Time-varying Commodity Portfolio Optimization

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Abstract

The commodity market is basically used for hedging against physical products. However, commodity portfolios alone can be an investment choice based on their return and risk characteristics. In order to analyze the interconnectedness among commodities, this study applies multivariate GARCH models of diagonal VECH, diagonal BEKK, and CCC to model the volatility among the selected commodity groups of metal, energy, and agriculture from 1991 to 2023. The empirical results show that the diagonal VECH model with student t distribution is the fittest model. By doing portfolio optimization based on the time-varying conditional statistics of the empirical results, the optimal commodity weights are obtained and then the performance is evaluated over time. Diversified portfolios of metal, energy, and agricultural commodities generally outperform portfolios focusing on mitigating extreme risk which allocates the majority of the proportion to energy commodities. This study emphasizes the importance of diversification in the commodity market to balance return and risk, and the need to adjust risk based on hedge ratios.

Keywords: commodity, portfolio optimization, time-varying volatility

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1. Introduction

Commodities are raw materials that can be classified into metal, energy, and agricultural products. Metal commodities include precious metals and industrial metals. Precious metals, such as gold and silver, have their intrinsic value and are often used for maintaining value over time, while industrial metals, such as copper and iron, are used for industrial purposes. Energy commodities often refer to crude oil and its refinements. Also, agricultural commodities generally include two types, which are crops and livestock commodities (Ielpo, 2018). These commodities' prices are easily influenced by various factors. For example, energy commodities are influenced by heating while agricultural commodities are affected by weather conditions and seasonal demand. In order to avoid the associated risk of these factors, the first commodity futures trading exchange was established in Japan for trading rice futures in 1730 (Dojima Rice Exchange, 2024). The original commodity trading of futures serves as a risk management tool for producers, allowing them to avoid the impact of price changes by transferring risk to speculators.

In modern times, the commodity market has become more like a financial market based on participants' motivation and strategies (Domanski & Heath, 2007). While the original commodity market only has two sides of participants called hedgers and speculators, a third group of transactors which can be viewed as financial investors has emerged and become increasingly important (Gilbert, 2008). This kind of investor views commodity derivatives as one kind of asset based on their return and risk nature. Financial investors often hold long positions and focus on investing in commodity futures. This is because commodity futures have the advantage of their liquidity and standardized contracts, along with minimal counterparty risk, despite other commodity trading ways such as physical commodity purchases, commodities stocks, commodities ETFs, and mutual funds (Ruano & Barros, 2022). Moreover, many investors use commodity futures as a way of hedging the stock market (Alshammari & Obeid, 2023), real estate risk (Raza et al., 2018), or Bitcoin (Joo & Park, 2024).

Focusing on the commodity market, this study aims to get the optimal commodity portfolio weights for financial investors by considering investment objectives and risk tolerance. Economically, the optimization of commodity portfolios facilitates effective investment allocation within the commodity market. Figure 1 presents the structure of the thesis. As for the empirical part of the analysis, this study begins by studying the interconnectedness among commodities, which are classified into three categories: metal, energy, and agricultural. This is done by plotting the returns and analyzing the descriptive statistics. Analyzing these data features is crucial for the subsequent analysis of applying models and constructing portfolios. The second part of the analysis starts by using the multivariate GARCH models, which can model the time-varying volatility during the period from 1991 to 2023. Next, this study builds seven portfolios of equal weight, mean-variance, minimum variance, minimum correlation, maximum Sharpe ratio, maximum Sortino ratio, and minimum CVaR, which can help financial investors get a comprehensive understanding of different investment strategies. By considering different short-sell opportunities and thus using the optimization process, the time-varying weights for different portfolios are obtained. Finally, based on the time-varying performance metrics of the seven optimal portfolios, investors can make investment decisions and adjust accordingly to reduce risk based on different market times.

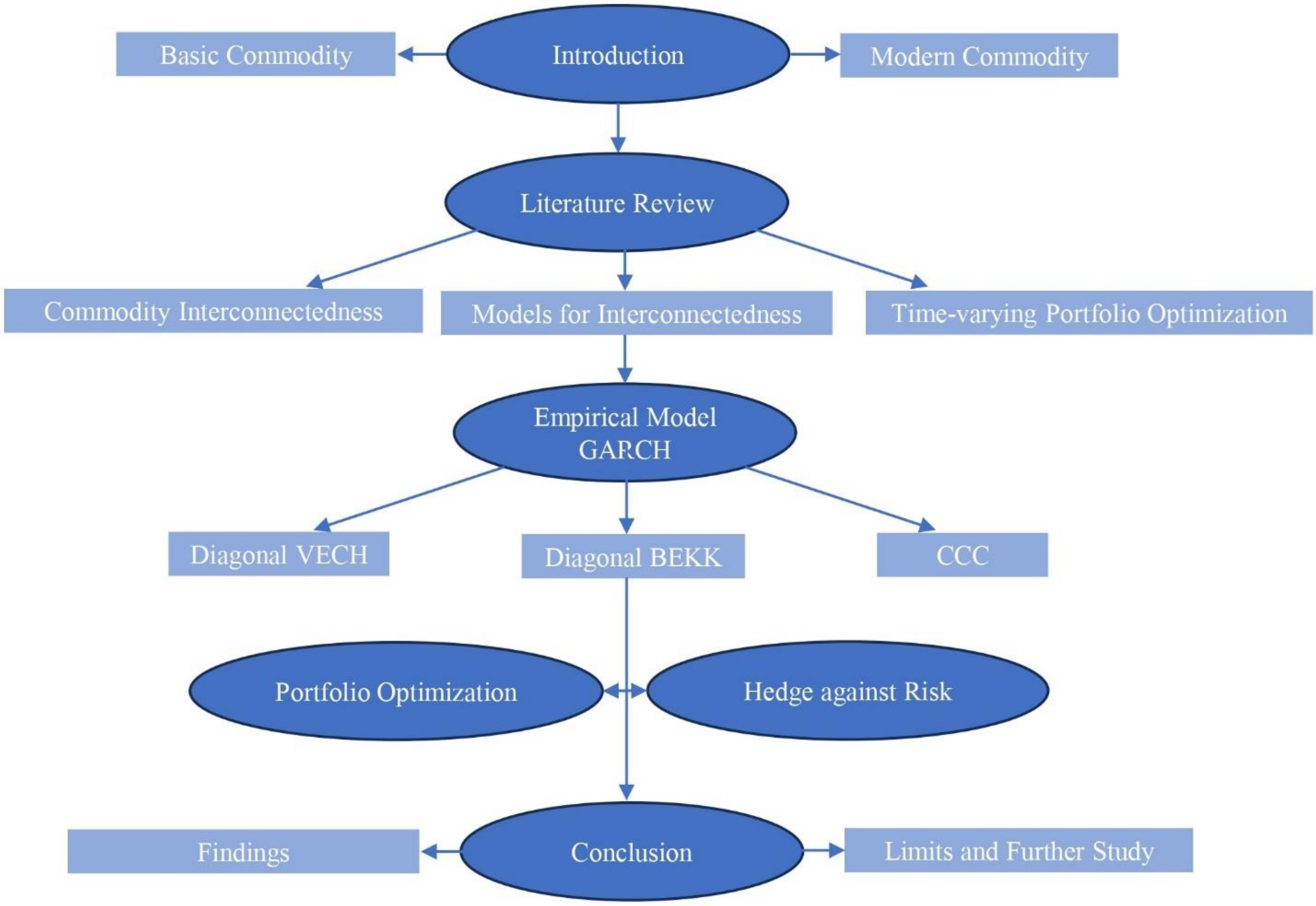

Figure 1: Thesis Structure

2. Literature Review

2.1 Interconnectedness among Commodities

Commodity portfolios can be used for hedging purposes, as different commodities have varying levels of risk and returns. Therefore, analyzing the interconnections between commodity markets and implementing portfolio optimization strategies can help investors optimize their portfolio allocations and mitigate risk. This study focuses on three commodity groups: metal, energy, and agricultural commodities. The aim is to explore the commodities' interrelationships, including metal-energy, metal-agricultural, and energy-agricultural connections, and facilitate the effectiveness of optimal portfolios.

2.1.1 Metal and Energy Commodities

Gold has been recognized as a safe haven asset in hedging against risk and serving as a reliable source of maintaining value during volatile times. Investors often use precious metals such as gold as a hedge to protect against currency depreciation during the slowing down of the economy which is indicated by the increasing oil price (Bedoui et al., 2023). Hammoudeh and Yuan (2008) find that gold and silver may be partial substitutes for hedgers under volatile times. Instead, copper is not suitable for hedging as the transitory volatility vanishes much more rapidly for copper commodities. Also, the comovement of metal and energy commodities becomes negative during financial booms, which brings out the possibility of portfolio diversification (Albulescu et al., 2020). In contrast, Reboredo (2013) finds that gold and oil show a strong positive dependence, suggesting that gold might not effectively hedge against oil price movements. However, they also find tail independence, which suggests that the hedge is effective under bad market times. Sari et al. (2010) also find that there are minor, but positive effects between precious metals and oil. Specifically, if the commodity price of silver increases, it may signal that the price of oil will also increase because silver is highly volatile like oil within days. Similarly, Rehman and Vo (2021) study the cointegration of precious metals and oil, and they find that precious metals and oil prices tend to move together in the long run under both bearish and normal market conditions. As a result, they suggest that investors should avoid building portfolios focusing solely on energy and precious metal commodities due to their high interdependence, especially during extreme market conditions.

2.1.2 Metal and Agricultural Commodities

A large number of researchers study the relationship between metal and agricultural commodities. By studying the dependence structure among energy, agriculture, and metal commodity markets, Albulescu et al. (2020) find asymmetric, high-tail, and low-tail dependence between commodity pairs. Specifically, they find that the relationship between metal and agricultural commodity prices is a V-shape local dependence, which means there are extreme co-movements in both lower and upper tails. This can be explained by the close linkage between metal and agricultural commodities. For example, when the agriculture demand increases, the demand for metal farming equipment will also increase. Similarly, Naeem et al. (2022) study the dynamic patterns of correlation between metal and agricultural commodities. They find that, before the global financial crisis in 2008, the correlation between the two commodities is low. However, there are high correlations during that financially volatile period, and the correlation still remains high after the global financial crisis. With the goal of analyzing investment allocation proportion in the commodity market, Hanif et al. (2023) study the time-varying relationship among commodity markets and find that the metal commodities portfolio is less risky compared to the agricultural commodities portfolio. According to their portfolio optimization results, investors in metal markets should allocate the majority of the commodities to gold, followed by aluminum, platinum, and lead. While for the agricultural market, the greatest amount should be allocated to timber, followed by wheat, cocoa, and soybeans.

2.1.3 Energy and Agricultural Commodities

In recent years, there has been significant cross-market linkage between energy and agricultural markets. This is due to the substitute of bio-fuel for the traditional fossil fuel. Studies show that energy-agricultural commodities connectedness is time-varying and thus there are times that exhibit limited causal effects, such as post-1980s (Shahzad et al., 2021) or during the COVID-19 pandemic times (Furuoka et al., 2023). By analyzing the systematic risk across markets, Kumar et al. (2021) recommend that investors consider choosing other commodities when the energy and agricultural commodity market collapse due to regime-switching risk spillovers. Moreover, when both oil and agricultural commodity markets are going up or declining, it is more likely to experience losses than gains simultaneously. As for the energy and agricultural commodity portfolios, researchers suggest that investors should allocate a lower proportion of energy commodities in order to achieve higher profits based on the performance of dynamic optimal portfolio weights for energy-agricultural commodities portfolios (Furuoka et al., 2023). Shiferaw (2019) studies the relationship between energy prices and agricultural commodity prices focusing on South Africa and finds that the cost of agricultural products is directly impacted by the volatile global oil prices. As a result, the changing oil price causes fluctuations in agricultural commodity prices. Recently, Miljkovic and Vatsa (2023) have shown that crop prices typically exhibit lags to oil prices, although there are times that the reverse is observed. Also, they group similar commodity price movements, which helps them identify two major clusters: one is oil and agricultural commodity prices, and the other is coal and natural gas prices.

2.2 Models for Interconnectedness

This study explores the interconnections among energy, metal, and agricultural commodity markets. Existing articles generally have four empirical methods: GARCH, copula, granger causality, and quantile methods.

2.2.1 GARCH Models

The GARCH model denotes the generalized autoregressive conditional heteroscedastic model, which requires the estimation of conditional variances, covariances, and correlations of multivariate time series (see de Almeida et al., 2018). A wide range of researchers use different GARCH models such as DCC (Shiferaw, 2019), Asymmetric DCC (Raza et al., 2018; Trabelsi et al., 2022), Beta-Skew-t-EGARCH (Gaete & Herrera, 2023), GARCH-MIDAS-X Framework (Yaya et al., 2022) to study the correlation and spillover among commodity markets or their interactions with other markets. The core of these models is to capture the volatility patterns and study how the volatility of one commodity impacts the volatility of others, but they might not be helpful in analyzing the relationship between different commodities. In order to analyze the dependence structure between agricultural commodities and energy prices, Shiferaw (2019) chooses eight commodities and applies the Bayesian multivariate GARCH model with skewness and heavy tails. The researcher finds that the DCC model, which is the dynamic conditional correlation model, with the error-skewed distribution assumption performed better than other competitive methods, such as the CCC model, which is the constant conditional correlation model. Additionally, Gaete and Herrera (2023) use the Beta-Skew-t-EGARCH model to capture the volatility between equities and commodities. By modeling the volatility and the standardized residuals, they can get the score-driven dynamics for solving the mean-variance optimization problem. Similarly, the DCC-MIDAS framework applied by (Yaya et al., 2022) helps investigate the conditional correlations and volatility between oil and precious metal prices. The DCC-MIDAS means the dynamic conditional correlation with mixed data sampling. This framework incorporates information from different frequencies of data, which is efficient when one variable is observed at a higher frequency than the others.

2.2.2 Copula Models

Copula models can characterize the tail dependence structure between returns or risk of different commodities (Albulescu et al., 2020; Kumar et al., 2021; Koirala et al., 2015; Adhikari & Putnam, 2020; Fousekis & Grigoriadis, 2017; Hanif et al., 2023). While copula models have flexibility in modeling different types of dependencies and are particularly effective for handling tail risk which is helpful for understanding how extreme movements in one commodity affect other commodities (see Albulescu et al., 2020), the models can not deal with volatility patterns efficiently. To model the asymmetric dependencies in the tails, Albulescu et al. (2020) use the Gumbel copula for upper-tail and Clayton copula for lower-tail dependence. Also, the rotated versions of both Gumbel and Clayton copulas are considered to capture different dependency structures. Similar to them, Koirala et al. (2015) use a mixture of Clayton and Gumbel copulas. For the dependence parameters estimation, the researchers use the maximum likelihood method. There exist various copula models. Kumar et al. (2021) apply a regime-switching copula approach to model the dependence between oil and agricultural commodity returns. The model considers two regimes of positive and negative correlation. By calculating the transition probabilities between positive and negative correlation regimes, the model helps capture the dynamic and asymmetric nature of market relationships, particularly under extreme conditions.

2.2.3 Granger Causality Test

The time-varying Granger causality test is another way to analyze how commodities impact each other's returns and volatility over different time periods. Similar to the Copula models, the Granger causality test is also not efficient at dealing with volatility. To identify the time-varying causal relationships, Mohamad and Fromentin (2023) apply the time-varying Granger causality methods to energy commodities and ethical investment indices. Additionally, in order to find the instability in causal relationships, forward expanding, rolling, and recursive evolving methods are also employed to incorporate the recursive estimation of Wald statistics from the Vector Autoregressive model, which helps in detecting changes in the intensity and direction of causality over time. Similarly, Shahzad et al. (2021) observe significant fluctuations in causal relationships as time changes and those variations are not always in line with stress time periods. Based on the basic Granger causality model, Meta-Granger addresses the heterogeneity by considering commodity type, sample period, and control variables (Wimmer et al., 2021). Focusing on the relationship between future and spot prices, Joseph et al. (2014) study the bi-directional causalities between futures and spot prices of gold, silver, and crude oil commodities. They use the frequency domain analysis to decompose the causality across different frequencies to capture the dynamics of causality over short, medium, and long-term periods.

2.2.4 Quantile Methods

Various quantile methods are available for assessing interconnectedness. While the quantile methods can capture nonlinearity in dependence between variables (Shafiullah et al., 2021), it does not address the time-varying nature of volatility. In order to examine the directional influence between crude oil and precious metal prices across different quantiles, Shafiullah et al. (2021) combines the Granger causality with the quantile method. The researchers first test for unit roots using the quantile unit root test as introduced by Galvao (2009), and then use the Kuriyama (2016) test to analyze the distributional aspects of quantile cointegration. Finally, they apply the Troster (2018) method to examine Granger causality in quantiles. These series of quantiles are helpful for enhancing robustness against outliers, nonlinearities, and distributional dynamics. Also, a widely used quantile method is the quantile-on-quantile regression, which is useful for estimating correlation in divergent return quantiles (Duan et al., 2023; Naeem et al., 2022). Duan et al. (2023) investigate the linkage between Shanghai crude oil futures prices and WTI crude oil futures prices. The researchers use non-parametric estimation to examine how the quantiles of independent variables affect the conditional quantiles of dependent variables. This quantile-on-quantile regression approach incorporates cross-validation to find a suitable bandwidth, which helps balance the errors and variance during the estimation and thus increases the robustness to capture dynamic patterns of information transmission between markets. Similar to them, Naeem et al. (2022) employ a bivariate quantile-on-quantile regression model to analyze the relationship between oil and gold prices on industrial metals and agricultural commodities across different time periods. The model captures the relationship during bearish, normal, and bullish market phases, representing lower, median, and higher quantiles, respectively.

2.2.5 Empirical Model Summary and Choice

For the models studying the dependence structure among different variables, they have different benefits and limits. The GARCH models can model and forecast the volatility of time-varying data, and numerous empirical studies have demonstrated the effectiveness of GARCH models for modeling the dependence structure. The Copula models are also good at modeling different types of dependencies, especially non-linear dependencies and tail dependence, but they are not designed for modeling volatility and can not be used for forecasting purposes. Similarly, while the Granger causality test and quantile methods can model the time-varying dependence, the volatility pattern can not be captured by these two models. As a result, while there are various models for studying the interconnectedness among commodities, this study chooses multivariate GARCH models, rather than Copula, Granger causality, and quantile models, for modeling the clustering volatility pattern of the commodities. This is due to the intrinsic characteristics of the commodity market, as the commodity products are sensitive to big events or seasonal changes. Details of the chosen multivariate GARCH models are described in the methodology part.

2.3 Time-varying Portfolio Optimization

GARCH models are widely used for portfolio optimization due to their effectiveness in modeling and forecasting volatility. Applying conditional statistics to the portfolio optimization process can help obtain the time-varying weights of different portfolio strategies, which is useful for rebalancing the portfolios over time. Some articles use the conditional covariance matrix as the input for the portfolio optimization process (Abdul Aziz et al., 2019; Siaw et al., 2017). According to Siaw et al. (2017), the covariance matrix is used to optimize the asset weights in portfolios. This includes forecasting the assets' one-day conditional standard deviations and dynamic correlation matrix. Similarly, by taking into account the time-varying nature of market volatility, Abdul Aziz et al. (2019) build five different optimization strategies: mean-variance, maximizing Sharpe ratio, mean-CVaR, and maximizing Sortino ratio. Additionally, The researchers also conduct out-of-sample diagnosis, and the result shows that the dynamic models outperform the static models. Different from them, Specht and Winke (2008) estimate the conditional covariance matrix with the Principal Components GARCH model and then calculate the VaR based on the time-varying statistics. Based on the estimation of conditional volatility and the resulting VaR, they can get the optimized portfolio weights. Apart from analyzing the heteroscedastic conditional variances for the optimal portfolios, Moreno et al. (2005) also use time-varying risk measures that help avoid returns below a certain target and accept returns above it. The applied risk measure is the Lower Partial Moment, which typically involves downside risk assessment (Price et al., 1982).

Apart from portfolio optimization, researchers also use conditional statistics for calculating hedge ratios, especially using the conditional volatility (Kroner & Sultan, 1993; Sadorsky, 2012). Different from the traditional static hedging models, Kroner and Sultan (1993) account for the dynamic nature of asset distributions, which typically involves continuously adjusting the hedge ratios based on the changing market conditions. This is achieved by utilizing the conditional volatility derived from the multivariate GARCH models. The optimal hedge ratio is represented as the divisor form of the numerator of covariance between the primary asset and the hedging asset and the denominator of the hedging asset's variance. Based on their study, Sadorsky (2012) applies the same hedge ratio calculation method for analyzing the correlations and volatility spillovers between commodity markets and stock markets. To improve the dynamic hedging efficiency, Kim and Park (2016) build the conditional shrinkage hedge ratio, which is the convex combination of the conditional hedge ratio from bivariate GARCH models and unconditional hedge ratios. This improvement can provide better risk reduction compared to traditional methods and provide more adaptability to fluctuating market conditions. Additionally, Harris and Shen (2003) also make improvements for the optimal hedge ratios. Their robust optimal hedge ratio uses both the rolling window approach and the exponentially weighted moving average approach, which can take into account the leptokurtosis of returns. Also, this robust optimal hedge ratio can lower the variance of the standard hedge ratio, which is useful for reducing transaction costs within dynamic hedging strategies. As for the evaluation of the efficiency of the hedge ratios, the performance is estimated by using the minimum variance hedge ratio framework introduced by Ederington (1979), which is the same performance evaluation method as the traditional static hedge ratios.

3. Methodology

This section presents the methodologies. First, this study considers using the normal and student t distribution to take into account the error features. Second, the multivariate GARCH models of diagonal VECH, diagonal BEKK, and CCC are constructed by using Eviews, which can model the time-varying volatility. Next, this study optimizes the investment weights based on seven portfolios by using Python. The selected seven portfolio strategies are equal weight, mean-variance, minimum variance, minimum correlation, maximum Sharpe ratio, maximum Sortino ratio, and minimum CVaR. By analyzing their performance, the fittest portfolio strategies over time can be found. Finally, with the goal of reducing risk within the commodity market, the hedge ratios between commodity pairs are calculated.

3.1 Data

This study uses the daily commodity closing price groups of metal, energy, and agriculture from 1991 to 2023 as shown in Appendix A of Table A1. There are 14 individual commodities, each with 6329 days of observations, including gold, silver, palladium, platinum, copper, WTI crude oil, Brent crude oil, natural gas, corn, cocoa, cotton, coffee, lean hogs, and soybeans. The price data is obtained from Refinitiv Eikon. The commodity products are traded on different commodity markets and each market has its own unique focus. The New York Mercantile Exchange serves as the primary trading market for palladium, platinum, WTI crude oil, and natural gas, focusing on the metal and energy sectors. Meanwhile, the Intercontinental Exchange mainly exchanges Brent crude oil, cocoa, cotton, and coffee, which is related to the global energy and agricultural markets. Corn and soybeans are the center products of the Chicago Board of Trade, which focuses on food industries. Additionally, the Chicago Mercantile Exchange concentrates on the lean hogs trading (see Haar & Gregoriou, 2021).

There are high correlations between commodity futures and spot prices (Gulley & Tilton, 2014), and particularly a strong unidirectional relationship from futures to spots for both metal and agricultural commodities (Joseph et al., 2014). However, the choice of commodity price is often the commodity futures rather than the spot price considering the absence of arbitrage (Rad et al., 2022). After acquiring the daily prices of the commodities of the chosen period, the continuously compounded daily returns are obtained by taking the difference in the log of two consecutive prices. The log returns can provide us with a normalized measure of returns and can help reduce the problem of non-stationarity or unit roots during the estimation. The daily return of the commodities is defined as:

$$R_t = ln\frac{P_t}{P_{t-1}} = lnP_t - lnP_{t-1} \tag{1}$$

where P_t is the closing price at time t. By taking the equally weighted average of the individual metal, energy, and agricultural products, the three groups of return data for different commodity markets are obtained for analysis. Through grouping, it is helpful to capture different market dynamics by studying the interconnectedness among these sectors using the multivariate GARCH models. The grouping approach can also identify common dependencies and risk factors across related commodities, which is useful for building optimal portfolios.

3.2 Descriptive Statistic

The data includes metal, energy, and agricultural commodity groups. Figure 2 shows the daily returns of the metal, energy, and agriculture commodities. Metal commodities experienced significant fluctuations around 2005. This might be caused by the development of emerging markets, such as China and India, which require a surge in demand for metals, especially copper, due to their industrial construction. For the energy commodities, the variations at the beginning of 1991 and 2020 might be due to the early 1990s recession and the COVID-19 pandemic, respectively. Additionally, the energy commodities show other significant variations around 2022. These are most likely attributed to the economic impact of the Russia-Ukraine War. The war's influence on global energy supplies, especially oil and natural gas, caused volatility in the energy markets. The agricultural commodities show a pattern of seasonal volatility. These variations are caused by seasonal factors such as weather conditions, which can have a significant impact on agricultural yields. In general, metal and agricultural commodities show more stability compared to energy commodities, while metal and energy commodities exhibit more outliers.

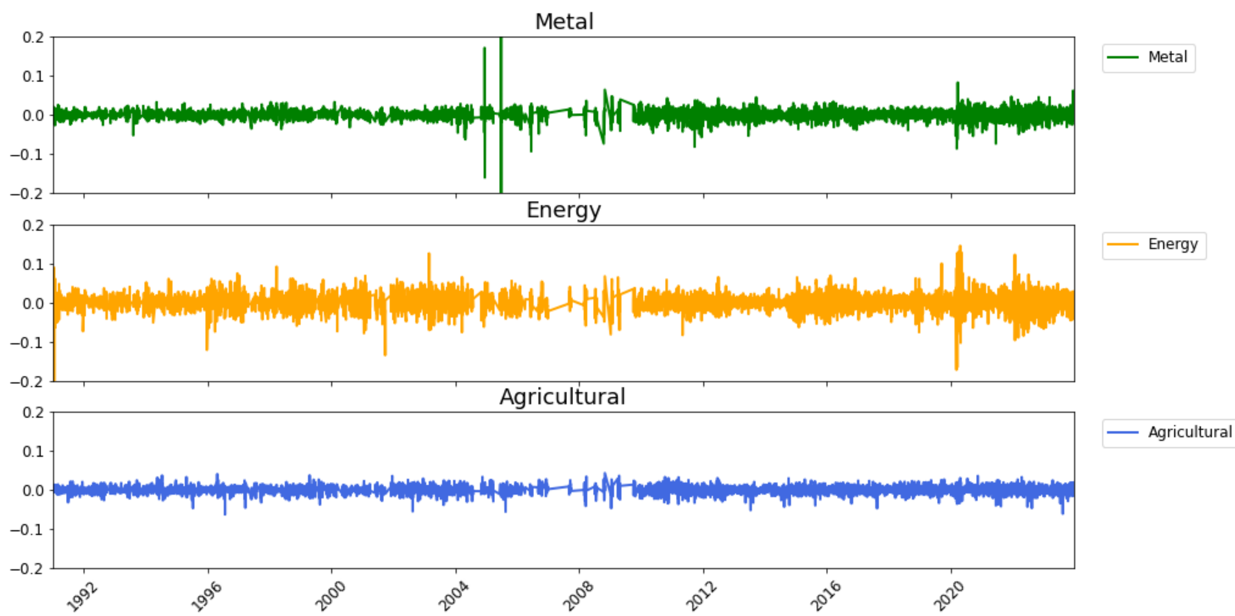

Figure 2: Commodity Return from 1991 to 2023

Table 1 presents descriptive statistics for the three commodity groups of metal, energy, and agriculture. For the three commodity groups, metal and energy commodities display higher returns, with energy having the highest at 0.02%, followed by metal at 0.01%, and agricultural with the lowest at 0.00%. Additionally, energy commodity shows the highest standard deviation of 2.07%, which corresponds to its highest mean return. Conversely, agricultural commodity displays the lowest volatility at 0.94%. All the commodities groups exhibit slightly negative skewness, which indicates that the data is left-skewed and has asymmetry features. Also, the metal commodity shows an extremely high kurtosis of 44.61, which suggests the presence of heavy tails and implies a higher risk of extreme values. Energy also has a high kurtosis of 11.27. Different from metal and energy commodities, agricultural commodities have a kurtosis of 2.63, which indicates that it is likely well approximated by a normal distribution.

Table 1: Descriptive Statistics

	Mean	**Std.Dev**	**Min.**	**Max.**	**Skewness**	**Kurtosis**
Metal	0.01%	1.26%	-22.38%	22.58%	-0.36	44.61
Energy	0.02%	2.07%	-29.55%	14.49%	-0.53	11.27
Agricultural	0.00%	0.94%	-6.30%	4.32%	-0.41	2.63

Note: This is the descriptive statistics of metal, energy, and agricultural commodity future returns. A skewness value around 0 indicates symmetry, with a negative skewness indicating left-skewed and a positive skewness indicating right-skewed. A kurtosis value around 3 indicates normal distribution, with a lower value indicating lighter tails and a higher value indicating heavy tails.

Some statistical tests are conducted for the three commodity group return data, which are shown in Table 2. This study employs Jarque-Bera tests, and the extremely high JB test statistics for all the commodities indicate that there is no normality for the dataset. Furthermore, the study applies the Augmented Dickey-Fuller test to test for stationarity. The significant negative values of the ADF statistics suggest that the data is stationary, indicating that it does not have significant trends over time. Similarly, the supplementary KPSS test, which is the Kwiatkowski-Phillips-Schmidt-Shin unit root test, can not reject the null hypothesis of stationary, indicating that the data series is centered around a deterministic trend. This study uses the Ljung-Box test to diagnose if there is autocorrelation for the commodities. The LBQ results for lag 15 show that metal and energy commodities have autocorrelation at a 10% significance level, while agricultural commodity has autocorrelation at a 1% significance level, indicating the presence of dependence over time for these commodities. To test if the GARCH models are fit for the dataset, this study conducts Engle's Lagrange Multiplier test (Engle, 1982) to see if the data has ARCH effects. The p-value of the LM test strongly rejects the null hypotheses, which show the conditional heteroscedasticity features of commodity returns. In summary, the data generally shows no normality, stationary, autocorrelation, and heteroscedasticity characteristics.

Table 2: Statistical Tests

	JB	**ADF**	**PP**	**KPSS**	**LBQ**	**LM**
Metal	524814.92***	-83.47***	-84.54***	0.02	22.74*	1952.31***
Energy	33814.35***	-80.36***	-80.35***	0.07	22.65*	407.00***
Agricultural	2011.31***	-28.13***	-75.08***	0.07	39.67***	93.77***

Note: These are the statistical tests for metal, energy, and agricultural commodity future returns. JB denotes the Jarque-Bera normality test, with H0: data is normally distributed. ADF denotes the Augmented Dickey-Fuller unit root test, with H0: data has a unit root, indicating non-stationary. PP denotes the Phillips-Perron unit root test, with H0: data has a unit root, indicating non-stationary. KPSS denotes the Kwiatkowski-Phillips-Schmidt-Shin unit root test, with H0: data is stationary around a deterministic trend. LBQ denotes the Ljung-Box test for autocorrelation using lag 15, with H0: data has no autocorrelation. LM denotes the White's Lagrange Multiplier test for Heteroskedasticity, with H0: data has no heteroskedasticity. The asterisk ***, **, and * indicate the significance at 99%, 95%, and 90% level.

The correlation analysis examines the relationships among metal, energy, and agricultural commodities. As shown in Table 3, The correlation between metal and energy commodities is 0.19, while the correlation between energy and agricultural commodities is 0.17. The correlation between metal and agricultural commodities shows the highest of 0.24. The relatively low correlation coefficients for all the commodity group pairs imply less comovement between different commodity groups. Also, the weak correlation indicates the possibility of diversification to mitigate risk. As for the hedging opportunity, the weak correlation offers potential for hedging strategies, but the effectiveness is limited. In general, the correlation between the commodity group pairs shows a positive and weak correlation.

Table 3: Commodity Correlation

	Metal	**Energy**	**Agricultural**
Metal	1.00	0.19	0.24
Energy	0.19	1.00	0.17
Agricultural	0.24	0.17	1.00

3.3 Multivariate GARCH Models

This study uses the normal distribution and student t distribution to model the error distribution of returns. In order to model the time-varying volatility, the multivariate GARCH models are applied to account for heteroscedasticity and assist in portfolio management. The multivariate GARCH models for studying the time-varying dynamics among commodities include the diagonal VECH, diagonal BEKK, and CCC, which denote the diagonal vectorized conditional heteroscedasticity, diagonal Baba, Engle, Kraft, and Kroner, and constant conditional correlation GARCH models, respectively. For setting up the models, the total number of assets is defined to be N, with each individual commodity denoted as i or j. The total length of the sample period is T, with each day represented as t. The order of the GARCH models is shown as p and q, which means the autocorrelation and moving average terms, respectively.

For the general framework of the multivariate GARCH models, the selected models of VECH, BEKK, and CCC all have the same form of the mean equation and the standardized residual equation, which are as follows:

$$\boldsymbol{R_t} = \boldsymbol{\mu_t} + \boldsymbol{\epsilon_t} \tag{2}$$

$$\boldsymbol{\epsilon_t} = \sqrt{\boldsymbol{H_t}} \times \boldsymbol{z_t} \tag{3}$$

In the mean equation (2), $\boldsymbol{R_t}$ is the actual commodity return for each commodity group, $\boldsymbol{\mu_t}$ is the expected conditional mean, and $\boldsymbol{\epsilon_t}$ is the residuals. $\boldsymbol{z_t}$ is a random variable that follows a normal or student t distribution. In the standardized residual equation (3). $\boldsymbol{H_t}$ is the $N \times N \times T$ conditional variance-covariance matrix.

For the estimation part of these GARCH models, this study uses the log-likelihood function, which is defined as follows:

$$l(\theta) = -\frac{T \times N}{2} log2\pi - \frac{1}{2}\sum_{t=1}^{T}(log|\boldsymbol{H_t}| + \boldsymbol{\epsilon_t H_t^{-1} \epsilon_t'}) \quad (4)$$

where θ is the parameters to be estimated. In order to obtain $\theta,$ the log-likelihood function $l(\theta)$ needs to be maximized. By using the numerical optimization method BFGS, which is the Broyden–Fletcher–Goldfarb–Shanno algorithm of the quasi-Newton method, this study can do non-linear optimizations by doing the approximation of the inverse Hessian matrix (Liu & Nocedal, 1989). Apart from that, the Marquardt Steps which is the Levenberg–Marquardt Algorithm is also used for adding the robustness of the optimization (Lourakis, 2005). The approach modifies the Gauss-Newton direction by adding a dampening parameter, which can provide a balance between the stability of gradient descent and the efficiency of the Gauss-Newton method. These methods are conducted by Eviews' built-in function automatically.

3.3.1 Diagonal VECH

The first multivariate GARCH model is the VECH model, which is useful for modeling variances and covariances among multiple time series data (Bollerslev et al., 1988). However, the VECH model has limits on its extremely large numbers of estimated parameters even under small dimensions, and the condition of positive-definite conditional covariance matrices needs to be fulfilled (de Almeida et al., 2018). As a result, the restricted VECH model of the diagonal VECH is more fitted in practice as it can deal with those limits (Bollerslev et al., 1988; de Almeida et al., 2018). The variance-covariance equation of the diagonal VECH model is as follows:

$$VECH(\boldsymbol{H_t}) = \boldsymbol{\omega} + \sum_{k=1}^{p} \boldsymbol{A_k} VECH(\boldsymbol{\epsilon_{t-k}\epsilon_{t-k}'}) + \sum_{l=1}^{q} \boldsymbol{B_l} VECH(\boldsymbol{H_{t-l}}) \quad (5)$$

where $\boldsymbol{H_t}$ is the conditional variance-covariance matrix of the residuals. In this equation, $VECH$ is the operator that stretches the lower triangular part of a matrix including the diagonal into a vector form. $\boldsymbol{\omega}$ is the vector with the dimension of $\frac{N\times(N+1)}{2}$, while $\boldsymbol{A_k}$ and $\boldsymbol{B_l}$ are the parameters of square $\frac{N\times(N+1)}{2}$ matrices to be estimated. For the diagonal restriction, $\boldsymbol{A_k}$ and $\boldsymbol{B_l}$ become diagonal square $\frac{N\times(N+1)}{2}$ matrices, which means that the off-diagonal elements are constrained to be zero. This indicates that one individual commodity's conditional variance will be determined only by its own autocorrelation and moving average, not by other commodities. The diagonal VECH model will estimate the total number of $(p + q + 1) \times \frac{N\times(N+1)}{2}$ parameters. For example, the number of estimated parameters of the diagonal VECH $(1, 1)$ with 3 commodities is $(1 + 1 + 1) \times \frac{3\times(3+1)}{2} = 18.$

3.3.2 Diagonal BEKK

The BEKK model is named the Baba, Engle, Kraft, and Kroner GARCH model (Engle & Kroner, 1995). This model is a more restricted form of the VECH model. The variance-covariance equation is shown as follows:

$$\boldsymbol{H_t} = \boldsymbol{\omega}\boldsymbol{\omega}' + \sum_{k=1}^{p} \boldsymbol{A'_k}\boldsymbol{\epsilon_{t-k}}\boldsymbol{\epsilon'_{t-k}}\boldsymbol{A_k} + \sum_{l=1}^{q} \boldsymbol{B'_l}\boldsymbol{H_{t-l}}\boldsymbol{B_l} \tag{6}$$

where $\boldsymbol{\omega}$, $\boldsymbol{A_k}$, and $\boldsymbol{B_l}$ are the $N \times N$ parameter matrices. The constant matrix $\boldsymbol{\omega}$ is the lower triangular matrix, which contains parameters with the number of $\frac{N\times(N+1)}{2}$. Similar to the diagonal VECH model, the diagonal BEKK model also has the diagonal form of $\boldsymbol{A_k}$ and $\boldsymbol{B_l}$. As the off-diagonal elements are constrained to be zero, $\boldsymbol{A_k}$ and $\boldsymbol{B_l}$ contains an equal number of estimated parameters of N. As a result, the total number of estimated parameters is $(p + q) \times N + \frac{N\times(N+1)}{2}$. For example, the number of estimated parameters of 3 commodities using BEKK $(1, 1)$ is $(1 + 1) \times 3 + \frac{3\times(3+1)}{2} = 12$.

3.3.3 CCC

The CCC model is the constant conditional correlation model where the conditional correlation is assumed to be constant while the conditional variances are time-varying (Bollerslev, 1990). The CCC model uses the univariate GARCH to get the conditional variance, the variance equation is defined as follows:

$$h_{i,t} = \omega_i + \sum_{k=1}^{p} a_{i,k}\epsilon_{i,t-k}^2 + \sum_{l=1}^{q} b_{i,l}h_{i,t-l} \tag{7}$$

where $h_{i,t}$ is the conditional variance of each individual commodity return at time t, which is the expected volatility based on information available at previous times. In this variance equation, ω_i ,$a_{i,k}$ and $b_{i,l}$ are the parameters to be estimated based on each commodity return separately, where ω_i measures the long-term average variance, $a_{i,k}$ captures the effect of lagged past shocks, and $b_{i,l}$ reflects the persistence of past conditional variances. $\epsilon_{i,t-k}^2$ are the squared innovation at time $t - k$. Based on the conditional variance equation (7), the conditional covariance equation (8) and the combination of the variance-covariance equation (10) are defined as follows:

$$h_{ij,t} = \rho_{ij}\sqrt{h_{i,t} \times h_{j,t}} \tag{8}$$

$$\boldsymbol{D_t} = diag\left\{\sqrt{h_{i,t}}\right\} \tag{9}$$

$$\boldsymbol{H_t} = \boldsymbol{D_t}\boldsymbol{R}\,\boldsymbol{D_t} \tag{10}$$

where $\boldsymbol{H_t}$ is the conditional variance-covariance matrix of residuals, $\boldsymbol{D_t}$ is the diagonal form of conditional variance. $\boldsymbol{R}$ is the constant correlation matrix of symmetric $N \times N$ with the elements of ρ_{ij}. As a result, the total parameters of the CCC model to be estimated is $(p+q+1) \times N + \frac{N\times(N-1)}{2}$. With 3 commodities' return data fitting into the CCC-GARCH $(1, 1)$ model, the estimated number of parameters is $(1+1+1) \times 3 + \frac{3\times(3-1)}{2} = 12$.

3.4 Time-varying Portfolio Optimization

By considering different risk and return objectives. This study constructs seven portfolios for optimization: equal weight, mean-variance, minimum variance, minimum correlation, maximum Sharpe ratio, maximum Sortino ratio, and minimum CVaR. The equal weight portfolio allocates the same weight for the three commodity groups and serves as the benchmark portfolio. Mean-variance portfolio balances the expected return against variance, while the minimum variance and minimum correlation focus on risk reduction and diversification, respectively. Also, maximum Sharpe ratio portfolios focus on risk-adjusted performance, while maximum Sortino ratio portfolio aims to deal with downside risk. Considering the extreme risk scenarios, the minimum CVaR portfolio seeks to minimize extreme risk.

3.4.1 Equal Weight Portfolio

The equal weight portfolio is built by setting $w_i = \frac{1}{N}$, where w_i is the weight of the commodity i, and N is the number of commodity types in the portfolio. This portfolio is not rebalanced through time, and it serves as a benchmark for all other portfolios.

3.4.2 Mean-Variance Portfolio

The basic portfolio optimization method is the mean-variance method developed by Markowitz (1952), in which investors seek to find the balance between maximum returns and minimum volatility of the portfolios. By assuming no short-selling opportunities, we can build a quadratic optimization process by using the time-varying variance-covariance matrix and the expected mean return (see Ghaemi Asl et al., 2024):

$$Max \left\{\boldsymbol{w^T E}(\boldsymbol{R}) - \frac{\gamma}{2}\boldsymbol{w^T H_t w}\right\}, subject\ to \sum_{i=1}^{N} w_{it} = 1\ and\ \mathrm{w}_{it} > 0 \tag{11}$$

where γ is the risk aversion coefficient, $\boldsymbol{H_t}$ is the time-varying conditional variance-covariance matrix with the dimension of $N \times N \times T$, $\boldsymbol{E}(\boldsymbol{R})$ is the fixed expected mean return for each commodity group calculated as the average of each commodity's historical data, and $\boldsymbol{w}$ is the optimal time-varying weight matrix, with elements of w_{it} as the weight of each commodity's proportion at time t.

3.4.3 Minimum Variance Portfolio

The minimum variance portfolio is built to reduce the investor's risk by minimizing the portfolio's variance. This is achieved by selecting the combination of the assets whose returns have the lowest overall variance, considering each individual commodity's volatility. During market volatile times, investors might prefer the minimum variance portfolio as their goal is to preserve their asset values by focusing on minimizing risk rather than maximizing returns. This portfolio can provide a conservative investment strategy that suits the risk tolerance level of certain investors. The function is defined as follows:

$$Min\ \{\boldsymbol{w^T H_t w}\}, subject\ to\ \sum_{i=1}^{N} w_{it} = 1\ and\ \mathrm{w}_{it} > 0 \tag{12}$$

where $\boldsymbol{H_t}$ is the time-varying conditional variance-covariance matrix.

3.4.4 Minimum Correlation Portfolio

This study creates the minimum correlation portfolio based on the time-varying conditional correlation matrix. The time-varying conditional correlation matrix is calculated based on the time-varying variance-covariance matrix. The correlation between assets has a significant influence on the risk and return of the portfolio. If assets are highly positively correlated, when one asset suffers losses, others may also be affected, thereby increasing the overall risk of the portfolio. Conversely, if assets exhibit a negative correlation or low correlation, it may provide some degree of hedging when some assets perform poorly, thereby reducing the overall risk of the portfolio. The calculation of the time-varying correlation equation and minimum correlation portfolio function are defined as follows:

$$\boldsymbol{H_t} = \boldsymbol{D_t R_t D_t} \tag{13}$$

$$Min\ \{\boldsymbol{w^T R_t w}\}, subject\ to\ \sum_{i=1}^{N} w_{it} = 1\ and\ \mathrm{w}_{it} > 0 \tag{14}$$

where $\boldsymbol{D_t}$ is the diagonal form of conditional volatility as shown in Equation (9), $\boldsymbol{R_t}$ is the time-varying conditional correlation matrix.

3.4.5 Maximum Sharpe Ratio Portfolio

The maximum Sharpe ratio aims to optimize the ratio of the portfolio's excess return to its volatility, thus maximizing risk-adjusted return. The function for the maximized Sharpe ratio with a fixed risk-free rate is as follows:

$$Max\ \left\{\frac{\boldsymbol{E}(\boldsymbol{R}) - R_f}{\sqrt{\boldsymbol{w^T H_t w}}}\right\}, subject\ to\ \sum_{i=1}^{N} w_{it} = 1\ and\ \mathrm{w}_{it} > 0 \tag{15}$$

where R_f is the fixed risk-free rate of 0.02%. By using the fixed risk-free rate, the consistency across different optimizations is guaranteed. $E(R)$ is the fixed expected mean return for each commodity group calculated as the average of each commodity's historical data and $\boldsymbol{H_t}$ is the time-varying conditional variance-covariance matrix.

3.4.6 Maximum Sortino Ratio Portfolio

The maximum Sortino ratio is similar to the maximum Sharpe ratio, but it focuses on downside risk rather than the total risk of assets, making it more suited for portfolios that aim to prevent losses rather than maximize risk-adjusted returns.

$$Max\left\{\frac{\boldsymbol{E}(\boldsymbol{R})-R_f}{\sqrt{\boldsymbol{w}^T\boldsymbol{S_t}\boldsymbol{w}}}\right\}, subject\ to\ \sum_{i=1}^{N} w_{it} = 1\ and\ \mathrm{w}_{it} > 0 \tag{16}$$

where $\boldsymbol{S_t}$ is the downside risk matrix, which considers only the variance-covariance of returns falling below the risk-free rate of 0.02%.

3.4.7 Minimum CVaR Portfolio

The minimum CVaR portfolio, which is the minimum conditional value-at-risk portfolio, focuses on reducing the extreme losses during the sample period. During the optimization process, VaR is calculated using the historical simulation as no assumptions of distribution are required for this non-parametric method. The Conditional VaR is also called the expected shortfall and it is calculated as the average losses above the VaR threshold. The minimum CVaR is calculated as follows:

$$VaR_\alpha = inf\{l \in \mathbb{R}: P(L > l) \leq 1-\alpha\} = inf\{l \in \mathbb{R}: F_L(l) \geq \alpha\} \tag{17}$$

$$CVaR_\alpha = E(R|R \leq VaR_\alpha) \tag{18}$$

$$Min\ \{CVaR_\alpha\}, subject\ to\ \sum_{i=1}^{N} w_{it} = 1\ and\ \mathrm{w}_{it} > 0 \tag{19}$$

where α represents the confidence level for calculating the VaR and is set to 95%. Also, the function inf represents the infimum of the set of numbers of losses l and the function F_L denotes the cumulative distribution function.

3.5 Hedge Ratios

The commodity market is highly volatile, which can be caused by a range of factors. For metal commodities, industrial metal prices can fluctuate due to changes in industry demand, mining production levels, and geopolitical events. Energy prices, such as oil and gas, are influenced by wars and heating. Agricultural commodities are particularly sensitive to seasonal changes and weather conditions, which will affect crop yields and livestock production. One effective way to reduce the above risk is hedging. By focusing on commodity futures, this study can analyze inter-commodity hedging. The primary benefit of inter-commodity hedging is diversification. By spreading risk across different but related commodities, investors can reduce the impact of price volatility in the commodity future market. This study uses the conditional volatility from the multivariate GARCH models to construct the hedge ratios (Kroner & Sultan, 1993).

$$\beta_{ij,t} = \frac{h_{ij,t}}{h_{j,t}} \qquad (20)$$

where $\beta_{ij,t}$ is the hedge ratio over time between primary asset i and hedging asset j. $h_{ij,t}$ are the conditional covariance between asset i and asset j, while $h_{j,t}$ is the conditional variance of the returns of asset j at time t. This hedge ratio normalizes the conditional covariance, which is useful for determining the size of the hedge position. When the conditional covariance between the two commodities is high and the variance of the hedging commodity is low, the hedge ratio will be higher. This means that investors need a larger position in the hedging commodity to effectively hedge the risk associated with the primary commodity.

4. Results

This section presents the empirical results by using the returns of the three commodity groups of metal, energy, and agriculture from 1991 to 2023. This study obtains the estimated parameters of the multivariate GARCH model: diagonal VECH, diagonal BEKK, and CCC. By using the time-varying volatility and correlation of these multivariate models, the portfolios can be optimized and thus the time-varying weights for the commodities groups are obtained. By calculating the portfolio performance of return, variance, and CVaR, this study compares the portfolios over time. Additionally, in order to hedge against the risk associated with the commodities, the hedge ratios are calculated for commodity pairs. In conclusion, the presented result will be the multivariate GARCH results, average optimal weights, portfolio performance, and hedge ratios.

4.1 Multivariate GARCH Models

As the data displays asymmetry and fat tail features, this study chooses different distributions of normal and student t distribution to fit the errors. The multivariate GARCH models that are used for comparison are diagonal VECH, diagonal BEKK, and CCC. The input data is the three commodity group data of metal, energy, and agriculture, each with a period of 6329 observations from 1991 to 2023.

4.1.1 GARCH Model Results

By inputting the three commodity group return data into the multivariate GARCH models of diagonal VECH, diagonal BEKK, and CCC, the estimated parameters obtained are displayed in the Appendix B of Tables B1, B2, and B3. Within the mean equation, the coefficient $C\ (i)$ denotes the estimated parameters, in which $C\ (1)$, $C\ (3)$, and $C\ (5)$ are the constant terms for each commodity group, and $C\ (2)$, $C\ (4)$, and $C\ (6)$ are lagged values of each commodity. For example, $C\ (6)$ of the agricultural commodity's constant term is significant at a 95% confidence level across all models, indicating a strong effect on the mean returns. As different GARCH models model the volatility in different ways, the parameters of the mean equation are different across models. For the variance equation, M denotes the constant terms, $A1$ is the ARCH term that captures the short-term effects, and $B1$ is the GARCH terms reflecting the long-term effects. For the CCC model, the $R\ (i,j)$ is the correlation coefficient between commodities. For example, $R\ (1,2)$ denotes the correlation between metal and energy commodities. While this study does not directly use the estimated parameters of the GARCH models for forecasting purposes, these parameters are helpful in analyzing how past returns and volatilities of one commodity affect the current period returns and volatilities of other commodities.

As shown in Table 4, the criteria used for comparison are the log-likelihood, AIC, and BIC. The AIC is Akaike Information Criterion and BIC is the Bayesian Information Criterion. Higher values of log-likelihood, along with lower values of AIC and BIC, indicate better goodness of fit. Under the normal distribution assumption, the diagonal VECH model estimates a total of 24 parameters and has the highest log-likelihood of 56886.42. It also has the lowest AIC and BIC values, indicating its best fit within the normal distribution assumption. The diagonal BEKK and CCC models have lower log-likelihoods, indicating less goodness of fit. When using the student t distribution, the results will get one more estimated parameter compared to the normal distribution.

This additional estimated parameter is the degree of freedom. Under this assumption, the diagonal VECH also achieves the highest log-likelihood of 57459.74 among other student t distribution models. Similarly, the diagonal BEKK and CCC models still have lower log-likelihood values, along with higher AIC and BIC values. Finally, by comparing the results of normal and student t distributions, it can be seen that the models assuming student t distribution generally show better goodness of fit compared to models assuming normal distribution. These results align with the data features of the high kurtosis observed in metal and energy commodities because the student t distribution is better at handling heavy tails and extreme values compared to the normal distribution.

Table 4: Multivariate GARCH (1, 1) model results

		Num_para	**Log-likelihood**	**AIC**	**BIC**
Norm	VECH	24	56886.42	-17.97	-17.95
	BEKK	18	56837.02	-17.96	-17.94
	CCC	18	56798.86	-17.95	-17.93
T	VECH	25	57459.74	-18.15	-18.13
	BEKK	19	57415.96	-18.14	-18.12
	CCC	19	57406.78	-18.14	-18.12

Note: Norm is the normal distribution, and t is the student t distribution. Num_para is the number of parameters to be estimated during the estimation process. AIC is Akaike Information Criterion and BIC is Bayesian Information Criterion. Higher values of log-likelihood, along with lower values of AIC and BIC, indicate better goodness of fit.

4.1.2 Diagnostic Tests

Table 5 presents the results of diagnostic tests for standardized residuals using the Ljung-Box test with 15 lags. The tests evaluate the correlation and covariance matrices of residuals under normal and student t distributions for the multivariate GARCH models. Across all models, the P-values are relatively high, with none below 0.10, indicating that there is no significant autocorrelation in the standardized residuals. Models under the student t distribution generally show slightly higher P-values, implying a marginally better fit in handling autocorrelation compared to those under the normal distribution. This supports the effectiveness of the models in capturing the autocorrelation present in the original data, validating their performance.

Table 5: Diagnostic Tests for Standardized Residuals

		Cor		**Cov**	
		Q-Stat	P-value	Q-Stat	P-value
Norm	VECH	151.80	0.15	150.42	0.17
	BEKK	154.36	0.12	153.38	0.13
	CCC	147.96	0.21	147.90	0.21
T	VECH	145.87	0.25	144.86	0.27
	BEKK	151.85	0.15	151.13	0.16
	CCC	141.67	0.33	141.71	0.33

Note: The table is the result of the Ljung-Box test with lags of 15. Norm is the normal distribution, and t is the student t distribution. Cor and Cov denote the square root of the correlation and covariance matrix of residuals.

4.2 Optimal Portfolio Weights

After using the multivariate GARCH models, the time-varying conditional covariance matrix $\boldsymbol{H_t}$ and correlation matrix $\boldsymbol{R_t}$ with the dimension of $N \times N \times T$ are obtained. Based on the loglikelihood, AIC, and BIC, the diagonal VECH model with student t distribution is the most suitable model for modeling commodity volatility and dependence. By applying the seven portfolio methods: equal weight, mean-variance, minimum variance, minimum correlation, maximum Sharpe ratio, maximum Sortino ratio, and CVaR, this study gets the time-varying weight for the 3 commodities to form portfolios, which means that the portfolios are rebalanced with a daily frequency. Specifically, the minimum CVaR portfolio uses a rolling window of size 250 and a a significane level of 95% to calculate the value-at-risk. By setting the mean-variance portfolio's risk version parameters of γ to be 1, which means an equal balance between return and risk, the average optimal portfolio weights of the three commodity groups are shown in Table 6.

Different portfolios are efficient for different investment objectives. The equal weight portfolio serves as the benchmark for all the portfolio strategies. The mean-variance portfolio optimizes the weight by maximizing the return and minimizing the risk. When not considering short-selling opportunities, the mean-variance portfolio's result of the optimal weight mainly focuses on allocating weights to metal and energy commodities, and this portfolio strategy has the best performance compared to others under the sample period from 1991 to 2023. With a focus on reducing the risk, the minimum variance portfolio allocates more weight to agricultural products. Also, considering the diversified portfolio, the minimum correlation portfolio approximately allocates equal weight to the three commodity groups. The performance of this portfolio strategy is similar to the minimum variance portfolio strategy. In contrast, other portfolio strategies of maximum Sharpe ratio and maximum Sortino ratio consider the risk-adjusted factor with maximum Sortino ratio portfolio focusing on the downside risk. Their results show that investors should allocate almost all the weights to energy commodities. However, these two portfolio strategies have higher variance and lower CVaR, with maximum Sortino ratio portfolio having negative average return. In order to manage the risk under extreme scenarios, minimum CVaR focuses on tail risk. This portfolio allocates the majority of the weight to energy commodities, which suggests that energy commodities might have relatively lower tail risk compared to metal and agricultural commodities. Similar to the maximum Sortino ratio portfolio, the minimum CVaR portfolio also has negative returns and higher volatility, indicating its poor performance.

When the investment objective is achieving higher returns while bearing higher risk, the short-selling opportunity is considered. The mean-variance, maximum Sharpe ratio, maximum Sortino ratio, and minimum CVaR portfolios have short-selling weights. Also, these strategies are all short-sell agricultural commodities. This suggests that agricultural commodities are underperformed compared to metals and energy commodities. In such cases, short-selling agricultural commodities allows the investor to benefit from the anticipated decline. Additionally, short-sell agricultural commodities can also be attributed to the seasonally predictable volatilities for the agricultural products. Regarding the portfolio performance of the short selling conditions, portfolio strategies of mean-variance and maximum Sharpe ratio archive higher return and lower volatility compared to no short selling opportunity conditions. However, the performance of short-selling maximum Sortino ratio and minimum CVaR ratio is generally worse than conditions without short-selling. This is because short selling can increase volatility and the chance of significant losses, which makes the portfolios more risky.

Table 6: Average Weights of Optimal Portfolios

	Equal Weight	Mean-Variance	Min Variance	Min Correlation	Sharpe Ratio	Sortino Ratio	Min CVaR
				No short Selling			
Weights							
Metal	33.33%	58.25%	36.62%	32.19%	0.41%	0.02%	17.37%
Energy	33.33%	39.73%	9.98%	34.51%	99.59%	99.98%	71.35%
Agricultural	33.33%	2.02%	53.41%	33.29%	0.00%	0.00%	11.27%
Performance							
Return	0.01%	0.02%	0.01%	0.01%	0.01%	-0.32%	-0.06%
Variance	0.01%	0.01%	0.01%	0.01%	0.04%	0.04%	0.04%
CVaR	-2.34%	-2.48%	-1.83%	-2.41%	-4.81%	-4.72%	-4.95%
				Short Selling			
Weights							
Metal	33.33%	95.62%	36.66%	32.17%	100.00%	99.93%	30.21%
Energy	33.33%	50.85%	9.88%	34.52%	100.00%	100.00%	84.58%
Agricultural	33.33%	-46.46%	53.46%	33.30%	-100.00%	-99.93%	-14.79%
Performance							
Return	0.01%	0.03%	0.01%	0.01%	0.03%	-0.33%	-0.10%
Variance	0.01%	0.02%	0.01%	0.01%	0.07%	0.06%	0.06%
CVaR	-2.34%	-3.23%	-1.83%	-2.41%	-5.73%	-5.65%	-5.82%

Note: The optimization process uses the conditional volatility and correlation of the diagonal VECH model with the student t distribution. The risk aversion parameters of the mean-variance portfolio are set to be 1, indicating an equal balance between return and risk. When calculating CVaR, the significance level is 95% and the rolling window size is 250.

As the seven portfolios' average results show two patterns of diversified and non-diversified characteristics, this thesis classifies the portfolios into two types: the first type of equal weight, mean-variance, minimum variance, minimum correlation, and the second type of maximum Sharpe ratio, maximum Sortino ratio, and minimum CVaR. The daily optimal weights without short-selling opportunities are shown in Figure 3 and Figure 4. For the diversified portfolios, it can be seen that different proportions are allocated to different portfolios based on investment preference, while the general feature is diversification. For example, the mean-variance portfolio allocates more weight to the energy portfolio. In contrast, in order to reduce risk, the minimum variance portfolio allocates more weight to agricultural commodities. For the non-diversified portfolios, the general allocation is primarily for the energy commodity due to the goal of adjusting for risk or considering minimizing extreme risk.

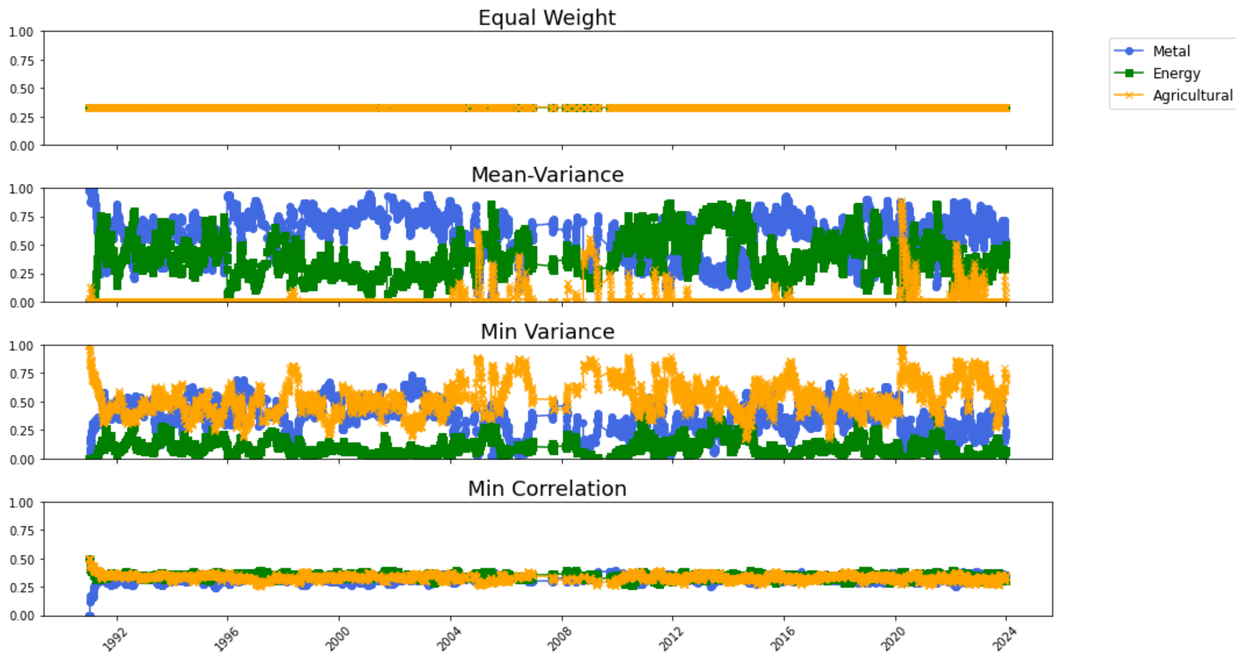

Figure 3: Optimal Daily Weights for Diversified Portfolios

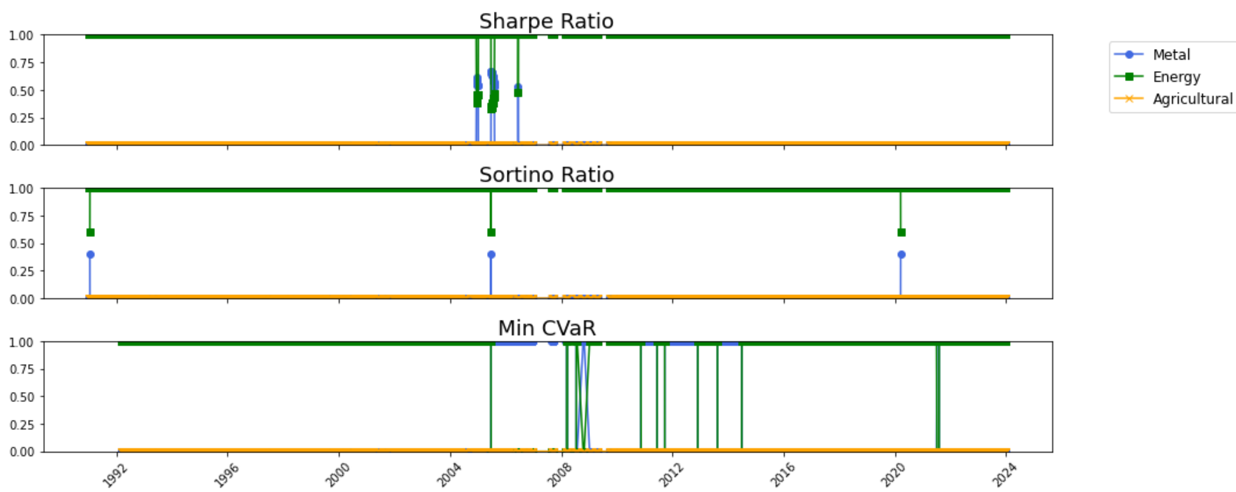

Figure 4: Optimal Daily Weights for Non-diversified Portfolios

Specifically, the mean-variance portfolio uses the risk aversion parameter γ, which is named gamma and measures the trade-off between return and risk. The results of the mean-variance portfolio with different risk aversion parameters are shown in Table 7. When the risk aversion parameter gamma is set to 0, it means that the investment goal is to maximize returns, and investors will no longer consider risk factors. The optimal weights are all allocated to the energy commodity group, which indicates that energy commodities have higher returns compared to metal and agricultural commodities. The risk aversion parameter of 0.5 suits risk-seeking investors who emphasize maximizing returns rather than minimizing risk. Additionally, an equal balance between return and risk is given by setting the gamma to 1, while risk-averse investors are represented as the gamma of 1.5. As the risk aversion level increases, which means that investors focus more on reducing risk, it can be seen that the proportion allocated to metal commodities is increasing, while the proportion to energy commodities is decreasing, with slight increases in the proportion of agricultural commodities. These results align with the characteristics of the commodities, as energy commodities are more volatile but have higher returns compared to metal and agricultural commodities.

Table 7: Risk Aversion Parameter Tuning

	γ=0	**γ=0.5**	**γ=1**	**γ=1.5**
Weights				
Metal	0.00%	42.92%	58.25%	61.23%
Energy	100.00%	56.73%	39.73%	32.90%
Agricultural	0.00%	0.35%	2.02%	5.87%
Performance				
Return	0.02%	0.02%	0.02%	0.02%
Variance	0.04%	0.02%	0.01%	0.01%
CVaR	-4.76%	-2.76%	-2.48%	-2.33%

Note: The optimization process uses the conditional volatility and correlation of the diagonal VECH model with the student t distribution. When calculating CVaR, the significance level is 95% and the rolling window size is 250. γ is the risk aversion parameter.

4.3 Portfolio Performance

In order to analyze how different portfolio strategies perform under different market times, the rolling window size of 250 is chosen to get the portfolio performance over time. Since the performance of the seven portfolios under short sell and no short sell conditions generally behaves in similar patterns, the plot of the performance without short-selling opportunity is drawn as the representation to see how portfolios behave under different market conditions. As shown in Figure 5, during the 1990s and the 2000s, there are significant fluctuations in the returns, which might be due to the early 1990s recessions and the Dot-com bubble at that time. Around 2008, there is a significant decline across all the portfolio strategies, which is caused by the Global Financial Crisis. Additionally, it can be seen that there are spikes in the return, variance, and CVaR around the 2020s. This is because, during the COVID-19 volatile times, investors have increased interest in the commodity market to hedge against inflation and economic instability. As a result, the higher returns in the commodity market are also associated with higher variance and extreme risk. Regarding the portfolio strategies, the equal weight, mean-variance, minimum variance, minimum correlation, and maximum Sharpe ratio portfolios generally show a stable behavior compared to other portfolio strategies. In contrast, the, maximum Sortino ratio, and minimum CVaR portfolios exhibit higher sensitivity to extreme market conditions.

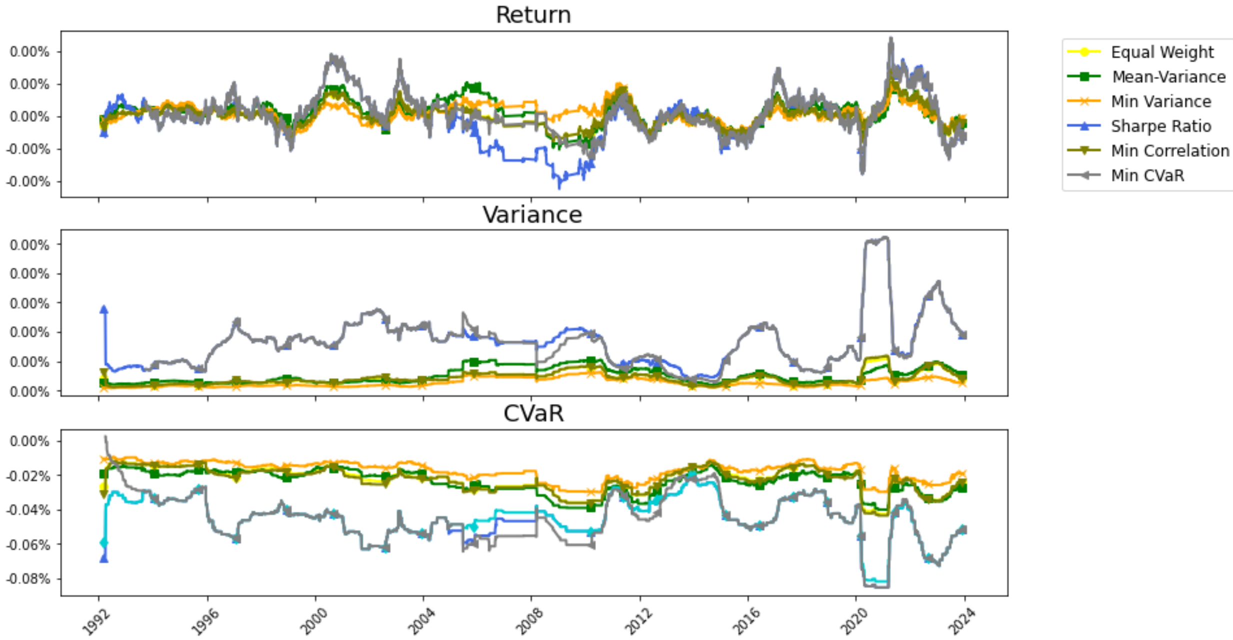

Figure 5: Portfolio Performance without Short-selling

4.4 Hedge against Risk

This study uses conditional volatility from the multivariate GARCH models of the three commodity groups to calculate hedge ratios. The hedge ratios are a measure of how much hedging asset is required to offset the risk of the primary asset, where the hedging asset holds a short position and the primary asset holds a long position. Figure 6 shows the hedge ratios for different commodity pairs from 1991 to 2023, with metal, energy, and agricultural commodities as the primary assets in each subplot. With metal commodities as the primary commodities, the hedge ratios show occasional fluctuations, which might be due to big events such as the early 1990s recession, the global financial crisis, and COVID-19. Also, the high hedge ratios imply a high correlation and similar volatility between the two commodities. Hedging ratios for energy commodities display the most significant fluctuations compared with the other two commodity groups. This implies that the energy market has higher risk and uncertainty. For hedging agricultural commodities, the hedge ratios remain relatively stable during the entire sample period. This indicates that the relationships between agricultural commodities with metal and energy are less volatile, indicating a lower correlation. Overall, the plot shows that the hedge ratios for metal and agricultural commodities are more stable compared to energy commodities. Stable hedge ratios indicate more predictable interconnectedness, which makes it easier to hedge risk. In contrast, volatile hedge ratios of energy commodities suggest that strategies for hedging should be more flexible and should be changing according to specific market situations.

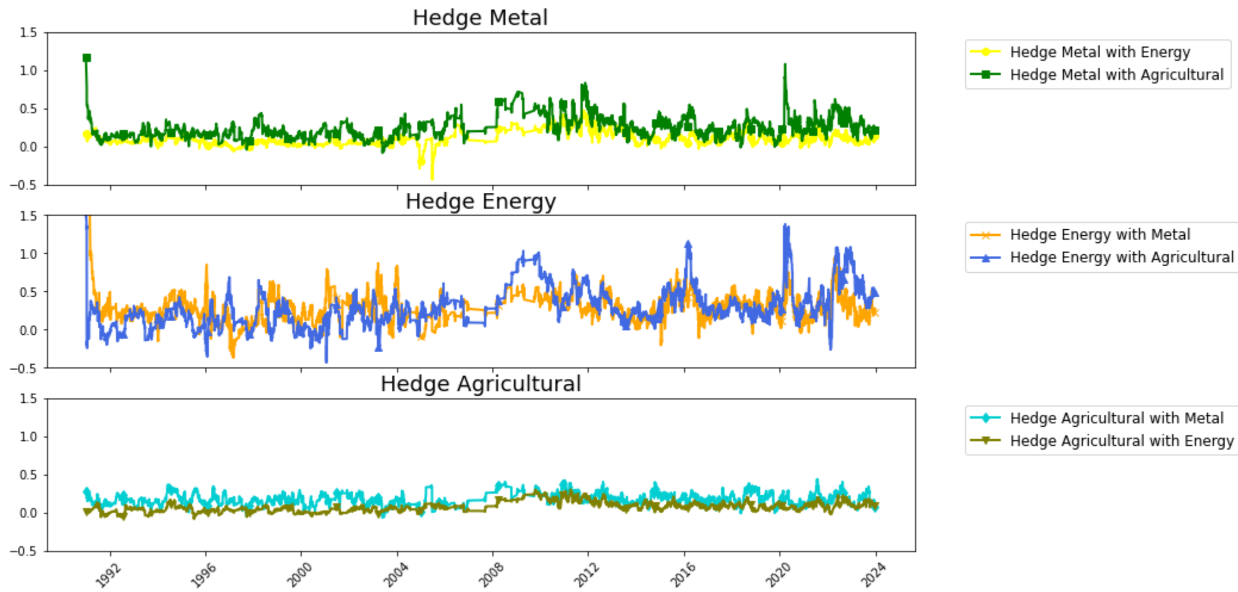

Figure 6: Hedge Ratios

Primary assets hold the long position while hedging assets hold the short position. As the energy commodity group bears the highest volatility and extreme risk, the subsequent analysis focuses on reducing its risk. As shown in Table 8, a $1 long position of energy commodities can be hedged by a $0.30 and a $0.29 short position of metal and agricultural commodities, respectively. Additionally, the cheapest hedging way is using energy commodities to hedge the agricultural commodities, with an average hedge ratio of 0.07.

Table 8: Hedge Ratio Descriptive Statistics

Primary Asset	**Hedging Asset**	**Mean**	**Std.Dev**	**Min**	**Max**
Metal	Energy	0.10	0.09	-0.44	0.47
	Agricultural	0.24	0.14	-0.09	1.16
Energy	Metal	0.30	0.27	-0.37	3.86
	Agricultural	0.29	0.26	-0.44	1.64
Agricultural	Metal	0.17	0.08	-0.07	0.44
	Energy	0.07	0.06	-0.09	0.30

Note: This table shows the hedge ratios between commodity pairs. The input data of conditional volatility to calculate the hedge ratios is obtained from the diagonal VECH model with the student t distribution.

4.5 Empirical Results Summary

Focusing on the commodity market, this study analyzes the dynamics of metal, energy, and agricultural commodity groups. This empirical analysis begins by exploring the statistical characteristics of the three commodity groups. All three commodity groups exhibit slightly negative skewness, indicating asymmetry. Moreover, metals and energy display high kurtosis, suggesting heavy tail distribution, while agricultural commodities demonstrate lighter tails and approximate normal distribution. Additionally, the statistical tests reveal that the commodity return data is stationary with autocorrelation and heteroscedasticity. Secondly, the multivariate GARCH models of diagonal VECH, diagonal BEKK, and CCC are applied to model the volatility of the three commodity groups. The empirical results reveal that the student t-distribution assumption of multivariate GARCH models outperforms the normal distribution assumption. Also, the diagonal VECH model with student t distribution has the highest goodness of fit.

By using the conditional volatility and conditional correlation from the multivariate GARCH models, this study conducts portfolio optimization of seven portfolios, which are the mean-variance, minimum variance, minimum correlation, maximum Sharpe ratio, maximum Sortinoz ratio, and minimum CVaR portfolio strategies. This study considers two conditions, which are the short selling condition and no short selling condition. If not considering short-selling opportunities, the mean-variance portfolio has the highest average return during the sample period and the lowest volatility, indicating its efficiency as an investment strategy compared to other portfolio strategies under the sample period. This portfolio approach allocates the majority of the proportion to metals, a relatively smaller amount of proportion to energy commodities, and the least minimal amount to agricultural commodities. In contrast, the minimum CVaR portfolio has the lowest average return and CVaR, along with the highest variance, indicating its inefficiencies during the sample period. When considering short-selling opportunities, the portfolio strategies of mean-variance, maximum Sharpe ratio, maximum Sortino ratio, and minimum CVaR all include short positions, particularly in agricultural commodities. Also, the average return performance of the strategies with short-selling opportunity, notably mean-variance portfolio and maximum Sharpe ratio, outperforms that without short-selling opportunities. Conversely, the maximum Sortino ratio and minimum CVaR strategies are less efficient compared to conditions without short-selling opportunities. Overall, short-selling increases variance across all the portfolio strategies. Considering the risk aversion levels for different types of investors, the analysis of the mean-variance portfolio with the tuning risk aversion parameters shows that the weights should be entirely allocated to energy commodities if investors do not take into account the risk factor. As the risk aversion extent increases, the proportion of metal commodities increases, while that of the energy commodity decreases, along with a slight increase in agricultural commodities.

The commodity future market is inherently volatile due to the fluctuating nature of the physical commodity market over time. Therefore, implementing hedging strategies to reduce risk is an efficient approach for conservative investors. This study calculates hedge ratios using conditional volatility from the fittest multivariate GARCH model of diagonal VECH with the student t distribution. The hedge ratio results indicate that the energy commodity group exhibits higher and more volatile hedge ratios, reflecting its strong correlation and volatility with metal and agricultural commodities. In contrast, the agricultural commodity group has stable hedge ratios, which shows that it has lower volatility and more predictable relationships with other asset classes. This stability suggests that agricultural commodities are less affected by extreme market events compared to energy commodities, making it easier to hedge effectively. Regarding the average statistics of the hedge ratios, a $1 long position of energy commodities can be hedged by a $0.30 short position of metal commodities and a $0.29 short position of agricultural commodities to reduce risk.

5. Conclusion

As financial investors in the commodity market increase, portfolio optimization for the commodity market becomes increasingly important (Gilbert, 2008). The commodity market is characterized by its high volatility and extreme sensitivity to market economic conditions (Ruano & Barros, 2022), which can lead to significant fluctuations in commodity future prices. As a result, diversification and hedging of commodity assets are crucial for achieving a balance between return and risk based on investors' investment strategies. Based on the empirical analysis of this thesis, energy commodities can provide higher returns but bear excessive risk. To achieve investment objectives tailored to higher risk aversion levels, adding more investment proportion of metal and agricultural commodities is crucial for achieving the desired goals. Also, the efficiency of diversification of energy, metal, and agricultural commodities is further confirmed. Overall, this study can help commodity market participants, especially financial investors, understand the dynamic interconnectedness among different commodity futures and build commodity portfolios with rebalancing directions.

This study has limits. As this study focuses solely on portfolio investment in the commodity futures market, the hedge of one commodity futures is simply the commodity future of the other. However, the commodity spot market can also provide the opportunity for hedging to reduce risk (Gulley & Tilton, 2014). As for further analysis, the spot price of different commodities can also be added to the empirical analysis of this study. This will increase the number of assets in the portfolios of different strategies, which will provide more opportunities for portfolio diversification and hedging. If the commodity spot prices are added for analysis, other factors such as the transaction costs and market liquidity might also need to be considered. Additionally, a wide range of researchers have studied the interaction between the commodity market and other markets, such as the stock market, real estate market, or Bitcoin market (Alshammari & Obeid, 2023; Joo & Park, 2024; Raza et al., 2018). These markets can also provide diversification or hedging possibilities for the commodity market.

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Appendixes

Appendix A

Table A1: Commodity Data

Group	Commodity	Market	Full Name
Metal	Gold	COMEX	COMEX Gold Composite Commodity Future Continuation 1
	Silver	COMEX	COMEX Silver Composite Commodity Future Continuation 1
	Palladium	NYMEX	NYMEX Palladium Electronic Commodity Future Continuation 1
	Platinum	NYMEX	NYMEX Platinum Electronic Commodity Future Continuation 1
	Copper	COMEX	COMEX Copper Composite Commodity Future Continuation 1
Energy	WTI Crude Oil	NYMEX	NYMEX Light Sweet Crude Oil (WTI) Electronic Energy Future Continuation 1
	Brent Crude Oil	ICE-Europe	ICE-Europe Brent Crude Electronic Energy Future Continuation 1
	Natural Gas	NYMEX	NYMEX Henry Hub Natural Gas Electronic Energy Future Continuation 1
Agriculture	Corn	CBoT	CBoT Corn Composite Commodity Future Continuation 1
	Cocoa	ICE-US	ICE-US Cocoa Futures Electronic Commodity Future Continuation 1
	Cotton	ICE-US	ICE-US Cotton No. 2 Futures Electronic Commodity Future Continuation 1
	Coffee	ICE-US	ICE-US Coffee Futures Electronic Commodity Future Continuation 1
	Lean Hogs	CME	CME Lean Hogs Electronic Commodity Future Continuation 1
	Soybeans	CBoT	CBoT Soybeans Composite Commodity Future Continuation 1

Note: The table includes 14 commodity futures grouped into metal, energy, and agricultural. Market denotes the place where the commodities are traded. COMEX is the Commodities Exchange, which is now part of the New York Mercantile Exchange. NYMEX is the New York Mercantile Exchange. ICE-Europe is the Intercontinental Exchange Europe. CBoT is the Chicago Board of Trade, which is now part of the Chicago Mercantile Exchange Group. ICE-US is the Intercontinental Exchange US. CME is the Chicago Mercantile Exchange. Continuation 1 means that the selected futures contract is the first continuation contract.

Appendix B

Table B1: Diagonal VECH

	Norm			T		
	Coef	z-Stat	P-value	Coef	z-Stat	P-value
Mean Equation						
C(1)	0.00	0.86	0.39	0.00	2.44	0.01
C(2)	0.00	0.03	0.97	0.00	-0.02	0.98
C(3)	0.00	1.45	0.15	0.00	2.55	0.01
C(4)	-0.02	-2.02	0.04	-0.02	-1.31	0.19
C(5)	0.00	0.13	0.90	0.00	1.60	0.11
C(6)	0.04	3.22	0.00	0.03	2.47	0.01
Variance Equation						
M(1,1)	0.00	7.65	0.00	0.00	7.49	0.00
M(1,2)	0.00	3.63	0.00	0.00	3.41	0.00
M(1,3)	0.00	3.25	0.00	0.00	3.24	0.00
M(2,2)	0.00	8.31	0.00	0.00	5.77	0.00
M(2,3)	0.00	2.42	0.02	0.00	2.34	0.02
M(3,3)	0.00	7.63	0.00	0.00	4.91	0.00
A1(1,1)	0.03	33.10	0.00	0.06	12.17	0.00
A1(1,2)	0.02	8.80	0.00	0.02	5.44	0.00
A1(1,3)	0.02	6.75	0.00	0.02	4.91	0.00
A1(2,2)	0.07	21.98	0.00	0.07	12.10	0.00
A1(2,3)	0.02	5.54	0.00	0.02	4.50	0.00
A1(3,3)	0.04	11.74	0.00	0.03	7.12	0.00
B1(1,1)	0.97	1721.36	0.00	0.92	154.10	0.00
B1(1,2)	0.96	326.64	0.00	0.95	129.10	0.00
B1(1,3)	0.97	262.50	0.00	0.95	100.32	0.00
B1(2,2)	0.92	283.73	0.00	0.92	154.66	0.00
B1(2,3)	0.97	232.89	0.00	0.97	152.00	0.00
B1(3,3)	0.94	188.95	0.00	0.94	110.27	0.00

Table B2: Diagonal BEKK

	Norm			T		
	Coef	z-Stat	P-value	Coef	z-Stat	P-value
Mean Equation						
C(1)	0.00	0.81	0.42	0.00	2.46	0.01
C(2)	0.00	-0.06	0.95	-0.01	-0.50	0.62
C(3)	0.00	1.38	0.17	0.00	2.45	0.01
C(4)	-0.02	-1.91	0.06	-0.02	-1.38	0.17
C(5)	0.00	-0.04	0.97	0.00	1.60	0.11
C(6)	0.04	3.27	0.00	0.03	2.45	0.01
Variance Equation						
M(1,1)	0.00	8.61	0.00	0.00	7.30	0.00
M(1,2)	0.00	3.47	0.00	0.00	2.78	0.01
M(1,3)	0.00	4.28	0.00	0.00	3.93	0.00
M(2,2)	0.00	8.52	0.00	0.00	6.44	0.00
M(2,3)	0.00	3.46	0.00	0.00	3.45	0.00
M(3,3)	0.00	7.55	0.00	0.00	4.91	0.00
A1(1,1)	0.16	67.16	0.00	0.17	25.41	0.00
A1(2,2)	0.25	45.60	0.00	0.25	24.67	0.00
A1(3,3)	0.16	22.67	0.00	0.15	14.33	0.00
B1(1,1)	0.99	3671.72	0.00	0.98	642.66	0.00
B1(2,2)	0.96	596.12	0.00	0.96	345.57	0.00
B1(3,3)	0.98	426.52	0.00	0.98	260.19	0.00

Table B3: CCC

	Norm			T		
	Coef	z-Stat	P-value	Coef	z-Stat	P-value
Mean Equation						
C(1)	0.00	0.75	0.46	0.00	2.41	0.02
C(2)	0.00	0.02	0.98	0.00	0.19	0.85
C(3)	0.00	1.57	0.12	0.00	2.64	0.01
C(4)	-0.02	-1.36	0.17	-0.01	-1.01	0.31
C(5)	0.00	-0.03	0.98	0.00	1.49	0.14
C(6)	0.04	3.40	0.00	0.03	2.53	0.01
Variance Equation						
M(1)	0.00	7.56	0.00	0.00	7.52	0.00
A1(1)	0.03	32.83	0.00	0.07	11.79	0.00
B1(1)	0.97	1690.00	0.00	0.90	119.71	0.00
M(2)	0.00	8.34	0.00	0.00	5.64	0.00
A1(2)	0.08	20.51	0.00	0.07	11.65	0.00
B1(2)	0.91	212.78	0.00	0.91	134.97	0.00
M(3)	0.00	7.14	0.00	0.00	4.66	0.00
A1(3)	0.04	11.50	0.00	0.04	6.84	0.00
B1(3)	0.94	161.04	0.00	0.94	97.47	0.00
R(1,2)	0.18	16.11	0.00	0.18	13.31	0.00
R(1,3)	0.22	19.53	0.00	0.22	16.19	0.00
R(2,3)	0.16	13.65	0.00	0.16	11.66	0.00

Declaration of Generative AI

I acknowledge the use of ChatGPT for understanding concepts and rephrasing sentences.