LUND UNIVERSITY

Plato's Philosophy of Mathematics:

ἀριθμητική (Arithmetic), λογιστική (*Logistikē*) and γεωμετρία (Geometry) as 'Spiritual' Mathematics

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Ancient Greek
SPVR01 Language and Linguistics: Degree Project – Master's (Two Years) Thesis, 30 credits

May 2024

Abstract

This thesis examines the nature and purpose of the Greek sciences ἀριθμητική, λογιστική and γεωμετρία in the texts of Plato. The statements of some other ancient authors are also mentioned, and the relevant modern research is consulted. ἀριθμός is at any instance, as Klein has already noted, ‘a definite number of definite objects’. In Plato’s philosophical ἀριθμητική, ἀριθμός seems to always consist of ‘the odd and even’, or it is the ‘multitude of the μονάδων/units’, just as in Euclid. Many of the key concepts of Plato’s mathematics appear to have a hierarchical order, or a duality (perhaps later called ‘προποδισμός’ process, progression). Plato seems to employ a peculiar ‘oracular/religious vocabulary’ which is only recognized in the original Greek sources. There is an obvious form of ‘spirituality’ in the entire philosophy of Plato’s mathematics. The source of the mathematical concepts, and of the ‘Forms’, is from a god (Prometheus?). The concept of the soul’s purification and ‘σωτηρία’ (salvation) is probably one of the ultimate purposes of Plato’s mathematics, along with the aim of reaching the ‘Good’ and ‘Being’. ἀριθμητική, λογιστική and γεωμετρία draw the soul towards ‘Truth’ (“πρὸς ἀλήθειαν”), and this is one of their purposes and an oft-mentioned theme by Plato. It is concluded that Plato’s mathematics is in its broadest extent an all-encompassing study of the very things (τῶν ὄντων) of nature and existence, in the background of a spiritual philosophy.

Keywords: ‘Plato’s philosophy of mathematics’, ‘ἀριθμητική’, ‘λογιστική’, ‘γεωμετρία’, ‘oracular vocabulary’, ‘spirituality’, ‘σωτηρία’, ‘ἀριθμός’, ‘ἀλήθεια’, ‘προποδισμός’,

Acknowledgements

Many thanks to Professor Christian Høgel who has supervised this endeavour, who has given a place to ancient mathematics with a new seminar group, and who allowed me as much freedom of thought as possible to complete this thesis.

I also want to thank Jordan Zlatev, Frida Mebius Önnerfors, Åsa Wikström, and all those involved in the MA Programme in Language and Linguistics. Thanks to Lund University for making this thesis possible.

Finally, I must thank my family who were there for me when no one else was.

Ad S. P. I.

“τί τὸ σοφώτατον; ἀριθμός”.

(One of the Pythagorean ἀκούσματα, in: Iamblichus, *Vita Pythagorae* 82)

“πάντων δὴ ἕνεκα τούτων οὐκ ἀφετέον τὸ μάθημα, ἀλλ᾽ οἱ ἄριστοι τὰς φύσεις παιδευτέοι ἐν αὐτῷ.”

(Plato, *Res Publica*, VII: 526c)

“In fact this question, of what *we* [the moderns] mean by *number*, is a greater stumbling-block than either the interpretation of Plato’s dialogues, or the analysis of Aristotle’s account of Plato’s philosophy. […] even if I did supply an account it would be of no use, since it would be no more than one view among many.” (P. Pritchard, 1995, 36)

“Therefore it cannot be emphasized too strongly that research into the history of ancient mathematics is impossible without a very careful and thorough investigation of its language. It should not be forgotten that mathematical thought and language were still very closely linked at that time. The mathematical symbols which we all rely on nowadays, could express practically nothing, they did not even exist, so words had to be used. Furthermore these words, even the ones which later acquired special mathematical meanings, were drawn mostly from everyday language or from the language of philosophy. So the characteristics of ancient mathematical thought, which sometimes provide a marked contrast to later ideas about mathematics, are only accessible to us through language.” (Á. Szabó, 1978, 24)

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Abbreviations

LSJ The *Liddell-Scott-Jones* Dictionary.

Liddell, H. G., Scott, R., Jones H. S., McKenzie, R., *A Greek-English Lexicon* (Oxford: Clarendon Press, 9th ed. 1953).

OCD The *Oxford Classical Dictionary.*

Eds. Hornblower, S., Spawforth, A., Eidinow, E., *The Oxford Classical Dictionary* (Oxford: Oxford University Press, 4th ed. 2012).

OED The *Oxford English Dictionary.*

Simpson, J., & Weiner, E., *The Oxford English Dictionary vol. 1–20* (Oxford: Clarendon Press, 2nd ed. 1989).

When abbreviating ancient authors and works I have used the *OCD* mainly, but whenever the *OCD* did not have an entry for a specific work, I have consulted the *LSJ* instead. If the neither the *OCD* nor the *LSJ* had an entry, I have cited the work in full. Whenever singular words are translated without a source cited, I have consulted the entries in the *LSJ*.

1. Introduction

1.1. Ancient and modern mathematics

1.1.1. Ancient Greek mathematics

This thesis seeks to gain more knowledge on Plato's views of the philosophy behind mathematics. Specifically, ἀριθμητική (arithmetic), λογιστική (*logistikē*)[1] and γεωμετρία (geometry) have the main attention since they are closest to our modern understanding of mathematics.

Greek mathematics has been called the basis of modern science, and one of its greatest contributions was the deductive method itself.[2] It is rather evident that the Platonists had quite a different view from us today of what mathematics is concerned with. ἀριθμητική was a section of mathematics dealing with the – sometimes metaphysical – principles and properties of numbers, and not only with the actual *calculation* which they rather seemed to refer to as λογιστική.[3] These sciences were considered so crucial that Plato classified them as some of the most important subjects in the education of his 'philosophical elect', the *guardians*.[4] λογιστική has hardly been transmitted to our age, although it is considered that some of the Greek calculation techniques were similar to our own.[5] While the art of calculating was also important in the training of the mathematician, it is rather clear that for Plato our modern sense of algebraic calculation was not similar nor as important as the peculiar notions of the nature of ἀριθμητική and γεωμετρία.[6] It is therefore of highest interest to the zealous student of ancient mathematics – and to the student of modern mathematics – to gain deeper insight into the actual roots of this science. Considering that mathematics still is the foundation of all modern science, it would evidently be valuable to consider its ancient Greek roots, which are also the roots of all later Western science. A famous example of the importance of mathematics in Platonic philosophy is the alleged inscription which was said to stand over the entrance to Plato's Academy: "ΑΓΕΩΜΕΤΡΗΤΟΣ ΜΗΔΕΙΣ

[1] I will either transliterate the noun λογιστική as '*logistikē*' or simply use the Greek word.
[2] R. Netz (2012).
[3] U. Rehmann, 'Arithmetic' (2020).
[4] Pl. *Resp*. VII, 525-526.
[5] 'Arithmetic', *Encyclopedia of Mathematics* (2020).
[6] See the subsection 5.1.1.

ΕΙΣΙΤΩ"; "Let no one who is 'ungeometrical' enter".[7] The architect Vitruvius also designated the most erudite men as those who "*efficiuntur mathematici*"; "are *accomplished*/*completed* as mathematicians". Vitruvius wrote that those who have become thoroughly skilled in all the important sciences ("*geometriam, astrologiam, musicen ceterasque disciplinas*") reach the ranks of the few geniuses such as Aristarchus of Samos, the Pythagoreans Philolaus and Archytas, Apollonius of Perga, and Archimedes.[8] We should note here how Vitruvius emphasizes that the mathematicians are those who have reached the highest pinnacle of learning and become "completed" or 'perfected' in their learning. This may also be an echo of the previous and similar Pythagorean hierarchical order of "listeners; mathematicians; natural philosophers".[9] It is important to mention that mathematics (or the general concept of 'μαθήματα', 'lessons') as a science comprised several subjects for the ancients (e.g. astronomy, music and harmonics), including for Plato, which today are considered separate ones.[10] For space and time limitations, this thesis will only focus on the concepts of ἀριθμητική, λογιστική, and γεωμετρία. This naturally also entails the consideration of what ἀριθμός really meant for our ancient authors, and a discussion on the mathematical notion of λογισμός in λογιστική. Plato's works are important because of the obvious influence of Platonism in all later Western thought.[11]

What must be acknowledged from the outset is the difficult nature of the subject. Not only might there be disagreements and different viewpoints among the ancient philosophers (and a development in Plato's own concepts of mathematics), but the question of what mathematics actually is, and what the nature of addition or division of 'ones' is, has been expressed as an unclarity already by a skeptical Socrates in the *Phaedo*.[12] Furthermore, there is need of primarily a literal reading and lucid analysis of the Greek primary sources.[13] Wedberg, for instance, only

[7] The earliest reference for this supposed inscription is a scholium on a manuscript of Aelius Aristides from the fourth century A.D. See Fowler's discussion (1999, p. 199-204) on the debate of whether this was inscribed already during Plato's time or not, and for the ambiguities of translating 'ΑΓΕΩΜΕΤΡΗΤΟΣ'. The scholium reads: "There had been inscribed at the front of the school of Plato. 'Let no one who is not a geometer enter'. [That is] in place of 'unfair' or 'unjust': for geometry pursues fairness and justice." D. H. Fowler (1999), 202:n9.
[8] Vitr. *De arch.* 1.1.17.
[9] See the subsection 5.1.1.
[10] *Ibid*.
[11] Cf. the famous statement by the philosopher Whitehead: "The safest general characterization of the European philosophical tradition is that it consists of a series of footnotes to Plato. I do not mean the systematic scheme of thought which scholars have doubtfully extracted from his writings. I allude to the wealth of general ideas scattered through them" (A.N. Whitehead, 1990, 39).
[12] Pl. *Phd.* 96e–97b.
[13] See the subsection 3.1.3. E.g.: "A translation can be 'accurate on the whole' and still give rise to a completely wrong

consulted English translations and Pritchard focuses a lot on modern researchers' opinions and tries to refute them, and his aim is partly to place Greek mathematics in its historical context.[14] Both are useful aims, but not for conclusively finding out what Plato really wrote, which is the aim of this thesis. Pritchard especially is valuable for this thesis since he has noticed the many discrepancies of scholarly theories on Plato's philosophy of mathematics, and Pritchard has done a great work in clarifying these misunderstandings and explaining Plato's thought quite concisely.[15] I will follow the refreshing practice of Pritchard[16] and either keep the noun 'ἀριθμός' as it stands, almost as a technical term, or transliterate it as simply '*arithmos*'. This is necessary in order to distinguish this ancient concept from our modern notion of 'number' as in the positive integers or any other number theories. The reasons for this will be stated and motivated in the chapter '4. Previous Research'.[17] Pritchard's insistence on this should not however be taken too far, since Plato himself obviously mentioned some of the first ten 'positive integers' in some of his passages on mathematics.[18] Also, Pritchard isn't so clear in all his assessments either, as we shall see. The relation of 'μονάς' and 'ἕν' with ἀριθμός will also be considered. The method employed is the usual linguistic and philological analysis, common in philology and in classics. Generally speaking, a hermeneutic Part-to-Whole dialectic is the methodical framework for the present thesis. A detailed philological analysis is imperative for understanding the source texts, this will be discussed in chapter '3. Method and materials'. 'Spirituality' and similar philosophical concepts are imbued in most of Plato's dialogues. 'Mathematics as a means for salvation' especially seems to be a central theme in Plato's mathematical philosophy.

ἀριθμός seems to be, generally, a *thing* (anything?) that can be measured or counted and grouped into several ἀριθμοί. It becomes an abstract idea, because it goes beyond what we today recognize as *number*, i.e. as in the integers 1–10. For this peculiarity, the Greeks (and others among the ancients) may have taken inspiration from nature itself by witnessing magnitudes and lines in trees or grass for instance and the dots and geometrical figures among the stars and

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interpretation of the text." Á. Szabó (1978), 17.
[14] See chapter 3.
[15] See chapter 3 and subsection 4.1.2.
[16] P. Pritchard, (1995), 15.
[17] See also the next subsection 1.1.2.
[18] E.g.: "**τὸ ἕν τε καὶ τὰ δύο καὶ τὰ τρία** διαγιγνώσκειν·" (Pl. *Resp.* VII, 522c); "οὐκοῦν, ἦ δ᾽ ὅς, **τὰ δέκα τῶν ὀκτὼ δυοῖν πλείω εἶναι,** καὶ διὰ ταύτην τὴν αἰτίαν ὑπερβάλλειν, φοβοῖο ἂν λέγειν, ἀλλὰ μὴ πλήθει καὶ διὰ τὸ πλῆθος;" (Pl. *Phd.* 101b). Bold emphasis is mine. Cf. also Pl. *Epin.* 990e–991b. See the subsection 5.3.1.

constellations.[19] In Plato's *Timaeus* 47a–c we are informed that eyesight has given man great benefits, such as the ability to see the all the astronomical phenomena (stars, planets, equinoxes, solstices) and thereby the notion of time was invented, and *even number itself* (ἀριθμός) and *philosophy*.[20] According to Chaeremon the Stoic, in Porphyrius' *De Abstinentia*, geometry originated with the Egyptian priest-initiates of rigorous asceticism.[21] The same priests who, like the Chaldean astrologers of Babylon, would also investigate the heavens for any signs from the gods (astronomy/astrology).[22] The seemingly Greek disregard of the higher and more complex order of the positive integers, especially the manipulation of number *à la* modern mathematics (e.g. fractions, irrationals, infinite series of numbers, infinitesimal calculus, etc.), may have its foundation in the Pythagorean and Platonic philosophical concept of the Limit and Unlimited. The Pythagorean opposites of "Limit and Unlimited, Odd and Even, One and Multitude […] Male and Female […] Light and Darkness, Good and Evil" etc., clearly identify the 'simple' (Limit, One, Male) with 'Good' and the 'complex (Unlimited, Multitude, Female) with 'Evil', although this should probably not be taken literally.[23] Now the 'dyad', also, was specifically identified by Aristotle with 'not-being' while Plato identifies 'being' (or 'essence', οὐσία) as one of the aims for practising mathematics.[24] Although both polarities of Limit and Unlimited, Finite and Infinite were needed for harmony to reign, the concept of 'Infinity' still is obviously identified with the Unlimited, the Indefinite, the Darkness, Evil, Female, all of which were in need of their opposites (Limit, Good, Male) for proper functioning. This Unlimitedness and Infinity, which in modern conception is considered as awe-inspiring in e.g. fractions, in irrational numbers, or in the indefinite expansion of the Universe, was then, probably, for the Greek philosophical mathematician *not* the ultimate and desirable form of 'Goodness'. Thus, fractions

[19] Cf. Pl. *Tht.* 198c where ἀριθμός seems to be something in itself but also derived from external things: [Σωκράτης.] "ἦ οὖν ὁ τοιοῦτος ἀριθμοῖ ἄν ποτέ τι ἢ αὐτὸς πρὸς αὑτὸν αὐτὰ **ἢ ἄλλο τι τῶν ἔξω ὅσα ἔχει ἀριθμόν;**". Bold emphasis is mine.
[20] See the subsection 5.1.1.
[21] Porph. *Abst.* 4.8.
[22] Cf. Maziarz & Greenwood (1968), 71: "If the Greeks were prompted to turn to **experience** for the basic notions of science and treatment of special mathematical problems, we could also find **empirical traces even in the abstract postulates** of their systems. **For example, the Euclidian *axioms* may be considered as general expressions of practical experiments with objects representing integers or with simple cases of mensuration**. Yet, in spite of the interest and value of the empirical tradition in Greek mathematics, which made itself felt in subsequent developments, the main characteristic of the Greek mind is its deliberate use of mental operations in systematizing knowledge." Bold emphasis is mine. Astronomy and astrology were often considered to be the same science during most of antiquity.
[23] "ἕτεροι δὲ τῶν αὐτῶν τούτων τὰς ἀρχὰς δέκα λέγουσιν εἶναι τὰς κατὰ συστοιχίαν λεγομένας, **πέρας καὶ ἄπειρον, περιττὸν καὶ ἄρτιον, ἓν καὶ πλῆθος, δεξιὸν καὶ ἀριστερόν, ἄρρεν καὶ θῆλυ, ἠρεμοῦν καὶ κινούμενον, εὐθὺ καὶ καμπύλον, φῶς καὶ σκότος, ἀγαθὸν καὶ κακόν, τετράγωνον καὶ ἑτερόμηκες**:". Arist. *Met.* I. 986a. Bold emphasis is mine.
[24] Arist. *Phys*. 192a:6 & Pl. *Resp*. VII, 523a.

or mathematical irrationality were probably not held in very high regard, or as being worthy of a very detailed study, for the philosophical mathematician.

Nonetheless, the general idea of ἀριθμός might have been simpler than we imagine it to be. Greek arithmetic and geometry primarily had *visible* things and shapes that were manipulated in the propositions and theorems. Plato later makes the important distinction that mathematics *for the philosophers* were only dealing with concepts of the *mind*, not physical and visible things.[25] The visible ἀριθμοί thus became images, or mere shadows, of the true intellectual ἀριθμοί.

We already see then that our modern concept of 'number' differs greatly from the ancient Greek concept of ἀριθμός.

1.1.2. Modern number theory and the vast difference from the Greek 'ἀριθμός'

On the ideas of post-renaissance number theory, we find an interesting characteristic of algebra which, as Pritchard states, shows that algebra neither presents to our imagination the 'things' under their mathematical concepts *nor* does it even *intend* such things. Rather, algebra simply works with symbols for the things "which stand for the property itself".[26] Commenting on the French 16th century mathematician Franciscus Vieta's algebraic procedure, the scholar Jakob Klein wrote:

> While every *arithmos* intends *immediately the things or the units themselves* whose number it happens to be, his letter sign intends *directly the general character of being a number* which belongs to every possible number, […] The letter sign designates the intentional object of a 'second intention' (intentio secunda), namely of a concept which itself directly intends another *concept* and not a *being.*[27]

[25] "ὅτι περὶ τούτων [ἀριθμῶν] λέγουσιν ὧν διανοηθῆναι μόνον ἐγχωρεῖ, ἄλλως δ᾽ οὐδαμῶς μεταχειρίζεσθαι δυνατόν" Pl. *Resp*. VII, 526a. See the subsection 5.2.1.
[26] P. Pritchard (1995), 41–42.
[27] J. Klein (1992), 174. Also cited by P. Pritchard (1995), 42. Cursive style is Klein's. Klein continued just after this sentence (174): "Furthermore – and this is the truly decisive turn – this general character of number or, what amounts to the same thing, this 'general number' in all its indeterminateness, that is, in its merely possible determinateness, is accorded a certain independence which permits it to be the subject of 'calculational' operations."

The modern notion of 'number' thus became a symbol for '*intentio secunda'*; that is, a symbol of a concept of a concept; a symbol of a second order concept.[28] Furthermore, the faculty of *imagination* has apparently been seen by some today as a "liability for the mathematician".[29] This is contrary to the concepts of the ancient and medieval mathematicians. Nicholas Oresme (14th c. A.D.), one of the most influential philosophers and mathematicians of the Middle Ages, considered his geometrical shapes and illustrations as *imaginations.* During the Middle Ages, geometry was closely correlated with imagination.[30] Pritchard explains:

> In the *Republic*, we find mathematical thinking distinguished from dialectic in the way that looking at images differs from looking at their originals; mathematicians are said to be 'dreaming of reality' (533c). […]. The ability to imagine solid figures in motion was an essential talent for Archytas, in his construction of the two mean proportionals, and for Eudoxus in his construction of the motions of the planets. They and all their contemporary mathematicians indeed had the ability to reason about abstract objects; but the kind of abstraction involved meant that their ability to imagine these abstractions was as important as their grasp of logic. On the other hand the thinking of the philosopher does not involve abstract objects of this kind, nor anything which can be imagined.[31]

The *imaginative* faculty thus provided the Greek mathematicians with a creativity that arguably can be said to be lacking in modern mathematicians.[32]

On 'abstraction', Bochner wrote that "the Greeks, for all their cleverness, were not able, or not yet able, to make abstractions which were more than idealizations from immediate actuality and 'external reality' […]. They did not make second abstractions from abstractions […] and such

[28] Klein claims that Vieta "became the true founder of modern mathematics" after continuing and altering the technique of Diophantus which together with Indian sources was crucial in forming the Arabic algebra (J. Klein, 1992, 4–5).
[29] P. Pritchard (1995), 43.
[30] P. Pritchard (1995), 43–44.
[31] P. Pritchard (1995), 45.
[32] Cf. Pritchard (1995), 46: "[…] the modern 'symbolic' mathematics, which does indeed contrive to subsume even logic itself as part of its subject matter, and by this token has some claim to being the highest knowledge, does not fit Kant's philosophy, while the ancient mathematics of 'direct' or 'imaginative' abstraction, properly subordinate to dialectic or logic, is the only mathematics to which Kant's theory properly applies."

abstractions of higher order as were made were operationally unproductive".[33] Bochner further argues that the modern notion of 'abstract' was unfamiliar to the ancient Greek philosopher.[34] One may ask, what exactly is this 'abstraction'?[35] And why are 'abstractions' of a second or perhaps even third or fourth order needed? One may wonder, if the basic definition of *number* itself has not been explained, if modern number theory and mathematics in general really is nothing more than '*a workable model'*, and not something which claims a basis in predictive reality, how can it be claimed that our mathematical science today is far more advanced than the ancient one? Are we instead to claim that our technological advancements prove our superiority, since modern mathematics is mainly applied in technology? Where is the evidence that the ancients were not capable of similar although not identical feats? Actually, we have the testimonies of Archimedes' engineering 'miracles', or the Pyramid of Giza itself which still stands before us as an architectural enigma. We must remind ourselves that the Antikythera Mechanism (ca. 2nd c. B.C. – 1st c. A.D), for instance, still hasn't even been replicated in its original detail let alone fully understood, and that its sophisticated gearwork would not appear again in engineering until at least a thousand years after.[36]

Finally, although this is a modern academic thesis, it must be acknowledged that the 'Neoplatonists', who considered themselves as the expounders of Plato's writings (and not any 'neo' group), already wrote several commentaries on Plato's texts.[37] Although they may have differed with Plato's extant texts, we cannot be certain that Plato's 'unwritten doctrines' were not commonly divulged amongst themselves.[38] Some of the most correct elucidators and commentators of Plato may therefore actually be the 'Neoplatonists', or Aristotle of course. Many scholars have tried to distance Plato both from the earlier Pythagoreans[39] and from the later Neoplatonists.[40] Such a strong focus on narrowing down Plato's teachings to himself becomes

[33] As cited by P. Pritchard (1995), 58:n23.
[34] As cited by P. Pritchard (1995), 58:n23.
[35] Perhaps, Bochner again, as cited by Pritchard (1995), 59:n26: "[…] the Greek employment of letters in lieu of numerals, propositions, and syllogisms falls within the general area of mathematical abstraction […] but it is not, or not yet, the kind of abstraction which constitutes the essence of the mathematics of today."
[36] M. Edmunds & P. Morgan, (2000), 17.
[37] E.g. Neoplatonic commentaries on *Grg.* and *Chrm.* in: J. Klein (1992), 7.
[38] See Arist. *Ph.* IV, 209b:11–17 for his mention of Plato's "unwritten teaching".
[39] M. A. Sutton (2019), 44-46.
[40] Proclus' *in Euc.* would be highly relevant as a comparison, or for understanding the Greek mathematical development chronologically, since it deals directly with the philosophy of mathematics and it was written during late antiquity at the end of the almost one millennium old Platonic tradition, around the closing of Plato's Academy. Thus, it not only stands as a

ridiculous in my opinion, since it is often forgotten by our modern and competent experts that we only possess mere fragments of all ancient Greek, Latin and even Eastern texts that were available to the ancients. More importantly, when it comes to the actual teachings of the Pythagoreans, unless this is also denied by our scholars for whatever reasons, it was commonly alleged during antiquity that the Pythagoreans were very secretive of their doctrines.[41] The Roman astrologer Julius Firmicus Maternus (4th c. A.D.) seems to claim the same for Plato.[42] It cannot then be denied that Plato may have been significantly influenced by Pythagorean sources.[43] More importantly, the fact that Plato altered or enhanced some of his views upon the relationship between mathematics and knowledge[44] (amongst other concepts) – when comparing his earlier and later works – almost nullifies any insistence on completely distancing Plato from his philosophical successors, such as the 'Middle-Platonists' or 'Neoplatonists'. On the general development of Greek mathematical thought, it is also reasonable to suppose that some notions (e.g. on geometry or the difference between *logistikē* and arithmetic) may have changed *after* Plato's time, since he informs us in the *Res Publica* that Greek geometry of solids ('stereometry') scarcely even existed during his lifetime.[45]

witness to most previous Platonic mathematical philosophy, but Proclus also explicitly expounds most of these previous doctrines. Iamblichus' *Comm.Math.* is also relevant since it similarly deals with mathematics and its philosophy. For the purpose of fulfilling the aim and research questions of this thesis, I will not analyze these works to any greater extent, the focus must remain on Plato's texts.

[41] *Vide*: Iambl. *VP* 75. The philosophy of the 'Hearers', the "ἀκούσματα", were to be guarded carefully as "θεῖα δόγματα" (*VP* 82); see also: M. A. Sutton (2019), 46.

[42] Firm. Mat., *Matheseos Libri VIII*, VII.1.

[43] Plato was even accused by some during ancient times for plagiarizing Epicharmus (Diog. Laert. *Vitae Philosophorum* 3.9.), so it stands as rather evident that he made use of earlier teachings. See also the subsection on 'γεωμετρία' for a discussion about all of this.

[44] "These general views about mathematics, as expressed in the seventh book of the Republic, do not represent Plato's final thought on the relation between mathematics and true knowledge" E. A. Maziarz & T. Greenwood (1968), 97.

[45] Pl. *Resp.* VII, 528b–c.

"Greece and her foundations are

Built below the tide of war,

Based on the crystàlline sea

Of thought and its eternity."

(Thomas Little Heath)[46]

[46] T. L. Heath (1921), ix.

2. Aim and research questions

The aim of this thesis is to elucidate what the ancient Greek sciences ἀριθμητική, λογιστική and γεωμετρία were and what their purpose was, according to Plato. The aim is not a detailed explanation of all concepts relating to these sciences, but rather a general overview. The focus is on Plato’s *philosophical* rather than *technical* concepts of mathematics.

1. What are Plato’s philosophical concepts of the nature and purpose of mathematics?
2. What is the purpose and nature of ἀριθμητική (arithmetic), λογιστική (*logistikē*) and γεωμετρία (geometry) according to Plato?
3. What are the concepts relating to ‘spirituality’ in Plato’s dialogues?
4. How can other ancient authors (e.g. Aristotle, Euclid, or Proclus) assist us in understanding Plato’s concepts?

3. Method and materials

3.1. Hermeneutic analysis of primary sources

3.1.1. Hermeneutics

All translations, unless otherwise noted, are mine. This thesis is a project within the philology and the philosophy of Greek mathematics. The focus is the passages of Plato's texts which deal with ἀριθμητική, λογιστική, and γεωμετρία. There will be a short mention on other subjects connected to these three, such as μετρητική or διαλεκτική, but these cannot be dwelled upon since the focus must be on answering the research questions. A short discussion and exposition of the kind of mathematics (and μαθήματα in general) requisite for Plato's *guardians* is given by Socrates in the *Res Publica* VII 522–528. Other texts that mention our three subjects are the *Philebus, Charmides, Phaedo, Gorgias, Protagoras, Theaetetus, Timaeus, Leges,* and *Epinomis*.[47] These are therefore considered in this thesis. Plato's mathematical philosophy in the *Philebus* probably "leads to his final views on forms and numbers."[48] I may not have covered every single instance where ἀριθμητική, λογιστική, and γεωμετρία, or ἀριθμός for that matter, is mentioned by Plato. I am rather confident though, that the most *relevant* passages in Plato are considered in this thesis, for the fulfillment of the aim and research questions. This textual collation and analysis are the qualitative parts of the method: a combination of primary sources (the ancient texts) and secondary sources (modern research).

The concepts of mathematics are read and interpreted in the original Greek, using any philological and linguistic analysis required. In all of this, secondary literature is consulted both as an aid to the concepts and for any relevant linguistic analysis of the texts. This thesis is inductively trying to find and systematize the ideas of the authors with a hermeneutic analysis of a Part-to-Whole dialectic. The texts are the focus and collecting and considering the relevant concepts in all the texts (*the parts*) will contribute to the formulation of a general view of the concepts themselves (*the whole*). The broader *whole* picture will then be a collation from Plato's extant texts and also of any other sources by ancient authors who could possibly explicate further

[47] And in *Cra.* 432 but very briefly. In *Phdr.* 274c it is the Egyptian god Thoth who has invented mathematics, astronomy, and writing. In *Hp. mi.* 367 Socrates discusses the possibility of true and false mathematics. The *Prm.* has a lengthy discussion on the 'The Forms'.
[48] Maziarz & Greenwood (1968), 117.

Plato's notions. The latter sources will not have any precedence over the former and they will not be given any greater focus since it is not often clear what was actually Plato's teachings and what was only *attributed* to Plato by others. Neither do issues of time and space allow us to dwell upon these other authors.

This broader picture must not necessarily be a conclusion, perhaps it does not aim to "establish generalizability;", but, "rather, it is to improve the researcher's interpretive vision".[49] Furthermore, as mentioned in Pollio et al., the hermeneutic approach in describing meaning should lead as much as possible to the elimination of the researcher's subjective predispositions so that the content of the texts may stand for itself, giving other analysts the possibility of arriving at similar interpretations. Yet, this is evidently a quite utopian demand, since the scientist cannot be removed entirely out of the research.[50] This entire analysis and interpretation is of course wholly reliant upon the ability and work of the researcher himself, as most academic work is. The Finnish linguist Esa Itkonen has addressed such a type of linguistic research, calling it "intuition" as a form of "conventionalized empathy", and classified it into three general steps: introspection, empathy and intuition.[51] Itkonen explains that: "1. I introspectively know […] 'Y' by X. 2. I emphatically know that also others […] meant 'Y' by X. 3. I intuitively know that X means 'Y' […]".[52] Itkonen further notes that the one who practices "empathy" in his research is "making the same hermeneutic effort as any historian who, in Collingwood's (1946) words, is 'rethinking people's thoughts' […]".[53]

49 R.H. Pollio et al. (2006), 51.
50 R. H. Pollio et. al. (2006), 37–38.
51 E. Itkonen (2008), 26–27.
52 E. Itkonen, (2008), 26.
53 E. Itkonen, (2008), 30.

3.1.2. ‘Spirituality’ as a concept in Plato’s dialogues

We will furthermore see how Plato’s mathematics is concerned with ‘spiritual’ concepts. The sense of ‘spiritual’ I gather from e.g. the entry in the *OED*:

> **Spiritual, adj. & n.**
> *Relating to or concerned with the human spirit or soul, esp. considered from a religious or moral standpoint.*[54]

That Plato’s mathematics is concerned with the human soul from a religious or moral standpoint will become evident in my analysis. The entry for ‘spirituality’ in the *Britannica Academic* is equally helpful:

> *Spirituality, the quality or state of being spiritual or of being attached to or concerned with religious questions and values broadly conceived. The term is also frequently used in a non- (or even anti-) religious sense to designate a preoccupation with or capacity for understanding fundamental moral, existential, or metaphysical questions, especially regarding the nature of the self (or soul, or person), the meaning of life, the nature of mind or consciousness, and the possibility of immortality.*[55]

“Religions questions” and “moral, existential, or metaphysical questions” concerned with the soul, life, mind and immortality are already general themes that exist in almost every Platonic dialogue. It will be quite evident how themes such as ‘the purification of the soul’, ‘God’, ‘understanding’, ‘The Good’ and other similar concepts are central to Plato’s philosophy of mathematics. One interesting key concept, which can only be pointed out clearly by reading the Greek source texts, is the use of the verbs χράομαι, παρακαλέω and μαντεύομαι in passages

[54] ‘Spiritual’, *OED* (2023).
[55] ‘Spirituality’, *Britannica Academic* (2024).

dealing with mathematics.[56] χράομαι could possibly have connotations of ‘consulting [a science]’, since its meaning is not only ‘to use’ but ‘to consult an oracle/god’. παρακαλέω could apart from ‘to summon’ also mean ‘to invoke [the gods]’, and μαντεύομαι (‘to divine, prophesy’ or ‘to consult an oracle, seek divinations’) is used by Socrates within these very discussions. Central themes like ‘the one and many, limit and unlimited’ or ‘Being’ are by Socrates said to have been gifts from the Gods, perhaps from a Promethean hero.[57]

This is not only interesting with Socrates’ ‘divine voice’ in mind, but the entire framework of Plato’s mathematics is given a sort of ‘oracular vocabulary’ or ‘oracular/religious’ semantics. There are even extant stories of the oracle at Delphi admonishing the Greeks for their neglect of mathematics.[58]

As for Plato’s use of the noun σωτηρία (‘deliverance, preservation’ or as later ‘salvation’) and the verb σῴζω (‘to save’), it seems to be a central idea in his dialogues. Stephen Menn has also recently drawn attention to the concept of σωτηρία in the dialogues. Menn concluded that:

> […] in a significant number of the passages where Plato speaks of a σωτήρ or σωτηρία or σῴζειν we can see that he is exploiting religious connotations of these terms, and competing with more traditional religious saviors and practices of salvation, or with earlier philosophers who were also drawing on those same religious connotations. And to this extent we can describe Plato’s concerns in these passages as religious.[59]

Menn points out however, that this is not used in an eschatological sense. Plato is not concerned with the end of the world, but with saving/salvation in the present world.[60]

Plato also seems to clearly relate ἀριθμός with the soul when Socrates states that: “ἀριθμητικὸς γὰρ ὢν τελέως ἄλλο τι πάντας ἀριθμοὺς ἐπίσταται; πάντων γὰρ ἀριθμῶν εἰσιν αὐτῷ ἐν τῇ ψυχῇ

[56] See especially subsection 5.3.1. for all of this.

[57] “θεῶν μὲν εἰς ἀνθρώπους δόσις, ὥς γε καταφαίνεται ἐμοί, ποθὲν ἐκ θεῶν ἐρρίφη διά τινος Προμηθέως ἅμα φανοτάτῳ τινὶ πυρί: καὶ οἱ μὲν παλαιοί, κρείττονες ἡμῶν καὶ ἐγγυτέρω θεῶν οἰκοῦντες, ταύτην φήμην παρέδοσαν, ὡς ἐξ ἑνὸς μὲν καὶ πολλῶν ὄντων τῶν ἀεὶ λεγομένων εἶναι, πέρας δὲ καὶ ἀπειρίαν ἐν αὑτοῖς σύμφυτον ἐχόντων.” Pl. *Phil.* 16c–d.

[58] Theo Sm. *Theonis Smyrnaei philosophi Platonici expositio rerum mathematicarum ad legendum Platonem utilium,* ii. Also discussed in E. A. Maziarz & T. Greenwood (1968), 80.

[59] S. Menn (2013), 192.

[60] S. Menn (2013), 192.

ἐπιστῆμαι."[61] "Doesn't a master arithmetician know all ἀριθμοί? For the knowledge of all ἀριθμοί reside within him, as knowledge in the soul."

The clear possibility of Plato's mathematics being 'spiritual' also brings it closer to later Neoplatonism.[62]

3.1.3. Textual problems of translation in philology that need special attention

Translating any ancient language is a task that must not be taken lightly. The flexible syntax of the classical languages often makes such a translation subject to several interpretations. When dealing with philosophical subjects, it is self-evident that this is the case. This has been explained by Kurt Lampe in his article on translating Greek philosophical texts. The 'illocutionary force' of a sentence is often equipped with a 'speakers' intentions'. On 'sentence meaning', a word such as 'cultivating' could mean that a person cultivates his personal attributes, if he is the 'cultivator', but it would not mean that he is a herb or a tree that cultivates itself.[63] The context of each sentence is therefore important to examine. Some philosophers may have used similar words in different meanings, without specifying it for different reasons (their audience might already have been conversant with their particular connotations). Also, the fact that Greek and Latin were languages that were in use for over 2500 years makes it obvious that they were subject to several different changes and developments in syntax, morphology, semantics, phonetics, etc. during this large time period. This has been studied thoroughly by modern scholars and needs no further explanation at this moment.

The Hungarian philologist Árpád Szabó discussed the issues of the interpretative methodology of texts in his seminal work from 1978[64]. He commended the work of van der Waerden[65] but was critical of his and many other scholars' reliance upon translations only. After a detailed study of a passage in Plato's Theaetetus (147c-148b) dealing with the technical term 'δύναμις' in which

[61] Pl. *Tht.* 198b.
[62] "[…] it must be stressed that Neoplatonism is predominately spiritual in nature." P. Remes (2008), 9.
[63] K. Lampe (2013), 138.
[64] Á. Szabó (1978), 15–17.
[65] B. L. Van der Waerden (1956).

Szabó found that it should rather be translated as "square" than "power", he concluded that "A translation can be 'accurate on the whole' and still give rise to a completely wrong interpretation of the text."[66] This is a very important reason for my insistence in this thesis on firstly translating and interpreting the primary sources before consulting any secondhand interpretations. Szabó further notes: "So it should be emphasized that as far as the history of mathematics is concerned, translations of the source materials are frequently unreliable, even when they are philologically excellent."[67] Now certainly, there is always the problematic element of the primary sources being obscure or simply hard to interpret (e.g. Pritchard's quote on the front page on interpreting Plato or Aristotle). It should be evident to all though, that any *difficulties* arising from secondhand interpretations confusing the meaning of the original texts is *significantly lessened* by focusing primarily on the primary sources. That is not to say that interpretations by scholars are not helpful, the opposite is true, but as long as these scholars have prioritized the original sources before their own ideas of those sources.

Since theories and opinions are slightly varied among modern scholars on what Plato envisioned ἀριθμητική and λογιστική to be, we will mainly focus on a proper translation of the primary sources so that the interpretation may be as neutral as possible. Plato's actual opinion on the ontology of mathematics – especially whether he posited mathematics as intermediate between sensibles and intelligibles as Proclus and Iamblichus did – is a question which has been much debated by scholars. We will discuss this in the next chapter.

[66] Á. Szabó (1978), 17.
[67] Á. Szabó (1978), 18.; cf. also: "I am convinced that the significance of many important facts about science in antiquity simply cannot be appreciated either historically or mathematically without using the philological precision I have attempted here." (1978), 23.

4. Previous research

4.1. Greek mathematics and its philosophy

4.1.1. The history of Greek mathematics

This thesis focuses on Plato’s *philosophical* concepts of mathematics and not the *technical* aspects of it. We will however briefly mention the most important scholarly works on the practical mathematics of the Greeks.[68]

The following short summary on the modern research of the history of Greek mathematics is based on Reviel Netz’ outline. Thomas Little Heath wrote several works on the history of Greek mathematics and translated several texts of important Greek mathematicians into English. Johan Ludvig Heiberg was a philologist whose research still stands out as imperative in Greek science. Otto Eduard Neugebauer was a mathematician and historian who explicated the practice of ancient mathematics. Later during the 20th century, the mathematician and scholar Wilbur Richard Knorr further elucidated the history and practice of ancient mathematics.[69]

T. L. Heath’s *A History of Greek Mathematics* is indispensable for understanding the practice of Greek Mathematics. Although James Gow and Gino Loria before him published similar works on the history of Greek mathematics, Heath incorporated all later and newer findings (e.g. Archimedes’ palimpsest) and arranged his two volumes according to mathematical subjects, and within each problem he arranged the exposition chronologically.[70] Although I have consulted Heath’s work, I have used it sparingly, since its focus is on the *technical* aspect of Greek mathematics while my thesis aims at understanding its *philosophy*. On the singular achievements of the Greeks in mathematics, Heath wrote: “For the mathematician the important consideration is that the foundations of mathematics and a great portion of its content are Greek. The Greeks laid down the first principles, invented the methods *ab initio*, and fixed the terminology. Mathematics in short is a Greek science, whatever new developments modern analysis has

[68] See the bibliography for full citations of the works mentioned here.
[69] R. Netz (2012).
[70] T. L. Heath, (1921), vi–vii.

brought or may bring."[71]

Knorr's *The Evolution of the Euclidean Elements: A Study of the Theory of Incommensurable Magnitudes and Its Significance for Early Greek Geometry* was perhaps his most important work during his unfortunately short life. Here Knorr focused on the development of incommensurability from pre-Euclidean times up until Euclid. In Knorr's *The Ancient Tradition of Geometric Problems,* the attention is on the 'analysis' method of Greek mathematics where solutions to problems were often reduced to others already solved. Knorr goes through e.g. the Geometers of Plato's Academy, the mathematicians during Euclid, Archimedes and his successors, Apollonius and others, focusing mostly on the 'three classical problems': the duplication of the cube, the quadrature of the circle, and trisecting the angle. Knorr states in the preface: "the present effort is conceived as an explanatory essay, intended to reveal the opportunities which the evidence available to us provides for an interpretation of the ancient field."[72] Stating that the three classical problems were considered impossible to solve by the Greek mathematicians is argued to be an anachronism.

The scholar Reviel Netz wrote an important work called *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*. In this book, Netz examines the practices of Greek mathematics, the method of deduction which had such widespread influence in western science, and a general overview of the history of science. The focus however is mostly on Euclid, Archimedes and Apollonius. In another very recent work of Netz, *A New History of Greek Mathematics,* we are given a general overview of the history of Greek mathematics. Netz' choice of title 'A New History…' seems rather odd in a sense, since this implies that Netz is giving new information (and what exactly?) on a topic that has been rather thoroughly researched already. I haven't perused this work fully, but the reviews seem mixed as of yet.[73]

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[71] T. L. Heath (1921), v.; cf T. L. Heath (1921), 1: "Not only are the range and the sum of what the Greek mathematicians actually accomplished wonderful in themselves; it is necessary to bear in mind that this mass of original work was done in an almost incredibly short space of time, and in spite of the comparative inadequacy (as it would seem to us) of the only methods at their disposal, namely those of pure geometry, supplemented, where necessary, by the ordinary arithmetical operations."

[72] W. R. Knorr (1986), vii.

[73] See e.g J. Timney (2023) & V. Blåsjö (2022). V. Blåsjö (2022): "Reviel Netz's New History of Greek Mathematics contains a number of factual errors, both mathematical and historical. Netz is dismissive of traditional scholarship in the field, but in some ways represents a step backwards with respect to that tradition. I argue against Netz's dismissal of many anecdotal historical testimonies as fabrications, and his 'ludic proof' theory." If Professor Blåsjö is correct, *it is beyond my comprehension how this work was even allowed to be released, while there are other works out there* (e.g. Maziarz &

Serafina Cuomo's *Ancient mathematics* in the book series 'Sciences of antiquity' is an extensive and accessible history of Greek mathematics with a survey of the extant evidence and its content.[74] An interesting part of mathematics which Cuomo discusses is the relation between mathematics and politics, in Aristotle especially, and the mathematical notions of number attributed to money and economy.

4.1.2. The philosophy of Greek mathematics

Many books and articles have been written on the history, practice and philosophy of ancient Greek mathematics. Not all are relevant for this thesis, and I may have overlooked to mention some.[75] However, the following short survey should cover the most important research which is in line with the aim of this thesis.

Plato (5th–4th c. B.C) stands evidently as an overshadowing figure above most later philosophers. Before Plato, Pythagoras (6th c. B.C.) and the Presocratics had their mathematical concepts, though no extensive works by them have survived (except for the later Neo-Pythagoreans). Not until Euclid (4th–3rd c. B.C.) do we find a substantially large work on the mathematical theorems and propositions that is still extant, although evidently it was based on almost all previous research in Greek mathematics.[76] Apart from Plato, the philosophy behind Greek mathematics has some other explanatory texts still extant, written by the later Neoplatonists (ca. 3rd–6th c. A.D.). Iamblichus (ca. 3rd c. A.D.) wrote several works on Pythagorean philosophy and mathematics (e.g. *On General Mathematical Science*). Proclus (ca. 6th c. A.D.) wrote *A commentary on the first book of Euclid's Elements* and much else noteworthy (e.g. *The Elements of Theology*, commentaries on Plato, and on Aristotle).

Although he may be considered outdated and perhaps even biased in many of his views (since he was an outspoken platonic philosopher), Thomas Taylor's (1758–1835) translations, commentaries, and dissertations still have academic merit worthy of regard. Taylor was probably

Greenwood, 1968) *that are neglected or even scorned by researchers.* The gap of knowledge seems to widen even further for ancient Greek mathematics.

74 S. Cuomo (2001).

75 E.g. R. S. Brumbaugh (1954); G. Schneider (2012); P. Couderc & L. Séchan (1949).

76 R. Netz (2012).

the first to translate the complete works of Plato and Aristotle into English,[77] certainly a remarkable and outstanding feat considering the state of Aristotle's texts and the elaborate prose of Plato. Glenn R. Morrow cites Taylor's works as being indispensable in his translation of Proclus' *In primum Euclidis librum commentarius*.[78] Taylor was accused of being ignorant in the Greek language, yet this is actually unsubstantiated, as the English librarian William Axon maintained.[79] And if Taylor would today be accused of having his work coloured and disfigured by his own beliefs, what scholar today can claim to be completely free from personal inclinations and bias in his academic work? How can we possibly even know for sure that some researchers are not adamant about conducting research in their own way and reaching conclusions that suit their own beliefs and biases? Surely, isn't research in all fields of science and the humanities riddled with all kinds of theories, the higher up we reach the ladder of academic research? Admittedly, though, Taylor's work may have had greater influence among scholars if he had been more objective and neutral in his commentaries. I have consulted Taylor's translations mainly as general reading, especially for the later Neoplatonists Proclus and Iamblichus. It is no more than fair to his merit though, that he is at least mentioned in a modern academic work.

The focus of this thesis is the writings of Plato, but a short mention will be made on the content of Aristotle's extant texts, who after all was one of Plato's close pupils and of course became vastly influential later on. In Aristotle's *Metaphysics*, mainly book I and XIII-XIV, we find his exposition and critique of Plato's (and others') mathematical philosophy. The scholar Anders Wedberg has already examined Aristotle's account of Plato's mathematics, which his essay *Plato's Philosophy of Mathematics* is largely focused on, although he partly examined Plato's texts also.[80] Wedberg posits "five groups of theories about the nature of mathematics" that are either Plato's own or ascribed to Plato by Aristotle. These are worthy of direct quotation, since they have a bearing upon our research questions:

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[77] 'THOMAS TAYLOR, The English Platonist, 1758 – 1835' (*The Prometheus Trust*).

[78] "The contributions of this indefatigable Platonist are so numerous and important that it is almost an act of impiety to presume to replace this early product of his industry with a new translation." G. R. Morrow (1970), xliv.

[79] "The sneers at his command of Greek are evidently absurd, for surely no man's mind was ever more thoroughly suffused with the very essence of Neo-Platonism. Whatever failure he may have made in unessential details would be more than compensated by the fidelity with which his sympathetic mind reproduced the spirit of the Pythagorean philosophers with whom he dwelt – apart from the noise and turmoil of the age in which he had been cast. His books remain a mighty monument of disinterested devotion to philosophic study. They were produced without regard to, and hopeless of, profit." W. E. A. Axon (1890), 14.

[80] A. Wedberg (1955), 9–10.

I. Theories which locate the objects of mathematics within a presupposed division of the universe;

II. Theories concerned with the (non-temporal) generation, within the realm of Ideas, of the so-called Ideal Numbers;

III. The theory that all Ideas are numbers;

IV. Those theories which deal with the explanation of the sensible world in terms of space and mathematical notions;

V. Views concerning the methodology of mathematics.[81]

Wedberg did not focus on groups II-IV at all, probably since they required a longer analysis, and since group I has the most conspicuous statements in the sources, as Wedberg says.[82] In this thesis the focus is upon ἀριθμητική, λογιστική, and γεωμετρία, but the five groups of theories may all be touched upon in connection to these three subjects.

In this thesis, I mention Plato's 'analogy of the undivided line' in the *Res Publica* VI. We will see how here, and probably elsewhere, Plato seems to mention the 'mathematical intermediates', and I believe that he does. This classification – Plato's supposition of *mathematical intermediate objects* between the intelligible and sensible entities – has been much debated among scholars. Wedberg seems to think that there are grounds for this in Plato's dialogues, and clearly of course in Aristotle's *Metaphysics*, but that the final verdict remains unanswered [on book VI of the *Res Publica*]:

[81] A. Wedberg (1955), 9–10.
[82] A. Wedberg (1955), 10.

> No doubt Plato's language here is obscure: in part it lends itself to an interpretation of the Aristotelian type, and in part it seems to contradict such interpretation. It may fairly be concluded that Plato had not quite made up his mind on the question whether or not there exists a class of ideal mathematical objects distinct from the mathematical Ideas: that he, so to speak, hesitated between the two opposite alternatives.[83]

On Plato's general philosophy of mathematics, Wedberg noted that "the most complete statements are those in the *Republic* and the *Philebus,* but even they would remain exceedingly enigmatic unless compared with statements in other dialogues".[84] Furthermore, Aristotle *probably* tells us that mathematics became more important in Plato's later life than during his early years.[85] The discrepancies between Aristotle's account and the *extant* writings of Plato have also been noted by several modern scholars.[86] This, of course, partly goes back to the differences between Plato's Academy and Aristotle's Peripatetics. This phenomenon is too lengthy and disputed to dwell upon at the moment. What should be mentioned though, is Aristotle's reference to Plato's 'unwritten teaching'.[87] This gives us a simple response to many of the 'discrepancies' between Plato's dialogues and Aristotle's account on Plato's philosophy.

A 'fallacy' of Wedberg may be argued thus: he does not discuss the original Greek sources at all. Wedberg mentions in the preface how he used the Loeb *translations* and sometimes made slight changes in them.[88] Further on, Wedberg admits that "the present essay is written from the point of view of a philosopher, not a philologian. In what concerns the philological interpretation of the texts I have had to rely largely on authorities".[89] A harsh but perhaps not unjustified reviewer would hence be cautious with any conclusions drawn by Wedberg. Indeed, he may be right in his logical assessments, but without any proper insight into the original language as it stands in the extant texts, he has no right to draw any definite conclusions (and he does not claim to do so

[83] A. Wedberg (1955), 14.
[84] A. Wedberg (1955), 15.
[85] Arist. *Met*. 1078b:9-12, in: A. Wedberg (1955), 18.
[86] A. Wedberg (1955) 12.; Á. Szabó (1978), 229 on Aristotle: "Yet the statements which he makes about axiomatics are often arbitrary and historically inaccurate; hence they need to be treated with some caution. […] seem almost to be products of his own imagination."
[87] Arist. *Physics* IV, 209b:11–17.
[88] A. Wedberg (1955), 5–6.
[89] A. Wedberg (1955), 19.

either).[90] Wedberg could, for instance, not have known whether his reading of a passage in Plato's works may have had other possible translations than those already provided, since he did not consult the original Greek texts by himself. Wedberg could neither have known whether Plato's language was ambiguous or clearer in some passages. These are only a few of the many problems faced by researchers who only consult translations, since translations must always remain *interpretations* of original works.

Jakob Klein's *Greek Mathematical Thought and the Origin of Algebra* is a pivotal work that often has been cited since its original publication in German, between 1934–1936.[91] Klein sought to investigate the origin of the formal mathematical language of the ancient Greek authors. Although Klein focuses partly on the concepts that this thesis deals with (e.g. ἀριθμητική and λογιστική), his ultimate aim was to understand the relationship between this original formal language and modern "mathematical *physics* today".[92] He therefore surveys also the Latin writings of e.g. the French mathematician Vieta and the philosopher Descartes. Moreover, Klein does not go into a thoroughly detailed linguistic examination of Plato's explicit passages on ἀριθμητική, λογιστική *and* γεωμετρία, as this thesis does. Klein's book has been diligently consulted by both Fowler and Pritchard, whose works we will describe below. On the question of what the concept 'ἀριθμός' pertains to and how Klein translated it in German, the English translator of Klein's book, Eva Brann, wrote: "[…] (*ἀριθμός*) is rendered in the German text as *Anzahl*: 'a number of [things],' to distinguish it from our modern *Zahl*: 'number.' […] *Anzahl,* like *Zahl,* has been rendered simply as 'number,' *although it is a chief object of this study to show that Greek* 'arithmos' *and modern 'number' do not mean the same thing,* that they differ in their *intentionality,* for the former intends *things,* i.e., a number of them, while the latter intends a *concept,* i.e., that of quantity".[93] According to Klein, it can be proven that "*arithmos never* means anything other than 'a definite number of definite objects'.[94] We will see how this definition of ἀριθμός as a general concept in the Greek language seems highly plausible, although the Greek philosophical concepts of ἀριθμός are a bit more elaborate and obscure. It seems to me as if Klein

[90] See: A. Wedberg (1955) 19–20. "[…] a juster title would be: 'Perhaps Plato's philosophy of mathematics'".
[91] J, Klein (1934–1936).
[92] J. Klein (1992), 4.
[93] J. Klein (1992), vii, translator's note (Eva Brann), cursive emphasis is Brann's.
[94] J. Klein (1992), 7.

is in favor of the Neoplatonic definitions (in *Gorgias* and *Charmides*)[95] of arithmetic and logistic where arithmetic "is concerned with the 'kinds' (εἴδη) of numbers" and the logistic "with their 'material' (ὕλη)",[96] for he states that: "Greek arithmetic is therefore originally nothing but the theory of the *eide* of numbers, while in the art of 'calculating', and therefore in theoretical logistic as well, these counted collections are considered only with reference to their 'material', their *hyle*, that is, with reference to the units as such. The possibility of theoretical logistic is therefore totally dependent on the mode of being which the pure units are conceived to have."[97]

Professor Edward A. Maziarz' and Professor Thomas Greenwood's *Greek Mathematical Philosophy*[98] *has very curiously been neglecte*d *by* Szabó, Fowler, Pritchard, Cuomo, White *and basically every other scholar on Greek mathematics* (!). As it looks now, the book is out of print (was it so from early on?) and Klein's *Greek Mathematical Thought* has gained all the attention (both were released 1968, in English editions). Are these some of the main reasons for the neglect? Maziarz' and Greenwood's book is a well-written and comprehensive survey of the interaction between Greek mathematics and philosophy from its earliest beginnings with the Pre-Socratics and Pythagoreans, through Plato and Aristotle, up until Euclid. I have consulted this book for a general overview of Greek mathematics, and I have applied its research and conclusions in some places in my thesis. As it seems to me, after perusing and consulting it, their splendid work is one of the landmarks in modern scholarship of both the *practice* and *philosophy* of ancient Greek mathematics. This will become clearer in my analysis. It is high time that their work receives the recognition which it very strangely has been denied.

The problem of whether Plato posited 'Mathematical Ideas' as intermediates between the 'Ideas' themselves and the sensible world is a still debated question, and one which Wedberg investigated. He claims that Aristotle thought so.[99] In connection to this, and verily on the nature of 'Number' itself, a quite simple and succinct explanation has been offered by a great philosopher and mystic of the 20th century. Perhaps this homeschooled scholar[100] should not be cited in an academic work as this, but since I will not be using any of his theories more than

[95] He does not specify exactly which Neoplatonic authors, perhaps since he only mentions them by way of introduction.
[96] J. Klein (1992), 7.
[97] J. Klein (1992), 7–8.
[98] E. A. Maziarz & T. Greenwood (1968).
[99] A. Wedberg (1955), 11.
[100] And a genius to say the least, if I'm allowed to be subjective for a short moment.

mentioning them here, I cannot see how this would work against our search for knowledge. Manly Palmer Hall claims in one of his many lectures that the Pythagorean doctrine made two general differences: *Numeration* and *Number*. “Numeration is number *in principio*. Numeration is a concept of number, but not number itself.”[101] We have the ‘Number’ three in three doves, for instance, and we have the *concept of number three* (i.e. ‘Numeration’) in the fact that three entities come together and form *one group consisting of three*. This example can be extended to basically anything that can be added in a ‘common group’.[102]

Árpád Szabó has already been shortly mentioned. Szabó’s work[103] dealt mainly with the early history of the deductive method in Greek mathematics; he claims that it was actually derived from the school of the Eleatic philosophy. In connection to this he examined the discovery of *linear incommensurability*, the concept of δύναμις as “square”, and the history of Euclid’s theory of proportions. We will see that for our purpose, apart from his invaluable comments and insistence on philological analysis, he has some important statements on Greek arithmetic and geometry.[104] Furthermore, Szabó argued that by linguistic analysis “it can be shown that all the technical terms of the geometrical theory of proportions have their origins in music.” The terms ‘ratio’ as λόγος and ‘sameness of ratio’ as ἀναλογία, for instance, entered into geometry only after they were discovered in music, according to Szabó, and his finding was only possible by

[101] M. P. Hall, *Seminar Series - The Pythagorean Theory of Number.*

[102] Cf. Á. Szabó (1978), 306: “It is relatively easy to maintain that numbers are ideal entities which have no bodies. Any given number (23, for example) is readily seen to be an abstraction once it has been distinguished from the objects which it counts. A higher level of abstraction is reached when properties (or sets) of numbers are considered instead of individual ones.”; cf. E. A. Maziarz & T. Greenwood (1968), 128–129: “As regards the function of duplication, it can be rightly assigned to the dyad rather than to the auto-dyad. Plato could not confuse the 2 in itself with the dyad. The expendable and contractile nature of the latter cannot be identified with the changeless and absolute character of the former without obliterating one of the most remarkable Platonic intuitions and introducing confusion into a theory otherwise relatively clear. [n23: This confusion of the dyad with 2 may account for some of Aristotle’s criticisms of the Platonic conceptions]. The distinction between the auto-dyad and the dyad becomes more significant if the first is identified with *twoness* and the second with *twiceness*. [n24: J. Cook Wilson, ‘On the Platonist Doctrine,’ *Classical Review*, 18 (1904), 247-260.]. If twoness is the actual essence of the mathematical number 2, twiceness is the ability of any mathematical number to proceed from itself to another number and to be integrated into measures and formulas. Hence, twiceness is neither a limit, nor a measure, nor a magnitude, nor a quantitative determinant of any kind. By its agency and its various functions (greater, less, doubling, halving), as well as by its combination with the one, twiceness produces all the real numbers.”; cf., also, Plotinus’ statements on everything participating in ‘The One’, sharing in ‘The One’ according to their own degree (Plotinus, *Enneades*, VI, 9 [9], 1).

[103] Á. Szabó (1978).

[104] E.g.: “My claim is that the construction of mathematics as a deductive system came about because of certain problems encountered in *geometry*. It is true that Eleatic doctrine can be applied more easily to arithmetic than to *geometry* and that the Greeks therefore regarded arithmetic as the superior science; however, this ranking was only a *theoretical* one. Euclid’s mathematics is predominantly geometrical in character; even his arithmetic takes a geometrical form. This should not surprise us in view of the fact that the problems which caused mathematicians to break with Eleatic philosophy came principally from *geometry* and the outcome of this break was a theoretical foundation for *geometry*.” Á. Szabó, (1978), 317.

linguistic analysis. It is thus proven that the discoveries in music of the Pythagoreans were so essential to Greek mathematics that coining phrases like ‘to Pythagoreanize’[105] could *almost* be dispensed with, since the Pythagoreans really seem to be the *fundamentum* of all later Greek mathematics.[106]

Paul Pritchard’s *Plato’s Philosophy of Mathematics*[107] is valuable and relevant in many ways, but unclear and hasty in some concerns. The conclusion of his 170-pages book is only ca. one page. Here he does not mention his key arguments for all his opinions and conclusions found throughout the work. What I have found most valuable in his book is that he insists on separating our modern mathematical notions of the positive integers from the ancient Greek idea of ‘*arithmos*’, and that he has pointed out and often corrected the confusions and inconsistencies of modern scholars on the Greek concept of ‘*arithmos*’. Here follows some examples of the unclarities noted by Pritchard in the scholarly literature. In one passage of Euripides,[108] Wilamowitz wrote that “ἀριθμός means ‘calculation’”, whereas as Pritchard wrote, ἄστρων ἀριθμός could only mean “an *arithmos* of Stars” i.e. “constellation”. Bond, even more confusingly, maintained that the aggregate (constellation) “may be considered as the sum or proportion (ἀριθμός) of its units”.[109] Also, Pritchard says: “We must be careful to distinguish the relation between an *arithmos* and the units which constitute it, from the relation between a number (a universal) and a set of which it is the number.”[110] On this, Pritchard wrote that Annas has confused these relations when she states that: “once more it seems that numbers are not being distinguished carefully enough from numbered groups.”[111] Annas claims that both Plato and Aristotle have confused ‘numbers’ with ‘numbered groups’, but Pritchard argues rather that neither of them are referring to ‘numbers’, but rather to ‘*arithmoi*’: “they speak only of *arithmoi*

[105] Coined by Michael J. White, see below.
[106] See the section 5.4. for a discussion on the similarities between Pythagoras’ purported doctrines and Plato’s.
[107] P. Pritchard (1995).
[108] καὶ τῷδ᾽ ἦν τούς τε κακοὺς ἂν
γνῶναι καὶ τοὺς ἀγαθούς,
ἴσον ἅτ᾽ ἐν νεφέλαισιν ἄ-
στρων ναύταις ἀριθμὸς πέλει. (Eur. *HF* 665–668).
[109] P. Pritchard (1995), 29–30.
[110] P. Pritchard (1995), 17.
[111] “Plato takes it as obvious that a number is a number *of* something; the plain man’s number is a number of shoes, so the philosopher’s number must be a number of pure units. Once more it seems that numbers are not being distinguished carefully enough from numbered groups.” As cited by P. Pritchard (1995), 17.

– which might reasonably be called ‘numbered groups’.”[112]

Tarán (and others before him), also, rather hastily assumed Plato’s Ideas of *arithmoi* in the *Phaedo* 101b–c to be the ‘natural numbers’, or as he wrote: “Plato’s ideal numbers are the hypostatization of the series of natural numbers.” Tarán argued that this concept of number was not understood by Plato’s contemporaries (e.g. Speusippus, Xenocrates, Aristotle), nor by the ancients at large, and that “the conceptual priority of the cardinal numbers” were later rediscovered during the 19th and 20th centuries, with modifications from Plato’s views.[113] These ideas are tempting to accept, if the ‘hypostasized’ natural numbers, equivalent to Plato’s ‘Ideal Numbers’, are different from the ‘natural numbers’ as we know them. But, as Pritchard argues, that if the ‘ideal *arithmoi’* cannot be manipulated as the natural numbers can (e.g. in addition, subtraction, or division), then how does Tarán reason by identifying the natural numbers with Aristotle’s ‘ἀσύμβλητοι ἀριθμοί’ which cannot be manipulated?[114] And, If I may add, on what grounds does Tarán claim that not only Speusippus, Xenocrates, and Aristotle, but “the ancients generally” did not understand Plato’s notion of Ideal Numbers? However, to come to Tarán’s aid, it is not clear if he literally identifies the natural numbers with Plato’s Ideal Numbers (as Pritchard claims), he rather calls Plato’s Ideal Numbers, as we just saw, “the hypostatization of the series of natural numbers”, where presumably the natural numbers assume another nature and function as Ideal Numbers (?).

Tarán furthermore assumed that the ideal numbers, for instance the ‘ideal two’ or ‘ideal three’, are not two or three units that could possibly be added, but that: “these numbers are just Twoness, Threeness, and Fiveness, each being a unity which is irreducibly itself and nothing else”.[115] As Pritchard counters, why would the ancients then have had the conception of e.g ‘Twoness’ as being a set of two units? On this, and Gallop’s translation of Plato’s Phaedo 104a-b involving ‘threeness’ and ‘fiveness’, Pritchard argues that: “whatever Plato means by ἡ τριάς etc. these must be things which can reasonably be said to be odd or even, which Threeness cannot be, unless it is by some extension of meaning.”[116] Here, however, it does not seem clear why Tarán’s and Gallop’s ‘Threeness’ cannot be odd but “by some extension of meaning” (as Pritchard

[112] P. Pritchard (1995), 17.
[113] P. Pritchard (1995), 33.
[114] P. Pritchard (1995), 34.
[115] P. Pritchard (1995), 34.
[116] P. Pritchard (1995), 34–35.

thinks) while still remaining ’Threeness’.[117]

Rather, it seems to me, that we are dealing with two different concepts of ἀριθμός, just as there seems to have been two different concepts of ‘ἕν’ in Plato’s works. I refer the reader to subsection 5.1.2. ‘The ἀριθμός, μονάς and ἕν before and after Plato’, of this thesis.

Finally, Wedberg maintained that both the common Greek and Pythagorean notion of number seemed imperfect to Plato, with his ‘Theory of Ideas’ in mind. Since numbers are said to be predicated on other things, Wedberg argues that Plato found this faulty, and that Plato’s criticism of this can be found in *Phaedo* 101b–d and in *Philebus*.[118] About this common Greek notion of number, Wedberg wrote: “The common Greek definition of number as ‘plurality of units’ tells us merely of what numbers are predicated, not what numbers are in themselves. Plato cannot have been satisfied with it.” Pritchard counters with:

> Surely a definition should not be found faulty on the grounds that it applies to the thing of which the term is predicated. On the contrary, this would seem to be a necessary condition for any definition to be sound. […] The difficulty is that *‘arithmos’* just *means* ‘plurality of units’, and not ‘number’ in the sense which Wedberg employs, that is to say, in some post-Renaissance sense. As for Plato’s alleged dissatisfaction with the common Greek notion, we have seen that there is nothing in the dialogues to suggest this.[119]

Many of the scholarly arguments, conclusions, and claims are hence rather hasty and unaccounted for. It seems that many of these researchers were so eager to explain the philosophy of ancient Greek mathematics that they stumbled over the ancients and gave their own personal reflections instead.

David Fowler’s *The mathematics of Plato’s Academy: A new reconstruction* examines both the technical and some of the philosophical aspects of the early Greek mathematics up until Euclid

[117] On the footnote to this, Pritchard wrote “for example, in the way called by Vlastos ‘Pauline predication’, e.g. ‘Charity is long-suffering’”. P. Pritchard (1995), 34:n11. But this statement does not seem to clarify anything, in my humble opinion.
[118] A. Wedberg (1955), 74–75. Wedberg does not specify in which exact paragraphs of Plato’s texts this can be found, but I assume that he refers to the passages in his appendices.
[119] P. Pritchard (1995), 60:n29.

and Archimedes.[120] Fowler maintains, in his new interpretation of Greek mathematics, that the concepts of ratio, for him the method of '*anthyphairesis*', was at the core of Greek mathematics. Greek mathematics was also different from the mathematics of other contemporary cultures, which was most conspicuous in the geometry of Euclid. Many chapters provide a very specialized knowledge of mathematics.

To 'Pythagoreanize' is a term coined by Michael J. White in his recent article *Plato and Mathematics*.[121] White means that an explanation of Plato's mathematical ontology as linked to Pythagorean notions of ethics and value is a viewpoint that 'Pythagoreanizes'. White cites the statement of the scholar M. F. Burnyeat as an example of this: "the content of mathematics is a constitutive part of ethical understanding"[122] in the idea that the actual mathematical theorems and axioms impart a peculiar form of value and ethics since, as Burnyeat succinctly maintained: "the goal of the mathematical curriculum is repeatedly said [by Plato] to be knowledge of the Good (526de, 530e, 531c, 532c)".[123]

As to the question of any possible connection between *technical mathematics* and *wisdom* (or 'Pythagoreanizing Platonism'), White concludes in his article that:

> There certainly is an aesthetic dimension to the way many mathematicians, particularly those who work in certain areas of 'pure mathematics', conceptualize their discipline. However, I am inclined to think that the aesthetic value that they discern is very much discipline-specific. It may well be true that there is a sense in which a mathematician such as John Nash has 'a beautiful mind.' But does it follow that his mind is therefore *kalos kai agathos,* 'noble and good,' either in the Platonic or some other, more common sense? Pythagoreanizing Platonism must confront the negative answer that I – and, I think, most of us – are inclined to give.

I think that White commits two mistakes here, as has been common in trying to understand the difference between ancient Greek and modern mathematics. Firstly, he confounds ancient Greek

[120] D. H. Fowler (1999).
[121] M. J. White (2006.
[122] M. J. White (2006), 234.
[123] M. J. White (2006), 234.

mathematics (whether technical or philosophical) with modern mathematics (by mentioning John Nash), and secondly, he seems to think that he has understood to the fullest extent the texts and teachings of Plato and the other ancient philosophers and mathematicians. We cannot therefore, in all earnestness, say that "most of us" agree with White, as he would have it. We will see how his first mistake can and has been mended (e.g. by Klein, Szabó, Pritchard and Fowler). But as for his second mistake, we must refer him to the famous saying of Socrates: "ὅτι ἃ μὴ οἶδα οὐδὲ οἴομαι εἰδέναι."[124]

[124] Pl. *Ap.* 21d.

5. Analysis

5.1. Mathematics in Platonism (and in 'Pythagoreanized' Platonism)

5.1.1. The hierarchical classification of mathematics during the classical period

μαθηματική[125] as a τέχνη – where it included the *quadrivium* of arithmetic, geometry, astronomy, and music – has as its earliest exponents the Pythagoreans. The term *'quadrivium'* was of course coined much later on by the Romans, but some scholars maintain that the Pythagoreans grouped together the four just mentioned subjects into an educational curriculum because of their similarities. As noted by Ian Mueller, the five subjects mentioned by Socrates in the *Res Publica* VII are easily reducible to four: Arithmetic, Geometry (and Stereometry), Astronomy, and Harmonics.[126] It is important to note however, that although the remaining subjects are three as in the *'trivium'* they are different from the later *trivium* of logic, rhetoric and grammar; they are music, gymnastics, and dialectic. Also, music and gymnastics are the first subjects to be taught to the guardians in Plato's state, and dialectic is the final and most important.[127] Mathematics as we know it today was classified as such much later (e.g. by comprising some form of arithmetic and geometry while excluding the rest of the *quadrivium*), and the process towards that reduction was begun by Aristotle.[128] The Roman author Aulus Gellius informs us that Pythagoras, after admitting someone into his fellowship, first ordered them to keep silent for at least two years during which time they were part of the *ἀκουστικοί* (*listeners*) and had to simply learn and keep quiet. When they had passed this stage, they were allowed to ask questions and inquire further into the Pythagorean knowledge, they were now admitted into the group of *μαθηματικοί*. Gellius tells us that for the ancient Greeks, 'μαθήματα' was *geometry, gnomonics, music*, "*ceterasque item disciplinas altiores* μαθήματα *veteres Graeci appellabant*".[129] Finally, they proceeded to the

[125] μάθημα, ατος, τό, ranged from the basic meaning of '*lesson'* to '*learning, knowledge* and *the mathematical sciences'* (*LSJ*).
[126] As cited by J. Furner (2021), 502; 508.
[127] Pl. *Resp.* VII, 521–533.
[128] J. Høyrup, & M. Folkerts (2006).
[129] Gell. *NA.* I, IX: 6. Gellius' mentioning of Greek mathematics is also (very) shortly referenced by the *New Pauly* article above (J. Høyrup, & M. Folkerts, 2006).

work of observing the world and the principles of nature, whence they were called φυσικοί.[130]

We will not focus too much on whether Plato did employ the *quadrivium* or not, but we will see how closely it resembles his ten-year education for his philosophical ruler(s), the φύλαξ (-ακος, ὁ; also ἡ, *watcher, guardian, keeper, protector*).[131]

Plato posited the most important παιδεία during the first five years of his young *guardians* as being *music* for the soul and *gymnastics* for the body.[132] For the next ten years, the five μαθήματα, in order of importance, were *arithmetic, plane geometry, solid geometry, astronomy, and harmonics.* After this, the philosophical art called *dialectic* was to be studied for five years; and for the last 15 years the guardian should become experienced in politics and military matters.[133] The guardian could then continue with philosophy and governance of the state. Some other ancient authors gave another classification attributed to Plato, quite similar to Aristotle's; the partition of knowledge into *theoretical*, *practical,* and *productive* sciences, with mathematics belonging to the first. These matters of classification have no greater bearing upon our aim though, neither does time and space allow for any further examination of this, so we will focus on Plato's supposedly own classification instead, in his extant writings.

Both Proclus and the author of the *Epinomis* (generally agreed by modern scholars as not being Plato's work[134]) agree about geometry having second place after arithmetic (and logistic?).[135] The *Epinomis* has been accused of having several discrepancies,[136] but this is actually not so clear in the case of e.g. Plato's *astronomy.* The idea of astronomy having preeminence is according to current scholarship a discrepancy which, amongst others, show that Plato was not the author. In the *Epinomis* it is stated that: "[…] ἀγνοεῖ τε ὅτι σοφώτατον ἀνάγκη τὸν ἀληθῶς ἀστρονόμον εἶναι".[137] The "wisest person is the astronomer", but not the one who practices it like Hesiod and others did, but rather he who examines the seven orbits, of the eight orbits ("[…] ἀλλὰ τὸν τῶν

130 Gell. *NA.* I, IX: 1–7.
131 The following exposition is based upon Pl. *Resp.* VII, 521–533 & J. Furner (2021). Plato uses the masculine and feminine form of the noun φύλαξ interchangeably throughout the *Res Publica*. e.g., cf. V, 461e & VII 522a.
132 Pl. *Resp*. II, 376e.
133 Pl. *Resp.* VII, 521–533.
134 J. M. Cooper & D. S. Hutchinson (1997), 1617.
135 Procl. *in Euc.* §48.9; Pl. *Epin.* 990c–d. Also cited by Á. Szabó (1978), 308.
136 J. M. Cooper & D. S. Hutchinson (1997), 1617.
137 Pl. *Epin.* 990a.

ὀκτὼ περιόδων τὰς ἑπτὰ περιόδους").[138] If we also turn to Plato's *Timaeus*, we actually find a similar statement on the high eminence of ἀστρονομία. Timaeus tells us that sight has been the cause of the greatest benefit for us; by seeing the stars, the sun and heaven we have come to understand the universe. More importantly: "νῦν δ᾽ ἡμέρα τε καὶ νὺξ ὀφθεῖσαι μῆνές τε καὶ ἐνιαυτῶν περίοδοι καὶ ἰσημερίαι καὶ τροπαὶ **μεμηχάνηνται μὲν ἀριθμόν**"; "observance of the periods of day and night, of months and years, and of equinoxes and solstices, **has invented [for us] ἀριθμός/number**".[139] Furthermore, this has *even brought us to philosophy itself*, and the great gift of sight is that by observing the periods of the mind in the universe we may apply them to the revolutions of our own understanding.[140]

There is the question if Plato agrees with what Timaeus is purported to have said in this dialogue, and there certainly is reason to believe so, since Plato obviously chose to include this statement in his text. Nevertheless, we see that although astronomy does not necessarily have the highest importance here, as in the *Epinomis*, still it has undoubtedly a very high rank since it is claimed that even *number itself* (ἀριθμός) has been invented because of ἀστρονομία. The gift of sight and the sciences that have followed have even led to *philosophy itself* (!). With all this in mind, and the ambiguity in Plato's *Res Publica* whether arithmetic or dialectic is most important – since the former is the beginning of education, but the latter is the end – shows how Plato's views on these matters might have changed during his life. Scholars agree at the moment that *Laws, Philebus, Sophist* and *Statesman* were some of the last works written by Plato.[141] As noted by E. A. Maziarz and T. Greenwood: "These general views about mathematics, as expressed in the seventh book of the *Republic*, do not represent Plato's final thought on the relation between mathematics and true knowledge", and concerning 'dialectic' as being "more effective than mathematics", in the *Res Publica* VII it will "lead to the proper apprehension of numbers and forms".[142] Although my thesis mainly deals with Plato's texts, it is ultimately unclear what his final opinions were, and we can but consult his *Letters* (some of which are considered spurious) and what his contemporaries (e.g. Aristotle) or what the later Neoplatonists wrote.

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138 "[…] ἀγνοεῖ τε ὅτι σοφώτατον ἀνάγκη τὸν ἀληθῶς ἀστρονόμον εἶναι, μὴ τὸν καθ' Ἡσίοδον ἀστρονομοῦντα καὶ πάντας τοὺς τοιούτους, οἷον δυσμάς τε καὶ ἀνατολὰς ἐπεσκεμμένον, ἀλλὰ τὸν τῶν ὀκτὼ περιόδων τὰς ἑπτὰ περιόδους, διεξιούσης τὸν αὑτῶν κύκλον ἑκάστης οὕτως ὡς οὐκ ἂν ῥᾳδίως ποτὲ πᾶσα φύσις ἱκανὴ γένοιτο θεωρῆσαι, μὴ θαυμαστῆς μετέχουσα φύσεως." (Pl. *Epin.* 990a–b).
139 Pl. *Ti.* 47a.
140 Pl. *Ti.* 47a–b.
141 J. M. Cooper & D. S. Hutchinson (1997), 398.
142 E. A. Maziarz & T. Greenwood (1968), 97.

It should also be mentioned that in the *Res Publica,* all previous learning is merely preparatory for διαλεκτική.[143]

5.1.2. The ἀριθμός, μονάς and ἕν before and after Plato.

In Plato's writings ἀριθμός, μονάς and ἕν are repeatedly mentioned in the passages dealing with ἀριθμητική and λογιστική. As we shall see in the subsections below, μονάς and ἕν are possibly interchangeable.[144]

According to Pritchard, ἀριθμός occurs three times in Homer, and "in every case the meaning is *collection of things falling under some description"*.[145] The examples show that either humans or animals are being counted: "an *arithmos* is something which can be counted.", e.g. "it denotes a set, the collection of seals on the beach".[146] After surveying more examples from Euripides and Aristophanes, Pritchard maintains that no difference is suggested between pre-Platonic habit and that of the mathematicians: "an *arithmos* is a collection of items answering to the same description".[147]

Fowler has succinctly described the difference between the Greek 'arithmetised' mathematics and the 'arithmetised' mathematics of the other cultures, such as the Babylonians and Egyptians. Both of the latter cultures employed the "regular numbers" in their calculations, with the Babylonians using e.g. fractions, irrational and infinitesimal numbers etc. Fowler claims that the different ancient cultures developed their peculiar forms of 'arithmetised mathematics'.[148] Although the Greeks had their techniques for the positive numbers, Fowler argues that up until

[143] "τὰ μὲν τοίνυν λογισμῶν τε καὶ γεωμετριῶν καὶ πάσης τῆς προπαιδείας, ἣν τῆς διαλεκτικῆς δεῖ προπαιδευθῆναι, παισὶν οὖσι χρὴ προβάλλειν, οὐχ ὡς ἐπάναγκες μαθεῖν τὸ σχῆμα τῆς διδαχῆς ποιουμένους." *Resp.* VII, 536d.
[144] See: J. Klein (1992), chapter 6 (pp. 46–61), for a wide-ranging discussion on ἀριθμός and μονάς in Platonic and Neoplatonic texts.
[145] P. Pritchard (1995), 27.
[146] P. Pritchard (1995), 27.
[147] P. Pritchard (1995), 29; cf. also J. Klein (1992), 51: "This is how the traditional 'classical' definitions of *arithmos* are to be understood; Eudoxus (Iamblichus, *in Nicom.* 10, 17 f.) 'A number is a *finite* multitude [of units]' (*ἀριθμός ἐστιν πλῆθος ὡρισμένον*) – cf. Aristotle, *Metaphysics* Δ 13, 1020 a 13: 'limited multitude' (*πλῆθος πεπερασμένον*); Eucl. (VII, Def. 2): 'The multitude composed *of units*' (*τὸ ἐκ μονάδων συγκείμενον πλῆθος*) […]."
[148] D. H. Fowler (1999), 9.

the second century B.C., early Greek mathematics was "completely non-arithmetised".[149]

For Fowler, ἀριθμητική is best understood as "number theory", where ἀριθμοί are manipulated in e.g. multiplication or division.[150] On the μονάς, the *Theologoumena Arithmeticae*, supposedly by the Neoplatonist Iamblichus, states: "ἀρτία τε οὖσα καὶ περιττὴ καὶ ἀρτιοπέριττος" (περὶ μονάδος, I: sentence 12); "the μονάς is even, and odd, and even-odd". As the footnote in the English translation by Robin Waterfield explains, when the μονάς is added to an even number, the result is odd, and when added to an odd number, the result is even. The μονάς has therefore the characteristic of both 'evenness' and 'oddness'.[151] When defining the difference between ἀριθμητική and λογιστική, Socrates explains the use of τὸ ἄρτιόν τε καὶ περιττόν (the even and the odd) by these two sciences.[152] Plato also conceptualized τὸ ἄρτιόν τε καὶ περιττόν as some form of ἀριθμοί.[153] Plato also mentions ἕν, δύο, τρία in the same sentence with ἄρτια καὶ περιττά and with ἀριθμεῖν, in *Leges* 818c.[154] We must remember that most of Plato's texts are *dialogues* and not scientific tracts dealing with technicalities. They were written for public dissemination. In any case, For the writer of the *Theologoumena Arithmeticae*, the μονάς is certainly connected to ἀριθμός, as it is so clearly in Euclid (as we shall see): "μονάς ἐστιν ἀρχὴ ἀριθμοῦ, θέσιν μὴ ἔχουσα."; "The μονάς is the source of ἀριθμός, it has no local position/spatiality."[155] Or, as Waterfield would have it translated: "The monad is the non-spatial source of number."[156]

How the μονάς is concretely related to ἀριθμοί seems to be explained in the same chapter.[157] When dealing with the "πλήθους σύστημα ἢ ὑποτομῆς μόριον" (e.g. the decad or a tenth) which "κατὰ μονάδα εἰδοποιεῖται" ("is endued with form by the μονάς"), it is said that: "καθ' ἕκαστον

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[149] D. H. Fowler (1999), 9–10.
[150] D. H. Fowler (1999), 15.
[151] Iambl. *Theologoumena Arithmeticae* 1; R. Waterfield (1988), 35: n2.
[152] Pl. *Grg.* 451a-c & *Chrm.* 166a. See the subsection 5.4.2. It is quite peculiar that the literal meaning of the adjective 'ἄρτιος' is 'complete, perfect of its kind, suitable' etc. when the mathematical meaning is 'even'; and 'περισσός' literally meant 'beyond the regular number or size, superfluous, excessive' etc. while the mathematical meaning is 'odd'. Why this would seem strange is because both Plato and Pythagorean teachings related 'ἄρτιος' (even) more with something indefinite and superfluous and 'περισσός' (odd) with something 'perfecting, completing' and setting a limit to the unlimited. See: Pl. *Phil.* 16c–d & Arist. *Met.* I, 986a.
[153] "κάλλιον δέ που καὶ μᾶλλον κατ' εἴδη καὶ δίχα διαιροῖτ' ἄν, **εἰ τὸν μὲν ἀριθμὸν ἀρτίῳ καὶ περιττῷ τις τέμνοι, τὸ δὲ αὖ τῶν ἀνθρώπων γένος ἄρρενι καὶ θήλει**". Pl. *Plt.* 262e. Bold emphasis is mine.
[154] "πολλοῦ δ' ἂν δεήσειεν ἄνθρωπός γε θεῖος γενέσθαι μήτε ἓν μήτε δύο μήτε τρία μήθ' ὅλως ἄρτια καὶ περιττὰ δυνάμενος γιγνώσκειν, μηδὲ ἀριθμεῖν τὸ παράπαν εἰδώς" (Pl. *Leg.* 818c).
[155] Iambl. *Theologoumena Arithmeticae*, I: 1.
[156] R. Waterfield (1988), 35.
[157] Iambl. *Theologoumena Arithmeticae*, I: 12–23.

δὲ τούτων εἴδει μὲν ἡ αὐτὴ μονάς, μεγέθει δὲ ἄλλη καὶ ἄλλη" ("in each of these the μονάς is the same in form, but different in magnitude"). The μονάς is hence said to forever remain the same in one aspect (εἴδει), but in another aspect (μεγέθει) it does simultaneously differ. Furthermore, the μονάς is even likened to "τὸν περὶ θεοῦ λόγον"; "the principle of God", literally showing the *theological* property of ἀριθμητική, as the title of the work suggests.

Pritchard argues that the scholar Egger's assertion that 'one' is an ἀριθμός is *not* proven by this statement in Herodotus: "καὶ ἓν κεράμιον οἰνηρὸν ἀριθμῷ κείμενον οὐκ ἔστι (ὡς λόγῳ εἰπεῖν) ἰδέσθαι."[158] Pritchard's motivation for this is the fact that Aristotle enumerates several different things that can be called 'ἓν', e.g. κατ᾽ ἀριθμόν, κατ᾽ εἶδος, κατὰ γένος.[159] Pritchard might have taken this theory too far, for even if Plato and Aristotle may have not denoted the 'unit' (μονάς) as an ἀριθμός,[160] Plato sometimes referred to the ἓν as a number and sometimes as something separate from ἀριθμός.[161] Moreover, if we are to speak plainly and refrain from illogical conclusions, how could the Greek mathematicians refer to the concept of 'one ἀριθμός' without having a similar notion as e.g. 'one finger' or 'one ox'? We need not conclude that 'ἀριθμός' is the same thing as 'the natural numbers' even if we count 1, 2, 3 ἀριθμοί, but it stands as evident, to me at least, that ἀριθμοί must be *something* that is *counted* or *calculated*.[162]

Let us briefly turn to Aristotle and see why Pritchard has exaggerated his theory. The same passage in Aristotle that we just mentioned is dealing with this topic, as stated by Aristotle in a passage before: "ἓν λέγεται τὸ μὲν κατὰ συμβεβηκὸς τὸ δὲ καθ᾽ αὑτό".[163] Aristotle thus begins this specific discussion by distinguishing between two concepts of 'ones': the ἓν which is called so *by chance*, and the ἓν which *is* ἓν by its *very nature*. Of the latter, he thereafter repeatedly gives examples of different things that "*are called one*" ("…ἓν λέγεται…").[164] Furthermore,

[158] Hdt. *Historiae*, 3,6. As cited by Pritchard (1995), 70.
[159] "ἔτι δὲ τὰ μὲν κατ᾽ ἀριθμόν ἐστιν ἕν, τὰ δὲ κατ᾽ εἶδος, τὰ δὲ κατὰ γένος, τὰ δὲ κατ᾽ ἀναλογίαν, ἀριθμῷ μὲν ὧν ἡ ὕλη μία, εἴδει δ᾽ ὧν ὁ λόγος εἷς, γένει δ᾽ ὧν τὸ αὐτὸ σχῆμα τῆς κατηγορίας, κατ᾽ ἀναλογίαν δὲ ὅσα ἔχει ὡς ἄλλο πρὸς ἄλλο." (Met. V, 1016b:31–35). Also as cited by Pritchard (1995), 70.
[160] Pritchard (1995), 70; 77.
[161] "πολλοῦ δ᾽ ἂν δεήσειεν ἄνθρωπός γε θεῖος γενέσθαι μήτε ἓν μήτε δύο μήτε τρία μήθ᾽ ὅλως ἄρτια καὶ περιττὰ δυνάμενος γιγνώσκειν, μηδὲ ἀριθμεῖν τὸ παράπαν εἰδώς" (Pl. *Leg.* 818c); "Τί οὖν; ἀριθμός τε καὶ τὸ ἓν ποτέρων δοκεῖ εἶναι;" (Pl. *Resp*. 524d). Also as cited by Pritchard (1995), 70–71.
[162] Cf. J. Klein (1992), 46: "The fundamental phenomenon which we should never lose sight of in determining the meaning of *arithmos (ἀριθμός)* is counting, or more exactly, the *counting-off*, of some number of things."
[163] Arist. *Met.* V, 1015b:16.
[164] E.g. "ἔτι ἄλλον τρόπον **ἓν λέγεται** τῷ τὸ ὑποκείμενον τῷ εἴδει εἶναι ἀδιάφορον" (*Met.* V, 1016a:17–18); "**λέγεται δ᾽ ἓν** καὶ ὧν τὸ γένος ἓν διαφέρον ταῖς ἀντικειμέναις διαφοραῖς" (*Met.* V, 1016a:24–25); "**ἔτι δὲ ἓν λέγεται** ὅσων ὁ λόγος ὁ τὸ τί

Aristotle states “τὸ δὲ ἑνὶ εἶναι ἀρχῇ τινί ἐστιν ἀριθμοῦ εἶναι”; “to be one is to be the source of number”.[165] And in the same passage, line 20: “ἀρχὴ οὖν τοῦ γνωστοῦ περὶ ἕκαστον τὸ ἕν.”; “the one is the source/beginning of each knowable [γένος]”. In this passage,[166] Aristotle seems to begin with Pythagorean examples, citing the μέτρον (“anything measured”, or the musical “*metre*”) and for the different types of ‘ones’ he begins with the δίεσις; “[musical] interval”, which W. D. Ross in his commentary mentions as referring to the smallest interval in music: the minor semitone for the Pythagorean Philolaus, and the three different δίεσις for Aristotle’s pupil Aristoxenus (enharmonic, chromatic, hemiolian).[167] Could it here be denied then, in an almost preposterous manner, that the μέτρον, or the διέσεις or any knowable thing or class (γένος) must not be counted as first ‘one’ and continuing with 2, 3 etc.? Pritchard would have this as inconclusive.[168]

It seems rather, as we just saw in Plato’s different statements of what the first number is, that we are dealing with two different notions of the ἓν: (1) as equivalent to the concept of any μονάς, and (2) as one ἀριθμός.[169] Continuing with the ἓν and the μονάς, Aristotle tells us in line 23–26 that: “πανταχοῦ δὲ τὸ ἓν ἢ τῷ ποσῷ ἢ τῷ εἴδει ἀδιαίρετον. τὸ μὲν οὖν κατὰ τὸ ποσὸν ἀδιαίρετον, τὸ μὲν πάντῃ καὶ ἄθετον λέγεται μονάς, τὸ δὲ πάντῃ καὶ θέσιν ἔχον στιγμή”; “But everywhere, the one is indivisible, either in quantity or in form. That which is indivisible in quantity, that which altogether has no position, is called a μονάς, but that which altogether has a position, is called a point [στιγμή]”.[170] Aristotle provides here (line 24–31) a hierarchical order of those

ἦν εἶναι λέγων ἀδιαίρετος πρὸς ἄλλον τὸν δηλοῦντα τί ἦν εἶναι τὸ πρᾶγμα (αὐτὸς γὰρ καθ᾽ αὑτὸν πᾶς λόγος διαιρετός)” (*Met*. V, 1016a:32–35). Bold emphasis is mine.

165 Arist. *Met.* V, 1016b:17–18.

166 “τὸ δὲ ἑνὶ εἶναι ἀρχῇ τινί ἐστιν ἀριθμοῦ εἶναι: τὸ γὰρ πρῶτον μέτρον ἀρχή, ᾧ γὰρ πρώτῳ γνωρίζομεν, τοῦτο πρῶτον μέτρον ἑκάστου γένους: ἀρχὴ οὖν τοῦ γνωστοῦ περὶ ἕκαστον τὸ ἕν. οὐ ταὐτὸ δὲ ἐν πᾶσι τοῖς γένεσι τὸ ἕν. ἔνθα μὲν γὰρ δίεσις ἔνθα δὲ τὸ φωνῆεν ἢ ἄφωνον: βάρους δὲ ἕτερον καὶ κινήσεως ἄλλο.” (Arist. *Met*. V, 1016b:17–23).

167 W.D. Ross (1924), 304:n22.

168 “The wider question, whether Plato or Aristotle ‘use 1 as a number’, cannot be answered as it stands.” (P. Pritchard, 1995, 77).

169 This may explain the apparent incongruity between the mathematicians and the Eleatics which Szabó would have: “The definition of ‘number’ (VII.2) as a ‘multitude composed of units’ marked a departure from Eleatic teaching. It is true that arithmeticians treated ‘numbers’ in much the same way that the Eleatics treated ‘Being’; they emphasized that numbers were ideal entities which did not have visible or tangible bodies and could only be apprehended by the understanding. Arithmetic, however, required the existence of a ‘plurality’ (or, at least, of an ‘ideal plurality’), whereas Eleatic philosophy admitted only the existence of the ‘One’. In an earlier chapter we saw that the Eleatic problem of ‘divisibility’ took on a new meaning when numbers came to be regarded as multiples of the ‘One’. This seems to have given rise to one of the most important and basic problems of pre-Euclidean arithmetic, namely the problem of divisibility of numbers.” (A. Szabó, 1978, 305). Cf. also: “[…] καὶ τῶν ἓν ἐκείνων ἕκαστον πάλιν ὡσαύτως, μέχριπερ ἂν τὸ κατ᾽ ἀρχὰς ἓν μὴ ὅτι ἓν καὶ πολλὰ καὶ ἄπειρά ἐστι μόνον ἴδῃ τις, ἀλλὰ καὶ ὁπόσα:” (Pl. *Phil.* 16d)

170 Also, on the difference between the monad and the point: “τὸ δὲ μηδαμῇ διαιρετὸν κατὰ τὸ ποσὸν στιγμὴ καὶ μονάς, ἡ

mathematical principles that are undivided in quantity (μονάς and στιγμή) and those that can be divided in one, two, or three dimensions (γραμμή, ἐπίπεδον, σῶμα).

It seems obvious then, that ἀριθμός is a countable thing, and that the ἓν, in one of its philosophical concepts, can be considered as a *unity residing in different γένος:* "οὐ ταὐτὸ δὲ ἐν πᾶσι τοῖς γένεσι τὸ ἕν".[171] This hearkens also to the previously mentioned passage in the *Theologoumena Arithmeticae*: "καθ' ἕκαστον δὲ τούτων εἴδει μὲν ἡ αὐτὴ μονάς, μεγέθει δὲ ἄλλη καὶ ἄλλη".

As to what ἀριθμός might have been concretely, or conceptualized as in mathematical practice, the Neoplatonist Proclus informs us that one classification (seemingly Geminus') of ἀριθμητική is thus: "τῆς δὲ ἀριθμητικῆς ὡσαύτως ἡ διαίρεσις εἴς τε τὴν τῶν γραμμικῶν ἀριθμῶν θεωρίαν καὶ τὴν τῶν ἐπιπέδων καὶ τὴν τῶν στερεῶν."; "Likewise ἀριθμητική is divided into the study of linear ἀριθμῶν, plane ἀριθμῶν, and solid ἀριθμῶν".[172] We may notice immediately here that number (ἀριθμός) has a geometrical character (lines, planes, solids) in its physical and practical conceptualization. Also, the 'study' (θεωρία) of these ἀριθμῶν is a noun with the primary meaning of "*sending of* θεωροί or *state-ambassadors* to the oracles or games, or, collectively, *the* θεωροί *themselves, embassy, mission*", but also of course '*viewing, beholding, contemplation, consideration*' etc. We will see how this peculiar 'oracular' semantics often recurs in both Plato and Aristotle.

Finally, for the μονάς and ἀριθμός with their roles and hierarchical 'hypostatization' in mathematics and geometry, this quotation may suffice, attributed to the Pythagoreans by Alexander Polyhistor in Diogenes Laertius' *Vitae Philosophorum*: "ἀρχὴν μὲν τῶν ἁπάντων μονάδα· ἐκ δὲ τῆς μονάδος ἀόριστον δυάδα ὡς ἂν ὕλην τῇ μονάδι αἰτίῳ ὄντι ὑποστῆναι· ἐκ δὲ τῆς μονάδος καὶ τῆς ἀορίστου δυάδος τοὺς ἀριθμούς· ἐκ δὲ τῶν ἀριθμῶν τὰ σημεῖα· ἐκ δὲ τούτων τὰς γραμμάς, ἐξ ὧν τὰ ἐπίπεδα σχήματα· ἐκ δὲ τῶν ἐπιπέδων τὰ στερεὰ σχήματα·"[173]

μὲν ἄθετος μονὰς ἡ δὲ θετὸς στιγμή." (*Met.* V, 1016b:29–31).

[171] Arist. *Met.* V, 1016b:21.

[172] Procl. *in Euc.* §39. Proclus continued just after this sentence with: "καὶ γὰρ τὰ εἴδη τοῦ ἀριθμοῦ καθ' αὑτὰ σκοπεῖ προϊόντα ἀπὸ μονάδος, καὶ τὰς γενέσεις τῶν ἐπιπέδων τῶν τε ὁμοίων καὶ τῶν ἀνομοίων, καὶ τὰς εἰς τρίτην αὔξην προόδους."

[173] Diog. Laert., *Vitae Philosophorum* VIII, 25. The quote continued with the elements, the world, etc: "ἐκ δὲ τούτων τὰ αἰσθητὰ σώματα, ὧν καὶ τὰ στοιχεῖα εἶναι τέτταρα, πῦρ, ὕδωρ, γῆν, ἀέρα· ἃ μεταβάλλειν καὶ τρέπεσθαι δι' ὅλων, καὶ γίγνεσθαι ἐξ αὐτῶν κόσμον ἔμψυχον, νοερόν, σφαιροειδῆ, μέσην περιέχοντα τὴν γῆν καὶ αὐτὴν σφαιροειδῆ καὶ

Paraphrased, Alexander Polyhistor told us that ‘the origin/principle of all things is the monad. From it the boundless dyad subsisted as matter for the monad which is its cause. From the monad and the boundless dyad came numbers (ἀριθμούς). From the ἀριθμῶν came the points, from these the lines, then the plane figures and then the solid figures.’

This Pythagorean ‘hierarchical classification’ seems to be the framework of all later Greek mathematics, as we shall see.

5.1.3. The two kinds of sciences; the common and the philosophical concept of ‘μονάς’

In the *Philebus*, agreed by scholars as being one of Plato’s last works,[174] Socrates makes a distinction first between those sciences that are more accurate (e.g. building) and those that are less accurate (e.g. music). Socrates then continues to establish a difference in all the sciences *themselves*. Beginning with the most accurate sciences, as ἀριθμητικήν, Socrates says that there is one kind of “τῶν πολλῶν”, and one of “τῶν φιλοσοφούντων”.[175] The basic difference is that the former calculate with any sorts of unequal ‘units’ while the latter will only do the same if it is agreed upon that all those ‘units’ have no difference between each other.[176] The word choice of Plato here for a ‘unit’ is “μονάς”.

5.1.4. μονάς

The μονάς can be a concept for “μονάδας ἀνίσους” such as “στρατόπεδα δύο καὶ βοῦς δύο” which is the concept used by τῶν πολλῶν; but τῶν φιλοσοφούντων on the other hand, as said, make no difference between the myriads of all μονάδας. When dealing with the former way, whether it is two armies or two oxen, or two things of the greatest or smallest size, these μονάδες are ἀνίσους according to Socrates. Perhaps, since they are simply not of the same size or type.

περιοικουμένην. εἶναι δὲ καὶ ἀντίποδας καὶ τὰ ἡμῖν κάτω ἐκείνοις ἄνω.” Also mentioned in: E. A. Maziarz & T. Greenwood (1968), 39.

174 J. M. Cooper & D. S. Hutchinson (1997), 398.

175 Pl. *Phlb.* 56d.

176 Pl. *Phlb.* 56d–e: “οὐ σμικρὸς ὅρος, ὦ Πρώταρχε. οἱ μὲν γάρ που μονάδας ἀνίσους καταριθμοῦνται τῶν περὶ ἀριθμόν, οἷον στρατόπεδα δύο καὶ βοῦς δύο καὶ δύο τὰ σμικρότατα ἢ καὶ τὰ πάντων μέγιστα: οἱ δ᾽ οὐκ ἄν ποτε αὐτοῖς συνακολουθήσειαν, εἰ μὴ μονάδα μονάδος ἑκάστης τῶν μυρίων μηδεμίαν ἄλλην ἄλλης διαφέρουσάν τις θήσει.”

The sentence here has an important connection between μονάς and ἀριθμός: "οἱ μὲν γάρ που μονάδας ἀνίσους καταριθμοῦνται τῶν περὶ ἀριθμόν"; lit. "some count unequal (unlike) *monads* concerning the things of *arithmos*". We will see in the subsection on Euclid also, how μονάς and ἀριθμός are connected. It seems as if Socrates means that by the very fact of counting two sorts of oxen or two sorts of whatever *thing*, the idea of all monads being equal is denied, since it is not recognized that the monads are all the same by the very nature of making such a calculation. Still, the calculation is possible by the philosopher's method as well, as long as no distinction is made between the μονάδες. Pritchard maintains that the μονάς in this *Philebus* passage is equivalent to the ἕν in the *Res Publica*. This is in contrast to the scholar Julia Annas' opinion who stated that Plato moved on from the concept of ἕν in the *Res Publica* to μονάς in *Philebus.*[177] Pritchard argues that Plato does not always use the same word to denote the same idea.[178]

In usual mathematical practice, all of this would probably mean that the μονάδες of e.g. all Euclid's propositions consist of a peculiar concept of monads/units which, though they can be manipulated in different geometrical and arithmetical calculations, *are still the exact same μονάδες*. This will be more evident in the analysis that follows on Plato's distinction between 'visible' and 'intellectual' mathematics, the latter which is only possible to "ὧν διανοηθῆναι μόνον"; "have in *mind*".[179]

Plato's division into a 'theoretical arithmetic/logistic' and 'practical arithmetic/logistic' has also been noted by Klein and Wedberg.[180]

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[177] As cited by P. Pritchard (1995), 21: n27.
[178] P. Pritchard (1995), 21: n27.
[179] Pl. *Resp.* VII, 526a.
[180] J. Klein (1992), 6; A. Wedberg (1955), 22.

5.1.5. **μετρητική: "mensuration": mathematics as a 'means to an end' for reaching σωτηρία (salvation)**

Although the examination of 'μετρητική' is not part of my thesis' research questions, there are some key concepts of Plato's mathematics that are mentioned in connection with this subject. It seems as if the general aim and purpose of Plato's mathematics is mentioned in these passages.

In *Protagoras* 356a–357b we can evince that these τέχναι are crucial for Plato's doctrine of the soul's salvation. Here we find a short mention and definition of the art called "μετρητική"; "mensuration".[181] Socrates explains that in order to solve the confusion of appearances when e.g. an object is seen as smaller in the distance and greater when it is in proximity, an 'art of measurement' is needed. With similar wording as in the *Res Publica,*[182] Socrates says "εἰ οὖν ἐν τούτῳ ἡμῖν ἦν **τὸ εὖ πράττειν**…"[183]; "if our wellbeing/good conduct consisted of this…" when explaining how one should avoid the misconceptions of sense. The "deliverance" or "salvation" ("σωτηρία") would come from the art of μετρητική rather than from the power of appearances: "τίς ἂν ἡμῖν σωτηρία ἐφάνη τοῦ βίου; ἆρα ἡ μετρητικὴ τέχνη ἢ ἡ τοῦ φαινομένου δύναμις;".[184] μετρητική would invalidate the "φάντασμα" of appearances, show the truth, give peace to the soul, let it remain in truth, and save [our] life.[185] μετρητική is the τέχνη of "ὑπερβολῆς τε καὶ ἐνδείας"; "excess and deficiency".[186] Although Stanley Lombardo's & Karen Bell's translation "the greater and the lesser"[187] may be correct, the context seems better suited for a literal translation, which they provided in the next sentence that deals with the definition of μετρητική: "ὑπερβολῆς τε καὶ ἐνδείας οὖσα καὶ ἰσότητος πρὸς ἀλλήλας σκέψις;"; "being the examination of excess and deficiency, and of their equality towards one another".[188] A more literal translation would be preferred since it shows clearly the context of Socrates' discussion: the avoidance of the 'wrong' form of pleasures and pains, i.e. considering the excess and deficiency of the pleasures and pains: "ἐπεὶ δὲ δὴ ἡδονῆς τε καὶ λύπης ἐν ὀρθῇ τῇ αἱρέσει ἐφάνη ἡμῖν ἡ σωτηρία τοῦ βίου οὖσα, τοῦ τε πλέονος καὶ ἐλάττονος καὶ μείζονος καὶ σμικροτέρου καὶ πορρωτέρω καὶ ἐγγυτέρω

[181] Also mentioned in *Phil.* 56e–57a, but without any deeper discussion on the art itself.
[182] Pl. *Resp*. VII, 519e–520a.
[183] Pl. *Prt.* 356d, text in bold emphasis is mine.
[184] Pl. *Prt.* 356d.
[185] Pl. *Prt.* 356d–e.
[186] Pl. *Prt.* 357a.
[187] In: J. M. Cooper & D. S. Hutchinson (1997), 785.
[188] Pl. *Prt.* 357b. Translation is mine.

[…]”[189] In Plato’s *Euthyphro*, Socrates investigates the meaning of piety. In the context of piety and justice, fear and shame, Socrates likens the discussion to the odd being a part of number and the even also likewise.[190] There is therefore, in Plato’s dialogues, a clear connection between human values (the soul’s virtues and vices; or philosophical principles in general) and mathematical number theory. Not that philosophical principles such as piety, justice, excess and deficiency are literally the counterparts of these odds and evens, but that mathematics seem to play a fundamental and intrinsic role to the soul’s principles.

Apart from the ethical considerations, we may also find spiritual concepts, especially in the *Protagoras*. “ἡ σωτηρία” and the verb with the same root “σῴζειν” is employed several times by Socrates in these passages when he discusses μετρητική and ἀριθμητική.[191] This ‘salvation’ by the ‘arts of measurement and arithmetic’ is connected to excess and deficiency, pleasure and pain.[192] ἀριθμητική is also singled out as being related to μετρητική and to this ‘salvation’: “[…]ἐπειδὴ δὲ περιττοῦ τε καὶ ἀρτίου, ἆρα ἄλλη τις ἢ ἀριθμητική;”.[193] There is moreover a peculiar mention of ‘λογισμός’ and ‘ἀριθμός’ as “σώιζει βροτούς” (saving mortals) in a fragment of the dramatist Epicharmus (ca. 5th – 4th c. B.C). The fragment goes as follows:

ὁ βίος ἀνθρώποις λογισμοῦ κἀριθμοῦ δεῖται πάνυ.

ζῶμεν [δὲ] ἀριθμῶι καὶ λογισμῶι. ταῦτα γὰρ σώιζει βροτούς.[194]

Life is altogether dependent on λογισμός and ἀριθμός for humans.

We live by λογισμός and ἀριθμός since these save mortals.

[189] Pl. *Prt.* 357a–b.
[190] Pl. *Euthphr.* 12c–d.
[191] Pl. *Prt.* 356d–357b.
[192] “τί δ᾽ εἰ ἐν τῇ τοῦ περιττοῦ καὶ ἀρτίου αἱρέσει ἡμῖν ἦν ἡ σωτηρία τοῦ βίου, ὁπότε τὸ πλέον ὀρθῶς ἔδει ἑλέσθαι καὶ ὁπότε τὸ ἔλαττον, ἢ αὐτὸ πρὸς ἑαυτὸ ἢ τὸ ἕτερον πρὸς τὸ ἕτερον, εἴτ᾽ ἐγγὺς εἴτε πόρρω εἴη; τί ἂν ἔσῳζεν ἡμῖν τὸν βίον; ἆρ᾽ ἂν οὐκ ἐπιστήμη; καὶ ἆρ᾽ ἂν οὐ μετρητική τις, ἐπειδήπερ ὑπερβολῆς τε καὶ ἐνδείας ἐστὶν ἡ τέχνη; ἐπειδὴ δὲ περιττοῦ τε καὶ ἀρτίου, ἆρα ἄλλη τις ἢ ἀριθμητική; Ὁμολογοῖεν ἂν ἡμῖν οἱ ἄνθρωποι ἢ οὔ;” Pl. *Prt.* 356e–357a.
[193] Pl. *Prt.* 357a.
[194] DK, Epicharmos (23), 56. Cf. also fragment 57:
“ὁ λόγος ἀνθρώπους κυβερνᾶι κατὰ τρόπον σώιζει τ᾽ ἀεί.
ἔστιν ἀνθρώπωι λογισμός, ἔστι καὶ θεῖος λόγος.
[…]
ὁ δέ γε ταῖς τέχναις ἁπάσαις συνέπεται θεῖος λόγος, […]
I found fragment 56 cited by P. Pritchard (1995) 81:n49, but he doesn’t mention the philosophical implications of “σώιζει βροτούς” and what the fragment’s contents really mean in a spiritual sense.

This shows how Plato may have been drawing on previous traditions for this notion of 'mathematics saving humans'. Epicharmus even juxtaposes 'ἀνθρώπωι λογισμός' with 'θεῖος λόγος' and says that "ὁ δέ γε ταῖς τέχναις ἁπάσαις συνέπεται θεῖος λόγος" in another fragment.[195] The spiritual, divine and religious character in the τέχναι may hence have been an idea that was current even before Plato.

The mathematical sciences for Plato seem therefore to be a sort of *means to an end*. By the means of at least μετρητική and ἀριθμητική, mere "appearances" vanish, and we are able to see "the truth" and we are "saved", as we have just read from all the above. One of the aims of these sciences seem therefore to be the saving of one's soul. ἀριθμητική, λογιστική, μετρητική, γεωμετρία and similar subjects are for the purification of a certain 'instrument' of the soul, as stated in another passage in the *Res Publica*.[196] The repetition of "σωτηρία" and "σῴζειν" give the passages just mentioned a clear spiritual or religious notion of 'the salvation of the soul' which of course sounds similar to Christian soteriology but must remain distinctly Platonic for us at this moment. μετρητική, moreover, bestows peace, truth, and life to the philosopher. All of this shows Plato's mathematics to be *a sort of spiritual way of life*, and not only a philosophical practice.

[195] See the note just before. Cf. also fragment B4 of the Pythagorean Archytas: "*Logismos*, when discovered, stops strife and increases concord; when it occurs, there is no excess of gain, but there is equality; for by this we settle our disputes. […] It is a rule and it prevents men from doing wrong […]." As cited by D. Fowler (1999), 150.
[196] Pl. *Resp*. VII, 527d–e. See the section 5.4. on γεωμετρία.

5.2. ἀριθμητική[197]

5.2.1. ἀριθμητική: the quantity of ἀριθμός, the religious connection (χρῆσθαι), and reaching νόησις, οὐσία and ἀλήθεια

In Plato's *Theaetetus,* Socrates gives us a short definition of what it means to 'ἀριθμεῖν' ("to number/count/*arithmetize*"): "τὸ δὲ ἀριθμεῖν γε οὐκ ἄλλο τι θήσομεν τοῦ σκοπεῖσθαι πόσος τις ἀριθμὸς τυγχάνει ὤν"; "We will posit counting/to arithmetize as nothing else than contemplating how many/how much there happens to be of ἀριθμὸς".[198] 'πόσος τις ἀριθμός' is translated by M. J. Levett & rev. Miles Burnyeat as "how large a number" and by Klein (or his English translator Eva Brann) as "how great a number".[199] Both these translations give 'πόσος' the connotation of 'largeness of magnitude' whereas the Greek word can in this context rather mean either that or 'quantity' of something, i.e. 'how many'.[200] Also, just as Klein translates, the 'τυγχάνει ὤν' furthermore indicates *specific cases* of calculating as in *how many of ἀριθμὸς there happens to be:* "how great a number happens to be [in a given case]."[201] Klein asserts here, and we agree that: "thus the *arithmos* indicates in each case a *definite number of definite things.* […]. It intends the *things* insofar as they are present in this number, and cannot, at least at first, be separated from the things at all."[202] We will return later to what ἀριθμός furthermore consists of, conceptually.[203]

Some examples of what that the mathematicians concretely were occupied with are given in

[197] J. Klein (1992), in his third chapter 'Logistic and arithmetic in Plato' (17–25) (and in the first part of his book generally) also surveys some of the following passages in Plato, but my examination is a bit more comprehensive and linguistically detailed, and I mention some things which he has overlooked, e.g. the use of 'χράω in med. + dat.' by Socrates in the *Resp.* VII, 523a, as 'consulting a science' in a divinatory or prophetic/oracular manner, or the use of the noun "ἡ σωτηρία" and the verb with the same root "σῴζειν" in *Prt.* 356d–357b and other places, to indicate the proper aim of these μαθήματα; the salvation of the soul.

[198] Pl. *Tht.* 198c.

[199] J. Levett & rev. Miles Burnyeat in J. M. Cooper & D. S. Hutchinson (1997), 219; J. Klein (1992), 46.

[200] Cf. *LSJ's* entry on πόσος: "1. of Number, *how many?* […] π. τις ἀριθμός; Pl.*Tht.*198c; […] 5. of Degree, *how great?* II. **ποσός**, ή, όν, indef. Adj. *of a certain quantity* or *magnitude*."

[201] J. Klein (1992), 46. More likely translated by Eva Brann from Klein's German into English.

[202] J. Klein (1992), 46.

[203] Cf. also the 'spurious' *Epinomis:* "διὸ μαθημάτων δέον ἂν εἴη: τὸ δὲ μέγιστόν τε καὶ πρῶτον καὶ ἀριθμῶν αὐτῶν ἀλλ᾽ οὐ σώματα ἐχόντων, ἀλλὰ ὅλης τῆς τοῦ περιττοῦ τε καὶ ἀρτίου γενέσεώς τε καὶ δυνάμεως, ὅσην παρέχεται πρὸς τὴν τῶν ὄντων φύσιν. ταῦτα δὲ μαθόντι τούτοις ἐφεξῆς ἐστιν ὃ καλοῦσι μὲν σφόδρα γελοῖον ὄνομα γεωμετρίαν, τῶν οὐκ ὄντων δὲ ὁμοίων ἀλλήλοις φύσει ἀριθμῶν ὁμοίωσις πρὸς τὴν τῶν ἐπιπέδων μοῖραν γεγονυῖά ἐστιν διαφανής: ὃ δὴ θαῦμα οὐκ ἀνθρώπινον ἀλλὰ γεγονὸς θεῖον φανερὸν ἂν γίγνοιτο τῷ δυναμένῳ συννοεῖν. μετὰ δὲ ταύτην τοὺς τρὶς ηὐξημένους καὶ τῇ στερεᾷ φύσει ὁμοίους: τοὺς δὲ ἀνομοίους αὖ γεγονότας ἑτέρᾳ τέχνῃ ὁμοιοῖ, ταύτῃ ἣν δὴ στερεομετρίαν ἐκάλεσαν οἱ προστυχεῖς αὐτῇ γεγονότες" *Epin.* 990c–e.

Plato's *Res Publica.* Socrates says that those who are engaged in geometry, counting, and similar endeavours[204] are: "ὑποθέμενοι τό τε περιττὸν καὶ τὸ ἄρτιον καὶ τὰ σχήματα καὶ γωνιῶν τριττὰ εἴδη καὶ ἄλλα τούτων ἀδελφὰ καθ' ἑκάστην μέθοδον […] ποιησάμενοι ὑποθέσεις αὐτά […]"; "they suppose/hypothesize the odd and the even, the figures, the three kinds of angles, and other things akin to these according to each pursuit/method […] making suppositions/hypotheses […]".[205]

Even today, these are the basic components of arithmetical mathematics and geometry. There are, however, several differences between modern and ancient Greek concepts, as already mentioned in the introduction.

In one sense, the purpose of μαθηματική was in Plato's reasoning concordant with the ultimate goal of his ideal state. Those who have reached the greatest μάθημα (note Plato's word choice); to know/perceive *the good* and through the education of the body and soul not linger there for their own sakes, but helping out the common citizens, τοὺς δεσμώτας *the prisoners* [in the cave], they must make sure that happiness and good deeds are shared by the city as a whole.[206] That not only "ἕν τι γένος" ("*one class/kind*") should "εὖ πράξει" (lit. "*do well"*) in the city, but that the entire city should share in this well-being. The law does not intend to form such people for their own selfish intents, but "ἵνα καταχρῆται αὐτὸς αὐτοῖς ἐπὶ τὸν σύνδεσμον τῆς πόλεως" ("*so that he [the law] may fully apply them for the bond/union of the city"*).[207]

Socrates begins his discussion in the *Res Publica* on the mathematical sciences important for the *guardian*, with some simple yet very revealing statements. The common thing ("τὸ κοινόν") which all arts, thoughts and sciences make use of, that which it is necessary to learn among the first things (in life, as today?), that simple thing ("τὸ φαῦλον τοῦτο"): "τὸ ἕν τε καὶ τὰ δύο καὶ τὰ τρία διαγιγνώσκειν·"; "to *distinguish* the one, the twos, and the threes". More specifically, Socrates says: "λέγω δὲ αὐτὸ ἐν κεφαλαίῳ ἀριθμόν τε καὶ λογισμόν"; "to sum up, I mean *number* (ἀριθμός) and *calculation* (λογισμός)".[208]

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[204] "οἱ περὶ τὰς γεωμετρίας τε καὶ λογισμοὺς καὶ τὰ τοιαῦτα πραγματευόμενοι" *Resp.* VI, 510c.
[205] Pl. *Resp.* VI, 510c.
[206] Pl. *Resp.* VII, 519c–e.
[207] Pl. *Resp.* VII, 519e–520a.
[208] Pl. *Resp.* VII, 522c. [Socr./Glauc.] "Οἷον τοῦτο τὸ κοινόν, ᾧ πᾶσαι προσχρῶνται τέχναι τε καὶ διάνοιαι καὶ ἐπιστῆμαι, ὃ καὶ παντὶ ἐν πρώτοις ἀνάγκη μανθάνειν. Τὸ ποῖον; ἔφη. Τὸ φαῦλον τοῦτο, ἦν δ' ἐγώ, τὸ ἕν τε καὶ τὰ δύο καὶ τὰ τρία

We must note here how Socrates refers to the simple methods of number and calculation, which everyone must learn first, and that he mentions in the same sentence 'τὸ ἕν τε καὶ τὰ δύο καὶ τὰ τρία' with 'ἀριθμόν τε καὶ λογισμόν'. Now whether ἀριθμός have more connotations than the first three integers (and Socrates does not specify if all the integers 1-10 are included), it is evident that the simplest Greek notion of ἀριθμόν τε καὶ λογισμόν include, as it does for us today, the one, two and three. The difference that could be noted here though, is that he uses the singular article for the one, 'τὸ' ἕν, but the plural articles for the 'twos' and 'threes', 'τὰ' δύο καὶ 'τὰ' τρία. It is unclear here why the 'two' and 'three' have the plural article 'τὰ' instead of the singular 'τὸ', but it seems to be because they simply denote plurality compared to the 'one'. As already stated elsewhere, the *Res Publica* was most likely not Plato's final statements on the nature of mathematics, so these notions might have developed, or there might already have been different notions of ἀριθμόν τε καὶ λογισμόν. What is clear though, is that we cannot completely separate the positive integers from Plato's mathematics, in the *Res Publica* at least, even if they were different from our modern notions of number.

Continuing with Socrates' train of thought in the same passage, the guardian, even in the context of a warrior, must be able to λογίζεσθαί τε καὶ ἀριθμεῖν, both for understanding how to arrange his troops and in order to become a 'proper human being' ("εἰ καὶ ὁτιοῦν μέλλει τάξεων ἐπαΐειν, μᾶλλον δ᾽ εἰ καὶ ἄνθρωπος ἔσεσθαι").[209]

λογίζεσθαί τε καὶ ἀριθμεῖν are some of the things that lead to τὴν νόησιν but they must be used correctly: "χρῆσθαι δ᾽ οὐδεὶς αὐτῷ ὀρθῶς, ἑλκτικῷ ὄντι παντάπασι πρὸς οὐσίαν".[210] Firstly, the use of the verb χράω in med.+dat. (Plato most likely employs the medial form since passive seems unlikely in this context) means according to the *LSJ* "*to consult a god* or *oracle*". The medial form χράομαι by itself could also possibly be translated as "*use*" (*LSJ* entry 'II.'), as G.M.A. Grube and rev. C.D.C. Reeve have translated it.[211] The former meaning is more tenable though, it seems, since both the medial voice and the dative case are present in Plato's sentence. Now we cannot know for sure which meaning Plato had in mind, or if he considered both as valid

διαγιγνώσκειν· λέγω δὲ αὐτὸ ἐν κεφαλαίῳ ἀριθμόν τε καὶ λογισμόν. ἢ οὐχ οὕτω περὶ τούτων ἔχει, ὡς πᾶσα τέχνη τε καὶ ἐπιστήμη ἀναγκάζεται αὐτῶν μέτοχος γίγνεσθαι; Καὶ μάλα, ἔφη."

[209] Pl. *Resp.* VII, 522e.

[210] Pl. *Resp.* VII, 523a. Debra Nails argues in her article from 1979 that 'οὐσία' was not equivalent to 'τὸ ὄν' and 'ὁ ἔστι' for Plato (D. Nails 1979, 71-77).

[211] J. M. Cooper & D. S. Hutchinson (1997), 1139.

here, but it is interesting nevertheless to note the 'religious' or oracular undertones here; the notion of *consulting* a science in the similar manner of *consulting divinities*. Secondly, perhaps 'Neoplatonically' (as in expounding different 'metaphysical layers and entities'),[212] the context shows itself in this whole passage: the world of τὴν νόησιν (*intelligence, understanding),* must be reached in the proper way by "*consulting"* the mathematical science in a way which "*is attractive altogether towards being/substance/essence (the things that are)*". There is hence a possibility of different 'metaphysical layers' which aren't so clearly spelled out in Plato's dialogues as in Neoplatonic writings.

But more importantly, Plato uses 'divinatory' language again in the passage just continuing after this. Socrates tells Glaucon that he will clarify what he means, so that they both may know if it is as Socrates "μαντεύομαι", i.e. literally if it is as he "*divines, prophesizes"* or even "*consults an oracle, seeks divinations*".[213] Socrates uses the same verb again, in a later passage where he 'divines, forebodes,' what he believes would happen to a child who practices dialectic wrongly, because of being brought up as a spoiled child by wealth and flatterers.[214] Another verb that may be used in a similarly religious way by Plato, but quite conjecturally now, is παρακαλέω which in its basic meaning is "*to summon*" but could also mean "*to invoke* (the gods)". In our context: "παρακαλοῦντα τὴν νόησιν εἰς ἐπίσκεψιν" and "πειρᾶται λογισμόν τε καὶ νόησιν ψυχὴ παρακαλοῦσα ἐπισκοπεῖν εἴτε […]".[215] Generally, it was used in the setting of *summoning* advisors in political or military affairs or summoning someone to trial or as a witness.[216]

There is another relevant passage in Plato's *Timaeus* dealing with θεός, χράω/χράομαι, the νοῦς of the heavens (in astronomy), the διανόησις of humans, and a natural λογισμός.[217] Discussing the great gifts that 'the god' has given us because of eyesight, Timaeus says: "θεὸν ἡμῖν ἀνευρεῖν

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212 Among the five characteristics of Neoplatonism marked out by Pauliina Remes are: "(ii) There is a proliferation of metaphysical layers and entities. Plato can be interpreted as postulating (in a more or less crude simplification) two aspects or levels of reality: one that is material, perceptible, temporal and changing, and another that is immaterial, intelligible, eternal and permanent. […] The Neoplatonists take this layered understanding of reality to be correct, but following Middle-Platonic authors and Plotinus they postulate yet further levels between the two, or, perhaps better, within the higher or the intelligible." The difference between Plato's scheme and the Neoplatonic is hence that the latter has more metaphysical complexity (it seems). P. Remes (2008), 7.
213 Pl. *Resp.* VII, 523a:5–8.
214 Pl. *Resp.* VII, 538a–b. [Socr./Glauc.] "ἦ βούλει ἐμοῦ μαντευομένου ἀκοῦσαι; Βούλομαι, ἔφη. Μαντεύομαι τοίνυν, εἶπον, […]"
215 Pl. *Resp.* VII, 523b–c & Pl. *Resp.* VII, 524b.
216 See the *LSJ*.
217 Pl. *Ti.* 47a–c.

δωρήσασθαί τε ὄψιν, ἵνα τὰς ἐν οὐρανῷ τοῦ νοῦ κατιδόντες περιόδους χρησαίμεθα ἐπὶ τὰς περιφορὰς τὰς τῆς παρ' ἡμῖν διανοήσεως".[218] The 'purpose' of eyesight is, amongst other things, to study the heavens (astronomy). By observing the periods of the mind of the heavens/universe, we are to use/apply them (χρησαίμεθα, aor. opt. med. 1. pl.) to the revolutions of our own thinking/understanding. Furthermore, "ἐκμαθόντες δὲ καὶ λογισμῶν κατὰ φύσιν ὀρθότητος μετασχόντες", we are to "thoroughly learn the calculations (λογισμῶν) according to nature, partaking in them correctly". Thereby, we mimic the true and unwavering revolutions of the god (of pure, not popular, astronomy) and set in order our own revolutions within. What can be evinced from this passage, in my opinion, is the ambivalent connotations and meanings of the verb 'χράω/χράομαι', ranging from 'proclaim', 'consult a god/oracle', 'to use, employ', and other meanings. What draws all these semantic inferences together (perhaps for an ancient Greek reader more conspicuously than for us?) is the context of θεός (spirituality/religion), χράω/χράομαι (consulting/using something), mind/intellect (νοῦς and διανόησις), and mathematics (λογισμός and the mathematical Kosmos). This fortifies my argument of the oracular references with mathematics in the *Res Publica*, as we just read. The fact that Socrates himself said that he was inspired by a daemonic/godlike voice[219] ("ὅτι μοι θεῖόν τι καὶ δαιμόνιον γίγνεται [φωνή]")[220] shows us, again, evidently how Plato has incorporated 'spirituality' into his dialogues. Socrates mentions also that dreams and oracular response/divination are gifts in connection to this, enjoined by 'the god' upon Socrates when he goes about in the city, examining if those who think they are wise actually are wise.[221]

The oracular interest in mathematics has a remarkable historical anecdote with Plato's philosophy. Theon of Smyrna (1st c. A.D.) narrated, on the authority of Eratosthenes, that when the Delians consulted the oracle at Delphi on how to remove a plague that was pestering them, they were ordered to construct an altar double the size of Apollo's ('The Delian Problem'). After much difficulty, they could not comprehend how a solid was to be made double from another solid (doubling the volume of the cube is an almost impossible construction in theoretical

[218] Pl. *Ti.* 47b.
[219] *'daemonic'* in the pre-Christian sense of the word (with 'ae' not 'e'), not the later '*demonic'* with evil connotations.
[220] Pl. *Ap.* 31c–d. Cf. also: "ὅτι ἀκούοντες χαίρουσιν ἐξεταζομένοις τοῖς οἰομένοις μὲν εἶναι σοφοῖς, οὖσι δ' οὔ. ἔστι γὰρ οὐκ ἀηδές. ἐμοὶ δὲ τοῦτο, ὡς ἐγώ φημι, **προστέτακται ὑπὸ τοῦ θεοῦ πράττειν καὶ ἐκ μαντείων καὶ ἐξ ἐνυπνίων καὶ παντὶ τρόπῳ ᾧπέρ τίς ποτε καὶ ἄλλη θεία μοῖρα ἀνθρώπῳ καὶ ὁτιοῦν προσέταξε πράττειν**. ταῦτα, ὦ ἄνδρες Ἀθηναῖοι, καὶ ἀληθῆ ἐστιν καὶ εὐέλεγκτα." (*Ap.* 33c), emphasis in bold is mine. Also in *Ap.* 31d–32c; 38a; 42a.
[221] *Ibid*.

mathematics). They went therefore to Plato, who informed them that the oracle did not divine for them to construct this double-altar, but the oracle simply presented this to reproach them since the Greeks were neglecting and trivializing mathematics and geometry.[222] Eratosthenes, Theon, Eutocius, and Plutarch are also reported to have noted this historical anecdote.[223]

This shows clearly how the relation between health and proper living (the plague that was haunting the Delians), mathematics, oracular religion, and philosophy, were closely intertwined in Plato's thinking. It therefore does not seem improper to interpret Plato's passages above, with χράομαι and μαντεύομαι, thus as we have suggested. According to E. A. Maziarz and T. Greenwood most scholars agree (at least when they were writing their book in the late 60's) that "Greek philosophy, mathematics, and science are at least partially derived from religion in its forms of myth, magic, and ritual. In fact, it has even been said that such thinkers as Plato and Aristotle attempted to have their own philosophical systems serve as the 'myth' for their contemporaries[1]"[224] This, of course, in relation to eastern and Egyptian influence, and the Greeks' own developments of philosophy and science.

How λογίζεσθαί τε καὶ ἀριθμεῖν may lead towards τὴν νόησιν and πρὸς οὐσίαν in the correct manner is further explained by Socrates to Glaucon. The idea is quite obscure and needs further attention, if we are to understand what Plato truly means by "χρῆσθαι […] αὐτῷ ὀρθῶς".[225] The sight, when gazing close by, immediately recognizes that a finger is a finger and not something opposite to a finger or something else.[226] But what about more detailed things like the bigness

[222] "Ἐρατοσθένης μὲν γὰρ ἐν τῷ ἐπιγραφομένῳ Πλατωνικῷ φησιν ὅτι, Δηλίοις τοῦ θεοῦ χρήσαντος ἐπὶ ἀπαλλαγῇ λοιμοῦ βωμὸν τοῦ ὄντος διπλασίονα κατασκευάσαι, πολλὴν ἀρχιτέκτοσιν ἐμπεσεῖν ἀπορίαν ζητοῦσιν ὅπως χρὴ στερεὸν στερεοῦ γενέσθαι διπλάσιον, ἀφικέσθαι τε πευσομένους περὶ τούτου Πλάτωνος. τὸν δὲ φάναι αὐτοῖς, ὡς ἄρα οὐ διπλασίου βωμοῦ ὁ θεὸς δεόμενος τοῦτο Δηλίοις ἐμαντεύσατο, προφέρων δὲ καὶ ὀνειδίζων τοῖς Ἕλλησιν ἀμελοῦσι μαθημάτων καὶ γεωμετρίας ὠλιγωρηκόσιν." Theo Sm. *Theonis Smyrnaei philosophi Platonici expositio rerum mathematicarum ad legendum Platonem utilium*, ii. Also discussed in: E. A. Maziarz & T. Greenwood (1968), 80.

[223] See: E. A. Maziarz & T. Greenwood (1968), 80, who mentioned these philosophers' accounts in their short survey on the Delian Problem.

[224] E. A. Maziarz & T. Greenwood (1968), vii–viii. Their footnote [1] cites: "Evert W. Beth, *The Foundations of Mathematics: A Study in the Philosophy of Science* (Amsterdam, 1959), pp. 34–36."

[225] The following is a short paraphrase of *Resp.* VII, 523a–524b.

[226] [Socr./Glauc.] " δάκτυλος μέν που αὐτῶν φαίνεται ὁμοίως ἕκαστος, καὶ ταύτῃ γε οὐδὲν διαφέρει, ἐάντε ἐν μέσῳ ὁρᾶται ἐάντ᾽ ἐπ᾽ ἐσχάτῳ, ἐάντε λευκὸς ἐάντε μέλας, ἐάντε παχὺς ἐάντε λεπτός, καὶ πᾶν ὅτι τοιοῦτον. ἐν πᾶσι γὰρ τούτοις οὐκ ἀναγκάζεται τῶν πολλῶν ἡ ψυχὴ τὴν νόησιν ἐπερέσθαι τί ποτ᾽ ἐστὶ δάκτυλος: οὐδαμοῦ γὰρ ἡ ὄψις αὐτῇ ἅμα ἐσήμηνεν τὸ δάκτυλον τοὐναντίον ἢ δάκτυλον εἶναι. οὐ γὰρ οὖν, ἔφη. οὐκοῦν, ἦν δ᾽ ἐγώ, εἰκότως τό γε τοιοῦτον νοήσεως οὐκ ἂν παρακλητικὸν οὐδ᾽ ἐγερτικὸν εἴη. εἰκότως. τί δὲ δή; τὸ μέγεθος αὐτῶν καὶ τὴν σμικρότητα ἡ ὄψις ἆρα ἱκανῶς ὁρᾷ, καὶ οὐδὲν αὐτῇ διαφέρει ἐν μέσῳ τινὰ αὐτῶν κεῖσθαι ἢ ἐπ᾽ ἐσχάτῳ; καὶ ὡσαύτως πάχος καὶ λεπτότητα ἢ μαλακότητα καὶ σκληρότητα ἡ ἁφή; καὶ αἱ ἄλλαι αἰσθήσεις ἆρ᾽ οὐκ ἐνδεῶς τὰ τοιαῦτα δηλοῦσιν; […]" *Resp.* VII, 523c–e.

and smallness of fingers (note Plato's word choice here: "*τὸ μέγεθος* αὐτῶν καὶ *τὴν σμικρότητα* ἡ ὄψις ἆρα ἱκανῶς ὁρᾷ")?[227] Sight does not distinguish that as clearly as the former notion. The same with the other senses, they do not reveal things such as hardness or softness with the same exactitude as the sight confirms that a finger is a finger and not a head or a foot or something else. The same faculty of sense must perceive both soft and hard things, e.g. when we touch a pillow or a chair.[228] What Plato seems to suggest is simply that the senses are unreliable when we seek exact answers to some things. And when this happens, when the senses prove themselves unreliable to us, we must "summon understanding", and this is what λογίζεσθαί τε καὶ ἀριθμεῖν is doing. This seems to be a sort of dialectic thinking over the things that are, a differentiating way of separating things *mentally* since they cannot be logically and fully understood *sensibly* by the five senses. Socrates explains: "εἰκότως ἄρα, ἦν δ᾽ ἐγώ, ἐν τοῖς τοιούτοις πρῶτον μὲν πειρᾶται λογισμόν τε καὶ νόησιν ψυχὴ παρακαλοῦσα ἐπισκοπεῖν εἴτε ἓν εἴτε δύο ἐστὶν ἕκαστα τῶν εἰσαγγελλομένων." That is to say, "the soul summons calculation and understanding in order to reflect over numbers", in this case whether the things 'announced' to the soul are one or actually two.

So, Plato's higher form of λογιστική τε καὶ ἀριθμητικὴ is about some kind of 'calculation' which aims for νόησιν and οὐσίαν, and they lead towards ἀλήθεια. For the warlike person, it is necessary to learn these subjects so that they may know how to form a battle array, and for the philosopher, it is necessary to escape from generation/creation and cling unto οὐσία with the help of these two subjects.[229] Socrates continues to explain to Glaucon that those who are to partake of the highest offices are not to engage in λογιστικήν[230] in a commonplace manner ("ἰδιωτικῶς"), but *in a manner by which they reach the vision of the very nature of numbers by the help of intellect/understanding*. Neither are these sciences to be used for buying and selling, like the tradesmen and their like do, *but for purposes of war and for the gentle turning away of the soul from generation towards truth and being*.[231] Socrates here implies two notions: that by the correct

[227] Pl. *Resp.* VII, 523e. μέγεθος is the same noun used in Greek mathematical works for "*magnitude"*.
[228] Pl. *Resp.* VII, 523a–524b.
[229] Pl. *Resp.* VII, 525a – 525b. "πολεμικῷ μὲν γὰρ διὰ τὰς τάξεις ἀναγκαῖον μαθεῖν ταῦτα, φιλοσόφῳ δὲ διὰ τὸ τῆς οὐσίας ἁπτέον εἶναι γενέσεως ἐξαναδύντι, ἢ μηδέποτε λογιστικῷ γενέσθαι."; cf. 525c:5.
[230] Plato writes specifically of λογιστική here, but in the next sentence he includes "ἀλλ᾽ ἕως ἂν ἐπὶ θέαν τῆς τῶν ἀριθμῶν φύσεως ἀφίκωνται τῇ νοήσει αὐτῇ", so we need not assume that Plato necessarily excludes ἀριθμητική here.
[231] Pl. *Resp.* VII, 525b–c. "ἀλλ᾽ ἕνεκα πολέμου τε καὶ αὐτῆς τῆς ψυχῆς ῥᾳστώνης μεταστροφῆς ἀπὸ γενέσεως ἐπ᾽ ἀλήθειάν τε καὶ οὐσίαν." As Professor Christian Høgel noted here (personal correspondence), Plato may have included war as a purpose for these sciences since this was in a context of his 'ideal state' which had to be defended during battles and wars

(1) *method* (not the vulgar one) and not for commercial (2) *purposes*, λογίζεσθαί τε καὶ ἀριθμεῖν are drawn from their material shells unto their true aims: *to understand the very nature of* ἀριθμός for *the final purpose of leaving generation and arriving at* ἀλήθεια *and* οὐσία. "τῆς ψυχῆς ῥᾳστώνης μεταστροφῆς ἀπὸ γενέσεως" is further hinted at when Socrates says that not numbers of visible or tangible bodies[232] should be discussed, but rather "αὐτῶν τῶν ἀριθμῶν"; "numbers themselves", while the soul is strongly led upwards.[233] The notion of the number one is given by Socrates as an example, and here we may note two things: the two different interpretations – or 'schools' of thought – of number itself, and the importance of "τὸ ἕν". If the number one is divided, 'they' multiply it, as Socrates informs us.[234] This *could* show that the other group, dealing with αὐτῶν τῶν ἀριθμῶν, are also allowed to multiply, and perhaps even divide, subtract e.t.c τὸ ἕν, *but* they are "εὐλαβούμενοι μή ποτε φανῇ τὸ ἓν μὴ ἓν ἀλλὰ πολλὰ μόρια."[235] In other words, they always seek to prove that τὸ ἕν cannot, ultimately, be changed from its 'oneness' into something else. However, care must be taken here not to interpret Socrates' words too literally. It is actually not stated that τὸ ἕν can be arithmetically manipulated in whatsoever way, only that if someone tries to divide it, the philosophers immediately multiply it back again, so that τὸ ἕν always remains the same.

When τὸ ἕν is always the same, undivided, even if there are several 'ones', the ἀριθμῶν[236] that are being discussed, are described in this manner by Socrates: "ὅτι περὶ τούτων λέγουσιν ὧν

by the guardians.

[232] Fowler wrote (D. H. Fowler, 1999, 107) that he ignored the later commentators, such as the Neoplatonists, when dealing with Plato's mathematical notions, since many of the Neoplatonists made λογιστική into a science that dealt exclusively with sensibles while Plato's theoretical sciences "do *not* concern themselves with sensibles". But Plato does still acknowledge the possibility of counting with sensibles, in the context of 'λογίζεσθαί τε καὶ ἀριθμεῖν', as we see in these passages and in VII, 522e where Socrates deems it necessary for the guardians to be able to count *troops* which really cannot be defined as much else than *sensible, visible things.* It does not seem therefore that Plato excludes the 'counting of sensibles' completely, but rather that the guardians are to focus more on "αὐτῶν τῶν ἀριθμῶν". The solution of this issue may, as noted already, be resolved by acknowledging that Plato's ideas on the details of these sciences developed and changed after the writing of his *Res Publica*.

[233] Pl. *Resp.* VII, 525d–e. "ὡς σφόδρα ἄνω ποι ἄγει τὴν ψυχὴν καὶ περὶ αὐτῶν τῶν ἀριθμῶν ἀναγκάζει διαλέγεσθαι, οὐδαμῇ ἀποδεχόμενον ἐάν τις αὐτῇ ὁρατὰ ἢ ἁπτὰ σώματα ἔχοντας ἀριθμοὺς προτεινόμενος διαλέγηται." Cf. Kleins discussion on this (J. Klein, 1992, 49–50) and especially (p. 50): "What is required is an object which has a purely noetic character and which exhibits at the same time all the essential characteristics of the *countable* as such. This requirement is exactly fulfilled by the 'pure' units, which are 'nonsensual', accessible only to the understanding, indistinguishable from one another, and resistant to all partition […]."

[234] Pl. *Resp.* VII, 525d–e.

[235] Pl. *Resp.* VII, 525e.

[236] Pl. *Resp.* VII, 526a: "τί οὖν οἴει, ὦ Γλαύκων, εἴ τις ἔροιτο αὐτούς: 'ὦ θαυμάσιοι, περὶ ποίων ἀριθμῶν διαλέγεσθε, ἐν οἷς τὸ ἓν οἷον ὑμεῖς ἀξιοῦτέ ἐστιν, ἴσον τε ἕκαστον πᾶν παντὶ καὶ οὐδὲ σμικρὸν διαφέρον, μόριόν τε ἔχον ἐν ἑαυτῷ οὐδέν;' τί ἂν οἴει αὐτοὺς ἀποκρίνασθαι;"

διανοηθῆναι μόνον ἐγχωρεῖ, ἄλλως δ᾽ οὐδαμῶς μεταχειρίζεσθαι δυνατόν" – "that they are talking about those [ἀριθμῶν] of which it is allowed to have in mind [lit.] only, and it is not possible at all to deal with them in another way".[237] The understanding of "τὸ ἕν" seems crucial in what Socrates defines as the proper way of λογίζεσθαί τε καὶ ἀριθμεῖν, just before these mentioned passages. Only when τὸ ἕν is seen to be two things at the same time – "ἅμα γὰρ ταὐτὸν ὡς ἕν τε ὁρῶμεν καὶ ὡς ἄπειρα τὸ πλῆθος"[238] does it stir up the soul to understanding. So, again, if something opposite ("ἐναντίωμα") to τὸ ἕν is seen by sight, then the soul must "κινοῦσα ἐν ἑαυτῇ τὴν ἔννοιαν", in order to understand the true nature of τὸ ἕν in any particular case. Therefore, Socrates concludes that "ἡ περὶ τὸ ἓν μάθησις"[239] would be among the things leading and turning the soul towards the vision of being ("ἐπὶ τὴν τοῦ ὄντος θέαν").[240] In the *Epinomis*, the author even claims, "in jest and in seriousness" that the one who studies the important sciences *correctly*, amongst them mathematics, he will after death "become one from many"; "ἐκ πολλῶν ἕνα γεγονότα".[241] This is a clear testimony to some form of 'spiritual mathematics', if we regard the *Epinomis* as authentic.

Socrates concludes the discussion on ἀριθμητική and λογιστική in the *Res Publica* with the recapitulation that they compel the soul, through understanding itself (or: '*The* Understanding), to be used towards the truth itself (or: *The* Truth).[242]

This peculiar discussion about opposites is also found in the *Phaedo* 100–107. It concerns Plato's doctrine of the Forms being only 'one thing'. When trying to prove the immortality of the soul,

[237] Pl. *Resp.* VII, 526a.
[238] Pl. *Resp.* VII, 525a.
[239] In connection to this, Szabó wrote: "It is no accident that the Eleatics often spoke as if *Being* (*τὸ ὄν*) and *the One* (*τὸ ἕν*) were interchangeable concepts. It is fair to say, therefore, that the Euclidean definition of 'unit' is nothing but a concise summary of the Eleatic doctrine of 'Being'. The definition was obtained by the same kind of indirect reasoning as Parmenides used to develop his theory of 'Being'. This is what Plato had in mind when he mentioned in passing the 'theory of the One' (*ἡ περὶ τὸ ἓν μάθησις*)." Á. Szabó (1968), 261.
[240] Pl. *Resp.* VII, 524e–525a: "[…] κινοῦσα ἐν ἑαυτῇ τὴν ἔννοιαν, καὶ ἀνερωτᾶν τί ποτέ ἐστιν αὐτὸ τὸ ἕν, καὶ οὕτω τῶν ἀγωγῶν ἂν εἴη καὶ μεταστρεπτικῶν ἐπὶ τὴν τοῦ ὄντος θέαν ἡ περὶ τὸ ἓν μάθησις." "ἡ περὶ τὸ ἓν μάθησις" as a nominative clause has been left untranslated in this last sentence by "G.M.A. Grube, rev. C.D.C. Reeve" in: J. M. Cooper & D. S. Hutchinson (1997), 1141.
[241] "τὸν δὲ σύμπαντα ταῦτα οὕτως εἰληφότα, τοῦτον λέγω τὸν ἀληθέστατα σοφώτατον: ὃν καὶ διισχυρίζομαι παίζων καὶ σπουδάζων ἅμα, ὅτε θανάτῳ τις τῶν τοιούτων τὴν αὑτοῦ μοῖραν ἀναπλήσει, σχεδὸν ἐάνπερ ἔτ᾽ ἀποθανὼν ᾖ, μήτε μεθέξειν ἔτι πολλῶν τότε καθάπερ νῦν αἰσθήσεων, μιᾶς τε μοίρας μετειληφότα μόνον καὶ ἐκ πολλῶν ἕνα γεγονότα, εὐδαίμονά τε ἔσεσθαι καὶ σοφώτατον ἅμα καὶ μακάριον, εἴτε τις ἐν ἠπείροις εἴτ᾽ ἐν νήσοις μακάριος ὢν ζῇ, κἀκεῖνον μεθέξειν τῆς τοιαύτης ἀεὶ τύχης, κεἴτε δημοσίᾳ τις ἐπιτηδεύσας ταῦτα εἴτε ἰδίᾳ διαβιῷ, τὰ αὐτὰ καὶ ὡσαύτως αὐτὸν πράξειν παρὰ θεῶν." *Ep.* 992b–c.
[242] Pl. *Resp.* VII, 526b: "ἐπειδὴ φαίνεταί γε προσαναγκάζον αὐτῇ τῇ νοήσει χρῆσθαι τὴν ψυχὴν ἐπ᾽ αὐτὴν τὴν ἀλήθειαν;"

Socrates embarks on an analysis of the forms (εἴδη), giving as examples 'The Beautiful' itself, 'The Good' itself, and 'The Great' itself.[243] Socrates sets out by proving that one wouldn't say that someone is taller than another man simply because he is 'one head' taller, and vice versa for the smaller person. *The reason is again, because it would lead to entertaining an opposite notion*, a contradiction, since one and the same reference of measure, the 'head', would at one time show something as being 'bigger' and at another time 'smaller'.[244] Rather, it is 'Bigness' or 'Smallness' itself, and nothing else, that would make something bigger or smaller.[245] It is noteworthy for us how this leads to Socrates having the same argument for numbers: "οὐκοῦν, ἦ δ' ὅς, τὰ δέκα τῶν ὀκτὼ δυοῖν πλείω εἶναι, καὶ διὰ ταύτην τὴν αἰτίαν ὑπερβάλλειν, φοβοῖο ἂν λέγειν, ἀλλὰ μὴ πλήθει καὶ διὰ τὸ πλῆθος;".[246] I.e. that it wouldn't be correct to say that the cause for 'ten' being greater than 'eight' is because it is larger by 'two'. Rather, the cause of it is the 'πλῆθος' itself, 'multitude/magnitude', that is to say the *concept* of πλῆθος. Socrates makes the same argument for cubits, stating that the μέγεθος itself, 'greatness/magnitude', is the cause for a cubit being bigger.[247] Socrates goes on to say that one would therefore avoid calling the addition of one with one as being 'addition', and the division of two as 'division'. Each thing that comes to be ("ἕκαστον γιγνόμενον") does so by partaking in its own particular 'οὐσία', 'essence/reality' ("τῆς ἰδίας οὐσίας"). 'δύο' ('two') comes to be δύο by participating in its particular οὐσία which is the 'δυάς' ('dyad' or 'Twoness'), and whatever becomes 'ἓν' does so by partaking in its own οὐσία which is the 'μονάς' ('monad' or 'Oneness').[248] We have here a clear identification of οὐσία with e.g. μονάς or δυάς. This has a bearing on Plato's previous insistence of λογιστική τε καὶ ἀριθμητικὴ leading towards οὐσία.

In what manner could Plato be claiming that the essence/being/substance (οὐσία) of things are understood by mathematics? If we turn to one of Plato's last works, the *Philebus,* where Socrates

[243] "ὑποθέμενος εἶναί τι καλὸν αὐτὸ καθ' αὑτὸ καὶ ἀγαθὸν καὶ μέγα καὶ τἆλλα πάντα". Pl. *Phd.* 100b.
[244] Cf. "ἐμοὶ γὰρ φαίνεται οὐ μόνον αὐτὸ τὸ μέγεθος οὐδέποτ' ἐθέλειν ἅμα μέγα καὶ σμικρὸν εἶναι, ἀλλὰ καὶ τὸ ἐν ἡμῖν μέγεθος οὐδέποτε προσδέχεσθαι τὸ σμικρὸν οὐδ' ἐθέλειν ὑπερέχεσθαι, ἀλλὰ δυοῖν τὸ ἕτερον, **ἢ φεύγειν καὶ ὑπεκχωρεῖν** ὅταν αὐτῷ προσίῃ τὸ ἐναντίον, τὸ σμικρόν, **ἢ προσελθόντος ἐκείνου ἀπολωλέναι**: ὑπομένον δὲ καὶ δεξάμενον τὴν σμικρότητα οὐκ ἐθέλειν εἶναι ἕτερον ἢ ὅπερ ἦν." *Phd.* 102d–e. Bold emphasis is mine.
[245] Pl. *Phd.* 100c–101b.
[246] Pl. *Phd.* 101b.
[247] "καὶ τὸ δίπηχυ τοῦ πηχυαίου ἡμίσει μεῖζον εἶναι ἀλλ' οὐ μεγέθει; ὁ αὐτὸς γάρ που φόβος." *Phd.* 101b.
[248] "τί δέ; ἑνὶ ἑνὸς προστεθέντος τὴν πρόσθεσιν αἰτίαν εἶναι τοῦ δύο γενέσθαι ἢ διασχισθέντος τὴν σχίσιν οὐκ εὐλαβοῖο ἂν λέγειν; καὶ μέγα ἂν βοῴης ὅτι οὐκ οἶσθα ἄλλως πως ἕκαστον γιγνόμενον ἢ μετασχὸν τῆς ἰδίας οὐσίας ἑκάστου οὗ ἂν μετάσχῃ, καὶ ἐν τούτοις οὐκ ἔχεις ἄλλην τινὰ αἰτίαν τοῦ δύο γενέσθαι ἀλλ' ἢ τὴν τῆς δυάδος μετάσχεσιν, καὶ δεῖν τούτου μετασχεῖν τὰ μέλλοντα δύο ἔσεσθαι, καὶ μονάδος ὃ ἂν μέλλῃ ἓν ἔσεσθαι, τὰς δὲ σχίσεις ταύτας καὶ προσθέσεις καὶ τὰς ἄλλας τὰς τοιαύτας κομψείας ἐῴης ἂν χαίρειν, παρεὶς ἀποκρίνασθαι τοῖς σεαυτοῦ σοφωτέροις:" *Phd.* 101b–c.

conveniently for us discusses with his interlocutors the meaning of ‘human good’ (again Plato begins by employing the verb ‘χρῆσθαι’ in the context of ‘τέχνη’ in these passages[249]) we read how Socrates claims that *as a gift from the Gods,* perhaps from *a Promethean hero,* the doctrine of ‘one and many, limit and unlimited’ *as the constituents of all that is said to exist* was given: “θεῶν μὲν εἰς ἀνθρώπους δόσις, ὥς γε καταφαίνεται ἐμοί, ποθὲν ἐκ θεῶν ἐρρίφη διά τινος Προμηθέως ἅμα φανοτάτῳ τινὶ πυρί:[250] καὶ οἱ μὲν παλαιοί, κρείττονες ἡμῶν καὶ ἐγγυτέρω θεῶν οἰκοῦντες, ταύτην φήμην παρέδοσαν, ὡς ἐξ ἑνὸς μὲν καὶ πολλῶν ὄντων τῶν ἀεὶ λεγομένων εἶναι, πέρας δὲ καὶ ἀπειρίαν ἐν αὑτοῖς σύμφυτον ἐχόντων.”[251]

As I argue in this thesis, Plato’s mathematics seems to be a spiritual science, seen as a gift from the Gods that would give salvation to the soul, by understanding Truth and reaching The Good. This becomes evident again in the above passage.[252] In the same work, Socrates even claims that the division of The Limit and The Unlimited was revealed by ‘a god’.[253] Furthermore, we have of the things that are (ὄντων) a basic division into ‘ἑνὸς μὲν καὶ πολλῶν’ (The One and The Many) and innate in these are ‘πέρας δὲ καὶ ἀπειρίαν’ (The Limit and The Unlimited). After taking music and vocal sound as examples,[254] Socrates later introduces a third form/kind as ‘the mixture’ of the forms (“εἰδῶν”) of Limit and Unlimited,[255] and a fourth kind (γένος) as ‘the cause of the mixture’.[256] In connection to these ‘classes’, Socrates discusses the nature of ‘hot and cold’, ‘strength and gentleness’, ‘more and less’, ‘equal and double’, ‘faster and slower’, ‘taller and shorter’ etc.[257] In the same context, Socrates mentions number (αριθμός).[258]

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[249] “ἣν δηλῶσαι μὲν οὐ πάνυ χαλεπόν, **χρῆσθαι δὲ παγχάλεπον**: πάντα γὰρ ὅσα **τέχνης** ἐχόμενα ἀνηυρέθη πώποτε διὰ ταύτης φανερὰ γέγονε. σκόπει δὲ ἣν λέγω.” *Phil*. 16c. Bold emphasis is mine.

[250] Cf. Aristotle’s statement on Parmenides positing Hot & Cold, **Fire & Earth** as **Being & Not-Being**: “Παρμενίδης δὲ […] δύο τὰς αἰτίας καὶ δύο τὰς ἀρχὰς πάλιν τίθησι, θερμὸν καὶ ψυχρόν, οἷον **πῦρ** καὶ γῆν λέγων: τούτων δὲ κατὰ μὲν τὸ ὂν τὸ θερμὸν τάττει θάτερον δὲ κατὰ τὸ μὴ ὄν.” Arist. *Met*. I: 986b–987a. Bold emphasis is mine.

[251] Pl. *Phil.* 16c–d.

[252] Cf. also: “[Σωκρ.] εἶεν: τὸ δὲ τρίτον τὸ μεικτὸν ἐκ τούτοιν ἀμφοῖν τίνα ἰδέαν φήσομεν ἔχειν; [Πρ.] σὺ καὶ ἐμοὶ φράσεις, ὡς οἶμαι. **[Σωκρ.] θεὸς μὲν οὖν, ἄνπερ γε ἐμαῖς εὐχαῖς ἐπήκοος γίγνηταί τις θεῶν.** [Πρ.] εὔχου δὴ καὶ σκόπει. **[Σωκρ.] σκοπῶ: καί μοι δοκεῖ τις, ὦ Πρώταρχε, αὐτῶν φίλος ἡμῖν νυνδὴ γεγονέναι.**” *Phil.* 25b.; “[Σωκρ.] ὕβριν γάρ που καὶ σύμπασαν πάντων πονηρίαν αὕτη **κατιδοῦσα ἡ θεός**, ὦ καλὲ Φίληβε, πέρας οὔτε ἡδονῶν οὐδὲν οὔτε πλησμονῶν ἐνὸν ἐν αὐτοῖς, **νόμον καὶ τάξιν πέρας ἔχοντ**᾽ **ἔθετο**: καὶ σὺ μὲν ἀποκναῖσαι φῂς αὐτήν, ἐγὼ δὲ τοὐναντίον **ἀποσῶσαι λέγω**. σοὶ δέ, ὦ Πρώταρχε, πῶς φαίνεται;” *Phil*. 26b–c. Bold emphasis is mine.

[253] Σωκράτης: “τὸν θεὸν ἐλέγομέν που τὸ μὲν ἄπειρον δεῖξαι τῶν ὄντων, τὸ δὲ πέρας;”. *Phil.* 23c.

[254] Pl. *Phil.* 17b–18d.

[255] “τούτω δὴ τῶν εἰδῶν τὰ δύο τιθώμεθα, τὸ δὲ τρίτον ἐξ ἀμφοῖν τούτοιν ἕν τι συμμισγόμενον.” *Phil.* 23c–d.

[256] “τετάρτου μοι γένους αὖ προσδεῖν φαίνεται. […] τῆς συμμείξεως τούτων πρὸς ἄλληλα τὴν αἰτίαν ὅρα, καὶ τίθει μοι πρὸς τρισὶν ἐκείνοις τέταρτον τοῦτο.” *Phil.* 23d.

[257] Pl. *Phil.* 23e–26c.

[258] “οὐκοῦν τὰ μὴ δεχόμενα ταῦτα, τούτων δὲ τὰ ἐναντία πάντα δεχόμενα, πρῶτον μὲν τὸ ἴσον καὶ ἰσότητα, μετὰ δὲ τὸ ἴσον τὸ διπλάσιον καὶ πᾶν ὅτιπερ ἂν **πρὸς ἀριθμὸν ἀριθμὸς ἢ μέτρον ᾖ πρὸς μέτρον**, ταῦτα σύμπαντα εἰς τὸ πέρας

It is very tempting to equate Plato's division here with the 'Ten Pythagorean Principles' mentioned by Aristotle as being a doctrine of at least *some* of the Pythagoreans, e.g.: "πέρας καὶ ἄπειρον, περιττὸν καὶ ἄρτιον, ἓν καὶ πλῆθος, δεξιὸν καὶ ἀριστερόν, ἄρρεν καὶ θῆλυ".[259] This seems tenable since Socrates in the *Politicus* proposes a division of arithmos into odd-even, and a division of humanity into male-female, just as the supposed Pythagoreans did.[260] With all this in mind we see that Plato's scope may not only be in line with Pythagorean thought (if 'unlimited' is identified with 'even', and 'limited' with 'odd'), but that the study of the 'even' and the 'odd' is more than simply a mathematical examination and calculation of numbers; it is rather an all-encompassing subject of the very things (τῶν ὄντων) of nature and existence.[261] For us today, it would be as if a form of [modern] mathematics, physics, and metaphysics, reinvented with ancient Greek garb, would coalesce into one grand subject, foremostly including [ancient] arithmetic, *logistikē*, geometry and perhaps also music/harmonics.[262] It seems reasonable to suggest then, that Plato's thought incorporated some form philosophy, or 'spirituality', in his ultimate aim for mathematics.

ἀπολογιζόμενοι καλῶς ἂν δοκοῖμεν δρᾶν τοῦτο. ἢ πῶς σὺ φῄς;" *Phil.* 25a–b.; τὴν τοῦ ἴσου καὶ διπλασίου, καὶ ὁπόση παύει πρὸς ἄλληλα τἀναντία διαφόρως ἔχοντα, σύμμετρα δὲ καὶ σύμφωνα ἐνθεῖσα **ἀριθμὸν** ἀπεργάζεται." *Phil.* 25d–e. Bold emphasis is mine.

[259] "ἕτεροι δὲ τῶν αὐτῶν τούτων τὰς ἀρχὰς δέκα λέγουσιν εἶναι τὰς κατὰ συστοιχίαν λεγομένας, **πέρας καὶ ἄπειρον, περιττὸν καὶ ἄρτιον, ἓν καὶ πλῆθος, δεξιὸν καὶ ἀριστερόν, ἄρρεν καὶ θῆλυ, ἠρεμοῦν καὶ κινούμενον, εὐθὺ καὶ καμπύλον, φῶς καὶ σκότος, ἀγαθὸν καὶ κακόν, τετράγωνον καὶ ἑτερόμηκες**:". Arist. *Met.* I: 986a. Bold emphasis is mine.

[260] "κάλλιον δέ που καὶ μᾶλλον κατ' εἴδη καὶ δίχα διαιροῖτ' ἄν, **εἰ τὸν μὲν ἀριθμὸν ἀρτίῳ καὶ περιττῷ τις τέμνοι, τὸ δὲ αὖ τῶν ἀνθρώπων γένος ἄρρενι καὶ θήλει**". Pl. *Plt.* 262e. See the note above for comparison. Bold emphasis is mine. Cf. Klein: "There can be no doubt that Plato's philosophy was decisively influenced by Pythagorean science, whatever the exact connection between Plato and the 'Pythagoreans' may have been. So too those definitions of arithmetic and logistic which were the basis of the preceding reflections seem to point to a Pythagorean origin." J. Klein (1992), 69.

[261] Cf. Wedberg's statement: "Plato's philosophical interpretation of the mathematics he knew is intimately related to his general theory of Ideas." A. Wedberg (1955), 26.

[262] Klein seems to agree with my assertion: "Thus the absence of any mention of either *arithmos* or *arithmoi* in the definitions of arithmetic and logistic in the *Gorgias* and in the *Charmides* not only expresses the fact that the multitude of arbitrarily chosen assemblages of monads is accessible to *episteme* only through the determinate *eide* which can always be found for the assemblages, but it also indicates that the characteristics of all possible kinds of numbers, beginning with the odd and the even, are to be found *indifferently in all countable things, be they objects of sense or 'pure' units.*" J. Klein (1992), 59.

5.2.2. Plato's epistemology and the two types of ἀριθμητική: how the dirt in the soul's wax-mould requires purification for true understanding

Szabó maintained that "it is well known that Plato distinguished between the world of becoming or the perceptible world (*ὁρατόν*) and the world of being (*νοητόν*)".[263] We will see in this subsection how such a division overlapped in Plato's epistemology, in his classification of all the sciences into two different types, in his metaphysics, and seemingly in his cosmology as well. This epistemology also has a clear bearing upon his doctrine of the soul's purification through the proper sciences, i.e. a form of 'spirituality' dealing with the salvation of the soul.

Although there is not enough space in this thesis to consider all implications of Plato's division between 'visible' and 'intelligible' numbers, it is important to shortly mention Plato's 'analogy of the divided line into two unequal sections', the "γραμμὴν δίχα τετμημένην λαβὼν ἄνισα τμήματα".[264] It deals with epistemological, metaphysical, psychological and several other concepts, but more importantly, it shows again how mathematics served as a model or perhaps even framework for the human psyche (its nature, virtue, habits, etc). In his discussion about the objects of knowledge and 'The Good', and its resemblance to the Sun in the visible world,[265] Socrates explains how there exists a division between two things: one part ruling over the intelligible class and place, and the other over the visible ("καὶ βασιλεύειν τὸ μὲν νοητοῦ γένους τε καὶ τόπου, τὸ δ᾽ αὖ ὁρατοῦ […], ἀλλ᾽ οὖν ἔχεις ταῦτα διττὰ εἴδη, ὁρατόν, νοητόν;").[266] This concept is likened to a divided line into two unequal sections, as just mentioned, and each section is divided a second time in the same ratio as before. Corresponding to these four sections of the line, Socrates posits four conditions/affections of the soul ("τέτταρα ταῦτα παθήματα ἐν τῇ ψυχῇ"): understanding (νόησις), thought (διάνοια), belief (πίστις), and likeness/imaging (εἰκασία).[267]

[263] Á. Szabó (1978), 308.
[264] Pl. *Resp.* VI, 509d.
[265] Pl. *Resp.* VI, 508a–509c.
[266] "νόησον τοίνυν, ἦν δ᾽ ἐγώ, ὥσπερ λέγομεν, δύο αὐτὼ εἶναι, καὶ βασιλεύειν τὸ μὲν νοητοῦ γένους τε καὶ τόπου, τὸ δ᾽ αὖ ὁρατοῦ, ἵνα μὴ οὐρανοῦ εἰπὼν δόξω σοι σοφίζεσθαι περὶ τὸ ὄνομα. ἀλλ᾽ οὖν ἔχεις ταῦτα διττὰ εἴδη, ὁρατόν, νοητόν;". VI, 509d.
[267] "καί μοι ἐπὶ τοῖς τέτταρσι τμήμασι τέτταρα ταῦτα παθήματα ἐν τῇ ψυχῇ γιγνόμενα λαβέ, νόησιν μὲν ἐπὶ τῷ ἀνωτάτω, διάνοιαν δὲ ἐπὶ τῷ δευτέρῳ, τῷ τρίτῳ δὲ πίστιν ἀπόδος καὶ τῷ τελευταίῳ εἰκασίαν, καὶ τάξον αὐτὰ ἀνὰ λόγον, ὥσπερ ἐφ᾽ οἷς ἐστιν ἀληθείας μετέχει, οὕτω ταῦτα σαφηνείας ἡγησάμενος μετέχειν." VI, 511d–e.

Now, it is notable for us how Socrates seems to claim that not only geometers but probably all mathematicians[268] are labouring with their subjects in the realm of διάνοια, employing the lower visible things of πίστις and εἰκασία in order to attempt an arrival at νόησις.[269] Those who are practicing geometry and similar habits/skills, i.e. the mathematicians, are in the habit of διάνοια, between δόξα (opinion) and νοῦς (mind): "διάνοιαν δὲ καλεῖν μοι δοκεῖς τὴν τῶν γεωμετρικῶν τε καὶ τὴν τῶν τοιούτων ἕξιν ἀλλ᾽ οὐ νοῦν, ὡς μεταξύ τι δόξης τε καὶ νοῦ τὴν διάνοιαν οὖσαν."[270] Here, Socrates seems to have added together πίστις and εἰκασία as being equivalent to δόξα, forming a trio instead of the previous quartet line: νοῦς, διάνοια, and δόξα. Nevertheless, for the previous quartet, Socrates concludes these passages with: "καὶ τάξον αὐτὰ ἀνὰ λόγον, ὥσπερ ἐφ᾽ οἷς ἐστιν ἀληθείας μετέχει, οὕτω ταῦτα σαφηνείας ἡγησάμενος μετέχειν". The four sections are to be arranged in a ratio so that they partake in as much of truth as they partake in of clearness. We see that 'truth', ἀλήθεια, is again at the core of not only the aim of mathematics, but also of the very existence of the world and the psyche of the human soul.

In *Theaeteus*, Plato's work that deals particularly with epistemology, Socrates claims that false judgment arises out of the wax in the soul being dirty, rugged and impure. In others, the wax is deep and smooth, and these men learn things easily about 'being' ("ὄντα").[271] Socrates evinces that although false judgment may arise out of the connection between perception and thought, it does actually exist within thought itself as well.[272] This seems connected to what Socrates says in the *Res Publica* about *all the sciences being discussed*, that by each one of these subjects there is an *'instrument'/'organ of sense' of the soul that is purified and rekindled anew*; an

[268] "οἶμαι γάρ σε εἰδέναι ὅτι οἱ περὶ τὰς γεωμετρίας τε καὶ λογισμοὺς καὶ τὰ τοιαῦτα πραγματευόμενοι […]". VI, 510c.
[269] "οὐκοῦν καὶ ὅτι τοῖς ὁρωμένοις εἴδεσι προσχρῶνται καὶ τοὺς λόγους περὶ αὐτῶν ποιοῦνται, οὐ περὶ τούτων διανοούμενοι, ἀλλ᾽ ἐκείνων πέρι οἷς ταῦτα ἔοικε, τοῦ τετραγώνου αὐτοῦ ἕνεκα τοὺς λόγους ποιούμενοι καὶ διαμέτρου αὐτῆς, ἀλλ᾽ οὐ ταύτης ἣν γράφουσιν, καὶ τἆλλα οὕτως, αὐτὰ μὲν ταῦτα ἃ πλάττουσίν τε καὶ γράφουσιν, ὧν καὶ σκιαὶ καὶ ἐν ὕδασιν εἰκόνες εἰσίν, τούτοις μὲν ὡς εἰκόσιν αὖ χρώμενοι, ζητοῦντες δὲ αὐτὰ ἐκεῖνα ἰδεῖν ἃ οὐκ ἂν ἄλλως ἴδοι τις ἢ τῇ διανοίᾳ." VI, 510d–511a.
[270] Pl. *Resp.* VI, 511d.
[271] Pl. *Tht.* 194c–195a. "ΣΩ. Ταῦτα τοίνυν φασὶν ἐνθένδε γίγνεσθαι**. ὅταν μὲν ὁ κηρός του ἐν τῇ ψυχῇ βαθύς τε καὶ πολὺς καὶ λεῖος καὶ μετρίως ὠργασμένος ᾖ**, […] καθαρὰ τὰ σημεῖα ἐγγιγνόμενα καὶ ἱκανῶς τοῦ βάθους ἔχοντα πολυχρόνιά τε γίγνεται καὶ εἰσὶν οἱ τοιοῦτοι πρῶτον μὲν εὐμαθεῖς, ἔπειτα μνήμονες, εἶτα οὐ παραλλάττουσι τῶν αἰσθήσεων τὰ σημεῖα ἀλλὰ δοξάζουσιν ἀληθῆ. σαφῆ γὰρ καὶ ἐν εὐρυχωρίᾳ ὄντα **ταχὺ διανέμουσιν ἐπὶ τὰ αὑτῶν ἕκαστα ἐκμαγεῖα, ἃ δὴ ὄντα καλεῖται, καὶ σοφοὶ δὴ οὗτοι καλοῦνται.** […] **Ὅταν τοίνυν λάσιόν του τὸ κέαρ ᾖ, ὃ δὴ ἐπῄνεσεν ὁ πάσσοφος ποιητής, ἢ ὅταν κοπρῶδες καὶ μὴ καθαροῦ τοῦ κηροῦ**, ἢ ὑγρὸν σφόδρα ἢ σκληρόν, ὧν μὲν ὑγρὸν εὐμαθεῖς μέν, ἐπιλήσμονες δὲ γίγνονται, ὧν δὲ σκληρόν, τἀναντία. […] **πάντες οὖν οὗτοι γίγνονται οἷοι δοξάζειν ψευδῆ**. ὅταν γάρ τι ὁρῶσιν ἢ ἀκούωσιν ἢ ἐπινοῶσιν, ἕκαστα ἀπονέμειν ταχὺ ἑκάστοις οὐ δυνάμενοι βραδεῖς τέ εἰσι καὶ ἀλλοτριονομοῦντες παρορῶσί τε καὶ παρακούουσι καὶ παρανοοῦσι πλεῖστα, **καὶ καλοῦνται αὖ οὗτοι ἐψευσμένοι τε δὴ τῶν ὄντων καὶ ἀμαθεῖς**." Bold emphasis is mine.
[272] Pl. *Tht.* 195d; 196c.

‘instrument/organ of the soul’ which has been destroyed and blinded by other pursuits/habits. Socrates uses the verb ‘σῴζω’ (“σωθῆναι”) here, in the context of the soul’s salvation, and the concept of ‘beholding the truth’ (“ἀλήθεια”) is mentioned.[273]

Returning to the *Theaetetus*, it is important to note here that for this possible ‘conclusion’ (possible, since there is no final verdict in this treatise of what knowledge really is) Socrates takes as an example the *numbers* five, seven, eleven and twelve. Socrates evidently refers to some form of ‘Ideal Numbers’, since he says that he is not referring to the contemplation of seven and five propositioned humans or anything of that sort, but: “ἀλλ᾽ αὐτὰ πέντε καὶ ἑπτά, ἅ φαμεν ἐκεῖ μνημεῖα ἐν τῷ ἐκμαγείῳ εἶναι καὶ ψευδῆ ἐν αὐτοῖς οὐκ εἶναι δοξάσαι”.[274] Socrates is speaking of those numbers which exist in the records of the [waxen] mould, i.e. in the soul, and it is not possible to imagine them having any falsehood. But Socrates and Theaetetus both agree that plenty of people have false judgments of these numbers, some think that the sum of them would be twelve, others would say eleven. Theaetetus says that when even larger numbers are involved, people are the more mistaken.[275] It is not clear here why and how someone would mistake five plus seven to be something else than twelve, but Plato’s point may perhaps be that very few have understood the ‘Twelveness’ of it? Maybe the meaning is that the even (twelve) is often mistaken for the odd (eleven), when reflecting over ‘The Even’ and ‘The Odd’ themselves?

This idea is further considered just after these passages. Socrates makes a distinction between “having” and “acquiring/possessing” knowledge.[276] A simile is made of someone hunting birds; he may “possess” them because he “has” them in an enclosure, but in another sense he does not “have” them completely, since he has only caught them in an area and thereby gained power over them.[277] When considering arithmetic, Socrates seems to refer to the idea of retaining knowledge about numbers in one’s memory, thereby “having” them only when needed in calculation, although still “possessing” them before.[278] He refers to ἀριθμητική as an example of not only “having” and “possessing” knowledge but more importantly as the possible confusion of a person

[273] “ὅτι ἐν τούτοις τοῖς μαθήμασιν ἑκάστου ὄργανόν τι ψυχῆς ἐκκαθαίρεταί τε καὶ ἀναζωπυρεῖται ἀπολλύμενον καὶ τυφλούμενον ὑπὸ τῶν ἄλλων ἐπιτηδευμάτων, κρεῖττον ὂν σωθῆναι μυρίων ὀμμάτων: μόνῳ γὰρ αὐτῷ ἀλήθεια ὁρᾶται.” Pl. *Resp*. VII, 527d–e. See the subsection ‘5.2.7. γεωμετρία’ for more on this.
[274] Pl. *Tht.* 196a.
[275] Pl. *Tht.* 196a–b.
[276] Pl. *Tht.* 197c: ”οὐ τοίνυν μοι ταὐτὸν φαίνεται τῷ **κεκτῆσθαι** τὸ **ἔχειν**.”
[277] Pl. *Tht.* 197c–d.
[278] Pl. *Tht.* 198d.

who believes he has complete knowledge of all numbers, but instead actually retaining a "false judgement" concerning this knowledge.[279] More concisely, Socrates says that one may have a knowledge of the number eleven *while believing that it is the number twelve. It is a matter of confusing different sets of knowledge/numbers,*[280] but from here the epistemological debate carries on much further than the scope of this thesis is examining.

In these passages, Socrates confirms, as in Plato's *Gorgias* and *Charmides,*[281] that ἡ ἀριθμητική τέχνη deals with the knowledge of all odd and even things: "ταύτην δὴ ὑπόλαβε θήραν ἐπιστημῶν ἀρτίου τε καὶ περιττοῦ παντός."[282] One of the reasons for dividing all the sciences into two separate parts may be stated in Plato's *Charmides*, 166a-c, where Socrates says that almost every science studies *something which is distinct from the science itself*. Socrates takes λογιστική as an example, claiming that although it studies the odd and the even ("τοῦ ἀρτίου καὶ τοῦ περιττοῦ"), the odd and the even are distinct from λογιστική itself.[283]

[279] Pl. *Tht.* 198e–199c.
[280] Pl. *Tht.* 199a–b. 199b: "…μὴ γὰρ ἔχειν τὴν ἐπιστήμην τούτου οἷόν τε, ἀλλ᾽ ἑτέραν ἀντ᾽ ἐκείνης…"
[281] *Grg.* 451a-c; *Chrm.* 166a.
[282] Pl. *Tht.* 198a.
[283] Pl. *Chrm.* 166a: "οὐκοῦν ἑτέρου ὄντος τοῦ περιττοῦ καὶ ἀρτίου αὐτῆς τῆς λογιστικῆς;"

5.2.3. Euclid, Plato, and mathematics: a possible hierarchical classification (‘προποδισμός’?) or duality of μονάς and ἀριθμός

If we are to take Proclus’ statement at face value, Euclid was of the same purpose/choice (προαίρεσις, lit. ‘*choosing* one thing *before* another’), as Plato (perhaps translates as “of the same philosophical sect”; “τῇ προαιρέσει δὲ Πλατωνικός ἐστι”), and he was familiar with the same philosophy, that is why he established the purpose of the *Elements* as being *the composition of the Platonic ‘σχημάτων’.*[284] According to modern scholars such as Klein and Zeuthen, Euclid’s Books VII, VIII, IX and XIII are essentially the works of the Platonists Theaetetus and Theodorus, members of Plato’s Academy.[285] It is with this possible systematization and unification of Platonic (and Pre-Socratic and Pythagorean) mathematical doctrine in Euclid’s *Elements* that we will briefly examine the definition of ἀριθμός in Euclid’s Book VII.

In Book VII of the *Elements,* **Definition 1**, it is stated: **“μονάς ἐστιν, καθ᾽ ἣν ἕκαστον τῶν ὄντων ἓν λέγεται.”** And in **Definition 2**: **“ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος.”** Definition 1 could be translated literally as “*The μονάς is that by which each of the things that are is called one.*” And Definition 2: “*ἀριθμός is the compounded*[286] *multitude of the μονάδων*”. I render ‘καθ᾽ ἣν’ (the pronoun ἣν must be referring to the feminine noun μονάς) simply as ‘by which’ according to *LSJ*s entry of ‘κατά’: “B. WITH Acc., […] II. *distributively,* of a whole divided into parts”. Not that μονάς is conceptually divided, but at least *distributively* making ‘ἕκαστον τῶν ὄντων’ to be *called* (λέγεται) one (ἓν). Perhaps, ‘καθ᾽ ἣν’ could also be rendered with “2. with or without signf. of motion, *on, over, throughout* a space” as in ‘the μονάς is that over which each of the things that are is called one’, but this might be too farfetched since it would probably imply motion.[287] The first definition is rendered by Heath as “an **unit** is that by virtue of which each of the things that exist is called one.”[288] Pritchard argues against Heath’s translation of ‘καθ᾽ ἣν’ as ‘by virtue of which’ and he prefers ‘in accordance with which’. Pritchard argues that suggesting “that the unit has some capacity or power to make things one” is

[284] “καὶ τῇ προαιρέσει δὲ Πλατωνικός ἐστι καὶ τῇ φιλοσοφίᾳ ταύτῃ οἰκεῖος, ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων Πλατωνικῶν σχημάτων σύστασιν.” Procl. *in Euc.* §68.
[285] J. Klein (1992), 43.
[286] Note: ‘σύγκειμαι’, lit. “lying together” (*LSJ*). Perhaps, therefore, not “compounded” as in “mixed”, but simply as in “arranged together”.
[287] But see the *LSJ*, same entry: “Geom., *at* a point, Euc.1.1,al.; τέμνειν [σφαῖραν] κ. κύκλον *in* a circle, *Archim.Aren.*1.17; also, *in the region of*, “οἱ κ. τὸν ἥλιον γινόμενοι ἀστέρες” Gem.12.7”
[288] T. L. Heath (1926).

"surely absurd" since a unit should be no more than a measure.[289] But Pritchard forgets that Euclidean/Platonic terminology is not fully understood by any living person, including Pritchard himself, so that 'the unit having a capacity to make things one' is possible in Plato's doctrine, as we shortly shall see. Pritchard also seems to contradict himself, or he writes very perplexingly, by first saying that "(3) κατά does not refer to participation. (4) The definition owes nothing to Platonic metaphysics; particularly"[290], but later admitting the opposite: "[…] the statement of Sextus Empiricus […] 'Pythagoras said that a first principle of things is the unit, by *participating* in which each of the things is called one' […] This sounds very like the Euclidean definition. *It can also plausibly be argued that it expresses Platonic doctrine* [cursive until here is mine] (it is certainly not *Pythagorean*)".[291]

Surely, Pritchard is too keen here on semantic wordplays to an almost bizarre level. Plato himself, as we have already seen in the *Phaedo*, evidently refers to 'participation' ('μετασχεῖν') of the one in the monad.[292] In order to avoid any reference to 'addition' or 'division', Socrates said that each thing that comes to be ("ἕκαστον γιγνόμενον" cf. with Euclid's "ἕκαστον τῶν ὄντων") does so by partaking in its own particular 'οὐσία', 'essence/reality' ("τῆς ἰδίας οὐσίας"). 'δύο' by participating in its particular οὐσία which is the 'δυάς', and 'ἓν' by partaking in its own οὐσία, the 'μονάς'. This passage in Plato is not mentioned by Pritchard in his contradictory statements.

As for 'τῶν ὄντων', we will later see how in Plato's *Timaeus*, in the threefold division of 'The All', the 'ὄν' as the 'model, source, and father' is said to be apprehended by 'νόησις' (intelligence, understanding).[293] In the *Philebus,* we also saw how Socrates claimed that the division of The Limit and The Unlimited was revealed by 'a god' (Prometheus?). The things that are (ὄντων) had a basic division into 'ἑνὸς μὲν καὶ πολλῶν' (The One and The Many, cf. Euclid's ἕν and πλῆθος) and cognate with these were 'πέρας δὲ καὶ ἀπειρίαν' (The Limit and The Unlimited).[294]

[289] Pritchard (1995), 13.
[290] Pritchard (1995), 12.
[291] Pritchard (1995), 13–14.
[292] "[…] καὶ ἐν τούτοις οὐκ ἔχεις ἄλλην τινὰ αἰτίαν τοῦ δύο γενέσθαι ἀλλ᾽ ἢ τὴν τῆς δυάδος μετάσχεσιν, καὶ δεῖν τούτου μετασχεῖν τὰ μέλλοντα δύο ἔσεσθαι, καὶ μονάδος ὃ ἂν μέλλῃ ἓν ἔσεσθαι […]" *Phd*. 101b–c. See the subsection: 5.2.1.
[293] See the subsection 5.2.7.
[294] *Phil*. 23c. See the subsection 5.2.1.

Returning to Euclid's definitions, a μονάς[295] seems to be that which in principle groups existing things into 'ones', and an ἀριθμός is the compounded multitude of these monads. Euclid actually seems to be explaining its abstract nature hierarchically, or for lack of a better term (and we must be excused for our 'Neoplatonic' influence here) *hypostasizing* it.[296] If Pritchard (contrary to Szabó, who claims that the Euclidean definition was a new concept of number), is correct that "the Euclidean definition, the definitions ascribed to pre-Euclidean mathematicians, the regular meaning of the word *arithmos* in Greek literature before Euclid, and the subject matter of Pythagorean arithmetic all relate to a single concept. An *arithmos* is a set of units",[297] then we see from the above how μονάς is simply at the higher rung of this conceptual hierarchical scale whereas ἀριθμός is just below that, in the general Platonic and Pythagorean philosophy.[298] The technical term for this might have been 'προποδισμός' (*process, progression*) among later authors.[299] Or perhaps, there is a duality-scheme here, as in the Pythagorean opposites mentioned by Aristotle, e.g. "πέρας καὶ ἄπειρον, περιττὸν καὶ ἄρτιον, **ἓν καὶ πλῆθος**"?[300] Euclid's μονάς is

[295] The literal meaning of μονάς is '*solitary, alone*' (*LSJ*). Note the interesting possible pseudo-wordplay in English of 'al-*one*'; 'All-One'.

[296] "Neoplatonic metaphysics is driven by certain dogmas and principles that all or most of the school's proponents adhered to. […] Most of these principles explicate and regulate the hierarchical ordering of the Neoplatonic metaphysics. The hierarchy results from Neoplatonists' interest, shared with Plato and Aristotle, in determining the priority and posteriority relations structuring reality. As they see it, the articulation of reality is the articulation of the relational patterns ordering being (O'Meara 1996)." P. Remes (2008), 42; cf. also: "Although the later Neoplatonists employ the term in much the same sense as contemporary research literature, Plotinus uses the term *hypostasis* for several kinds of entities that are immaterial and independent […]". P. Remes (2008), 48.

[297] P. Pritchard (1995), 25. Pritchard continues: "But what of the Euclidean representation of *arithmoi* by line segments? It seems that this is determined by the Euclidean style of proof, rather than being a reflection of a different notion of number. The line segments are preferred because they suppress the visual aspects of the proof; also, greater generality is achievable since even and odd numbers are not distinguishable in this form of diagram." See also Pritchard (1995), 14–15, where he argues that Plato's mathematical philosophy, especially the concept of *arithmos,* was consistent with contemporary practice.

[298] Szabó also notes the similarity between the Euclidean definition and the Pythagorean: "Cf. Sextus Empiricus. *Adversus math.* X.260–1: ὁ Πυθαγόρας ἀρχὴν ἔφησεν εἶναι τῶν ὄντων τὴν μονάδα, ἧς κατὰ μετοχὲν ἕκαστον τῶν ὄντων ἓν λέγεται. The concluding words of this quotation are almost the same as Euclid's definition of 'unit'." Á. Szabó (1978), 261:n149.

[299] Cf. J. Klein (1992), 52, on the 'definitions of the series of numbers' among ancient authors: "'a progression of multitude beginning from the unit and a recession ceasing with the unit' (*προποδισμὸς πλήθους ἀπὸ μονάδος ἀρχόμενος καὶ ἀναποδισμὸς εἰς μονάδα καταλήγων* – Theon 18, 3 ff. […] Thus also Domninus (413, 5 ff.): 'The whole realm of number is a progress from the unit to the infinite by means of the excess of one unit [of each successive number over the preceding].' (*ὁ δὲ σύμπας ἀριθμός ἐστι προκοπὴ ἀπὸ μονάδος κατὰ μονάδος ὑπεροχὴν ἄχρις ἀπείρου*.)" etc. and: "The truth is that the unit can be spoken of as a 'multitude' only improperly, confusedly (*συγκεχυμένως* – Iamblichus 11, 7). The unit is rather that permanently same and irreducible basic element which is met with in all counting – and thus in every number. To determine a number means to count off in sequence the given single units, be they single objects of sense, single events within the soul, or single 'pure' units. What is countable must, *insofar* as it is countable, be articulated in such a way that the units in question are similar to one another (cf. P. 46) and yet separated and clearly 'determined' (*διωρισμένα*). This means that the single units possess similarity and perfect wholeness insofar as they are units of *counting.*" I.e., one would count 'one apple, two apples, three apples' etc and never distinguish the *matter* of the *counting* which is an *apple* and nothing else, while the *quantity* of apples may differ. It is therefore a rather simple concept.

evidently connected to **ἕν**, and Euclid's ἀριθμός is connected to **πλῆθος**, all the while the μονάς is playing a crucial role in both definitions.

Nevertheless, it is interesting to note how Plato several times refers to 'τὸ πλῆθος' in connection with his mathematics and metaphysical speculations, as we have already seen.[301] It is referred to in the same sentence with 'ἕν', when τὸ ἕν is confounded and seen to be two things at the same time, "ἅμα γὰρ ταὐτὸν ὡς ἕν τε ὁρῶμεν καὶ ὡς ἄπειρα τὸ πλῆθος".[302] In the *Phaedo* 101b, we are informed that the cause of 'ten' being larger than 'eight' by 'two' is not because of the number 'two', but rather *because of the concept of* πλῆθος itself. In *one* sense, therefore, πλῆθος was a concept in itself for Plato, distinct from ἀριθμός, as 'multitude' in itself. Later, we will also see how πλῆθος is intimately connected to λογιστική and how it involves Plato's definition on the difference between ἀριθμητική and λογιστική in Plato's Gorgias 451b.[303]

Is there then a possibility that Euclid's Definition 1 is centered around Plato's ἀριθμητική,[304] and the Definition 2 is concerned with λογιστική? Perhaps, but that would require much further research into the mathematical relation between Euclid and Plato, and it would go beyond the scope of this thesis.

On the question of the philosophers' prohibition on mathematically manipulating τὸ ἕν,[305] we may turn το books VII, VIII and IX where Euclid dealt explicitly with μονάδες and ἀριθμοί. We may interestingly note that never does Euclid allow the μονάς to be multiplied, subtracted, added or divided, although the same is allowed for ἀριθμοί. The unit is often used to simply "measure any number" as in VII, prop. 15.[306] where it is symbolized as the shortest line among larger lines which are numbers, or it is conceptualized *as something which is begun with* and out of which larger numbers (lines) are drawn in proportion, with the unit only *supposed* and not drawn (e.g.

[300] "ἕτεροι δὲ τῶν αὐτῶν τούτων τὰς ἀρχὰς δέκα λέγουσιν εἶναι τὰς κατὰ συστοιχίαν λεγομένας, **πέρας καὶ ἄπειρον, περιττὸν καὶ ἄρτιον, ἓν καὶ πλῆθος, δεξιὸν καὶ ἀριστερόν, ἄρρεν καὶ θῆλυ, ἠρεμοῦν καὶ κινούμενον, εὐθὺ καὶ καμπύλον, φῶς καὶ σκότος, ἀγαθὸν καὶ κακόν, τετράγωνον καὶ ἑτερόμηκες**:". Arist. *Met.* I: 986a. Bold emphasis is mine.
[301] See the subsection 5.3.1.
[302] Pl. *Resp*. VII, 525a.
[303] "διαφέρει δὲ τοσοῦτον, ὅτι καὶ πρὸς αὑτὰ καὶ πρὸς ἄλληλα πῶς ἔχει πλήθους ἐπισκοπεῖ τὸ περιττὸν καὶ τὸ ἄρτιον ἡ λογιστική"; also in *Chrm.* 166a. See the subsection 5.4.2.
[304] Which in *Gorg.* 451b is defined as: "τῶν περὶ τὸ ἄρτιόν τε καὶ περιττὸν [γνῶσις], ὅσα ἂν ἑκάτερα τυγχάνῃ ὄντα".
[305] Pl. *Resp.* VII, 525e. See the subsection 5.2.1.
[306] "ἐὰν μονὰς ἀριθμόν τινα μετρῇ".

IX, prop. 8–10).[307] Klein also confirms that *fractional parts* of the *unit* are avoided in Books VII, VIII and IX, while ‘parts of *number’* are allowed.[308]

Since the unit is e.g. ‘measuring’ any number or ‘commencing’ a series of numbers in proportion, this may also show how beginning with μονάς and continuing/ending with ἀριθμός again infers a sort of hierarchical classification *or* a juxtaposed duality of e.g. πέρας καὶ ἄπειρον.

[307] IX, 8: "Ἐὰν ἀπὸ μονάδος ὁποσοιοῦν ἀριθμοὶ ἑξῆς ἀνάλογον ὦσιν"; IX, 9: "Ἐὰν ἀπὸ μονάδος ὁποσοιοῦν ἑξῆς κατὰ τὸ συνεχὲς ἀριθμοὶ ἀνάλογον ὦσιν".

[308] "[…] it is understandable that Books VII, VIII, and IX consistently avoid the introduction of fractional parts of the *unit* of calculation, while they certainly use the notion of the part or parts of a *number*, as previously defined (VII, Defs. 3 and 4; cf. especially VII, 37 and 38)." J. Klein (1992), 43.

5.3. λογιστική

5.3.1. λογιστική and the ambiguity of 'λογιστικός'/'λογισμός'

Since Plato mentions ἀριθμητική and λογιστική together in the *Res Publica* without making any distinction between them, it is not completely clear whether λογιστική was a 'different' subject. We will see however, in the next subsection, how there was some form of difference between them. Still, it is important to note that Plato generally refers to 'λογισμοί' and 'λογιστικοί' when speaking of those versed in calculation, and seems to include ἀριθμητική, or at least ἀριθμός, in these arguments.[309] There is also the ambiguity of the meaning employed by Plato, since as the *LSJ* entry has both "*skilled* or *practised in calculating*" and "*endued with reason, rational, reasonable*" for the adjective 'λογιστικός'. Similarly, the noun 'λογισμός' can mean "*counting, calculation*" but also "II. without reference to number, *calculation, reasoning* […] III. *reasoning power*".[310] The latter meaning of 'rational' for 'λογιστικός' may for instance be translated in *Resp*. VII, 525b.[311] Hence, one could argue that 'to calculate' is in the Greek language equivalent to 'being rational'.

Fowler has drawn attention to three main issues with Plato's different accounts of λογιστική. (1) Plato seems to both refer to ἀριθμητική and λογιστική as they were parts of the same study[312] while also making a distinction between them in some of his other works. (2) λογιστική seems to have an inherent meaning of practical common calculation. (3) Around the first century B.C. a theoretical λογιστική referred to an artificial calculation of numbered sets such as counting pebbles or cattle. The neo-platonic writers, perhaps by way of definitions like the one by astronomer Geminus, seem to use this particular meaning for λογιστική.[313]

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[309] See e.g.: Pl. *Resp*. VII, 525b, 525d and 526b:5.
[310] Cf. Fowler: "The references to *logistikos* (the lemma form of *logistikē*) in Brandwood, *A Word Index to Plato,* divide between about twenty-five where the context is explicitly mathematical and about ten with the more general sense of 'intellectual principle' or 'reason'; while *logismos* (excluding ten instances found in the pseudo-Platonic *Definitions*, all of them non-mathematical) divide between about thirty explicitly mathematical references and forty usages with the more general sense of 'rational discussion' or 'reason'." D. H. Fowler (1999), 148.
[311] "ὧν ζητοῦμεν ἄρα, ὡς ἔοικε, μαθημάτων ἂν εἴη: πολεμικῷ μὲν γὰρ διὰ τὰς τάξεις ἀναγκαῖον μαθεῖν ταῦτα, φιλοσόφῳ δὲ διὰ τὸ τῆς οὐσίας ἁπτέον εἶναι γενέσεως ἐξαναδύντι, **ἢ μηδέποτε λογιστικῷ γενέσθαι**." *Resp*. VII, 525b. Bold emphasis is mine.
[312] As we have seen in the section 'ἀριθμητική'.
[313] D. H. Fowler (1999), 106–107. On the 'confusion' of defining these two subjects, cf. also Klein's statement on Euclid's Books VII, VIII, and IX: "It is true that in these books 'arithmetic' and 'logistic' matter can hardly be separated, but the 'logistic' constituent undoubtedly predominates and is here understood precisely as 'arithmetic'; it is obviously this fact which permits the later 'arithmetical tradition' (*ἀριθμητικὴ παράδοσις*) *to include the theory of relations as well.*" J. Klein

The issue with (1) may be explained partly by the dialogue form of the *Res Publica* – where Socrates mentions all subjects necessary for the *guardians*, but does not explain them in detail, since the core of the discussion here is a general overview of education – and partly by the already mentioned fact that Plato may have altered or developed some of his concepts. In the *Res Publica,* we have also seen how "λογίζεσθαί τε καὶ ἀριθμεῖν"[314] is divided into counting visible things (e.g. military troops) and intelligible things (τὸ ἕν which cannot be manipulated). All sciences are divided into a 'common' method of calculation and the 'philosophers' method, *so that the phrase* 'λογίζεσθαί τε καὶ ἀριθμεῖν' *is used in two different senses.*

5.3.2. The difference between ἀριθμητική and λογιστική: the former considers quantity (simple) and the latter multitude (complex)

We can understand the basic nature of ἀριθμητική and λογιστική by the statements in the *Gorgias* and *Charmides* where Socrates informs us of the distinction between them. I think it is possible to assume that Plato had a sort of hierarchical scheme in mind where ἀριθμητική considered the simplest forms of ἀριθμοί and λογιστική the more complex relations between the ἀριθμοί, at least in these two dialogues where the definitions are stated clearly.[315]

In the *Gorgias* 451a-c, Socrates provides us with a short and concise definition of both ἀριθμητική and λογιστική. The context is within a discussion about rhetoric and persuasion and Socrates just gives a passing example of the nature and difference of ἀριθμητική and λογιστική in this overarching dialogue on the art of persuasion. It is thus important to first note how this is not a dialogue on mathematics in the setting of an ideal state, as in the *Res Publica,* or any other similar context, but the main focus is rhetoric. Plato's description may therefore not be so exhaustive.

(1992), 43–44; "A further indication of these difficulties may be seen in the fact that Plato (*Statesman* 259 E) refers the knowledge of the 'difference among numbers' (*τὴν ἐν τοῖς ἀριθμοῖς διαφορὰν*) to logistic, although this might as well be said to be the business of arithmetic (cf. 258 D, also *Republic* 587 D)." J. Klein (1992), 39.

[314] Pl. *Resp.* VII, 522e.

[315] Cf. also Klein: "Arithmetic deals with numbers insofar as these present assemblages whose unity is rooted in the unity of a certain *eidos*, although this fact usually remains hidden from the person immersed in the practical activity of counting. As a theoretical discipline, at least, arithmetic studies each quantity and each multitude of monads which falls under a particular *eidos* only indirectly. 'Logistic', on the other hand, be it practical or theoretical, aims of necessity – insofar as it is concerned with the mutual relations of numbers – directly at the 'quantity', at the multitude, of those things which are in each case related to one another or computed, i.e., at the 'material' which underlies each relation or calculation." J. Klein (1992), 60.

In the following passage in the *Gorgias* 451b, "ἡ ἀριθμητικὴ τέχνη" is that which just like rhetoric has its authority/influence ("τὸ κῦρος") through dialogue/speech/reasoning ("διὰ λόγου"). Here we are subtly reminded of another important concept for Plato; the preeminence of oral instruction over written.[316] If the questioner would continue to ask about what sort of things ("τῶν περὶ τί;") does ἀριθμητικὴ concern itself with, Socrates would respond "τῶν περὶ τὸ ἄρτιόν τε καὶ περιττὸν [γνῶσις], ὅσα ἂν ἑκάτερα τυγχάνῃ ὄντα"; "The knowledge of the even and the odd, *how much* (or *how many*) there happens to exist of both."[317] If the same question was posed for λογιστική, Socrates' response would be that just as ἀριθμητικὴ, it deals with the even and odd, but: "διαφέρει δὲ τοσοῦτον, ὅτι καὶ πρὸς αὑτὰ καὶ πρὸς ἄλληλα πῶς ἔχει πλήθους ἐπισκοπεῖ τὸ περιττὸν καὶ τὸ ἄρτιον ἡ λογιστική"; "but it differs thus much – λογιστική examines the odd and the even *in terms of how multitude exists both in themselves and between each other".*[318]

In Plato's *Charmides,* which has already been briefly mentioned, there is a definition of λογιστική with a very similar wording: "οἷον ἡ λογιστική ἐστίν που τοῦ ἀρτίου καὶ τοῦ περιττοῦ, πλήθους ὅπως ἔχει πρὸς αὑτὰ καὶ πρὸς ἄλληλα: ἦ γάρ;".[319] "Such as λογιστική is the art of the even and the odd, how *multitude* (πλῆθος) relates both to themselves and to each other, am I right?" This confirms the possibility that Plato was dealing with already pre-determined definitions of some of these mathematical concepts, and that these definitions were quite fixed. But it probably remains difficult to establish whether these definitions were exclusively of Plato's academy, or if they also were found in general Greek mathematical thought. Furthermore, in Plato's *Politicus,* Socrates informs us that it is indeed ἀριθμός which is divided into odd and even, just as the human 'race' could be divided into males and females.[320]

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[316] Cf. *Phdr.* 274d–275e, for Plato's story of the Egyptian god Thoth's invention of writing and the king Thamus/Ammons negative opinion of it, and for Socrates' argument against it.

[317] Pl. *Grg.* 451b. For "[γνῶσις]", Burnet wrote in the critical apparatus "**b** 4 γνῶσις secl. Bekker"

[318] On the μονάς, the *Theologoumena Arithmeticae,* supposedly by Iamblichus, states: "ἀρτία τε οὖσα καὶ περιττὴ καὶ ἀρτιοπέριττος" (περὶ μονάδος, I: sentence 12); "the μονάς is even, and odd, and even-odd"; cf. Pl. *Phd.* 105c: "οὐδ' ᾧ ἂν ἀριθμῷ τί ἐγγένηται περιττὸς ἔσται, οὐκ ἐρῶ ᾧ ἂν περιττότης, ἀλλ' ᾧ ἂν μονάς". Plato's *Theaetetus* confirms ἀριθμητικὴ as being the science of the knowledge of the odd and even: "[Σωκρ.] […] ἀριθμητικὴν μὲν γὰρ λέγεις τέχνην; [Θεαίτ.] ναί. [Σωκρ.] ταύτην δὴ ὑπόλαβε θήραν ἐπιστημῶν ἀρτίου τε καὶ περιττοῦ παντός." (*Tht.* 198a).

[319] Pl. *Chrm.* 166a. Rosamond Kent Sprague has curiously translated the last part as "… how many they are in themselves and with respect to other numbers" (in: Morrow (1997), 652), but "πρὸς ἄλληλα" can more or less mean nothing but "in respect of one another" (cf. D. J. Zeyl's "…in relation to each other", for *Grg.* 451b in: Morrow (1997), 797). And since Burnet's critical apparatus mentions no alternative reading, I cannot see how it can be translated otherwise. Sprague's "other numbers" show succinctly how there is a gap in scholarly understanding of Plato's mathematics, since Socrates is not even speaking of the modern notion of 'numbers' here, but of the 'odds' and 'evens'.

[320] "κάλλιον δέ που καὶ μᾶλλον κατ' εἴδη καὶ δίχα διαιροῖτ' ἄν, εἰ τὸν μὲν ἀριθμὸν ἀρτίῳ καὶ περιττῷ τις τέμνοι, τὸ δὲ αὖ τῶν ἀνθρώπων γένος ἄρρενι καὶ θήλει". Pl. *Plt.* 262e.

Now the word πλήθους (sg. gen. of πλῆθος, εος, τό,) used in both passages can according to the *LSJ* be translated as not only *multitude*, but also as *quantity, magnitude* or *amount.* All the entries in the *LSJ* denote[321] that it generally refers to a *large amount of something*, often used for the common masses of the people (at least as early as Homer and Herodotus). All of this indicates that the πλῆθος of the ἄρτιόν and περιττόν is a sort of *quantitative mass* or a *great multitude of the different 'Evens' and 'Odds'.* Brill's *Etymological Dictionary of Greek* (ed. R. S. P. Beekes) states on the *etymology* of ἀριθμός (translated there as "number; payment") that it is: "a derivation in -θμο- from the root of νήριτος 'countless'."[322] We have already seen how the Euclidean definition identifies ἀριθμός with πλῆθος, and how Plato often mentions πλῆθος in mathematical discussions.[323] Now the *LSJ* entries give: "**νήριτος**, ον, = νήριθμος, *countless, immense*" and "**νήριθμος**, ον, = ἀνάριθμος, *countless*." With 'νήριτος' we have for instance in the Iliad (2.632): "[…] Νήριτον εἰνοσίφυλλον" ("the Neritos of quivering foliage") as a proper name of a mountain in Ithaca. In this particular instance of the very creative Homeric vocabulary, we may notice an interesting semantic wordplay. We have 'immensity' for the mountain itself and the 'multitudinous' meaning for the 'countless' foliage which the mountain is 'dressed' in. We could also interpret the mountain itself, already named as '*countless, immense*', as being likened to something composed of a '*countless*' (νήριτος) quantity; a *multitude* and great *quantity/magnitude* (πλῆθος) of ἀριθμός; and by this very '*indefinite*' composition, forming an '*immensity*' to the eye of the beholder.

This shows how ἀριθμός and πλῆθος are often likened to, or equivalent with, a 'great quantity', 'multitude' or 'countlessness'. That Plato uses the word πλῆθος for λογιστική specifically does not necessarily mean that Euclid's fifth book deals with λογιστική only (see his two Definitions we mentioned and the concept of πλῆθος in them), but such a *possibility* does exist.

To shortly summarize before we consider the modern scholarly ideas on this, the difference seems to be that ἀριθμητική deals with a '*higher form of numerical concept*' or a '*greater*

[321] Always having in mind, of course, that detailed scholarly evaluations of these terms and their etymology is to be preferred over literal readings of dictionaries.

[322] Note the interesting conjecture on the comparison with the latin *'rītus':* "A derivation in -θμο- from the root of νήριτος 'countless'. Outside Greek, there are comparable words in Germanic: ON *rím* [n.] 'account', OHG *rīm* [m.] 'row, number', and in Celtic: OIr. *rím* 'number'. Probably, Lat. *rītus* 'religious observance, rite' is related too (< * *h2rei-ti-*)." In: R. S. P. Beekes (2010).

[323] (Eucl.) "ἀριθμὸς δὲ τὸ ἐκ μονάδων συγκείμενον πλῆθος." See the subsection 5.2.3.

simplicity' of the ἄρτιόν and περιττὸν whereas λογιστική has as its scope the *multitude* of the ἄρτιόν and περιττόν. As we have seen in the *Gorgias* and *Charmides*, Socrates says that ἀριθμητική would deal with '*how much',* or '*how many'*, there happens to exist of the even and odd. λογιστική, on the other hand, is that which considers the even and odd *in terms of how multitude exists both in themselves and between each other*. Interpreting the meaning of λογιστική with no more words than these of Socrates, it seems to be closer to mathematical combinations (e.g. addition, subtraction etc) than ἀριθμητική is, since λογιστική is concerned with a greater multitude (πλῆθος) of the different odds and evens and their combinations. ἀριθμητική considers the quantity of the odd and the even, whereas λογιστική goes further to a different 'stage' or 'level'[324] of the odd and even where their *multitude* is considered (1) in the odd itself, (2) in the even itself; and (3) in the different exchanges between the odd and even. It does not seem improbable that in this definition of λογιστική Plato would have allowed some sort of positive integers to be manipulated in mathematical addition or division, especially since he mentions "λογίζεσθαί τε καὶ ἀριθμεῖν" in the same vein in the previously mentioned passage of the *Res Publica,*[325] where one of those subjects or both would indicate some sort of strategic calculation of e.g. military troops. As for the division into two different kinds of all subjects – an ἀριθμητική and λογιστική that respectively deal with either sensibles or intelligibles – it seems evident that the odd and the even, whether considered in their '*simplicity*' or '*multitude*', can either be examined in sensible things (like military troops) or in intelligible things (like τὸ ἕν).[326]

Klein maintains that the "strangely elaborate formulation" of ἀριθμητική and λογιστική in the *Gorgias* and *Charmides* is due to the application of a definition on subjects that usually are consulted in practical things.[327] On the difference between 'theoretical logistic' and 'theoretical arithmetic', Klein writes: "according to this definition, theoretical logistic would have to include primarily knowledge concerning all those *relations,* i.e., ratios (*λόγοι*) among 'pure' units, on which the success of any calculation depends, while knowledge of these 'pure' *numbers*

[324] Or, Neoplatonically, 'ὑπόστασις'.
[325] Pl. *Resp.* VII, 522e.
[326] On the difference between ἀριθμητική and λογιστική Pritchard wrote: "These two branches of mathematics have the same objects, that is, *arithmoi*. They differ in that *arithmētikē* studies *arithmoi* by themselves, while *logistikē* studies their relations with one another in respect of quantity. For example, it would be the business of *arithmētikē* to know that 10 is a triangular *arithmos*, but of *logistikē* to know that 2 and 6 are in the same ratio as 5 and 15." P. Pritchard (1995), 73.
[327] J. Klein (1992), 24.

themselves would be reserved for theoretical arithmetic."[328] This is a succinct definition on the difference of these two subjects, and I think it shows again how Plato had a hierarchical classification in mind, where the 'simpler' theoretical notions of ἀριθμός stand at the higher rung of the scale and the more complex relations and ratios (λόγοι) are at the next, lower stage.

According to Klein, Diophantus' *Arithmetic* exhibits Plato's 'theoretical logistic'. But the relation between Plato's λογιστική and Diophantus' *Arithmetica* seems a bit more complex than that. Diophantus' work deals with numbers of monads, yet these numbers can be expressed in fractions. The possibility here of dividing the unit is, according to Klein, contrary to the Neoplatonic notions of the monad, and closer to a *Peripatetic ontology.* Still, Klein maintains that as to the problems formulated in the *Arithmetica,* they are unmistakeably similar to those discussed in the Platonic definition of logistic.[329] The 'theoretical logistic' of Diophantus is "founded on a Peripatetic theory of number relations".[330] This must not be interpreted as dealing with the modern notion of equations and their different types of solutions, but on *the relations* between *quadratic* (τετράγωνοι) and *cubic* (κύβοι) numbers and their *roots* (πλευραί, lit. "sides"). Klein further claims that this shows how the *Arithmetica* is very similar to Euclid's 'arithmetical' books: VII, VIII, IX.[331] Klein's statement on the *Arithmetica* as a work considering the *relations* between these numbers is indeed similar to what we just read in Plato's definition of λογιστική: that which considers the even and *odd in terms of how multitude exists both in themselves and between each other.* The emphasis in λογιστική is hence on the *relation* between a *multitude of numbers*. This lends a hand to the theory that Plato's λογιστική indeed deals with some form of calculation of ἀριθμός.

Amongst the later commentators during antiquity, there seems to have been slight differences of opinion on what the difference was between ἀριθμητική and λογιστική. Klein shortly surveys the views of the Neoplatonist Proclus; a *Charmides scholium;* the view of Olympiodorus; and the opinion of a *Gorgias scholium.*[332] The ambiguity, according to Klein, resides in the original ideas of Plato himself.

[328] J. Klein (1992), 24.
[329] J. Klein (1992), 9; 129–133.
[330] J. Klein (1992), 135.
[331] J. Klein (1992), 135.
[332] J. Klein (1992), 11–16.

Fowler maintains that in mathematics and science during the first half of the fourth century B.C., especially in Plato and Aristotle, we find "the frequent appeal to the idea of *logos* as ratio, and the use of the derived words that may even have been coined by Plato and his associates: *logistikē,* the art (i.e. *technē,* understood) of *logos,* […] and *logismos*."[333] As I've pointed out already, they have different meanings, but Fowler claims: "while these words are used in a range of contexts and with a range of meanings that may be irrelevant to my mathematical enquiry here, a very substantial number of explicitly technical occurrences remain."[334] Although Fowler agrees with Klein that Plato's 'theoretical logistic' seems identical with 'the theory of ratios and proportions', Fowler believes that Klein is still clinging to the modern conceptualization that the ratio of two numbers must be regarded as a *fraction*¸ which of course goes against Plato's statement of 'equal units' in *Res Publica,* VII 525e.[335] Fowler argues thus: "I propose, then, that we should conceive of *logistikē* (*technē*) and *logismos* as 'ratio theory'."[336] Still, is Plato's '**πλήθους** ὅπως ἔχει πρὸς αὑτὰ καὶ πρὸς ἄλληλα' of the odd and the even ἀριθμοί such a narrow scope of study; nothing else but 'ratios'? We already saw how 'πλῆθος' could mean a great mass of quantity or multitude, or a great magnitude. Also, as Fowler mentions,[337] for the later Neoplatonists (e.g. Proclus) λογιστική simply became the science of *all sensible* (αἰσθητῶν) ἀριθμοί.[338] Would the later Neoplatonic λογιστική really have drifted so far so as to encompass the general study of *all sensible numbers,* from originally only considering the *ratios* of numbers? That may be a possibility since, as mentioned, Plato's mathematics developed even during his time, but still, the correct interpretation of Plato's 'πλῆθος' must first be ascertained and that seems to me quite ambiguous.

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[333] D. H. Fowler (1999), 105.
[334] D. H. Fowler (1999), 105.
[335] D. H. Fowler (1999), 108.
[336] D. H. Fowler (1999), 109.
[337] D. H. Fowler (1999), 106–107.
[338] "οἶδ' αὖ ὁ λογιστικὸς αὐτὰ καθ' ἑαυτὰ θεωρεῖ τὰ πάθη τῶν ἀριθμῶν, ἀλλ' ἐπὶ τῶν αἰσθητῶν, ὅθεν καὶ τὴν ἐπωνυμίαν αὐτοῖς ἀπὸ τῶν μετρουμένων τίθεται, μηλίτας καλῶν τινας καὶ φιαλίτας." *In Euc.* §40.

5.4. γεωμετρία

5.4.1 γεωμετρία: beholding the form of the good (τὴν τοῦ ἀγαθοῦ ἰδέαν), the soul contemplating being/essence (οὐσία), and the soul's salvation through purification

"τὸ μὲν γὰρ ὅλον, καθάπερ εἴρηται νυνδή, σφαιροειδὲς ὄν"[339]

Just as we saw how the Greek ἀριθμός was related to λογισμός and the science of λογιστική, ἀριθμός was also conceptualized in geometrical forms. Indeed, it seems that to a certain extent all ἀριθμοί were so, to our modern understanding at least. We saw this both in Plato's *Res Publica* and in Proclus' definitions.[340] On the discussion about irrational numbers and incommensurability, we have in the *Theaetetus* a clear example of this. Theaetetus says: "Τὸν ἀριθμὸν πάντα δίχα διελάβομεν"; "We divided ἀριθμός altogether in two ways".[341] The first class is of the numbers/ἀριθμοί that are equal, having been multiplied by equals. They are likened to the square in shape and are called 'square' or 'equilateral' (lit. 'four-angled' and 'equal-sided'; "τετράγωνόν τε καὶ ἰσόπλευρον").[342] The second class consists of the numbers/ἀριθμοί that are multiplied by a greater and a lesser number. It is always encompassed by a greater and lesser side. It is likened to an oblong shape and called an oblong number ("τῷ προμήκει αὖ σχήματι ἀπεικάσαντες προμήκη ἀριθμὸν ἐκαλέσαμεν").[343] The adjective προμήκης literally means "prolonged, elongated", which again refers to a visible geometrical figure with unequal lines. J. M. Cooper's note to this passage is telling: "Greek mathematicians did not recognize irrational *numbers* but treated of irrational quantities as geometrical entities […]".[344] E. A. Maziarz and T. Greenwood discussed the relation between Greek arithmetic and geometry during their ancient

[339] Pl. *Ti.* 63a.
[340] "Socrates says that those who are engaged in geometry, counting, and similar endeavours are: 'ὑποθέμενοι τό τε περιττὸν καὶ τὸ ἄρτιον καὶ τὰ σχήματα καὶ γωνιῶν τριττὰ εἴδη καὶ ἄλλα τούτων ἀδελφὰ καθ' ἑκάστην μέθοδον […] ποιησάμενοι ὑποθέσεις αὐτά […]'; 'they suppose/hypothesize the odd and the even, the figures, the three kinds of angles, and other things akin to these according to each pursuit/method […] making suppositions/hypotheses […].'" Pl. *Resp.* VI, 510c., see the subsection 5.3.1.; "As to what ἀριθμός might have been concretely, or conceptualized as in mathematical practice, the Neoplatonist Proclus informs us that one classification (seemingly Geminus') of ἀριθμητική is thus: 'τῆς δὲ ἀριθμητικῆς ὡσαύτως ἡ διαίρεσις εἴς τε τὴν τῶν γραμμικῶν ἀριθμῶν θεωρίαν καὶ τὴν τῶν ἐπιπέδων καὶ τὴν τῶν στερεῶν.'; 'Likewise ἀριθμητική is divided into the study of linear ἀριθμῶν, plane ἀριθμῶν, and solid ἀριθμῶν.'" Procl. *in Euc.* §39.
[341] Pl. *Tht.* 147e.
[342] "[ΘΕΑΙ.] Τὸν ἀριθμὸν πάντα δίχα διελάβομεν· τὸν μὲν δυνάμενον ἴσον ἰσάκις γίγνεσθαι τῷ τετραγώνῳ τὸ σχῆμα ἀπεικάσαντες τετράγωνόν τε καὶ ἰσόπλευρον προσείπομεν." *Tht.* 147e.
[343] "[ΘΕΑΙ.] Τὸν τοίνυν μεταξὺ τούτου, ὧν καὶ τὰ τρία καὶ τὰ πέντε καὶ πᾶς ὃς ἀδύνατος ἴσος ἰσάκις γενέσθαι, ἀλλ' ἢ πλείων ἐλαττονάκις ἢ ἐλάττων πλεονάκις γίγνεται, μείζων δὲ καὶ ἐλάττων ἀεὶ πλευρὰ αὐτὸν περιλαμβάνει, τῷ προμήκει αὖ σχήματι ἀπεικάσαντες προμήκη ἀριθμὸν ἐκαλέσαμεν." *Tht.* 147e–148a.
[344] J. M. Cooper & D. S. Hutchinson, *Plato: Complete Works* (1997), 164.

developments, but I must refer to the footnote for this long quote.[345]

The preference of ἀριθμητική over γεωμετρία might have been due to the relation of the latter with incommensurability and irrational numbers: "*Epinomis* implies that it is the business of geometry to discover similar series for all quadratic surds."[346]

In Plato's *Res Publica* 526c–527c Socrates mentions the third subject as being γεωμετρία, after ἀριθμητική and λογιστική. It is important to clarify here that for Socrates, it is the *second* subject in rank,[347] so that ἀριθμητική and λογιστική are considered as a single subject. Geometry, at this stage of development in Platonic (and Greek?) mathematics is only concerned with *plane* surfaces: "τὴν μὲν γάρ που τοῦ ἐπιπέδου πραγματείαν γεωμετρίαν ἐτίθης".[348] Astronomy deals with the *revolving* (περιφορά) solids, and that which to us is simply another part of geometry – stereometry – is called by Socrates as that subject which deals with *solids* (στερεός). Stereometry moreover, according to Socrates, deals with the dimension of cubes and of depth.[349] Both interlocutors curiously claim that Stereometry hasn't yet been developed enough as a proper subject during the time of their 4th c. B.C Athens. Socrates says that the reasons for this is that no (Greek?) city values it enough, and there is no superintendent (ἐπιστάτης) of it who would be heeded by anyone.[350]

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[345] "Although geometry can always represent the motions of the heavenly bodies, its processes are less satisfactory to reason [for the Platonist], as they are more pictorial than those of arithmetic. When the discovery of irrational lines had placed geometry in a higher position than arithmetic, there was no choice but to apply geometry to astronomical problems. Even in generalizing the Pythagorean theory of proportion, Eudoxus used geometrical rather than arithmetical concepts; but if geometry is considered only an illustration of the arithmetic of the quadratic and cubic surds, this parallelism of arithmetic with geometry preserved the primacy of arithmetic over all the sciences. Plato may have worked out a numerical interpretation of the discoveries of Theaetetus and Eudoxus, thus reverting to the earlier priority of arithmetic over geometry. This is implied in Epinomis, which goes beyond the mathematical considerations of the Republic. 'The first and most important 'study' is of numbers in themselves; not of corporeal numbers, but of the whole genesis of the odd and even, and the extent of their influence on the nature of things.' [Epinomis 990c]. There comes next geometry and stereometry, which permit 'an evident likening of numbers unlike one another by nature.' According to the following fragment of Archytas, this view was predominant at the time: 'In respect of wisdom, arithmetic surpasses all the other arts and especially geometry, seeing it can treat the objects it wishes to study in a clearer way. Where geometry fails, arithmetic completes its demonstration in the same way even with regard to figures, if there is such a thing as the study of figures.' [Diels, Vors., 47 B 4, III, p. 438]." E. A. Maziarz & T. Greenwood (1968), 111–112.

[346] E. A. Maziarz & T. Greenwood (1968), 122.

[347] "Δεύτερον δὴ τοῦτο τιθῶμεν μάθημα τοῖς νέοις;" (Pl. *Resp.* VII, 527c).

[348] Pl. *Resp.* VII, 528d.

[349] "Μετὰ ἐπίπεδον, ἦν δ' ἐγώ, ἐν περιφορᾷ ὂν ἤδη στερεὸν λαβόντες, πρὶν αὐτὸ καθ' αὑτὸ λαβεῖν· ὀρθῶς δὲ ἔχει ἑξῆς μετὰ δευτέραν αὔξην τρίτην λαμβάνειν. ἔστι δέ που τοῦτο περὶ τὴν τῶν κύβων αὔξην καὶ τὸ βάθους μετέχον." (*Resp.* VII, 528a–b)

[350] Pl. *Resp.* VII, 528b–c.

Just as with ἀριθμητική and λογιστική, γεωμετρία is an appropriate subject for the *guardian* who is to be skilled in both war and philosophy.[351] Yet, Socrates continues, even a small part of γεωμετρίας τε καὶ λογισμοῦ would suffice for matters of war. It is rather much more important to find out if the more advanced part of geometry tends towards *that which makes it easier to behold the form of the good* ("πρὸς τὸ ποιεῖν κατιδεῖν ῥᾷον τὴν τοῦ ἀγαθοῦ ἰδέαν").[352] The things that aim towards that are those which compel the soul to turn around to that place in which the most blessed things of being are, the place which the soul must by any means behold.[353] Socrates states that if geometry compels the soul to contemplate ("θεάσασθαι") being/essence ("οὐσίαν"), it is befitting, but if it compels the soul to contemplate generation/becoming ("γένεσιν"), it is not befitting.[354] Now, Socrates further claims that anyone with just a little knowledge of γεωμετρία would be able to see how it is the opposite of what its practitioners claim it to be. The words they use rather refer to practical things, doing things in general or as in transactions of business ("πράττοντές τε καὶ πράξεως"), such as "squaring", "applying", or "adding".[355] The subject of geometry is (should? "ἐπιτηδευόμενον") rather be pursued for knowledge itself ("γνώσεως ἕνεκα").[356] Geometry, furthermore, is the knowledge of what always exists, not of what is created and destroyed: "Ὡς τοῦ ἀεὶ ὄντος γνώσεως, ἀλλ' οὐ τοῦ ποτέ τι γιγνομένου καὶ ἀπολλυμένου."[357] I choose here to translate "γιγνομένου καὶ ἀπολλυμένου" as "created and destroyed" since the latter verb often denotes *destruction, death, killing.*

Socrates' interlocutor agrees with his last statement and Socrates then makes the same claim for γεωμετρία as for ἀριθμητική and λογιστική, with similar wordings: it would be attractive towards the truth for the soul ("Ὁλκὸν ἄρα, ὦ γενναῖε, ψυχῆς πρὸς ἀλήθειαν εἴη ἂν…") and it would be effecting/causing philosophic thought, maintaining upwards that which we now unnecessarily

[351] "Ὅσον μέν, ἔφη, πρὸς τὰ πολεμικὰ αὐτοῦ τείνει, δῆλον ὅτι προσήκει· πρὸς γὰρ τὰς στρατοπεδεύσεις καὶ καταλήψεις χωρίων καὶ συναγωγὰς καὶ ἐκτάσεις στρατιᾶς, καὶ ὅσα δὴ ἄλλα σχηματίζουσι τὰ στρατόπεδα ἐν αὐταῖς τε ταῖς μάχαις καὶ πορείαις, διαφέροι ἂν αὐτὸς αὑτοῦ γεωμετρικὸς καὶ μὴ ὤν." Pl. *Resp.* VII, 526d.
[352] Pl. *Resp.* VII, 526d–e.
[353] "Ἀλλ' οὖν δή, εἶπον, πρὸς μὲν τὰ τοιαῦτα καὶ βραχύ τι ἂν ἐξαρκοῖ γεωμετρίας τε καὶ λογισμοῦ μόριον· τὸ δὲ πολὺ αὐτῆς καὶ πορρωτέρω προϊὸν σκοπεῖσθαι δεῖ εἴ τι πρὸς ἐκεῖνο τείνει, πρὸς τὸ ποιεῖν κατιδεῖν ῥᾷον τὴν τοῦ ἀγαθοῦ ἰδέαν. τείνει δέ, φαμέν, πάντα αὐτόσε, ὅσα ἀναγκάζει ψυχὴν εἰς ἐκεῖνον τὸν τόπον μεταστρέφεσθαι ἐν ᾧ ἐστι τὸ εὐδαιμονέστατον τοῦ ὄντος, ὃ δεῖ αὐτὴν παντὶ τρόπῳ ἰδεῖν." (*Resp.* VII, 526d–e).
[354] "Οὐκοῦν εἰ μὲν οὐσίαν ἀναγκάζει θεάσασθαι, προσήκει, εἰ δὲ γένεσιν, οὐ προσήκει." (*Resp.* VII, 526e).
[355] Pl. *Resp.* VII, 527a.
[356] "τὸ δ' ἐστί που πᾶν τὸ μάθημα γνώσεως ἕνεκα ἐπιτηδευόμενον." (*Resp.* VII, 527b).
[357] Pl. *Resp.* VII, 527b.

have downwards.[358] Socrates also says that there is an enormous difference between someone who as understood γεωμετρία and someone who has not. He who has grasped γεωμετρία has a better understanding of all subjects compared to him who hasn't studied γεωμετρία.[359]

Socrates continues to make a general statement for all these subjects that they are discussing. He claims that it is difficult to believe, as the philosophers do: "ὅτι ἐν τούτοις τοῖς μαθήμασιν ἑκάστου ὄργανόν τι ψυχῆς ἐκκαθαίρεταί τε καὶ ἀναζωπυρεῖται ἀπολλύμενον καὶ τυφλούμενον ὑπὸ τῶν ἄλλων ἐπιτηδευμάτων, κρεῖττον ὂν σωθῆναι μυρίων ὀμμάτων: μόνῳ γὰρ αὐτῷ ἀλήθεια ὁρᾶται."[360] By each one of these subjects, says Socrates, there is an *'instrument'/'organ of sense' of the soul that is purified and rekindled anew*; an 'instrument/organ of the soul' which has been destroyed and blinded by other pursuits/habits. This 'instrument/organ of the soul' is more worthy to save than an infinite amount of eyes are*, for it is only through this instrument that truth can be beheld.* Again, Socrates uses the verb 'σῴζω' ("σωθῆναι") in the context of the soul's salvation, and the concept of 'beholding the truth' ("ἀλήθεια") is again mentioned, but now as an aim of *all* the subjects discussed in these passages of the *Res Publica,* and not only of the mathematical subjects.

The Neoplatonist Iamblichus tells us that Pythagoras titled geometry as 'ἱστορία' (lit. "inquiry").[361] According to Szabó, who is drawing on previous research, "*ἱστορίη*, […] was reserved for empirical knowledge which had been acquired by observation". Szabó further maintains that it can be inferred from Iamblichus' statement that geometry was *initially* concerned with practical and experimental science: "*ἱστορίη* rather than a true *mathema*".[362] This would be a consistent and logical explanation to why Plato, as we have seen, considered stereometry to be underdeveloped during his time. It is furthermore noteworthy how Iamblichus claims that everything about (Greek?) geometry was actually invented by Pythagoras, when mentioning the story of how Hippasus perished at sea because of his impiety in divulging how to

[358] "Ὁλκὸν ἄρα, ὦ γενναῖε, ψυχῆς πρὸς ἀλήθειαν εἴη ἂν καὶ ἀπεργαστικὸν φιλοσόφου διανοίας πρὸς τὸ ἄνω σχεῖν ἃ νῦν κάτω οὐ δέον ἔχομεν." (*Resp*. VII, 527b).
[359] Pl. *Resp*. VII, 527c.
[360] Pl. *Resp*. VII, 527d–e.
[361] "ἐκαλεῖτο δὲ ἡ γεωμετρία πρὸς Πυθαγόρου ἱστορία" (Iambl. *Vita Pythagorae* 89). In: Á. Szabó (1978), 307.
[362] Á. Szabó (1978), 308. Cf. Also Proclus' statement that although as Aristotle said, that the sciences have fallen and arisen several times during countless cycles, and that they will do so again, the study of number and geometry originated from basic necessities, the former by the Phoenicians in trade, and the latter by the Egyptians in measuring lands. (*In Euc.* §64–65.)

construct a sphere from twelve pentagons.[363] Pythagoras' purported preeminence in knowledge and wisdom, and the accompanying lack of evidence for him being the originator for much of the Greek knowledge, is easily explained by the tradition which claimed that Pythagoras was safekeeping his wisdom from the unworthy by rigorous disciplines of virtue and integrity. In Iamblichus' *Vita Pythagorae*, divulging Pythagoras' teachings to the unworthy is likened to revealing the secret teachings of the Eleusinian goddesses to the profane. Only those who have first purified their souls are admitted to the Pythagorean lore.[364] The 'purification of the soul' and its connection to the 'μαθήματα' is a concept that is also mentioned by Plato's Socrates, as we have just seen. The modern scholarly endeavor to distance Plato as a lone inventor of doctrines away from his predecessors, such as the Pythagoreans, hence becomes increasingly futile, if not childish and unaccounted for. Where similarities exist, it should be acknowledged, and where there are none it should be accepted.[365] It is always possible though, to dismiss later writers as Iamblichus or Proclus as having merely 'Pythagoreanized' Plato's dialogues, and to claim that a scholar writing over 2,000 years after these ancient authors, when ancient Greek is 'frozen in time', knows better.

It is evident that geometric and numeric conceptions are at the center of the creation of the world in Plato's *Timaeus*, although the sciences themselves are not discussed.[366] For instance, the shapes of the four elements are given by 'the god' using *forms* and *numbers*: "οὕτω δὴ τότε πεφυκότα ταῦτα πρῶτον διεσχηματίσατο εἴδεσί τε καὶ ἀριθμοῖς." Also, the four elements were originally composed of *triangles*, and the five 'Platonic' *solids* are constructed by specific *plane surfaces*, also composed of *triangles*.[367] Inferences about Plato's conception of mathematics could be made from the entire *Timaeus*, but that would not be as clear as when the sciences themselves are considered (eg. in *Resp*. and *Gorg*.), and it would probably go beyond the scope of my thesis.

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[363] Iambl. *VP* 88–89.
[364] *Vide*: Iambl. *VP* 75. The philosophy of the '*Hearers'*, the "ἀκούσματα", were to be guarded carefully as "θεῖα δόγματα" (*VP* 82).
[365] E.g.: "[…] ὡς οἵ τε Πυθαγόρειοί φασι καὶ ἡμεῖς, ὦ Γλαύκων, συγχωροῦμεν" (*Resp*. VII, 530d). cf. Maziarz & Greenwood (1968), 96, that Plato disagrees with (some?) of the Pythagoreans in *Resp.* VII, 531b–c.
[366] E.g.: "πότερον οὖν ὀρθῶς ἕνα οὐρανὸν προσειρήκαμεν" (*Ti.* 31a); "σωματοειδὲς δὲ δὴ καὶ ὁρατὸν ἁπτόν τε δεῖ τὸ γενόμενον εἶναι, χωρισθὲν δὲ πυρὸς οὐδὲν ἄν ποτε ὁρατὸν γένοιτο, οὐδὲ ἁπτὸν **ἄνευ τινὸς στερεοῦ**, **στερεὸν δὲ οὐκ ἄνευ γῆς** […] δεσμῶν δὲ κάλλιστος ὃς ἂν αὑτὸν καὶ τὰ συνδούμενα ὅτι μάλιστα ἓν ποιῇ, τοῦτο δὲ πέφυκεν **ἀναλογία κάλλιστα** ἀποτελεῖν. ὁπόταν γὰρ **ἀριθμῶν τριῶν** εἴτε **ὄγκων** εἴτε **δυνάμεων** ὡντινωνοῦν ᾖ τὸ μέσον, ὅτιπερ τὸ πρῶτον πρὸς αὑτό […]" (*Ti.* 31b–32a). Bold emphasis is mine.
[367] *Ti.* 53d–55d. Cf. also E. A. Maziarz & T. Greenwood (1968), 87.

In Plato’s *Timaeus* we are given a threefold division of the Kosmos (lit. ‘the all’: “τοῦ παντὸς”). The earlier two forms/kinds mentioned (“δύο εἴδη”) were (1) the intelligible and eternal model (παράδειγμα), and (2) the imitation of the model, having generation and visibility. The third is introduced as the ‘receptacle of generation, as a wet-nurse’.[368] Again, they are referred to as ‘that which comes into being’, ‘that into which it comes to be’, and ‘the source of the becoming, from which it is modeled’. Interestingly, the receptacle is likened to a ‘mother’, the source a ‘father’, and the nature in between them as ‘child/offspring’.[369] They are finally summarized as: “ὄν τε καὶ χώραν καὶ γένεσιν”; “that which is/being, space, and generation”.[370]

We have thus, (1) as the model, source, and father, <u>‘being’ (ὄν);</u> (2) as the receptacle and wet-nurse of generation, the mother, <u>‘space’ (χώρα);</u> and (3) the visible imitation of the model, generated, becoming, the child, <u>‘generation/creation’ (γένεσις).</u>

The ‘ὄν’ is said to be apprehended by ‘νόησις’ (intelligence, understanding), the ‘γένεσις’ by “δόξῃ μετ᾽ αἰσθήσεως περιληπτόν” (“opinion, involving sense perception”), and the ‘χώρα’ by “μετ᾽ ἀναισθησίας ἁπτὸν λογισμῷ τινι νόθῳ” (“by a sort of bastard/counterfeit calculation/reasoning, with no sense perception”).[371] As noted by Szabó, two passages in Plato’s *Res Publica* seem to identify ‘bastard reasoning’ as being occupied with geometry.[372] We have already mentioned one of these passages, where the trio (also the quartet) of νοῦς, διάνοια, and δόξα are the subsections of Plato’s ‘analogy of the divided line’, serving as metaphysical, cosmological and psychological frameworks.[373] The faculty of the geometers, and indeed of *all mathematicians*, is ‘διάνοια’.[374]

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[368] “τότε μὲν γὰρ δύο εἴδη διειλόμεθα, νῦν δὲ τρίτον ἄλλο γένος ἡμῖν δηλωτέον. τὰ μὲν γὰρ δύο ἱκανὰ ἦν ἐπὶ τοῖς ἔμπροσθεν λεχθεῖσιν, ἓν μὲν ὡς παραδείγματος εἶδος ὑποτεθέν, νοητὸν καὶ ἀεὶ κατὰ ταὐτὰ ὄν, μίμημα δὲ παραδείγματος δεύτερον, γένεσιν ἔχον καὶ ὁρατόν. τρίτον δὲ […] τίν᾽ οὖν ἔχον δύναμιν καὶ φύσιν αὐτὸ ὑποληπτέον; τοιάνδε μάλιστα: πάσης εἶναι γενέσεως ὑποδοχὴν αὐτὴν οἷον τιθήνην.” *Ti.* 48e–49a.
[369] “ἐν δ᾽ οὖν τῷ παρόντι χρὴ γένη διανοηθῆναι τριττά, τὸ μὲν γιγνόμενον, τὸ δ᾽ ἐν ᾧ γίγνεται, τὸ δ᾽ ὅθεν ἀφομοιούμενον φύεται τὸ γιγνόμενον. καὶ δὴ καὶ προσεικάσαι πρέπει τὸ μὲν δεχόμενον μητρί, τὸ δ᾽ ὅθεν πατρί, τὴν δὲ μεταξὺ τούτων φύσιν ἐκγόνῳ”. *Ti.* 50c–d.
[370] “οὗτος μὲν οὖν δὴ παρὰ τῆς ἐμῆς ψήφου λογισθεὶς ἐν κεφαλαίῳ δεδόσθω λόγος, ὄν τε καὶ χώραν καὶ γένεσιν εἶναι, τρία τριχῇ, καὶ πρὶν οὐρανὸν γενέσθαι” *Ti.* 52d.
[371] Pl. *Ti.* 52a–b.
[372] Pl. *Resp.* VI, 511d–e & VII. 533e–534a, as cited by Szabó (1978), 311.
[373] See the subsection 5.2.2.
[374] “οἶμαι γάρ σε εἰδέναι ὅτι οἱ περὶ τὰς γεωμετρίας τε καὶ λογισμοὺς **καὶ τὰ τοιαῦτα πραγματευόμενοι** […]”. VI, 510c; “ἃς ἐπιστήμας μὲν πολλάκις προσείπομεν διὰ τὸ ἔθος, δέονται δὲ ὀνόματος ἄλλου, ἐναργεστέρου μὲν ἢ δόξης, ἀμυδροτέρου δὲ ἢ ἐπιστήμης—**διάνοιαν δὲ αὐτὴν ἔν γε τῷ πρόσθεν που ὡρισάμεθα**”, VII, 533d. Bold emphasis is mine. Szabó refers only to the geometers as employing διάνοια, not all mathematicians, which Socrates evidently says.

On a further note, Szabó claims that Plato's idea that geometers use διάνοια, which lies between δόξα and νοῦς, is a notion that was developed out of the speculations of the Eleatics.[375] Szabó explains that originally, the notion of 'space' was denied by the Eleatics, since the phenomena of the visible world (e.g. motion, change, generation) were inconsistent. Later, Plato conceptualized 'space' as having a *dual* nature; partly eternal and partly partaking of generation. It is because of this, and Socrates' statement in the *Res Publica*[376] of the geometers using a 'wrong' and practical type of language to describe their activities, that Plato refers to a certain 'bastard reasoning' for geometry.[377] Szabó further remarks: "after starting out as a kind of *ἱστορίη* whose chief tool was the sense of *sight*, geometry became the science of *space itself*."[378]

On what geometry was concretely for Plato, Wedberg maintains that "geometry was for Plato those parts of what is now known as Euclidean geometry that had been developed in his days. According to Heath, most of the theories brought together by Euclid in the *Elements* existed already in Plato's time."[379]

[375] Á. Szabó (1978), 311.
[376] *Resp.* VII, 527a.
[377] Á. Szabó (1978), 311–312.
[378] Á. Szabó (1978), 313.
[379] A. Wedberg (1955), 22; T. L. Heath (1921), 217.

5.5. Plato's ἀριθμητική and γεωμετρία according to Aristotle

In the context of studying 'first principles' ("ἐπεὶ δὲ τὰς ἀρχὰς καὶ τὰς ἀκροτάτας αἰτίας ζητοῦμεν"), Aristotle stated that there is a science which studies 'Being' itself' ("Ἔστιν ἐπιστήμη τις ἣ θεωρεῖ τὸ ὂν ᾗ ὂν καὶ τὰ τούτῳ ὑπάρχοντα καθ' αὑτό").[380] This science (dialectic?) is completely distinct from all other sciences, including mathematics. In the general sense, according to Aristotle, the mathematical sciences are of those who study only 'a portion of being (τὸ ὄν)': "οὐδεμία γὰρ τῶν ἄλλων ἐπισκοπεῖ καθόλου περὶ τοῦ ὄντος ᾗ ὄν, ἀλλὰ μέρος αὐτοῦ τι ἀποτεμόμεναι περὶ τούτου θεωροῦσι τὸ συμβεβηκός, οἷον αἱ μαθηματικαὶ τῶν ἐπιστημῶν."[381] Aristotle literally writes that "they cut off a part of Being (τὸ ὄν) and consider its attributes".

It is interesting to note how Aristotle's choice of verb, 'θεωρέω', originally meant 'to be a θεωρός', i.e. '*to be sent to consult an oracle*', but later (?) seems to have included '*to behold, inspect, contemplate, consider*' etc. Similarly the noun 'θεωρός' literally meant '*envoy sent to consult an oracle*', but also 'title of a *magistrate*', and '*spectator* […] *one who travels to see* men and things'.[382] This has another bearing upon Plato's obvious use of 'oracular' or 'religious' terminology which we have previously seen. It should also be mentioned how Aristotle (in the same passage) speaks of '*those who seek the elements of Being/of things existing*' ('οἱ τὰ στοιχεῖα τῶν ὄντων ζητοῦντες') as being the same investigators who, by this, were seeking the first principles ('τὰς ἀρχὰς').[383] The '*Elements*' are of course the title of Euclid's famous thirteen books, although I cannot linger on this point any further, and prove whether this is exactly what Aristotle is referring to. Aristotle does state elsewhere however, that "of the mathematical things that are (beings) there are *first principles, elements, and causes*" ("καὶ τῶν μαθηματικῶν εἰσὶν ἀρχαὶ καὶ στοιχεῖα καὶ αἴτια").[384]

[380] Arist. *Met.* 1003a, 21–30.
[381] Arist. *Met.* 1003a, 23–27.
[382] See the entries in the *LSJ*.
[383] "εἰ οὖν καὶ οἱ τὰ στοιχεῖα τῶν ὄντων ζητοῦντες ταύτας τὰς ἀρχὰς ἐζήτουν, ἀνάγκη καὶ τὰ στοιχεῖα τοῦ ὄντος εἶναι μὴ κατὰ συμβεβηκὸς ἀλλ' ᾗ ὄν· διὸ καὶ ἡμῖν τοῦ ὄντος ᾗ ὂν τὰς πρώτας αἰτίας ληπτέον." *Met.* 1003a, 28–32.
[384] Arist. *Met.* 1025b, 4–5.

As already stated, Wedberg examined Plato's philosophy of mathematics mainly from Aristotle's writings.[385] Two concepts that stand out and must be mentioned, on Aristotle's opinion of Plato's mathematics, is (1) "*the teaching that there are three fundamental types of entities, viz. the Ideas or Forms, the intermediate objects of mathematics, and sensible things*", and (2) that there is a distinction between 'Ideal Numbers' and 'Mathematical Numbers' where "*there is a relation of 'priority' among the Ideal Numbers, by which they are ordered in a series that runs parallel to the series of Mathematical Numbers, ordered according to size*."[386]

The first point is stated by Aristotle thus: "ὥσπερ Πλάτων τά τε εἴδη καὶ τὰ μαθηματικὰ δύο οὐσίας, τρίτην δὲ τὴν τῶν αἰσθητῶν σωμάτων οὐσίαν".[387] He specifically calls these three entities three different 'οὐσίαι'. Book VII of Aristotle's *Metaphysics* actually opens with the discussion on what 'τὸ ὄν' and 'οὐσία' is. Although we cannot dwell any further upon Aristotle's own claims, he states that τὸ ὄν is referred to in many senses, *but the primary sense of 'what'* τὸ ὄν *is, is referred to as* οὐσία.[388]

The second point is according to Wedberg exemplified in this passage, where Aristotle differs between countable/comparable (συμβλητός) and uncountable/uncomparable (ἀσύμβλητος) ἀριθμοί: "ἢ τὰς μὲν συμβλητὰς τὰς δὲ μή […] διὸ καὶ ὁ μὲν μαθηματικὸς ἀριθμεῖται μετὰ τὸ ἓν δύο, πρὸς τῷ ἔμπροσθεν ἑνὶ ἄλλο ἕν, καὶ τὰ τρία πρὸς τοῖς δυσὶ τούτοις ἄλλο ἕν, καὶ ὁ λοιπὸς δὲ ὡσαύτως· οὗτος δὲ μετὰ τὸ ἓν δύο ἕτερα ἄνευ τοῦ ἑνὸς τοῦ πρώτου, καὶ ἡ τριὰς ἄνευ τῆς δυάδος, ὁμοίως δὲ καὶ ὁ ἄλλος ἀριθμός·"[389]

It is important to note though, how Aristotle does not mention Plato here as agreeing with this teaching, although Wedberg maintains that the same idea is referred to in another passage where it clearly is Plato's teaching.[390] We should also note here how Aristotle defines the

[385] See the subsection 4.1.2. Also: "Thus, an interpretation or a reconstruction of Plato's philosophy of mathematics will here be offered that, in all main points, agrees with Aristotle's exposition. […] The most complete statements [of Plato] are those in the *Republic* and *Philebus*, but even they would remain exceedingly enigmatic unless compared with statements in other dialogues." A. Wedberg (1955), 15.

[386] A. Wedberg (1955), 84 & 121.

[387] Arist. *Met.* 1028b, 20–22.

[388] "Τὸ ὂν λέγεται πολλαχῶς, καθάπερ διειλόμεθα πρότερον ἐν τοῖς περὶ τοῦ ποσαχῶς· σημαίνει γὰρ τὸ μὲν τί ἐστι καὶ τόδε τι, τὸ δὲ ποιὸν ἢ ποσὸν ἢ τῶν ἄλλων ἕκαστον τῶν οὕτω κατηγορουμένων. **τοσαυταχῶς δὲ λεγομένου τοῦ ὄντος φανερὸν ὅτι τούτων πρῶτον ὂν τὸ τί ἐστιν, ὅπερ σημαίνει τὴν οὐσίαν**" *Met.* 1028a, 10–15. Bold emphasis is mine.

[389] Arist. *Met.* 1080a, 24–35.

[390] A. Wedberg (1955), 117 & 121; *Met.* 1080b, 11–15: "οἱ μὲν οὖν ἀμφοτέρους φασὶν εἶναι τοὺς ἀριθμούς, τὸν μὲν ἔχοντα τὸ πρότερον καὶ ὕστερον τὰς ἰδέας, τὸν δὲ μαθηματικὸν παρὰ τὰς ἰδέας καὶ τὰ αἰσθητά, καὶ χωριστοὺς ἀμφοτέρους τῶν

mathematician counting (‘μαθηματικὸς ἀριθμεῖται’) as ‘adding one upon one(s)’. Two is made up of one added to one, three is made up of two with another one etc, so that just as Wedberg observed, the mathematical numbers are “*made up of certain ideal ‘units’, or ‘1s’.*”, i.e. “A Mathematical Number is an aggregate of units”.[391]

This second point seems to agree with my assertion in this thesis, that Plato had a ‘hierarchical’ classification in mind, or a ‘duality’ such as expressed by Wedberg above: ‘a relation of priority’ in a ‘parallel series’. The first point also exemplifies my argument, since the ‘Ideas’ obviously hold a priority over the ‘intermediate world of mathematics’ and especially over the ‘sensible world’.

The first point, whether Plato’s mathematical objects were intermediate between sensibles and intelligibles, has been a question of debate among several scholars. Wedberg argued that Aristotle’s scheme of mathematical intermediates is corroborated by many statements in Plato’s dialogues, at least for arithmetic, but perhaps not for geometry.[392] Pritchard believes that the intermediates is a mistaken notion by scholars, at least from the supposed inferences in the *Res Publica,* or he seems to say that this is inconclusive both in Plato and Aristotle.[393] White also seems to think that the evidence for the mathematical objects as ‘ontological intermediates’ is inconclusive.[394] I think that there is strong evidence for the intermediates though, especially in the *Res Publica,* in the passage we have discussed where the divided line between the visible and intelligible world is mentioned.[395] The question is rather if he includes all mathematicians or only geometers here (“καὶ τὰ τοιαῦτα πραγματευόμενοι” and “τὴν τῶν γεωμετρικῶν τε **καὶ τὴν τῶν τοιούτων** ἕξιν”).[396] We stated that clearly: “not only geometers but all mathematicians [“καὶ τὰ τοιαῦτα πραγματευόμενοι”] are labouring with their subjects in the realm of διάνοια, employing the lower visible things of πίστις and εἰκασία in order to attempt an arrival at νόησις.” Socrates even made a trio division, after the quartet: διάνοια, between δόξα (opinion) and νοῦς (mind).[397]

αἰσθητῶν·”

[391] A. Wedberg (1955), 118.
[392] A. Wedberg (1955), 12–13.
[393] P. Pritchard (1995), 89 & 156–157.
[394] M. J. White (2006), 239–240.
[395] Pl. *Resp.* VI, 509d etc. See the subsection 5.2.2.
[396] Pl. *Resp.* VI, 510c & 511d.
[397] “διάνοιαν δὲ καλεῖν μοι δοκεῖς τὴν τῶν γεωμετρικῶν τε καὶ τὴν τῶν τοιούτων ἕξιν ἀλλ᾽ οὐ νοῦν, ὡς μεταξύ τι δόξης τε καὶ νοῦ τὴν διάνοιαν οὖσαν.” Pl. *Resp.* VI, 511d.

We see again, how a detailed examination of the original Greek passage, with a quite literal translation, evinces much more than what previous scholars have claimed.[398] Furthermore, the "μετασχὸν τῆς ἰδίας οὐσίας" in *Phaedo* 101c, where 'δύο' is participating in its particular οὐσία which is the 'δυάς', and 'ἕν' is participating in its οὐσία, the 'μονάς', perhaps means that 'the *partaking* of its particular essence' is the 'intermediate state' of mathematics? That is to say, this particular 'partaking' becomes a new intermediate state?

[398] Pritchards own conjecture against this does not hold, in my opinion, for he is merely applying his own personal conviction upon the Greek text: "The subsequent division of each section does not introduce new 'sub-faculties', (shadows and reflections are surely seen by the same faculty as sees their originals) but rather distinguishes two ways in which the same faculty can be directed at its proper object." P. Pritchard (1995), 96.

6. Conclusion

As we have read in the analysis, Plato's ἀριθμητική is conceptualized in several levels. Plato first makes a distinction between the sciences that are more accurate (e.g. building) and those that are less accurate (e.g. music). ἀριθμητική is of the former. There is furthermore a distinction within each and every science. The basic distinction between the two types of ἀριθμητική is that the common (τῶν πολλῶν) ἀριθμητική calculates with any sorts of 'unequal units' (μονάδας ἀνίσους) while the philosopher's (τῶν φιλοσοφούντων) ἀριθμητική will only do the same if it is agreed upon that all those 'units' (μονάδες) have no difference between each other. The philosopher's μονάδες can probably not be mathematically manipulated. It seems as if Plato's μονάς is, in a sense, equivalent to the ἕν in his dialogues. I argue in this thesis that the ἕν could possibly denote two different concepts, both in Plato and in most other Greek mathematical philosophy: (1) as equivalent to the concept of any μονάς, and (2) as one ἀριθμός. All these key notions in Plato's mathematics seem to have a hierarchical order (perhaps later called as 'προποδισμός' *process, progression*?) if we regard both Plato's dialogues and the similarities of that with Aristotle's *Metaphysics* (at least), Euclid's two Definitions in the *Elements* Book VII, and other ancient authors. The μονάς and ἕν seem to take a higher order than ἀριθμός and πλῆθος. Plato seems to agree with Euclid that the 'ἕν' is participating ('μετασχεῖν') in its own οὐσία, the 'μονάς', where Plato identifies the οὐσία of ἕν as μονάς or the οὐσία of δύο as being δυάς (dyad/twoness). μονάς seems to be that which groups existing things into 'ones', and ἀριθμός is the compounded multitude of these μονάδες.

The Greek 'arithmetised' mathematics was most likely different from the 'arithmetised' mathematics of other contemporary cultures, as Fowler maintains. 'τὸ δὲ ἀριθμεῖν' (to number/arithmetise), in Plato, is in short nothing but considering the quantity of number/*arithmos* ('πόσος τις ἀριθμός'). These ἀριθμοί seem to include some concept of the positive integers 1–10, yet probably differing from our modern notion of these ten numbers. In the *Epinomis* and *Timaeus*, ἀστρονομία seems to be one of the most important sciences for man's discovery of ἀριθμός and even for philosophy itself. Since the *Res Publica* seems not to have been Plato's final thoughts on mathematics, all other dialogues and perhaps even his *Letters* must also be considered. An ἀριθμός is divided into odd and even (περιττόν and ἄρτιον), just as the human 'race' could be divided into males and females. ἀριθμός could generally in most cases be said to denote, as Paul Pritchard wrote: "*a collection of things falling under some description*". As Klein also wrote, generally speaking "*arithmos never means anything other than 'a definite number of definite objects'*." It is often though, in pure mathematical speculations, as Euclid's Definition 2 states: '*the compounded multitude of the μονάδων/units'*, or a '*compounded multitude*' of anything being counted. An ἀριθμός can also be conceptualized in geometric figures, such as "linear ἀριθμῶν, plane ἀριθμῶν, and solid ἀριθμῶν" (Proclus).

As for the purpose of ἀριθμητική, it and the other sciences seem to be a sort of 'means to an end' where the goal (or purpose) is to reach the greatest μάθημα which is to perceive 'The Good' and to receive some form of salvation ('σωτηρία'). By the means of μετρητική ('mensuration') and most likely by ἀριθμητική in connection to this, these sciences would "invalidate the 'φάντασμα' of appearances, show the truth, give peace to the soul, let it remain in truth, and save [our] life" (Plato). Plato's philosophical λογιστική τε καὶ ἀριθμητικὴ is about some form of 'calculation' aimed at the recurring central themes of νόησις, οὐσία and ἀλήθεια. Plato maybe drew from earlier sources for this, such as Epicharmus who wrote that 'λογισμός' and 'ἀριθμός' "σώιζει βροτούς". This is all in connection to the 'spirituality' in Plato's mathematics. There seems to be some kind of 'religious' or 'oracular' undertone in Plato's language about ἀριθμητική and perhaps about mathematics in general; the notion of *consulting a science* in the similar manner of *consulting divinities*. This may have a connection to Socrates' 'divine/daemonic voice'. The oracle at Delphi is also ascribed an interest in mathematics where it is described as something which would save people from evil. The 'oracular' vocabulary is possibly also found in Aristotle's passages on mathematics. Aristotle employs the verb 'θεωρέω' when describing what the mathematicians are studying. The verb originally meant 'to be a θεωρός', i.e. 'to be sent to consult an oracle', but also included the meaning of 'to behold, inspect, contemplate, consider'. In Plato's *Timaeus* we find the central concepts of θεός, χράομαι, the νοῦς of the heavens (in astronomy), the διανόησις of humans, and a natural λογισμός, all of which also could relate to some form of spirituality.

ἀριθμητική and λογιστική are mentioned without any greater distinction between them in the *Res Publica*, but in the *Gorgias* and *Charmides* the distinction is clearly that ἀριθμητική is the knowledge of the *simple quantity (how much or how many)* of the even and the odd (the ἄρτιον and περιττόν), whereas λογιστική considers *how multitude* (πλῆθος) *exists both in the even and the odd respectively, and in the relations between the even and the odd*. ἀριθμητική examines a 'higher form of numerical concept' or a 'greater simplicity' (*the simple quantity itself*) of the ἄρτιον and περιττόν while λογιστική considers the multitude (*complexity*) of the ἄρτιον and περιττόν. ἀριθμός and πλῆθος are often likened to a 'great quantity', a 'multitude' or 'countlessness'. Plato uses the word πλῆθος for λογιστική specifically, and the term is often mentioned in his general mathematical speculations. Fowler and Klein believe that Plato's 'theoretical logistic' seems identical with 'the theory of ratios and proportions', but it doesn't seem that Plato's 'πλήθους ὅπως ἔχει πρὸς αὑτὰ καὶ πρὸς ἄλληλα' of the odd and the even ἀριθμοί is nothing else but the study of 'ratio'. There is furthermore an obvious ambiguity in the meanings of the adjective 'λογιστικός' and the noun 'λογισμός' (*calculation* or *reasoning*). Perhaps, 'to calculate' is in Greek equivalent to 'being rational'.

ἀριθμητική and λογιστική help the soul to turn away from generation (another of its purposes) ("τῆς ψυχῆς ῥᾳστώνης μεταστροφῆς ἀπὸ γενέσεως") when studying by the philosopher's method the 'numbers themselves' ("αὐτῶν τῶν ἀριθμῶν") which include units that cannot be divided and ἀριθμῶν which can be understood by mind alone ("διανοηθῆναι μόνον"). Whether the Greek *arithmos* is altogether different from the positive integers, or if the integers are part of *arithmos*, that does not matter for this: it stands as evident that Plato's philosophical *arithmos* is concerned with thinking, intellect, and mind only, and not with visible/material phenomena. 'ἡ περὶ τὸ ἓν μάθησις' has a crucial role in this, where it leads the soul towards the vision of being (ἐπὶ τὴν τοῦ ὄντος θέαν). In Plato's epistemology, metaphysics, cosmology and other concepts he distinguishes between the perceptible world (ὁρατόν) and the world of being/intellect (νοητόν). This is most clearly seen in Plato's 'analogy of the divided line into two unequal sections'. As noted before by Szabó, this doctrine has an implication in Plato's mathematics as well. It seems that *not only geometers but probably all mathematicians* are labouring with their subjects in the 'realm' (or condition of the soul) of διάνοια, employing the lower visible things of πίστις and εἰκασία in order to reach νόησις. The mathematicians are furthermore said to employ διάνοια (thought), intermediate between δόξα (opinion) and νοῦς (mind). The idea that false judgment of 'being' arises "out of the wax in the soul being impure" (*Theaetetus*) seems related to what is stated in the *Res Publica*, about *all the sciences being discussed*, that by each one of these subjects (including our three subjects examined in this thesis) there is an '*instrument'/'organ of sense' of the soul that is purified and rekindled anew*. Here we find again the use of 'σῴζω' ("σωθῆναι") for the soul's salvation, and the concept of 'beholding the truth' ("ἀλήθεια"). This contemplation seems to be aimed at the 'Ideal Numbers' which are discussed about in the *Theaetetus* and which are said to exist without any false conceptions in the soul (spirituality again).

There is a concept of 'πλῆθος' (multitude) connected with ἀριθμός and τὸ ἕν. 'Proper understanding', and therefore implying a 'salvation' for the soul's understanding, is e.g. that the 'πλῆθος' itself, 'multitude/magnitude', i.e. the *concept* of πλῆθος, is responsible for an increase in quantity from e.g. eight to ten, it is not the number two in itself. This entire scheme is also related to Plato's doctrine of The Forms (εἴδη), in the *Phaedo*, where there is a clear identification of οὐσία with e.g. μονάς (monad/oneness) or δυάς (dyad/twoness). This has a bearing on Plato's other statements about λογιστική τε καὶ ἀριθμητικὴ leading towards οὐσία. The spirituality is again apparent here where as "a gift from the Gods" (probably Prometheus), "the doctrine of 'one and many, limit and unlimited' (ἑνὸς μὲν καὶ πολλῶν; πέρας δὲ καὶ ἀπειρίαν) as the constituents of all that is said to exist was given". The doctrine of opposites seems similar to that given by Aristotle as purported to be by *some* of the Pythagoreans (e.g. "πέρας καὶ ἄπειρον, περιττὸν καὶ ἄρτιον, ἓν καὶ πλῆθος").

γεωμετρία was during Plato's time concerned with *plane* surfaces only, since it is claimed that stereometry (study of *solids*) was hardly developed during this time. ἀριθμητική, λογιστική and γεωμετρία were subjects for the *guardian* who was to be skilled in both war and philosophy. Socrates states that geometry should compel the soul to contemplate ("θεάσασθαι") being/essence ("οὐσίαν"), and not to contemplate generation/becoming ("γένεσιν"). γεωμετρία is the knowledge of what always exists (this is its nature), not of what is created ('becoming') and destroyed: "Ὡς τοῦ ἀεὶ ὄντος γνώσεως, ἀλλ' οὐ τοῦ ποτέ τι γιγνομένου καὶ ἀπολλυμένου". γεωμετρία should assist us in beholding *the form of The Good* ("τὴν τοῦ ἀγαθοῦ ἰδέαν"). Just as was stated for ἀριθμητική and λογιστική, γεωμετρία draws the soul towards 'truth' ("πρὸς ἀλήθειαν"), and this is one of its purposes. γεωμετρία is singled out in the *Res Publica* as one of those subjects which *purify an instrument of the soul by which 'Truth' is beheld*. The 'purification of the soul' was also a central doctrine of Pythagoras according to Iamblichus.

Geometric and numeric conceptions play a crucial role in the creation of the world in Plato's *Timaeus*. For instance, the shapes of the four elements are given by 'the god' using forms and numbers ("εἴδεσί τε καὶ ἀριθμοῖς"), the four elements were composed of *triangles*, and the five 'Platonic' *solids* are constructed by specific *plane surfaces*, also composed of *triangles*. In the *Timaeus*, there is outlined a threefold division of the Kosmos (lit. 'the all': "τοῦ παντὸς"). We have (1) the model, source, and father, 'being' (ὄν); (2) the receptacle and wet-nurse of generation, the mother, 'space' (χώρα); and (3) the visible imitation of the model, generated, becoming, the child, 'generation/creation' (γένεσις). The 'ὄν' is understood by 'νόησις' (intelligence, understanding), the 'γένεσις' by "δόξῃ μετ᾽ αἰσθήσεως περιληπτόν" ("opinion, involving sense perception"), and the 'χώρα' by "μετ᾽ ἀναισθησίας ἁπτὸν λογισμῷ τινι νόθῳ" ("by a sort of bastard/counterfeit calculation/reasoning, with no sense perception"). Szabó noted that two passages in Plato's *Res Publica* seem to identify 'bastard reasoning' as being occupied with geometry. One of these passages is the one previously mentioned, where the trio of νοῦς, διάνοια, and δόξα are the subsections of Plato's 'analogy of the divided line', showing the metaphysical, cosmological and psychological frameworks. The faculty of the geometers, and probably of all mathematicians, is 'διάνοια'. Aristotle, as shown by Wedberg, also believed that Plato classified an 'intermediate state of mathematical objects' between 'the Ideas or Forms' and 'sensible things'. I believe that there is sufficient evidence for this in Plato's *Res Publica*, in the discussion on the 'analogy of the divided line', where διάνοια is related to the mathematicians, between δόξα (opinion) and νοῦς (mind). The distinction between 'Ideal Numbers' and 'Mathematical Numbers', argued by Aristotle to exist in Plato's doctrine, also shows Plato's possible 'hierarchical' classification, or 'duality' as Wedberg argued ('a relation of priority' in a 'parallel series'). This also points to my previously mentioned argument that the ἓν was conceptualized in two different ways: (1) as equivalent to the concept of any μονάς, and (2) as one ἀριθμός.

Plato's mathematics thus, in one sense, incorporates the study of an all-encompassing framework of the very things (τῶν ὄντων) of nature and existence, in the background of a spiritual philosophy as an aim. In Plato's 'analogy of the divided line' we also saw how 'truth' (ἀλήθεια) is at the core of not only the aim of mathematics, but also of the very existence of the world and the psyche of the human soul.

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