Lund University
Department of Economics

Master thesis, September 2011
Course: NEKM01

Is inflation mean-reverting?

A fractional integration approach.

Author
Niclas Lavesson

Supervisor
Fredrik NG Andersson

Abstract

This paper investigates whether inflation series are mean-reverting. Traditional unit root tests (ADF and KPSS tests) are conducted in the paper. These tests indicate that inflation contains a unit root. It is however well-known that traditional unit root tests have difficulties to distinguish between a unit root process and a fractionally integrated process (see e.g. Diebold & Rudebusch, 1991). Hence, in this thesis, fractional integration estimators are used in order to investigate if inflation is better modeled as a fractionally integrated process.

The main finding of this paper is that inflation with a high certainty is fractionally integrated and mean-reverting. Another important finding is that the persistence in inflation series likely was lower during the Bretton-Woods era relative the years after the system collapsed. Moreover, the findings in the paper do not crucially depend on which fractional integration estimator that are used.

Keywords: inflation, fractional integration, mean-reversion, unit root

Table of contents

1. Introduction

Inflation is often modeled as a non-stationary unit root process (see e.g. Ball & Cecchetti, 1990; Brunner & Hess, 1993; MacDonald & Murphy, 1989). If inflation contains a unit root, a shock to the inflation series has a permanent effect (Greene, 2003). In such case, successful inflation targeting in order to control the path of inflation is difficult to achieve for the Central Banks (see e.g. Gregoriou & Kontonikas, 2006). To avoid misalignments from a desired inflation level the Central Banks need appropriate policy rules. The design and implementation of economic policy rules depends on characteristics of the inflation series (Dias & Marques, 2010). Knowledge about the integration order and the time it takes for inflation series to revert back to its mean value after a disturbance provides policymakers valuable information. The purpose of this paper is therefore to investigate the statistical properties of inflation and primarily if inflation is mean-reverting.

Since inflation is a key macroeconomic variable an increased understanding about the inflation process is undoubtedly important. Inflation forecasts, for instance, can be improved by an increased knowledge about the time series properties of inflation (see e.g. Hendry & Hubrich, 2006; Hubrich, 2005). Moreover, economic frameworks are built upon assumptions of inflation series. For instance, the long run Fisher hypothesis states that inflation and nominal interest rates are cointegrated (Fisher, 1930). In order for variables to be cointegrated they must have the same integration order (Greene, 2003). In Jensen (2009) it is shown that the nominal interest rates and inflation series have different integration orders. Furthermore, Jensen (2009) shows that inflation is mean-reverting and not a unit root process. As a consequence, the long-run Fisher hypothesis cannot be tested (Jensen, 2009).

According to traditional integration literature a time series is modeled as either a stationary process or simply a non-stationary unit root process (Brockwell & Davis, 2002). Fractional integration literature provides a somewhat less restricted definition of the time series properties (see e.g. Baillie, 1996; Granger, 1980; Granger & Joyeux, 1980; Hosking, 1981). According to fractional integration literature a time series can take any integration order. Hence, a time series is not restricted to be modeled as either a stationary process or a non-stationary unit root process. For instance, a time series can be non-stationary but not necessarily contain a unit root. Such time series is referred to as a fractionally integrated process with long memory or simply a long memory process (Granger, 1980). The distinction between a unit root process and a long memory process is that the latter is highly persistent but mean-reverting in the long run. In other words, a shock eventually dissipates and the time series returns to its pre-shock mean value. As a result, the Central Banks can use appropriate economic tools to maintain price stability and thus control the inflation level to some extent.

Several papers in the past conclude that inflation is a unit root process (see e.g. Ball & Cecchetti, 1990; Brunner & Hess, 1993; MacDonald & Murphy, 1989). A vast majority of the studies referred to use traditional unit root tests such as the tests of Dickey-Fuller (1979), Kwiatkowski-Phillips-Schmidt-Shin (1992), Phillips-Perron (1988) to mention some. The conclusion that inflation is an integrated unit root process is however challenged by empirical observations and earlier studies (see e.g. Barsky, 1987; Brimmer, 2002; Culver & Papell, 1997; Dittmar et al., 1999; Mishkin & Posen, 1997; Rose, 1988; Svensson, 1999). Findings in Barsky (1987), Culver & Papell (1997) and Rose (1988) indicate that inflation is a stationary process. According to Brimmer (2002) a number of Central Banks have been successful in inflation targeting in order to control the inflation rate. Similar results as in Brimmer (2002) are found in Mishkin & Posen (1997). Clearly, there is some evidence that Central Banks actively can affect the inflation rate with economic policy.

The view that inflation is a unit root process can be due to shortcomings of the traditional unit root tests as these tests do not involve the case of fractionally integrated series (see e.g. Dickey & Fuller, 1979; Kwiatkowski et al., 1992; Phillips & Perron, 1988). Several papers have pointed out weaknesses of the traditional unit root tests (see e.g. Blough, 1992; Cochrane, 1991; Diebold & Rudebusch, 1991; Hassler & Wolters, 1994; Leybourne & Newbold, 1999). The one or two most important findings in these papers are that the power and size of traditional unit root tests are poor. The tests' weak power implies that the statistical tests cannot distinguish between a unit root process and a fractionally integrated series with long memory (Baillie, 1996). As a consequence, a mean-reverting time series is incorrectly considered as a unit root process.

Methods that involve fractionally integrated series have been used in numerous studies since around 1980s (Baillie, 1996). For instance, Coleman (2010) investigates asymmetries in monetary unions. According to Coleman (2010), inflation series are expected to be more persistent in some countries of a monetary union. Therefore a common inflation shock has a larger effect in these member countries as well. As a consequence, joint policy decisions have different effects in each country. For similar studies as Coleman (2010), see e.g. Hofmann & Remsperger (2005); Mooslechner & Schuerz (1999) and Batini (2006).

A more general result found in papers using fractional integration methods is that inflation series for individual countries are mean-reverting (see e.g. Choudhry, 2001; Coleman, 2010; Jensen, 2009; Zagaglia, 2009). In these studies, there are no unambiguous findings whether inflation processes are stationary or not since this partly depends on which time period investigated, the type of estimator used and upon the country being examined. In this paper the main result is in accordance with findings in earlier studies since the analysis shows that inflation likely is a mean-reverting process. Moreover, findings in the paper indicate that inflation was less persistent during the Bretton-Woods era in comparison with the years after the system collapsed.

In order to answer whether inflation is mean-reverting, the analysis of the paper begins with performing traditional unit root tests. These tests are conducted to examine whether inflation can be modeled as an integrated unit root process. The unit root tests used in the paper are the tests of Dickey-Fuller (1979) and Kwiatkowski-Phillips-Schmidt-Shin (1992); henceforth called the ADF and KPSS test. Next, fractional integration estimators are used to examine if inflation is better modeled as a fractionally integrated process. Fractional integration methods mainly operate in the frequency domain. Analyzing a time series in the frequency domain has its advantages over traditional time series analysis. The time series of interest can be analyzed in more detail in the frequency domain than what is possible in the time domain; cycles and irregularities, for instance, are easier found when viewing the process in the frequency domain (Brandes et al., 1968; Chatfield, 1996). There are a vast variety of fractional integration estimators available and the ones used in this paper are the Geweke Porter-Hudak (1983) estimator, Robinson (1995) estimator and Jensen's (1999) Wavelet OLS estimator. Hereafter called the GPH, Robinson and WOLS estimator respectively. In this paper, the countries under investigation are Canada, Germany, Norway, Sweden, the United Kingdom and the United States.

This study extends the existing literature by employing fractional integration methods to, in the context, longer time series of monthly inflation data. The main contribution of the paper is that the somewhat newly developed fractional integration methods are used when analyzing inflation. Since the fractional integration methods are developed in later years they are not yet widely employed. Hence, the amount of earlier research employing fractional integration methods is rather limited. The lack of earlier research is seemingly true when considering the WOLS estimator and analysis of the time series properties of inflation.

The remainder of this paper is organized as follows. In the next section, some econometric methodology is provided. Fractionally integrated series are described and the methods used in the paper and some properties of these are described as well. In section 3, the data material used in the analysis is presented. Several plots over the inflation series as well as some summary statistics are provided. Moreover, possible explanations behind the countries' inflation patterns are discussed in section 3. The analysis of the paper is found in section 4. In the analysis, traditional unit roots tests are conducted and the fractional integration parameter is estimated using the fractional integration estimators. The conclusions and some discussion about the results in the paper are found in section 5.

2. Estimators of the fractional integration order

In section 2.1 fractionally integrated series are described. An explanation of the properties of the fractionally integrated series is provided as well. As the fractional integration estimators mainly not operate in the time domain, theory of spectral estimation as well as wavelet estimation are described in section 2.2 and 2.3.

The fractional integration estimators are described in the following sub-sections. In section 2.2.2 and 2.2.3 the spectral estimators (GPH and Robinson) are described. The GPH and Robinson estimators are referred to as spectral estimators, or equivalently frequency domain estimators, since these operate only in the frequency domain. The WOLS estimator is described in section 2.3.1. Since the WOLS estimator operates in both the time domain and the frequency domain it is not a pure spectral estimator. Hence, this estimator is rather referred to as a wavelet estimator.

2.1 Fractionally integrated series

A time series is integrated of order d if the series can be represented as an invertible autoregressive moving average (ARMA) process after being differenced d times. When d is equal to zero the series is stationary, while d equals to one defines a non-stationary unit root process. In the latter case, the series needs to be differenced one time (since d is equal to one) to get an invertible ARMA representation and thus a stationary process (Brockwell & Davis, 2002).

In traditional unit root literature the value of d is restricted to integer values (i.e. 0 or 1). Fractionally integrated processes can be seen as generalizations of integrated processes since the parameter d also takes non-integer values. The simplest form of a fractional integration process is the fractional white noise described in Granger (1980), Granger & Joyeux (1980) and Hosking (1981) and is defined as:

$$(1-L)^d x_t = \varepsilon_t \tag{1}$$

where L is the lag operator, ε_t is a white noise process (i.e. an independently and identically distributed (iid) process with zero mean and the variance σ_ε^2) and x_t is a fractionally integrated process of order d. The parameter d is the fractional integration parameter (FI-parameter in Table 1), or equivalently the memory parameter. Properties of the fractional integration parameter d are summarized in Table 1.

Table 1 – Properties of the fractional integration parameter (*d*).

Memory	FI-parameter (d)	Mean-reverting	Variance	Characteristics
No	$d=0$	Yes	Finite	Covariance stationary
Short	$0<d<0.50$	Yes	Finite	Covariance stationary
Long	$0.50 \leq d < 1$	Yes	Infinite	Covariance non-stationary
Infinite	$d \geq 1$	No	Infinite	Covariance non-stationary

Sources: Granger (1980); Granger & Joyeux (1980); Hosking (1981); Tkacz (2001)

A process with no memory is mean-reverting as well as finite in variance. In other words, the process is stationary. Equation (1) reveals that stationarity occurs when the fractional integration parameter is equal to zero. When d is equal to zero, Equation (1) translates into $x_t = \varepsilon_t$. Since ε_t by assumption is a white noise process, the time series x_t is a white noise process as well when the memory parameter equals zero.

Time series with short memory has a value of the fractional integration parameter within the interval ($0 < d < 0.50$). A series with short memory has the same properties as a process with no memory except that the mean-reversion process is slower. Once a shock hits the process, the series reverts back to the pre-shock mean value later in time. Hence, any shock eventually dissolves when the time series has short memory.

Long memory time series differ from the stationary alternatives covered so far. The long memory process is still stationary in its mean value and is hence mean-reverting. However, since the fractional integration parameter lies within the span ($0.50 \leq d < 1$) covariance non-stationarity is implied (Granger, 1980; Granger & Joyeux, 1980; Hosking, 1981; Tkacz, 2001). The property of covariance non-stationarity arises due to the fact that the variance no longer is finite. In all, the process reverts back to the mean level after a shock or disturbance even if it possibly takes longer time in comparison with the no memory and short memory time series.

The time series has the property of infinite memory when $d \geq 1$. Infinite memory implies that the series is non-stationary in both mean and variance with no tendency moving back to any deterministic component (Tkacz, 2001). In other words, a series with infinite memory contains a unit root. From a statistical point of view, a series with a unit root is somewhat problematic. For instance, the variance becomes explosive as the number of observations increases to infinity (Greene, 2003). Moreover, since a time series with infinite memory is not mean-reverting such process randomly drifts away in any direction. To sum up, any disturbance to a process with infinite memory is neither predictable nor transitory.

2.2 Spectral estimation theory

In pure spectral analysis, or equivalently frequency domain analysis, the time series is expressed as frequencies in the frequency domain instead of observations in the time domain. The different time horizons in an economy are represented by frequency bands in the frequency domain. The long run horizon is represented by frequencies in the vicinity of the zero frequency. Equivalently, the medium and short run of the time series such as oscillations or business fluctuations are represented by higher frequencies (Andersson, 2008).

An orthonormal transformation is the procedure of translating a time series from one domain into another. In frequency domain analysis the time series is translated from the time domain into the frequency domain (Andersson, 2008). It should be possible to use an orthonormal transformation to the transformed series as well, ending up with the original series in the original domain (Percival & Walden, 2000). None of the statistical properties of the time series change even though the process has been subject to an orthonormal transformation. The mean, variance, autocovariance function and other time series properties are exactly the same as before the transformation. The difference is that these properties are expressed in another domain (Andersson, 2008).

When expressing a time series in the frequency domain, the orthonormal transformation is the Fourier transformation. Basis functions in such transformation consist of sine and cosine functions (Percival & Walden, 2000). Consequently, the time series is expressed as a sum of sinusoidal components in spectral analysis:

$$x_t = \int_{-\pi}^{\pi} \left(a_f \, cos(ft) + b_f \, sin(ft)\right) df \quad (2)$$

In Equation (2), f is the frequency while a_f and b_f represent the amplitudes of the basis functions at different frequencies in the interval $f \in (-\pi, \pi)$. Frequencies with relatively high values of a_f and b_f, or in other words frequencies with large amplitudes, contribute more in explaining the variation in the time series (Chatfield, 1996). The variance of the time series as well can be represented in spectral terms (Andersson, 2008; Chatfield, 1996):

$$Var(x) = \int_{-\pi}^{\pi} s_x(f) df \quad (3)$$

In Equation (3), the function $s_x(f)$ is called the spectral density or simply the spectrum of the time series. This function describes how much of the variation in the time series each frequency in the frequency interval accounts for (Chatfield, 1996). Equation (3) reveals that all variation in the time series is covered within the frequency interval. The variance is expressed as an integral of the spectrum in the interval $f \in (-\pi, \pi)$ and since 2π completes a cycle all variation of the time series is covered.

There is a connection between the economic time horizons and the spectrum of the time series. A large concentration of the variance around the zero frequency of the spectrum is an indication of a long run component in the time series. Such component could be a trend (Chatfield, 1996). A more intuitive interpretation is that frequencies concerning the long run play a significant role in explaining the process. Similarly, a large concentration at higher frequencies of the spectrum means that medium and short run information dominates the process. If the frequencies in the spectrum are evenly distributed (the spectrum is flat) this means that no time horizon dominates over another in explaining the series; thus all the time horizons are equally important when explaining the variation of the process. Hence, when the spectrum is flat, this corresponds to a white noise process (Baillie, 1996).

2.2.1 The Geweke Porter-Hudak (1983) estimator

The GPH estimator is built upon the spectral density of a time series (Geweke & Porter-Hudak, 1983). Since the estimator is based on the ordinary least squares (OLS) of the log spectrum, it is a spectral regression estimator. The spectrum of an economic time series is given by:

$$s_x(f) = \left\{4\,sin^2\left(\frac{f}{2}\right)\right\}^{-d} s_\varepsilon(f) \qquad (4)$$

In Equation (4), $s_\varepsilon(f)$ is the spectral density of the white noise process ε_t in Equation (1). As the spectrum in Equation (4) is unknown it is estimated by the periodogram (Chatfield, 1996). For simplicity, the terms periodogram and spectrum are used interchangeably hereafter. The periodogram evaluated at the Fourier frequencies $f_i = \frac{2\pi i}{T}$ is denoted $I(f_i)$ for $i = 1,\ldots,m$ where m denotes the number of Fourier frequencies (measured from the spectrum's origin) used in the OLS regression (Geweke & Porter-Hudak, 1983). T denotes the number of observations in the time series. Taking the logarithm of $I(f_i)$ and rewriting it as a log periodogram regression model gives:

$$log(I(f_i)) = \beta - d\,log\left\{4\,sin^2\left(\frac{f_i}{2}\right)\right\} + u_i \qquad (5)$$

where $u_i \sim iid(0, \frac{\pi^2}{6})$ is the disturbance term (Baillie, 1996).

When running a regression on Equation (5) the estimate $\hat{d}_{GPH}$ is obtained. The estimate of the memory parameter can also be obtained using the expression (see Boutahar et al., 2007):

$$\hat{d}_{GPH} = \frac{\sum_{i=1}^{m}(Y_i - \bar{Y})\,log(I(f_i))}{\sum_{i=1}^{m}(Y_i - \bar{Y})^2} \qquad (6)$$

where $Y_j = -\,log\left\{4\,sin^2\left(\frac{f_i}{2}\right)\right\}$ and $\bar{Y} = \frac{1}{m}\sum_{i=1}^{m} Y_i$.

Given that the value of m is chosen correctly, the coefficient $\hat{d}_{GPH}$ is an estimate of the slope of the periodogram in the vicinity of the zero frequency (Greene, 2003). The GPH estimator utilizes that frequencies close to zero in the spectrum concern the long run horizon of the process. Since a non-stationary process is dominated by long run information, a large estimate of d is an indication that the time series is non-stationary. Note that this result corresponds to what Table 1 tells. For instance, an estimate of $\hat{d}_{GPH}$ which is greater than or equal to one is a strong indication that the time series is a non-stationary unit root process (infinite memory); (see Table 1). At the other extreme, the estimated memory parameter $\hat{d}_{GPH}$ is zero. In such case, the time series has no long run behavior. Or in other words, the time series has no memory (see Table 1).

The choice of how many frequencies m to include in the log periodogram regression is not straightforward. The optimal choice of m covers only the frequencies concerning the long run. Inclusion of too many frequencies creates difficulties distinguishing between long run behavior and the fluctuations which arise at medium and higher frequencies (Geweke & Porter-Hudak, 1983). Moreover, the estimate $\hat{d}_{GPH}$ becomes biased if medium and high frequencies are included in the estimation procedure (Geweke & Porter-Hudak, 1983).

The value of m is expressed as a function $m = T^{\alpha}$ where $\alpha = 0.50$ is most commonly used (Geweke & Porter-Hudak, 1983; Greene, 2003). Nevertheless, $\alpha = 0.55, 0.60 \dots 0.80$ can be used as well (see e.g. Crato & de Lima, 1994; Kumar & Okimoto, 2007). In various papers different choices of α works as a tool to determine the robustness of the results (see e.g. Cheung, 1993; Choudhry, 2001).

The properties of the fractional integration parameter have been extensively examined in the past. In a paper by Robinson (1995) an alternative estimator to the GPH estimator is suggested.

2.2.2 The Robinson (1995) estimator

Robinson (1995) points out some shortcomings of the GPH estimator. Most importantly, it is shown that the asymptotic properties of the GPH estimator are poor (Robinson, 1995). Agiakloglou et al. (1993) as well as Robinson (1995) show that the GPH estimator is biased. Robinson (1995) presents a way to lessen the extent of bias by omitting the l first frequencies of the spectrum, hence yielding a consistent estimator (Boutahar et al., 2007; Robinson, 1995).

Since the intuition behind the Robinson and GPH estimator is the same, the Robinson estimator is most easily described in the same form as Equation (6); (see Boutahar et al., 2007):

$$\hat{d}_{ROB} = \frac{\sum_{i=l+1}^{m}(Y_i - \bar{Y})\, log\big(I(f_i)\big)}{\sum_{i=l+1}^{m}(Y_i - \bar{Y})^2}, \;\; 0 \le l < m < T \qquad (7)$$

where $\bar{Y} = \frac{1}{m-l}\sum_{i=l+1}^{m} Y_i$ and l corresponds to the l first frequencies of the spectrum.

Since there is no optimal way to choose the values of l and m problems can arise when employing spectral estimators such as GPH and Robinson (Boutahar et al., 2007). However, an advantage with the spectral estimators used in this paper is that the estimates follow a normal distribution. As a consequence, standard normal theory can be used in the analysis.

2.3 Wavelet estimation theory

The major disadvantage with analysis performed exclusively in the frequency domain is that no time resolution is considered. In the Fourier transformation, the basis functions are periodic since sine and cosine curves are fluctuating with constant amplitude. Such functions have no starting or ending point in time. A Fourier transformation is thus considered to be a global transformation. As a consequence, the underlying time series must be stationary when using the Fourier transformation (Andersson, 2008). Since economic time series rarely consist of smooth and recurring cycles, analysis exclusively performed in the frequency domain is often not an attractive choice for economists (Kennedy, 2008).

An alternative to spectral analysis is wavelet analysis. Wavelet analysis is not exclusively performed in either the time domain or the frequency domain. Instead, both time and frequency resolution are combined (Andersson, 2008). The orthonormal transform used in wavelet analysis is the wavelet transformation. In this paper, the discrete wavelet transformation (DWT) is of interest since inflation data is discrete. In comparison with the Fourier transformation, there is no constraint using sine and cosine curves as basis functions in the DWT. The basis functions in the DWT rather consist of wavelet functions (or simply wavelets). By definition, wavelets are small waves that have a definite starting point in time. Each wave has a certain number of oscillations and persists to a given point in time (Crowley, 2007). The property of a certain starting point and ending point in time guarantees that both time and frequency resolution are combined in the DWT (Andersson, 2008). The DWT can be considered as a local transformation. Hence, the underlying time series in wavelet analysis need not to be stationary in order to achieve a valid analysis (Andersson, 2008).

The central building block when approximating a time series is the mother wavelet. The mother wavelet is a function that takes weighted averages between neighboring observations; simply a weighted difference operator (Crowley, 2007). In the DWT process, the mother wavelet is used to create variants of itself to sequentially cover larger subsets of the data material. Thus, by stretching and translating the wavelet, the complicated structure of the process is divided into smaller less complicated components (Gençay et al., 2002). The translations of the wavelet function yield wavelet coefficients that are located at different scales. Each scale represents a certain frequency band of the time series (Crowley, 2007). Wavelet coefficients associated with low scales represent higher frequency behavior in the time series, such as rapid oscillations. Higher scales represent lower frequencies of the process (Kennedy, 2008).

The time series is represented as weighted sums of wavelets after the series has been transformed by the DWT. The observations are modeled using contributions from every scale. So an observation is approximated by a high frequency wavelet, a wavelet of half that frequency, a wavelet of a quarter of the highest frequency wavelet until the lowest frequency contribution is included (Kennedy, 2008). The strength of wavelet analysis is that the wavelet coefficients are allowed to change depending on which time period considered. Therefore, unusual features which have occurred at different time periods can be captured (Kennedy, 2008).

Wavelet functions exist in many different types and shapes; the Haar wavelet, Daubechies wavelet, Mexican Hat wavelet and Morlet wavelet to mention some (see Crowley, 2007; Gençay et al., 2002; Percival & Walden, 2000 for further reference). Hence the choice of mother wavelet is not straightforward and it is solely up to the researcher to determine which type to use. However, irrespective of the wavelet type, the data sample in wavelet analysis must be a power of two (Kennedy, 2008). This restriction on the sample size arises since the mother wavelet is a difference operator and is translated in steps of two in the DWT.

In this paper, the simple Haar wavelet is used as well as the slightly more detailed Daubechies (4) wavelet. However, the Haar wavelet is not considered as a good choice since it is discontinuous and has no signs of smoothness (Crowley, 2007). A longer wavelet (i.e. involves more observations in each translation) captures the correct frequencies better since it is smoother (Andersson, 2008). Hence, the Daubechies (4) wavelet is used in the paper as well. The approach using wavelets of different lengths and shapes can be considered as a technique to check the robustness of the result (see e.g. Tkacz, 2001; Zagaglia, 2009).

2.3.1 Jensen's (1999) Wavelet OLS estimator

Jensen (1999) proposed an estimator that can be seen as an alternative to the spectral estimators described in the previous sections. As the name suggests, the Jensen's WOLS is built upon wavelet theory and OLS (see Jensen, 1999). Jensen (1999) utilizes that the variance of the wavelet coefficients is a regularization of the spectrum of the time series (Percival & Walden, 2000). When the GPH and Robinson estimators are considered, there exist a log-linear relationship between the periodogram and the different frequencies (Geweke & Porter-Hudak, 1983; Robinson, 1995). In a similar fashion, Jensen (1999) shows that the variance of the wavelet coefficients is log-linearly related to the different frequency bands (scales). If the estimated variance of the wavelet coefficients $\bar{R}(j)$ at scale j is expressed in log terms and rewritten into a regression model the following expression evolves:

$$log\,\bar{R}(j) = log\,\sigma_{\varepsilon}^{2} - d\,log\,2^{2j} + v_j \quad (8)$$

where σ_{ε}^{2} is the variance of the white noise process ε_t in Equation (1) and v_j represents the disturbance term at scale $j = 1,\ldots J$ where J is the highest scale.

Equation (8) is estimated using OLS. The obtained slope coefficient represents the estimate of the fractional integration parameter $\hat{d}_{WOLS}$. An advantage with the WOLS estimator is that the estimates of the fractional integration parameter follow a normal distribution (Jensen, 1999). Jensen (1999) shows that the WOLS estimator is an effective and consistent estimator of the fractional integration parameter.

3. Data

All data used in the analysis are observations of monthly CPI inflation. In the data set, the inflation rate for the current month is calculated as the percentage change from the previous year and corresponding month (12 month percentage change). The data for all countries, except for the United States, originates from the *OECD Statistics Database* (http://stats.oecd.org). For the United States, the data is obtained from the *U.S Department of Labor: Bureau of Labor Statistics* (http://www.bls.gov/data/).

For Germany, Norway, Sweden and the United Kingdom the monthly inflation data spans January 1956 to April 2011. For Canada there is slightly more data available since monthly inflation data exist from January 1950. The data availability for the United States is ample. The data material for the United States stretches over the period January 1914 until April 2011.

Figure 1 shows the inflation rate in the United States during 1914M1-2011M4. The variability of the inflation rate is salient until the 1950s. This finding is expected since several crises occurred from 1914 until the middle of 1950s. The Great Depression and the World Wars are some major crises that can be mentioned. From the middle of 1950s until the end of the data material in 2011M4, the inflation rate is fairly stable; an exception is the years from the middle of 1960s until the middle of 1980s.

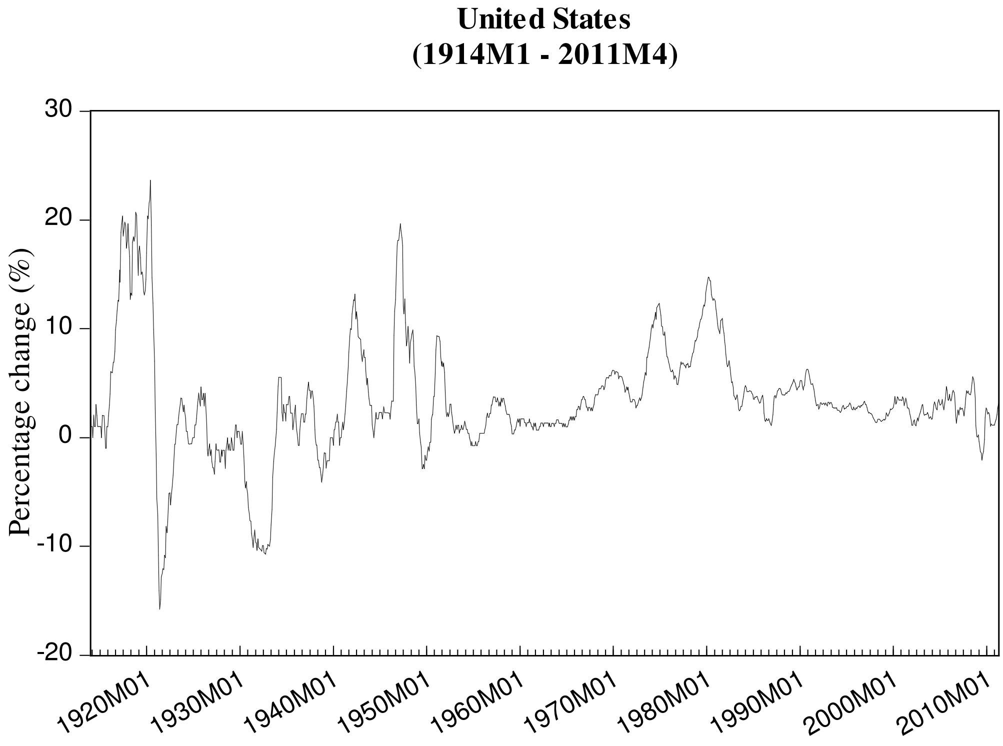

Figure 1: Annual inflation rates (12 month percentage change) – United States

The Vietnam War occurred 1967 and might be an explanation to why the inflation rate in the United States increased sub-sequent years. During the 1970-1980s, the Oil Crises and the collapse of the Bretton Woods system are possible explanations to the high inflation rates. After year 2000, some variation in the inflation rate is seen in Figure 1. Likely, this inflation variability is explained by fluctuations in the United States economy such as the IT boom in the late 1990s and early 2000s as well as the financial crisis in the late 2000s.

As revealed by Figure 2, a similar pattern in inflation rates as in the United States is observed for Canada. The Canadian inflation rate is characterized by high variation in the vicinity of the 1950s. The variable inflation rate most likely occurs due to aftereffects of the World War II (WWII). As for the United States, the years between 1970 and 1980 are marked by high inflation rates. Since the middle of 1990s, the Canadian inflation is fairly stable.

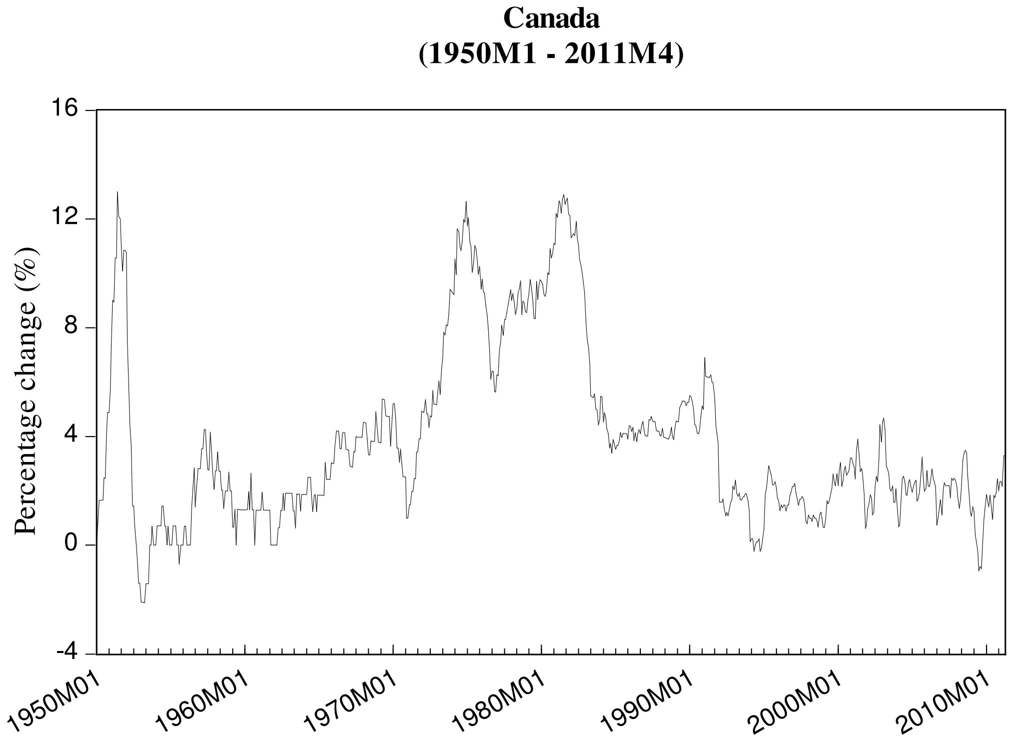

Figure 2: Annual inflation rates (12 month percentage change) – Canada

Figure 3 shows the inflation series for Germany, Norway, Sweden and the United Kingdom. Common for these countries, is that the years around 1970s to the 1990s are characterized by high and variable inflation. The inflation rate for the United Kingdom is fairly stable (except during 1970-1990); such finding is not true for Germany, Norway and Sweden. In the case of Germany, several explanations can be sought to the certain inflation pattern; possible explanations are the aftermaths of WWII as well as the unification of East and West Germany. Regarding Norway and Sweden, the inflation series are quite similar. A distinct feature is the peak around 1990 in the Swedish series. The peak in inflation is probable, to some extent, explained by the change of exchange rate regime from a pegged to a floating system.

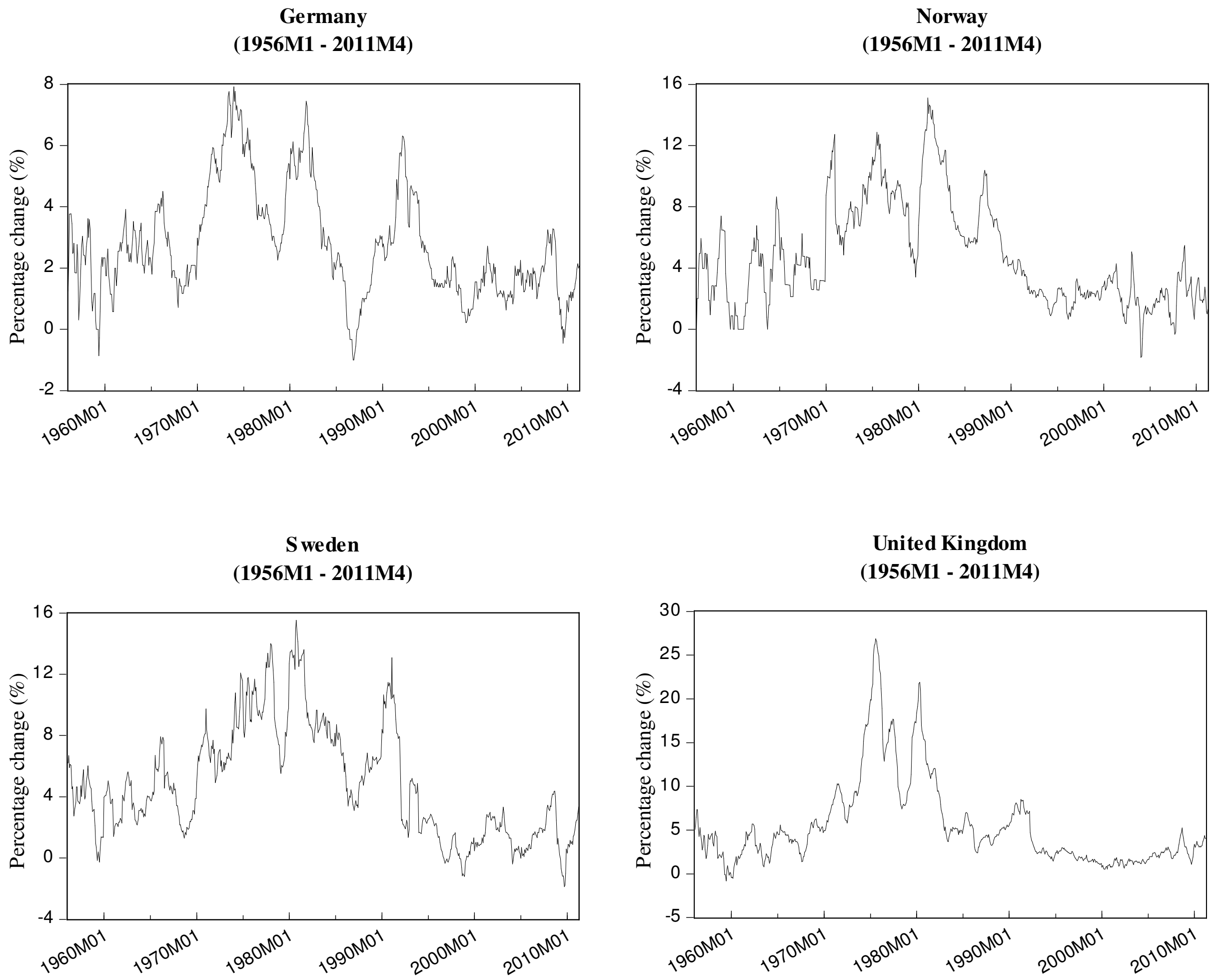

Figure 3: Annual inflation rates (12 month percentage change) – other countries

Table 2 provides some summary statistics of the inflation rates. The United Kingdom experienced the highest mean inflation while Germany had the lowest rate; compare 5.45% against 2.76%. In Norway and Sweden, the mean inflation rates as well as the standard deviations are similar. Regarding the standard deviations, the United States and the United Kingdom historically have relatively high values in comparison to the other countries according to Table 2.

Table 2 – Summary statistics

	# obs.	Mean	Std.dev	Maximum	Minimum
United States	1168	3.37	5.19	23.67	-15.79
Canada	736	3.82	3.25	13.01	-2.11
Germany	664	2.76	1.79	7.92	-1.00
Norway	664	4.77	3.35	15.12	-1.83
Sweden	664	4.83	3.65	15.53	-1.87
United Kingdom	664	5.45	5.05	26.87	-0.82

* The values in the table are calculated as percentages and the calculations are based on the observations (1914M1-2011M4) for the United States, (1950M1-2011M4) for Canada and (1956M1-2011M4) for the others.
Source: Author's calculations.

Table 2 reveals that the highest inflation rates occurred for the United Kingdom and the United States. For the United Kingdom, the inflation rate reached its maximum value of 26.87% in 1975M8. The maximum inflation was 23.67% in the United States and this value occurred in 1920M6.

Sweden, Norway and Canada historically have similar maximum inflation rates (15.53%, 15.12% and 13.01% respectively). In Sweden and Norway, the highest inflation rates occurred 1980M10 and 1981M1 while in Canada the highest value took place in 1956M6. In Germany, the highest inflation of 7.92% occurred 1973M12. The minimum inflation rate has been considerably lower in the United States than in the other countries. The smallest value of the inflation rate occurred 1921M6 and was -15.79%. For the other countries, the minimum inflation rates are fairly the same ranging from -2.11% to -0.82%.

There are numerous explanations to why the inflation rate is highly variable and differs between the countries. Only a few explanations have been discussed so far. Inflation uncertainty and inflation expectations are indeed other possible explanations to certain inflation patterns (see e.g. Ball & Cecchetti, 1990). Different inflation rates among countries can probably be explained by economic structures as well (see e.g. Coleman, 2010). Economies have different consumer patterns and produce different goods. For instance, a small country with a large public sector is most likely not as affected by an oil shock as a large and mainly manufacturing country.

It is also reasonable to believe that all of the examined countries were affected by the breakdown of the Bretton Woods system. During the Bretton Woods era the US dollar was convertible to gold at a fixed rate. Additionally, the member economies of the Bretton Woods system were connected to the dollar at a fixed rate. Implicitly, all currencies were pegged to a gold reserve (Cohen, 2001). During the years 1971-1973 the currencies of the member economies were allowed to float independently since the US dollar lost its credibility; eventually the Bretton Woods system disbanded in 1973M2. Consequently, a large number of countries were forced to change their exchange rate regime at this time (Cohen, 2001).

Evidently several other explanations to the inflation patterns could be mentioned than those already discussed. However, the inflation series of the countries in this study share some common characteristics. Mainly, the data description shows that inflation often was high and unstable during 1970-1980s. For some countries the high and variable rates of inflation remained until the early 1990s as well.

4. Analysis

In section 4.1, traditional unit root tests are conducted to examine if inflation is modeled as an integrated unit root process. In order to answer whether traditional unit root tests model inflation incorrectly and foremost if inflation is a mean-reverting process, fractional integration methods are used in section 4.2.

4.1 Traditional unit root tests

In order to avoid that the traditional unit root tests are influenced of structural breaks or periods of instability, several sub-time periods are tested in the analysis. A natural breakpoint is to test before and after the collapse of the Bretton Woods system in February 1973 (Cohen, 2001). The rationale of analyzing before and after February 1973 arises since many countries changed exchange rate regime when the Bretton Woods system collapsed; hence, a structural break occurred at this point in time.

Many unit root tests are built upon asymptotic theory and it has been shown that size distortions often are a problem; especially in small samples. Therefore many observations are required to obtain reliable estimates. For further reference, see e.g. Blough (1992); Cochrane (1991) and Diebold & Rudebusch (1991). As the unit root tests require many observations the number of sub-samples considered in the paper is limited. All countries in the study are tested before and after the collapse of the Bretton Woods system. However, since there are many observations of the United States inflation series, additional time periods are tested for this country. For the United States breakpoints around the time of the WWII are considered as well as the structural break at the time of the Bretton Woods collapse.

The result of several ADF tests is presented in Table 3. The unit root null hypothesis of the ADF tests is in most cases not rejected at any reasonable level of significance. In the case of the United States, the result is somewhat ambiguous since the null hypothesis can be rejected at some times (depends on which time period tested). When considering the United States, inclusion of a trend in the test does not affect the result considerably. The null hypothesis of a unit root can be rejected at 10% level for Canada during the period before the collapse of the Bretton Woods system (1950M1-1973M2). However, the unit root hypothesis is only rejected in the ADF test with an intercept as deterministic component.

Table 3: ADF tests

Country	Sample period	ADF test (*intercept*)		ADF test (*intercept+trend*)	
		Value	Prob.	Value	Prob.
United States	1914M1 - 2011M4	-5.536	0.000***	-5.559	0.000***
	1914M1 - 1939M9	-2.403	0.142	-2.879	0.171
	1914M1 - 1945M8	-2.749	0.067*	-2.869	0.174
	1945M9 - 1973M2	-4.674	0.000***	-4.406	0.003***
	1945M9 - 2011M4	-5.309	0.000***	-5.223	0.000***
	1973M3 - 2011M4	-2.779	0.062*	-3.353	0.059*
Canada	1950M1 - 2011M4	-2.320	0.166	-2.306	0.430
	1950M1 - 1973M2	-2.829	0.056*	-3.128	0.102
	1973M3 - 2011M4	-1.601	0.481	-1.830	0.688
Germany	1956M1 - 2011M4	-2.403	0.141	-2.623	0.270
	1956M1 - 1973M2	-1.732	0.414	-2.397	0.380
	1973M3 - 2011M4	-2.487	0.119	-2.543	0.307
Norway	1956M1 - 2011M4	-2.094	0.247	-2.423	0.367
	1956M1 - 1973M2	-2.134	0.232	-3.111	0.107
	1973M3 - 2011M4	-1.538	0.513	-2.711	0.233
Sweden	1956M1 - 2011M4	-1.957	0.306	-2.191	0.493
	1956M1 - 1973M2	-2.042	0.269	-2.724	0.228
	1973M3 - 2011M4	-1.664	0.449	-2.705	0.235
United Kingdom	1956M1 - 2011M4	-2.129	0.233	-2.310	0.428
	1956M1 - 1973M2	-0.673	0.850	-2.460	0.348
	1973M3 - 2011M4	-1.931	0.318	-2.212	0.481

*** 1% significance ** 5% significance * 10% significance
Source: Author's calculations.
Note that the ADF test has the null hypothesis of a unit root in the time series (see Dickey & Fuller, 1979).

Mostly, there is a straightforward interpretation of the results from the ADF tests. In general, when considering all countries in the study, the null hypothesis of a unit root is mostly not rejected. Hence, the ADF tests indicate that inflation is a unit root process. Consequently, the inflation series would not return to its mean value after an inflation shock.

To lend some credibility to the results found in Table 3 additional KPSS tests are performed. As can be seen in Table 4 the null hypothesis of the KPSS test (that the time series is stationary) can be rejected in most cases. As previously, when considering the ADF tests, the result for the United States is not explicit. The result of the KPSS tests seems to depend on which data sample that is used in the testing procedure. Regarding Canada, there are some indications of stationarity before the Bretton Woods system collapsed. The KPSS test with only an intercept fails to reject the null hypothesis that the Canadian inflation series is stationary. A similar result as for Canada also holds for Norway and Sweden. The null hypothesis cannot be rejected before the Bretton Woods system collapsed. The latter statement is at least true when considering the KPSS tests with an intercept and a trend for Norway and Sweden.

Table 4: KPSS tests

		KPSS test (*intercept*)	KPSS test (*intercept+trend*)
Country	**Sample period**	**Value**	**Value**
United States	1914M1 - 2011M4	0.190	0.128*
	1914M1 - 1939M9	0.592**	0.133*
	1914M1 - 1945M8	0.342	0.225***
	1945M9 - 1973M2	0.292	0.233***
	1945M9 - 2011M4	0.280	0.263***
	1973M3 - 2011M4	1.563***	0.194**
Canada	1950M1 - 2011M4	0.483**	0.460***
	1950M1 - 1973M2	0.221	0.150**
	1973M3 - 2011M4	1.957***	0.318***
Germany	1956M1 - 2011M4	0.600**	0.267***
	1956M1 - 1973M2	0.673**	0.154**
	1973M3 - 2011M4	1.327***	0.145*
Norway	1956M1 - 2011M4	0.874***	0.516***
	1956M1 - 1973M2	0.702**	0.097
	1973M3 - 2011M4	2.025***	0.231***
Sweden	1956M1 - 2011M4	0.933***	0.524***
	1956M1 - 1973M2	0.411*	0.110
	1973M3 - 2011M4	2.060***	0.162**
United Kingdom	1956M1 - 2011M4	0.667**	0.407***
	1956M1 - 1973M2	0.898***	0.203**
	1973M3 - 2011M4	1.756***	0.349***

*** 1% significance ** 5% significance * 10% significance
Source: The author's calculations.
Note that the null hypothesis of the KPSS test is that the time series is stationary (see Kwiatkowski et al., 1992). Hence, the time series does not contain a unit root under the null hypothesis.

The overall result from the KPSS tests is that the inflation series is considered as non-stationary process. With only a few exceptions, the null hypothesis of stationarity is rejected. In total, the result of the traditional unit root tests indicates that inflation is a unit root process. The few cases when there are any indications of stationarity occur for the United States or time periods before the collapse of the Bretton Woods system when other countries are considered. From this point of view, the Bretton Woods system had stabilizing effects on the inflation rates of some member countries. Such conjecture is although taken with precaution since the result of stationarity is uncommon in the analysis. Furthermore, there are likely other explanations as well to why the tests sometimes indicate stationarity during the Bretton-Woods era.

According to the tests performed so far it is more the rule than the exception that inflation is considered as a unit root process. The finding of a unit root in inflation is in accordance with several other studies using similar techniques as the ones employed so far in this paper (see e.g. Ball & Cecchetti, 1990; Brunner & Hess, 1993; MacDonald & Murphy, 1989).

4.2 Fractional integration estimates

The analysis of the fractional integration estimates begins with examining the United States and Canada. Since there are more observations for the United States and Canada, these countries can be analyzed more thoroughly in comparison with the other countries in the study. In a later stage of this section, the remaining countries are examined in a similar fashion as the United States and Canada but concerning other time periods.

The analyzed time periods differ to some extent depending on which estimator that is used in the analysis. For the spectral estimators (GPH and Robinson) the same time periods are tested. However, since the sample size must be a power of two when using WOLS, the analyzed time periods differ for this estimator. Hence, the possibility to compare estimates of d between the estimators vanishes to some extent. Such analysis is informative in order to distinguish certain patterns during specific time periods. Since the main purpose of the paper is to investigate whether inflation is mean-reverting, the matter of comparing different time periods is of less importance in the paper.

Table 5 reveals that the estimates fluctuate considerably depending on the value of α. The estimates using $T^{0.60}$ frequencies are of less importance in the analysis since the obtained estimates are unrealistic. Several of the estimated memory parameters in Table 5 are greater than one. Such fractional integration orders imply an infinite and explosive variance (Greene, 2003). Series with infinite variance are characterized by an increasing variance as the number of observations increase. An inspection of the data material in Figure 1 and Figure 2 shows no obvious tendencies of infinite variance. Furthermore, the spectral estimates based on $\alpha = 0.60$ are unrealistic in comparison with the WOLS estimates as well. The value of α need not to be specified for the WOLS estimator; hence, this estimator does not suffer from misspecification due to an invalid choice of α.

Table 5: Fractional integration estimates – United States and Canada

	GPH		Robinson	
United States	$T^{0.50}$	$T^{0.60}$	$T^{0.50}$	$T^{0.60}$
1914M1 - 1939M9	0.817$^{\Psi}$ (0.164)	1.392$^{\Psi}$ (0.157)	0.815$^{\Psi}$ (0.164)	1.387$^{\Psi}$ (0.155)
1914M1 - 1945M8	0.809$^{\Psi}$ (0.183)	1.491$^{\Psi}$ (0.189)	0.808$^{\Psi}$ (0.182)	1.487$^{\Psi}$ (0.187)
1914M1 - 2011M4	0.343 (0.118)	0.703 (0.094)	0.344 (0.114)	0.702 (0.094)
1945M9 - 1973M2	0.641$^{\Psi}$ (0.237)	1.273$^{\Psi}$ (0.198)	0.615$^{\Psi}$ (0.222)	1.269$^{\Psi}$ (0.190)
1945M9 - 2011M4	0.559 (0.137)	0.738 (0.087)	0.552 (0.132)	0.757 (0.087)
1950M1 - 1973M2	0.542 (0.088)	1.204$^{\Psi}$ (0.142)	0.647 (0.124)	1.200$^{\Psi}$ (0.141)
1950M1 - 2011M4	0.735$^{\Psi}$ (0.136)	0.918$^{\Psi}$ (0.103)	0.735$^{\Psi}$ (0.135)	0.928$^{\Psi}$ (0.101)
1956M1 - 1973M2	1.022$^{\Psi}$ (0.150)	0.984$^{\Psi}$ (0.109)	1.054$^{\Psi}$ (0.140)	0.950$^{\Psi}$ (0.106)
1956M1 - 2011M4	0.867$^{\Psi}$ (0.151)	1.082$^{\Psi}$ (0.104)	0.866$^{\Psi}$ (0.150)	1.079$^{\Psi}$ (0.103)
1973M3 - 2011M4	1.000$^{\Psi}$ (0.127)	1.366$^{\Psi}$ (0.132)	0.999$^{\Psi}$ (0.126)	1.363$^{\Psi}$ (0.131)
Canada				
1950M1 - 1973M2	0.862$^{\Psi}$ (0.135)	1.193$^{\Psi}$ (0.162)	0.947$^{\Psi}$ (0.274)	1.188$^{\Psi}$ (0.161)
1950M1 - 2011M4	0.606 (0.284)	0.965$^{\Psi}$ (0.106)	0.605 (0.135)	0.971$^{\Psi}$ (0.104)
1956M1 - 1973M2	0.468 (0.192)	0.568 (0.116)	0.441 (0.185)	0.554 (0.111)
1956M1 - 2011M4	0.931$^{\Psi}$ (0.111)	0.952$^{\Psi}$ (0.090)	0.930$^{\Psi}$ (0.110)	0.950$^{\Psi}$ (0.090)
1973M3 - 2011M4	0.983$^{\Psi}$ (0.159)	1.354$^{\Psi}$ (0.146)	0.981$^{\Psi}$ (0.158)	1.351$^{\Psi}$ (0.145)

	WOLS	
United States	Haar	Daubechies-4
1914M1 - 1999M4	0.594 (0.109)	0.746 (0.092)
1926M1 - 2011M4	0.715 (0.066)	0.664 (0.131)
1914M1 - 1956M8	0.647 (0.126)	0.820$^{\Psi}$ (0.098)
1956M9 - 1999M4	0.810$^{\Psi}$ (0.115)	0.974$^{\Psi}$ (0.068)
1914M1 - 1935M4	0.858$^{\Psi}$ (0.083)	0.970$^{\Psi}$ (0.097)
1935M5 - 1956M8	0.670 (0.149)	0.814$^{\Psi}$ (0.121)
1956M9 - 1977M12	0.952$^{\Psi}$ (0.072)	0.987$^{\Psi}$ (0.056)
1978M1 - 1999M4	1.001$^{\Psi}$ (0.077)	0.912$^{\Psi}$ (0.151)
Canada		
1950M1 - 1992M8	0.808 (0.060)	0.846$^{\Psi}$ (0.086)
1968M9 - 2011M4	0.719$^{\Psi}$ (0.183)	0.910$^{\Psi}$ (0.058)
1950M1 - 1971M4	0.485 (0.182)	0.725 (0.129)
1971M5 - 1992M8	0.918$^{\Psi}$ (0.089)	0.937$^{\Psi}$ (0.097)

(Ψ) means that there is a unit root within two standard deviations of the estimated value; a unit root within two standard deviations is equivalent with not rejecting the hypothesis $H_0: d = 1$.

The values within the parentheses are the standard errors associated with the estimated fractional integration parameters.

Since the sample size for the WOLS estimator must be a power of two, the same samples cannot be used as for the spectral estimators. The time periods when employing the WOLS estimator is chosen as follows. First, the highest number of observations as a power of two is used to calculate the memory parameter in the beginning and end of the sample. For the United States $2^{10} = 1024$ observations are the highest number possible since there in total are 1168 observations. The first 1024 observations are used (1914M1-1999M4), and then the last 1024 observations (1926M1-2011M4). Second, the next highest number of observations as a power of two is used. In this case $2^9 = 512$ observations. Henceforth, a recursive method is employed instead of testing the observations in the beginning and the end of the sample. The observations 1-512 are used in the first calculation (1914M1-1956M8), following after this, observation 513-1024 is used (1956M9-1977M12) until no more time periods of the same length can be used. The sample size is trimmed further, recursively testing the third highest number of observations as a power of two, $2^8 = 256$ observations and so on.

In Table 5, it is examined whether a unit root is present within two standard deviations of the estimated fractional integration parameter. Examining if a unit root exists within two standard deviations of the estimated d is the same as testing the unit root null hypothesis $H_0: d = 1$ at 95% confidence level against the alternative hypothesis $H_1: d < 1$. Such relationship arises since the confidence interval is the inverse of the test statistic used in hypothesis testing (Davidson & MacKinnon, 2009).

In total, 54 relevant estimates of d are presented in Table 5 (i.e. excluding the estimates using $T^{0.60}$ frequencies). According to Table 5, there is a unit root within two standard deviations of the estimated value of d for 35 out of 54 of the relevant estimates. Equivalently, the null hypothesis of a unit root is not rejected in 35 cases. From this point of view, there is minor evidence against $d = 1$ since the null hypothesis seldom is rejected. However, the fact that almost all estimates of the memory parameter in Table 5 are smaller than one (50 out of 54 estimates) is a strong indication that the true parameter value of d in fact is smaller than one. It is reasonable to assume that the estimates of d in Table 5 are close to the true parameter values due to unbiasedness of the estimators (the GPH estimates are likely unbiased since these are similar to the unbiased Robinson estimates). Hence, the lack of significant values is likely a matter of sample size rather than unreliable estimates. For instance, if very long data samples were used significant estimates would likely occur.

Figure 1 reveals that there is a clear spike in the United States inflation series during the WWII (1939-1945). When observations for WWII are included in the data sample, no remarkable changes of the estimates follow. The GPH-estimate of d is 0.817 when observations for WWII not are included in the data sample. Including observations covering WWII as well returns an estimate of d equal to 0.809. The corresponding estimates of d for the Robinson estimator are respectively 0.815 and 0.808.

However, the memory parameter estimates likely are affected of where and when in an inflation cycle the data sample begins and ends. To illustrate the importance of when the sample begins and ends, the time periods 1950M1-1973M2 and 1956M1-1973M2 for the United States can be taken as an example. As for the years 1939-1945, the United States inflation series during 1950-1956 is characterized by a large inflation spike (see Figure 1). Including 1950M1-1956M1 in the estimations (i.e. using the sample 1950M1-1973M2) yields a considerably smaller value of the estimated fractional integration parameter. Table 5 reveals that the estimated fractional integration order is almost twice as large during 1956M1-1973M2 (independent of which spectral estimator that is used).

It is difficult to draw any general conclusions about the magnitude of the fractional integration parameter during the pre-Bretton Woods era (samples ending at 1973M2 in Table 5). Some patterns can however be distinguished. The estimated memory parameters range from 0.468 to 1.022 when the GPH estimates are considered. The corresponding values for the Robinson estimator are 0.441 and 1.054. Hence the estimates of d differ substantially. Several of the d estimates are however found within the interval between 0.40 and 0.70. Values within this span correspond to relatively low integration orders. Comparatively, the inflation series is not highly persistent; the effect of a shock dissipates relatively fast under these circumstances, especially for time series with a value of d that is smaller than 0.50.

The period after the Bretton Woods collapse is characterized by large estimates of the fractional integration parameter; see the values for the United States and Canada during 1973M3-2011M4. The high fractional integration orders after 1973M2 are likely caused by the collapse of the Bretton Woods system and the oil crises. However, other explanations can be sought as well. For instance, in Brunner & Hess (1993) it is concluded that high levels of inflation are less predictable. Hence, it could be the case that poor inflation forecasts caused misaligned economic decisions which added to the effect of high and variable inflation rates.

Since the spectral estimators are sensitive to structural breaks it is difficult to draw any general conclusion about the magnitude of d; especially when considering longer data sets. The estimates of the memory parameter are affected since many breaks and other disturbances occur during longer time spans. When the WOLS estimator is used structural breaks are not a problem. In Table 5, the memory parameter estimates generated by the WOLS estimator are almost exclusively smaller than one. The only case when the estimated d is greater than one in Table 5 occurs for the United States in 1978M1-1999M4. Since the wavelet estimator not heavily depend on the data samples and structural breaks, the WOLS estimates of d can be considered as more appropriate than the spectral estimates. The overall result of the WOLS estimates for the United States and Canada is that inflation likely is mean-reverting. Only one out of 24 WOLS estimates in Table 5 is greater than or equal to one. In all, the estimates of d are between 0.60 and 0.95.

Some certain patterns can be discerned when the different time periods are compared with each other. Nearly all of the estimated memory parameters for time periods ending in 1973M2 are smaller than memory estimates obtained from the sample 1973M3-2011M4. This result is true both for the United States and Canada. A similar result is seen when the WOLS estimates are considered. Samples ending in the vicinity of the Bretton Woods collapse yield smaller estimates of *d* than the corresponding estimates based on data after the collapse. More generally, the WOLS estimates based on observations early in the data material are smaller than the estimates obtained when using observations in the end of the data material. This result indicates that inflation seems to be less persistent in the beginning of the data material.

In Table 6, the estimates of the fractional integration parameter for the remaining countries in the study are found. As before the estimates using $T^{0.60}$ frequencies are excluded from the analysis since unrealistic estimates appear due to an invalid choice of α. In total, 56 relevant estimates of *d* are available in Table 6. Six of 56 estimated values of *d* are greater than or equal to one. So, the estimates of the memory parameter are mostly smaller than one and thus the inflation series are considered as mean-reverting. Just as when analyzing the United States and Canada, only a few of the *d* estimates are significantly different from one. By the same rationale as before an increased sample size would likely create significant estimates.

Considering the spectral estimates of *d* before the collapse of the Bretton Woods system, the inflation series for Germany, Sweden and the United Kingdom have the characteristics of *d* being between 0.80 and 0.90. Norway has a slightly different property since the estimated memory parameter is slightly smaller ($d \approx 0.60$). Due to the plots of the time series in Figure 3, the similar values of *d* are expected. The inflation series for Germany, Norway, Sweden and the United Kingdom are alike during 1956M1-1973M2.

The post Bretton Woods era 1973M3-2011M4 for the countries in Table 6 is characterized by somewhat large spread in the estimates of the fractional integration parameter. The estimate of *d* for Germany is greater than one, irrespective of which spectral estimator considered. In the case of the United Kingdom, the estimate of *d* is slightly smaller than one, no matter if the GPH or Robinson estimator is used. Norway and Sweden are similar in the sense that the estimated memory parameters take values between 0.80 and 0.90. Figure 3 shows that the inflation series for the latter countries are quite similar. In addition, Table 2 shows that the inflation processes for Norway and Sweden share the same characteristics with respect to the mean inflation rate, standard deviations and maximum/minimum values in inflation. The result that the estimated memory parameters not differ to any greater extent between Norway and Sweden is hence expected.

Table 6: Fractional integration estimates – other countries

	GPH		Robinson			WOLS	
Germany	$T^{0.50}$	$T^{0.60}$	$T^{0.50}$	$T^{0.60}$	**Germany**	Haar	Daubechies-4
1956M1 - 2011M4	0.828$^{\Psi}$ (0.142)	0.881$^{\Psi}$ (0.112)	0.823$^{\Psi}$ (0.131)	0.837$^{\Psi}$ (0.112)	1956M1 - 1998M8	0.648 (0.119)	0.816 (0.051)
1956M1 - 1973M2	0.850$^{\Psi}$ (0.115)	0.969$^{\Psi}$ (0.092)	0.850$^{\Psi}$ (0.115)	0.967$^{\Psi}$ (0.092)	1968M9 - 2011M4	0.834 (0.062)	0.880$^{\Psi}$ (0.072)
1973M3 - 2011M4	1.035$^{\Psi}$ (0.233)	1.035$^{\Psi}$ (0.233)	1.034$^{\Psi}$ (0.233)	1.034$^{\Psi}$ (0.233)	1956M1 - 1977M4	0.709 (0.122)	0.724 (0.066)
***					1977M5 - 1998M8	0.910$^{\Psi}$ (0.073)	0.983$^{\Psi}$ (0.079)
Norway					**Norway**		
1956M1 - 2011M4	0.818$^{\Psi}$ (0.262)	0.973$^{\Psi}$ (0.160)	0.783$^{\Psi}$ (0.241)	0.885$^{\Psi}$ (0.169)	1956M1 - 1998M8	0.654 (0.143)	0.823 (0.049)
1956M1 - 1973M2	0.605 (0.170)	0.775 (0.103)	0.605 (0.170)	0.774 (0.103)	1968M9 - 2011M4	0.722$^{\Psi}$ (0.160)	0.882 (0.037)
1973M3 - 2011M4	0.798$^{\Psi}$ (0.138)	1.089$^{\Psi}$ (0.118)	0.797$^{\Psi}$ (0.138)	1.086$^{\Psi}$ (0.118)	1956M1 - 1977M4	0.780 (0.083)	0.743 (0.069)
***					1977M5 - 1998M8	0.870$^{\Psi}$ (0.082)	0.870$^{\Psi}$ (0.091)
Sweden					**Sweden**		
1956M1 - 2011M4	0.862$^{\Psi}$ (0.238)	1.056$^{\Psi}$ (0.196)	1.011$^{\Psi}$ (0.255)	1.027$^{\Psi}$ (0.187)	1956M1 - 1998M8	0.740 (0.096)	0.805 (0.050)
1956M1 - 1973M2	0.778$^{\Psi}$ (0.179)	1.058$^{\Psi}$ (0.133)	0.778$^{\Psi}$ (0.179)	1.056$^{\Psi}$ (0.133)	1968M9 - 2011M4	0.807 (0.091)	0.872$^{\Psi}$ (0.078)
1973M3 - 2011M4	0.855$^{\Psi}$ (0.149)	1.386$^{\Psi}$ (0.146)	0.854$^{\Psi}$ (0.149)	1.382$^{\Psi}$ (0.145)	1956M1 - 1977M4	0.788$^{\Psi}$ (0.170)	0.719 (0.077)
***					1977M5 - 1998M8	0.741 (0.074)	0.777 (0.078)
United Kingdom					**United Kingdom**		
1956M1 - 2011M4	1.180$^{\Psi}$ (0.193)	1.181$^{\Psi}$ (0.164)	1.130$^{\Psi}$ (0.181)	1.087$^{\Psi}$ (0.173)	1956M1 - 1998M8	0.650$^{\Psi}$ (0.191)	0.929$^{\Psi}$ (0.077)
1956M1 - 1973M2	0.795$^{\Psi}$ (0.105)	1.127$^{\Psi}$ (0.121)	0.794$^{\Psi}$ (0.105)	1.125$^{\Psi}$ (0.121)	1968M9 - 2011M4	0.908$^{\Psi}$ (0.065)	0.912$^{\Psi}$ (0.132)
1973M3 - 2011M4	0.981$^{\Psi}$ (0.212)	1.222$^{\Psi}$ (0.134)	0.979$^{\Psi}$ (0.212)	1.219$^{\Psi}$ (0.134)	1956M1 - 1977M4	0.820 (0.052)	0.761 (0.042)
***					1977M5 - 1998M8	0.739$^{\Psi}$ (0.245)	1.010$^{\Psi}$ (0.053)

(Ψ) means that there is a unit root within two standard deviations of the estimated value; a unit root within two standard deviations is equivalent with not rejecting the hypothesis $H_0: d = 1$.

The values within the parentheses are the standard errors associated with the estimated fractional integration parameters.

Since the sample size for the WOLS estimator must be a power of two, the same time periods cannot be used as for the spectral estimators. The largest possible number of observations as a power of two is used when calculating the fractional integration parameter. The largest possible number is $2^9 = 512$ observations. Just as for the United States and Canada (see Table 5) the beginning and the end of the sample is used in the calculations; see (1956M1-1998M8) and (1968M9-2011M4). Hereafter, a recursive procedure begins and the sample size is reduced into smaller parts; in this case recursively using $2^9 = 256$ observations.

The GPH and Robinson estimates during 1956M1-2011M4 are relatively similar. An exception occurs for Sweden; compare $\hat{d}_{GPH} = 0.862$ and $\hat{d}_{ROB} = 1.011$. This exception likely occurs since the GPH estimator sometimes gives biased estimates. Therefore, the Robinson estimate can be considered as more reliable and hence indicates a unit root in the Swedish inflation series during 1956M1-2011M4. As was stated when analyzing the United States and Canada it is however difficult to establish the magnitude of d when using the spectral estimators and considering longer time periods.

The WOLS estimates for Germany, Norway, Sweden and the United Kingdom are widely spread. The most common values, irrespective of time period, are found within the span between 0.60 and 0.95. Some comparisons can be done with the WOLS estimates of the United States and Canada. In general, the WOLS estimates more or less range within the same intervals for all countries in the study. No clear pattern regarding the magnitude of d can be distinguished either. The magnitude of d to a large extent depends on which time period tested, wavelet used and country examined.

There is some evidence that the fractional integration parameter for the countries in Table 6 was smaller during the Bretton Woods era than after the system dissolved. Concerning the spectral estimators, the obtained estimates of d when using the sample 1956M1-1973M2 are in all cases smaller than the corresponding estimates from the sample 1973M3-2011M4. Clearly the same result as for the United States and Canada holds for Norway, Germany, Sweden and the United Kingdom.

Since WOLS estimates not are affected by structural breaks, the time periods 1956M1-1977M4 and 1977M5-1998M8 can be considered as time periods before and after the Bretton Woods collapse. No matter which wavelet that is considered the estimates of the memory parameter in general are smaller during 1956M1-1977M4. A similar result is seen when considering the samples 1956M1-1998M8 and 1968M9-2011M4; the estimates are mostly smaller during the former time period. It clearly seems to be the case that the degree of memory was lower during the years in the beginning of the data material; in some sense inflation was closer to a stationary process earlier in history.

Previous studies show that the degree of memory is higher in countries with historically high and variable inflation rates (see e.g. Hofmann & Remsperger, 2005). In Hofmann & Remsperger (2005) evidence is also found that the persistence in inflation is close to zero in countries which experienced low and stable inflation rates. Since it is common that the fractional integration order is fairly high for all countries and time periods examined in this paper, no such result is found; the value of the fractional integration parameter is often between 0.80 and 0.90. One possible interpretation to why none of the estimated memory parameters are even close to zero is that all countries in this study experienced high and unstable inflation rates in the past. Another interpretation is that the methods used in this paper differ from the ones used in Hofmann & Remsperger (2005). In addition, somewhat other countries are studied in Hofmann & Remsperger (2005). In any case, the connection between past inflation rates and future inflation persistence has to be examined further to be able to draw any conclusions.

The main result found in the analysis of the fractional integration parameters is that inflation likely is mean-reverting. This result is in accordance with what recent studies using similar estimators found (see e.g. Choudhry, 2001; Coleman, 2010; Jensen, 2009; Zagaglia, 2009). Furthermore, as stated in Kumar & Okimoto (2007), it is difficult to compare time periods within and between studies. However, the analysis of the fractional integration estimates reveals that inflation series likely had lower memory during the Bretton Woods era than after the collapse of the system. More generally, inflation series had shorter memory in the past in comparison to subsamples ending 2011M4.

5. Conclusions

In this paper traditional unit root tests (ADF and KPSS tests) show that inflation series contain a unit root. However, traditional unit root tests have poor power against fractional integration alternatives (see e.g. Diebold & Rudebusch, 1991). Thus, the result that inflation is a unit root process is questioned. In this thesis it is therefore investigated whether inflation is better modeled as a fractionally integrated and mean-reverting process. The fractional integration estimators used in the paper are the Geweke Porter-Hudak (1983) estimator, Robinson (1995) estimator and the Jensen's (1999) Wavelet OLS estimator. The countries included in the study are Canada, Germany, Norway, Sweden, the United Kingdom and the United States. These countries are considered as stable since they have not experienced persistent instability in macroeconomic variables. Thus, in this paper, there is in some sense high internal validity when considering industrialized OECD countries.

The analysis of the fractional integration estimates reveals that these mostly are smaller than one. The main result of the paper is hence that inflation series with a high probability not contain a unit root and therefore is a fractionally integrated and mean-reverting process. The estimated value of the memory parameter is often relatively high; values between 0.80 and 0.90 are commonly found in the analysis. Hence, inflation has long memory and is a highly persistent process; such process is close to a unit root process. Even though the estimates of the fractional integration parameter rarely are significant, a vast amount takes a value smaller than one. Since the estimates of the memory parameter are unbiased many estimates smaller than one is a strong enough indication that the true parameter value likely is smaller than one as well. The lack of significant results is rather a matter of too small sample sizes.

An important finding in the paper is that the memory parameter is smaller during the Bretton Woods era in comparison with the years after the breakdown of the system. More generally, inflation seems to have been less persistent earlier in history. The magnitude of the memory parameter is an important but unanswered question in the analysis. The estimates of the memory parameter are sensitive to different samples and to some extent depend on when the sampling started and ended. For instance, it is shown in the paper that the estimated memory parameter becomes almost twice as large for the United States depending on where the data sample begins. In order to establish the magnitude of the fractional integration parameter more sophisticated methods are required (for instance methods using cross-sectional observations). Better estimation methods will be developed as sure as more observations become available. Therefore research on inflation series and its characteristics likely has a bright future.

REFERENCES

Agiakloglou, C., Newbold, P & Wohar, M (1993), "Bias in an estimator of the fractional difference parameter", *Journal of Time Series analysis*, Vol.14, No. 3; 235-246

Andersson, F.N.G (2008), "Wavelet analysis of economic time series", *Lund Economic Studies*, No.149

Baillie, R.T (1996), "Long memory processes and fractional integration in econometrics", *Journal of Econometrics*, Vol.73; 5-59

Ball, L. & Cecchetti S.G (1990), "Inflation and uncertainty at short and long horizons", *Brookings Papers on Economic Activity*, Vol. 1990, No.1; 215-254

Barsky, R.B (1987), "The Fisher hypothesis and the forecastability and persistence of inflation", *Journal of Monetary Economics*, Vol.19; 3-24

Batini, N (2006), "Euro area inflation persistence", *Empirical Economics*, Vol.31; 977-1002

Blough, S.R (1992), "The relationship between power and level for generic unit root tests in finite samples", *Journal of Applied Econometrics,* Vol.7, No.3; 295-308

Boutahar, M., Marimoutou, V & Nouira, L (2007), "Estimation methods of the long-memory parameter: Monte Carlo analysis and application", *Journal of Applied Statistics*, Vol.34, No.3; 261-301

Brandes, O., Farley, J., Hinich, M. & Zackrisson, U (1968), "The time domain and the frequency domain in time series analysis", *The Swedish Journal of Economics*, Vol.70, No.1; 25-42

Brimmer, A.F (2002), "Central banks and inflation targeting in perspective", *North American Journal of Economics and Finance*, Vol.13; 93-97

Brockwell, P.J & Davis, R.A (2002), *Introduction to time series and forecasting*, 2ED, Springer-Verlag, New York

Brunner, A.D & Hess, G.D (1993), "Are higher levels of inflation less predictable? A state-dependent conditional heteroscedasticity approach", *Journal of Business & Economic Statistics*, Vol.11, No.2; 187-197

Chatfield, C (1996), *The analysis of time series – an introduction*, 5ED, Chapman & Hall/CRC, New York

Cheung, Y-W (1993), "Long memory in foreign-exchange rates", *Journal of Business & Economic Statistics*, Vol.11, No.1; 93-101

Choudhry, T (2001), "Inflation and rates of return on stocks: Evidence from high inflation countries", *Journal of International Financial Markets, Institutions and Money,* Vol.11; 75-96

Cochrane, J.H (1991), “A critique of the application of unit root tests”, *Journal of Economic Dynamics and Control*, Vol.15; 275-284

Cohen, B.J (2001), “Bretton Woods system” in Jones, R.J.B (ed), *Routledge Encyclopedia of International Political Economy*, Routledge, London

Coleman, S (2010), “Inflation persistence in the Franc zone: evidence from disaggregated prices”, *Journal of Macroeconomics,* Vol. 32; 426-442

Crato, N & de Lima, P.J.F (1994), ”Long-range dependence in the conditional variance of stock returns”, *Economic Letters,* Vol.45; 281-285

Crowley, P.M (2007), “A guide to wavelets for economists”, *Journal of Economic Surveys*, Vol. 21, No.2; 207-265

Culver, S.E & Papell, D.H (1997), “Is there a unit root in the inflation rate?”, *Journal of Applied Econometrics*, Vol. 12; 435-444

Davidson, R & MacKinnon, J.G (2009), *Econometric theory and methods*, International Edition, Oxford University Press, New York

Dias, D.A & Marques, C.R (2010), “Using mean reversion as a measure of persistence”, *Economic Modeling*, Vol.27, No.1; 262-273

Dickey, D.A & Fuller, W.A (1979), “Distribution of the estimators for autoregressive time series with a unit root”, *Journal of the American Statistical Association*, Vol.74, No.366; 427-431

Diebold, F.X & Rudebusch, G.D (1991), “On the power of Dickey-Fuller tests against fractional alternatives”, *Economic Letters*, Vol.35; 155-160

Dittmar, R., Gavin, W.T & Kydland, F.E (1999), ”The inflation-output variability tradeoff and price-level targets”, *in Federal Reserve Bank of St. Louis Review*, Vol.81, No1; 23-32

Fisher, I (1930), *The theory of interest*, MacMillan, New York

Gençay, R., Selçuk, F & Brandon, W (2002), *An introduction to wavelets and other filtering methods in finance and economics,* Academic Press, San Diego

Geweke, J & Porter-Hudak, S (1983), “The estimation and application of long memory time series models”, *Journal of Time Series analysis*, Vol.4, No. 4; 221-238

Granger, C.W.J (1980), “Long memory relationships and the aggregation of dynamic models”, *Journal of Econometrics,* Vol.14; 227-238

Granger, C.W.J & Joyeux, R (1980), “An introduction to long-memory time series models and fractional differencing”, *Journal of Time Series analysis*, Vol.1, No.1; 15-29

Greene, W.H (2003), *Econometric Analysis*, 5ED, Prentice Hall, Upper Saddle River, New Jersey

Gregoriou, A & Kontonikas, A (2006), "Inflation targeting and the stationarity of inflation: New results from an ESTAR unit root test", *Bulletin of Economic Research*, Vol.54, No.4; 309-322

Hassler, U & Wolters, J (1994), "On the power of unit root tests against fractional alternatives", *Economic Letters*, Vol.45; 1-5

Hendry, D.F & Hubrich, K (2006), "Forecasting economic aggregates by disaggregates", *Working Papers Series*, No.589, European Central Bank

Hofmann, B & Remsperger, H (2005), "Inflation differentials among the Euro area countries: Potential causes and consequences", Journal of Asian Economics, Vol.16; 403-419

Hosking, J.R.M (1981), "Fractional differencing", *Biometrika*, Vol.68, No.1; 165-176

Hubrich, K (2005), "Forecasting euro area inflation: Does aggregating forecasts by HICP component improve forecast accuracy?", *International Journal of Forecasting*, Vol.21; 119-136

Jensen, M (1999), "Using wavelets to obtain a consistent ordinary least squares estimator of the long-memory parameter", *Journal of Forecasting,* Vol. 18; 17-32

Jensen, M (2009), "The long-run Fisher effect – can it be tested?", *Journal of Money, Credit and Banking*, Vol. 41, No. 1; 221-231

Kennedy, P (2008), *A guide to econometrics*, 6ED, Blackwell Publishing, United Kingdom

Kumar, M.S & Okimoto, T (2007), "Dynamics of persistence in international inflation rates" *Journal of Money, Credit and Banking*, Vol. 39, No. 6; 1457-1479

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P & Shin, Y (1992), "Testing the null hypothesis of stationarity against the alternative of a unit root", *Journal of Econometrics*, Vol.54; 159-178

Leybourne, S & Newbold, P (1999), "On the size properties of Phillips-Perron tests", *Journal of Time Series analysis*, Vol.20, No. 1; 51-61

MacDonald, R & Murphy, P.D (1989), "Testing for the long run relationship between nominal interest rates and inflation using cointegration techniques", *Applied Economics*, Vol.21, No.4; 439-447

Mishkin, F.S & Posen, A.S (1997), "Inflation targeting: lessons from four countries", *NBER Working Paper,* No.6126, August, 1997

Mooslechner, P & Schuerz, M (1999), "International macroeconomic policy coordination: any lessons for EMU?", *Empirica*, Vol.26; 171-199

OECD, Organization for Economic Cooperation and Development, In *OECD Statistics Database*. Database name: Key short-term economic indicators. Dataset: Consumer prices:

All items. Parameters: Percentage change on the same period of the previous year. Found at http://stats.oecd.org. Date generated: 2011/05/17

Percival, D.B & Walden, A.T (2000), *Wavelet methods for time series analysis*, Cambridge University Press, New York

Phillips, P.C.B & Perron, P (1988), "Testing for a unit root in time series regression", *Biometrika*, Vol.75, No.2; 335-346

Robinson, P.M (1995), "Log-periodogram regression of time series with long range dependence", *The Annals of Statistics,* Vol.23, No. 3; 1048-1072

Rose, A.K (1988), "Is the real interest rate stable?", *The Journal of Finance*, Vol. 43, No.5; 1095-1112

Svensson, L.E.O (1999), "Price-level targeting versus inflation targeting: a free lunch?", *Journal of Money, Credit and Banking*, Vol.31, No.3; 277-295

Tkacz, G (2001), "Estimating the fractional order of integration of interest rates using a wavelet OLS estimator", *Studies in Nonlinear Dynamics and Econometrics,* Vol.5, No.1; 19-32

U.S Department of Labor, Bureau of Labor Statistics, Database name: Consumer Price Index – All Urban Consumers (Current series). Dataset: U.S. All items, 1982-84 = 100 – CUUR0000SA0. Parameters: 12-Month percentage change, year range 1914 – 2011. Found at http://www.bls.gov/data/. Date generated: 2011/05/17. Last updated: 2011/05/13.

Zagaglia, P (2009), "Fractional integration of inflation rates: a note", *Applied Economic Letters,* Vol.16; 1103-1105