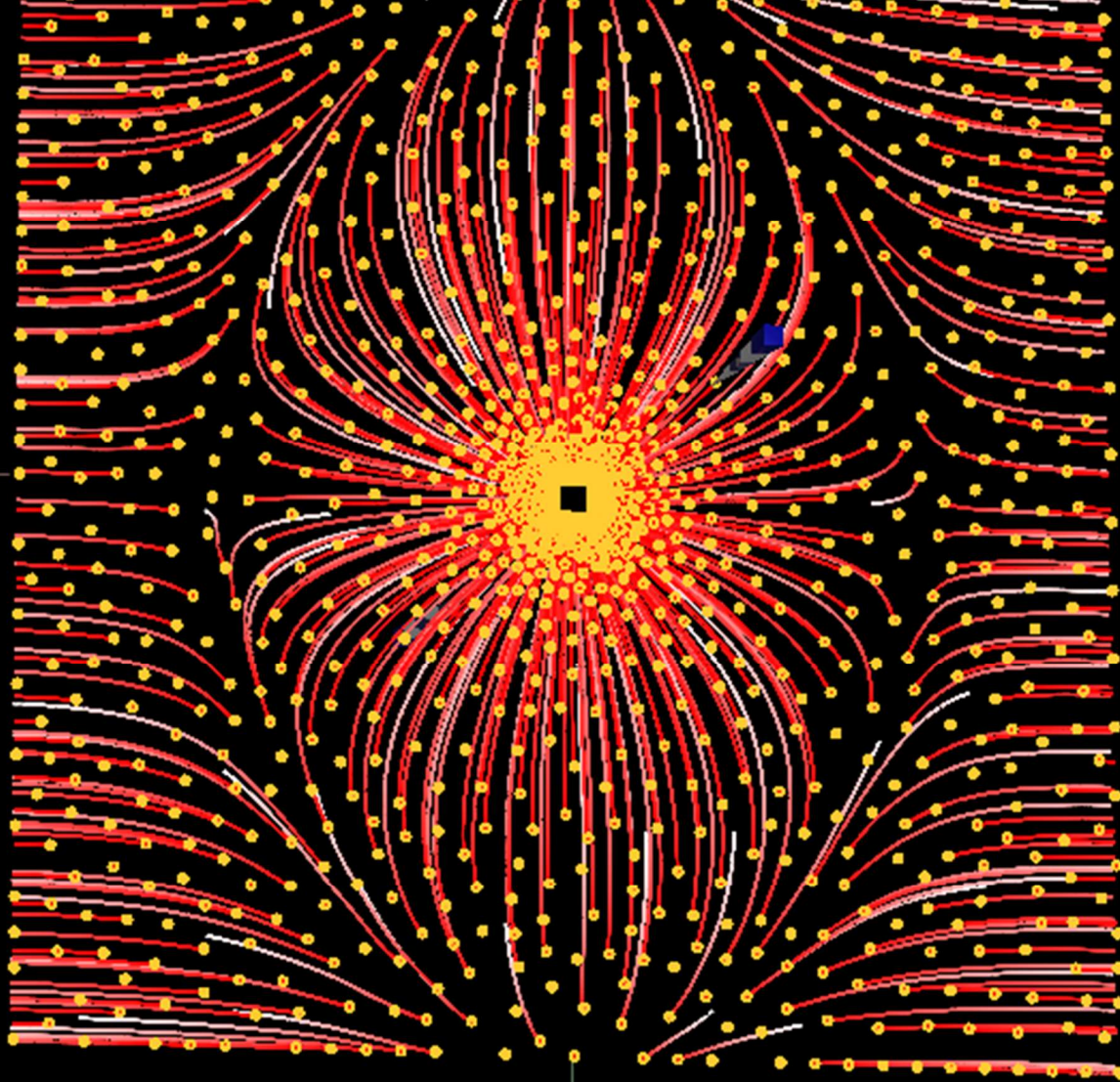




Comparative study between numerical and analytical groundwater modelling

Comparing three-dimensional FEFLOW modelling with analytical one-dimensional modelling

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Comparative study between numerical and analytical groundwater modelling
Comparing three-dimensional FEFLOW modelling with analytical one-dimensional modelling
Jämförelsestudie mellan numerisk och analytisk grundvattenmodellering
Jämförande av tredimensionella FEFLOW modeller med endimensionella analytiska modeller

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Abstract

This study compares analytical and numerical models for groundwater modelling. The focus of comparison is to investigate the change of the analytical results when there is only a partial penetration of the aquifer. Two methods of comparison are used, the first one consisting of calculating a radial distance from a shaft to the point where the drawdown is 0.3 meters with both models. The second method used was by comparing the radius of influence, which was determined through the numerical model as the radius inside which all groundwater recharge was transported to the shaft. The second method gave the best results as the models become more similar when examining the radius of influence through the groundwater recharge flow direction. There was only a slight difference in result, between the numerical and analytical method when there was a partial penetration with this method. The difference was deemed to be inside the margin of insecurity. However, this study has not verified that the analytical model can be used in all cases of partial penetration. It does however give an indication that it might be possible to use it for soils with high hydraulic conductivity, such as sand.

Sammanfattning

Studien jämför analytiska och numeriska metoder för grundvattenmodellering. Fokuset för jämförelsen är att undersöka hur de analytiska resultaten förändras när akviferen endast är delvis penetrerad. Två metoder användes för att jämföra resultaten. Den första metoden utgick ifrån det radiella avståndet till punkten där avsänkningen är 0,3 meter. Den andra metoden utgick ifrån influensradien inom vilken all påfyllning av grundvatten färdas till schaktet. Den andra metoden gav bäst resultat, då modellerna är mer lika varandra när influensradie bestäms efter området där det nybildade grundvattnet flödar mot schaktet. Det var en liten skillnad i resultat mellan den numeriska och analytiska modellen när de jämfördes med denna metod. Skillnaden mellan resultaten bedöms vara inom osäkerhetsspannet. Det bör noteras att studien inte har verifierat att analytiska modeller kan användas vid alla fall av delvis penetration av akviferer. Däremot ger den en indikation på att det skulle kunna vara möjligt att använda analytiska modeller för en delvis penetration i jord med hög hydraulisk konduktivitet, så som sand.

Preface

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1 Introduction

Across the world roughly one and a half percent of all the water is groundwater (U.S. Geological Survey, n.d.). Considering the large amount of groundwater across the planet the understanding of its functions, variations, and transportation is very important. For the construction business the groundwater has a large impact, since it can generate large costs in pumping as well as large problems, such as settlements in the adjacent area and damage to ecosystems (Jakeman et al., 2016). There are different methods to model groundwater. In the consulting business it is preferred to model using one-dimensional analytical models as far as possible, since they are much cheaper due to their simplicity. It can also be more efficient to use an analytical model as they do not require the same amount of data to create a model, due to their generalizing of the sites' properties (El-Rawy et al., 2020). However, in some cases a more advanced three-dimensional numerical model must be used to acquire reliable results. One advantage with the 3D-models, is that more parameters can be altered and varied to take the uncertainties into account and create a statistical model of the probabilities of each scenario (Kahe et al., 2021). However, the problem with these three-dimensional models is that they require substantial amounts of data to be useful and are very time-consuming in creating.

As with all models, a groundwater model is an attempt to represent the real world to understand how it functions and what consequences a change in the system would generate. A numerical model can be much more detailed and accurate than an analytical one, but it is very dependent on the input data. A highly complex and advanced model based on poor input data will only give poor output data (Cashman & Preene, 2001). Moreover, the one of the most important part of the input data, the hydraulic conductivity is quite difficult to determine exactly over large areas. For example, a test to determine the hydraulic conductivity on the Chemin Roul site in France which has an area of 1 150 m² was determined to be between $0.5 \cdot 10^{-5}$ to $9.6 \cdot 10^{-5}$ m/s from testing over a 5 year period of time (Peyraube et al., 2023), despite the long time of testing the highest estimated value was almost 20 times larger than the lowest estimated value. This illustrates how there can be large uncertainties in the hydraulic conductivity.

The analytical models' simplification is mainly made through some assumptions for the aquifer. For the cylindrical analytical models which will be used in this study it is assumed that (Krešić, 2007):

1. There is full penetration from the well and the entire aquifer provides the well with water.
2. There is a constant pump rate.
3. The flow towards the well is horizontal

4. The aquifer is isotropic and homogeneous and has a constant thickness and a horizontal bottom.
5. After a certain time period there is no increase in drawdown and steady-state conditions are reached.

1.1 Why there is a need for modelling

There are four main reason to model groundwater prior to a project according to Gustafson (2009).

1. Buildability

Buildability analysis aims to study if there is a need for eventual reinforcements or other actions to ensure that facilities built on the area are safe and durable.

2. Inner environment

Inner environment studies are done to examine the safeness of the project. For example if there could be problems with the working environment during the construction or if a recently built underground surface can be used safely after construction.

3. Effects on the surroundings

By studying the effects on the surroundings, the aim is to understand how the area around a planned structure, such as a shaft, a tunnel, etc. will be affected. There may be many factors that will affect the surroundings, from the most obvious how the groundwater will be drawn down around the structure to how eventual pollutants, such as chromates in cement, in the structure can spread to the surrounding area.

4. Durability

Durability analysis is simply done to examine how durable the structure will be against the surrounding groundwater.

1.2 Negative consequences of a lowered groundwater surface

One common consequence on the surroundings when the groundwater level is lowered, is subsidence. If there are buildings in the vicinity, they could experience damage from the subsidence caused (Goh et al., 2020). In most cases the subsidence tend to be so small that it can be neglected as no damage is created on the surrounding buildings (Cashman & Preene, 2001). However, for the cases where damage does occur it can vary from purely aesthetic problems to larger structural problems. There are several different causes for this problem. One is a large groundwater flow to the shaft providing enough force to transports soil particles from the surroundings into the shaft. At lower flowrates sand and

silt particles can still be transported with the groundwater, and if not filtered out can create sub-surface erosion. Sub-surface erosion decreases the stability of the surrounding ground. The final cause of subsidence is that a lowering of the groundwater will decrease the pore pressure in the soil and increase the effective stress in the soil particles, causing them to consolidate more (Cashman & Preene, 2001).

Another immediate problem is when groundwater is used as a water source, and lowering the groundwater surface will reduce the amount of available water for the people and industries who rely on groundwater supply. The disruption of the groundwater balance can also lead to inflow of unwanted pollutants to the surrounding extraction wells. Pollutants from industries or landfills can reach the wells, and if the area is close to the ocean saline water can contaminate the aquifer. Hazardous materials used in the construction process itself can also come to contaminate the surrounding aquifer (Cashman & Preene, 2001).

Continuing, the water discharged into the shaft must be properly taken care of. A high flow could for example cause erosion problems, and eventual impurities in the extracted groundwater could contaminate the disposal area (Cashman & Preene, 2001).

All of the above mentioned problems can be avoided by thoroughly studying and modelling the current and planned hydrogeological groundwater conditions in the area.

1.2.1 Example of problems with insufficient groundwater modelling

One example of the problems that may be caused by an insufficient modelling is the construction of the Hallandsås tunnel (Edling et al., 1998). The construction of the tunnel was started in 1993 with a tunnel boring machine. The geology of the ridge was not sufficiently studied and the boring machine used was not suitable for the project. This led to a change in method to a drilling and blasting method. As the project went on the groundwater inflow into the tunnel showed to be extremely high and it was impossible to only drain the allowed 33 litres per second, and a toxic sealant had to be used to attempt to limit the inflow to the tunnel from the surrounding groundwater. A few years after the start of the project, villages surrounding the project suffered severe water shortage in some wells due to the seepage into the tunnel. Moreover, the toxic sealant poisoned some of the water in the area causing the agricultural products from the areas near the project to be banned (Sjölander-Lindqvist, 2004).

It can be interpreted from (Edling et al., 1998) that the problems caused through the construction of the tunnel could probably have been avoided if the pre-studies of the groundwater in the area had been more carefully and accurately performed, including more groundwater modelling.

All the four main reasons for groundwater modelling, previously mentioned in chapter 1.1, were valid as the methods chosen to build the tunnel were poor, the environment in the tunnel was poor due to the large leakages of water, there were large negative effects

on the surroundings and many of the sealants used were not durable enough in the construction.

1.3 Previous studies

From previous work it has been examined that some conditions make the use of an analytical model useless, since the model will differ too much from the real-world groundwater system it attempts to model. Some of these aquifer conditions according to (Powers et al., 2007) are:

- Stratified aquifers, when the aquifer is layered and the conditions in each layer vary from one another the use of an estimated mean value for the hydraulic conductivity will not be able to mirror the groundwater system. In other words, the main assumption that the aquifer system is homogenous and isotropic is far from being fulfilled causing the large difference from the real-world aquifer.
- Anisotropy and/or vertical flow in the aquifer. Both the Dupuit and Thiem method assume isotropic condition and neglect vertical flow.
- Nearby or varying boundary conditions, if there are boundary conditions that affect the aquifer that are either very close to the investigated site or if they are irregular there is a need for numerical models.
- Partial penetration, if the aquifer is only partially penetrated by the shaft/well the flow to the shaft/well will be different from the analytical methods as a vertical flow will occur from the layer below the bottom of the shaft/well (Powers et al., 2007).

One study comparing analytical results with numerical results for an unconfined aquifer was conducted by El-Hames (2020). In that study a long-stretched unconfined aquifer was numerically modelled with the software GMS, and its results were compared to a modified version of the Dupuit equation. The modification made was that it used the hydraulic gradient instead of the hydraulic conductivity to estimate radius of influence. The result from the study showed a very small difference in result between the numerical and analytical model.

Barker & Herbert (1992) adjusted for an additional head loss, caused by partial penetration through adding an empirical equation to their analytical results. Which showed somewhat efficient for when the well penetration surpassed at least 20 % of the aquifer thickness. The formula they used included a logarithmic dependency on the penetration rate.

1.4 Purpose

This study aims to evaluate the feasibility of using analytical models for groundwater modelling in situations where a shaft is not fully penetrating an unconfined aquifer, instead of using numerical models. This will be done through comparing the results from numerical and analytical models in terms of drawdown and radius of influence, using idealised conditions.

In civil engineering and hydrogeology there is a pursuit to increase efficiency by decreasing the required input data and computational time. Analytical models can be made quite quickly and do not require the same amount of input data as the numerical ones do, which means that if an analytical model can provide accurate results it is beneficial to use.

Furthermore, this report examines different methods of comparison between analytical and numerical results for radial flow to a shaft, with a main focus on how a partial penetration of the shaft influences the results.

This study also focuses on the modelling for what in chapter 1.1 is referred to as reason 3 - how projects affect their surroundings, specifically how groundwater levels surrounding the project will be affected.

1.4.1 Delimitations

This study focuses solely on how the results change when a shaft partially penetrates an aquifer compared to full penetration. All other assumptions required for the analytical models will be fulfilled, and their potential effects will not be examined. Additionally, the required pumping rate to maintain steady-state conditions is excluded from the scope of this study.

The study assumes idealised and homogeneous aquifer conditions. Factors such as heterogeneity, anisotropy, or time-dependent changes in recharge or boundary conditions are not considered in this study.

2 Method

2.1 Groundwater recharge

The formation of groundwater is created mainly through hydrological cycle, where precipitation infiltrate through the ground and refills the groundwater aquifers. Parts of the precipitation is transported back to the atmosphere through evaporation and the vegetations transpiration (summarized as evapotranspiration (E)). Groundwater can also be transported in the soil and some groundwater can runoff to bodies of surface water. Although, it should be noted that some of the water directly runoff from the surface immediately after precipitation (Gustafson, 2009). The general budget equation for the hydrological cycle is written as:

$$P - E = R + \frac{dM}{dt}$$

Where P is precipitation

E is evapotranspiration

R is runoff

$\frac{dM}{dt}$ is the change in storage over time.

For the long steady-state conditions the change in the storage can be neglected, giving the equation as

$$\bar{P} - \bar{E} = \bar{R}$$

(Gustafson, 2009)

2.2 Analytical modelling

Analytical models are based on Darcy's law (named after civil engineer Henry Darcy) (Cashman & Preene, 2001). Darcy discovered that the flowrate through an intergranular porous media is in direct relation to the cross-sectional area of the media (Cashman & Preene, 2001; Fetter, 2014). Where a constant multiplied to the area and the hydraulic gradient gives the flow rate. The constant is the hydraulic conductivity. Darcy's law can be written as (Fetter, 2014; Krešić, 2007; Strack, 2017; Verruijt, 1970)

$$Q = A \cdot K \cdot \frac{\Delta h}{l} \tag{1}$$

Q is the flow (L^3/T)

A is the cross-sectional area (L^2)

K the hydraulic conductivity (L/T)

Δh the difference in hydraulic head (L)

l the length (L)

($\frac{\Delta h}{l} = i$, where i is the hydraulic gradient).

The one-dimensional velocity can be calculated using the flow rate and cross-sectional area.

$$v = \frac{Q}{A} \quad (2)$$

Two of the most common analytical models are the Thiem equation for steady state flow to a well in a confined aquifer and the Dupuit method for steady state flow to a well in an unconfined aquifer. Both are very common methods of modelling groundwater and are explained in the large majority of literature regarding groundwater modelling such as (Fetter, 2014; Krešić, 2007).

2.2.1 Steady state flow to a well in a confined aquifer

The Thiem equation is used to model steady state flow to a well in a confined aquifer. In the numerical model unconfined conditions are present. The reason the Thiem method is used in this report is to examine if the effects of a shaft only partially penetrating an unconfined aquifer will cause the inflow to the shaft, the drawdown, and/or the radius of influence to be more similar to the Thiem method than the Dupuit method.

For a confined aquifer, Thiem's equation can be used to determine aquifer properties. For it to be valid the following assumptions/conditions must be fulfilled (Krešić, 2007) :

1. There is full penetration from the well and the entire aquifer provides the well with water.
2. There is a constant pump rate.
3. The flow towards the well is horizontal radial and laminar, the equipotential lines are radial and the flowlines parallel along every radial cross-section.
4. The aquifer is isotropic and homogeneous and has a constant thickness and a horizontal bottom.
5. After a certain time period there is no increase in drawdown and steady-state conditions are reached.

By starting from Darcy's law, the flow rate can be determined as:

$$Q = Av \quad (3)$$

Where A is the cross-sectional area and v the horizontal flow velocity. The cross-sectional area is that of the cylinder (see Figure 1, in chapter 2.3.3) that is modelled:

$$A = 2\pi r b \quad (4)$$

r is the radius and b the thickness of the aquifer.

The flow velocity is determined as:

$$v = K \cdot \frac{dh}{dr} \quad (5)$$

Where K is the hydraulic conductivity and $\frac{dh}{dr}$ the hydraulic gradient.

By inserting the equations for the area and the velocity the flow rate can now be written as:

$$Q = 2\pi r b K \cdot \frac{dh}{dr} \quad (6)$$

This is a differential equation and for it to be solved dh and dr must be isolated on different sides from each other:

$$dh = \frac{Q}{2\pi b K} \cdot \frac{dr}{r} \quad (7)$$

The hydraulic conductivity multiplied with the aquifer thickness is the transmissivity and can therefore be written as:

$$bK = T \rightarrow dh = \frac{Q}{2\pi T} \cdot \frac{dr}{r} \quad (8)$$

This equation can be integrated by using the following boundary conditions:

1. At the distance of the well radius (r_w) the hydraulic head is the water level in the well (h_w)
2. At the arbitrary distance r , the hydraulic head is h_r .

Written mathematically as follows:

$$\int_{h_w}^{h_r} dh = \frac{Q}{2\pi T} \int_{r_w}^r \frac{dr}{r} \quad (9)$$

The integral is then solved:

$$h_r - h_w = \frac{Q}{2\pi T} \cdot \ln\left(\frac{r}{r_w}\right) \quad (10)$$

From this the hydraulic head at the arbitrary distance r can be determined with:

$$h_r = h_w + \frac{Q}{2\pi T} \cdot \ln\left(\frac{r}{r_w}\right) \quad (11)$$

To add the drawdown to the equation, the boundary conditions are changed to:

1. At the arbitrary distance r , the hydraulic head is h_r .
2. At the distance of the radius of influence, R , where no drawdown is recorded the hydraulic head is H , the initial hydraulic head.

$$\int_{h_r}^H dh = \frac{Q}{2\pi T} \int_r^R \frac{dr}{r} \quad (12)$$

$$H - h_r = \frac{Q}{2\pi T} \cdot \ln\left(\frac{R}{r}\right) \rightarrow H - h_r = s_r \quad (13)$$

$$s_r = \frac{Q}{2\pi T} \ln\left(\frac{R}{r}\right) \quad (14)$$

Considering the logarithmic rules equation 14 can be rewritten as follows:

$$s_r = \frac{Q}{2\pi T} \cdot \ln(R) - \frac{Q}{2\pi T} \cdot \ln(r) \quad (15)$$

(Krešić, 2007)

2.2.2 Flow toward a well inside an unconfined aquifer

In an unconfined aquifer the saturated zones thickness varies, in difference to a confined aquifer. This since the groundwater level equals to the thickness of the saturated zone. By using Dupuit's hypothesis, the flow rate towards the well can be calculated. However, Dupuit's hypothesis uses the following large simplifications:

1. All equipotential lines are assumed to be vertical.
2. The velocity is constant perpendicular to the equipotential lines.
3. All flow is assumed to be horizontal.
4. There is full penetration of the aquifer by the well and the entire aquifer provides the well with water.
5. There is a constant pump rate.
6. The aquifer is isotropic and homogeneous and has a constant thickness and a horizontal bottom.
7. After a certain time period there is no increase in drawdown and steady-state conditions are reached.

From Darcy's law it is given that the flow rate is:

$$Q = Av \quad (3)$$

For the cylindric flow towards a well the cross-sectional area for the flow can be determined as:

$$A = 2\pi r h \quad (16)$$

The flow velocity is according to Darcy's law:

$$v = K \frac{dh}{dr} \quad (5)$$

By combining these equations, the radial flow rate can be written as:

$$Q_r = 2\pi r h K \frac{dh}{dr} \quad (17)$$

The inflow from precipitation straight into the shaft is written as

$$Q_{inf} = \pi r^2 w \quad (18)$$

Where w is the groundwater recharge per area unit.

The total flow is set as

$$Q_{tot} = 2\pi r h K \frac{dh}{dr} + \pi r^2 w \quad (19)$$

Multiplying with dr and dividing by r

$$\frac{Q_{tot} dr}{r} = 2\pi h K dh + \pi r w dr \quad (20)$$

Following by isolating for hydraulic head

$$h \cdot dh = \frac{Q_{tot}}{2\pi K} \frac{dr}{r} - \frac{rw}{2K} dr \quad (21)$$

When integrated the equation is read as:

$$\int_{h_w}^h h dh = \frac{Q_{tot}}{2\pi K} \int_{r_w}^r \frac{dr}{r} - \frac{w}{2K} \int_{r_w}^r r dr \quad (22)$$

The primitive function is

$$\left[\frac{h^2}{2} \right]_{h_w}^h = \frac{Q_{tot}}{2\pi K} \cdot [\ln(r)]_{r_w}^r - \frac{w}{2K} \left[\frac{r^2}{2} \right]_{r_w}^r \quad (23)$$

Rewritten as

$$\frac{h^2 - h_w^2}{2} = \frac{Q_{tot}}{2\pi K} (\ln(r) - \ln(r_w)) - \frac{w}{2K} \left(\frac{r^2 - r_w^2}{2} \right) \quad (24)$$

Both sides are multiplied by 2 and the equation is rewritten to solve the hydraulic head h at the arbitrary distance r

$$h^2 = h_w^2 + \frac{Q_{tot}}{\pi K} (\ln(r) - \ln(r_w)) - \frac{w}{K} \left(\frac{r^2 - r_w^2}{2} \right) \quad (25)$$

Giving

$$h = \sqrt{h_w^2 + \left(\frac{Q_{tot}}{\pi K} (\ln(r) - \ln(r_w)) - \frac{w}{K} \left(\frac{r^2 - r_w^2}{2} \right) \right)} \quad (26)$$

Solving for the radius of influence R_0 where the hydraulic head is H_0 and R_0 is much larger than r_w

$$R_0 \gg r_w$$

$$H_0^2 - h_w^2 = \frac{Q}{\pi K} \ln\left(\frac{R_0}{r_w}\right) - \frac{w}{2K} \cdot R_0^2 \quad (27)$$

In order for steady state condition the groundwater recharge has to be in equilibrium with the outflow

$$Q_{tot} = Q_{inf}$$

$$Q_{inf} = \pi R_0^2 w \rightarrow R_0 = \sqrt{\frac{Q_{inf}}{\pi w}} \quad (28)$$

Inserted to the previous equation

$$H_0^2 - h_w^2 = \frac{Q_{inf}}{\pi K} \left(\ln\left(\frac{\sqrt{\frac{Q_{inf}}{\pi w}}}{r_w}\right) \right) - \frac{w}{2K} \cdot \frac{Q_{inf}}{\pi w} \quad (29)$$

Simplified to

$$H_0^2 - h_w^2 = \frac{Q_{inf}}{\pi K} \left(\ln\left(\frac{\sqrt{\frac{Q_{inf}}{\pi w}}}{r_w}\right) \right) - \frac{Q_{inf}}{2\pi K} \quad (30)$$

Simplifying more

$$H_0^2 - h_w^2 = \frac{Q_{inf}}{\pi K} \left(\ln\left(\frac{\sqrt{\frac{Q_{inf}}{\pi w}}}{r_w}\right) - \frac{1}{2} \right) \quad (31)$$

$$\ln(\sqrt{e}) = \frac{1}{2}$$

$$H_0^2 - h_w^2 = \frac{Q_{inf}}{\pi K} \ln\left(\frac{\frac{\sqrt{\frac{Q_{inf}}{\pi w}}}{r_w}}{\frac{\sqrt{e}}{1}}\right) \quad (32)$$

$$H_0^2 - h_w^2 = \frac{Q_{inf}}{\pi K} \ln\left(\frac{\sqrt{\frac{Q_{inf}}{\pi w e}}}{r_w}\right) \quad (33)$$

The equation can be written to solve for radial distance to the arbitrary point where the hydraulic head is h_a

$$H_0^2 - h_a^2 = \frac{Q_{inf}}{\pi K} (\ln(R_0) - \ln(r_a)) - \frac{w}{2K} \left(\frac{R_0^2 - r_a^2}{2} \right) \quad (34a)$$

(Krešić, 2007; Todd & Mays, 2005)

It should be noted that this method is non-linear.

2.3 Usage of analytical modelling

Both analytical methods, Thiem and Dupuit, were initially used to determine the radius of influence. Under steady state conditions, the inflow to the aquifer must equal the outflow from the shaft. The flow is dependent on the radius of influence as it is calculated through the groundwater recharge per area unit, multiplied by the model's circular area, shown in equation 18.

$$Q = \pi r^2 w \quad (18)$$

To be able to determine the radius of influence, the flow must first be determined. The flow is calculated iteratively. Initially a small radius is assumed and is increased with each iteration, which increases the circular area and thus the flow. The calculated flow is then used to calculate a radius of influence with the analytical model. The iterations continue until the iterated radius is within 0.05 meters from the radius of influence calculated analytically with the iterated flow for Thiem's equation (equation 35a) and until the initial hydraulic head is within 0.05 m of the hydraulic head calculated with the radius of influence for Dupuit's method (equation 34b).

2.3.1 Thiem's equation

$$s_r = \frac{Q}{2\pi T} \ln\left(\frac{R_0}{r}\right) \quad (14)$$

$$r_w \cdot e^{\left(\frac{2 \cdot \pi \cdot T}{Q} s_r\right)} = R_0 \quad (35a)$$

$$r = \frac{R_0}{e^{\left(\frac{2 \cdot \pi \cdot T}{Q} s_r\right)}} \quad (35b)$$

Where R_0 is the radius influence (L)

s_r is the drawdown, set as 0.3 m (L)

r_w is the radius of the shaft, set as 5 m (L)

r is the radial distance from the center of the shaft to the point where the drawdown is 0.3 m (L)

Q is the flow (L^3/T)

w is the aquifer recharge per area unit. ($L^3/L^2 \cdot T$)

Equation 35a is used to verify that the radius of influence assumed in the flow calculations is the same as the radius of influence obtained when calculating from the flow. The drawdown is set as difference in hydraulic head between the shaft bottom and the initial hydraulic head. As mentioned above a margin of error of 0.05 m between the two radii is considered acceptable. With a verified radius of influence

the values were inserted into equation 35b, with the drawdown set to 0.30 m and the radial distance to 0.30 m drawdown, r , was determined.

2.3.2 Dupuit's method

$$H_0^2 - H_w^2 = \frac{Q}{\pi K} (\ln(R_0) - \ln(r_w)) - \frac{w}{2K} \left(\frac{R_0^2 - r_w^2}{2} \right) \quad (34b)$$

$$H_0^2 - H_a^2 = \frac{Q}{\pi K} (\ln(R_0) - \ln(r_a)) - \frac{w}{2K} \left(\frac{R_0^2 - r_a^2}{2} \right) \quad (34a)$$

H_0 is the initial hydraulic head (L)

H_a is the hydraulic head at the arbitrary distance r_a (L)

H_w is the hydraulic head at the shaft (L)

Q is the pumping rate from the shaft (L^3/T)

w is the aquifer recharge per area unit ($L^3/L^2 \cdot T$)

R_0 is the radius of influence (L)

r_w is the radius of the shaft (L)

To verify the radius of influence equation 34b is used. The squared value of the initial hydraulic head subtracted with the hydraulic head at the shaft is calculated iteratively with an increasing radius of influence and flow, until it is within 0.05 m from the set initial hydraulic head.

Since the Dupuit method is non-linear and includes the radius of influence in two parts of the equation it is preferred to use the initial hydraulic head instead as a verification parameter rather than the radius of influence. Once the radius of influence and the flow are determined equation 34a is used to determine the radial distance to 0.30 m drawdown. This calculation is also performed iteratively where a small radial distance to 0.30 m drawdown is assumed and then is increased with each iteration until H_a is 0.30 m lower than H_0 .

2.3.3 Conceptual models of the analytical method

To clarify how the analytical approach is implemented conceptually figures 1-4 illustrate the configuration of the model

Figure 1 shows how the analytical model is conceptually modelled.

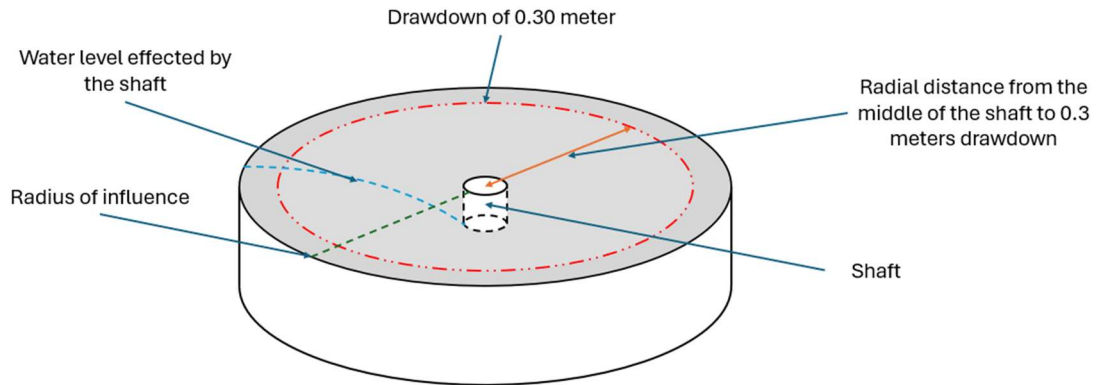


Figure 1 Conceptual model of the analytical method, with the shaft fully penetrating the aquifer.

However, when the shaft does not fully penetrate the aquifer and there is a thickness below there are different ways to model it. For the Thiem method it is not important as it only calculates the difference in hydraulic head between the shaft bottom and the initial hydraulic head before shafting. For the Dupuit method the layer below can either be ignored where the initial hydraulic head will be put as the difference between the shaft bottom and the actual initial head (shown in figure 2). It may also be modelled with both the shaft bottom and the actual initial hydraulic head. This is referred to as thickness below the shaft in figure 3 and is what will be the hydraulic head inside the shaft.

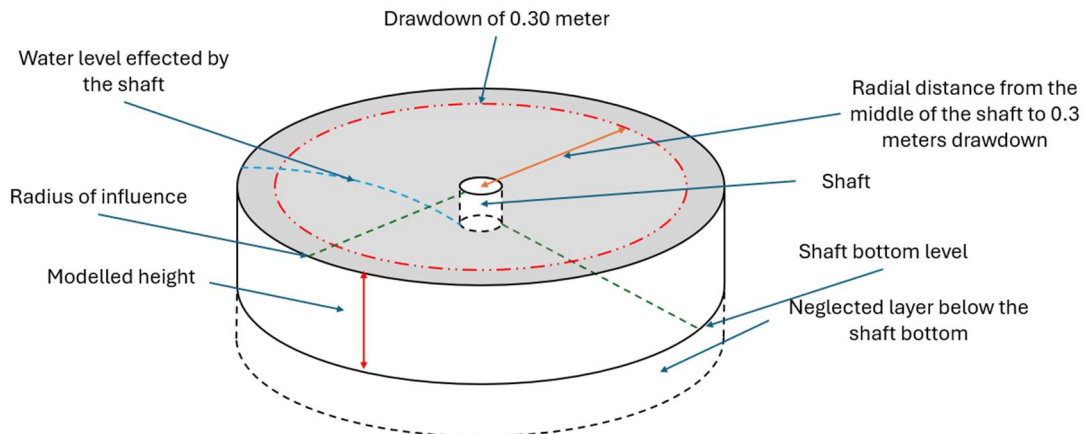


Figure 2 Conceptual model of the analytical method when the layer below the partial penetration is neglected.

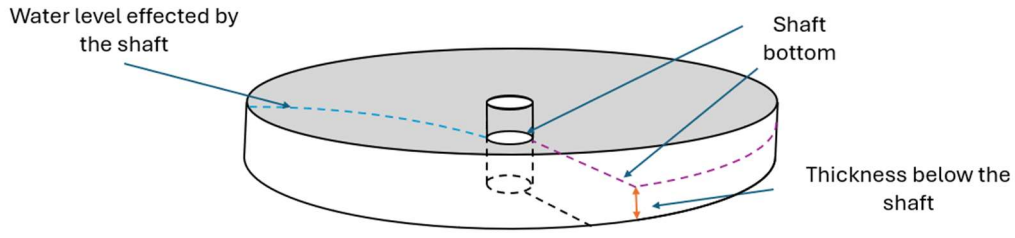


Figure 3 Conceptual model of the analytic method when the layer below the partial penetration is included.

For the analytical models anything placed outside the radius of influence is not included in the model, this is schematically shown in figure 4.

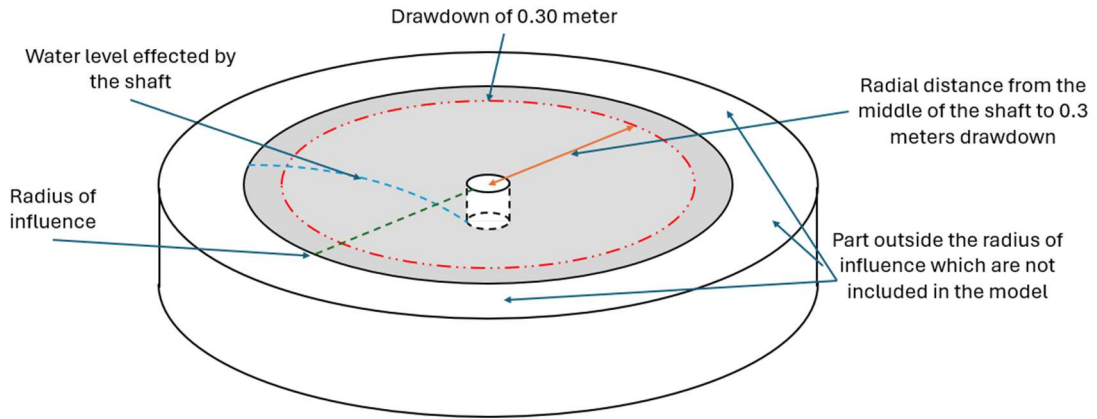


Figure 4 Schematic picture showing how parts outside the radius of influence are neglected.

2.3.4 Variations in analytical models

The aim of this report is to examine how to manage to get a result from the analytical model which is similar to the numerical one. For this reason, some variations will be made to the analytical models. As mentioned in chapter 1 there can be a large variation in the estimated hydraulic conductivity and the actual hydraulic conductivity. For this reason, the hydraulic conductivity will be varied in the analytical models, as it is possible that it deviates from the actual hydraulic conductivity. An error with a factor 10 is assumed and the value for the hydraulic conductivity in the numerical model is set as the mean value. The numerical models' hydraulic conductivities were not varied since the task would have been too time consuming. To find the lowest and highest value the conductivity is multiplied by 9/11 and then this deviation value is subtracted to the initial conductivity to give the lowest value and vice versa for the highest value.

$$K \pm \frac{9K}{11} = K_{max,min} \quad (36)$$

$$\frac{K - \frac{9K}{11}}{K + \frac{9K}{11}} = \frac{\frac{2K}{11}}{\frac{20K}{11}} = 0.1 \quad (37)$$

2.4 Numerical modelling with FEFLOW

FEFLOW stands for Finite-Element subsurface FLOW and transport system. It can be used to generate both two dimensional as well as three dimensional models. FEFLOW can generate both triangular and rectangular FE-meshes (Trefry & Muffels, 2007). In this thesis report triangular meshes were built.

One of the differences between the numerical and analytical modelling methods is that the numerical model is always restrained by its boundary conditions. Whereas an analytical model can stretch to infinity (Strack, 2017).

2.4.1 Theory FEFLOW

FEFLOW's modelling is based on the Richard's equation. The basic principle of the Richard's equation is to combine the soil moisture content curve with the function for relative hydraulic conductivity (Févotte et al., 2024). This is performed through determining the soil moisture in each element depending on the hydraulic head in the surrounding nodes. Following this the hydraulic conductivity is determined through the moisture content in each element. One of the most common ways to approximate this relationship is with the Van Genuchten equation. It is a rather simple method which depends on six parameters, which can be determined with the least square approximation method and measured points. However, the relationship between pressure, soil moisture content and hydraulic conductivity can be more complicated, requiring a more accurate approximation. One method for this is through empirical spline curves, which are cubical or polynomial (Wesseling et al., 2008).

The Richard's equation is written as:

$$\partial_t \vartheta(\psi) - \nabla \cdot (K(\theta(\psi))(\nabla \psi + \nabla z)) = g$$

∂_t is the time derivate

$\nabla \cdot$ is the divergence operator

∇ is the gradient operator

ψ is the normalized pressure head

z is the vertical coordinate

g is the source term

ϑ is the active pore volume

ϑ is related to θ as follows:

$$\vartheta(\psi) := \theta(\psi) + \frac{S_s}{\theta_s} \int_{-\infty}^{\psi} \theta(s) ds$$

S_s is a material parameter ≥ 0

θ_s is a material parameter ≥ 0

The hydraulic conductivity $K(\psi) = K_s K_r(\psi)$. In this expression K_s is the saturated hydraulic conductivity and K_r is the relative permeability depending on the relative saturation varying from 0 to 1.

θ and K_r are functions given by constitutive relations.

(Dolejší et al., 2024)

The Van Genuchten model, mentioned earlier can be written as:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha h)^n]^m}$$

θ is the soil moisture content

θ_r is the residual soil moisture content

θ_s is the saturated soil moisture content

α and n are scale parameter used to fit the curve.

h is the soil water potential

$m = 1 - \frac{1}{n}$ and is lower than 1 and above zero.

By measuring data from the aquifer the parameters in the Van Genuchten equation can be estimated with the least square method as following.

$$\min f = \sum_{i=1}^k \left(\hat{\theta}_i(h) - \theta_i(h) \right)^2$$

$\hat{\theta}_i$ is the measured soil water content

θ_i is the calculated soil water content from the van Genuchten equation.

k is the number of measurements.

(Chen et al., 2016)

For the saturated flow the element nodes are moved with the groundwater table during the modelling, meaning that saturated flow only occurs in fully saturated elements. (FEFLOW 7.3 Documentation - Flow Description, n.d.)

2.5 Usage of numerical modelling

In the numerical models built for this study, spline curves were used to set the relationship between relative soil moisture content s and relative hydraulic conductivity K_r . Where the lowest unsaturated relative conductivity was set to $K_r(s) = 0.05$. This was set for the values $s \leq 0.4$, meaning that the soil only is 40 % saturated. Besides this the spline

curve follows the program's standard values. The program's initial value for dry zones where $s = 0$ was $Kr(0) = 0$. This was increased to 0.005 due to the program having convergence problems when the hydraulic conductivity in some nodes was 0.

Each model was built with a large square shape containing a smaller square shaped shaft in the middle of the model. A hydraulic head was set as a boundary condition for the eastern and western border of the model, the boundary was set to the same value as the initial hydraulic head. The inflow was set to 150 mm/y. Using these initial conditions the model was run without a shaft to model the initial water level in each node before the shaft. Here already there is a difference between the methods as the analytical models assume a flat groundwater table and in the numerical it will have an undulating shape before the shaft is built. Once the shaft is placed all the elements inside the shaft are set as inactive in the model. The nodes inside and immediately surrounding the shaft are given a constant head boundary condition with the same level as the height from the bottom of the model to the shaft bottom. Then the model is rerun with the shaft in place and the difference between the groundwater level before and after the introduction of the shaft is used to acquire the drawdown.

For practical reasons the results from the numerical and analytical models which are compared, are set to compare at what radial distance the drawdown is 0.30 meters. This is quite common for construction projects as a drawdown which is smaller than 0.3 meters could be the result of a natural variation in the groundwater levels as well as it is very unlikely to cause damage for its surroundings (Lissel, 2017). A smaller test will also be made where the radius of influence is compared.

Following this, nodal values for the drawdown were extracted for each node and placed inside a GIS-map. The values were then interpolated linearly and then the interpolated values were placed in three categories in the map. Green category meaning no drawdown, yellow category a drawdown of 0.3 m or less and a red category meaning a drawdown larger than 0.3 m. The measurement of the radius of influence was made in QGIS by extracting the distance in the east-west direction between the two starts of the yellow zone in the height of the centre of the shaft. This was then divided by two to acquire the radius.

The figures 5 and 6 show the conceptual model of the numerical model.

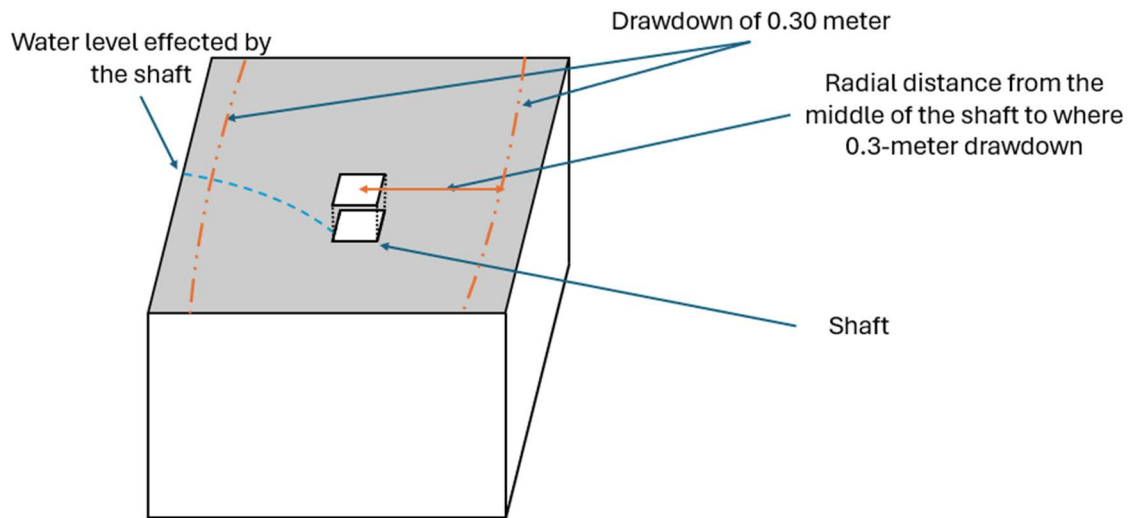


Figure 5 Conceptual model of the numerical method shown in three dimensions

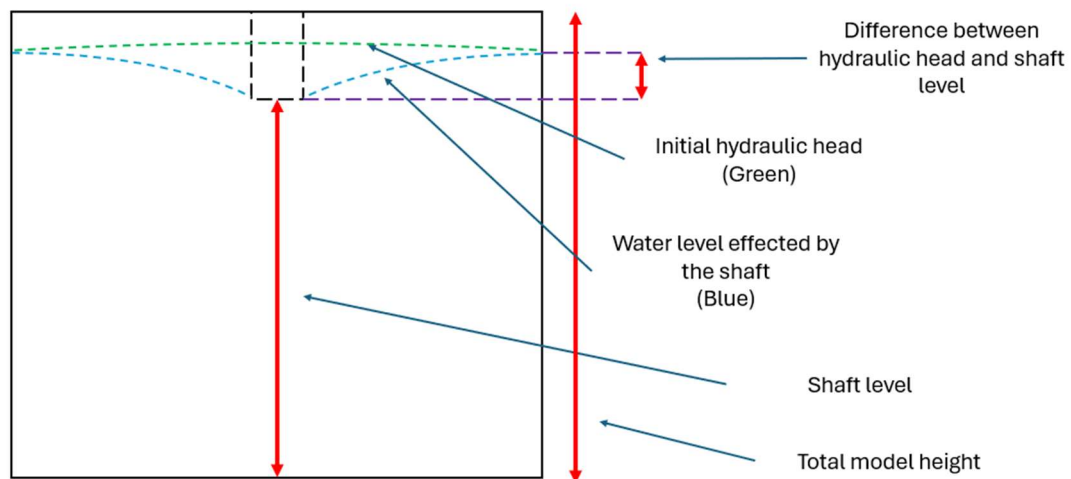


Figure 6 Conceptual model of the numerical method shown in two dimensions

The second method of comparison is computed by examining the flowlines from the top layer of the model. The flow lines show how the inflow from the top layer of the model will move. If they travel towards the shaft they are inside the radius of influence and if they are outside, they are not. From the flow lines the radius of influence can be modelled to be the radius inside which the recharge travels into the shaft. The model only has boundary conditions for 2 of the 4 boundaries, to decrease the effects of the boundary conditions on the result. Due to this the influence area will have an oval shape. The shortest radius of the oval shape is what is measured. Figure 7 shows an example of how the flowlines are seen in FEFLOW.

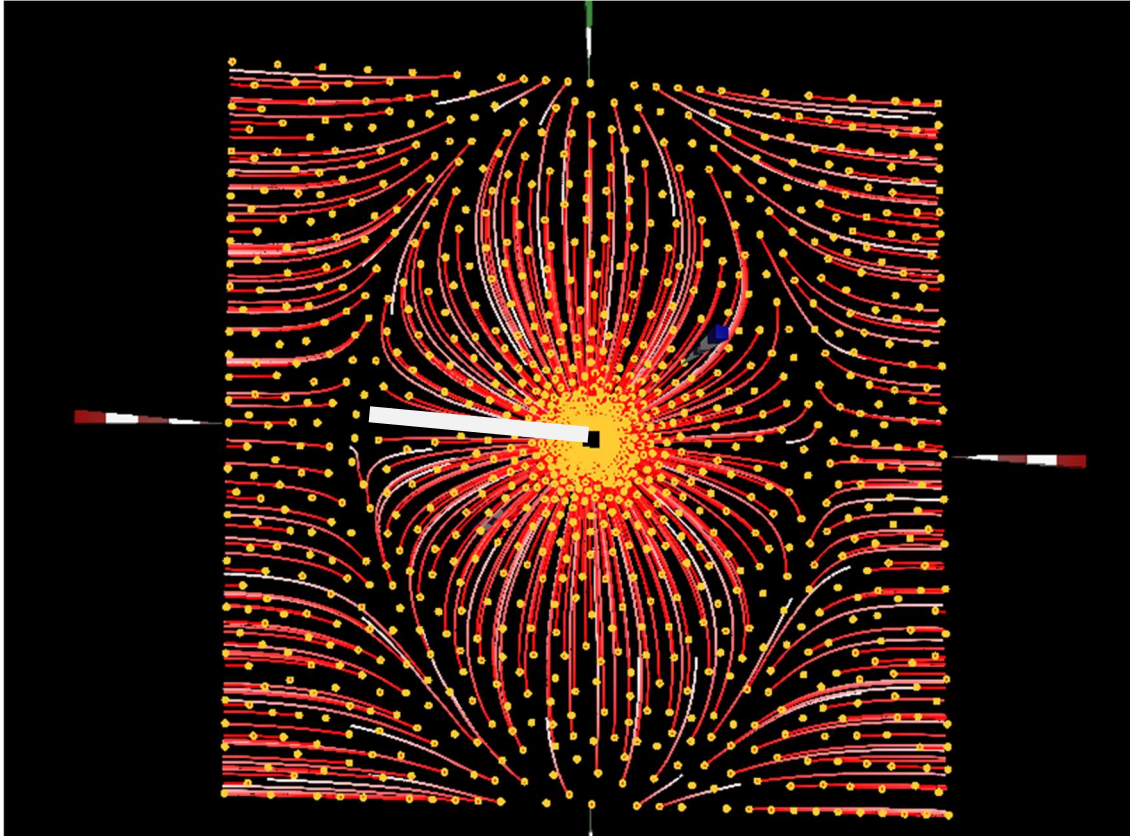


Figure 7 How the flowlines are seen in FEFLOW

In figure 7 above, there is an oval shape with lines from the nodes (yellow dots) to the shaft. Outside this oval shape the lines go between the nodes and the edges of the model. It is the shortest radial distance from the centre of the shaft to the edge of this oval that is the radius of influence. Since there is not an infinite number of nodes in the model, there is an intermediate zone between where the water travels to the shaft and where it travels to the edges. The white line in the middle shows how this distance was estimated.

2.5.1 Model with domain 100x100 m and hydraulic conductivity 0.1 m/d

This model was built as a square which was 100x100m. In the middle of this box a square shaft was placed sized 10x10 m. The hydraulic conductivity was set to 0.1 m/d. The initial hydraulic head was varied between 8 and 3 m and height from the bottom of the model ($h=0\text{m}$) was varied between 0-5 m. The total height of the model was consistently 2 m higher than the initial hydraulic head. The groundwater recharge per area unit was set as $150\text{mm}/\text{y}$. The models built with these conditions were given the following parameters in table 1.

Table 1 Parameters in model with domain 100x100m and hydraulic conductivity 0.1 m/d. Parameter definitions are shown in figure 6.

No.	Water level pre-shafting	Shaft level	Total model height
1	8	5	10
2	5	2	7
3	4	1	6
4	3	0	5
5	5	1	7
6	4	0	6
7	5	0	7
8	8	2	10
9	6	0	8
10	8	1	10
11	7	0	9
12	8	0	10

The initial results of this model appeared unreasonable, and it was believed that the boundary conditions affected them largely, showing a need to increase the model domain to 300x300m while maintaining the shaft size of 10x10 m.

When the domain of the model was increased without changing the recharge per area unit or the hydraulic conductivity, the hydraulic head in the middle of the model became very large before the shaft was placed. The hydraulic head was higher than the model height at some points. Such high pressures are unreasonable for an unconfined aquifer as the water surface is free. Thus, the enlargement of the model required a higher hydraulic conductivity, and it was therefore set to 1 m/d.

2.5.2 Varying conductivity

For quality assessment of the results the larger 300x300 model was built once more with a higher conductivity. The conductivity was set to 8.64 m/d for this test. The groundwater recharge per area unit was set as 150mm/y. For this test a control on the result from the drawdown was conducted by calculating the drawdown in two different methods for the numerical model. The first method was the same as for the previous models, the model was run without the shaft to model the water level pre-shafting. The nodal values for the hydraulic head after the shafting were then subtracted from the nodal values of the water level pre-shafting. The other method was conducted by subtracting the nodal values after the shafting from the set value for initial hydraulic head in the numerical model. Meaning that the water surface was modelled to be flat in the numerical model before the shafting.

The hydraulic conductivity is an important factor for the result from the model. The conductivity values chosen correspond to probable real-world values. Below follows a table which shows typical values for hydraulic conductivity for different soil types. (Maurya et al., 2016)

Table 2 Typical hydraulic conductivity for different soil types

Soil type	Hydraulic conductivity, K [m/d]	Hydraulic conductivity, K [m/s]
Gravel	$86.4 < K$	$10^{-3} < K$
Sand	$8.64 \cdot 10^{-1} < K < 86.4$	$10^{-5} < K < 10^{-3}$
Silty sand	$8.64 \cdot 10^{-3} < K < 8.64 \cdot 10^{-1}$	$10^{-7} < K < 10^{-5}$
Silt, Silty clay	$8.64 \cdot 10^{-5} < K < 8.64 \cdot 10^{-3}$	$10^{-9} < K < 10^{-7}$
Clay	$K < 8.64 \cdot 10^{-5}$	$K < 10^{-9}$

3 Results

The results are presented in this chapter in tables and in figures. In general, there is a large deviation between the results from the numerical modelling and the analytical modelling.

3.1 Results presented in tables

3.1.1 Model with domain 100x100 m and hydraulic conductivity 0.1 m/d

The results from the first model type had the analytical result very close to or even further away than the boundary condition in the numerical model. Due to this these results are not reliable for a comparison and are therefore presented in the appendix.

3.1.2 Model with increased domain to 300x300m and hydraulic conductivity 0.1 m/d

When the model was enlarged to make the catchment area 300x300 m whilst maintaining the size and position of the shaft the difference in result between the analytical and numerical result was smaller, than in the previous model. This can be seen in tables 3, 4 & 5. The parameters in tables 3, 4 & 5 are shown in figures 1, 5 & 6. The difference between the three tables is that they show results for different analytical methods. In table 3, the Dupuit method was modelled assuming the bottom layer below the shaft was neglected, as shown in figure 2. In table 4 the Dupuit method was modelled according to figure 3 where the bottom layer below the shaft was included. Table 5 shows the results from the Thiem modelling. For this analysis the result is the radial distance to the point where the drawdown is 0.3 m.

The smallest difference between the results from the analytical and numerical methods was obtained when the Dupuit method was used with the layer below the shaft included in the model. However, the numerical results were all around 120-130 meter, despite large differences in hydraulic head and shaft level.

Table 3 Results from model enlarged to 300x300m compared with Dupuit's method. The layer below the shaft bottom is neglected in the analytical method. The numerical and analytical result show the radial distance from the centre of the shaft to where there is 0.3-meter drawdown.

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	126.5	39	87.5	23.5
5	2	7	3	119.5	39	80.5	30.5
4	1	6	3	120	39	81	30
3	0	5	3	119.5	39	80.5	30.5
8	4	10	4	124	61	63	26
5	1	7	4	123	61	62	27
4	0	6	4	122	61	61	28
8	3	10	5	127.5	84	43.5	22.5
5	0	7	5	125	84	41	25
8	2	10	6	129.5	110	19.5	20.5
6	0	8	6	127.5	110	17.5	22.5
8	1	10	7	131	139	-8	19
7	0	9	7	129.5	139	-9.5	20.5
8	0	10	8	132	172	-40	18

Table 4 Results from model enlarged to 300x300m compared with Dupuit's method. The layer below the shaft bottom is included in the analytical method. The numerical and analytical result show the radial distance from the centre of the shaft to where there is 0.3-meter drawdown.

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	126.5	92	34.5	32.5
5	2	7	3	119.5	67	52.5	30.5
4	1	6	3	120	56	64	30
3	0	5	3	119.5	39	80.5	30.5
8	4	10	4	124	118	6	26
5	1	7	4	123	80	43	27
4	0	6	4	122	61	61	28
8	3	10	5	127.5	140	-12.5	22.5
5	0	7	5	125	84	41	25
8	2	10	6	129.5	157	-27.5	20.5
6	0	8	6	127.5	110	17.5	22.5
8	1	10	7	131	168	-37	19
7	0	9	7	129.5	139	-9.5	20.5
8	0	10	8	132	172	-40	18

Table 5 Results from model enlarged to 300x300m compared with Thiem's equation. The numerical and analytical result show the radial distance from the centre of the shaft to where there is 0.3-meter drawdown

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	126.5	86	40.5	23.5
5	2	7	3	119.5	86	33.5	30.5
4	1	6	3	120	86	34	30
3	0	5	3	119.5	86	33.5	30.5
8	4	10	4	124	117	7	26
5	1	7	4	123	117	6	27
4	0	6	4	122	117	5	28
8	3	10	5	127.5	148	-20.5	22.5
5	0	7	5	125	148	-23	25
8	2	10	6	129.5	179	-49.5	20.5
6	0	8	6	127.5	179	-51.5	22.5
8	1	10	7	131	209	-78	19
7	0	9	7	129.5	209	-79.5	20.5
8	0	10	8	132	239	107	18

3.2 Graphical presentations of results

3.2.1 Numerical results for model with domain size 300x300m and hydraulic conductivity 0.1 m/d

The numerical results for the radial distance to 0.3-meter drawdown generally appear to correlate with the difference between the shaft level and the initial head, with one notable exception. When the initial head was 8 meters and the shaft level was 5 meters, the resulting radial distance was 126.5 m, marked in red in figure 8, deviating from the observed trend.

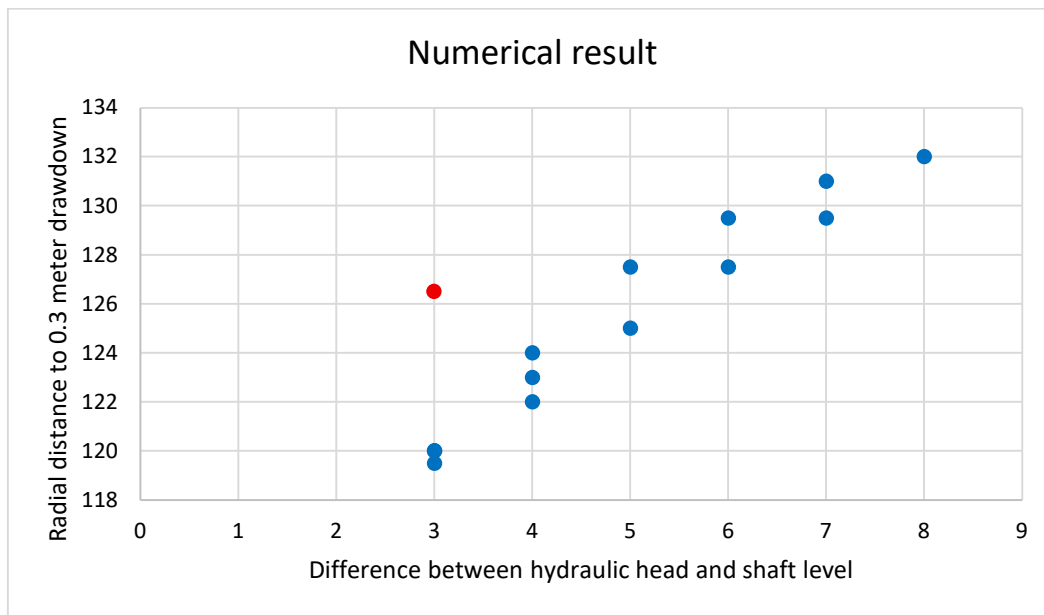


Figure 8 Results from numerical model with domain 300x300 m, where the distance from the middle of the shaft to where there is 0.3 m drawdown is compared to the difference between hydraulic head and shaft level. The red point is a result that is considered to deviate from the trend.

When the numeric result on the radial distance to 0.3-meter drawdown is compared to the initial head there is a tendency on an increase in radial distance with an increased initial head. However, the results from when the difference between initial head and shaft level was 3 meters and the initial head was 8 and 5 meters (marked in red in figure 9) were neglected a correlation appears more likely. This is presented in figure 9,

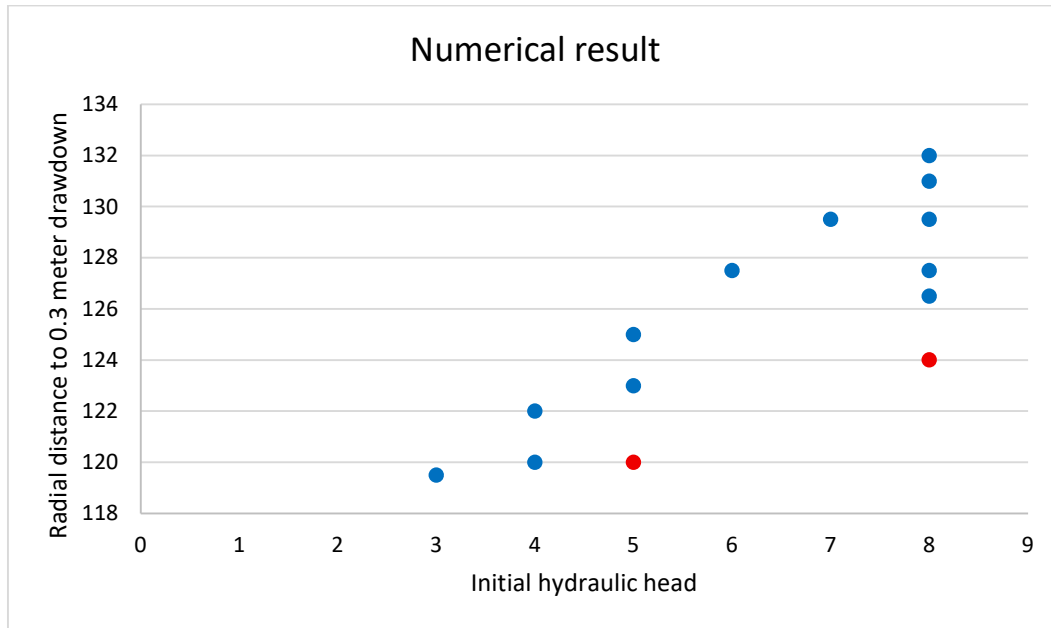


Figure 9 Results from numerical model with domain 300x300m and conductivity 0.1 m/d, where the distance from the middle of the well to where there is 0.3 m drawdown is compared to the initial head. The red points are results that are considered to deviate from the trend

3.2.2 Model with increased domain to 300x300m and increased hydraulic conductivity to 8.64 m/d

The results are presented with a varying hydraulic conductivity for the Dupuit method. The analytical results are presented as lines to ease the comparison between the numerical and analytical methods. Despite the lines drawn, the results for the analytical models are only made for the same values on the X-axis as the numerical model. The yellow dots show the results for what is referred to as numeric flat, which means that the initial water level was set as completely flat in the FEFLOW-model. The results shown in figures 10 & 11, show results for when the analytical Dupuit method neglects the layer below the shaft as is shown in figure 2.

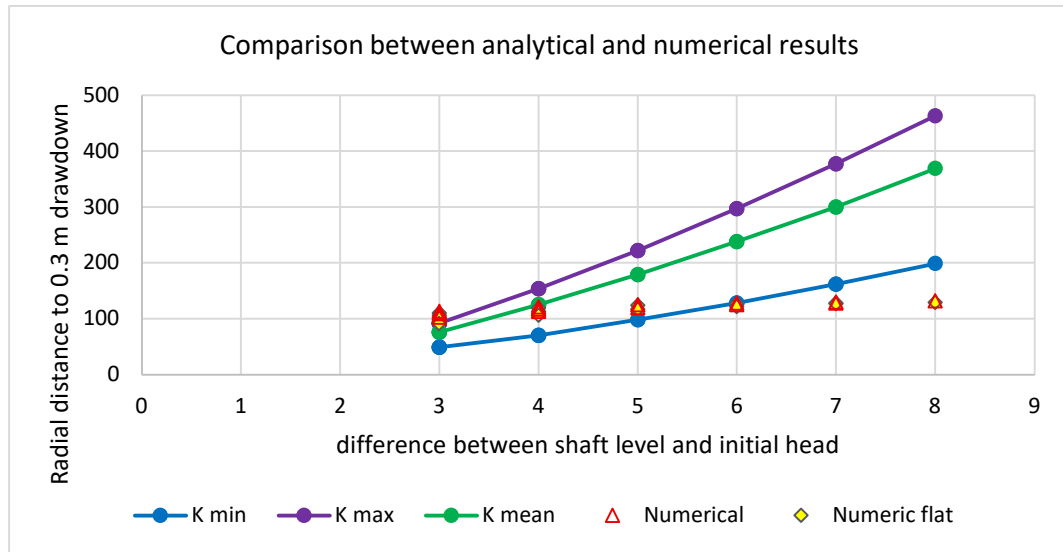


Figure 10 Results from the analytical Dupuit models where the layer below the shaft was neglected and numerical results, plotted against the difference between hydraulic head and shaft level. The points mark out the numerical results and the lines the analytical results.

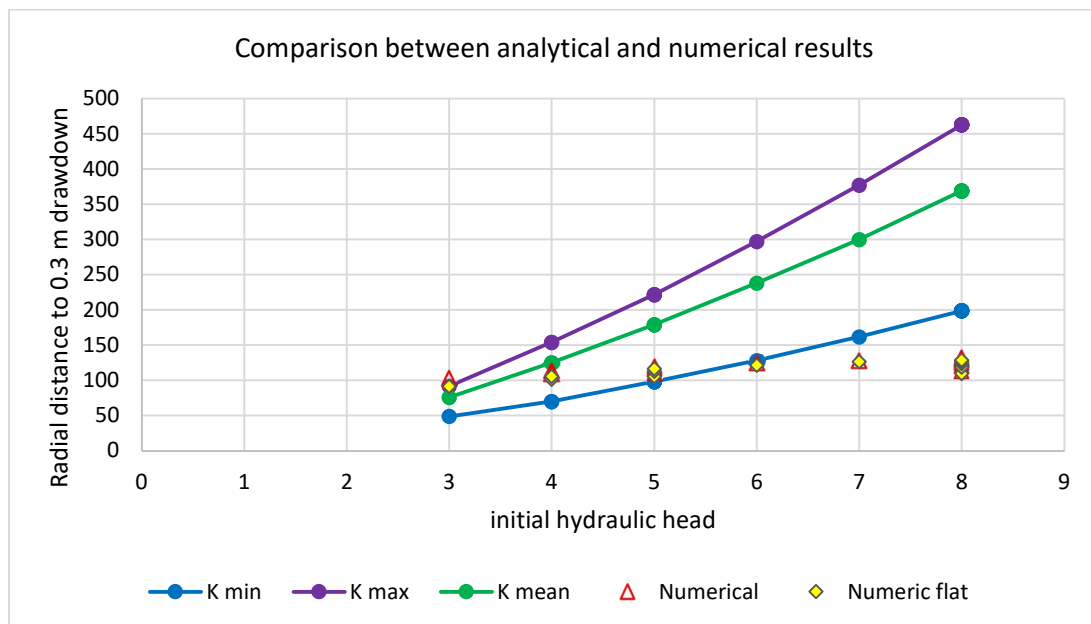


Figure 11 Results from the analytical Dupuit models where the layer below the shaft was neglected and numerical results, plotted against the initial hydraulic head.

In the next two figures 12 and 13, the analytical results are computed whilst including the layer below the shaft as in figure 3. This differs from the results shown in figures 10 and 11, where the layer below the shaft was neglected as shown in figure 2. Due to this the results from the analytical model vary much more than the numerical model. For this reason, the lines in the figures 12 and 13 below show the numerical results and the dots the analytical results.

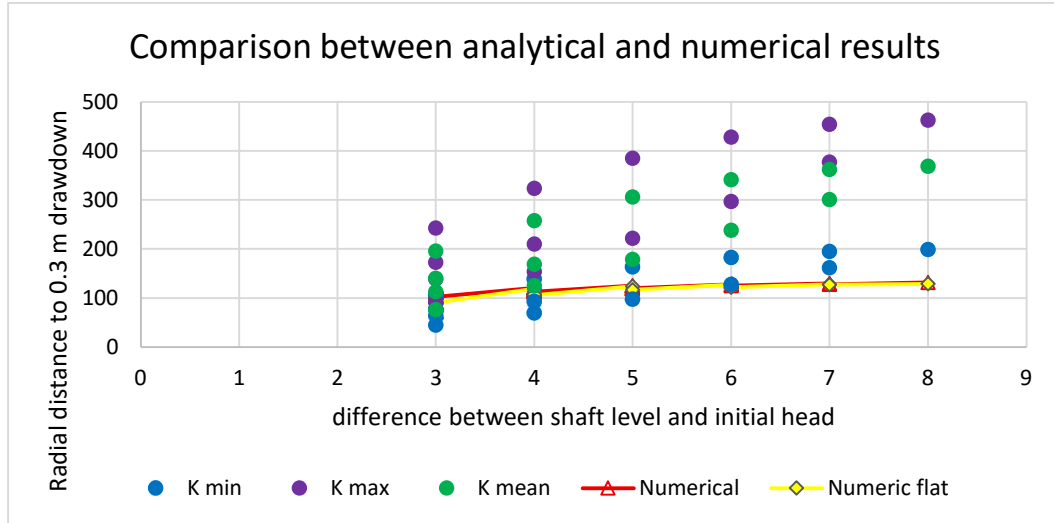


Figure 12 Results from the analytical Dupuit models where the layer below the shaft was included and numerical results, plotted against the difference between hydraulic head and shaft level. The points mark out the analytical results and the lines mark out the numerical results.

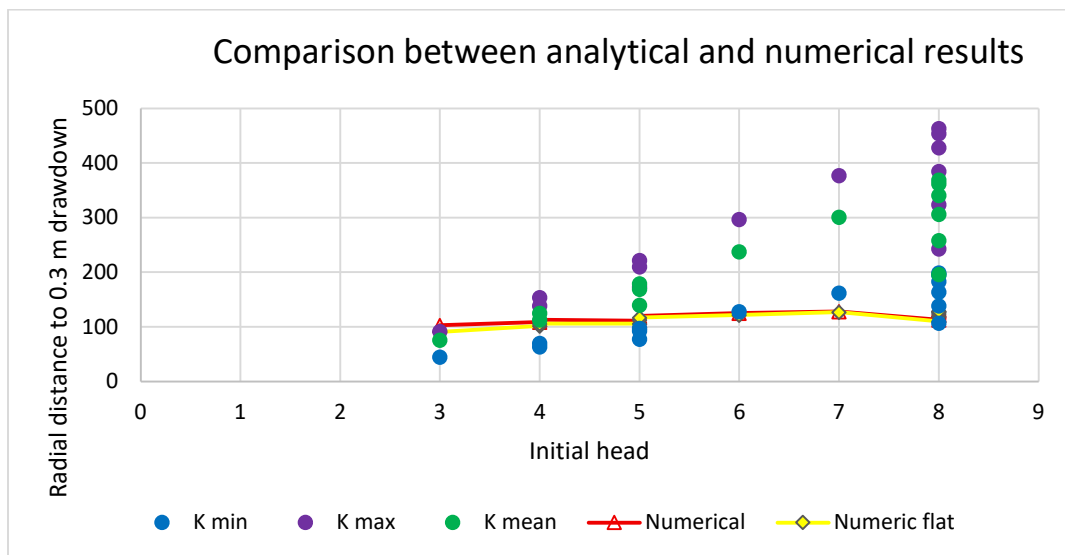


Figure 13 Results from the analytical Dupuit models where the layer below the shaft was included and numerical results, plotted against the initial head. The points mark out the analytical results and the lines mark out the numerical results.

The following figures 14-17 show the difference in result, when calculating the radial distance to 0.3 m drawdown, between the analytical and numerical models. This comparison is made for the Dupuit method and the Thiem method, as well as when the numerical model has a flat initial surface and when it has a normal surface. In figures 14 and 15 the difference between the numerical and analytical result is plotted against the difference between water level pre-shafting and the shaft level. In figures 16 and 17 the difference between the numerical and analytical result is plotted against the shaft level. The results are valid for the model with domain size 300x300-meters and a hydraulic conductivity of 8.64 m/d.

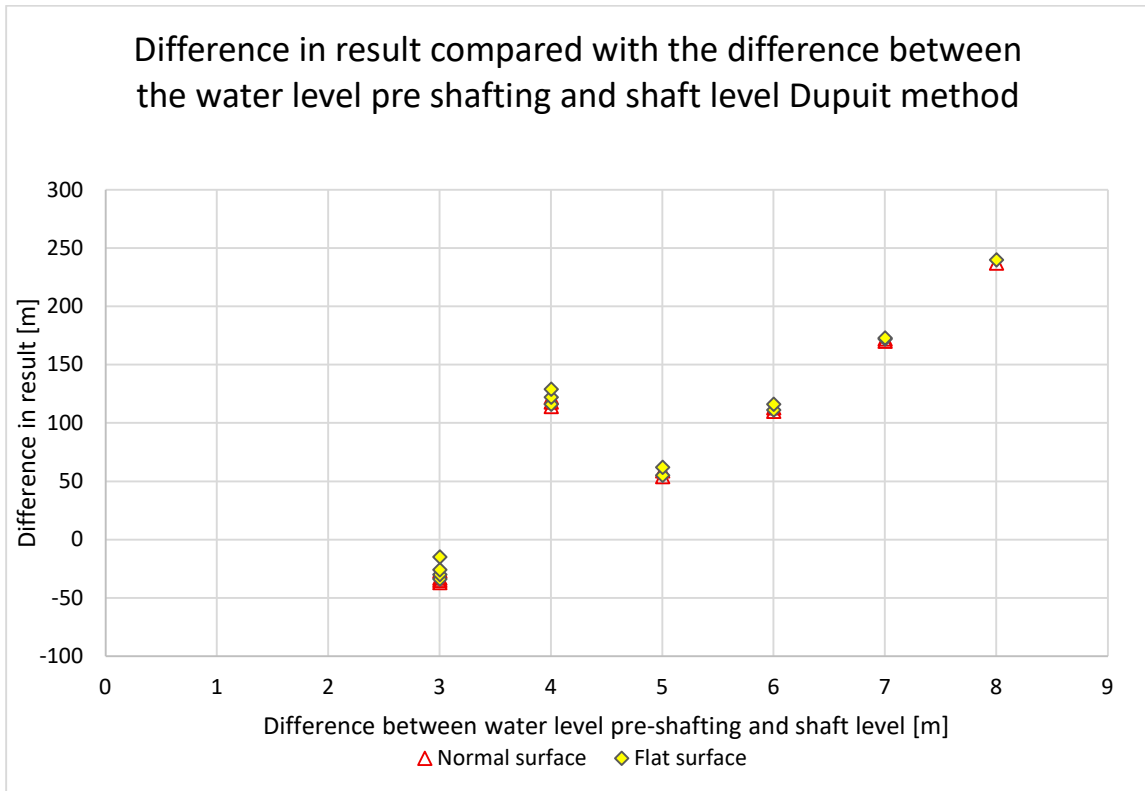


Figure 14 Difference in results between the numerical model and Dupuit equation for model with domain 300x300m and conductivity 8.64 m/d

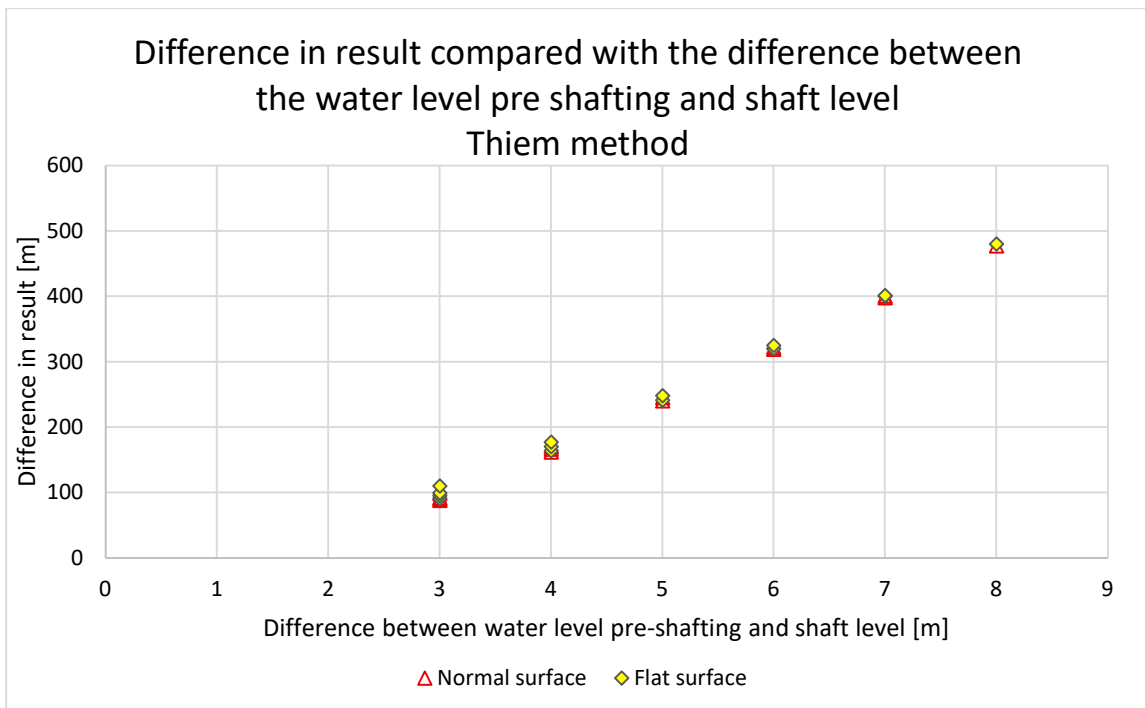


Figure 15 Difference in results between the numerical model and Thiem's equation for model with domain 300x300m and conductivity 8.64 m/d.

Below the difference in result between numeric and analytic model, for the radial distance to 0.3 m drawdown, is shown compared to the thickness of the layer below (figures 16 & 17). There does not appear to be any systematic correlation between neither of the analytical models and the numerical one. It should be noted that for the Thiem comparison the Y-axis is logarithmic scale.

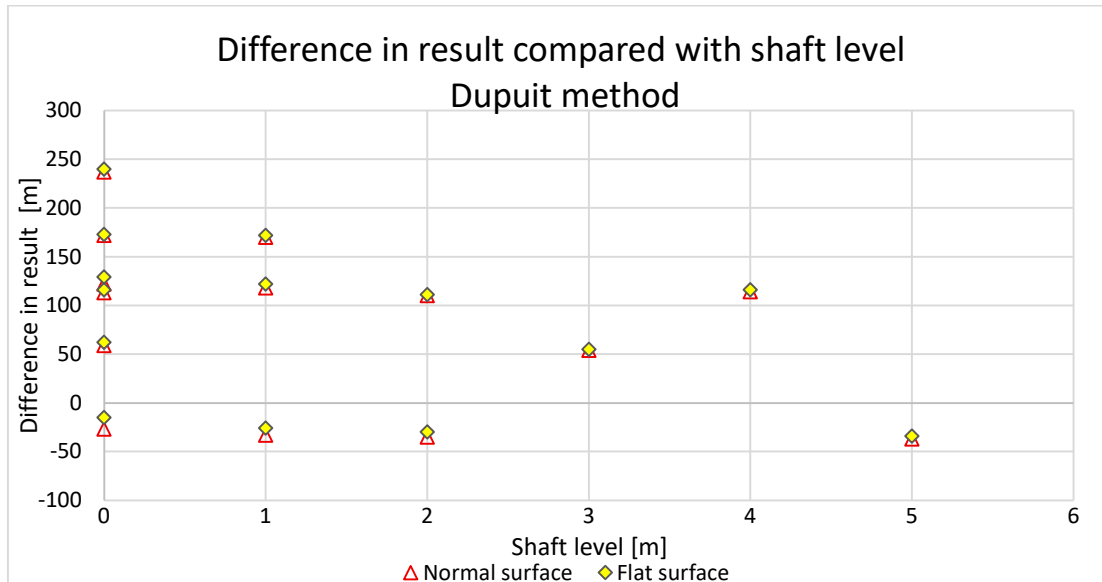


Figure 16 Difference in results between the numerical model and Dupuits method for model with increased domain to 300x300m and increased conductivity to 8.64 m/d

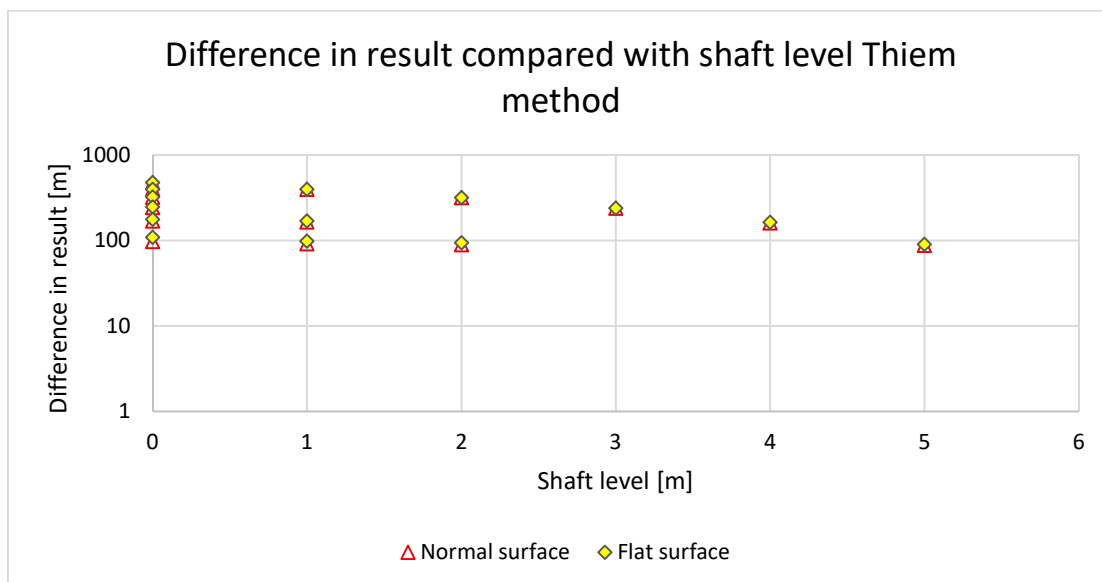


Figure 17 Difference in results between the numerical model and Thiem's method for model with increased domain to 300x300m and increased conductivity to 8.64 m/d

The following figure 18 shows values for the numerical method when the difference between the initial head and the shaft level is 3 meters and how an increased thickness varies the results. It aims to show how the radius of influence increases with an increased

thickness below the shaft bottom, whilst maintaining the difference between the initial head and the shaft level. This conducted with a hydraulic conductivity of 8.64 m/d.

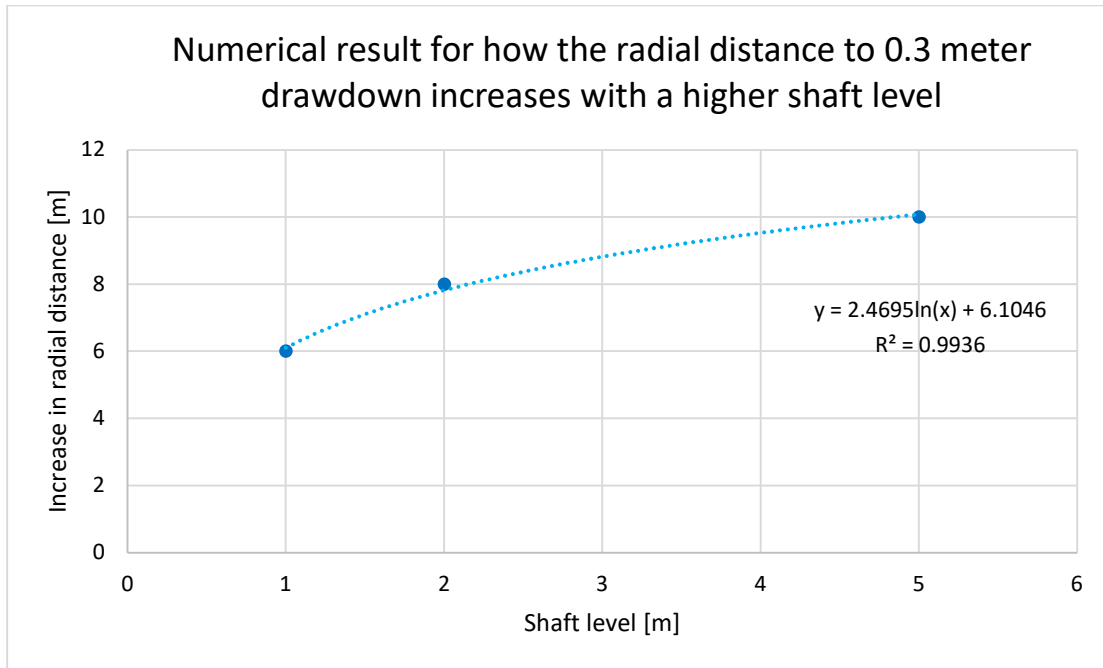


Figure 18 How the radius of influence increases with the shaft level when the difference between initial water level and shaft level is 3 meters

Another comparison is made where both the numeric and numeric flat models' results are compared to the initial hydraulic head. All the values are for when the difference between initial head and shaft level is 3 m. The logarithmic curves deviate more from the measured values than it did in figure 18. However, the logarithmic curve still provides the best overall fit to the data points, despite the increased distance from the values. This is shown in figure 19. In figure 18 the increase in radial distance to 0.30 m drawdown was shown on the Y-axis and in figure 19 the radial distance to 0.30 m drawdown is shown in the Y-axis.

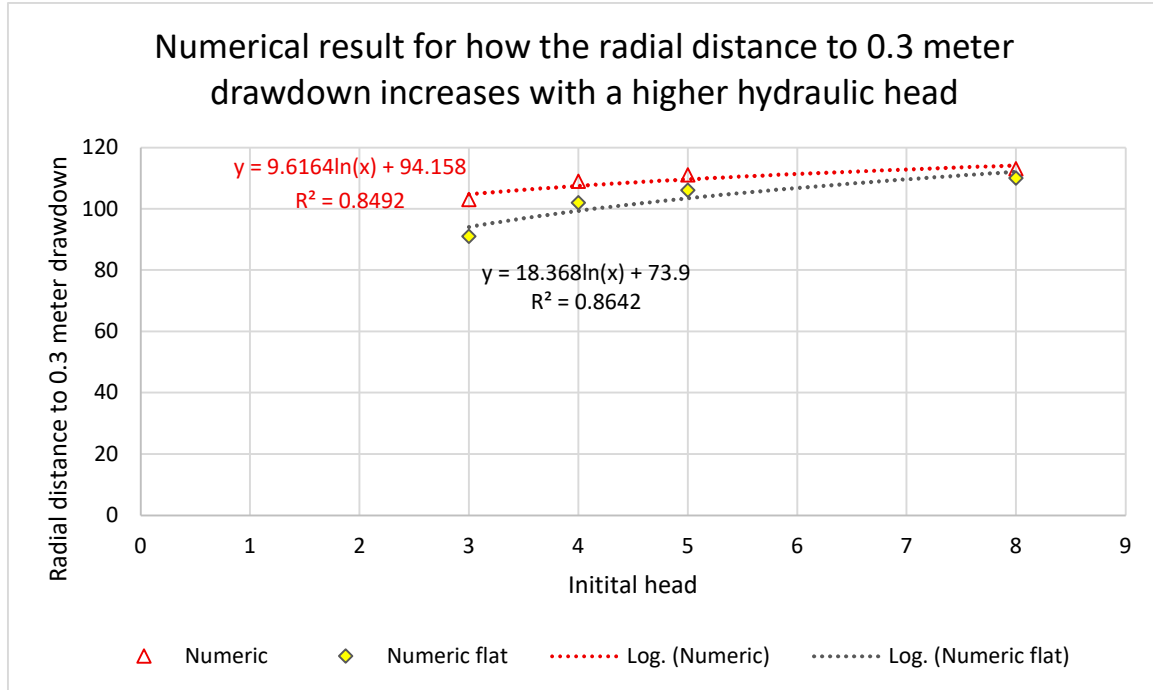


Figure 19 Numerical results for when the difference between shaft level and initial head is 3 meters.

3.2.3 Flowline method comparison

The flowline method compared the results of the radius of influence instead of the previously compared radial distance to 0.3 m drawdown. The flowline method had a reduced difference between the results when the layer below the shaft decreased, when the numeric and Dupuit method were compared. This is shown in figure 20. However, when the numeric and Thiem method were compared there was no correlation between the layer below and the difference in result, shown in figure 21.

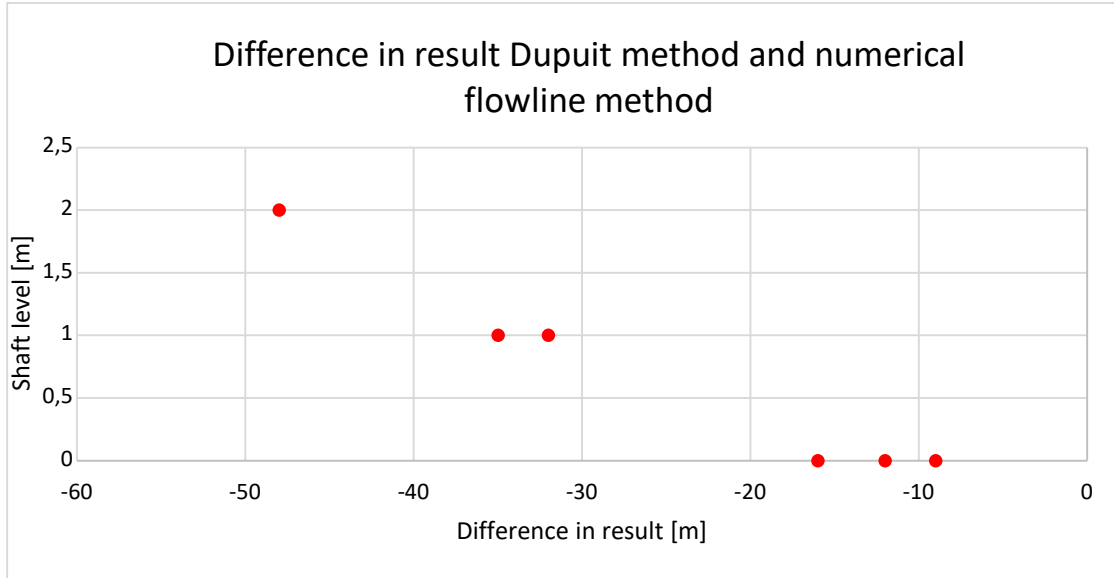


Figure 20 Difference in results between the numerical model and Dupuit's method for model with increased domain to 300x300m.

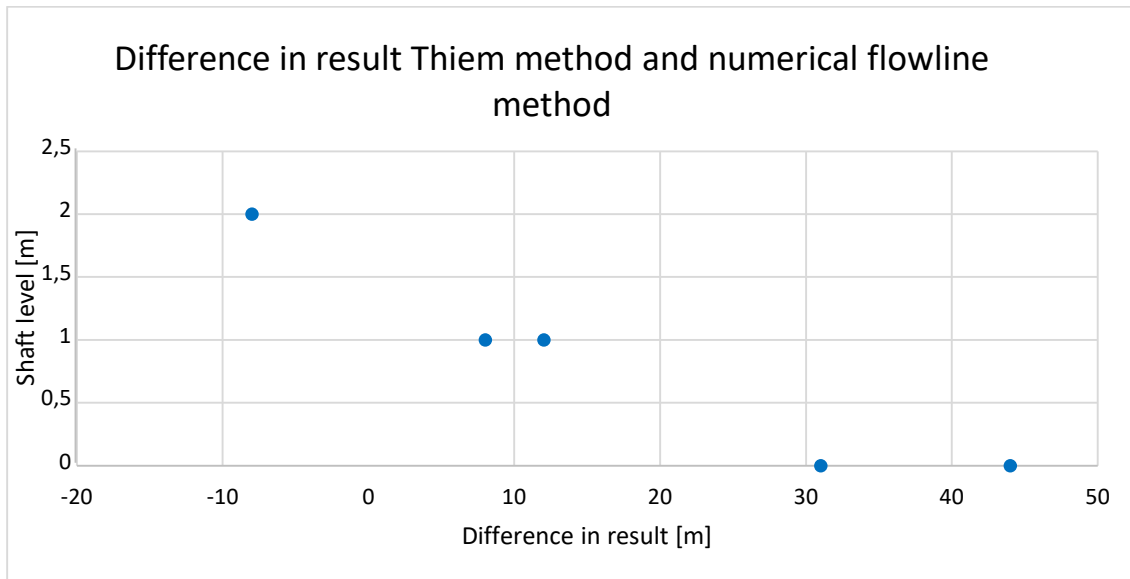


Figure 21 Difference in results between the numerical model and Thiem method for model with increased domain to 300x300m.

Figures 22 and 23 compare the results from the flowline method with the radius of influence obtained using the Dupuit method. When the radius of influence was plotted against the shaft level, no clear correlation between the two methods was observed, and the numerical results occasionally fell outside the analytical range, even when accounting for a factor of ten uncertainty in hydraulic conductivity. However, when the radius of influence was compared to the initial head instead, the numerical results showed strong agreement with the analytical values. Notably, the alignment was not only within the uncertainty range but also closely matched the mean value of the hydraulic conductivity.

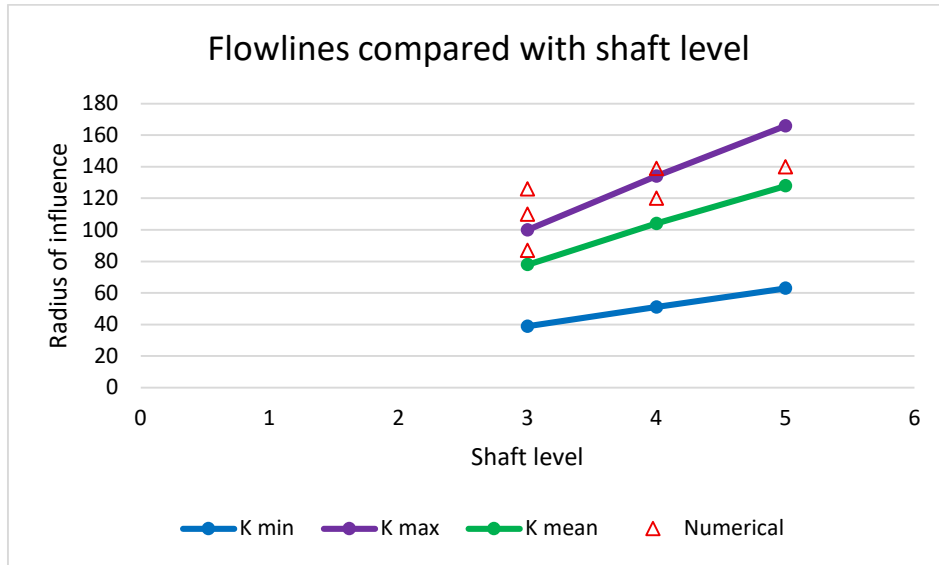


Figure 22 Results of radius of influence with the flow lines method and with the Dupuit method, where the Dupuit method was modelled with insecurity for the hydraulic conductivity (factor 10 difference between K_{min} and K_{max}).

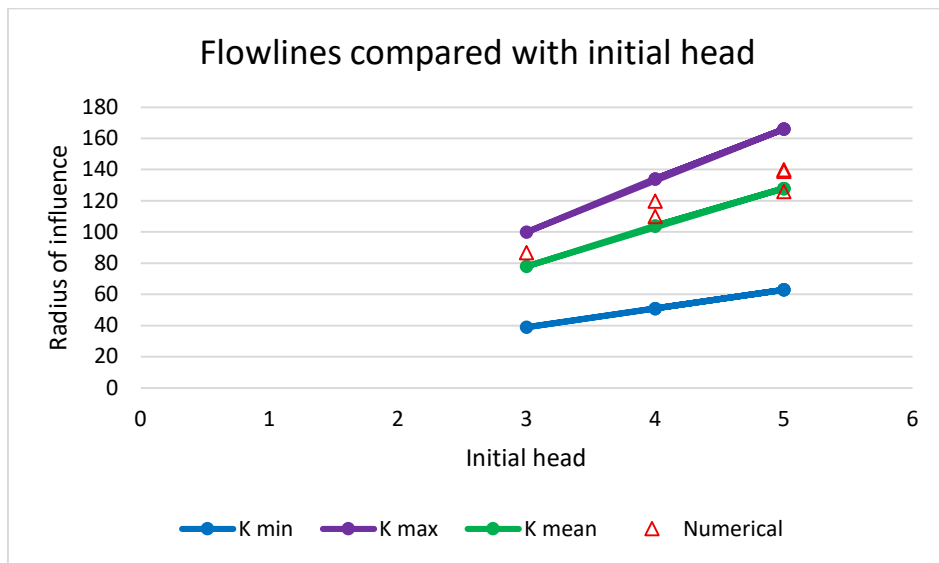


Figure 23 Results of radius of influence with the flow lines method and with the Dupuit method, where the Dupuit method was modelled with insecurity for the hydraulic conductivity (factor 10 difference between K_{min} and K_{max}).

In figure 24, the results from the numerical method using the mean hydraulic conductivity and the maximum hydraulic conductivity models are compared. The Y-axis shows their results whilst the X-axis show the model number which is explained in table 3 below the figure. In the analytical models the initial hydraulic head and the shaft level were both included. The result for the lowest hydraulic conductivity is not shown since the numerical result is placed in between the results for the mean conductivity and maximum conductivity.

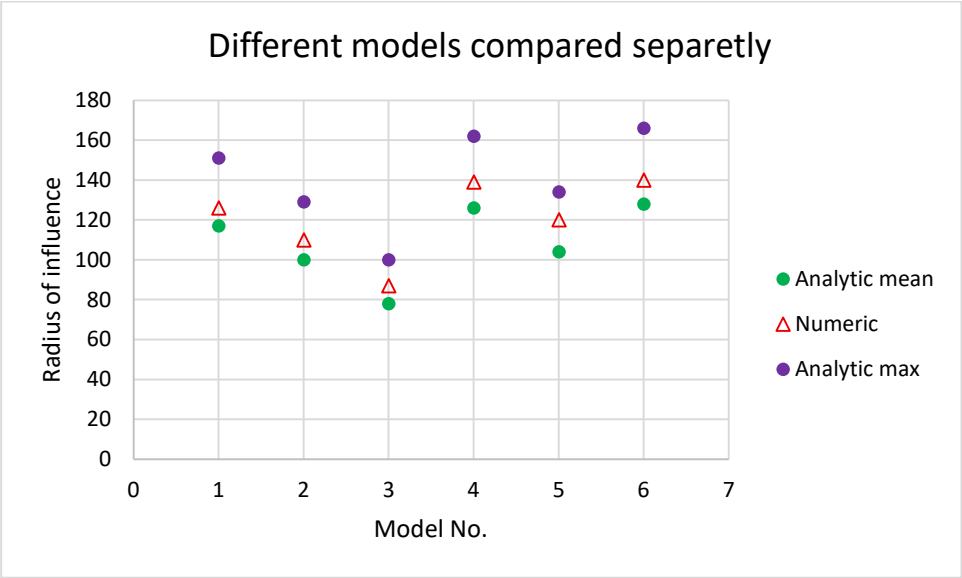


Figure 24 Results of radius of influence for different models.

In table 3 the analytical results are shown as K-max and K-mean, which indicates if the result is from the mean hydraulic conductivity or the maximum conductivity. The result comparison at the end is a comparison between the numerical result and the mean conductivity result.

Table 3 Data for the models presented in figure 24

Model No.	Head	Shaft level	Difference between hydraulic head and shaft level	Numeric result	Analytical result (K-max)	Analytical result (K-mean)	difference in result
1	5	2	3	126	151	117	-9
2	4	1	3	110	129	100	-10
3	3	0	3	87	100	78	-9
4	5	1	4	139	162	126	-13
5	4	0	4	120	134	104	-16
6	5	0	5	140	166	128	-12

Figure 25 presents the relationship between initial hydraulic head and radius of influence for the three cases where the difference between hydraulic head and shaft level is 3 meters in the flow line method. Note that the X-axis is for initial hydraulic head and not for shaft level, which was presented in figure 18 in chapter 3.2.3.

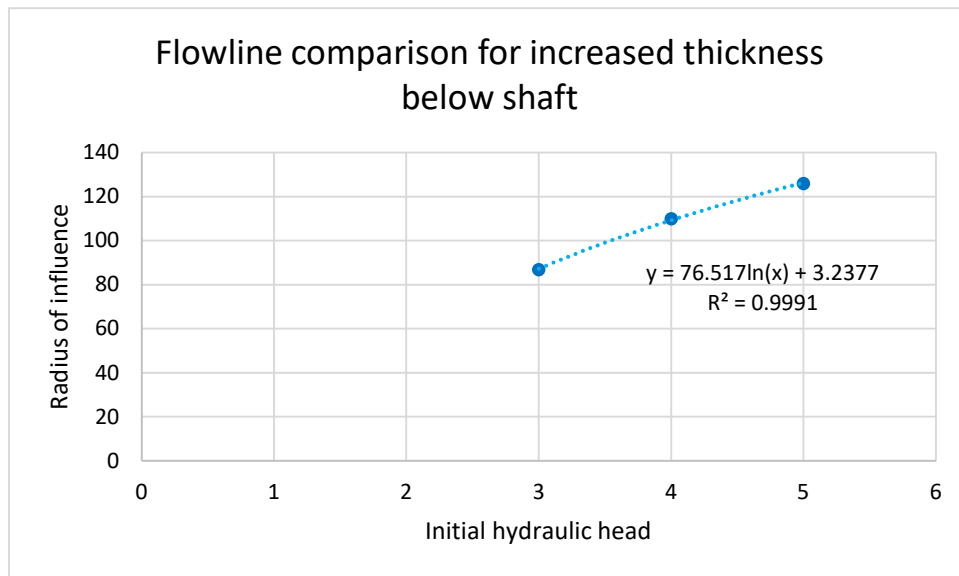


Figure 25 Flow line method results for when the difference between initial hydraulic head and shaft level is 3 meters.

3.3 Summary of results

In the results the least difference between the analytical and numerical method was when the Dupuit method was used and the hydraulic head inside the shaft was set to the same as in the numerical model. When the radius of influence was determined through the flowline method the results were even more similar.

4 Discussion

4.1 Interpretation of different model results

4.1.1 Interpretation of the results from model with domain 100x100 m and hydraulic conductivity 0.1 m/d

The most apparent influence on the numerical model's radial distance to a 0.3 m drawdown observed in this model, is the significant effect of the boundary conditions on the results. This became more apparent when the domain size was increased. As the calculated drawdown consistently approaches the location of the boundary, it can be argued that the boundary conditions may be constraining the outcomes, thereby limiting the reliability of the numerical method. Moreover, the analytical model's radial distance to a 0.3 m drawdown, was in some cases larger than the distance from the numerical model's center to the boundary condition. Meaning that it was impossible for the numerical model to acquire the same result as the analytical model.

4.1.2 Combined interpretation of the results from the models with domain 300x300 m

From the different graphs it can clearly be seen that of the two analytical methods, the Dupuit method consistently produced results most similar to those of the numerical method, despite the aquifer only being partially penetrated. When the difference between the initial hydraulic head and the shaft level remained constant, the numerical results for the radial distance to 0.30 m drawdown and the radius of influence varied as the absolute values of the initial hydraulic head and shaft level varied. Since the Thiem method is linear it only takes the difference between the initial hydraulic head and the shaft level into account. Leading to that there is no difference as the initial hydraulic head varies if the difference between the shaft level and the initial hydraulic head remains the same.

Similar to the results from the model with domain 100x100 m, the boundary conditions seemed to have a large impact for the other results as well. This is most likely what causes the very large differences in results for the model with increased hydraulic conductivity. The analytically modelled radial distance to 0.3 m drawdown was sometimes much larger than the distance from the centre of the numerical model to the boundary conditions, meaning that it would be impossible for the numerical model to have a similar result as the analytical one.

However, when the domain size of the model is increased to solve the issue with the analytically calculated radial distance to 0.3 m drawdown exceeding the distance from the centre of the numerical model to its boundary, the hydraulic head in the middle of the numeric model become greater than its total height. To mitigate the high hydraulic head

in the middle of the model, a higher hydraulic conductivity is required, which increases the analytically modelled radial distance to 0.3 m drawdown, calculated from the analytical model. Considering this issue the most viable solution is studying the flow lines, to determine a radius of influence instead of the area with drawdown.

4.2 Comparing numerical results with analytical results

The two different methods with which the results were compared in this report was either through calculating the radial distance to where there was a 0.3-meter drawdown or by examining the radius of influence in the numerical method by studying the radius of the zone inside which all recharged groundwater travelled to the shaft. In the second method the numerical result was compared with the analytical result of the radius of influence, which is the same as the radial distance to where the drawdown is 0 meters.

It should be noted that for the numerical model the radial distance to 0.3 m drawdown was consistently greater than the radius of influence determined through the flow lines. In contrast, the analytical distance to the 0.3-meter drawdown compared to its radius of influence, is always shorter. In the study by El-Hames (2020), the numerical radius of influence was defined as the distance from the centre of the shaft to the point with highest hydraulic head in the aquifer. This generated quite similar results between the numerical and the analytical model.

As mentioned in chapter 4.1.2. the most similar results between the numerical and analytical method occurred when the Dupuit method was compared to the numerical flow line method. With no in situ comparison to determine what an actual result would be, it is hard to determine whether the similarity in result is purely coincidental or if the comparison of the radius of influence through flowlines compared with the radius of influence through Dupuit's method is a superior method of comparing analytical and numerical results.

However, considering how the analytical Dupuit model functions, it assumes that outside of the radius of influence no water will flow towards the shaft. Similarly, when determining the radius of influence using the flow line method, any recharge occurring outside the radius of influence does not contribute to flow toward the shaft. Intuitively, this is a more appropriate basis for comparison, as the same flow behavior is being assessed in both the numerical and analytical models.

4.3 Effects of a well not fully penetrating the aquifer

From the flowline studies an added thickness below the shaft increased the radius of influence substantially and should not be overlooked. Since there were few results with this method it is difficult to determine how the correlation between the increased size of the layer below and the increase of radius of influence is shaped.

However, for the model with higher conductivity, the results were compared as both the initial hydraulic head and shaft level were increased while maintaining a constant difference of 3 m between them. If one compares the increase of the distance to where the drawdown is 0.30 meters, they seem to follow a logarithmic curve. However, the only data that can be used from this study are three points which is far from sufficient to either prove or disprove that the increase of this radius follows a logarithmic shape. It could be possible to acquire an empirical addition to the equations that take the layer below into account. In the introduction a study with Bangladeshi testing wells by Barker & Herbert, (1992) was mentioned where a method was used which added an empirical equation to the Thiem equation to adapt to a thickness below the bottom of the shaft. This addition was not used in this report as it was not sufficiently prevalent in scientific literature to be reliable.

The analytical result from the Thiem method did not vary with the increased aquifer thickness as the method has a linear dependency on the difference between the initial hydraulic head and the shaft level. In contrast, the Dupuit method varied with the increased thickness as it depends on the squared values of the hydraulic head and shaft level. When the Dupuit method was compared to the flow line-method, the difference between the analytically and numerically determined radii of influence appeared to be quite consistent as the initial parameters were varied. This is particularly seen in figure 23.

4.4 Best methods of comparison

From the study it appears that the first numerical method to determine a radial distance to a drawdown of 0.3 meters mainly depends on the radial distance to the boundary conditions rather than the shaft level or the thickness of the model. This also caused a fundamental difference between the numerical and analytical model as the analytical model's boundary is the radius of influence, inside which all the aquifer recharge will flow into the shaft. Whereas the numerical model was given a pre-determined domain size which can limit the possibility of modelling the groundwater situation properly.

Continuing analysing this fundamental model difference, the flowline method comparison evaluates the same radius as the analytical model. Moreover, if a comparison is to be made between a numerical and analytical model the same parameter should be calculated from both for a comparison to be relevant. Unfortunately, there was not sufficient time to compute more comparisons with the flow line method. But from the data acquired, when the same input was given into both methods the difference in result between the analytical and numerical results did not seem to have a relevant change when a thickness below the shaft was added.

4.5 Possible errors

As always, when using models there will be errors. The errors are caused by the fact that a model is an attempt to replicate the real world. The parameters throughout the modelled

area are (almost) never as homogenous and isotropic as they are modelled. In this model there was no real area that was replicated, but an idealised hypothetical scenario created to fulfil as many of the assumptions required for the analytical model. However, the initial flat groundwater surface was never fulfilled in the numerical, since the model is constantly refilled from precipitation which causes an undulating shape of the groundwater surface. This means to that it is impossible, even with an idealised case, to recreate the assumptions of an analytical model. Due to this the comparison between the two models was never as accurate as desired.

For the model set up with increased domain to 300x300 meters, there were some very high head values in the middle for some numerical models. For example, when the initial head was intended to be set as 5 m it reached above 5.8 m in the centre of the model. These higher head values in the middle deviate from the intended flat groundwater surface and probably contribute to the results sometimes decreasing as the shaft level increases. To adjust for this, the inflow should probably be set lower, or the hydraulic conductivity be increased. Moreover, when the same model was remade with a higher conductivity the results appeared much more reasonable.

There is also a significant simplification in the numerical model where the hydraulic head boundary condition was set as the same value for the entire shaft. There is a possible of seepage throughout the shaft and the pressure inside the shaft is atmospheric pressure. Another way to model this which could have been better would have been to give the walls the hydraulic head boundary condition as its height. Meaning that instead of giving all the walls the same value as the shaft, they should vary with the height of the walls.

Despite the results being slightly closer to the boundary conditions for the higher conductivity it is only a marginal difference, and it could be that the boundary conditions create a too large effect on the numeric model. Thus, it could make the results unreliable.

As mentioned in chapter 4.4. a fundamental difference between the analytical and numerical model is their spatial limitations. A numerical model's domain extent is determined by the modeller's decisions and generally extends beyond the radius of influence, whereas the analytical model is limited by its radius of influence. For the same reason a numerical model can also be made too small limiting its ability to accurately model the groundwater situation in the aquifer. Furthermore, the analytical model does not stretch further in height than between the initial hydraulic head and the shaft level. Thus, any thickness below is not modelled, and its effects are thereby not accounted for.

Of course, there is also always a human factor during the measurements. While measuring the radius of influence or the radial distance to 0.3 m drawdown, an assessment has to be made by the modeller to determine where either the radius of influence is as well as the point with 0.3 m drawdown.

5 Conclusion

This study investigated the feasibility of using analytical models, for groundwater modelling in cases where a shaft only partially penetrates an unconfined aquifer, under idealised and homogenous conditions. The comparison between the analytical Dupuit method and the numerical flowline method showed a consistent relationship for the radius of influence, even when the aquifer was only partially penetrated. Provided that the aquifer bottom was used as the reference height.

However, the analysis was limited to testing a few conditions, with high hydraulic conductivity and steady-state assumptions. Although the results indicate that the radius of influence from the Dupuit method provides an accurate result under certain conditions, further studies are needed to assess the reliability of the analytical models in a wider range of hydraulic conductivities and aquifer properties.

Based on the results, the Dupuit method is considered applicable for determining the radius of influence in a homogenous, isotropic aquifer with a hydraulic conductivity close to 1 m/d. Further studies are needed to verify its applicability in other hydraulic conductivities.

6 Future work

A continuation of this work should be made to verify the correlation for more conditions. This study had quite high hydraulic conductivities. Future research should investigate larger variations in hydraulic conductivity and include thicker layers below the shaft. For comparing the results from numerical and analytical models, the radius of influence from the numerical flowline method and the radius of influence from the analytical Dupuit method (using the aquifer bottom as the reference height of 0 meters) should be used.

Moreover, to continue the studies in this area it would be beneficial to have a real-world example to compare with. Along with the results from the models such an example would be used to compare and examine which one is more accurate depending on the conditions surrounding it. The problem is that the models are made on ideal conditions which are extremely unlikely to be found in the real world. Especially on such a large scale. However, it could be possible to build a scaled model of the aquifers and test its result compared to the results from the calculated models. However, there are probably errors in the built model as well. Meaning that several models should be built to increase the probability that random errors does not impact the final result.

There are several other assumptions that were not tested in this study that could have been studied to examine what assumptions are the most critical for the analytical models to give a valid result. A study where several assumptions are not fulfilled in different combinations should also be made to examine whether some assumptions must be fulfilled at the same time for accurate results to be acquired.

However, it could still be possible that modelling the aquifer analytically with the shaft level as reference height 0 meters and adding an empirical function provides better results for other aquifer conditions, for example if the hydraulic conductivity is much lower or if the unpenetrated part of the aquifer is much larger than in this study. This empirical equation should be logarithmical.

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APPENDIX

Results presented in tables

Tables A and B show the results from the models with domain 100x100 m and hydraulic conductivity 0.1m/d. In table A the Dupuit method is used for determining the radial distance to 0.3 m drawdown. For some of the model's the Dupuit equation did not converge a solution. It is modelled according to figure 2 where the thickness below the shaft is neglected. Table B show the results with the Thiem method as analytical model.

Table A Results from model compared with Dupuit's method

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	38	21	-14	12
5	2	7	3	43	21	-19	7
4	1	6	3	42.5	21	-18.5	7.5
3	0	5	3	42	21	-18	8
5	1	7	4	44	31	-13	6
4	0	6	4	43	31	-12	7
5	0	7	5	44.5	43	-5.5	5.5
8	2	10	6	45	N/A	N/A	5
6	0	8	6	44.5	N/A	N/A	5.5
8	1	10	7	45.5	N/A	N/A	4.5
7	0	9	7	45	N/A	N/A	5
8	0	10	8	45.5	N/A	N/A	4.5

Table B Results from model compared with Thiem's equation

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	38	36	-2	12
5	2	7	3	43	36	-7	7
4	1	6	3	42.5	36	-6.5	7.5
3	0	5	3	42	36	-6	8
5	1	7	4	44	47	3	6
4	0	6	4	43	47	4	7
5	0	7	5	44.5	58	13.5	5.5
8	2	10	6	45	69	24	5
6	0	8	6	44.5	69	24.5	5.5
8	1	10	7	45.5	80	34.5	4.5
7	0	9	7	45	80	35	5
8	0	10	8	45.5	91	45.5	4.5

Tables C and D presents the results from the model with domain 300x300 m and hydraulic conductivity 8.64 m/d. Table C show the results for the Dupuit method as analytical method where the thickness below the shaft is neglected according to figure 2. Table D show the results when the Thiem method is used as analytical model.

Table C Results from model (enlarged to 300x300m, with hydraulic conductivity 8.64 m/d) compared with Dupuit's equation

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	113	76	-37	37
8	4	10	4	121	235	114	29
8	3	10	5	125	179	54	25
8	2	10	6	128	238	110	22
8	1	10	7	130	300	170	20
8	0	10	8	132	369	237	18
7	0	9	7	128	300	172	22
6	0	8	6	125	238	113	25
5	2	7	3	111	76	-35	39
5	1	7	4	117	235	118	33
5	0	7	5	120	179	59	30
4	1	6	3	109	76	-33	41
4	0	6	4	113	235	122	37
3	0	5	3	103	76	-27	47

Table D Results from model (enlarged to 300x300m, with hydraulic conductivity 8.64 m/d) compared with Thiem's equation

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	113	201	88	37
8	4	10	4	121	283	162	29
8	3	10	5	125	365	240	25
8	2	10	6	128	447	319	22
8	1	10	7	130	528	398	20
8	0	10	8	132	609	477	18
7	0	9	7	128	528	400	22
6	0	8	6	125	447	322	25
5	2	7	3	111	201	90	39
5	1	7	4	117	283	166	33
5	0	7	5	120	365	245	30
4	1	6	3	109	201	92	41
4	0	6	4	113	283	170	37
3	0	5	3	103	201	98	47

Tables E and F presents the results from the model with domain 300x300 m and hydraulic conductivity 8.64 m/d. For the numerical model the initial water surface was set to be completely flat. Table E show the results for the Dupuit method as analytical method where the thickness below the shaft is neglected according to figure 2. Table F show the results when the Thiem method is used as analytical model.

Table E Results from model (enlarged to 300x300m, with hydraulic conductivity 8.64 m/d) compared with Dupuit's equation, and a flat initial water surface in the numerical model

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	110	76	-34	40
8	4	10	4	119	235	116	31
8	3	10	5	124	179	55	26
8	2	10	6	127	238	111	23
8	1	10	7	128	300	172	22
8	0	10	8	129	369	240	21
7	0	9	7	127	300	173	23
6	0	8	6	122	238	116	28
5	2	7	3	106	76	-30	44
5	1	7	4	113	235	122	37
5	0	7	5	117	179	62	33
4	1	6	3	102	76	-26	48
4	0	6	4	106	235	129	44
3	0	5	3	91	76	-15	59

Table F Results from model (enlarged to 300x300m, with hydraulic conductivity 8.64 m/d) compared with Thiem's equation, and a flat initial water surface in the numerical model

Water level pre-shafting	Shaft level	Total model height Numeric	Difference between hydraulic head and shaft level	Numerical result	Analytical result	Difference in result between the two methods	Numerical result's distance to boundary condition
8	5	10	3	110	201	91	40
8	4	10	4	119	283	164	31
8	3	10	5	124	365	241	26
8	2	10	6	127	447	320	23
8	1	10	7	128	528	400	22
8	0	10	8	129	609	480	21
7	0	9	7	127	528	401	23
6	0	8	6	122	447	325	28
5	2	7	3	106	201	95	44
5	1	7	4	113	283	170	37
5	0	7	5	117	365	248	33
4	1	6	3	102	201	99	48
4	0	6	4	106	283	177	44
3	0	5	3	91	201	110	59