# Systemic Risk and Default Contagion in Financial Networks: Identifying Systemically Important Banks

Oscar Clarke Nilsson

Ossian Relander

LUND UNIVERSITY

Department of Automatic Control

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Department of Automatic Control
Lund University
Box 118
SE-221 00 LUND
Sweden


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# Abstract

This thesis uses real data from the Bank of International Settlements to create financial networks for the interbank market using five different methods for network reconstruction. The goal is to analyze how defaults propagate to assess the importance of banks and to examine how the network's structure affects the system's vulnerability. By applying network theory, communicability theory, and the DebtRank algorithm, we aim to identify which banks are the most vulnerable and which propagate the largest losses to the system. We also investigate how DebtRank correlates with centrality and communicability measures. Our results will be compared to the Basel Committee's annual assessment of global systemically important banks.

Our findings show small differences between the network reconstruction methods. The most noticeable difference is that the minimum density method produces more resilient networks when equity is low. In contrast, the small-world method results in networks with slightly higher losses, especially when equity is in the middle range. Additionally, our results indicate that JP Morgan is the most systemically important bank in most scenarios, matching the Basel Committee's conclusions. However, we believe our methods overestimate the importance of some of the largest Chinese banks. We also show that PageRank and impact diffusion have the highest correlation with DebtRank impact. Finally, we conclude that Katz centrality and impact susceptibility show a strong correlation with DebtRank vulnerability.

## Keywords

DebtRank, network dynamics, graph theory, centrality, interbank market, financial networks, shock propagation, financial contagion

# Acknowledgements

We would like to express our gratitude to our advisors, Giacomo Como and Emma Tegling, for their support and guidance throughout this process. Their insights and expertise have been crucial in the successful completion of this thesis. We would also like to thank our family and friends for their constant encouragement.

# Contents

# Notation List

| **Symbol** | **Description** |
|---|---|
| $\mathcal{G}$ | Graph representing the network |
| $\mathcal{V}$ | Set of nodes in a graph |
| $\mathcal{E}$ | Set of links in a graph |
| $W$ | Weight matrix of a graph |
| $P$ | Normalized weight matrix |
| $A$ | Interbank assets matrix |
| $A_i$ | Interbank assets of bank $i$ |
| $AD_i$ | Asset deficit of bank $i$ |
| $L$ | Interbank liabilities matrix |
| $L_i$ | Interbank liabilities of bank $i$ |
| $LD_i$ | Liabilities deficit of bank $i$ |
| $E_i$ | Equity of bank $i$ |
| $V$ | Vulnerability matrix |
| $h_i$ | Cumulative relative equity loss of bank $i$ |
| $\Lambda_{ij}$ | Reduced interbank leverage matrix |
| $C_B(i)$ | Betweenness centrality of node $i$ |
| $C_C(i)$ | Closeness centrality of node $i$ |
| $C_{in}(i)$ | In-degree centrality of node $i$ |
| $C_{out}(i)$ | Out-degree centrality of node $i$ |
| $C_E(i)$ | Eigenvector centrality of node $i$ |
| $C_I(i)$ | Invariant distribution centrality of node $i$ |
| $C_K(i)$ | Katz centrality of node $i$ |
| $C_{PR}(i)$ | PageRank centrality of node $i$ |
| $C_{BP}(i)$ | Bonacich centrality of node $i$ |
| $S_i$ | Impact susceptibility of bank $i$ |
| $I_i$ | Impact diffusion of bank $i$ |
| $F$ | Impact fluidity of the network |

# 1

# Introduction

In today's interconnected world, the global financial system is more complex and tightly woven than ever before. Characterized by its dense structure and intricate interconnections, it has repeatedly demonstrated a surprising fragility. The collapse of the Lehman Brothers bank during the devastating financial crisis of 2007-2008, and the more recent challenges faced by Silicon Valley Bank, First Republic Bank and Signature Bank, underscores the systemic risks that lie within the system. They also highlight how severe the damage can be. The recent bank disturbances raises the question of systemic importance, suggesting that influence and connectivity of a financial institution can be as critical as its size. This was illustrated during last year's bank crisis, when the relatively smaller American banks contributed to the collapse of Switzerland's second largest Bank, Credit Suisse [Ozili, 2024]. All these events serve as a reminder of the necessity for ongoing research in order to ensure stability within the financial system.

This thesis explores the intricate world of global financial networks, more specifically, the interbank market. Through the application of network dynamics theory we seek to gain a deeper understanding of its complexities. This approach offers insights into the network's structure and its resilience against stresses such as defaults. By first modeling the connections and monetary flows within these systems and then simulating financial stress on the models, we can predict and potentially prevent the propagation of crises.

Our thesis spans several key concepts within network dynamics, including measures of centrality and the generation of random graphs. Furthermore, we explore the relatively new algorithm, DebtRank, which is specifically designed for interbank markets. The analysis also includes metrics from communicability, which have recently been modified to better fit the context of financial networks. The models rely on both empirical data and relevant theory from network dynamics to provide a robust foundation for analysis

## 1.1 Purpose

The main purpose of this thesis is to better understand the interbank market. By creating different networks using real data from the largest banks in the world we want to determine: how the construction of the network and its structure affects stress and default propagation, which bank's are the most systemically important and how DebtRank metrics correlate with centrality and communicability measures.

## 1.2 Limitations

Although this thesis uses real data, it estimates the size of each interbank loan using methods for creating random graphs. It also treats the interbank market as an isolated system, whereas, in reality, the interbank market is interconnected with many other financial markets and networks. Graph theory offers numerous theoretical concepts for network analysis, but this thesis focuses on only a select few. These limitations are due to both the lack of available data and the constrained time frame of 20 weeks.

## 1.3 Previous work

There is an increasing amount of literature that aims to better understand financial systems. In recent years, the framework of network dynamics has proven to be especially useful in order to capture the complexity of these networks. Many well-established concepts have been expanded and refined to better suit the field of finance. Our thesis builds upon several key papers, including the following:

- In the study [Battiston et al., 2012] the DebtRank algorithm is introduced, a way to assess the systemic impact of financial institutions. By applying DebtRank to data on major US banks, the authors provide a method to quantify the potential propagation effects in financial networks. This highlights how more central nodes can have a greater influence on systemic stability.

- Expanding on the concepts from their previous paper, the colleagues improve their DebtRank methodology in the study [Battiston et al., 2015]. It is their refined version first presented here that will be used in our thesis.

- In the study [Estrada and Hatano, 2008] the authors introduce the concept of communicability in networks. Their metric, based on the exponential of the weight matrix, gives a new perspective on how nodes communicate with each other by considering not only direct neighbors but longer paths too.

- In the study [Silva et al., 2015] the authors explore the dynamics of financial contagion through network analysis, focusing on how shocks can propagate through the banking sector. In their paper, they introduce key concepts that we will employ in our analysis such as impact susceptibility and impact diffusion. They expand on existing literature that aims to use network communicability in the context of finance.

- The study [Anand et al., 2017] addresses the significant challenge of reconstructing financial networks from incomplete data. In their study, they use a method called minimum density to overcome these challenges. In our thesis, we will employ this method among others, using our data.
- In the study [Upper and Worms, 2002] the authors analyze the bilateral exposures in the German interbank market to evaluate the potential risk of contagion. They, like many others struggle with the lack of data. In their paper, they use a method called maximum entropy to reconstruct the financial network. This method will also be used by us in this thesis.

## 1.4 Outline

This thesis is structured as follows:

- **Chapter 2:** This chapter presents the fundamental concepts of graph theory used in our analysis, including basic notations, topology, centrality measures, random graphs, and communicability theory.
- **Chapter 3:** We introduce network theory in the context of finance. This includes a description of the financial network model, how lending and borrowing are represented in the graph, and the definition of default. We also present contagion metrics derived from DebtRank and discuss the application of communicability in financial models.
- **Chapter 4:** Our methods for network reconstruction are presented. We detail each of the five methods, highlighting their differences and unique aspects.
- **Chapter 5:** This chapter provides information about our data and the methodology used in our thesis.
- **Chapter 6:** Here, we present and discuss our results and compare them with the Basel Committee's.
- **Chapter 7:** This final chapter includes further analysis, suggestions for future work, and our conclusions.

# 2

# Background on Graph Theory

In this chapter we will go through all the theory from graph theory and network dynamics which we believe is necessary for our thesis. This includes basic concepts, centrality measures and communicability in graphs. A few of the theoretical concepts will be illustrated with a simple example.

## 2.1 Networks as Graphs

Nodes interconnected by links form a network. A road map offers an intuitive example: cities serve as nodes, and roads as the connecting links. Many real-world phenomena can be modeled as networks, where nodes might represent individuals, financial institutions, or web pages, and links depict the relationships or interactions between them. The mathematical concept of which all networks stand on is called graph theory. Here, a network is described as a graph $\mathcal{G}$, which consist of nodes $\mathcal{V}$, links $\mathcal{E}$ and a weight matrix $W$, formally expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, W)$. The set $\mathcal{V}$ contains the nodes, $\mathcal{V} = (1, 2, \dots, n)$ where $n$ is the number of nodes in the graph. The set $\mathcal{E}$ contains the links, each link $e = (i, j)$ determines whether or not there is a connection between node $i$ and $j$. The weight matrix $W$ assigns values to the links, which can have various interpretations depending on the context, such as the capacity of a road to handle traffic. A graph where all weights are limited to 0 and 1 is known as an unweighted graph. [Como and Fagnani, 2021]

Another important matrix in graph theory is the normalized weight matrix $P$. The normalized weight matrix adjusts the link weights of a graph to ensure that the sum of the weights for each row equals 1. It is defined by normalizing each entry of the weight matrix $W$ by the sum of the weights of all links connected to the corresponding node. The formula for the normalized weight matrix $P$ is given by:

$$P_{ij} = \frac{W_{ij}}{\sum_k W_{ik}}$$

Furthermore, a graph can be either directed or undirected. In a directed graph, links have a direction, meaning that a link from $i$ to $j$ does not imply a link from $j$ to $i$. In an undirected graph, the links do not have a direction, meaning that a link from $i$ to $j$ also goes from $j$ to $i$. [Como and Fagnani, 2021]

If there exists a link between two nodes they are referred to as neighbours. In the context of directed graphs, there are two types of neighbours: in-neighbours and out-neighbours. This distinction becomes redundant in undirected graphs. [Como and Fagnani, 2021]

A graph is said to be strongly connected if there exists a path between every pair of nodes. A graph is said to be unilaterally connected if it contains a directed path between every pair of nodes. A graph is said to be weakly connected if it is directed and replacing the directed links with undirected ones create a strongly connected graph. A graph can also be disconnected but this thesis will not deal with those. [Como and Fagnani, 2021]

## 2.2 Example of Graphs

### Basic Graphs

A graph's topology can be extremely complex, but it can also be quite simple. Some of the most basic graphs can be seen below. Analyzing these can be helpful in order to understand the more complicated graphs. A graph formed from a subset of nodes and links of $\mathcal{G}$ is called a subgraph of $\mathcal{G}$. The simple graphs seen below are often found as subgraphs in more complex graphs.

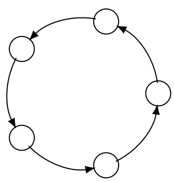

**Figure 2.1** Directed ring graph with 5 nodes.

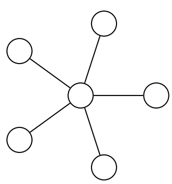

**Figure 2.2** Star graph with 1 central node and 5 peripheral nodes.

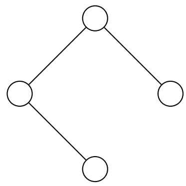

**Figure 2.3** Tree graph with 1 root and 3 children nodes.

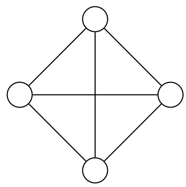

**Figure 2.4** Complete graph with 4 nodes.

## Random Graphs

Often, the exact topology of a network is unknown. In such cases, it is common to generate random graphs with certain parameters. Various methods exist for this purpose. One popular method for undirected graphs is the Erdős-Rényi random graph. This method relies on two parameters: the number of nodes and the probability that any given pair of nodes is connected by a link. It is important to note that each generated graph will differ, as they are created at random based on these parameters. In this thesis, we will employ five different methods to generate graphs. These methods will be discussed in detail in Chapter 4. [Como and Fagnani, 2021]

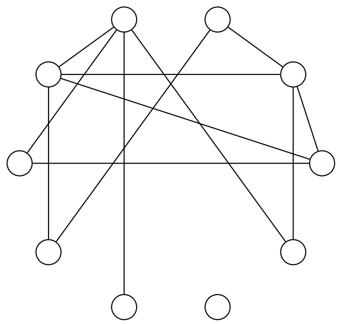

**Figure 2.5** Erdős-Rényi graph $G(10,0.3)$

## 2.3 Network Centrality

In graph theory, network centrality aims to quantify the centrality of a node within a graph. Determining centrality is crucial for understanding the importance of a node. There are many types of centrality measures and we will focus on some of the most commonly used variants. Some of these measures can be calculated using both unweighted and weighted links; in such cases, we will specify which version has been used.

### Betweenness Centrality

This is a measure which uses the concept of shortest path to determine a node's importance. Between all nodes in the graph there exists a shortest path. The centrality is defined as the number of these that pass through the node. For unweighted graphs the shortest path is simply the minimum number of links between two nodes. For weighted graphs it is the minimum sum of weights on the links between two nodes. The betweenness centrality $C_B$ for node $i \in V$ in an unweighted network is given by Equation 2.1

$$C_B(i) = \sum_{j,k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}} \tag{2.1}$$

where $i \neq j$, $i, j \neq k$, $\sigma_{jk}(i)$ is the number of shortest paths that pass node $i$ and $\sigma_{jk}$ is the number of shortest paths between node $j$ and $k$. Betweenness centrality in a weighted network is calculated similarly to the unweighted scenario, with the distinction that the calculation of the shortest path incorporates the weights of the edges. The weighted version of betweenness centrality will be used throughout our thesis. [Layton and Watters, 2016].

### Closeness Centrality

Closeness also uses shortest paths to determine importance. Here, the node's shortest path to all other nodes is calculated and the average value is computed to give the farness. The closeness is defined as the reciprocal of the farness. Closeness centrality is calculated according to Equation 2.2

$$C_C(i) = \frac{1}{\sum_j d(i,j)} \tag{2.2}$$

or, the more commonly used normalized closeness centrality

$$C_C(i) = \frac{N-1}{\sum_j d(i,j)} \tag{2.3}$$

where $d(i,j)$ is the shortest path between $i$ and $j$ and $N$ is the number of nodes in the network. Similarly to the case of betweenness centrality, closeness centrality can also be calculated taking the weights of the network into account. In this case, the shortest path between two adjacent nodes, might not always be through the direct link that connects them but through another node(s) in the network, all dependent on the weights of course. In the context of a weighted graph, $(d(i,j))$ represents not the shortest path in terms of the number of links, but rather the path that minimizes the total weight. The weighted version of closeness centrality will be used throughout our thesis.[Rubio, 2023b]

### Degree Centrality

Degree centrality is one of the simplest ways of defining centrality where the node's importance only depends on its in- or out-degree. In the directed and unweighted network, the in-degree of the node *i* is simply the number of links pointing towards *i* and the out-degree of *i* is the number of links originating from *i*. In the context of directed and weighted networks, the in-degree of node *i* is defined as the sum of all weights of the links pointing towards *i*. The out-degree of node *i* is defined as the sum of all weights of the links originating from *i*. The in- and out- degree centrality is calculated according to Equation 2.4 and 2.5, respectively

$$C_{in}(i) = deg_{in}(i) \tag{2.4}$$

$$C_{out}(i) = deg_{out}(i). \tag{2.5}$$

For the normalized degree centrality you simply divide 2.4 and 2.5 with $N-1$ where $N$ is the number of vertices in the network. [Rubio, 2023a]

### Eigenvector Centrality

This definition is an extension of the degree centrality. Rather than treating connectivity to all nodes the same, the eigenvector centrality values connections to nodes with high centrality more than connections to nodes with low centrality. In other words, the relative importance $C_E$ of node *i* should be proportional to the sum of the relative importance of its in-neighbours

$$C_E(i) \propto \sum_{j \in V} W_{j,i} C_E(j) \tag{2.6}$$

If we let the proportionality constant be $1/\lambda$ the relative importance of the node *i* can be calculated as

$$C_E(i) = \frac{1}{\lambda} \sum_{j \in V} W_{j,i} C_E(j) \tag{2.7}$$

which can be extended to vector format

$$W^T C_E = \lambda C_E. \tag{2.8}$$

We recognise Equation 2.8 which says that $C_E$ is an eigenvector of $W^T$ with eigenvalue $\lambda$. The eigenvalue $\lambda$ is usually set to the largest eigenvalue of $W$ as this choice, by Perron-Frobenius theorem, guarantees a positive and unique eigenvector under the condition that the graph is strongly connected. [Como and Fagnani, 2021]

### Invariant Distribution Centrality

Invariant distribution centrality is similar to eigenvector centrality, but instead of using the adjacency matrix, the normalized weight matrix $P$ is used. This is matrix, is essentially the same as a transition probability matrix for a random walk. Invariant distribution centrality thus quantifies the likelihood of ending up at a particular node in a long-term random walk on the network. The invariant distribution centrality of a node $i$ is given by:

$$C_I(i) = \lim_{t \to \infty} p_{ij}^{(t)}$$

where $p_{ij}^{(t)}$ is the probability of transitioning from node $j$ to node $i$ in $t$ steps. In the context of a Markov chain, this is typically solved by finding the eigenvector corresponding to the eigenvalue of 1 of the transition probability matrix $P$. [Como and Fagnani, 2021]

### Katz Centrality

Katz centrality extends the idea of eigenvector centrality by taking into account all paths that lead into a given node, not just direct neighbours. Longer paths are then exponentially penalized by incorporating a decay factor. The Katz centrality for a node $i$ in a network with weight matrix $W$ is defined as:

$$C_K(i) = \alpha \sum_{j=1}^{n} W_{ij} C_K(j) + \beta$$

where $C_K(i)$ represents the centrality of node $i$, $\alpha$ is a decay factor that penalizes contributions from distant nodes and $\beta$ typically takes a value of 1 to initiate the influence from each node. Note that, $\alpha$ must be less than the reciprocal of the largest eigenvalue of $W$ for convergence. In vector form, Katz centrality can be expressed as:

$$C_K = (I - \alpha W)^{-1} \beta \mathbf{1}$$

where $I$ is the identity matrix, and $\mathbf{1}$ is a vector of all ones. [Como and Fagnani, 2021]

### Bonacich Centrality

Bonacich centrality is a way to measure the influence of nodes in a network based on both their direct connections and their connections to other highly connected nodes. The centrality vector $C_{BP}$ is given by the formula:

$$C_{BP} = \alpha(I - \beta W)^{-1} W \mathbf{1} \tag{2.9}$$

where $I$ is the identity matrix, $W$ is the weight matrix, $\beta$ is a parameter that modulates the influence of paths within the network, and $\mathbf{1}$ is a vector of all ones. The coefficient $\alpha$ functions as a scaling parameter and is adjusted to ensure that the sum of squared scores $C_{BP}^2$ matches the number of nodes. The value of $\beta$ must be less than the absolute value of the reciprocal of the largest eigenvalue of $W$ to ensure convergence. [Bonacich, 1987]

### PageRank Centrality

PageRank centrality can be described as a variant of Bonacich centrality, where the influence of a node within a network is adjusted by a damping factor. PageRank was originally developed by Google's founders for their search engine to rank web pages and has since been used in various network analysis scenarios.

The PageRank of a node $i$, is defined as follows:

$$C_{PR}(i) = \frac{1-d}{N} + d \sum_{j \in M(i)} \frac{C_{PR}(j)}{deg_{out}(j)}$$

where $C_{PR}(i)$ represents the PageRank of node $i$. The constant $d$ is the damping factor, typically set between 0.8 and 0.9. The constant $N$ is the total number of nodes. $M(i)$ are nodes that link to $i$, indicating the direct influence on $i$. $deg_{out}(j)$ is the number of outbound links from node $j$, which adjusts the contribution of $j$ to the centrality of $i$. [Brin and Page, 1998]

## 2.4 A Simple Network Example

The purpose of this section is to make it easier for the reader to understand the various centrality measures. We have created a simple graph and calculated all all centrality measures previously mentioned on this graph.

### Network Visualization

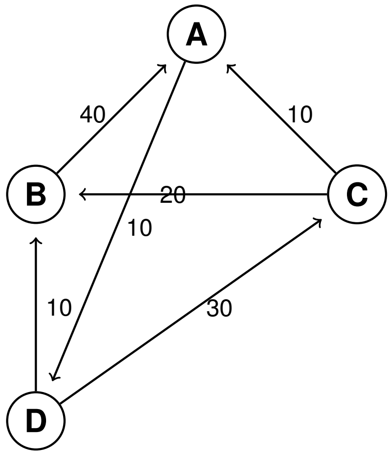


**Figure 2.6** Network consisting of four nodes and weighted links.

### Centrality for each node

| Centrality Measure | A | B | C | D |
|---|---|---|---|---|
| Out Degree | 1.0000 | 1.0000 | 2.0000 | 2.0000 |
| Betweenness | 3.0000 | 0 | 1.0000 | 3.0000 |
| Closeness | 0.0143 | 0.0059 | 0.0200 | 0.0125 |
| Eigenvector | 0.9166 | 1.0000 | 0.8404 | 0.7302 |
| Katz | 17.6959 | 27.7848 | 28.7240 | 44.1217 |
| PageRank | 0.2851 | 0.2193 | 0.2159 | 0.2798 |
| Bonacich | 0.7011 | 0.6129 | 1.2126 | 1.2893 |
| Invariant distribution | 0.3077 | 0.2308 | 0.1538 | 0.3077 |

**Table 2.1** Centrality for each node

## 2.5 Communicability

Concepts closely related to communicability have been researched for decades within the field of graph theory and network analysis. The modern formalization of communicability, as it is understood today, significantly owes to the contributions of [Estrada and Hatano, 2008]. In their paper they proposed a way of analyzing communicability by using the exponential of the weight matrix. This allows them to not only capture the shortest path, but all possible paths between nodes. The motivation for this is firstly that communication between nodes do not always follow the shortest path and secondly that shortest paths can fail to capture the nuances in the network, like bottlenecks for instance. The communicability concepts aims to increase the understanding and quantify how easily information travels through a network. Communicability quantifies how easily node $i \in \mathcal{V}$ can communicate with node $j \in \mathcal{V}$ by considering both the shortest path and random walks with varying length between the nodes. The authors define communicability from node $i$ to $j$ as

$$G_{ij} = \frac{1}{s!}S_{ij} + \sum_{k>s} \frac{1}{k!}W_{ij}^{(k)} = e_{ij}^{W} \tag{2.10}$$

where $S_{ij}$ is the number of shortest paths with length $s$ between nodes $i$ and $j$. Furthermore, $W_{ij}^{(k)}$ is the number of walks of length $k > s$ connecting $i$ to $j$. Note that for $k = 1$, $W_{ij}$ is the element $(i, j)$ of the weight matrix. The intention is to decrease the contribution to communicability as the length of the walk increases, which is why the first and second terms in Equation 2.10 are divided by $s!$ and $k!$, respectively. [Estrada and Hatano, 2008]

# 3

# Financial Network Models

In this chapter we will explain and go through how graph theory and network dynamics can be applied in the context of finance. More specifically, we will explain how a financial network model is defined, how communicability can be calculated and how stress can be initiated in these systems. Many of the concepts we go through here will be illustrated with a simple example.

## 3.1 The Financial Model

The theory of network dynamics is commonly used in a financial context. In financial networks, each node represents a financial institution, such as a bank, and the links between them represent financial relationships such as assets and liabilities. This network-based perspective is useful for understanding the dependencies and the potential for systemic risk within the financial system.

- **Nodes (Financial Institutions):** Each node in the network corresponds to a financial institution. These engage in various activities such as lending and borrowing money to each other. Every institution is characterized by its balance sheet, which includes assets, liabilities, and equity.
- **Links (Assets and Liabilities):** The links between the nodes represent financial relationships. **Assets:** When a bank lends money to another bank, the lending bank has an asset in the form of the loan amount. This is represented as a directed link from the lending bank to the borrowing bank. **Liabilities:** Conversely, when a bank borrows money, it incurs a liability. This is represented as a directed link from the borrowing bank to the lending bank.
- **Equity (Buffer):** Each financial institution has an equity value, which serves as its buffer against financial distress. Equity is the difference between a bank's total assets and total liabilities. It represents the institution's ability to absorb losses, providing a cushion that helps prevent default during times of financial stress. Equity is used as a proxy for default, and a bank is considered to be in default once its equity becomes negative.

## 3.2 Communicability in Financial Networks

Applying the theory of communicability in financial networks was done in the paper [Silva et al., 2015]. Here, the authors introduce three measures that we will focus on; impact susceptibility, impact diffusion and impact fluidity. In order to use these measures they create a vulnerability matrix. This matrix is created by calculating each banks vulnerability index which is simply the liabilities from one bank to another divided by the lending banks equity. The vulnerability matrix $V$ is defined as:

$$V_{ij} = \frac{L_{ij}}{E_j} \tag{3.1}$$

In other words, when $V_{ij} > 1$ the liability from $i$ to $j$ surpasses the equity of $j$. In this case, a default at bank $i$ will lead to a subsequent default of bank $j$. In the other case when $V_{ij} < 1$, bank $j$ can absorb the direct default of bank $i$, but not necessarily if default is spread indirectly to $j$. The vulnerability matrix is used as a weight matrix to create a new graph. In this graph, a link represents a potential direct contagion path. Taking higher powers of $V$ gives longer contagion paths, for example $V$ power of $k$ quantifies the of contagion paths of length $k$. [Silva et al., 2015]

### Impact Susceptibility

Impact susceptibility is a measure built on the theory of communicability. It aims to understand how likely a financial institution is to be impacted by a randomly occurring default originating from somewhere in the network. Impact susceptibility, therefore, measures a banks potential for contagion from the other banks. The impact susceptibility uses the communicability in Equation 2.10 to capture the dynamics that nodes can communicate not only through the shortest path, but also through other paths of longer length. The impact susceptibility $S_j$ of bank $j$ is defined as

$$S_j(G(V)) = \begin{cases} \frac{1}{k_j^{in}(V)} \sum_{i \in \mathcal{V}, i \neq j} G_{ij}(V), & \text{when } k_j^{in}(V) > 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.2}$$

where $k_j^{in}(\bar{V})$ is the number of direct neighbours $j$ such that $G_{ij}(V) > 0$. Multiplying a bank's value by its equity calculates the impact susceptibility as the total loss, rather than as a fraction.

[Silva et al., 2015]

### Impact Diffusion

Impact diffusion aims give the opposite perspective from impact susceptibility. While impact susceptibility measures how vulnerable a bank is, impact diffusion aims to measure the potential harm a bank could cause the other banks. The measure quantifies how large the impact from a bank is on the network to determine its importance. The bank $j$'s influence on diffusion can be analyzed by observing how its removal affects the difference in communicability of the vulnerability matrix. This is done by removing all the links that originate from $j$ making it a sink node and effectively removing the bank's ability to spread contagion. The logic behind impact diffusion approach is as follows. If $j$ plays an important role in spreading contagion, then its exclusion should significantly reduce the networks ability to spread contagion. On the other hand, if $j$ is non-important in spreading contagion, then, excluding $j$ should only have a minor effect on the networks ability to spread contagion. The impact diffusion $I_j(V)$ that $j$ exerts on the network is defined as

$$I_j(V) = \frac{1}{k_j^{out}(V)} \sum_{i \in \mathcal{V}} \sum_{k \in \mathcal{V}, k \neq i} \left[ G_{ik}(V) - G_{ik}\left(V^{(j-)}\right) \right], \tag{3.3}$$

where $k_j^{(out)}$ is the number of nodes that are directly connected that will default in the event of a default of $j$. The modified vulnerability matrix where all links originating from $j$ are removed is denoted $V^{(j-)}$. The summation term, $\left[ G_{ik}(V) - G_{ik}\left(V^{(j-)}\right) \right]$, can be interpreted as the communicability index of paths from $i$ to $k$ through $j$. The summation term is the difference in communicability between the original vulnerability network and the modified vulnerability network which effectively is the communicability between $i$ and $k$ that includes $j$ on the path. In conclusion, Equation 3.3 measures the decrease in communicability that occurs when $j$'s ability to diffuse impact is disabled. A high impact diffusion implies that the bank has the ability to propagate losses to many other banks in the network and the opposite for a low impact diffusion. However, impact diffusion does not distinguish between important and non important banks and does consequently not quantify the harm that these banks can cause to the network. This implies that a bank might spread losses to many banks, but those banks are of less importance. Conversely, a bank might spread losses to only a few, but important banks. By multiplying the final matrix with the equity vector, we address this issue so that the impact diffusion is adjusted to account for the banks' importance. [Silva et al., 2015]

## Impact Fluidity

Impact fluidity is the global equivalent of impact susceptibility. Impact fluidity is defined as how easily losses propagate through the network. Contagion propagation is more likely for networks with high impact fluidity. This is due to the impact fluidity's close relationship to impact susceptibility. The network's impact fluidity is calculated as the average impact susceptibility for all nodes in the network as Equation 3.4 shows.

$$F(V) = \frac{1}{N} \sum_{j \in V} S_j(V), \tag{3.4}$$

When a majority of the banks have a high impact susceptibility, it is more likely for defaults to propagate throughout the network which results in a high impact fluidity value, $F(V)$. Conversely, when a majority of the banks have a low impact susceptibility it is more likely that defaults remain localized near their origin which leads to a low impact fluidity value, $F(V)$. In conclusion, this measure shows how potentially easy an impact can travel in the network. [Silva et al., 2015]

## 3.3 A Simple Network Example

This serves as an example to illustrate how the metrics: impact susceptibility and impact diffusion are calculated. When we do the calculations on our larger, more complicated networks, we do it in the same way.

### Network Visualization

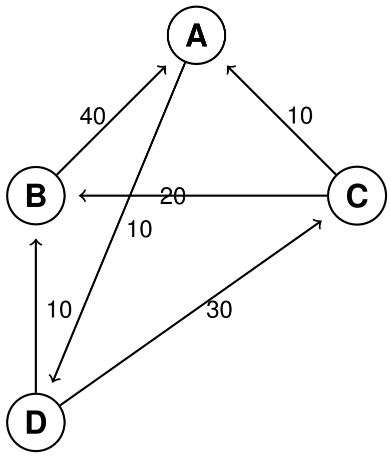


**Figure 3.1** Financial network where each bank has an equity of 100.

### Impact Susceptibility Calculations

**Asset Matrix** where $A_{ij}$ is an asset for $i$ and a liability for $j$.

$$A = \begin{pmatrix} 0 & 0 & 0 & 10 \\ 40 & 0 & 0 & 0 \\ 10 & 20 & 0 & 0 \\ 0 & 10 & 30 & 0 \end{pmatrix}$$

**Liability Matrix** where $L$ is the transpose of A

$$L = \begin{pmatrix} 0 & 40 & 10 & 0 \\ 0 & 0 & 20 & 10 \\ 0 & 0 & 0 & 30 \\ 10 & 0 & 0 & 0 \end{pmatrix}$$

**Equity Vector**

$$E = (100, 100, 100, 100)$$

**Vulnerability Matrix** where $V_{ij} = \frac{L_{ij}}{E_j}$

$$V = \begin{pmatrix} 0 & 0.4 & 0.1 & 0 \\ 0 & 0 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0.3 \\ 0.1 & 0 & 0 & 0 \end{pmatrix}$$

**Exponential of Vulnerability Matrix**

$$e^V = \begin{pmatrix} 1.0013 & 0.4001 & 0.1400 & 0.0390 \\ 0.0060 & 1.0008 & 0.2002 & 0.1300 \\ 0.0150 & 0.0020 & 1.0006 & 0.3001 \\ 0.1000 & 0.0200 & 0.0063 & 1.0013 \end{pmatrix}$$

**Impact Susceptibility**

$$S_i = \sum_{j \neq i} (e^V)_{ji} E_i$$

**Impact Susceptibility for Each Bank**

$$\begin{aligned} S_A &= (0.0060 + 0.0150 + 0.1000) \cdot 100 \\ &= 0.1210 \cdot 100 = 12.1037 \\ S_B &= (0.4001 + 0.0020 + 0.0200) \cdot 100 \\ &= 0.4221 \cdot 100 = 42.2127 \\ S_C &= (0.1400 + 0.2002 + 0.0063) \cdot 100 \\ &= 0.3465 \cdot 100 = 34.6599 \\ S_D &= (0.0390 + 0.1300 + 0.3001) \cdot 100 \\ &= 0.4691 \cdot 100 = 46.9133 \end{aligned}$$

**Impact Susceptibility Vector**

$$S(G(V)) = \begin{pmatrix} 12.1037 & 42.2127 & 34.6599 & 46.9133. \end{pmatrix}$$

## Impact Diffusion Calculations

**Asset Matrix** where $A_{ij}$ is an asset for $i$ and a liability for $j$.

$$A = \begin{pmatrix} 0 & 0 & 0 & 10 \\ 40 & 0 & 0 & 0 \\ 10 & 20 & 0 & 0 \\ 0 & 10 & 30 & 0 \end{pmatrix}$$

**Liability Matrix** where $L$ is the transpose of A

$$L = \begin{pmatrix} 0 & 40 & 10 & 0 \\ 0 & 0 & 20 & 10 \\ 0 & 0 & 0 & 30 \\ 10 & 0 & 0 & 0 \end{pmatrix}$$

**Equity Vector**

$$E = (100, 100, 100, 100)$$

**Vulnerability Matrix** where $V_{ij} = \frac{L_{ij}}{E_j}$

$$V = \begin{pmatrix} 0 & 0.4 & 0.1 & 0 \\ 0 & 0 & 0.2 & 0.1 \\ 0 & 0 & 0 & 0.3 \\ 0.1 & 0 & 0 & 0 \end{pmatrix}$$

**Exponential of Vulnerability Matrix**

$$e^V = \begin{pmatrix} 1.0013 & 0.4001 & 0.1400 & 0.0390 \\ 0.0060 & 1.0008 & 0.2002 & 0.1300 \\ 0.0150 & 0.0020 & 1.0006 & 0.3001 \\ 0.1000 & 0.0200 & 0.0063 & 1.0013 \end{pmatrix}$$

**Exponential of Vulnerability Matrix with** $V_{Aj} = 0$

$$e^{V^{A-}} = \begin{pmatrix} 1.0000 & 0 & 0.0 & 0.00 \\ 0.0060 & 1 & 0.2 & 0.13 \\ 0.0150 & 0 & 1.0 & 0.30 \\ 0.1000 & 0 & 0.0 & 1.00 \end{pmatrix}$$

**Difference between** $e^V$ **and** $e^{V^{A-}}$

$$e^V - e^{V^{A-}} = \begin{pmatrix} 0.0000 & 0.4001 & 0.1400 & 0.0390 \\ 0.0000 & 0.0000 & 0.0002 & 0.0000 \\ 0.0000 & 0.0020 & 0.0000 & 0.0001 \\ 0.0000 & 0.0200 & 0.0063 & 0.0000 \end{pmatrix}$$

**Difference matrix multiplied with $E$**

$$(57.9165, 0.0265, 0.2095, 2.6368)$$

**Impact diffusion of $A$**

$$I_A(V) = 60.7893$$

**Impact diffusion vector**

$$I(V) = \begin{pmatrix} 60.7893 & 40.1778 & 36.7237 & 14.9893. \end{pmatrix}$$

## 3.4 DebtRank

DebtRank is an algorithm designed for assessing systemic risk, which in this context refers to the risk of losses and defaults in a financial system. The algorithm was introduced in the paper [Battiston et al., 2012]. The concept was then further developed in the paper [Battiston et al., 2015]. This refined version will be used in our thesis and is presented below.

### Model Description

The financial system is represented with a weighted directed matrix where each node is a bank. A link $A_{ij}$ connecting the nodes $i, j$ represent an interbank loan from the creditor $i$ to debtor $j$. Each node is characterized by its balance sheet, containing the banks assets and liabilities. An asset is considered an interbank asset when it corresponds to a link in the network, a principle that also applies to liabilities. Assets and liabilities not represented in the network are referred to as external. The total amount of interbank assets and liabilities for bank $i$ is equal to $A_i = \sum_j A_{ij}$ and $L_i = \sum_j A_{ji}$, respectively. A bank $i$'s equity is defined through the balance sheet identity, being the difference between total assets and total liabilities

$$E_i = A_i - L_i + A_i^E - L_i^E \tag{3.5}$$

where $A_i^E$ $L_i^E$ are the external assets and liabilities of the bank $i$. Once a bank's liabilities surpass its assets, i.e equity becomes negative, it cannot meet its claims and consequently is insolvent. Insolvency is used as a proxy for default and once a bank has defaulted it cannot meet any of its claims. A bank that has not yet defaulted up until time $t$ is said to be active and belongs to the set $\mathcal{A}(t)$, which formally is defined according to Equation 3.6

$$\mathcal{A}(t) = \{i : E_i(t) > 0\}. \tag{3.6}$$

We want to extend the balance sheet identity 3.5 to hold over time t. In order to do so we first make the assumption that the assets are valued mark-to-market while liabilities are valued at their face value. The motivation behind this is that if a bank $j$ is under stress it is less likely to be able to meet its claims towards bank $i$ which consequently lowers the value of the interbank assets $A_{ij}$. The stress propagation dynamics from borrowers to lender follows the assumption that the relative change in the borrowers $j$'s equity is equal to the relative change of the lender $i$'s asset $A_{ij}$ with the delay of one time unit.

$$\frac{A_{ij}(t+1)}{A_{ij}(t)} = \frac{E_j(t)}{E_j(t-1)} \implies A_{ij}(t+1) = \begin{cases} A_{ij}(t)\frac{E_j(t)}{E_j(t-1)} \text{ if } i \in \mathcal{A}(t) \\ A_{ij}(t) = 0 \text{ if } i \notin \mathcal{A}(t) \end{cases} \tag{3.7}$$

The interbank liability from $j$ to $i$ $A_{ji}$, however, remains unchanged even though $j$ is under stress. As mentioned, once a bank $i$ has defaulted, it cannot meet any of its claims, effectively setting the liabilities of $i$, $A_{ji}$, to zero. The balance sheet identity for bank $i$ over time $t$ can therefore be formulated according to Equation 3.8.

$$E_i(t) = A_i^E(t) + L_i^E(t) + \sum_{j \in \mathcal{A}(t-1)} A_{ij}(t) - \sum_j A_{ji}(t) \tag{3.8}$$

Note that the information about a bank's default reaches the other banks with a one time unit delay, which is why we sum over the active banks $\mathcal{A}$ at time $t-1$. Given the fact that external assets and liabilities as well as interbank liabilities remain constant the evolution of equity is solely determined by the evolution of interbank assets according to Equation 3.9.

$$\begin{aligned} E_i(t+1) - E_i(t) &= \sum_{j \in \mathcal{A}(t)} A_{ij}(t+1) - \sum_{j \in \mathcal{A}(t-1)} A_{ij}(t) \\ &= \sum_{j \in \mathcal{A}(t-1)} [A_{ij}(t+1) - A_{ij}(t)] - \underbrace{\sum_{j \in \mathcal{A}(t-1) \setminus \mathcal{A}(t)} A_{ij}(t+1)}_{=0} \end{aligned} \tag{3.9}$$

The last term in Equation 3.9 is the sum over the set of banks being active at $t-1$ and non-active banks at t which consequently makes the interbank assets being valued at zero at time $t+1$. Using Equation 3.7 recursively we can rewrite the equity evolution as seen in Equation 3.10

$$\begin{aligned} E_i(t+1) - E_i(t) &= \sum_{j \in \mathcal{A}(t-1)} \frac{A_{ij}(t)}{E_j(t-1)} [E_j(t) - E_j(t-1)] \\ &= \sum_{j \in \mathcal{A}(t-1)} \frac{A_{ij}(0)}{E_j(0)} [E_j(t) - E_j(t-1)] \end{aligned} \tag{3.10}$$

Defining the leverage matrix $\tilde{\Lambda}$ as

$$\tilde{\Lambda}_{ij} = \begin{cases} \frac{A_{ij}(0)}{E_j(0)} & \text{if } j \in \mathcal{A}(t-1) \\ 0 & \text{if } j \notin \mathcal{A}(t-1) \end{cases} \tag{3.11}$$

the equity evolution can be further rewritten as per Equation 3.12.

$$E_i(t+1) = \max\left[0, E_i(t) + \sum_{j \in \mathcal{A}(t-1)} \tilde{\Lambda}_{ij}\left[E_j(t) - E_j(t-1)\right]\right] \tag{3.12}$$

By letting $h_i(t+1) = \frac{E_i(0)-E_i(t+1)}{E_i(0)}$ denote the cumulative relative equity loss it can be rewritten according to Equation 3.13

$$\begin{aligned} h_i(t+1) &= \frac{E_i(0) - E_i(t+1)}{E_i(0)} \\ &= \frac{E_i(0) - \max\left[0, E_i(t) + \sum_{j \in \mathcal{A}(t-1)} \tilde{\Lambda}_{ij}\left[E_j(t) - E_j(t-1)\right]\right]}{E_i(0)} \\ &= \min\left[\frac{E_i(0)}{E_i(0)}, \frac{E_i(0) - E_i(t) - \sum_{j \in \mathcal{A}(t-1)} \tilde{\Lambda}_{ij}\left[E_j(t) - E_j(t-1)\right]}{E_i(0)}\right] \\ &= \min\left[1, h_i(t) - \sum \underbrace{\frac{\tilde{\Lambda}_{ij} E_j(0)}{E_i(0)}}_{=\Lambda_{ij}} \left[\frac{E_j(t) - E_j(t-1)}{E_j(0)}\right]\right] \\ &= \min\left[1, h_i(t) + \sum \Lambda_{ij}\left[h_j(t) - h_j(t-1)\right]\right] \end{aligned} \tag{3.13}$$

where $\Lambda_{ij} = \frac{\tilde{\Lambda}_{ij} E_j(0)}{E_i(0)}$ is the reduced interbank leverage matrix. In conclusion, we get the evolution of cumulative relative equity loss as seen in Equation 3.14

$$h_i(t+1) = \min\left[1, h_i(t) + \sum \Lambda_{ij}\left[h_j(t) - h_j(t-1)\right]\right] \tag{3.14}$$

## 3.5 Centrality based on DebtRank

The DebtRank algorithm does not automatically assign a centrality score to the banks; it is an algorithm for stress propagation. However, the outcomes of applying DebtRank can be used to construct a ranking. We do this by introducing the initial distress to each of the banks one at a time. It then iterates as the DebtRank algorithm states until a steady state has been reached. At this point, each of the other banks will have a value between 0 and 1, stating the proportion of their equity loss. We then multiply this value with the bank's initial equity and sum over all banks. This gives the total loss for the system caused by a specific bank. We call this aggregate figure DebtRank impact, where higher scores indicate a greater systemic importance of the bank. To gain a different perspective, one can calculate each bank's vulnerability by summing its loss each time one of the other banks is subjected to initial stress. This sum gives a DebtRank vulnerability for each bank.

## 3.6 A Simple Network Example

The purpose of this section is to make it easier for the reader to understand the DebtRank model and how the measures from the algorithm are derived.

### Network Visualization

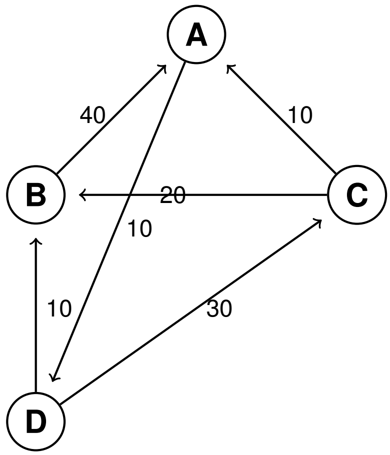


**Figure 3.2** Financial network where each bank has an equity of 100.

### DebtRank Calculation

**Reduced Interbank Leverage Matrix**

$$\Lambda = \begin{pmatrix} 0 & 0 & 0 & \frac{10}{100} \\ \frac{40}{100} & 0 & 0 & 0 \\ \frac{10}{100} & \frac{20}{100} & 0 & 0 \\ 0 & \frac{10}{100} & \frac{30}{100} & 0 \end{pmatrix}$$

**Step 1**

$h_A(1) = 1$

$$h_B(1) = \min\left[1, h_B(0) + \frac{40}{100} \cdot (h_A(0) - 0) + 0 \cdot (h_C(0) - 0) + 0 \cdot (h_D(0) - 0)\right] = 0.4$$

$$h_C(1) = \min\left[1, h_C(0) + \frac{10}{100} \cdot (h_A(0) - 0) + \frac{20}{100} \cdot (h_B(0) - 0) + 0 \cdot (h_D(0) - 0)\right] = 0.1$$

$$h_D(1) = \min\left[1, h_D(0) + 0 \cdot (h_A(0) - 0) + \frac{10}{100} \cdot (h_B(0) - 1) + \frac{30}{100} \cdot (h_c(0) - 1)\right] = 0$$

**Step 2**

$$
\begin{aligned}
h_A(2) &= 1 \\
h_B(2) &= \min\left[1, h_B(1) + \frac{40}{100}\cdot(h_A(1)-h_A(0))\right] = 0.4 \\
h_C(2) &= \min\left[1, h_C(1) + \frac{10}{100}\cdot(h_A(1)-h_A(0)) + \frac{20}{100}\cdot(h_B(1)-h_B(0))\right] = 0.18 \\
h_D(2) &= \min\left[1, h_D(1) + \frac{10}{100}\cdot(h_B(1)-h_B(0)) + \frac{30}{100}\cdot(h_C(1)-h_C(0))\right] = 0.07
\end{aligned}
$$

**Step 3**

$$
\begin{aligned}
h_A(3) &= 1 \\
h_B(3) &= \min\left[1, h_B(2) + \frac{40}{100}\cdot(h_A(2)-h_A(1))\right] = 0.4 \\
h_C(3) &= \min\left[1, h_C(2) + \frac{10}{100}\cdot(h_A(2)-h_A(1)) + \frac{20}{100}\cdot(h_B(2)-h_B(1))\right] = 0.18 \\
h_D(3) &= \min\left[1, h_D(2) + \frac{10}{100}\cdot(h_B(2)-h_B(1)) + \frac{30}{100}\cdot(h_C(2)-h_C(1))\right] = 0.094
\end{aligned}
$$

**Final Conditions**

$$
\begin{aligned}
h_A(3) &= 1 \\
h_B(3) &= 0.4 \\
h_C(3) &= 0.18 \\
h_D(3) &= 0.094
\end{aligned}
$$

## Final values for all initial conditions

**Scenario 1: Initial Condition at Node A**

| Initial Values | Final Values | |
|---|---|---|
| $h_A(0) = 1$ | $h_A(3)$ | $= 1$ |
| $h_B(0) = 0$ | $h_B(3)$ | $= 0.4$ |
| $h_C(0) = 0$ | $h_C(3)$ | $= 0.18$ |
| $h_D(0) = 0$ | $h_D(3)$ | $= 0.094$ |

**Scenario 2: Initial Condition at Node B**

| Initial Values | Final Values | |
|---|---|---|
| $h_A(0) = 0$ | $h_A(3)$ | $= 0.01603$ |
| $h_B(0) = 1$ | $h_B(3)$ | $= 1$ |
| $h_C(0) = 0$ | $h_C(3)$ | $= 0.20160$ |
| $h_D(0) = 0$ | $h_D(3)$ | $= 0.16048$ |

---

**Scenario 3: Initial Condition at Node C**

| Initial Values | Final Values | |
|---|---|---|
| $h_A(0) = 0$ | $h_A(3)$ | $= 0.03012$ |
| $h_B(0) = 0$ | $h_B(3)$ | $= 0.01200$ |
| $h_C(0) = 1$ | $h_C(3)$ | $= 1$ |
| $h_D(0) = 0$ | $h_D(3)$ | $= 0.30120$ |

---

**Scenario 4: Initial Condition at Node D**

| Initial Values | Final Values | |
|---|---|---|
| $h_A(0) = 0$ | $h_A(3)$ | $= 0.10000$ |
| $h_B(0) = 0$ | $h_B(3)$ | $= 0.04000$ |
| $h_C(0) = 0$ | $h_C(3)$ | $= 0.01800$ |
| $h_D(0) = 1$ | $h_D(3)$ | $= 1$ |

---

**DebtRank Impact**

$$\text{DebtRank Impact (A)} = \frac{0.4 + 0.18 + 0.094}{3} \cdot 100 = 22.467$$

$$\text{DebtRank Impact (B)} = \frac{0.01603 + 0.20160 + 0.16048}{3} \cdot 100 = 12.670$$

$$\text{DebtRank Impact (C)} = \frac{0.03012 + 0.01200 + 0.30120}{3} \cdot 100 = 11.444$$

$$\text{DebtRank Impact (D)} = \frac{10.000 + 0.04000 + 0.01800)}{3} \cdot 100 = 5.267$$

**DebtRank Vulnerability**

$$\text{DebtRank Vulnerability (A)} = \frac{0.01603 + 0.03012 + 0.10000}{3} \cdot 100 = 4.872$$
$$\text{DebtRank Vulnerability (B)} = \frac{0.4 + 0.01200 + 0.04000}{3} \cdot 100 = 15.067$$
$$\text{DebtRank Vulnerability (C)} = \frac{0.18 + 0.20160 + 0.01800}{3} \cdot 100 = 13.387$$
$$\text{DebtRank Vulnerability (D)} = \frac{0.094 + 0.16048 + 0.30120}{3} \cdot 100 = 18.523$$

## Comments

In this simple example, we observe that bank A causes the largest additional losses in the network, followed by banks B, C, and D. Conversely, bank D is the most financially vulnerable, followed by banks B, C, and finally A.

Comparing the DebtRank impact with the weighted in-degree,

$$C_{in} = \big(50, 30, 30, 10\big)$$

we see a clear correlation. Similarly, comparing the DebtRank vulnerability with the weighted out-degree,

$$C_{out} = \big(10, 40, 30, 40\big)$$

we again observe a clear correlation.

# 4

# Methods for Network Reconstruction

In the complex world of financial networks, the lack of data presents a significant challenge for analyzing and modeling. To overcome this problem we turn to the theory of random graphs. By using both established and new methods we are able to reconstruct these unknown networks in different ways. Here we present five different methods which we believe each give a unique perspective.

## 4.1 Maximum Entropy Method

The maximum entropy method, abbreviated as ME, maximises the entropy while satisfying the given constraints of each institution's total interbank assets and liabilities. Entropy in this context means disorder or uncertainty, more precisely it means that the graph's structure is less predictable. By creating the most unbiased graph possible one assures that the model is driven by data rather than by preconceived notions of the graph's characteristics. It is especially useful in financial network analysis, where assumptions can lead to simplified models that fail to capture nuances within the system.

Maximizing the entropy is not the same as maximising the number of links, which one might be mistaken to believe. Instead this method provides the framework of creating an unbiased graph. Any complexity or connectivity is a reflection of the financial reality.

Again, we let $W$ denote the interbank matrix with $W_{ij} \geq 0$ being the bilateral exposures between bank $i$ and $j$ and $W_{ij} = 0$ when $i = j$. Furthermore we denote the interbank assets and liabilities as $A_i = \sum_j W_{ij}$ and $L_i = \sum_j W_{ji}$, respectively. Implementing this method involves solving an optimization problem. The entropy we aim to maximize is mathematically represented by the following Equation 4.1.

$$\begin{aligned}
\text{minimize} \quad & \sum_{i}^{N}\sum_{j\neq i}^{N} W_{ij}\ln\left(\frac{W_{ij}}{A_i L_j}\right) \\
\text{subject to} \quad & \sum_{j=1}^{N} W_{ij} = A_i,\ \forall i = 1\ldots N, \\
& \sum_{j=1}^{N} W_{ji} = L_i,,\ \forall i = 1\ldots N, \\
& W_{ij} \geq 0\ \forall (i,j)
\end{aligned} \tag{4.1}$$

The optimization problem 4.1 can be solved numerically with the RAS-algorithm [Upper and Worms, 2002].

When using this framework on our data, the result is a highly interconnected and complex graph. It is important to note that this complexity, evidenced by high degree distribution, is a reflection of our underlying constraints and not an inherent feature of the method. A graph constructed with the ME-method can be seen in Figure 4.1

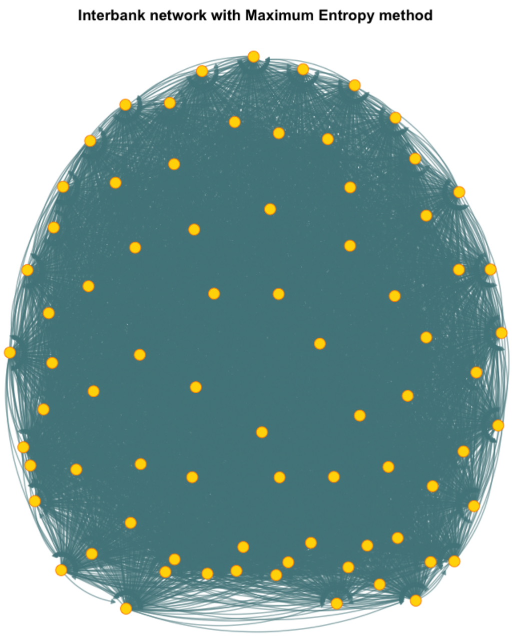


**Figure 4.1** Graph of maximum entropy network.

## 4.2 Minimum Density Method

The minimum density method, abbreviated as MD, minimises the density while satisfying the given constraints of each institution's total interbank assets and liabilities. Minimal density in this context simply means creating a graph with as few connections as possible. The method is based on the idea that a simpler graph, one with fewer but more significant links, can provide a clearer understanding of the essential relationship between nodes. The method identifies the core connections that are needed to represent the financial flows.

The mathematical implementation of the method is an optimization problem. We let $W$ denote the interbank matrix with $W_{ij} \geq 0$ being the bilateral exposure between bank $i$ and $j$. Furthermore we denote the interbank assets and liabilities as $A_i = \sum_j W_{ij}$ and $L_i = \sum_j W_{ji}$, respectively. Note that banks are not allowed to lend to themselves and therefore $W_{ij} = 0$ where $i = j$. By iteratively removing links that are redundant the objective is to minimize the number of links which in mathematical terms can be expressed according to Equation 4.2.

$$\begin{aligned} \text{minimize} \quad & \sum_{i}^{N} \sum_{j \neq i}^{N} c \cdot \mathbf{1}_{[W_{ij}>0]} \\ \text{subject to} \quad & \sum_{j=1}^{N} W_{ij} = A_i, \ \forall i = 1 \ldots N, \\ & \sum_{j=1}^{N} W_{ji} = L_i, , \ \forall i = 1 \ldots N, \\ & W_{ij} \geq 0 \ \forall i, j, \end{aligned} \tag{4.2}$$

However, the optimization problem is computationally expensive to solve. A network that minimizes the number of link given the constraints in 4.2 is generated through the MD-algorithm according to the following steps.

1. Compute current deficits:
    - Asset deficit $AD_i = (\sum_j W_{ij} - A_i)$
    - Liabilities deficit $LD_i = \sum_j W_{ji} - L_i$
2. Select link $(i, j)$ according to probability $Q_{ij} \propto \max\left\{\frac{AD_i}{LD_j}, \frac{LD_i}{AD_j}\right\} \forall i, j$.
3. Load exposure $W_{ij} = \min\left\{AD_i, LD_j\right\}$ and accept $W_{ij}$ if $f(W' = W + W_{ij}) \geq f(W)$.

4. Iterate until deficits are zero and thus satisfies the following equations

$$\sum\sum W_{ij} = \sum A_i \tag{4.3}$$

$$\sum_j W_{ij} = A_i \,\forall i \tag{4.4}$$

$$\sum_j W_{ji} = L_i \,\forall i \tag{4.5}$$

The function $f$ is a smooth function which increases as the number of links decrease, assuming that the interbank constraints are satisfied. [Anand et al., 2017].

The result is an aesthetically pleasing and low-density graph which can be seen in Figure 4.2


**Figure 4.2** Graph of minimum density network.

## 4.3 Preferential Attachment Method

This method, abbreviated as PA, is a nuanced adaptation of the well-known preferential attachment method, which is widely recognized for its ability to generate networks mimicking the ones seen in the real world. In the classical model the network evolves by adding new nodes, where the likelihood of forming a connection to an existing node is proportional to the number of connections that node already has. This mechanism naturally leads to "rich get richer" dynamics. Nodes with many connections will grow their connectivity at a faster rate than nodes with fewer connections.

The unweighted PA- matrix is generated by introducing 20 new links with each new node. The choice of adding 20 new links per node is caused by the need to meet the interbank assets and liabilities constraints, which is often unachievable with a lower number of links. Furthermore, this model is very sensitive to the order in which the nodes are introduced to the system. Because the topography of the resulting graph is heavily influenced by the order, it is important that one chooses it wisely. Recognizing this, our modified approach randomizes the order in which they are introduced. This guarantees that the order does not affect our result. The method is still, as all our methods are, in compliance with our given constraints of each bank's total interbank assets and liabilities. The weights are distributed on the unweighted PA-matrix using the RAS-algorithm.

The result is an interesting network where a few banks play a more crucial role. Due to our constraints, the network has a high degree distribution. A graph constructed with the PA-method can be seen in Figure 4.3

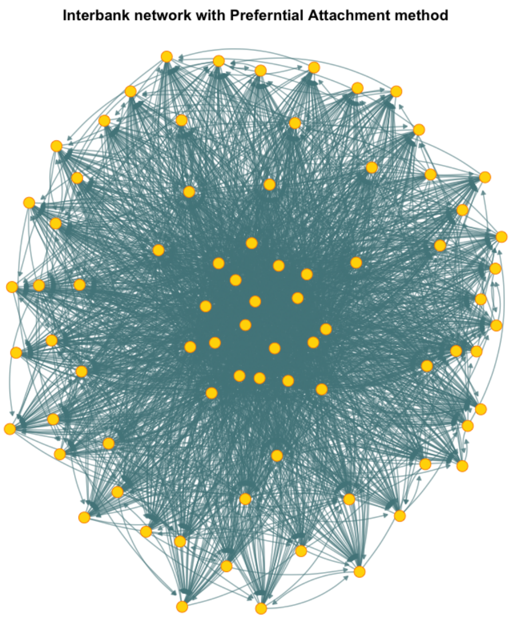


**Figure 4.3** Graph of preferential attachment network.

## 4.4 Small World Method

The small world method, abbreviated as SW, acknowledges the prevalent clustering characteristics observed in many real-world networks. This recurring phenomenon is well documented in network studies and this method aims to capture this by assuming geographical clustering.

We implement this concept by assigning a number to each bank based on its continent: AMER, EUR, or ASIA. This number determines the likelihood of a connection between the banks. Banks sharing the same number are connected with the probability $p = 1$, ensuring that all banks on the same continent form a strongly connected network. If the banks do not share the same number, a link is drawn with probability $p = 0.25$. This much lower value reflects the lower probability of connection between geographically distant banks. An unweighted matrix was created using these probabilities. Note that we still do not allow self exposures and therefore all matrix entries $(i, j)$ where $i = j$ were set to 0. The weighted matrix is then created using the RAS-algorithm under the same given constraints of total interbank assets and liabilities. The resulting network shows geographical clustering. However, due to the necessity of complying to the given constraints, the extent of clustering is moderate and the degree distribution is high. A graph constructed with the SW-method can be seen in Figure 4.4.

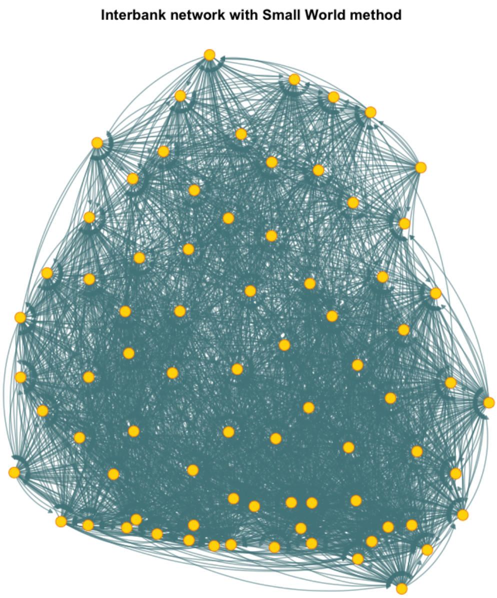


**Figure 4.4** Graph of small world network.

## 4.5 Random Link Equal Probability Method

The random link equal probability method, abbreviated as RL, unlike the previous two, is not inspired by previous work. Instead it is constructed by us under the assumption that there exists a constant, quantifiable likelihood of one bank extending credit to another. The method still complies to the constraint of total interbank assets and liabilities. One advantage of this method is that it does not assume any qualities of a certain bank, instead it treats all banks equally when creating the network. Given the absence of empirical data we chose a probability of 40 percent. Firstly, an unweighted matrix was generated where the probability of a matrix entry of 1 and 0 was 40 and 60 percent, respectively. A matrix entry of 1 at element $(i, j)$ represented an loan from $i$ to $j$ and a 0 meant no exposure between the parties. Since we do not allow self exposures, all matrix entries $(i, j)$ when $i = j$ were set to zero. Secondly, we distribute the interbank assets and liabilities between the banks that where given a 1 in the unweighted matrix. The interbank assets and liabilities where iteratively distributed using the RAS-algorithm until the total interbank assets and liabilities constraints where satisfied. The constraints are satisfied when the matrix's row sum equal the total interbank assets and matrix's column sum equal the total interbank liabilities. The result is a network which, in terms of degree distribution lies somewhere between the ME-network and the MD-network. A graph constructed with the RL-method can be seen in Figure 4.5

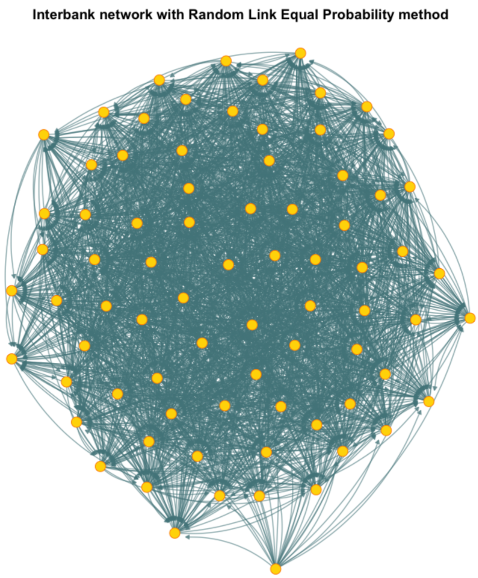


**Figure 4.5** Graph of random link equal probability network.

## 4.6 RAS Iterative Algorithm

The RAS algorithm itself is not an algorithm for estimating interbank networks, but it can be used in conjunction with methods such as the ME-method and PA-method to yield interbank networks. The algorithm is an iterative method for adjusting the rows and columns of a non-negative matrix to satisfy specific constraints. It adjusts the matrix so that the sum of the rows and columns meets predefined totals while preserving the ratios of the entries as much as possible. The algorithm follows these steps:

1. The matrix $A$ is the matrix to be adjusted and $R$ and $S$ are identity matrices initially. The goal is to find $R$ and $S$ such that the matrix multiplication RAS satisfies the predefined matrix row and column sums $r$ and $s$.
2. For each row, calculate the row scaling factors
$$R_{ii} = \frac{r_i}{\sum_j^N A_{ij} S_{jj}} \tag{4.6}$$
Apply the row scaling and update $A = RA$
3. For each column, calculate the column scaling factors
$$S_{ii} = \frac{s_i}{\sum_i^N A_{ij} R_{ii}} \tag{4.7}$$
Apply the column scaling and update $A = AS$
4. Repeat the steps 2-3 until convergence or until the row- and column sums of the matrix RAS are sufficiently close to the target row- and column sums r and s.

[Trinh and Phong, 2013]

## 4.7 Monte Carlo Simulation

Apart from the maximum entropy, the resulting networks from the methods above will vary slightly with each application due to the stochastic nature of these methods. Consequently, there is no single, definitive graph. To enhance the robustness and reliability of our analysis, we employ Monte Carlo Simulation for each method. Monte Carlo Simulation is a powerful computational technique that utilizes random sampling to approximate solutions, making it ideal for dealing with the inherent randomness in our methods. By running the methods 100 times, we ensure that our results are not dependent on any single, potentially atypical graph simulation. As a result, the metrics we calculate on the network represent average values from 100 network reconstruction simulations for each method. This approach provides a more robust statistical basis for our conclusions. The number of simulations are limited to 100 due to long computation times. [IBM, 2020].

# 5

# Data and Methodology

In this chapter we will firstly discuss the data used in our model. Secondly, we will explain Spearman's rank correlation which we use to compare our results. Finally, we will explain step by step our methodology.

## 5.1 Our data

The data used in this thesis is sourced from The Bank of International Settlements. We believe this is the most reliable and accurate information available. However, the data set did not provide us with all the information we needed. Therefore, we had to rely on certain assumptions. We selected data on the 76 largest banks in the world. This data included each banks total interbank assets, total interbank liabilities and total exposures at the end of 2020 [Basel Committee on Banking Supervision, 2023]. These figures were then distributed between the banks in five different ways as seen in Chapter 4. The total of the interbank assets and liabilities in our data set does not equal each other, as the figures are part of a much larger network in reality. In our model we need them to do so, therefore we introduce a node denoted as 'Other'. The connections between this node and the other banks are assigned to ensure that the total interbank assets and liabilities align correctly. Additionally, we collected data on each bank's total exposures. This data was used for the purpose of calculating each banks equity.

Our main challenge was to find precise estimates of the bank's equity, which in our model represents the bank's buffer. The Liquidity Coverage Ratio or LCR, mandated by Basel III, states that a bank is required to hold high-quality liquid assets sufficient to cover the total net cash outflows over a 30-day stressed period [BBVA, 2023]. Although there are many regulatory frameworks such as Basel's LCR that provide useful information, it was difficult to translate them into concrete and reliable numbers. This is both due to lack of data but also because the regulatory framework is customized for the real world which is more complex and dynamic than that of our model. By assuming that banks maintain a buffer ranging from 10 to 15 percent of their total exposures, we obtained each banks equity. By estimating equity in this way, we believe they sufficiently reflect reality while also aligning with the rest of our model to provide meaningful results. The banks' total exposures was taken from the same dataset from The Bank of International Settlements.

## 5.2 Spearman's Rank Correlation

Spearman's rank correlation is often preferred over Pearson's correlation coefficient in specific statistical analyses because it focuses on identifying monotonic relationships rather than strictly linear ones. Pearson's correlation assesses the degree of linear relationship between two variables, which means it calculates how well a linear equation can describe the relationship between them. This method assumes that the change in one variable directly corresponds to a proportional change in the other, which is ideal for variables that increase or decrease at a consistent rate relative to each other.

However, in our case, we do not assume a linear relationships but rather seek monotonic relationships between our variables. A monotonic relationship is one where the variables tend to move in the same direction (either increasing or decreasing), but not necessarily at a constant rate. In these cases, Spearman's rank correlation is more appropriate because it is designed to measure the strength and direction of a monotonic relationship. Spearman's method works by converting the data values into ranks and then calculating Pearson's correlation coefficient for these ranks. This approach allows Spearman's rank correlation to capture relationships where the variables increase or decrease together but not necessarily at a constant or proportional rate. Thus, it provides a more general assessment of association in cases where the assumption of linearity is too rigid or not evident in the data. [Networks, 2023]

## 5.3 Step by Step

For each of the five methods for network reconstruction, the following steps are performed:

1. **Network Creation**: A network consisting of 77 banks is created as the methods state in Chapter 4.

2. **Centrality Metrics Calculation**: The centrality metrics introduced in Chapter 2 are calculated for each bank.

3. **Stress Simulation**: The DebtRank algorithm is used to simulate stress on the network. In each scenario, a different bank is subjected to initial stress, with equity levels varying between 10% and 15% of total exposures.

4. **DebtRank Calculation**: The results from each simulation are used to calculate the DebtRank impact and DebtRank vulnerability for each bank.

Steps 1 to 4 are repeated 100 times to perform a Monte Carlo simulation.

5. **Correlation Analysis**: Spearman's rank correlations are used to determine how the pre-calculated centrality metrics correlate with the DebtRank metrics.

6. **Importance Assessment**: The DebtRank impact is used to assess which banks are the most critical to each network under varying equity levels.

# 6

# Results and Analysis

In this chapter we will go through and analyze our results. First we will show how the selected centrality measures correlate with DebtRank for each network. We will then compare the five different reconstruction methods with each other, both in regards to additional losses and additional defaults. Finally, we will show our rankings of most important banks and highlight similarities and differences with the Basel Committee's annual assessment.

## 6.1 Correlation between DebtRank and Selected Measures

### DebtRank Impact

We aim to investigate whether the network centrality measures presented in 2.3 are effective predictors of DebtRank impact. Specifically, we want to determine if a bank with a high centrality also causes significant losses in the system in the event of a default, thereby identifying systemically important banks. While impact diffusion, as presented in 3.2, is not strictly a centrality measure, it indicates a bank's ability to spread losses, making it relevant for our comparison. Given the significant role of interbank liabilities in driving financial stress during crises, we will also compare their size with DebtRank impact. Note that a bank's liabilities are equivalent to its weighted in-degree. The correlation will indicate the extent to which the simulation outcome can be explained by the properties of the liabilities. Specifically, we will compare network centrality, impact diffusion and interbank liabilities with DebtRank impact.

To evaluate our measure's predictive ability in identifying systemically important banks, we will compare the previously mentioned measures with DebtRank impact from 100 simulations per equity level. The only exception is the ME-network, where we will compare DebtRank impact from a single simulation per equity level. This comparison will be conducted by calculating Spearman's rank correlation between each measure and DebtRank impact, which will indicate the strength of their monotonic relationship.

### DebtRank Vulnerability

A central bank in the network could be vulnerable to other banks within the network. Such a central bank might be susceptible to financial contagion and, as a result, suffer significant losses. Therefore, we investigate whether high centrality is associated with large losses for that particular bank. The centrality measures that will be considered are the ones presented in 2.3. We will also consider the impact susceptibility as presented in 3.2, as it aims to capture how susceptible a bank is to financial contagion. In contrast to interbank liabilities, the size of a bank's interbank assets determines its susceptibility to financial contagion. Therefore, we will include interbank assets in the comparison with DebtRank vulnerability. The correlation will indicate the extent to which the simulation outcome can be explained by the properties of the assets. Note that a banks interbank assets is equivalent to its weighted out-degree. Specifically, we will compare network centrality, impact susceptibility and interbank assets with DebtRank vulnerability.

To evaluate are measure's predictive ability in identifyling systemically vulnerable banks, we will compare the previously mentioned measures with DebtRank vulnerability from 100 simulations per each equity level. Again, for the ME-network, only one simulation per equity level will be executed. The comparison will again be conducted by calculating Spearman's rank correlation and the result which will indicate the strength of their monotonic relationship.

### Maximum Entropy

The ME-method aims to maximize the entropy in the network, resulting in a network where all nodes are directly linked to each other with the exception of the 'Other'-bank. This implies that the out-degree centrality is equal for all nodes except the 'Other'-bank. Consequently, this measure is not useful in this context, as calculating the correlation with a constant vector is meaningless. Similarly, the betweenness centrality becomes redundant because the shortest paths between the nodes are, with a few exceptions, the direct link. This results in a betweenness centrality of almost only zeros for all nodes, making it irrelevant for the ME-network and thus excluded from our analysis.

As shown in Figure 6.1, PageRank perfectly correlates with the DebtRank impact when equity exceeds 11% of total exposures. Both eigenvector centrality and Katz centrality exhibit a positive, although slightly weaker, monotonic relationship with DebtRank impact. In contrast, Bonacich Power centrality demonstrates negative correlation with DebtRank impact. Interbank liabilities and impact diffusion also correlate almost perfectly with DebtRank impact, particularly as equity increases. These correlations remain relatively constant across equity levels until equity decreases to 10% and 11% of total exposures. This likely occurs because the financial network becomes increasingly unstable as equity decreases.

All the correlation values for the centrality measures and DebtRank vulnerability remain relatively constant across the equity levels as Figure 6.2 shows. Katz

Centrality, impact susceptibility, and interbank assets exhibit a perfect positive correlation with DebtRank vulnerability. Conversely, closeness centrality shows a perfect negative correlation with DebtRank vulnerability.

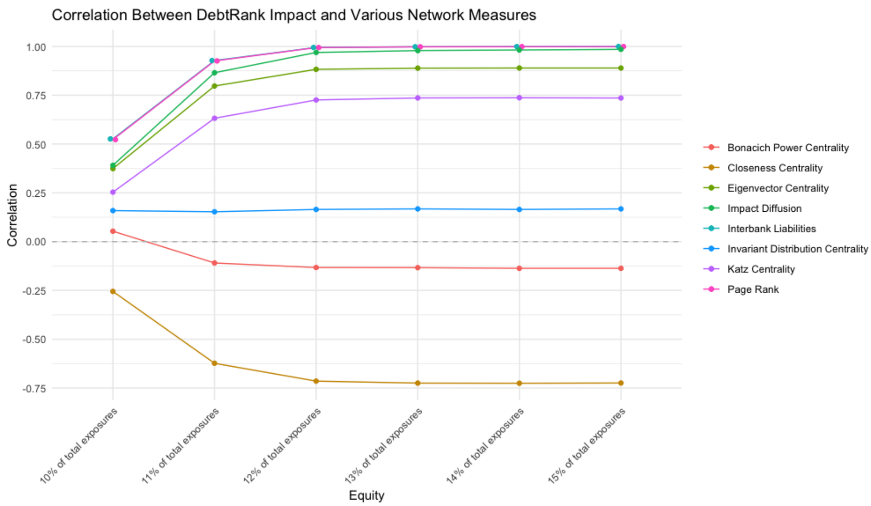


**Figure 6.1** Spearman's correlation between various network measures and DebtRank impact for equity between 10 and 15 percent of total exposures simulated with one instance of the maximum entropy model.

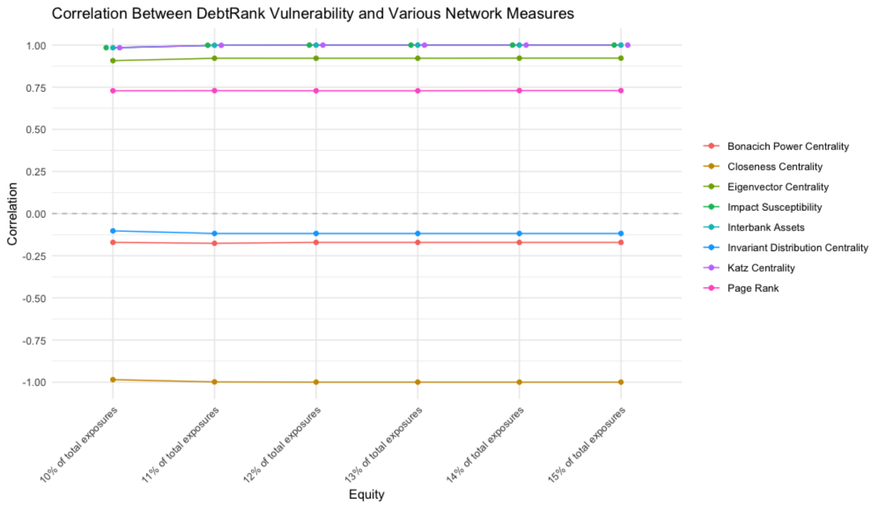


**Figure 6.2** Spearman's correlation between various network measures and DebtRank vulnerability for equity between 10 and 15 percent of total exposures simulated with one instance of the maximum entropy model.

## Minimum Density

Unlike the ME-method, the MD-method results in a network with varying node degrees. Therefore, out-degree centrality and betweenness centrality will be included in the subsequent analysis.

As with the ME-network, PageRank stands out as the centrality measure with the strongest monotonic relationship to DebtRank impact, as shown in Figure 6.3. The strength of the PageRank correlation remains relatively constant for equity levels above 11% but decreases for lower levels. Invariant distribution centrality shows the second strongest correlation with DebtRank impact, though it is weaker than PageRank, with values around 0.75. In-degree centrality also exhibits a strong correlation with DebtRank impact. Although not as pronounced as in the ME-network, interbank liabilities correlate strongly with DebtRank impact, particularly as equity increases. None of the remaining centrality measures display a particularly strong monotonic relationship with DebtRank impact, however, they all share the characteristic of increasing strength as equity levels rise. Impact diffusion also demonstrates a strong monotonic relationship, equalling PageRank at the higher end of the equity spectrum.

Impact susceptibility, followed by interbank assets exhibits the strongest correlation with DebtRank vulnerability. Katz centrality and out-degree centrality shows the strongest relationship with DebtRank vulnerability among the centrality measures, with an average score slightly greater than 0.6. The average correlation strengths are relatively constant across the equity spectrum.

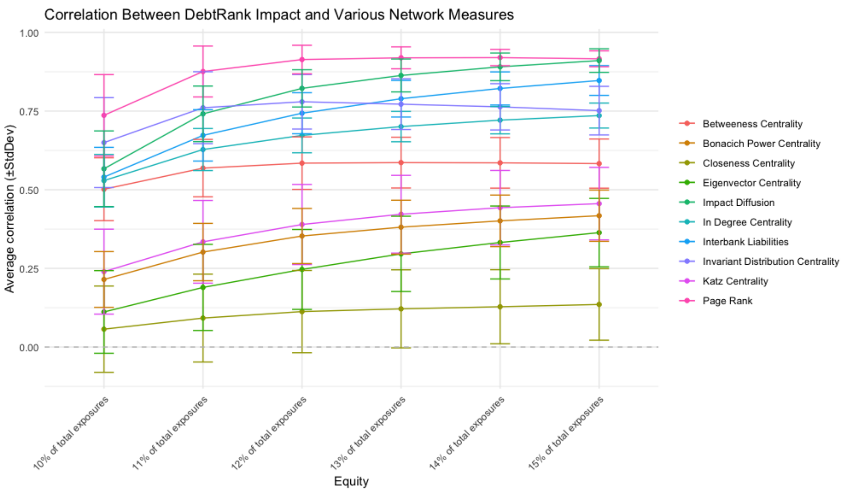


**Figure 6.3** Average Spearman's correlation between various network measures and DebtRank impact for equity between 10 and 15 percent of total exposures simulated with 100 different network instances of the minimum density model.

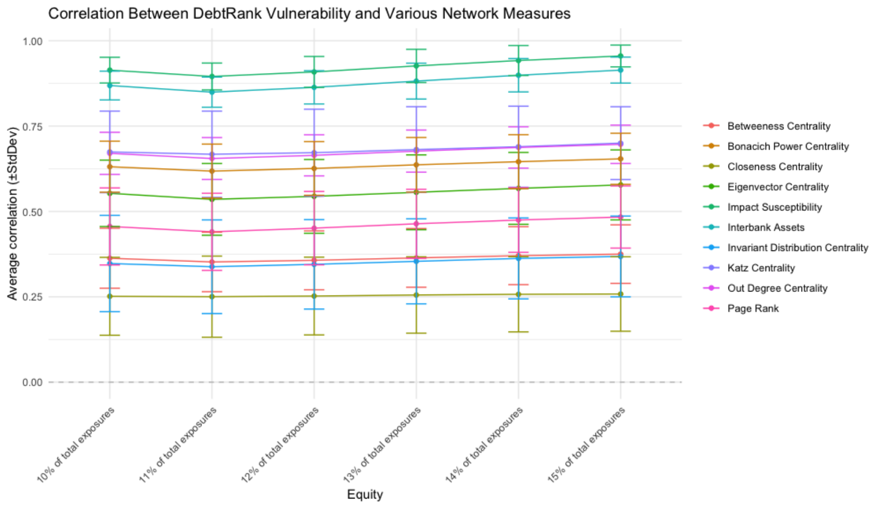


**Figure 6.4** Average Spearman's correlation between various network measures and DebtRank vulnerability for equity between 10 and 15 of total exposures percent simulated with 100 different network instances of the minimum density model.

## Preferential Attachment

In the PA-networks, PageRank displays the most consistent and robust relationship with DebtRank impact, as illustrated in Figure 6.5. When equity exceed 11% of total exposures, the correlation between DebtRank impact and PageRank is nearly 1, with small standard deviations. However, as equity drops below 11%, the correlation decreases and standard deviation increases. Among other centrality measures, Katz and eigenvector centrality follow PageRank in showing a relatively strong monotonic relationship. Impact diffusion and interbank liabilities also demonstrates a strong correlation with DebtRank impact, maintaining values near 1 for equity above 11%, but this correlation significantly weakens when equity is lower.

When examining DebtRank vulnerability, impact susceptibility and interbank assets exhibits perfect postive correlation. Katz centrality shows the highest correlation among the centrality measures, but it remains just below 1. Across various equity levels, all the centrality measures, as well as impact susceptibility, exhibit relatively stable values. The variances for all these measures and across different equity levels are relatively small.

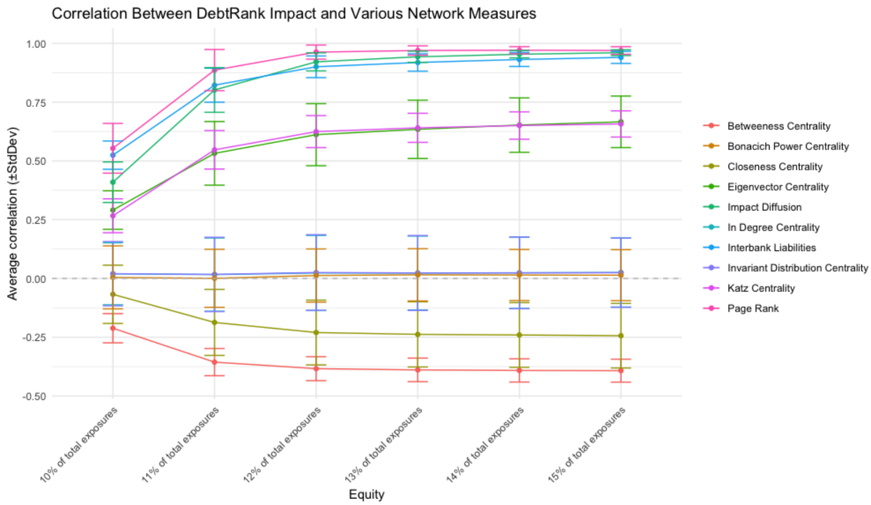


**Figure 6.5** Average Spearman's correlation between various network measures and DebtRank impact for equity between 10 and 15 percent of total exposures simulated with 100 different network instances of the preferential attachment model.

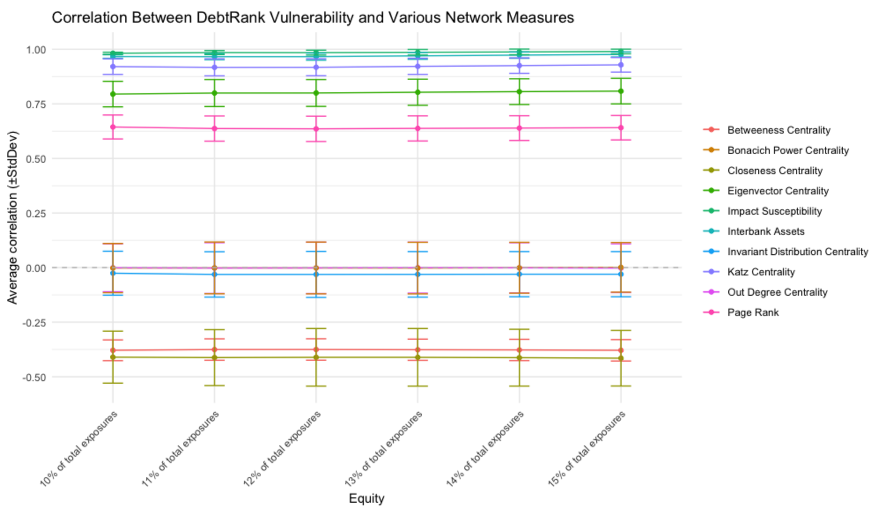


**Figure 6.6** Average Spearman's correlation between various network measures and DebtRank vulnerability for equity between 10 and 15 percent of total exposures simulated with 100 different network instances of the preferential attachment model.

## Small world

Similar to the previously considered networks, PageRank exhibits the strongest monotonic relationship with DebtRank impact for the SW-network, as shown in Figure 6.7. The correlation between DebtRank impact and PageRank is close to 1 with small standard deviations for equity levels greater than 11% of total exposures. As in the previous cases, PageRank's correlation decreases as equity decreases below 11% of total exposures. Katz centrality, followed by eigenvector centrality, shows the strongest monotonic relationship after PageRank among the other centrality measures. Interbank liabilities and impact diffusion also demonstrate strong monotonic relationships with DebtRank impact, with values close to 1 for equity levels above 11% of total exposures, but these correlations decrease dramatically below that level.

Impact susceptibility, interbank assets and Katz centrality all show a positive perfect correlation with DebtRank vulnerability. Eigenvector followed by PageRank show the second and third strongest correlation with DebtRank vulnerability, respectively. All of the considered network measures and their variances as well as impact susceptibility remain relatively constant for levels of equity.

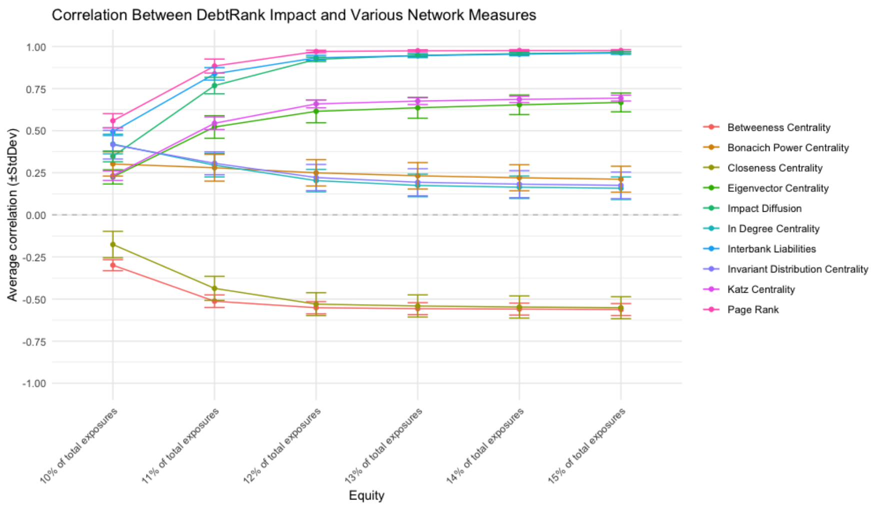


**Figure 6.7** Average Spearman's correlation between various network measures and DebtRank impact for equity between 10 and 15 percent of total exposures simulated with 100 different network instances of the small world model.

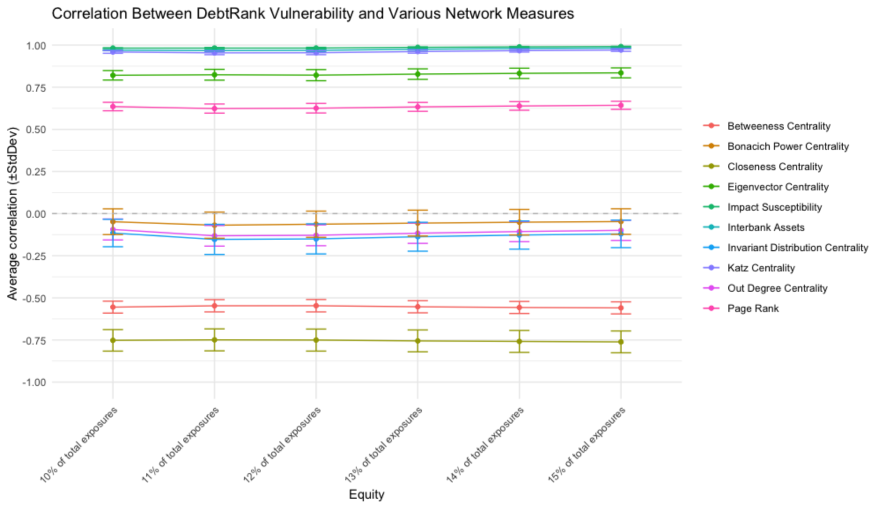


**Figure 6.8** Average Spearman's correlation between various network measures and DebtRank vulnerability for equity between 10 and 15 percent of total exposures simulated with 100 different network instances of the small world model.

## Random Link Equal Probability

As with all the other network reconstruction methods, PageRank shows the strongest monotonic relationship with DebtRank impact for the RL-network with values very close to 1. Among the other centrality measures, eigenvector followed by Katz centrality show the strongest correlation, with correlation values around 0.75. Impact diffusion and interbank liabilities correlate strongly with DebtRank impact with values close to 1. For all of the previously mentioned measures the strength of the correlation decreases as the equity decreases, particularly for 10 and 11 % of total exposures. None of the other centrality measures correlate particularly strongly with DebtRank impact.

Impact susceptibility, Katz centrality and interbank assets exhibit a perfect positive correlation with DebtRank vulnerability. Eigenvector centrality followed by PageRank exhibit the second and third strongest monotonic relationship with DebtRank vulnerability, respectively. Closeness correlates negatively with DebtRank vulnerability. None of the other centrality measures correlates particularly strongly with Debtrank vulnerability.

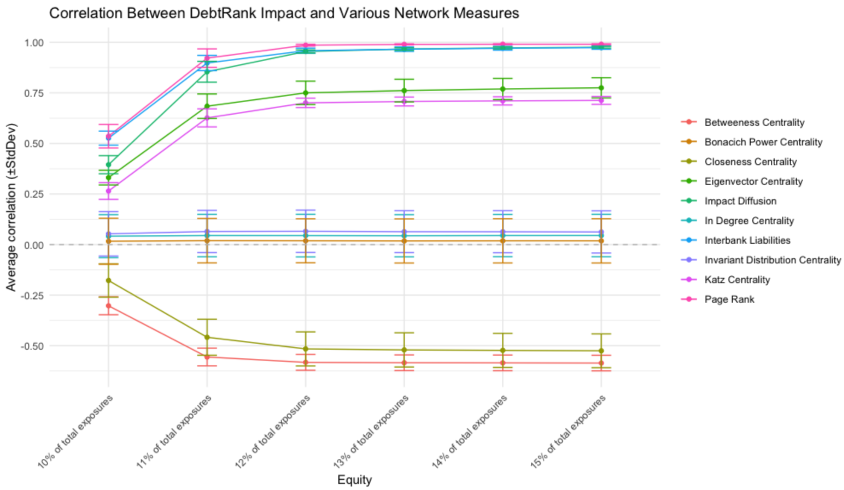


**Figure 6.9** Average Spearman's correlation between various network measures and DebtRank impact for equity between 10 and 15 percent of total exposures simulated with 100 different network instances of the random link equal probability model.

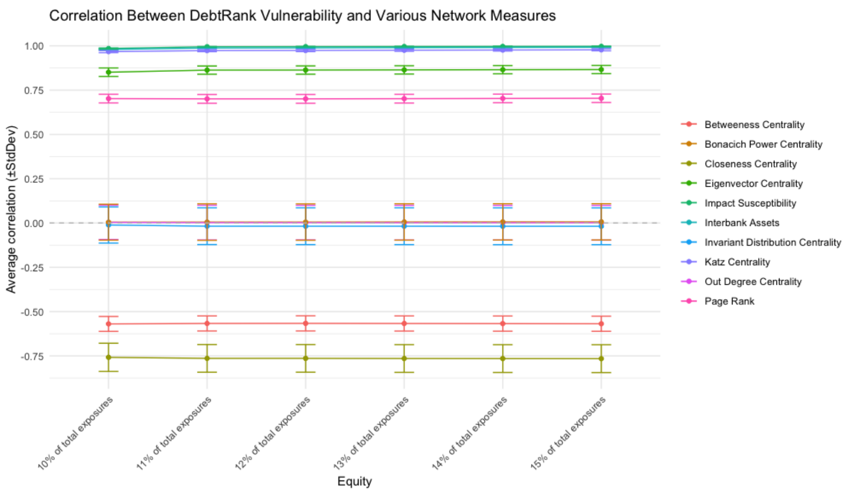


**Figure 6.10** Average Spearman's correlation between various network measures and DebtRank vulnerability for equity between 10 and 15 percent of total exposures simulated with 100 different network instances of the random link equal probability model.

## 6.2 Comparison between Reconstruction Methods

In this analysis, we aim to evaluate how different network reconstruction models perform regarding the amount of additional loss and the number of additional defaults. We believe it is valuable to focus on the middle range of equity because high buffers result in minimal system changes, while low buffers cause excessive system disruptions. To better set the results in perspective, density has been calculated for a graph by each method. The table is seen below.

| Method | Density |
|---|---|
| Minimum Density (MD) | 0.0259 |
| Maximum Entropy (ME) | 0.9742 |
| Preferential Attachment (PA) | 0.4432 |
| Small-World (SW) | 0.4924 |
| Random Link Equal Probability (RL) | 0.3896 |

**Table 6.1** Comparison of densities for different methods

### Losses and Defaults

**Additional loss** When the banks' equity is low, the MD-method creates a significantly more robust system compared to the others. Losses are better absorbed and do not propagate as efficiently. The other four methods, being denser, perform similarly to each other. At medium levels, the SW-network performs the worst, while the other networks, including the MD-network, perform similarly. At high equity levels, all models perform almost identically. Focusing on the middle range of the spectrum indicates that the MD-network is slightly superior, while the SW-network is clearly inferior. The RL-network, the ME-network and the PA-network give additional losses between the previously mentioned networks. Overall, the differences between the models are smaller than expected. See Figure 6.11 for reference.

**Additional defaults** When the banks' equity is 10%, the MD-method again creates a significantly more resilient network, resulting in far fewer defaults compared to the other methods. The ME-network performs the worst in this scenario. At equity levels of 12% and above, the MD-network is actually slightly worse than the others. As the equity increases towards 15%, the number of defaults decreases and approaches zero for all models. See Figure 6.12 for reference.

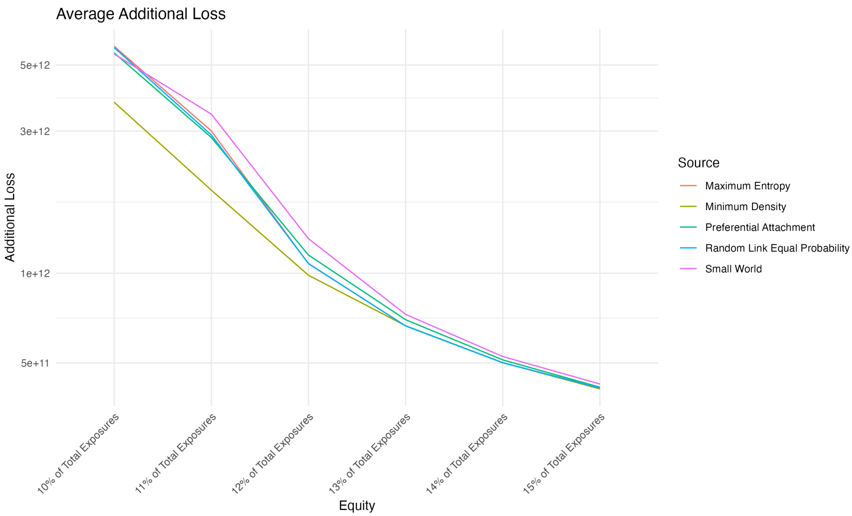


**Figure 6.11** Additional loss for each network reconstruction model over the equity range 10% to 15%.

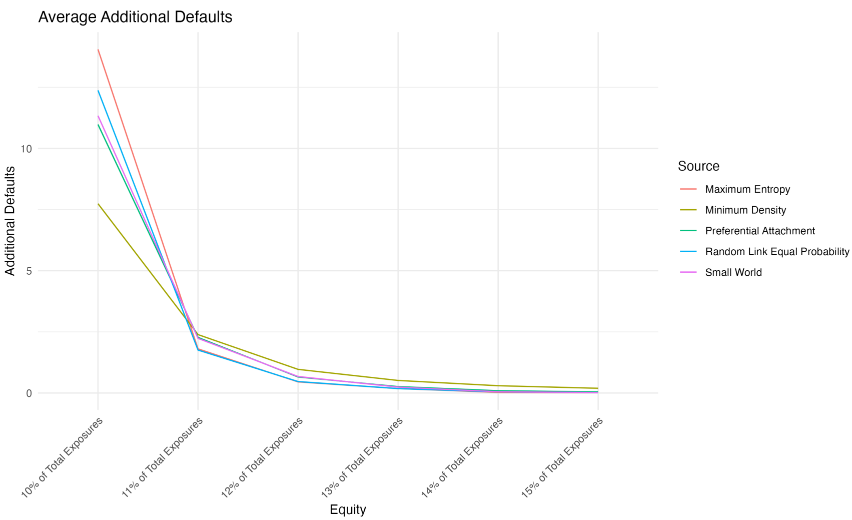


**Figure 6.12** Additional defaults for each network reconstruction model over the equity range 10% to 15%.

### Network Fluidity

We have calculated the impact fluidity of each network. The theory suggests that higher impact fluidity should increase losses, as banks are more vulnerable in highly fluid networks. The impact fluidity for each scenario is shown below. However, the results do not align with expectations. Although the MD-networks perform the best for low buffers and show the lowest impact fluidity for each buffer as anticipated, predicting resilience based on this metric is not possible. This difficulty is illustrated clearly when comparing their values. For example, at 10% equity, the MD-networks have the same impact fluidity as the SW-networks at 13% equity, despite significant differences in additional losses in both scenarios.

| Metric | MD | ME | PA | SW | RL |
|---|---|---|---|---|---|
| Equity 10% | $1.83 \times 10^{11}$ | $2.12 \times 10^{11}$ | $2.06 \times 10^{11}$ | $2.11 \times 10^{11}$ | $2.11 \times 10^{11}$ |
| Equity 11% | $1.79 \times 10^{11}$ | $2.00 \times 10^{11}$ | $1.95 \times 10^{11}$ | $1.99 \times 10^{11}$ | $1.99 \times 10^{11}$ |
| Equity 12% | $1.75 \times 10^{11}$ | $1.91 \times 10^{11}$ | $1.87 \times 10^{11}$ | $1.90 \times 10^{11}$ | $1.90 \times 10^{11}$ |
| Equity 13% | $1.72 \times 10^{11}$ | $1.83 \times 10^{11}$ | $1.80 \times 10^{11}$ | $1.83 \times 10^{11}$ | $1.83 \times 10^{11}$ |
| Equity 14% | $1.69 \times 10^{11}$ | $1.77 \times 10^{11}$ | $1.74 \times 10^{11}$ | $1.77 \times 10^{11}$ | $1.77 \times 10^{11}$ |
| Equity 15% | $1.66 \times 10^{11}$ | $1.72 \times 10^{11}$ | $1.70 \times 10^{11}$ | $1.72 \times 10^{11}$ | $1.72 \times 10^{11}$ |

**Table 6.2** Network fluidity for each scenario

## 6.3 Most Central Banks

In this section, we identify the most central banks by determining which cause the largest additional losses in the system upon default. We will rank the top 10 banks for each scenario. Each scenario consists of a specific reconstruction model and a specified equity level. Finally, we will merge all tables into one final rank.

### Maximum Entropy

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | ICBC | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan |
| 2 | JP Morgan | ICBC | ICBC | ICBC | ICBC | ICBC |
| 3 | China Construction | China Construction | China Construction | China Construction | China Construction | Bank of China |
| 4 | Agricultural Bank | Agricultural Bank | Bank of China | Bank of China | Bank of China | China Construction |
| 5 | MUFG | Bank of China | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong |
| 6 | HSBC | MUFG | MUFG | MUFG | MUFG | MUFG |
| 7 | Citigroup | HSBC | HSBC | HSBC | HSBC | HSBC |
| 8 | Shanghai Pudong | Shanghai Pudong | Agricultural Bank | Citigroup | Société Générale | Société Générale |
| 9 | BNP Paribas | Citigroup | Citigroup | Agricultural Bank | Citigroup | BNY Mellon |
| 10 | Bank of China | BNP Paribas | Société Générale | Société Générale | Agricultural Bank | Citigroup |

**Table 6.3** Average additional losses in decreasing order by equity for the ME-method.

## Minimum Density

| Rank | 10% of Total exposures | 11 % of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan |
| 2 | ICBC | ICBC | ICBC | ICBC | ICBC | ICBC |
| 3 | China Construction | China Construction | China Construction | China Construction | China Construction | China Construction |
| 4 | Agricultural Bank | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong |
| 5 | Shanghai Pudong | Citigroup | MUFG | MUFG | MUFG | MUFG |
| 6 | HSBC | Agricultural Bank | Citigroup | Bank of China | Bank of China | Bank of China |
| 7 | Citigroup | MUFG | Agricultural Bank | Société Générale | Société Générale | Société Générale |
| 8 | MUFG | HSBC | HSBC | Citigroup | Agricultural Bank | BNY Mellon |
| 9 | Société Générale | Bank of China | Bank of China | Agricultural Bank | BNY Mellon | Agricultural Bank |
| 10 | Bank of China | Société Générale | Société Générale | HSBC | Citigroup | Citigroup |

**Table 6.4** Average additional loss in decreasing order by equity for the MD-method.

## Preferential Attachment

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | ICBC | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan |
| 2 | JP Morgan | ICBC | ICBC | ICBC | ICBC | ICBC |
| 3 | China Construction | China Construction | China Construction | China Construction | China Construction | Bank of China |
| 4 | Agricultural Bank | Shanghai Pudong | Bank of China | Bank of China | Bank of China | China Construction |
| 5 | HSBC | Agricultural Bank | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong |
| 6 | MUFG | Bank of China | Agricultural Bank | Agricultural Bank | Citigroup | Citigroup |
| 7 | Citigroup | Citigroup | Citigroup | Citigroup | Agricultural Bank | Société Générale |
| 8 | Shanghai Pudong | MUFG | MUFG | Société Générale | Société Générale | Agricultural Bank |
| 9 | BNP Paribas | HSBC | HSBC | MUFG | MUFG | BNY Mellon |
| 10 | Bank of China | BNP Paribas | Société Générale | HSBC | BNY Mellon | MUFG |

**Table 6.5** Average additional losses in decreasing order by equity for the PA-method.

## Small World

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | ICBC | ICBC | ICBC | ICBC | ICBC | ICBC |
| 2 | China Construction | China Construction | China Construction | HSBC | HSBC | JP Morgan |
| 3 | Agricultural Bank | Bank of China | JP Morgan | JP Morgan | JP Morgan | HSBC |
| 4 | MUFG | Agricultural Bank | HSBC | Société Générale | Société Générale | Société Générale |
| 5 | Bank of China | JP Morgan | Bank of China | BNP Paribas | BNP Paribas | Bank of China |
| 6 | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong | China Construction | Bank of China | BNP Paribas |
| 7 | JP Morgan | MUFG | Agricultural Bank | Bank of China | China Construction | China Construction |
| 8 | Mizuho | HSBC | BNP Paribas | Shanghai Pudong | UBS | UBS |
| 9 | China Minsheng | BNP Paribas | MUFG | UBS | Shanghai Pudong | Shanghai Pudong |
| 10 | HSBC | Industrial Bank | Société Générale | MUFG | Unicredit | Unicredit |

**Table 6.6** Average additional losses in decreasing order by equity for the SW-method.

## Random Link Equal Probability

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan |
| 2 | ICBC | ICBC | ICBC | ICBC | ICBC | ICBC |
| 3 | China Construction | China Construction | China Construction | China Construction | China Construction | China Construction |
| 4 | Agricultural Bank | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong | Shanghai Pudong |
| 5 | Shanghai Pudong | Citigroup | MUFG | MUFG | MUFG | MUFG |
| 6 | HSBC | Agricultural Bank | Citigroup | Bank of China | Bank of China | Bank of China |
| 7 | Citigroup | MUFG | Agricultural Bank | Société Générale | Société Générale | Société Générale |
| 8 | MUFG | HSBC | HSBC | Citigroup | Agricultural Bank | BNY Mellon |
| 9 | Société Générale | Bank of China | Bank of China | Agricultural Bank | BNY Mellon | Agricultural Bank |
| 10 | Bank of China | Société Générale | Société Générale | HSBC | Citigroup | Citigroup |

**Table 6.7** Average additional losses in decreasing order by equity for the RL-method.

## Final Rank and Comparison with the Basel Committee

By comparing 6.3 - 6.7, we observe small differences in the top-ranking banks. JP Morgan consistently ranks as the most important bank, and the five Chinese banks — ICBC, China Construction, Shanghai Pudong, and Bank of China — frequently follow. An interesting result is that in the SW-networks, Chinese banks are more central compared to other networks. Table 6.6 shows that this method is the only one where JP Morgan is consistently less important, with ICBC being the most important. This is likely because the SW-method enhances the centrality of banks in key geographic areas, such as Asia, by requiring banks from the same continent to have links. Based on these results, we constructed a final ranking, weighted to emphasize the middle range of equity. The final ranking is shown in Table 6.8.

**Table 6.8** Our final rank of the most important banks.

| Bank | Rank |
|---|---|
| JP Morgan | 1 |
| ICBC | 2 |
| China Construction | 3 |
| Shanghai Pudong | 4 |
| Bank of China | 5 |
| HSBC | 6 |
| MUFG | 7 |
| China Agriculture | 8 |
| Citigroup | 9 |
| Societe Generale | 10 |

Since 2014, the Basel Committee has conducted annual assessments of the interbank market, aiming to quantify and rank the systemic importance of banks. Each bank is evaluated across five categorizes: size, interconnectedness, substitutability, complexity and cross-jurisdictional activity. The overall score is derived from an average of these categories. The methodology used by the Basel Committee is both more complex and more sophisticated than that of our thesis [Banking Supervision, 2018]. Nevertheless, our primary goal is the same, to determine each bank's importance. Therefore we wanted to compare our findings with their conclusions. The Basel Committee's results are presented in Tables 6.9-6.14 and are taken from Bank of International Settlements. [Bank for International Settlements, 2021]. We have highlighted in yellow the name of those banks that are included in our final ranking seen in Table 6.8. When comparing our results with the Basel Commitee's we notice similarities. Our top 10 list includes many of the same banks which are included in their list. The similarities in the ranking are especially prominent when comparing Basel's list sorted by the categories size and interconnectedness. Here, 8 out of 10 banks are the same as in our list. In the other categories, only between 3 and 5 banks are the same as our list. Considerably less than the categories size and interconnectedness. The similarities and differences are shown in Table 6.9 - 6.14.

**Table 6.9** 10 highest overall scores in Basel's annual assessment. 6 out of 10 banks are the same as in our final list.

| Bank | Overall score |
|---|---|
| JP Morgan | 441 |
| Citi Group | 377 |
| HSBC | 369 |
| BNP Paribas | 333 |
| ICBC | 303 |
| MUFG | 292 |
| Bank of America | 291 |
| Bank of China | 287 |
| Deutsche Bank | 262 |
| Barclays | 250 |

**Table 6.10** 10 highest size scores in Basel's annual assessment. 8 out of 10 banks are the same as in our final list.

| Bank | Size score |
|---|---|
| ICBC | 517 |
| Agricultural Bank | 433 |
| China Construction | 433 |
| JP Morgan | 396 |
| Bank of China | 379 |
| MUFG | 356 |
| Bank of America | 315 |
| HSBC | 282 |
| Citigroup | 274 |
| BNP Paribas | 266 |

**Table 6.11** 10 highest interconnectedness scores in Basel's annual assessment. 8 out of 10 banks are the same as in our final list.

| Bank | Interconn. score |
|---|---|
| JP Morgan | 441 |
| ICBC | 389 |
| Bank of China | 341 |
| Agricultural Bank | 277 |
| Citigroup | 274 |
| MUFG | 253 |
| HSBC | 250 |
| SMFG | 246 |
| Bank of America | 239 |
| China Construction | 232 |

**Table 6.12** 10 highest complexity scores in Basel's annual assessment. Only 3 banks are the same as in our final list.

| Bank | Complexity score |
|---|---|
| JP Morgan | 498 |
| Goldman Sachs | 419 |
| Morgan Stanley | 414 |
| Barclays | 391 |
| Deutsche Bank | 390 |
| Citigroup | 360 |
| Mizuho | 354 |
| Bank of America | 308 |
| MUFG | 281 |
| BNP Paribas | 275 |

**Table 6.13** 10 highest cross-jurisdictional scores in Basel's annual assessment. 5 out of 10 of the banks are the same as in our final list.

| Bank | C. Jurist. score |
|---|---|
| HSBC | 704 |
| BNP Paribas | 623 |
| Citigroup | 475 |
| Santander | 469 |
| ING Bank | 382 |
| JP Morgan | 371 |
| MUFG | 366 |
| Bank of China | 297 |
| Barclays | 281 |
| Deutsche Bank | 280 |

**Table 6.14** 10 highest substitutability scores in Basel's annual assessment. 5 out of 10 banks are the same as in our final list.

| Bank | Substi. score |
|---|---|
| Citigroup | 500 |
| State Street | 500 |
| JP Morgan | 500 |
| BNY Mellon | 500 |
| Bank of America | 432 |
| Deutsche Bank | 354 |
| HSBC | 340 |
| BNP Paribas | 281 |
| China Construction | 248 |
| Bank of China | 246 |

# 7

# Conclusions

## 7.1 Our work

### A Brief Recapitulation

This thesis uses aggregate data from the Bank of International Settlements to create financial networks. These networks are created in five different ways. They all represent different scenarios of how the interbank market could be. The goal is to analyze how defaults propagate to assess the importance of banks and to examine how the network's structure affects the system's vulnerability. We apply network theory, communicability theory, and the DebtRank algorithm to identify which banks are the most vulnerable and which propagate the largest losses to the system. We also investigate how DebtRank correlates with centrality and communicability measures.

### DebtRank Impact

Among the considered centrality measures, PageRank most accurately predicts the DebtRank impact across all network models. As equity increases PageRank becomes even more effective and the correlation variance is reduced. Following PageRank, eigenvector and Katz centrality also show strong correlation which can be explained by their theoretical similarities with PageRank. Each of these metrics are rooted in the concept of iterative computation, where the importance of a node is determined by the importance of its neighbours, reflecting the principle that connections to high-scoring nodes contribute more to a node's score. Therefore, all three metrics capture global network centrality rather than just local centrality. This means they evaluate the influence of a node within the context of the entire network, considering the broader connectivity patterns and the cumulative importance of all nodes. Unlike local centrality measures, which focus on a node's immediate neighbourhood, these global measures provide a more comprehensive view of a node's significance in the overall structure of the network. However, one could argue that Bonacich centrality also share these similarities and should therefore correlate strongly with DebtRank impact as well, which is not the case. A possible explanation to this could be the value of the decay rate $\beta$ which is used when calculating Bonacich centrality. The parameter regulates the influence of distant nodes and could be the reason why the centrality differs a lot form PageRank, eigenvector and Katz centrality. None of the other centrality measures show any particularly convincing correlations with DebtRank impact. See Figures 6.1, 6.3, 6.5, 6.7, and 6.9 for reference.

The ranking of impact diffusion closely predicts the ranking of DebtRank impact, particularly in the higher range of the equity spectrum. The correlation between impact diffusion and DebtRank impact is strong across all five network models, although it seems as if it is increasingly strong in high density networks. This is clearly illustrated when comparing the ME-network with the MD-network in Figures 6.2 and 6.3. The correlation decreases for lower equity which can be explained by an increasingly sensitive and unstable system. This argument is strengthened by the increase of variance of the correlation for the lower equity. In conclusion, impact diffusion accurately predicts the ranking of the DebtRank impact and thus also the systemically important banks. We believe this high correlation is interesting as it shows that the DebtRank simulation is largely a matter of communicability between banks.

Interbank liabilities correlate strongly with DebtRank impact across all network models. The naive predictor, interbank liabilities, accurately predicts the systemically important banks in our setting. This shows that banks with higher interbank liabilities are more likely to propagate financial stress throughout the network. By focusing on interbank liabilities, we can identify potential points of failure within the financial system. Consequently, monitoring and managing these liabilities is crucial for maintaining financial stability and mitigating systemic risk.

### DebtRank Vulnerability

A few of the considered centrality measures accurately predicts DebtRank vulnerability. Among the considered centrality measures, Katz centrality correlates the strongest with DebtRank vulnerability. This correlation is very strong and we therefore believe Katz centrality can be useful to evaluate which bank's are the most vulnerable. Eigenvector centrality, followed by PageRank, also strongly correlates with DebtRank vulnerability. However, these correlations are consistently weaker than Katz centrality, making them less effective than Katz. The average correlation values remain relatively constant for different equity levels although the variances are relatively large in some cases. No large differences of average correlation can be seen between the networks either. See figures 6.2, 6.4, 6.6, 6.8, and 6.10 for reference.

The ranking of impact susceptibility does accurately predict the ranking of DebtRank vulnerability. The correlation scores are consistent across the different equity levels and ranges between 0.95 and 1 with small differences between the networks. The variances of the correlation scores are relatively low which suggest a stable mean. Impact susceptibility aims to measure how susceptible a bank is to financial contagion and does capture the dynamics as anticipated in our setting.

Interbank assets correlate strongly with DebtRank vulnerability. This strong correlation shows that banks with higher interbank assets are more vulnerable to financial contagion. By accurately predicting a bank's susceptibility to financial distress, interbank assets serve as a crucial metric for assessing systemic risk. Monitoring these assets allows for better identification of banks that may act as contagion points in a crisis. Consequently, managing interbank assets is essential for enhancing the resilience of the financial network and preventing widespread financial instability.

## Evaluation of Network Reconstruction Methods

Our main conclusion is that more defaults occur in dense networks compared to low-density networks when equity is low. This is highlighted by the fact that the MD-networks outperform all other networks at the 10% level, while the ME-networks experience the highest number of defaults. As equity increases, these differences diminish. At high equity levels, the situation reverses. Although the differences are small, the MD-networks actually perform the worst in terms of the number of defaults. We believe this is because, in the MD-network, the stress is concentrated on a few banks. When buffers are low, this concentration helps contain the overall network stress, making the MD-network perform better. However, when buffers are high, the same concentration causes these few banks to default more easily, leading to worse performance in terms of defaults.

Regarding additional loss, the MD-networks perform the best for low equity, as expected, while the other networks perform similarly. In the middle range of equity, the SW-network is inferior to the others. At higher buffer levels, again, all differences diminish. Interestingly, the network aiming to capture geographic clustering appears to be less robust. We believe this is due to the significant number of important banks from China, which are highly interconnected in this scenario. As seen in Table 6.1 the three methods - PA, SW, and RL - create networks with similar density. Despite this similarity, the SW-networks perform slightly worse in terms of additional loss. This suggests that density is not the only influencing factor. As previously mentioned, we believe the poorer performance of the SW-network is due to the increased centrality of the large Chinese banks which in turn causes larger losses in the system.

It is important to realize, however, that the analysis above focuses on systemic risk. The MD-method results in stronger networks for low equity but does not necessarily provide more support for each individual bank. In fact, the less dense the network, the greater the impact on a few key nodes. Therefore, even though the overall damage to the network is lower, the damage to a few banks is likely to be more severe.

An important takeaway from our analysis is the practical implications of the relatively small differences observed between networks. The difference in loss and number of defaults across different equity levels are bigger than those between the networks, even when considering extreme scenarios like the MD-network. In the pursuit of a resilient financial network, we believe that while network structure is important, it is secondary to the size of a bank's equity. A purely fictional network with greater variance in banks' assets, liabilities, and equity might better highlight the importance of network structure than our models do.

Finally, we conclude that network fluidity is ineffective at predicting future losses in a network under financial distress. The measure does capture communicability on a global level but fails to incorporate the impact of changes in equity in a sufficient way.

### Evaluation of Banks

JP Morgan is the most important bank, followed by five Chinese banks. Our results show clear similarities with the Basel Committee's, especially in the categories size and interconnectedness. Our results differ more when comparing with the other categories; complexity, substitutability and cross-jurisdictional activity. Our model does not include these variables and therefore it is expected that our ranking differs more in these aspects. When comparing it is evident that our model favors Chinese banks more than the Basel Committee's assessment. We believe this discrepancy arises because our model captures the size and high interconnectivity of Chinese banks. However, it does not necessarily incorporate other important aspects, such as their complexity and cross-jurisdictional activity, which the Basel Committee considers. One possible way of improving the model, to capture the lower cross-jurisdictional activity from the Chinese banks, is to create a network where the probability of links between Chinese banks and non-Chinese banks are lower.

## 7.2 Future Work

There are many possible ways one could further analyze the interbank market. There are an infinite amount of parameters that can be varied which makes the field broad. Future work should focus on a few areas: more accurate methods for network reconstruction, more advanced financial models, more nuanced ways of initiating distress in the model and deeper integration between finance and network theory.

We believe network reconstruction is an area which deserves its own focus. As the lack of data serves as a challenge, making realistic networks is the first important step in being able to do any further analysis. In our network reconstruction we chose 5 different methods and only included interbank assets and interbank liabilities. In a future paper one could include more data thus making the networks more complex and closer to reality. In the paper [Anand et al., 2017] the authors focus on network reconstruction. They use maximum entropy, minimum density and 5 other methods. Their goal is to asses how well each of these methods are for reconstruction. This paper is a good example of how one could focus on the important subject of filling the gap between partial data and robust models. One could back test how well the reconstructed network resemble reality by simulating a real event on the network and draw comparisons to reality.

Another angle in future work would be to focus on simpler, non-realistic network models and see how metrics such as DebtRank turn out. By constructing networks from theory instead of trying to resemble reality, one could find valuable insights in how defaults propagate. If the models are simpler it might be easier to integrate more concepts from network theory with finance. These insights can then help to understand complex scenarios that closer mimics the real world.

In our thesis all initial shocks are set to 1 so that the first bank defaults. It would be interesting to see how smaller shocks are absorbed by the system, this could give a more nuanced and maybe realistic view on the systems vulnerabilities. In reality it is unlikely that there is only one isolated shock in a system. It would be interesting to initiate numerous shocks in different parts of the system and see what happens. Future works could focus on varying the magnitude of the shocks and how they are initiated to give a more complex understanding.

The financial model in this thesis is rather simple. Equity is defined as the difference between assets and liabilities. In future work one could use more complex models. These could include more financial instruments such as bonds, options, commodities, futures and forwards. It could include proactive risk management in the form of hedging. This opens up for new interesting questions: What happens in the interbank market if the oil price goes down?

We also believe back testing can be a powerful tool in this area of science. Simulating an actual historic event on a model can help to validate its quality. If available, this could be done by collecting data on the interbank market before and after the COVID-19-crash in 2020. Let the initial stress in your model match the stress caused on numerous days in March of 2020, including the black Monday II when the Dow Jones index plummeted almost 13 percent. Evaluate how well your model matches how the real interbank market was affected. Try to do necessary adjustments to optimize the model. This could imply increasing a banks exposure to the stock market etc. Similar back testing could be done for the time when Credit Suisse defaulted. The lack of data can in many cases provide a challenge. If one can overcome this, the concept of back testing can be useful.

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# 8

# Appendix

| Rank | 10% of Total exposures | 11 % of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan |
| 2 | ICBC | ICBC | ICBC | ICBC | ICBC | ICBC |
| 3 | China Construction | China Construction | MUFG | MUFG | MUFG | China Construction |
| 4 | MUFG | MUFG | Agricultural Bank | China Construction | Agricultural Bank | Shanghai Pudong |
| 5 | Citigroup | Agricultural Bank | China Construction | Bank of China | China Construction | MUFG |
| 6 | Agricultural Bank | Citigroup | Bank of China | Agricultural Bank | Bank of China | Bank of China |
| 7 | HSBC | Shanghai Pudong | HSBC | Shanghai Pudong | Shanghai Pudong | Agricultural Bank |
| 8 | Shanghai Pudong | HSBC | Shanghai Pudong | HSBC | Unicredit | Société Générale |
| 9 | Bank of China | Bank of China | Société Générale | Citigroup | Société Générale | Unicredit |
| 10 | BNP Paribas | Société Générale | Citigroup | Société Générale | UBS | HSBC |

**Table 8.1** Average number of additional defaults in decreasing order by equity for the minimum density method.

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | ICBC | JP Morgan | ICBC | UBS | ICBC | JP Morgan |
| 2 | JP Morgan | China Construction | JP Morgan | Agricultural Bank | JP Morgan | ANZ |
| 3 | ANZ | ICBC | Credit Suisse | Bank of China | ANZ | Commonwealth |
| 4 | Commonwealth | ANZ | UBS | China Construction | Commonwealth | National Australia Bank |
| 5 | Westpac | Commonwealth | Agricultural Bank | ICBC | National Australia Bank | Westpac |
| 6 | RBC | National Australia Bank | Bank of China | Industrial Bank | Westpac | CIBC |
| 7 | Agricultural Bank | Westpac | BoComm | Shanghai Pudong | CIBC | Bank of Montreal |
| 8 | Bank of Beijing | Bank of Montreal | China Construction | BNP Paribas | Bank of Montreal | Bank of Nova Scotia |
| 9 | Bank of China | Bank of Nova Scotia | China Minsheng | Société Générale | Bank of Nova Scotia | RBC |
| 10 | BoComm | RBC | CITIC | HSBC | RBC | Toronto Dominion |

**Table 8.2** Average number of additional defaults in decreasing order by equity for the maximum entropy method.

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | ICBC | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan |
| 2 | China Construction | ICBC | ICBC | ICBC | ICBC | ICBC |
| 3 | JP Morgan | China Construction | China Construction | Bank of China | Bank of China | Bank of China |
| 4 | Agricultural Bank | Bank of China | Bank of China | China Construction | China Construction | China Construction |
| 5 | Shanghai Pudong | Shanghai Pudong | Citigroup | Shanghai Pudong | Shanghai Pudong | Citigroup |
| 6 | HSBC | Agricultural Bank | Agricultural Bank | Société Générale | Citigroup | MUFG |
| 7 | MUFG | Citigroup | Shanghai Pudong | MUFG | Société Générale | BNY Mellon |
| 8 | Citigroup | HSBC | MUFG | Citigroup | MUFG | UBS |
| 9 | Bank of China | MUFG | BNP Paribas | HSBC | BNY Mellon | Shanghai Pudong |
| 10 | BNP Paribas | BNP Paribas | Société Générale | Agricultural Bank | HSBC | BNP Paribas |

**Table 8.3** Average number of additional defaults in decreasing order by equity for the preferential attachment method.

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | ICBC | ICBC | HSBC | HSBC | JP Morgan | JP Morgan |
| 2 | China Construction | China Construction | ICBC | ICBC | Citigroup | ICBC |
| 3 | Agricultural Bank | HSBC | Société Générale | Société Générale | ICBC | Société Générale |
| 4 | MUFG | JP Morgan | BNP Paribas | BNP Paribas | HSBC | Citigroup |
| 5 | Shanghai Pudong | BNP Paribas | China Construction | UBS | BNY Mellon | ANZ |
| 6 | Bank of China | Agricultural Bank | Crédit Agricole | China Construction | Société Générale | Commonwealth |
| 7 | Mizuho | Bank of China | Barclays | Citigroup | Bank of China | National Australia Bank |
| 8 | JP Morgan | MUFG | Bank of China | JP Morgan | China Construction | Westpac |
| 9 | HSBC | Shanghai Pudong | Shanghai Pudong | BNY Mellon | BNP Paribas | CIBC |
| 10 | China Minsheng | Santander | UBS | Bank of China | UBS | Bank of Montreal |

**Table 8.4** Average number of additional defaults in decreasing order by equity for the small world method.

| Rank | 10% of Total exposures | 11% of Total exposures | 12% of Total exposures | 13% of Total exposures | 14% of Total exposures | 15% of Total exposures |
|---|---|---|---|---|---|---|
| 1 | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan | JP Morgan |
| 2 | ICBC | ICBC | ICBC | ICBC | ICBC | ICBC |
| 3 | China Construction | China Construction | MUFG | MUFG | MUFG | China Construction |
| 4 | MUFG | MUFG | Agricultural Bank | China Construction | Agricultural Bank | Shanghai Pudong |
| 5 | Citigroup | Agricultural Bank | China Construction | Bank of China | China Construction | MUFG |
| 6 | Agricultural Bank | Citigroup | Bank of China | Agricultural Bank | Bank of China | Bank of China |
| 7 | HSBC | Shanghai Pudong | HSBC | Shanghai Pudong | Shanghai Pudong | Agricultural Bank |
| 8 | Shanghai Pudong | HSBC | Shanghai Pudong | HSBC | Unicredit | Société Générale |
| 9 | Bank of China | Bank of China | Société Générale | Citigroup | Société Générale | Unicredit |
| 10 | BNP Paribas | Société Générale | Citigroup | Société Générale | UBS | HSBC |

**Table 8.5** Average number of additional defaults in decreasing order by equity for the random link equal probability method.

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| *Author(s)*<br>Oscar Clarke Nilsson<br>Ossian Relander | *Supervisor*<br>Emma Tegling, Dept. of Automatic Control, Lund University<br>Giacomo Como, Dept. of Automatic Control, Lund University, Sweden (examiner) |

*Title and subtitle*

Systemic Risk and Default Contagion in Financial Networks: Identifying Systemically Important Banks

*Abstract*

This thesis uses real data from the Bank of International Settlements to create financial networks for the interbank market using five different methods for network reconstruction. The goal is to analyze how defaults propagate to assess the importance of banks and to examine how the network's structure affects the system's vulnerability. By applying network theory, communicability theory, and the DebtRank algorithm, we aim to identify which banks are the most vulnerable and which propagate the largest losses to the system. We also investigate how DebtRank correlates with centrality and communicability measures. Our results will be compared to the Basel Committee's annual assessment of global systemically important banks.

Our findings show small differences between the network reconstruction methods. The most noticeable difference is that the minimum density method produces more resilient networks when equity is low. In contrast, the small-world method results in networks with slightly higher losses, especially when equity is in the middle range. Additionally, our results indicate that JP Morgan is the most systemically important bank in most scenarios, matching the Basel Committee's conclusions. However, we believe our methods overestimate the importance of some of the largest Chinese banks. We also show that PageRank and impact diffusion have the highest correlation with DebtRank impact. Finally, we conclude that Katz centrality and impact susceptibility show a strong correlation with DebtRank vulnerability.

*Keywords*

DebtRank, network dynamics, graph theory, centrality, interbank market, financial networks, shock propagation, financial contagion

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