample periods considered.

Moving on to model 2, the independent variable is significant (at a 5% level) both in the overall period and in the 2012-2016 subsample period (in this case at a 1% level). In all scenarios, the sign is the same and the magnitude is similar in two out of the three subsample periods. The constant term is not significant for the overall period, only for the 2012-2016 subsample period, with the sign differing between the overall sample and some of the subsample periods and indicating a low level of robustness.

In model 3, the long term and short term interest rate variables show a low level of robustness, with varying signs, magnitude and significance, contributing to an idea of spuriousness. The same can be said for the exchange rate and exit channel variables. The per capita GDP variable is significant at a 5% level, with a similar sign and magnitude in both the overall sample and the 2017-2021 subsample period, in addition to also having significance (although only at a 10% level). However, the coefficient in the 2007-2011 subsample period is omitted (due to multicollinearity) and the coefficient in the 2012-2016 subsample period is higher in magnitude, while also being significant at a 10% level. Relative to the constant term, the sign is the same in both the overall sample and two of the three subsample periods, with the coefficient for the 2012-2016 subsample period being significant (at a 10% level).

In model 4, the per capita GDP variable shows a high level of robustness, with the same sign and magnitude found in both the overall sample and in the three subsample periods. Significance at a 1% level is found in the overall sample, with significance at a 1% level found in the 2012-2016 subsample period and significance at a 5% level found in the 2017-2021 subsample period. Relative to the constant term, the sign is the same in both the overall sample and two of the three subsample periods, with the coefficient being significant in the 2012-2016 subsample period.

In particular, the variation found in some of the macroeconomic variables regarding magnitude, signs and significance of the coefficients between the overall sample and the subsample periods may indicate the presence of non-stationary effects, which could follow from the lack of stationarity found in table 3, for example, for the short term interest rate and exit channel variables.

5.4.2 Specification Test

As previously mentioned, the specification robustness test will be based on comparing the coefficients from the pooled OLS estimation with those from a pooled OLS estimation with time fixed effects. Results can be found in table 11 (Appendix A.4).

From the pooled OLS estimation, the coefficients for the independent variable in model

1 are similar (POLS = 0.588**; TFE = 1.736*), having the same sign and comparable magnitudes, as well as both having significance, although lower in the pooled OLS with time fixed effects case (only at a 10% level). In the constant term, values differ significantly (POLS = 0.042**; TFE = - 0.085), with no significance found in the time fixed effects case. This therefore indicates that the coefficients for the constant term are not robust.

The Refinitiv VC index variable found in model 2 also presents similar values in both estimations (POLS = 0.554**; TFE = 2.450*), having the same sign and magnitude, as well as similar levels of significance. Once again, significance is lower in the pooled OLS with time fixed effects case (only at a 10% level). Here, although the constant terms are similar (POLS = -0.0297; TFE = -0.434), they both are non-significant.

In the case of model 3, the lack of robustness found for the long term interest rate variable persists, with different signs and no significance found in the pooled OLS with time fixed effects estimation (POLS = 0.0314; TFE = -0.539). This is also true in the case of the short term interest rate variable (POLS = -0.0392; TFE = 0.295). The per capita GDP variable has similar coefficients in both estimations, adding to the idea of robustness previously found (POLS = 5.663**; TFE = 4.889). There is, however, no significance in the pooled OLS with time fixed effects case.

The exchange rate variable has differents signs in both coefficients, in addition to being non-significant. (POLS = 0.536; TFE = -0.405). The same can be said for the exit channel variable (POLS = -0.128; TFE = 3.626). Lastly, considering the constant term in model 3, even though coefficients have the same sign, they have different magnitudes and are both non-significant (POLS = -0.0678; TFE = -0.556), in line with what was found for model 2.

Model 4 mimics the conclusions found for model 3 relative to both the per capita GDP variable and the constant term. The per capita GDP variable is found to be robust, since the two coefficients have the same sign and similar magnitude, as well as both having significance (although lower in the case of the time fixed effects model) (POLS = 4.769***; TFE = 8.165*)

The constant term has the same sign in both scenarios. However, it is non-significant in both specifications, with diverging magnitudes (POLS = -0.0458; TFE = -0.230). In general, this adds to the lack of robustness seen in the previous models.

5.5 Model Selection

To measure model performance and select the model which produces the most accurate adjustment factors, I focused on an approach of measuring errors produced in out of sample forecasts relative to actual values.

5.5.1 Out of Sample Forecast Evaluation

In this section, I re-estimated the four models in log form for a subsample between the years 2007 and 2017 and compared the predictions for the 2018-2021 period with the actual values found in the dataset. Two accuracy metrics are presented: root mean square errors (RMSE) and Theil's U. Additionally, RMSE are calculated for the naive prediction. This procedure is then repeated using a rolling window, by progressively including an additional year in the subsample.

The estimation and performance statistics are made resorting to the XTOOS statistical package (Ugarte Ruiz 2019), and results can be found in Table 9.[18] Column 1 presents the last year included in the rolling window for the model estimation. Column 2 reports the root mean square errors (RMSE) of the model predictions relative to the actual values observed in the comparison period and Column 3 calculates the RMSE between the actual values and the naive prediction. Column 4 reports the relative performance expressed by Theil's U. (Theil 1971)[19] Additional results by subcategory can be found in appendixes A.5, A.6, A.7 and A.8.[20]

The analysis of model performance is based on three aspects: (i) the comparison of the RMSE obtained from the models and from the naive prediction, (ii) the comparison of the RMSE obtained between the four models and (iii) an analysis of Theil's U between the four models.

The comparison of RMSE between the models' predictions and the naive predictions allows for a performance assessment since it can be justified that, if the RMSE are lower in the estimated models than in the naive prediction, it is true that there is an improvement relative to the naive prediction. At the same time, a comparative assessment between the RMSE in the four estimated models, everything else kept the same, allows for a selection of the best model in terms of forecasting ability. The analysis of Theil's U measures model performance through the estimation of the errors between predicted and actual values relative to the actual values. A value below 1 improves on the naive forecast, with proximity to 0 determining the quality of the model. Consequently, this also allows for the comparison between the performance of the four estimated models.

Looking at model 1 results in table 9 and using the "Summary" row as reference, the root mean square errors (RMSE) of the out of sample estimates improve on those from the naive forecast (RMSE Alt) ($0.612 < 0.727$). This is corroborated by the results from the U-Theil statistic, which are consistently below one (U-Theil = 0.841), corresponding to a 15.9% improvement relative to the naive prediction.

[18] The corresponding Stata command is: xtoos_t (...), indate(2017) cdate(2021) met(reg) vce(cluster id).

[19] Since Theil (1971) is unavailable online, I would suggest Brown & Rozeff (1978) as an alternative source.

[20] For further details on the calculation of RMSE and Theil's U see appendix A.9.

Table 9: Out of Sample Forecast - Pooled OLS - Performance Metrics

	Model 1				*Model 2*			
	RMSE	RMSE_Alt	U-Theil	N	RMSE	RMSE_Alt	U-Theil	N
2017	0,534	0,555	0,962	64	0,533	0,555	0,961	64
2018	0,595	0,712	0,836	48	0,597	0,712	0,839	48
2019	0,663	0,761	0,871	32	0,661	0,761	0,869	32
2020	0,813	1,174	0,693	16	0,819	1,174	0,698	16
Summary	0,612	0,727	0,841	160	0,613	0,727	0,842	160

	Model 3				*Model 4*			
	RMSE	RMSE_Alt	U-Theil	N	RMSE	RMSE_Alt	U-Theil	N
2017	0,503	0,555	0,906	64	0,512	0,555	0,923	64
2018	0,570	0,712	0,800	48	0,570	0,712	0,801	48
2019	0,622	0,761	0,817	32	0,616	0,761	0,809	32
2020	0,767	1,174	0,653	16	0,736	1,174	0,627	16
Summary	0,578	0,727	0,795	160	0,577	0,727	0,793	160

The results relative to the out of sample forecast categorized by industry and stage of financing, which can be found in table 13 (see appendix A.5), indicate that the model forecast is more accurate than the naive prediction across all subcategories, since it has both lower RMSE values than the naive prediction and U-Theil values below 1. At the same time, considering industry categorization, the model is the most accurate when estimating the adjustment factor in valuations for "Biotech and Pharma" (U-Theil = 0.726) and "Consumer Tech" (U-Theil = 0.749), with worse results in the "Enterprise Tech" (U-Theil = 0.929) and "Fintech" (U-Theil = 0.939) segments. Regarding stage of financing, the model is found to be more accurate when applied to the "Seed" (U-Theil = 0.698) and "Late-Stage" (U-Theil = 0.729) rounds of financing.

Models 2 and 3 mimic the overall conclusions for model 1, with the root mean square errors (RMSE) of the model improving on those from the naive prediction (RMSE Alt). At the same time, the results from the U-Theil statistic are also consistently below one. Specifically for model 2, the U-Theil result of 0.842 shows a 15.8% improvement relative to the naive prediction.

Analysing model 2 results relative to the out of sample forecast in the different subcategories, present in table 14 (see appendix A.6), the RMSE improve on those from the naive forecast (RMSE Alt), with results from the U-Theil statistic being consistently below one. In line with the conclusions from the estimation of model 1, the results for the forecast are overall more accurate for the "Biotech and Pharma" (U-Theil = 0.736) and "Consumer Tech" (U-Theil = 0.809) sectors, with worse results being produced for the "Enterprise Tech" (U-Theil = 0.898) and "Fintech" (U-Theil = 0.929) sectors. Conclusions are also similar when analysing the different stages of financing, with lower prediction errors being obtained in "Seed" (U-Theil = 0.737) and "Late-Stage" (U-Theil = 0.689) financing.

Considering model 3, the U-Theil result of 0.795 indicates a 20.5% improvement relative to the naive prediction. Table 15, which can be found in appendix A.7, shows lower prediction errors in the "Biotech and Pharma" (U-Theil = 0.678) and "Consumer Tech" (U-Theil = 0.636) sectors, with higher prediction errors being obtained for the "Enterprise Tech" (U-Theil = 0.809) and "Fintech" (U-Theil = 0.905). In the categorization by stage of financing, the model is the most accurate when considering the "Seed" (U-Theil = 0.659) and "Late-Stage" (U-Theil = 0.680) rounds of financing, with the highest prediction errors being obtained in the "Angel" (U-Theil = 0.807) and the "Early-Stage" (U-Theil = 0.877) categories.

Model 4 presents a RMSE result of 0.577, with the U-Theil result of 0.793 indicating a 20.7% improvement relative to the naive prediction. Table 16, which can be found in appendix A.8, also shows lower prediction errors in the "Biotech and Pharma" (U-Theil = 0.687) and "Consumer Tech" (U-Theil = 0.652) sectors. In the categorization by stage of financing, the model is the most accurate when considering the "Seed" (U-Theil = 0.446) and "Late-Stage" (U-Theil = 0.642) rounds of financing.

A between-models comparison reveals that macro-based models present the lowest prediction errors, with model 4 having both the lowest summary RMSE (0.577) and U-Theil (0.793) results and indicating its predictions are closer to the actual values in the dataset. Model 3 is found to be the second best, with slightly higher RMSE (0.578) and U-Theil (0.795) results. The studied market-based models present marginally higher prediction errors, with model 1 having a summary RMSE of 0.612 and a summary U-Theil of 0.841 and model 2 having a summary RMSE of 0.613 and a summary U-Theil of 0.842.

Looking at tables 13, 14, 15 and 16 the conclusions are generally the same, with models 3 and 4 slightly outperforming models 1 and 2. In the industry categorization, model 3 has the lowest prediction errors in the "Biotech and Pharma" (U-Theil = 0.678) and "Consumer Tech" (U-Theil = 0.636) categories, with model 4 having the lowest prediction errors in the "Enterprise Tech" (U-Theil = 0.718) and "Fintech" (U-Theil = 0.897) categories. In the categorization by stage of financing, model 3 has the lowest prediction errors in the "Angel" (U-Theil = 0.807) stage, with model 4 having the lowest prediction errors in the "Seed" (U-Theil = 0.446), "Early-Stage" (U-Theil = 0.779) and "Late-Stage" (U-Theil = 0.642) categories.

6 Discussion

In this section, I will reason on the results found for model estimation (section 5.2) and selection (section 5.5), framing them in the context of previous research.

Starting by the results in model 1, the natural log of the returns on the Russell 2000 index, which corresponds to the relative change in the Russell 2000 index, is found to positively affect the natural log of the adjustment factor, meaning an increase in the index leads to an increase in valuations in any given year. The coefficient for the overall sample is also found to be significant (at a 5% level) and robust. This finding is in line with Gompers & Lerner (1997), who suggest that there might be significant positive correlation between venture capital activity and public market prices.

Considering model 2, the natural log of the returns on the Refinitiv VC index, that is, the relative change in the Refinitiv VC index, is also found to positively affect the natural log of the adjustment factor, so that an increase in the index leads to an increase in valuations in any given year. Here too, the coefficient for the overall sample is found to be significant (at a 5% level) and robust. Furthermore, the results are similar to those found in Refinitiv (2021, p.9), which also present for the sectorial market index variable a positive coefficient with similar magnitude.

The results from model 3 find significance (at a 5% level) and robustness in the per capita GDP variable, therefore introducing that the relative change in per capita GDP is found to positively affect the natural log of the adjustment factor. This aligns with some of the qualitative intuitions which had been defined, such as the initial work by Gompers et al. (1998), who suggest that GDP may be positively associated with greater VC activity, and Füss & Schweizer (2012), who suggest that an increase in GDP drives up valuations through increases in the equilibrium levels of demand and supply in an economy.

Regarding the absolute change in the long and short term interest rate variables, the lack of both significance and robustness found in the estimation results means it cannot be inferred that an absolute change in interest rates affects the relative change in valuations. Because of this, the idea proposed by Füss & Schweizer (2012) that higher long term interest rates may lead to a decrease in the search of traditional lending by entrepreneurs, which would instead rely on venture capital funding, cannot be expanded to affect equilibrium valuations by means of an increase in VC funding demand. The same can be said regarding the impact of short term interest rates through the effect on economic activity.

When considering the exchange rate variable, results are also non-significant for the overall sample, meaning the suggestion found in Alba et al. (2010) that a more valuable USD would attract higher levels of FDI, therefore driving up the supply of capital and equilibrium valuations cannot be confirmed. Furthermore, the exit channel, measured by the relative change in the S&P 500 index, is found to be non-significant for the overall sample. This clarifies the importance using market indexes which replicate start-up firms' fundamental properties or industries. At the same time, it is not possible to discern a significant effect between general public market quality and the behaviour of the series of valuations.

Model 4 is an extension of model 3 built based on the principle of parsimony, and therefore the same conclusion can be taken regarding the per capita GDP variable, which is significant at a 1% level and robust. It follows that, here too, the relative change in per capita GDP positively affects the natural log of the adjustment factor.

The discussion on the constant term is centered around the results found in Refinitiv (2021, p.9), which suggests that the constant should have a negative value since "firms with delayed rounds or delayed reporting tend to have worse returns than those with frequent events and immediately reported returns". However, across models and estimation techniques, coefficients are typically without any level of significance.

The only signficant result for the constant term in the overall sample is found for model 1 in the pooled OLS case. However, this result is not found to be robust, since its coefficient differs when compared with the pooled OLS with time fixed effects case. In total, the idea proposed in Refinitiv (2021, p.9) cannot be directly corroborated from the results obainted.

Reasoning on the central thesis of this work, that is, which approach yields the best adjustment factors, it can be said that the adjustment of valuations in start-up firms using both macroeconomic and financial indicators generally improves on naive predictions, since it generates lower prediction errors. Overall, models based on macroeconomic variables also seem to improve on the traditional market-based approach. The parsimonious model based on per capita GDP reports the lowest prediction errors, followed by the complete macro-based model. Approaches based on market indexes, such as those found in Damodaran (2009) and Refinitiv (2021), report marginally higher prediction errors. Between the models based on market indexes, my results show a slight advantage for a small cap market index, when considering the overall sample, with mixed results between the small cap market index (Russell 2000 index) and the sectorial market index (Refinitiv VC index) when considering subcategories. It is also important to evaluate these conclusions in context of the results found for the pooled OLS with time fixed effects case.[21] Not only are prediction errors higher in each model relative to the pooled OLS estimation, but only models 1 and 3 achieve an overall improvement relative to the naive prediction (11,4% and 19,9% respectively), with model 2 roughly equaling the errors of the naive prediction and model 4 reporting markedly higher overall prediction errors.

[21]See table 12 in appendix A.4.

7 Conclusion

This paper is directed at the creation and evaluation of adjustment factors which can be used to calculate the timing adjustments of start-up valuations. This is achieved through a pooled OLS estimation of the relative change in valuations based on macroeconomic and financial variables. In total four models are estimated, allowing for a comparison between the current market-based literature proposals and the idea introduced in this paper of macro-based adjustment factors.

The results allow for some general conclusions about the behaviour of valuations relative to macroeconomic and financial variables. Notably, in models 1 and 2, which are based on the Russell 2000 index and the Refinitiv VC index respectively, the market index variable is found to be significant and have a positive coefficient, indicating positive correlation between the behaviour of market indexes and that of private company valuations, aligning with some of the earlier work by Gompers & Lerner (1997).

Regarding models 3 and 4, the per capita GDP variable, which is significant and robust for the overall period, has a positive coefficient, following from the idea introduced in Gompers et al. (1998) that higher increases in GDP may be linked to higher levels of VC activity. Considering the remaining variables studied, none are significant for the overall sample, with some significance, often not robust, found in subsample periods for the long and short term interest rate, exchange rate and exit channel variables.

Across models, the constant term is only found to be significant in the small cap market index model, in which case it has a positive coefficient. However, the results found are not robust. In remaining cases, the estimated coefficient was found to be non-significant, meaning the results found in Refinitiv (2021, p.9) cannot be verified.

Using prediction errors for an out-of-sample forecast to measure model performance, it can be seen that the four models are overall more accurate than the naive prediction. A comparison suggests that macro-based models slightly improve on the traditional market-based approach, with model 4, which uses the per capita GDP variable as a single covariate, producing the lowest overall errors for the entire sample, as well as in most subcategories. Between models based on market indexes, model 1 has overall lower prediction errors, although results are similar to those present for model 2. Introducing time fixed effects in the regression generally worsens prediction results, with prediction errors generally improving on the naive prediction only in the case of models 1 and 3.

7.1 Limitations and Further Research

It is also important to discuss some of the limitations found throughout the paper. First, the dataset in use for company valuations contains only some of the industries which could be considered for the analysis. This is compounded by the fact that only summary statistics are available, decreasing within-category variance and resulting in reduced predictive power of the models.

Additionally, there are some concerns relating to the applicability of the adjustment factors produced. The use of yearly data for the estimation does not provide the necessary granularity when calculating valuations in practice, since the multiples used as a reference for the valuation of a company usually report to at most a few months before a given event. In this case, this choice was based on the available series of company valuations. However, monthly or quarterly data, which is generally available for macroeconomic indicators and financial indexes, would be more appropriate.

Furthermore, the dataset on company valuations is based on the median of valuations produced in a given year (in USD M), instead of focusing on valuation multiples. This may cause bias in predictions, since it means the forecast does not account for the effects of between-year median revenue variation on valuation multiples.

Lastly, future research could focus on alternatives to linear regression as an estimation method, by building on the application of techniques such as machine learning to financial forecasting, as presented in Wasserbacher & Spindler (2022). Methods such as these have already been shown to outperform traditional OLS regressions in peer firm selection and relative valuation in public market settings (Geertsema & Lu 2022), leaving space for expansion in their use under private market constraints.

References

Alba, J. D., Wang, P. & Park, D. (2010), 'The impact of exchange rate on FDI and the interdependence of FDI over time', *The Singapore Economic Review* **55**(04), 733–747.
URL: *https://doi.org/10.1142/S0217590810004024*

Alford, A. W. (1992), 'The effect of the set of comparable firms on the accuracy of the price-earnings valuation method', *Journal of Accounting Research* **30**(1), 94–108.
URL: *http://www.jstor.org/stable/2491093*

Asquith, P., Mikhail, M. B. & Au, A. S. (2005), 'Information content of equity analyst reports', *Journal of Financial Economics* **75**(2), 245–282.
URL: *https://www.sciencedirect.com/science/article/pii/S0304405X04001369*

Baltagi, B. H. (2008), *Econometric Analysis of Panel Data*, Vol. 4, Springer.

Bancel, F., Martinaud, B. & Philippe, H. (2021), 'The seven deadly sins of start-up valuation', *Journal of Applied Corporate Finance* **33**(3), 125–129.
URL: *https://doi.org/10.1111/jacf.12469*

Barg, J. A., Drobetz, W. & Momtaz, P. P. (2021), 'Valuing start-up firms: A reverse-engineering approach for fair-value multiples from venture capital transactions', *Finance Research Letters* **43**, 102008.
URL: *https://doi.org/10.1016/j.frl.2021.102008*

Berkus, D. (2016), 'After 20 years: Updating the Berkus Method of valuation'.
URL: *https://berkonomics.com/?p=2752*

Black, B. S. & Gilson, R. J. (1998), 'Venture capital and the structure of capital markets: banks versus stock markets', *Journal of Financial Economics* **47**(3), 243–277.
URL: *https://doi.org/10.1016/S0304-405X(97)00045-7*

Black, F. & Scholes, M. (1973), 'The pricing of options and corporate liabilities', *Journal of Political Economy* **81**(3), 637–654.
URL: *https://doi.org/10.1086/260062*

Breusch, T. S. & Pagan, A. R. (1979), 'A simple test for heteroscedasticity and random coefficient variation', *Econometrica* **47**(5), 1287–1294.
URL: *http://www.jstor.org/stable/1911963*

Breusch, T. S. & Pagan, A. R. (1980), 'The Lagrange Multiplier Test and its Applications to Model Specification in Econometrics', *The Review of Economic Studies* **47**(1), 239–253.
URL: *https://doi.org/10.2307/2297111*

Brown, L. D. & Rozeff, M. S. (1978), 'The superiority of analyst forecasts as measures of expectations: Evidence from earnings', *The Journal of Finance* **33**(1), 1–16.
URL: *https://doi.org/10.2307/2326346*

Butters, J. (2023), 'Factset Earnings Insight Feb 17, 2023'.
URL: *https://insight.factset.com/sp-500-earnings-season-update-february-17-2023*

Capital IQ (2023*a*), 'Russell 2000 Index'. Accessed: June 6, 2023.
URL: *https://www.capitaliq.com/CIQDotNet/Charting4/ModernBuilder.aspx ?companyId=2667223*

Capital IQ (2023*b*), 'S&P 500'. Accessed: May 9, 2023.
URL: *https://www.capitaliq.com/CIQDotNet/Index/IndexWidgetTearsheet.aspx ?companyId=2668699*

Choi, I. (2001), 'Unit root tests for panel data', *Journal of International Money and Finance* **20**(2), 249–272.
URL: *https://www.sciencedirect.com/science/article/pii/S0261560600000486*

D'Agostino, R. B. & Belanger, A. (1990), 'A suggestion for using powerful and informative tests of normality', *The American Statistician* **44**(4), 316–321.
URL: *http://www.jstor.org/stable/2684359*

Damodaran, A. (2009), 'Valuing young, start-up and growth companies: estimation issues and valuation challenges', *Available at SSRN 1418687* .
URL: *https://pages.stern.nyu.edu/ adamodar/pdfiles/papers/younggrowth.pdf*

Depeyrot, T. & Heap, S. (2018), 'Hacking Software's Rule of 40'.
URL: *https://www.bain.com/contentassets/b514f7b986c949a0b9c24d3748fb4fef/bain-brief-hacking-softwares-rule-of-40.pdf*

Fama, E. F. & French, K. R. (1992), 'The cross-section of expected stock returns', *the Journal of Finance* **47**(2), 427–465.
URL: *https://doi.org/10.1111/j.1540-6261.1992.tb04398.x*

FRED (2023*a*), '10-Year Treasury Constant Maturity Rate'. Accessed: May 6, 2023.
URL: *https://fred.stlouisfed.org/series/GS10*

FRED (2023*b*), '3-Month Treasury Bill: Secondary Market Rate'. Accessed: May 6, 2023.
URL: *https://fred.stlouisfed.org/series/TB3MS*

FRED (2023*c*), 'Per Capita Gross Domestic Product'. Accessed: May 6, 2023.
URL: *https://fred.stlouisfed.org/series/A939RC0Q052SBEA*

FRED (2023*d*), 'U.S. / Euro Foreign Exchange Rate'. Accessed: May 6, 2023.
URL: *https://fred.stlouisfed.org/series/DEXUSEU*

Füss, R. & Schweizer, D. (2012), 'Short and long-term interactions between venture capital returns and the macroeconomy: evidence for the united states', *Review of Quantitative Finance and Accounting* **38**, 391–410.
URL: *https://doi.org/10.1007/s11156-011-0233-4*

Geertsema, P. & Lu, H. (2022), 'Relative valuation with machine learning', *Journal of Accounting Research* **61**(1), 329–376.
URL: *https://doi.org/10.1111/1475-679X.12464*

Gompers, P. A. & Lerner, J. (1997), 'Risk and reward in private equity investments: The challenge of performance assessment', *The Journal of Private Equity* **1**(2), 5–12.
URL: *http://www.jstor.org/stable/43503183*

Gompers, P. A., Lerner, J., Blair, M. M. & Hellmann, T. (1998), 'What drives venture capital fundraising?', *Brookings Papers on Economic Activity. Microeconomics* **1998**, 149–204.
URL: *https://doi.org/10.3386/w6906*

Gompers, P. & Lerner, J. (2001), 'The venture capital revolution', *Journal of Economic Perspectives* **15**(2), 145–168.
URL: *https://www.aeaweb.org/articles?id=10.1257/jep.15.2.145*

Granlund, M. & Taipaleenmäki, J. (2005), 'Management control and controllership in new economy firms — a life cycle perspective', *Management Accounting Research* **16**(1), 21–57.
URL: *https://doi.org/10.1016/j.mar.2004.09.003*

Hausman, J. A. (1978), 'Specification tests in econometrics', *Econometrica* pp. 1251–1271.
URL: *https://doi.org/10.2307/1913827*

Herrmann, V. & Richter, F. (2003), 'Pricing with performance-controlled multiples', *Schmalenbach Business Review* **55**, 194–219.
URL: *https://doi.org/10.1007/BF03396674*

Kennedy, P. (1998), *A Guide to Econometrics*, 4th edn, John Wiley & Sons.

Kenney, J. F. & Keeping, E. (1962), 'Root Mean Square', *Mathematics of Statistics* **1**, 59–60.

Knight, F. H. (1921), *Risk, Uncertainty and Profit*, Vol. 31, Houghton Mifflin.

Korteweg, A. & Sorensen, M. (2010), 'Risk and return characteristics of venture capital-backed entrepreneurial companies', *The Review of Financial Studies* **23**(10), 3738–3772.
URL: *https://doi.org/10.1093/rfs/hhq050*

KPMG (2022), 'Venture Pulse Report Q4 2021'.
URL: *kpmg.com/xx/en/home/campaigns/2022/01/q4-venture-pulse-report-global.html*

Liu, J., Nissim, D. & Thomas, J. (2002), 'Equity valuation using multiples', *Journal of Accounting Research* **40**(1), 135–172.
URL: *https://doi.org/10.1111/1475-679X.00042*

Midway, S. (2022), 'Data Analysis in R - Random Effects'.
URL: *https://www.bookdown.org/steve_midway/DAR/*

Montani, D., Gervasio, D. & Pulcini, A. (2020), 'Startup company valuation: The state of art and future trends', *International Business Research* **13**(9), 31–45.
URL: *https://doi.org/10.5539/ibr.v13n9p31*

Neter, J., Kutner, M. et al. (2005), *Applied Linear Statistical Models*, 5th edn, Irwin Chicago.

Payne, B. (2011), 'Scorecard Valuation Methodology'.
URL: *seedspot.org/wp-content/uploads/2021/02/Scorecard-Valuation-Methodology.pdf*

Pinto, J. E., Robinson, T. R. & Stowe, J. D. (2019), 'Equity valuation: A survey of professional practice', *Review of Financial Economics* **37**(2), 219–233.
URL: *https://doi.org/10.1002/rfe.1040*

Pitchbook (2022), 'Q3 2022 U.S. VC valuations report'. Accessed: May 2, 2023.
URL: *https://pitchbook.com/news/reports/q3-2022-us-vc-valuations-report*

Plenborg, T. & Pimentel, R. C. (2016), 'Best practices in applying multiples for valuation purposes', *The Journal of Private Equity* **19**(3), 55–64.
URL: *http://www.jstor.org/stable/44397547*

Poterba, J. M. (1989), 'Venture capital and capital gains taxation', *Tax Policy and the Economy* **3**, 47–67.
URL: *http://www.jstor.org/stable/20061783*

Psaradakis, Z., Ravn, M. O. & Sola, M. (2005), 'Markov switching causality and the money–output relationship', *Journal of Applied Econometrics* **20**(5), 665–683.
URL: *https://doi.org/10.1002/jae.819*

Refinitiv (2021), *Refinitiv Venture Capital Research Index Methodology*, Refinitiv, New York, NY.
URL: *https://www.refinitiv.com/en/financial-data/indices/venture-capital-index*

Refinitiv Eikon (2023), 'Refinitiv Venture Capital Index'. Accessed: May 8, 2023.

Roberts, M. R. & Whited, T. M. (2013), Endogeneity in empirical corporate finance, *in* 'Handbook of the Economics of Finance', Vol. 2, Elsevier, pp. 493–572.
URL: *https://doi.org/10.1016/B978-0-44-453594-8.00007-0*

Romer, D. (2012), *Advanced Macroeconomics*, 4th edn, McGraw-Hill Irwin, New York.

Scherlis, D. R. & Sahlman, W. A. (1987), *A Method for Valuing High-risk, Long-term Investments: The Venture Capital Method*, Harvard Business School Publishing.

Schmidt, J. (2022), 'Private Company Valuation'. Accessed: May 3, 2023.
URL: *https://www.corporatefinanceinstitute.com/resources/valuation/private-company-valuation/*

Theil, H. (1971), *Applied Economic Forecasting*, North-Holland Pub. Co.

Trigeorgis, L. (1993), 'The nature of option interactions and the valuation of investments with multiple real options', *The Journal of Financial and Quantitative Analysis* **28**(1), 1–20.
URL: *https://doi.org/10.2307/2331148*

Ugarte Ruiz, A. (2019), 'XTOOS: Stata module for evaluating the out-of-sample prediction performance of panel-data models', Statistical Software Components S458710, Boston College Department of Economics, revised 03 Apr 2023.

Wasserbacher, H. & Spindler, M. (2022), 'Machine learning for financial forecasting, planning and analysis: recent developments and pitfalls', *Digital Finance* **4**(1), 63–88.
URL: *https://doi.org/10.1007/s42521-021-00046-2*

A Appendix

A.1 Median Valuations by Year

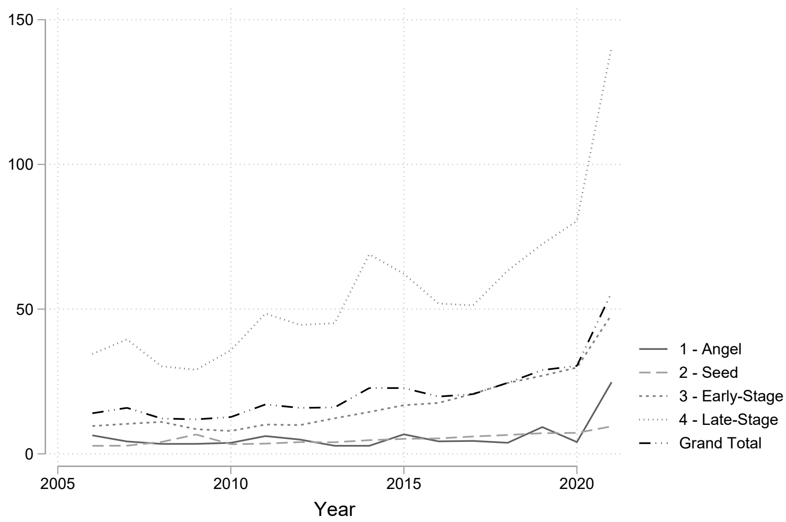

Figure 3: Median Valuations by Year

(USD M)	1 - Angel	2 - Seed	3 - Early-Stage	4 - Late-Stage	Grand Total
2006	6,382	2,768	9,604	34,510	14,019
2007	4,279	2,836	10,372	39,551	15,892
2008	3,429	4,117	11,039	30,238	12,206
2009	3,446	6,689	8,515	29,045	11,924
2010	3,807	3,327	7,904	35,808	12,712
2011	6,138	3,530	10,133	48,465	17,066
2012	4,929	4,072	9,933	44,560	15,874
2013	2,788	4,003	12,291	45,126	16,052
2014	2,776	4,726	14,469	68,955	22,731
2015	6,743	5,178	16,799	62,249	22,742
2016	4,318	5,338	17,592	51,942	19,797
2017	4,463	6,000	20,750	51,292	20,626
2018	3,827	6,519	24,500	63,318	24,541
2019	9,250	7,125	27,000	72,500	28,969
2020	4,069	7,250	29,812	80,438	30,392
2021	24,738	9,490	48,063	140,500	55,698

A.2 Macroeconomic and Financial Variables

Table 10: Complete Dataset - Macroeconomic and Financial Variables

Year	LTIR *(%)*	STIR *(%)*	GDPpc *(USD)*	USD_EUR_EX *(EUR)*	EXIT *(USD)*	RUSS_2000 *(USD)*	VC_INDEX *(USD)*
2006	4,79	4,73	46233,00	0,80	1310,46	735,13	934,08
2007	4,63	4,35	47974,75	0,73	1477,19	804,50	1148,15
2008	3,67	1,37	48499,00	0,68	1220,04	655,37	1051,01
2009	3,26	0,15	47122,50	0,72	948,05	521,68	985,08
2010	3,21	0,14	48569,25	0,75	1139,97	672,47	1276,83
2011	2,79	0,05	49951,00	0,72	1267,64	770,93	1552,68
2012	1,80	0,09	51644,25	0,78	1379,35	806,40	1819,38
2013	2,35	0,06	53115,50	0,75	1643,80	1008,52	2178,35
2014	2,54	0,03	54913,00	0,75	1931,38	1151,68	3031,59
2015	2,14	0,05	56520,00	0,90	2061,07	1205,62	3741,58
2016	1,84	0,32	57591,50	0,90	2094,65	1171,70	4000,39
2017	2,33	0,93	59588,00	0,88	2449,08	1423,42	5635,62
2018	2,91	1,94	62448,50	0,85	2746,21	1590,72	8057,60
2019	2,14	2,06	64689,00	0,89	2913,36	1546,18	9483,52
2020	0,89	0,37	63475,50	0,88	3217,86	1523,91	14049,63
2021	1,44	0,04	70150,25	0,85	4273,41	2242,91	21304,28
Mean	2,67	1,04	55155,31	0,80	2004,59	1114,45	5015,61
Max	4,79	4,73	70150,25	0,90	4273,41	2242,91	21304,28
Min	0,89	0,03	46233,00	0,68	948,05	521,68	934,08
Std Dev	1,06	1,53	7269,16	0,08	913,35	460,74	5720,96

A.3 Hausman Specification Test

```
                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |       fe           re         Difference       Std. err.
-------------+----------------------------------------------------------------
     ln_russ |   .5914646     .5882768        .0031878        .0450259
------------------------------------------------------------------------------
                           b = Consistent under H0 and Ha; obtained from xtreg.
            B = Inconsistent under Ha, efficient under H0; obtained from xtreg.

Test of H0: Difference in coefficients not systematic

    chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
            =   0.01
Prob > chi2 = 0.9436
```

Figure 4: Hausman Specification Test - Model 1

```
                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |       fe           re         Difference       Std. err.
-------------+----------------------------------------------------------------
    ln_refvc |   .5579731     .5536481         .004325        .0494284
------------------------------------------------------------------------------
                           b = Consistent under H0 and Ha; obtained from xtreg.
            B = Inconsistent under Ha, efficient under H0; obtained from xtreg.

Test of H0: Difference in coefficients not systematic

    chi2(1) = (b-B)'[(V_b-V_B)^(-1)](b-B)
            =   0.01
Prob > chi2 = 0.9303
```

Figure 5: Hausman Specification Test - Model 2

```
                 ---- Coefficients ----
             |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
             |       fe           re         Difference       Std. err.
-------------+----------------------------------------------------------------
        ltir |   .0317857     .0314255        .0003601        .0178942
        stir |  -.0387967    -.0392301        .0004334        .0118431
    ln_gdppc |   5.643515     5.662689       -.0191738        .3862267
     ln_exch |    .543937     .5361652        .0077718        .1270977
     ln_exit |  -.1239824    -.1277595        .0037771        .0844259
------------------------------------------------------------------------------
                           b = Consistent under H0 and Ha; obtained from xtreg.
            B = Inconsistent under Ha, efficient under H0; obtained from xtreg.

Test of H0: Difference in coefficients not systematic

    chi2(5) = (b-B)'[(V_b-V_B)^(-1)](b-B)
            =   0.01
Prob > chi2 = 1.0000
```

Figure 6: Hausman Specification Test - Model 3

A.4 Robustness Check - Pooled OLS with Time FE

The pooled OLS with time fixed effects estimation adds a time specific effect as part of its error term. This allows the model to capture the factors common to all entities in a given time period which affect the dependent variable. It can generally be represented as:

$$\ln(Adj)_{i,t} = \beta_1 + \beta_2 x_{i,tK} + ... + \beta_K x_{i,tK} + \mu_i + \lambda_t + v_{i,t} \tag{22}$$

In this case, λ_t corresponds to the time fixed effect and the unobservable individual specific effect μ_i is set to 0, as determined in subsection 5.2. The results from the estimation of this specification for the four models considered can be found in table 11. Coefficients relative to the time specific effects are omitted since they are not used in the robustness check.[22]

Table 11: Estimation - Pooled OLS with Time FE

	(1)	(2)	(3)	(4)
ln_adj	Model 1	Model 2	Model 3	Model 4
ln_russ	1.736*			
	(0.912)			
ln_refvc		2.450*		
		(1.287)		
ltir			-0.539	
			(3.010)	
stir			0.295	
			(1.992)	
ln_gdppc			4.889	8.165*
			(8.419)	(4.289)
ln_exch			-0.405	
			(12.91)	
ln_exit			3.626	
			(18.97)	
Constant	-0.0850	-0.434	-0.556	-0.230
	(0.302)	(0.470)	(2.667)	(0.370)
Observations	236	236	236	236

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 12 presents the performance metrics obtained in the out of sample estimation of the four models from the pooled OLS with time fixed effects configuration, built according to table 9. Comparing tables 9 and 12, the overall results from pooled OLS with time fixed effects estimations are found to be worse than those from the pooled OLS estimations in

[22] The corresponding Stata command is: reg (...) i.time, vce(cluster id).

each of the four cases, only showing improvement relative to the naive prediction, when considering the "Summary" row, in the case of models 1 and 3.[23]

Table 12: Out of Sample Forecast - Pooled OLS with Time FE - Performance Metrics

Model 1

	RMSE	RMSE_Alt	U-Theil	N
2017	0,523	0,555	0,942	64
2018	0,554	0,712	0,778	48
2019	0,922	0,761	1,212	32
2020	0,662	1,174	0,564	16
Summary	0,644	0,727	0,886	160

Model 2

	RMSE	RMSE_Alt	U-Theil	N
2017	0,536	0,555	0,965	64
2018	0,602	0,712	0,845	48
2019	1,224	0,761	1,609	32
2020	0,975	1,174	0,830	16
Summary	0,786	0,727	1,081	160

Model 3

	RMSE	RMSE_Alt	U-Theil	N
2017	0,611	0,555	1,101	64
2018	0,541	0,712	0,760	48
2019	0,578	0,761	0,759	32
2020	0,663	1,174	0,565	16
Summary	0,590	0,727	0,811	160

Model 4

	RMSE	RMSE_Alt	U-Theil	N
2017	0,862	0,555	1,552	64
2018	0,567	0,712	0,795	48
2019	6,751	0,761	8,872	32
2020	0,732	1,174	0,623	16
Summary	3,092	0,727	4,251	160

[23]The corresponding Stata command is: xtoos_t (...) i.time, indate(2017) cdate(2021) met(reg) vce(cluster id).

A.5 Model 1

A.5.1 Out of Sample Forecast - Graph

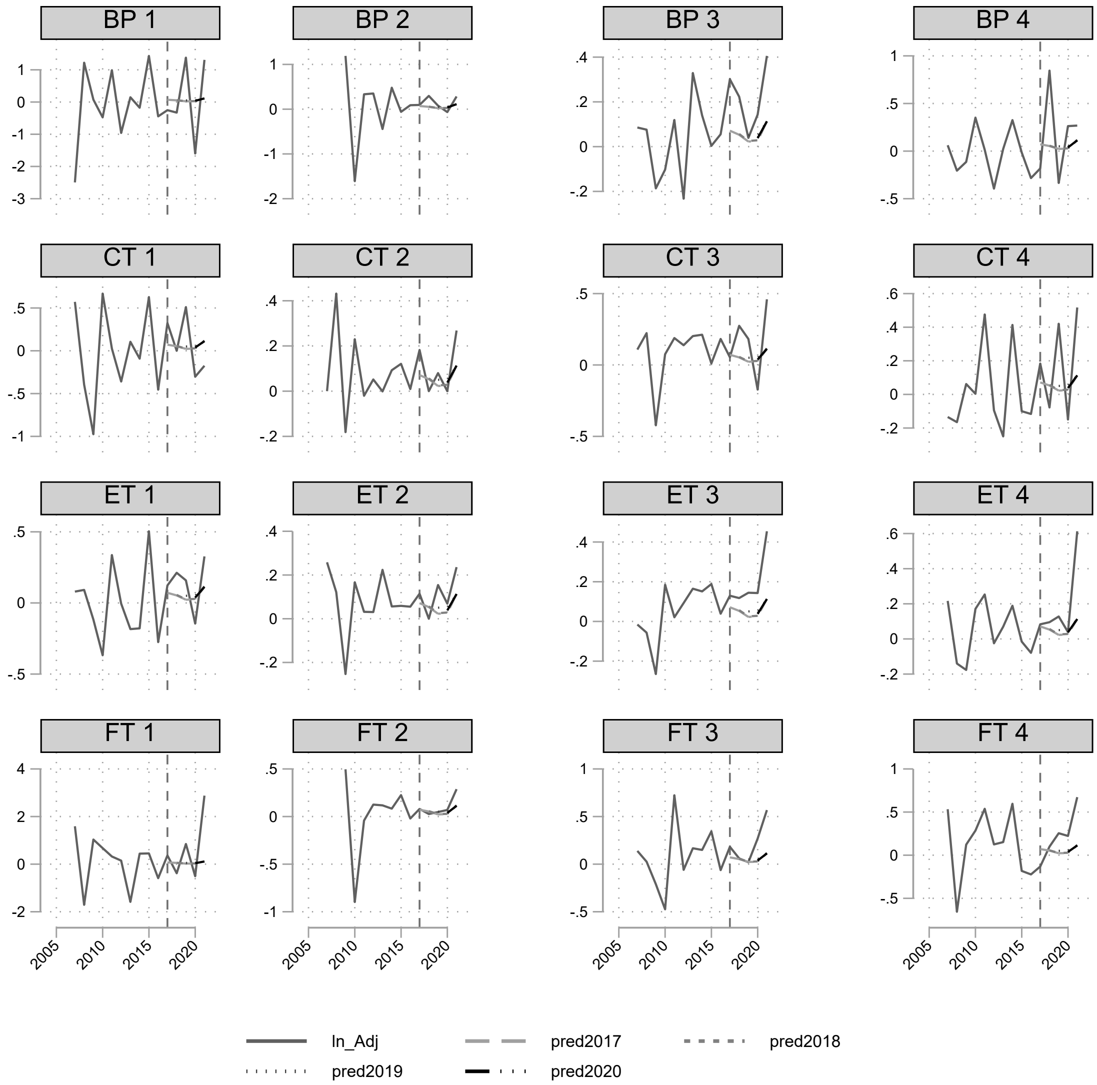

Figure 7: Out of Sample Forecast - Model 1 - Pooled OLS

A.5.2 Out of Sample Forecast by Industry and Stage of Financing

Table 13: Out of Sample Forecast - Model 1 - Pooled OLS - Performance Metrics

	RMSE	RMSE_Alt	U-Theil	N	RMSE	RMSE_Alt	U-Theil	N
	Biotech and Pharma				Consumer Tech			
2017	0,667	0,733	0,909	16	0,258	0,306	0,845	16
2018	0,721	0,891	0,809	12	0,287	0,331	0,867	12
2019	0,739	1,106	0,668	8	0,277	0,463	0,599	8
2020	0,626	1,469	0,426	4	0,313	0,483	0,647	4
Summary	0,694	0,956	0,726	40	0,276	0,369	0,749	40
	Enterprise Tech				Fintech			
2017	0,188	0,185	1,021	16	0,771	0,754	1,023	16
2018	0,208	0,226	0,918	12	0,879	1,037	0,847	12
2019	0,245	0,243	1,007	8	1,037	0,905	1,145	8
2020	0,327	0,411	0,794	4	1,431	1,718	0,833	4
Summary	0,223	0,240	0,929	40	0,943	1,004	0,939	40
	1 - Angel				2 - Seed			
2017	0,981	0,990	0,991	16	0,112	0,129	0,871	16
2018	1,118	1,288	0,868	12	0,103	0,185	0,557	12
2019	1,246	1,417	0,879	8	0,122	0,149	0,819	8
2020	1,517	2,245	0,676	4	0,157	0,259	0,606	4
Summary	1,140	1,340	0,851	40	0,117	0,168	0,698	40
	3 - Early-Stage				4 - Late-Stage			
2017	0,220	0,208	1,057	16	0,342	0,439	0,780	16
2018	0,237	0,247	0,961	12	0,317	0,525	0,603	12
2019	0,285	0,318	0,897	8	0,330	0,429	0,769	8
2020	0,364	0,405	0,898	4	0,433	0,494	0,877	4
Summary	0,256	0,269	0,954	40	0,343	0,470	0,729	40

A.6 Model 2

A.6.1 Out of Sample Forecast - Graph

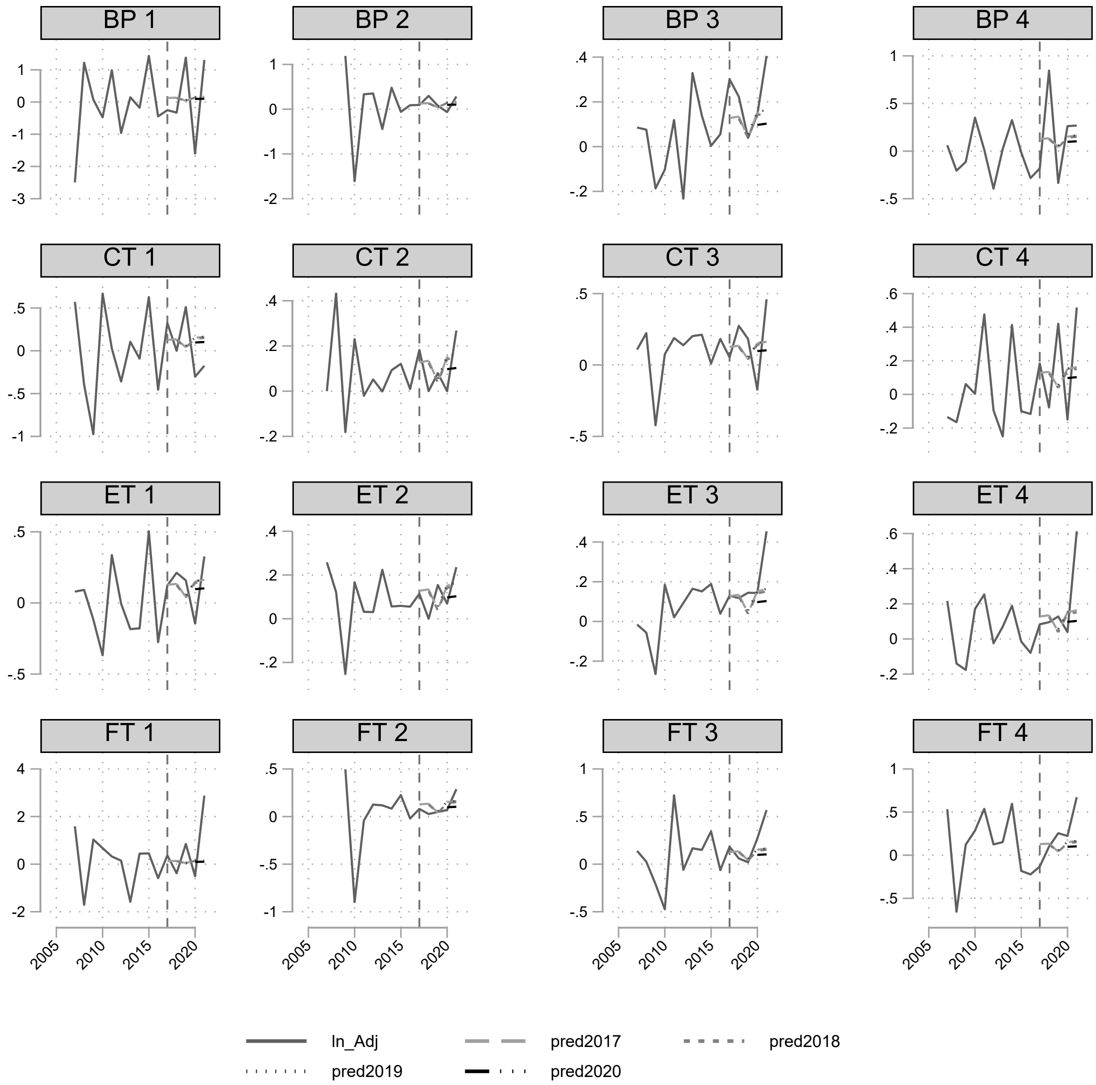

Figure 8: Out of Sample Forecast - Model 2 - Pooled OLS

A.6.2 Out of Sample Forecast by Industry and Stage of Financing

Table 14: Out of Sample Forecast - Model 2 - Pooled OLS - Performance Metrics

	RMSE	RMSE_Alt	U-Theil	N	RMSE	RMSE_Alt	U-Theil	N
	Biotech and Pharma				Consumer Tech			
2017	0,671	0,733	0,915	16	0,280	0,306	0,915	16
2018	0,733	0,891	0,823	12	0,308	0,331	0,933	12
2019	0,752	1,106	0,680	8	0,310	0,463	0,671	8
2020	0,634	1,469	0,431	4	0,318	0,483	0,658	4
Summary	0,703	0,956	0,736	40	0,299	0,369	0,809	40
	Enterprise Tech				Fintech			
2017	0,176	0,185	0,956	16	0,761	0,754	1,009	16
2018	0,202	0,226	0,891	12	0,868	1,037	0,837	12
2019	0,231	0,243	0,950	8	1,016	0,905	1,123	8
2020	0,336	0,411	0,817	4	1,439	1,718	0,837	4
Summary	0,216	0,240	0,898	40	0,933	1,004	0,929	40
	1 - Angel				2 - Seed			
2017	0,994	0,990	1,004	16	0,117	0,129	0,911	16
2018	1,130	1,288	0,878	12	0,110	0,185	0,596	12
2019	1,256	1,417	0,886	8	0,128	0,149	0,864	8
2020	1,524	2,245	0,679	4	0,167	0,259	0,645	4
Summary	1,151	1,340	0,859	40	0,124	0,168	0,737	40
	3 - Early-Stage				4 - Late-Stage			
2017	0,190	0,208	0,913	16	0,317	0,439	0,723	16
2018	0,218	0,247	0,884	12	0,301	0,525	0,573	12
2019	0,253	0,318	0,797	8	0,299	0,429	0,698	8
2020	0,374	0,405	0,924	4	0,443	0,494	0,897	4
Summary	0,236	0,269	0,877	40	0,324	0,470	0,689	40

A.7 Model 3

A.7.1 Out of Sample Forecast - Graph

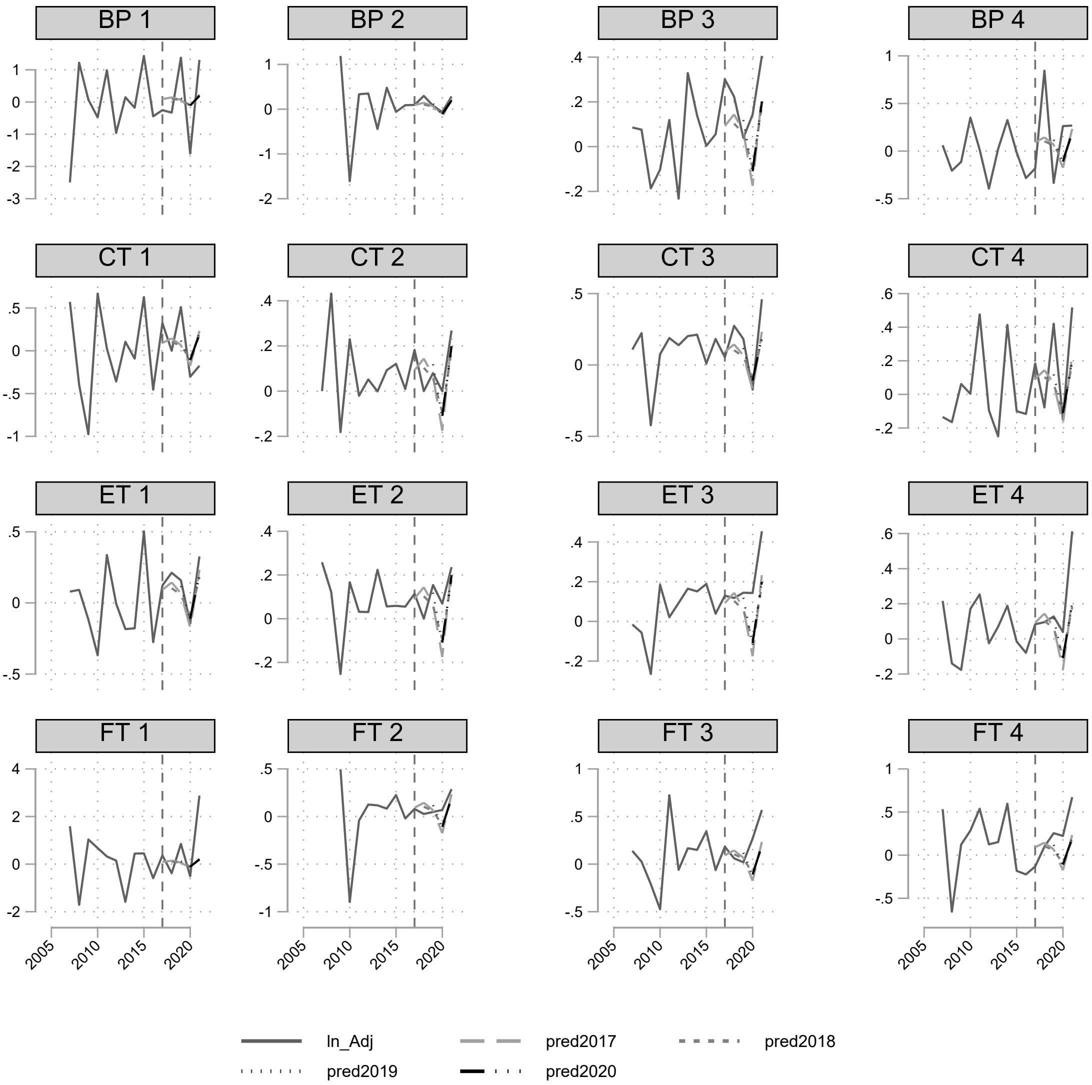

Figure 9: Out of Sample Forecast - Model 3 - Pooled OLS

A.7.2 Out of Sample Forecast by Industry and Stage of Financing

Table 15: Out of Sample Forecast - Model 3 - Pooled OLS - Performance Metrics

	RMSE	RMSE_Alt	U-Theil	N	RMSE	RMSE_Alt	U-Theil	N
	Biotech and Pharma				Consumer Tech			
2017	0,620	0,733	0,845	16	0,222	0,306	0,727	16
2018	0,688	0,891	0,772	12	0,248	0,331	0,750	12
2019	0,677	1,106	0,612	8	0,213	0,463	0,460	8
2020	0,567	1,469	0,386	4	0,280	0,483	0,580	4
Summary	0,648	0,956	0,678	40	0,235	0,369	0,636	40
	Enterprise Tech				Fintech			
2017	0,170	0,185	0,923	16	0,741	0,754	0,983	16
2018	0,187	0,226	0,825	12	0,855	1,037	0,824	12
2019	0,216	0,243	0,889	8	0,997	0,905	1,101	8
2020	0,252	0,411	0,613	4	1,373	1,718	0,799	4
Summary	0,194	0,240	0,809	40	0,909	1,004	0,905	40
	1 - Angel				2 - Seed			
2017	0,920	0,990	0,929	16	0,126	0,129	0,982	16
2018	1,070	1,288	0,831	12	0,093	0,185	0,505	12
2019	1,167	1,417	0,823	8	0,115	0,149	0,775	8
2020	1,463	2,245	0,652	4	0,073	0,259	0,281	4
Summary	1,081	1,340	0,807	40	0,110	0,168	0,659	40
	3 - Early-Stage				4 - Late-Stage			
2017	0,208	0,208	1,000	16	0,325	0,439	0,739	16
2018	0,228	0,247	0,924	12	0,306	0,525	0,583	12
2019	0,271	0,318	0,855	8	0,311	0,429	0,726	8
2020	0,279	0,405	0,690	4	0,354	0,494	0,717	4
Summary	0,236	0,269	0,877	40	0,320	0,470	0,680	40

A.8 Model 4

A.8.1 Out of Sample Forecast - Graph

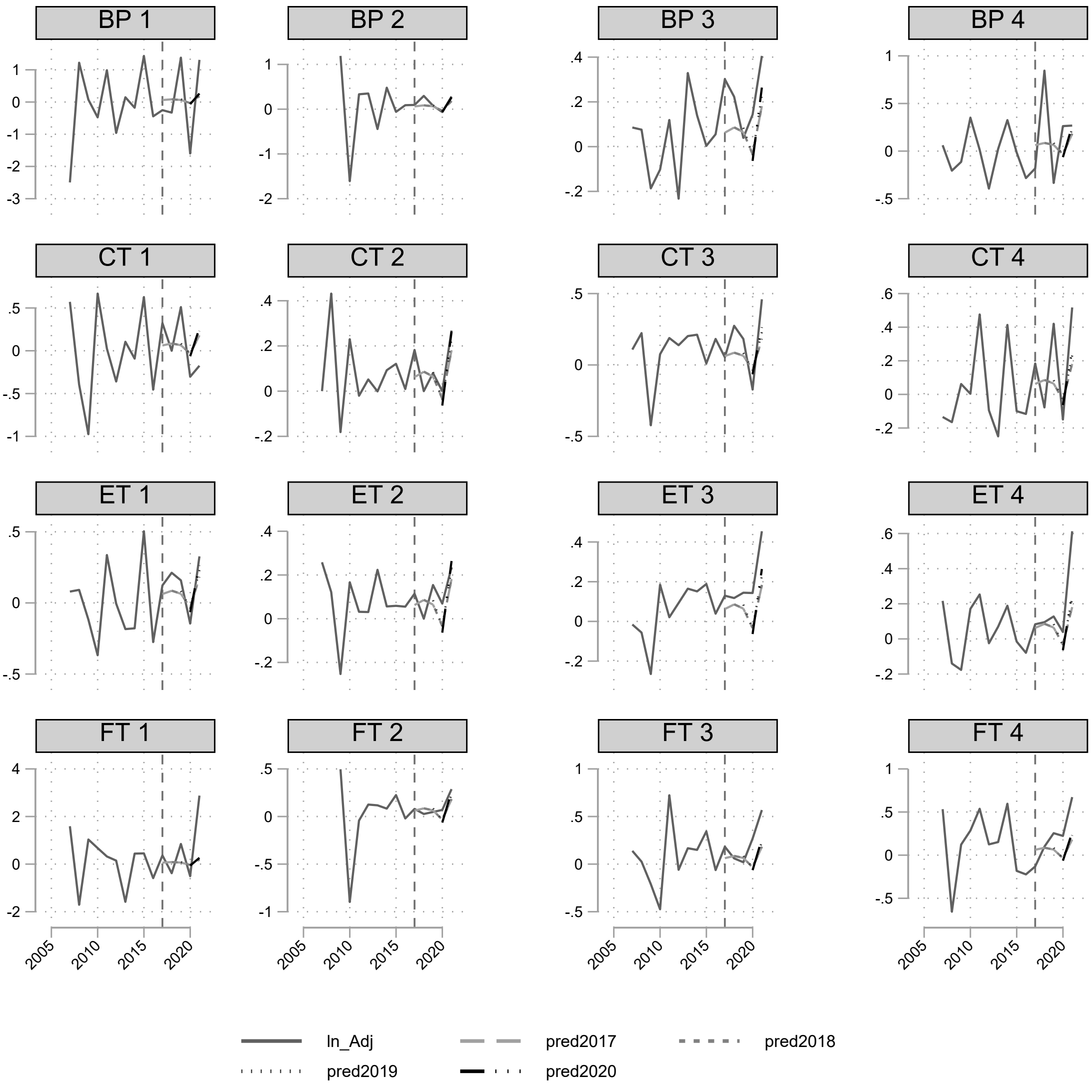

Figure 10: Out of Sample Forecast - Model 4 - Pooled OLS

A.8.2 Out of Sample Forecast by Industry and Stage of Financing

Table 16: Out of Sample Forecast - Model 4 - Pooled OLS - Performance Metrics

	RMSE	RMSE_Alt	U-Theil	N	RMSE	RMSE_Alt	U-Theil	N
	Biotech and Pharma				Consumer Tech			
2017	0,642	0,733	0,876	16	0,230	0,306	0,754	16
2018	0,694	0,891	0,779	12	0,253	0,331	0,765	12
2019	0,685	1,106	0,619	8	0,224	0,463	0,485	8
2020	0,528	1,469	0,359	4	0,272	0,483	0,562	4
Summary	0,657	0,956	0,687	40	0,241	0,369	0,652	40
	Enterprise Tech				Fintech			
2017	0,157	0,185	0,853	16	0,748	0,754	0,992	16
2018	0,175	0,226	0,772	12	0,851	1,037	0,821	12
2019	0,182	0,243	0,750	8	0,982	0,905	1,084	8
2020	0,202	0,411	0,491	4	1,332	1,718	0,775	4
Summary	0,173	0,240	0,718	40	0,901	1,004	0,897	40
	1 - Angel				2 - Seed			
2017	0,949	0,990	0,959	16	0,089	0,129	0,692	16
2018	1,079	1,288	0,838	12	0,072	0,185	0,389	12
2019	1,176	1,417	0,830	8	0,064	0,149	0,429	8
2020	1,425	2,245	0,635	4	0,021	0,259	0,081	4
Summary	1,091	1,340	0,814	40	0,075	0,168	0,446	40
	3 - Early-Stage				4 - Late-Stage			
2017	0,194	0,208	0,932	16	0,321	0,439	0,732	16
2018	0,212	0,247	0,860	12	0,294	0,525	0,560	12
2019	0,230	0,318	0,724	8	0,274	0,429	0,639	8
2020	0,216	0,405	0,534	4	0,297	0,494	0,602	4
Summary	0,209	0,269	0,779	40	0,302	0,470	0,642	40

A.9 Out of Sample Forecast - Performance Metrics

Root Mean Square Error (RMSE)

The RMSE is the standard deviation of the residuals. Its formula is given by the square root of:

$$RMSE^2 = \frac{\sum_t^T (P_t - A_t)^2}{T} \tag{23}$$

Where:

1. A_t corresponds to the actual value in period t;

2. P_t corresponds to the predicted value in period t;

3. T corresponds to the sample size;

The closer this value is to 0, the more accurate the model is. Since the result of this metric depends on the absolute values of the variables studied, it is not possible to predetermine what values would indicate an accurate model. However, comparing different models would allow for the selection of the better choice, that is, the choice which produces the lowest value.[24]

Theil's U

Theil's U is given by the square root of:

$$U^2 = \frac{\sum_t^T (P_t - A_t)^2}{\sum_t^T A_t^2} \tag{24}$$

The results can be interpreted as such:

- $U_{ij} = 0$ indicates the prediction is perfect;

- $0 < U_{ij} < 1$ indicates the prediction is more accurate than the no-change prediction;

- $U_{ij} > 1$ indicates the prediction is less accurate than the no-change prediction;

[24] See, for example, Kenney & Keeping (1962).

A.10 Methodology Summary - Refinitiv VC Research Index

This serves to present the methodology for the Refinitiv Venture Capital Research Index, which measures the performance of the U.S. VC industry through the aggregation of "venture-funded private company values". (Refinitiv 2021) Currently, in all aspects of the methodology which require adjustment of company valuations by time, a sectorial market returns index is used.

Interpolation between valuation events

In the cases where there exist 2 positive valuation events, the monthly interpolation of the valuation of the company in the months between the 2 valuation events is made according to the following formula:

$$\gamma = \frac{\log \frac{v_T/M_T}{V_t/M_t}}{T-t} \tag{25}$$

Where:

1. t is the time of the most recent valuation event;
2. T is the time of the subsequent valuation event;
3. M is an industry specific public market index;
4. v is the pre-money value;
5. V is the post-money value;

Extrapolation after last funding round

The extrapolation after the last funding round of companies until the "present" is made using a linear regression with the monthly return of a market index for the respective sector as a covariate. This goes in line with the best practice proposed by Damodaran (2009).

$$r_C = \alpha + \beta * r_M \tag{26}$$

Where:

1. r_C is the log of the monthly return for the firm
2. r_M is the log of the monthly return for the sectorial market index

Producing the following results:

$$r_C = -0.0122633 + 1.195972 * r_M \tag{27}$$

Consequently, the estimated value of r_C is applied to estimate the difference in valuation between periods s ("present") and $s-1$.

$$V_s = V_{s-1} * e^{r_C} \tag{28}$$

A.11 Distribution of Residuals - Pooled OLS

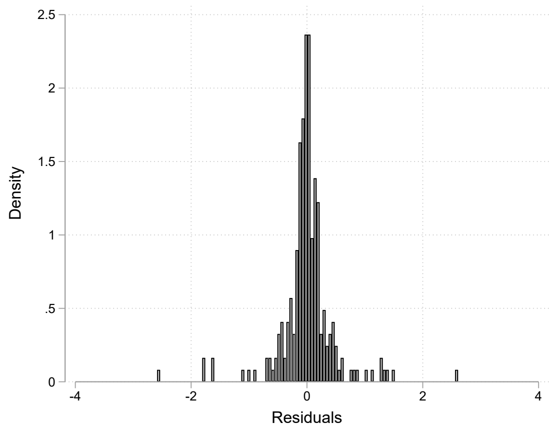

Figure 11: Distribution of Residuals - Model 1 - Pooled OLS

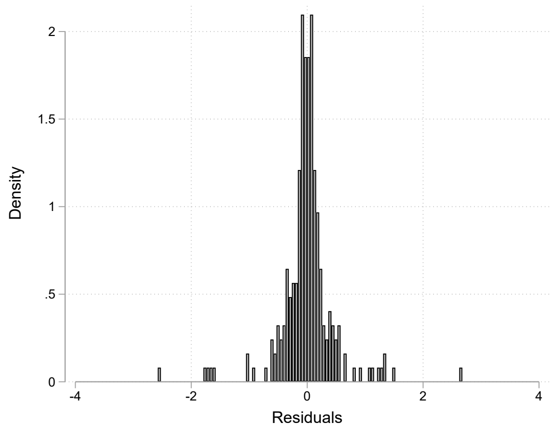

Figure 12: Distribution of Residuals - Model 2 - Pooled OLS

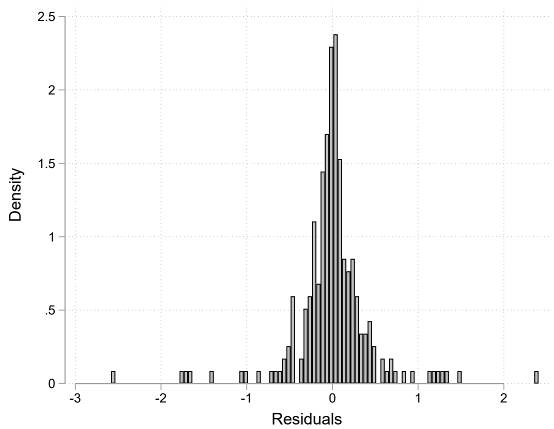

Figure 13: Distribution of Residuals - Model 3 - Pooled OLS

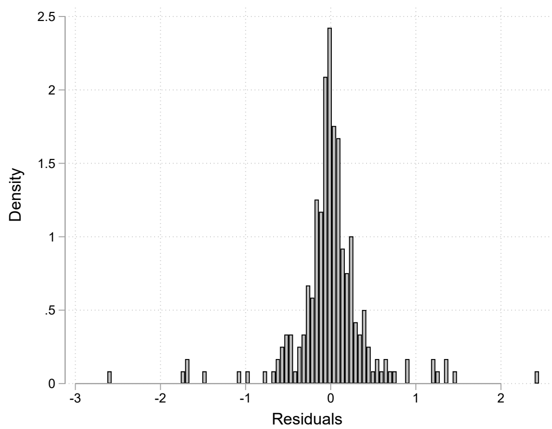

Figure 14: Distribution of Residuals - Model 4 - Pooled OLS