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SIMPLIFYING QUANTUM GRAVITY CALCULATIONS

With examples of scalar-graviton and graviton-graviton scattering

Safi Rafie-Zinedine

Department of Astronomy and Theoretical Physics, Lund University

Master thesis supervised by Prof. Johan Bijnens

This work is dedicated to the memory

of my friend Rabeeh Zinedine.

Abstract

The Einstein-Hilbert Lagrangian for gravity is non-renormalizable at loop level. However, it can be treated in the effective field theory framework which means that gravity as an effective theory can be renormalized when a proper expansion of the effective Lagrangian is made. At the same time, the Feynman rules for gravity are very complicated, although the resulting amplitudes do not have the same complications. Therefore, in this thesis we want to simplify the Feynman rules as much as possible by using the most general parameterized gauge condition, adding all possible parameterized total derivative terms and redefining the gravitational, ghosts and scalar fields in a general parameterization way. By choosing the parameters in a specific way, we obtain simplified Feynman rules, especially the triple and quadruple graviton vertices are simplified. In addition, we verify our simplified rules by calculating the amplitudes of scalar-graviton and graviton-graviton scattering at tree level using the simplified and standard Feynman rules. Finally, we show the utility of these simplified rules by calculating some one-loop diagrams for scalar-graviton scattering and comparing to the standard Feynman rules.

Popular Science Description

In physics, the story of gravity is still incomplete. It began under an apple tree when Newton started his journey to discover his laws about the gravitational force, but these laws were not enough to describe all the gravitational phenomena. To describe gravity in a more accurate way, Einstein came with the theory of general relativity. This theory treats the gravitational force as a consequence of the curvature of spacetime. This should be compared with the other forces in nature (the electromagnetic, weak and strong force) which are described by the standard model of particle physics. This model treats the forces as a consequence of exchanging particles which are called quanta. There have been many attempts in the last century to study gravity as a quantized theory, quantum gravity, where the exchanged particles are called gravitons. Quantum gravity is still not fully understood because of many obstacles, one of them being its complicated calculations.

The purpose of this thesis is to address this latter problem of complicated calculations, following the belief that nature should be described in a beautiful and simple mathematical way. Moreover, a simplified form with fewer terms that contribute to gravitational effects can lead to a deeper understanding of gravity. To treat this problem, we want to find mathematical tools that can simplify the math of the theory without changing the information that it contains. Fortunately, in quantum physics such tools exist as field redefinition which means that we can redefine the gravitons in order to find a simpler expression that can describe exchanging these particles. As a result of applying these mathematical tools, we successfully simplify the math that describes the gravitational interaction between particles. In particular, we show that the interaction between three gravitons can be reduced from 40 to just 4 terms, and the interaction between four gravitons can be reduced from 113 to 12 terms. Finally, we verify our simplification by comparing the results for physical processes using the standard approach and using our simplified approach.

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Contents

1 Introduction

In physics, there have been many attempts to study the gravitational field as a quantized field in order to unify the gravitational force, which is described by General Relativity (GR), with the other forces in nature, which are described by the Standard Model (SM) using the Quantum Field Theory (QFT) framework [1, 2]. These attempts have met various obstacles, either the lack of experimental abilities to explore sufficiently high energy or the lack of needed mathematical tools that allow the study of gravity as a quantized theory, quantum gravity. The reason behind the latter problem is that the Einstein-Hilbert Lagrangian of quantum gravity diverges at loop level so we consider it as a non-renormalizable theory [3, 4]. Thus, such a quantum theory for gravity has not been solved yet, and a full unified theory has not been found yet.

Even so, it is still possible to construct and solve a renormalizable effective theory for gravity order by order by using the Effective Field Theory (EFT) framework [3, 5]. In this framework, we can study gravity at a particular loop level by expanding the Lagrangian in the energy expansion up to the relevant terms for this loop level. To determine which terms are relevant, we use Weinberg's power counting theorem [3]. These new terms in the expansion contain new parameters which can absorb the divergences from the loop diagrams, and at the same time can be measured experimentally. Therefore, gravity can be renormalized at loop level when we make the proper expansion of the Lagrangian.

At the same time, the Einstein–Hilbert Lagrangian is very complicated and leads to equally complicated Feynman rules as given in [6], which we will call the standard Feynman rules. These rules give lengthy complicated calculations, but they also lead to scattering amplitudes that are simple in general [7, 8]. Therefore, there is a strong indication that manipulating this Lagrangian can lead to a simpler form which still gives the same scattering amplitudes. In addition, when the Lagrangian contains fewer terms, it is possible to understand the math in the theory better. Consequently, there has been some efforts to simplify these rules as in [9], where a parameterized metric field was used.

As starting point for the simplification approach in this thesis, we derive the Lagrangian for gravity, the Einstein–Hilbert Lagrangian, by using an analogy with the Yang-Mills (YM) gauge theory. Then we construct the most general effective Lagrangian by using Effective Field Theory (EFT). After that, we set out to simplify the Feynman rules as much as possible by manipulating the Lagrangian for gravity using the three freedoms [1, 3, 10]: choosing a gauge, adding total derivative terms to the Lagrangian and re-parameterization of the fields (gravitational, scalar, ghost fields). In other words, we choose the most general parameterized gauge condition, and we add a parameterization of all possible total derivative terms. Then, we redefine the gravitational, ghosts and scalar fields using a general parameterization. As a result of parameterizing the previous freedoms, we efficiently reduce the problem of simplifying Feynman rules as much as possible to solve a system of linear equations for eight sets of parameters.

In addition, for comparison and verification purposes, we perform the calculations in two approaches. In the standard calculations, we use the de Donder gauge to obtain the standard Feynman rules as shown in App. B, which agree with those in [6]. In the simplified calculations, we use the three freedoms that we mentioned before, and then choose the parameters in order to obtain Feynman rules as simple as possible, especially the triple and quadruple graviton vertices.

To check our simplified rules, we compare the resulting amplitudes of scalar-graviton and graviton-graviton scattering at tree level using the standard rules and using the simplified ones [7, 8, 11]. Furthermore, to show the utility of these simplified rules, we compare the standard and simplified calculations of some one-loop diagrams for scalar-graviton scattering, where we use the dimensional regularization scheme with the Passarino-Veltman method to calculate the loop integrals [12, 13]. Finally, since we are interested in the calculations up to one-loop level for scalar-graviton scattering, we only simplified the lowest order vertices.

In this thesis, we start in Sec. 2 by discussing the theoretical background for deriving the Lagrangian for gravity and show the tools needed for manipulating the Lagrangian and calculating scattering processes. Then in Sec. 3 we calculate the Feynman rules in the standard and simplified way. After that, in Sec. 4 we use the Feynman rules to calculate the amplitudes of scalar-graviton and graviton-graviton scattering at tree level in both ways. Moreover, in Sec. 5 we show the usefulness of our Feynman rules by comparing calculations of some one-loop diagrams for scalar-graviton scattering using both approaches. Finally, the conclusions of our work is in Sec. 6.

Before we start, it is important to mention that we follow the conventions in [3] throughout this thesis, such as the metric signature $(+,-,-,-)$ and the natural units $c = \hbar = 1$. In addition, since the calculations of Feynman rules and the amplitudes are extremely lengthy to do by hand, we use the FORM program [14, 15] to perform them.

2 Theoretical Background

In this section, we start by discussing our motivation behind choosing spin-2 for the graviton [1, 2, 3, 4]. Then, we give the derivation of the Einstein–Hilbert Lagrangian using an analogy with YM theory [2, 3]. After that, using the EFT framework, we build the most general effective Lagrangian for gravity which is needed to study gravity at one-loop level [3, 5]. In addition, we discuss the three freedoms that we use to simplify Feynman rules: choosing the gauge [1, 3], adding total derivative terms [1], and field redefinitions [10].

2.1 Spin of Graviton

As in this thesis we want to quantize the gravitational field, let us start by discussing the possible spins for its quanta, which are called gravitons [3]. Since the graviton is a boson, its spin has to be an integer number. In this case, there are three possibilities: spin-0, spin-1 and spin-2 while higher spins are not consistent with QFT for fundamental particles with interactions [4]. Firstly, if we start with a spin-0 particle as the Higgs boson, the scalar-scalar scattering via a spin-0 graviton, as shown in Fig. 1, leads to a Newtonian gravitational potential with the bare mass as the source [1, 3]. However, we know from GR that the bare mass of an object is not the only source of the gravitational field [2].

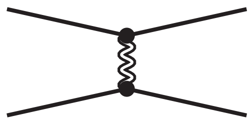


Figure 1: Scalar-Scalar scattering at tree level.

Secondly, with a spin-1 particle as the photon, the scalar-scalar scattering leads to an attractive or repulsive potential, so spin–1 particles are not appropriate to describe the gravitational force [3]. Finally, in the case of a spin-2 particle, the scalar-scalar scattering leads to a gravitational potential with the energy-momentum tensor $\mathcal{T}_{\mu\nu}$ as the source for gravity, which is consistent with GR [2, 3].

From the above discussion, it is clear that the graviton should be a spin-2 boson in order to be consistent with GR and QFT, and later on we will see more indications that the graviton should be a spin-2 particle.

2.2 Analogy with Yang-Mills Theory

The main idea of a Yang-Mills (YM) theory is to search for an appropriate global symmetry which is relevant to the force that we study. Then, we convert this symmetry to a local or gauge symmetry, where this conversion is called gauging the symmetry. After that, to preserve the local gauge invariance, we need to insert a new field which will be the field of the gauge boson for this force.

For example, consider the following, matter Lagrangian for a massive real scalar field ϕ:

$$\mathcal{L}_{\text{Matter}} = \frac{1}{2}\eta^{ab}\partial_a\phi\partial_b\phi - \frac{1}{2}m^2\phi^2\,, \tag{2.1}$$

where η^{ab} is the Minkowski metric, and m is the mass. This Lagrangian is invariant under the global translational symmetry

$$y^a \quad \rightarrow \quad y'^a = y^a + d^a\,. \tag{2.2}$$

Then, we convert this global translational symmetry to a local translational symmetry, where the latter is called the General Coordinate Transformations (GCT),

$$x^\mu \quad \rightarrow \quad x'^\mu = x^\mu + d^\mu(x)\,, \tag{2.3}$$

where Lorentz indices $a, b, \cdots$ in flat space-time have been replaced by world indices $\mu, \nu, \cdots$ in curved space-time. In addition, we need to replace the Minkowski metric in flat space η_{ab} by the metric field in curved space $g_{\mu\nu}$ in order to make the Lagrangian Eq. (2.1) invariant under GCT Eq. (2.3). Thus, the Lagrangian becomes

$$\mathcal{L}_{\text{Matter}} = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\,, \tag{2.4}$$

which contains a new field, the metric field $g^{\mu\nu}$, that represents the spin-2 gauge boson for gravity (i.e., the graviton).

As with other gauge theories, we need to introduce a kinetic term for the gravitational field $g^{\mu\nu}$. We search for a quantity similar to the YM field strengh tensor $F_{\mu\nu}$, which we recall is related to the commutator of covariant derivatives $[D_\mu, D_\nu]$. For gravity, with a vector field V^β, this commutator can be written as [3]

$$[D_\mu, D_\nu]V^\beta = R_{\mu\nu\alpha}{}^{\beta}V^\alpha\,, \tag{2.5}$$

where $D_\nu V_\beta = \partial_\nu V_\beta - \Gamma^\gamma_{\beta\nu}V_\gamma$ is the covariant derivative, $R_{\mu\nu\alpha}{}^{\beta}$ is the Riemann tensor given by

$$R_{\mu\nu\alpha}{}^{\beta} = \partial_\mu\Gamma_{\nu\alpha}{}^{\beta} - \partial_\nu\Gamma_{\mu\alpha}{}^{\beta} + \Gamma_{\mu\sigma}{}^{\beta}\Gamma_{\nu\alpha}{}^{\sigma} - \Gamma_{\nu\sigma}{}^{\beta}\Gamma_{\mu\alpha}{}^{\sigma}\,, \tag{2.6}$$

and $\Gamma_{\nu\alpha}{}^{\beta}$ is the Christoffel symbol given by

$$\Gamma_{\nu\alpha}{}^{\beta} = \frac{1}{2}g^{\beta\rho}(\partial_\nu g_{\rho\alpha} + \partial_\alpha g_{\rho\nu} - \partial_\rho g_{\nu\alpha})\,. \tag{2.7}$$

In addition, from the Riemann tensor we can also get the Ricci tensor $R_{\nu\alpha}$ and the scalar curvature R as

$$R_{\nu\alpha} = R_{\mu\nu\alpha}{}^{\mu}\,, \tag{2.8}$$

$$R = g^{\nu\alpha}R_{\nu\alpha} = g^{\nu\alpha}R_{\mu\nu\alpha}{}^{\mu}\,. \tag{2.9}$$

We also use the standard expansion of the weak gravitational field around the Minkowski metric $\eta_{\mu\nu}$, that is used to raise or lower indices, given by

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}\,, \tag{2.10}$$

where $h_{\mu\nu}$ is the canonical quantized gravitational field, and κ is the Newtonian strength of gravitational interactions.

In this situation, we have three quantities: $R_{\mu\nu\alpha\beta}$, $R_{\nu\alpha}$ and R that are related to the commutator of covariant derivatives. One of them can be chosen as a field strength tensor for the gravitational field to build the kinetic term in analogy with $\mathcal{L}_{(\text{Kin, YM})} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$. To do this, we use that the YM field strengh tensor is antisymmetric, $F_{\mu\nu} = -F_{\nu\mu}$, and has one partial derivative $F_{\mu\nu} \sim \partial A$, while the Riemann tensor $R_{\mu\nu\alpha\beta} = R_{\alpha\beta\mu\nu}$ and the Ricci tensor $R_{\nu\alpha} = R_{\alpha\nu}$ do not have the same symmetry property and have two partial derivatives $R_{\mu\nu\alpha\beta}, R_{\mu\nu} \sim \partial\partial h$. In addition, the kinetic term should be Lorentz invariant and GCT invariant. So, the simplest combinations for the kinetic term are $R \sim \partial\partial h$ as well as $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}, R_{\mu\nu}R^{\mu\nu} \sim \partial\partial\partial\partial h$. It follows that R is the most relevant one of them for weak-field since it has only two partial derivatives while the other have four. Thus, we can write the kinetic term for the gravitational field in terms of the scalar curvature as the Einstein-Hilbert Lagrangian for gravity [3]

$$\mathcal{L}_{\text{Gravity}} = -\frac{2}{\kappa^2} R \,. \tag{2.11}$$

In addition, when we move from flat to curved space, there is a correction of the measure d^4y in the action $\mathcal{S}$ as

$$d^4y = \sqrt{-\det(g_{\mu\nu})}\, d^4x = \sqrt{-g}\, d^4x \,, \tag{2.12}$$

where $\sqrt{-g}$ is the square root of the determinant of the metric tensor which is given by

$$\begin{aligned}\sqrt{-g} = \sqrt{-\det(g_{\mu\nu})} &= \Big(-\det(\eta_{\mu\lambda}) \det\big(\delta^\lambda_\nu + \kappa h^\lambda{}_\nu + \cdots\big)\Big)^{1/2} = \Big(e^{\mathrm{tr}\left(\ln\left(\delta^\lambda_\nu + \kappa h^\lambda{}_\nu + \cdots\right)\right)}\Big)^{1/2} \\ &= \sum_{i=0}^{\infty} \frac{1}{i!} \Big(\frac{1}{2} \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} (\kappa h^\lambda{}_\lambda + \cdots)^j \Big)^i \,, \end{aligned} \tag{2.13}$$

and we have used that the metric tensor can be written as $g_{\mu\nu} = \eta_{\mu\lambda}(\delta^\lambda_\nu + \kappa h^\lambda{}_\nu + \cdots)$. In this thesis, we take the expansion in i and j up to four which is relevant to the lowest order vertices as we will see later.

Summarizing, the actions of matter and gravity become

$$\mathcal{S}_{\text{Matter}} = \int d^4x \sqrt{-g} \Big(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \Big) \,, \tag{2.14}$$

$$\mathcal{S}_{\text{Gravity}} = \int d^4x \sqrt{-g} \Big(-\frac{2}{\kappa^2} R \Big) \,. \tag{2.15}$$

From the matter action, the variation with respect to $g^{\mu\nu}$ is

$$\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{S}_{\text{Matter}}}{\delta g^{\mu\nu}} = \mathcal{T}_{\mu\nu} \,, \tag{2.16}$$

where $\mathcal{T}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu}(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - m^2\phi^2)$ is the Energy-Momentum Tensor (EMT). Thus, the EMT is the conserved current that follows from GCT which is consistent with GR, where EMT is the source for gravity. This is an indication that we are dealing with the correct symmetry for gravity. Moreover, the total action is

$$\mathcal{S}_{\text{Total}} = \int dx^4 \sqrt{-g} \Big(-\frac{2}{\kappa^2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu\phi\partial_\nu\phi - \frac{1}{2} m^2 \phi^2 \Big) \,. \tag{2.17}$$

From this action, the equation of motion for $g^{\mu\nu}$ will be

$$\delta\mathcal{S} = \int dx^4 \sqrt{-g} \Big(-\frac{2}{\kappa^2} R_{\mu\nu} + \frac{2}{\kappa^2}\frac{1}{2} g_{\mu\nu} R + \frac{1}{2} \mathcal{T}_{\mu\nu} \Big) \delta g^{\mu\nu} = 0 \,, \tag{2.18}$$

$$\Rightarrow \qquad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\kappa^2}{4} \mathcal{T}_{\mu\nu} \,, \tag{2.19}$$

This equation can be recognized from GR as Einstein's field equation with $\frac{\kappa^2}{4} = 8\pi G$, where G is the gravitational constant [2]. Reaching Einstein's equation is another indication that we are gauging the correct global symmetry for the gravitational force.

However, when we calculate scattering at loop level with this Lagrangian Eq. (2.11), we get UV divergences. Therefore, we need to construct gravity as an effective field theory in order to study this theory at loop level.

2.3 Effective Field Theory

Effective Field Theory (EFT) is a way to study physics in a particular range of energy while neglecting the physics at higher energy. This can be done if the contributions from high energy are small when the theory is studied in the low energy range. For example, we can study the hydrogen atom using the Schrödinger equation, neglecting the quark and gluon interactions inside the nucleus. Mathematically, it means that we need to expand and organize the Lagrangian according to the dimension of the energy operators, where in the case of gravity, the energy operators are just partial derivatives. This expansion is called the energy expansion of the Lagrangian, and it separates the terms which are relevant at high energy, from the terms which are relevant at low energy. In other words, the effective Lagrangian can be written in the energy expansion as

$$\mathcal{L}_{\mathsf{eff}} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots , \tag{2.20}$$

where the term $\mathcal{L}_0$ does not contain any energy operator $\mathcal{O}(E^0)$, and it is just a constant. The term $\mathcal{L}_1$ contains energy operators of dimension one $\mathcal{O}(E^1)$ and so on.

In the case of gravity, the quantities that can be used to construct the effective Lagrangian are: $R_{\mu\nu\alpha\beta}$, $R_{\nu\alpha}$ and R, which as already mentioned are related to the commutator of the covariant derivative Eq. (2.5). However, each of these quantities $(R_{\mu\nu\alpha\beta}, R_{\nu\alpha}, R \sim \partial\partial h)$ contain two partial derivatives, which as already mentioned are energy operators of the gravitational field. Therefore, only even energy dimensions are possible in the energy expansion

$$\mathcal{L}_{\mathsf{eff}} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots . \tag{2.21}$$

At the same time, since in this thesis we are only interested in studying gravity up to one-loop order, it is useful to use Weinberg's power counting theorem [3]. This theorem can tell us how many terms in the energy expansion should be taken into account in order that the theory can be renormalized at one-loop. According to this theorem, the energy dimension $\mathcal{D}$ of a diagram with N_L loops and N_n vertices arising from the effective Lagrangian terms that contain n derivatives is given by

$$\mathcal{D} = 2 + \sum_n N_n(n-2) + 2N_L . \tag{2.22}$$

In our case, we want to calculate the energy dimension $\mathcal{D}$ for gravity with $n = 2$ up to one-loop order $N_L = 1$. This gives $\mathcal{D} = 4$, which means that gravity can be renormalized at one-loop, if we take into account terms up to energy dimension four $\mathcal{O}(E^4)$ in the energy expansion of the Lagrangian.

Let us discuss the possible terms that can be inserted into the effective Lagrangian for gravity in more detail. First, the terms should be GCT invariant and Lorentz invariant. Then we need to organize the terms according to the energy dimension as follows:

- $\mathcal{L}_0$: This is only a constant such as the Cosmological constant Λ.
- $\mathcal{L}_2$: The only possible term is $R \sim \partial\partial$.
- $\mathcal{L}_4$: There are three possible combinations: R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \sim \partial\partial\partial\partial$.
- $\mathcal{L}_6$: There are four possible combinations: R^3, $R\,R_{\mu\nu}R^{\mu\nu}$, $R\,R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$, $R^{\mu\nu}R^{\alpha\beta}R_{\mu\nu\alpha\beta} \sim \partial\partial\partial\partial\partial\partial$.

Thus, the most general effective Lagrangian in the energy expansion up to energy dimension four is given by

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 + \cdots \\ &= -\Lambda - \frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \cdots ,\end{aligned} \tag{2.23}$$

where c_i are numerical coefficients of the higher order corrections, which are called counter terms. In addition, Λ is experimentally very small so we will neglect it as the following.

When we use the κ term $(-\frac{2}{\kappa^2}R)$ for one-loop calculations, we get UV divergences which are absorbed by c_i. The finite parts of c_i have to be determined experimentally. Thus, the theory can be renormalized at one-loop. In addition, when we calculate scalar-graviton scattering up to one-loop order from the above Lagrangian, the κ term contributes to one-loop diagrams while the κ and c_i terms contribute to tree level diagrams.

Finally, let us discuss locality of the theory [3]. EFT is local if the higher order operators in the energy expansion are taken into account. However, if we neglect these higher order operators, the theory becomes non-local at high energy whereas locality will be restored at low energy. For example, in the full theory, the one-loop diagram as shown in Fig. 2a is local at large energy. On the other hand, in its corresponding EFT and after neglecting the higher order operators, this diagram becomes non-local. However, locality will be restored at low energy where this loop will be reduced to a vertex as shown in Fig. 2b. In addition, using EFT the diagram becomes easier to calculate by reducing this loop to a vertex.

Figure 2: (a) one-loop diagram at high energy in a full theory. (b) its corresponding vertex diagram in its EFT at low energy when locality is restored.

2.4 Lagrangians for Fixing Gauge and Ghosts

Since the Lagrangian for gravity has more degrees of freedom than its gauge boson, we need to fix the gauge and introduce ghosts in order to get rid of the extra degrees of freedom. The gauge boson in our case is a spin-2 massless graviton, which has a transverse, traceless, symmetric polarization tensor, with two degrees of freedom.

Another way to see why we need to fix the gauge and introduce ghosts is by considering the following GCT, where $\xi^\mu(x)$ is an infinitesimal translation,

$$x^\mu \;\rightarrow x'^\mu = x^\mu - \xi^\mu(x)\,. \tag{2.24}$$

In this case, the metric $g_{\mu\nu}$ transforms as

$$g_{\mu\nu} \;\rightarrow g'_{\mu\nu}(x') = g_{\alpha\beta}(x)\Big(\frac{\partial x^\alpha}{\partial x'^\mu}\Big)\Big(\frac{\partial x^\beta}{\partial x'^\nu}\Big) = g_{\alpha\beta}(x)\Big(\delta^\alpha_\mu + \partial_\mu \xi^\alpha(x)\Big)\Big(\delta^\beta_\nu + \partial_\nu \xi^\beta(x)\Big)\,. \tag{2.25}$$

We also consider the weak gravitational field expansion, Eq. (2.10), so the transformation of the gravitational field $h_{\mu\nu}$, the gauge transformation, is

$$h_{\mu\nu} \;\rightarrow h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu(x) + h_{\mu\sigma}\partial_\nu \xi^\sigma(x) + h_{\nu\sigma}\partial_\mu \xi^\sigma(x) + \xi^\sigma(x)\partial_\sigma h_{\mu\nu}\,. \tag{2.26}$$

However, since the Lagrangian for gravity is invariant under this transformation, we get a redundancy in the description of the physical system. This redundancy can also be seen from the path integral formulation of the generating functional

$$\mathcal{Z} = \int D[h]\;\exp(i\,S(h)) = \int D[h]\;\exp\Big(i\int d^4x\mathcal{L}(h)\Big)\,, \tag{2.27}$$

where the measure $\int D[h]$ is performed over all configurations of h, including those configurations that are equivalent under the gauge transformation, Eq. (2.26).

Because of this, we need to choose a gauge condition to build two Lagrangians which together can remove this redundancy. The first Lagrangian is to fix the gauge, and the second Lagrangian is to correct the first one depending on the choice of the gauge condition. In other words, we need to insert a Lagrangian for fixing the gauge $\mathcal{L}_{\text{FG}}$ and a Lagrangian for ghosts $\mathcal{L}_{\text{GH}}$ into the Lagrangian for gravity in order to remove this redundancy. In the path integral formalism, this means that using $\mathcal{L}_{\text{FG}}$ and $\mathcal{L}_{\text{GH}}$ will correct the measure $\int D[h]$ to be performed over the correct configurations of h. Next, we will follow the Faddeev-Popov method to derive the Lagrangian for fixing the gauge $\mathcal{L}_{\text{FG}}$ and the Lagrangian for ghosts $\mathcal{L}_{\text{GH}}$.

The Faddeev-Popov method depends on multiplying the generating functional by two identities and then performing the integral in order to get a new generating functional with two new terms, where one of them is related to $\mathcal{L}_{\text{FG}}$ and the other is related to $\mathcal{L}_{\text{GH}}$.

To do that, we use the following identity, always taking into account the gauge transformation Eq. (2.26),

$$1 = \int D[\xi]\;\delta\Big(\mathcal{C}_\mu(h) - F_\mu(x)\Big)\;\Delta(h)\,, \tag{2.28}$$

where $\mathcal{C}_\mu(h) = F_\mu(x)$ is the gauge condition, and $\Delta(h) = \det\left(\frac{\partial \mathcal{C}_\alpha(h)}{\partial \xi_\beta}\right)$ is Faddeev-Popov determinant.

Since the generating functional Eq. (2.27) does not depend on ξ, it is possible to insert the above identity into it as

$$\mathcal{Z} = \int D[h]\; D[\xi]\; \delta\Big(\mathcal{C}_\mu(h) - F_\mu(x)\Big)\; \Delta(h)\; \exp(iS)\,. \tag{2.29}$$

We also use the following identity

$$1 = N \int D[F]\; \exp\Big(-\frac{i}{2\epsilon}\int d^4x F_\nu(x)F^\nu(x)\Big)\,, \tag{2.30}$$

where N is a normalization constant, and ϵ is a parameter.

Again, since the generating functional Eq. (2.29) does not depend on $F(x)$, it is also possible to insert the above identity into it as

$$\mathcal{Z} = N \int D[F]\; D[h]\; D[\xi]\; \delta\Big(\mathcal{C}_\mu(h) - F_\mu(x)\Big)\; \Delta(h)\; \exp\Big(iS - \frac{i}{2\epsilon}\int d^4x F_\nu(x)F^\nu(x)\Big)\,. \tag{2.31}$$

Then, performing the integral over $\xi, F(x)$ yields

$$\mathcal{Z} = N' \int D[h]\; \Delta(h)\; \exp\Big(iS - \frac{i}{2\epsilon}\int d^4x\; \mathcal{C}_\nu(h)\mathcal{C}^\nu(h)\Big)\,, \tag{2.32}$$

where N' is a new normalization constant. However, the Faddeev-Popov determinant $\Delta(h)$ can be written in terms of an artificial vector fermion field χ_β, the ghost field, and an anti-fermion field $\bar{\chi}_\alpha$, the anti-ghost field, as

$$\Delta(h) = \det\left(\frac{\partial \mathcal{C}_\alpha(h)}{\partial \xi_\beta}\right) = \int D[\bar{\chi}]D[\chi]\exp\Big(i\int d^4x\; \bar{\chi}^\alpha\, \frac{\partial \mathcal{C}_\alpha(h)}{\partial \xi_\beta}\, \chi^\beta\Big)\,. \tag{2.33}$$

Thus, the generating functional becomes

$$\mathcal{Z} = N' \int D[h]\; D[\bar{\chi}]\; D[\chi]\; \exp\left(iS + i\int d^4x\Big(\bar{\chi}^\alpha\, \frac{\partial \mathcal{C}_\alpha(h)}{\partial \xi_\beta}\, \chi^\beta\Big) - i\int d^4x\Big(\frac{1}{2\epsilon}\mathcal{C}_\nu(h)\mathcal{C}^\nu(h)\Big)\right),$$

where we have two new terms:

- Lagrangian for ghost fields:

$$\mathcal{L}_{\text{ghost}}(\bar{\chi}, \chi, h) = \bar{\chi}^\alpha\, \frac{\partial \mathcal{C}_\alpha(h)}{\partial \xi_\beta}\, \chi^\beta\,. \tag{2.34}$$

- Lagrangian for fixing gauge:

$$\mathcal{L}_{\text{FG}}(h) = \frac{1}{2\epsilon}\mathcal{C}_\nu(h)\mathcal{C}^\nu(h)\,, \tag{2.35}$$

  where ϵ is a parameter that can be chosen differently for different gauges. However, Feynman–'t Hooft gauge, $\epsilon = 1$ will be used throughout the thesis.

Finally, we calculate $\mathcal{L}_{\rm FG}$ and $\mathcal{L}_{\rm ghost}$ in two approaches. In the standard approach, we use the de Donder (harmonic) gauge condition

$$\mathcal{C}_\mu(h) = \kappa\Big[\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h_\nu{}^\nu\Big]\,. \tag{2.36}$$

In the simplified approach, we use a general parameterized gauge condition which contains all possible combinations for $(\partial h,\, \partial hh,\, \partial hhh)$ as given below

$$\begin{aligned}
\mathcal{C}_\mu(h) = {}& \kappa\Big[b_1\partial^\nu h_{\mu\nu} + b_2\partial_\mu h_\nu{}^\nu\Big] \\
&+ \kappa^2\Big[b_3\partial_\mu h_\nu{}^\nu h_\alpha{}^\alpha + b_4\partial_\mu h^{\nu\alpha}h_{\nu\alpha} + b_5\partial^\nu h_{\mu\nu}h_\alpha{}^\alpha + b_6\partial_\nu h_{\mu\alpha}h^{\nu\alpha} \\
&\qquad + b_7\partial_\nu h^{\nu\alpha}h_{\mu\alpha} + b_8\partial^\nu h_\alpha{}^\alpha h_{\mu\nu}\Big] \\
&+ \kappa^3\Big[b_9\partial_\mu h_\nu{}^\nu h_\alpha{}^\alpha h_\beta{}^\beta + b_{10}\partial_\mu h_\nu{}^\nu h^{\alpha\beta}h_{\alpha\beta} + b_{11}\partial_\mu h^{\nu\alpha}h_{\nu\alpha}h_\beta{}^\beta + b_{12}\partial_\mu h^{\nu\alpha}h_\alpha{}^\beta h_{\beta\nu} \\
&\qquad + b_{13}\partial^\nu h_{\mu\nu}h_\alpha{}^\alpha h_\beta{}^\beta + b_{14}\partial^\nu h_{\mu\nu}h^{\alpha\beta}h_{\alpha\beta} + b_{15}\partial_\nu h_{\mu\alpha}h^{\nu\alpha}h_\beta{}^\beta + b_{16}\partial^\nu h_{\mu\alpha}h^{\alpha\beta}h_{\beta\nu} \\
&\qquad + b_{17}\partial_\nu h^{\nu\alpha}h_{\mu\alpha}h_\beta{}^\beta + b_{18}\partial^\nu h^{\alpha\beta}h_{\mu\alpha}h_{\nu\beta} + b_{19}\partial^\nu h_{\nu\alpha}h_{\mu\beta}h^{\alpha\beta} + b_{20}\partial_\alpha h_\nu{}^\nu h_{\mu\beta}h^{\alpha\beta} \\
&\qquad + b_{21}\partial^\nu h_\alpha{}^\alpha h_{\mu\nu}h_\beta{}^\beta + b_{22}\partial^\nu h^{\alpha\beta}h_{\mu\nu}h_{\alpha\beta}\Big] + \cdots\,,
\end{aligned} \tag{2.37}$$

where b_i are the parameters that will be chosen later to simplify the Feynman rules as much as possible. In addition, note that we consider only up to three powers of h which are relevant to the lowest order vertices, that we need to calculate scalar-graviton scattering to one-loop as will be shown later.

2.5 Total Derivative Lagrangians

In general, a transformation of the fields is a symmetry transformation if the Lagrangian changes by a total derivative [1]. This means that adding total derivative terms to the Lagrangian does not change the physics that it contains. We can see this from the principle of least action. Adding a total derivative term to the Lagrangian

$$\mathcal{L} \qquad \rightarrow \qquad \tilde{\mathcal{L}} = \mathcal{L} + \partial_\mu F^\mu(h)\,, \tag{2.38}$$

the variation of the action remains zero. Explicitly,

$$\delta\tilde{S} = \delta\int d^4x\tilde{\mathcal{L}} = \delta\int d^4x\big(\mathcal{L} + \partial_\mu F^\mu(h)\big) = \delta\int d^4x\mathcal{L} + \delta\int d^4x\partial_\mu F^\mu(h) = 0\,, \tag{2.39}$$

where $\delta S = \delta\int d^4x\mathcal{L} = 0$, and the infinitesimal variation of the total derivative part vanishes by the assumption that F vanishes at the boundary of integration.

In other words, adding total derivative terms to the Lagrangian is equivalent to doing integration by parts. For example,

$$\begin{aligned}
\int d^4x\, \phi\, \partial_\mu\partial^\mu\phi &= \int d^4x\, \partial_\mu(\phi\, \partial^\mu\phi) - \int d^4x\, \partial_\mu\phi\, \partial^\mu\phi = \phi\, \partial^\mu\phi\Big|_{\delta V} - \int d^4x\, \partial_\mu\phi\, \partial^\mu\phi \\
&= -\int d^4x\, \partial_\mu\phi\, \partial^\mu\phi\,.
\end{aligned} \tag{2.40}$$

In addition, the freedom of adding total derivative terms can also be seen from applying momentum conservation in a vertex in momentum space. For example, if we consider the term $\phi\,\phi\,\partial^\mu\partial_\mu\phi$ and then add the total derivative $-\partial^\mu(\phi\,\phi\,\partial_\mu\phi)$, we get

$$\begin{aligned} &\phi\,\phi\,\partial^\mu\partial_\mu\phi = \phi\,\phi\,\partial^\mu\partial_\mu\phi - \partial^\mu(\phi\,\phi\,\partial_\mu\phi) = -2\phi\,\partial^\mu\phi\,\partial_\mu\phi\,, \\ \Rightarrow \qquad &\phi\,\phi\,\partial^\mu\partial_\mu\phi = -2\phi\,\partial^\mu\phi\,\partial_\mu\phi\,. \end{aligned} \tag{2.41}$$

These partial derivatives give momenta in momentum space. So, assuming that all the momenta in this vertex are ingoing, equation (2.41) can be represented in momentum space as

$$2(p_1^2+p_2^2+p_3^2) = -4(p_1\cdot p_2 + p_1\cdot p_3 + p_2\cdot p_3)\,. \tag{2.42}$$

This result also agrees with momentum conservation in this vertex

$$p_1+p_2+p_3=0\,,\ \Rightarrow\ (p_1+p_2+p_3)^2=0\,,\ \Rightarrow\ p_1^2+p_2^2+p_3^2 = -2(p_1\cdot p_2+p_1\cdot p_3+p_2\cdot p_3)\,.$$

In this thesis, we use this freedom by adding a parameterization of all possible total derivative terms which are relevant to the lowest order vertices. This means that we use the expansion up to four h for the gravitational field,

$$\begin{aligned}
\mathcal{L}_{\rm TD}(h) = \frac{1}{\kappa^2}\partial^\mu\Bigg[&\kappa\Big[a_1\partial_\mu h_\nu{}^\nu + a_2\partial^\nu h_{\mu\nu}\Big] + \kappa^2\Big[a_3\partial_\mu h_\alpha{}^\alpha h_\beta{}^\beta + a_4\partial_\mu h^{\alpha\nu}h_{\alpha\nu} + a_5\partial^\alpha h_{\mu\alpha}h_\nu{}^\nu \\
&+ a_6\partial_\alpha h_{\mu\nu}h^{\alpha\nu} + a_7 h_{\mu\nu}\partial_\alpha h^{\alpha\nu} + a_8 h_{\mu\alpha}\partial^\alpha h_\nu{}^\nu\Big] + \kappa^3\Big[a_9\partial_\mu h_\nu{}^\nu h_\alpha{}^\alpha h_\beta{}^\beta \\
&+ a_{10}\partial_\mu h_\nu{}^\nu h^{\alpha\beta}h_{\alpha\beta} + a_{11}\partial_\mu h_{\nu\alpha}h^{\nu\alpha}h_\beta{}^\beta + a_{12}\partial_\mu h^{\nu\alpha}h_\nu{}^\beta h_{\alpha\beta} + a_{13}\partial^\nu h_{\mu\nu}h_\alpha{}^\alpha h_\beta{}^\beta \\
&+ a_{14}\partial^\nu h_{\mu\nu}h^{\alpha\beta}h_{\alpha\beta} + a_{15}\partial_\nu h_{\mu\alpha}h^{\nu\alpha}h_\beta{}^\beta + a_{16}\partial^\nu h_{\mu\alpha}h_{\nu\beta}h^{\alpha\beta} + a_{17}\partial_\nu h^{\nu\alpha}h_{\mu\alpha}h_\beta{}^\beta \\
&+ a_{18}\partial_\nu h^{\nu\alpha}h_{\mu\beta}h_\alpha{}^\beta + a_{19}\partial^\nu h_\alpha{}^\alpha h_{\mu\nu}h_\beta{}^\beta + a_{20}\partial^\nu h^{\alpha\beta}h_{\mu\nu}h_{\alpha\beta} + a_{21}\partial_\nu h_\alpha{}^\alpha h_{\mu\beta}h^{\nu\beta} \\
&+ a_{22}\partial^\nu h^{\alpha\beta}h_{\mu\alpha}h_{\nu\beta}\Big] + \kappa^4\Big[a_{23}\partial_\mu h_\alpha{}^\alpha h_\beta{}^\beta h_\gamma{}^\gamma h_\delta{}^\delta + a_{24}\partial_\mu h_\alpha{}^\alpha h_\beta{}^\beta h^{\gamma\delta}h_{\gamma\delta} \\
&+ a_{25}\partial_\mu h_\alpha{}^\alpha h^{\beta\gamma}h_\gamma{}^\delta h_{\delta\beta} + a_{26}\partial^\alpha h_{\mu\alpha}h_\beta{}^\beta h_\gamma{}^\gamma h_\delta{}^\delta + a_{27}\partial^\alpha h_{\mu\alpha}h_\beta{}^\beta h^{\gamma\delta}h_{\gamma\delta} \\
&+ a_{28}\partial^\alpha h_{\mu\alpha}h^{\beta\gamma}h_\gamma{}^\delta h_{\delta\beta} + a_{29}\partial_\mu h_{\alpha\beta}h^{\alpha\beta}h_\gamma{}^\gamma h_\delta{}^\delta + a_{30}\partial_\mu h_{\alpha\beta}h^{\alpha\beta}h_{\gamma\delta}h^{\gamma\delta} \\
&+ a_{31}\partial_\alpha h_{\mu\beta}h^{\alpha\beta}h_\gamma{}^\gamma h_\delta{}^\delta + a_{32}\partial_\alpha h_{\mu\beta}h^{\alpha\beta}h^{\gamma\delta}h_{\gamma\delta} + a_{33}\partial^\beta h_\alpha{}^\alpha h_{\mu\beta}h_\gamma{}^\gamma h_\delta{}^\delta \\
&+ a_{34}\partial^\beta h_\alpha{}^\alpha h_{\mu\beta}h^{\gamma\delta}h_{\gamma\delta} + a_{35}\partial^\beta h^{\alpha\gamma}h_{\mu\beta}h_{\alpha\gamma}h_\delta{}^\delta + a_{36}\partial^\beta h^{\alpha\gamma}h_{\mu\beta}h_\gamma{}^\delta h_{\delta\alpha} \\
&+ a_{37}\partial_\alpha h^{\beta\alpha}h_{\mu\beta}h_\gamma{}^\gamma h_\delta{}^\delta + a_{38}\partial_\alpha h^{\beta\alpha}h_{\mu\beta}h^{\gamma\delta}h_{\gamma\delta} + a_{39}\partial^\alpha h^{\beta\gamma}h_{\mu\beta}h_{\alpha\gamma}h_\delta{}^\delta \\
&+ a_{40}\partial^\alpha h^{\beta\gamma}h_{\mu\beta}h_\gamma{}^\delta h_{\delta\alpha} + a_{41}\partial_\gamma h_\alpha{}^\alpha h_{\mu\beta}h^{\beta\gamma}h_\delta{}^\delta + a_{42}\partial_\gamma h^{\alpha\delta}h_{\mu\beta}h^{\beta\gamma}h_{\alpha\delta} \\
&+ a_{43}\partial^\alpha h_{\gamma\alpha}h_{\mu\beta}h^{\beta\gamma}h_\delta{}^\delta + a_{44}\partial_\alpha h_{\gamma\delta}h_{\mu\beta}h^{\beta\gamma}h^{\alpha\delta} + a_{45}\partial_\alpha h^{\delta\alpha}h_{\mu\beta}h^{\beta\gamma}h_{\gamma\delta} \\
&+ a_{46}\partial^\delta h_\alpha{}^\alpha h_{\mu\beta}h^{\beta\gamma}h_{\gamma\delta} + a_{47}\partial^\alpha h_{\mu\beta}h_{\alpha\gamma}h^{\beta\gamma}h_\delta{}^\delta + a_{48}\partial_\alpha h_{\mu\beta}h^{\alpha\gamma}h^{\beta\delta}h_{\gamma\delta} \\
&+ a_{49}\partial_\mu h^{\alpha\beta}h_\alpha{}^\gamma h_{\beta\gamma}h_\delta{}^\delta + a_{50}\partial_\mu h^{\alpha\beta}h_\alpha{}^\gamma h_\beta{}^\delta h_{\gamma\delta}\Big]\Bigg] + \cdots\,,
\end{aligned} \tag{2.43}$$

up to three h for the scalar field

$$\begin{aligned}\mathcal{L}_{\text{TD}}(\phi, h) = \partial^\mu \Big[& d_1 \phi \partial_\mu \phi + \kappa \Big[d_2 \phi^2 \partial_\mu h_\nu{}^\nu + d_3 \phi^2 \partial^\nu h_{\nu\mu} + d_4 \phi \partial_\mu \phi h_\nu{}^\nu + d_5 \phi \partial^\nu \phi h_{\nu\mu} \Big] \\
& + \kappa^2 \Big[d_6 \phi^2 \partial_\mu h_\nu{}^\nu h_\alpha{}^\alpha + d_7 \phi^2 \partial_\mu h^{\nu\alpha} h_{\nu\alpha} + d_8 \phi^2 \partial^\nu h_{\mu\nu} h_\alpha{}^\alpha + d_9 \phi^2 \partial_\nu h_{\mu\alpha} h^{\nu\alpha} \\
& + d_{10} \phi^2 \partial_\nu h^{\nu\alpha} h_{\mu\alpha} + d_{11} \phi^2 \partial^\nu h_\alpha{}^\alpha h_{\mu\nu} + d_{12} \phi \partial_\mu \phi h_\nu{}^\nu h_\alpha{}^\alpha + d_{13} \phi \partial_\mu \phi h^{\nu\alpha} h_{\nu\alpha} \\
& + d_{14} \phi \partial^\nu \phi h_{\mu\nu} h_\alpha{}^\alpha + d_{15} \phi \partial_\nu \phi h_{\mu\alpha} h^{\nu\alpha} \Big] + \kappa^3 \Big[d_{16} \phi^2 \partial_\mu h^{\nu\alpha} h_{\nu\alpha} h_\beta{}^\beta \\
& + d_{17} \phi^2 \partial^\nu h_{\mu\nu} h_\alpha{}^\alpha h_\beta{}^\beta + d_{18} \phi^2 \partial_\nu h_{\mu\alpha} h^{\nu\alpha} h_\beta{}^\beta + d_{19} \phi^2 \partial_\nu h^{\nu\alpha} h_{\mu\alpha} h_\beta{}^\beta \\
& + d_{20} \phi^2 \partial^\nu h_\alpha{}^\alpha h_{\mu\nu} h_\beta{}^\beta + d_{21} \phi \partial_\mu \phi h_\nu{}^\nu h_\alpha{}^\alpha h_\beta{}^\beta + d_{22} \phi \partial_\mu \phi h^{\nu\alpha} h_{\nu\alpha} h_\beta{}^\beta \\
& + d_{23} \phi \partial^\nu \phi h_{\mu\nu} h_\alpha{}^\alpha h_\beta{}^\beta + d_{24} \phi \partial_\nu \phi h_{\mu\alpha} h^{\nu\alpha} h_\beta{}^\beta + d_{25} \phi^2 \partial_\mu h_\nu{}^\nu h_\alpha{}^\alpha h_\beta{}^\beta \Big] \Big] + \cdots ,\end{aligned} \tag{2.44}$$

and up to two h for the ghost and antighost fields

$$\begin{aligned}\mathcal{L}_{\text{TD}}(\chi, \bar{\chi}, h) = \partial^\mu \Big[& h_1 \bar{\chi}^\nu \partial_\mu \chi_\nu + \kappa \Big[h_2 h^{\nu\alpha} \bar{\chi}_\nu \partial_\mu \chi_\alpha + h_3 \bar{\chi}_\nu \partial_\alpha \chi_\mu h^{\nu\alpha} + h_4 \bar{\chi}^\nu \partial_\mu \chi_\nu h_\alpha{}^\alpha \\
& + h_5 \bar{\chi}^\nu \partial_\nu \chi_\mu h_\alpha{}^\alpha + h_6 \bar{\chi}^\nu \partial^\alpha \chi_\alpha h_{\mu\nu} + h_7 \bar{\chi}^\nu \partial^\alpha \chi_\nu h_{\mu\alpha} + h_8 \bar{\chi}_\nu \partial^\nu \chi^\alpha h_{\mu\alpha} \\
& + h_9 \bar{\chi}_\mu \partial^\nu \chi_\nu h_\alpha{}^\alpha + h_{10} \bar{\chi}_\mu \partial^\nu \chi^\alpha h_{\nu\alpha} + h_{11} \bar{\chi}^\nu \partial^\alpha \chi_\nu h_{\mu\alpha} + h_{12} \bar{\chi}^\nu \partial^\alpha \chi_\alpha h_{\nu\mu} \\
& + h_{13} \bar{\chi}_\nu \partial_\mu \chi_\alpha h^{\nu\alpha} + h_{14} \bar{\chi}_\nu \partial_\mu \chi^\nu h_\alpha{}^\alpha + h_{15} \bar{\chi}_\mu \partial_\nu \chi_\alpha h^{\nu\alpha} \Big] \\
& + \kappa^2 \Big[h_{20} \bar{\chi}_\mu \partial^\nu \chi_\nu h_\alpha{}^\alpha h_\beta{}^\beta + h_{21} \bar{\chi}_\mu \partial_\nu \chi_\alpha h^{\nu\alpha} h_\beta{}^\beta + h_{22} \bar{\chi}_\mu \partial^\nu \chi_\alpha h_{\nu\beta} h^{\alpha\beta} \\
& + h_{23} \bar{\chi}_\nu \partial_\mu \chi^\nu h_\alpha{}^\alpha h_\beta{}^\beta + h_{24} \bar{\chi}_\nu \partial_\mu \chi^\nu h^{\alpha\beta} h_{\alpha\beta} + h_{25} \bar{\chi}_\nu \partial_\mu \chi_\alpha h^{\nu\alpha} h_\beta{}^\beta \\
& + h_{26} \bar{\chi}^\nu \partial_\mu \chi_\alpha h_{\nu\beta} h^{\alpha\beta} + h_{27} \bar{\chi}^\nu \partial^\alpha \chi_\nu h_{\mu\alpha} h_\beta{}^\beta + h_{28} \bar{\chi}^\nu \partial_\alpha \chi_\nu h_{\mu\beta} h^{\alpha\beta} \\
& + h_{29} \bar{\chi}_\nu \partial_\alpha \chi_\mu h^{\nu\alpha} h_\beta{}^\beta + h_{30} \bar{\chi}^\nu \partial_\alpha \chi_\mu h_{\nu\beta} h^{\alpha\beta} + h_{31} \bar{\chi}^\nu \partial^\alpha \chi_\alpha h_{\nu\mu} h_\beta{}^\beta \\
& + h_{32} \bar{\chi}_\nu \partial^\alpha \chi_\alpha h^{\nu\beta} h_{\mu\beta} + h_{33} \bar{\chi}^\nu \partial_\alpha \chi_\beta h_{\nu\mu} h^{\alpha\beta} + h_{34} \bar{\chi}^\nu \partial^\alpha \chi^\beta h_{\nu\alpha} h_{\mu\beta} \\
& + h_{35} \bar{\chi}_\nu \partial^\alpha \chi_\beta h^{\nu\beta} h_{\mu\alpha} + h_{36} \bar{\chi}_\mu \partial^\nu \chi_\nu h^{\alpha\beta} h_{\alpha\beta} \Big] \Big] + \cdots ,\end{aligned} \tag{2.45}$$

where a_i, d_i, h_i are parameters that we choose later to simplify the Feynman rules. Mainly, we use them to get rid of terms that have second order derivatives of the fields (e.g., $\partial\partial h$) as we show later in more detail.

2.6 Field Redefinition

The field redefinition freedom follows from the equivalence theorem [10] which states that the S-matrix in quantum field theory remains unchanged under reparameterization of the field operators. We can illustrate this theorem by taking a scalar field ϕ as an example, where the generating functional is given by

$$\mathcal{Z} = \int D[\phi] \, \exp\left(i \int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi)\right). \tag{2.46}$$

Now, redefining the scalar field $\phi \to \tilde{\phi} = \phi + a_1\phi^2 + a_2\phi^3 + \cdots$, we get

$$\mathcal{Z} = \int D[\tilde{\phi}] \, \exp\left(i \int d^4x \, \mathcal{L}(\tilde{\phi}, \partial_\mu \tilde{\phi})\right). \tag{2.47}$$

This redefinition is allowed as long as the Jacobian of the integral is essentially one [10]. Similarly, we can also redefine the other fields: $h_{\mu\nu}$, χ, and $\bar{\chi}$.

Now let us explain how the field redefinition can simplify the Lagrangian. Consider the following field redefinition for the gravitational field

$$h_{\mu\nu} \;\to\; h'_{\mu\nu} = h_{\mu\nu} + \kappa\Big[a_1 h_{\mu\gamma}h_\nu{}^\gamma + a_2 h_{\mu\nu}h_\gamma{}^\gamma\Big] + \cdots . \tag{2.48}$$

Inserting this into $h_{\mu\nu}\,\partial^\mu h^{\nu\alpha}\partial_\alpha h_\beta{}^\beta$, which is part of the triple graviton vertex, as an illustration gives

$$\begin{aligned} h_{\mu\nu}\,\partial^\mu h^{\nu\alpha}\partial_\alpha h_\beta{}^\beta \quad \to \quad & h_{\mu\nu}\,\partial^\mu h^{\nu\alpha}\partial_\alpha h_\beta{}^\beta \;+\; a_1\,\kappa\, h_{\mu\gamma}h_\nu{}^\gamma\,\partial^\mu h^{\nu\alpha}\partial_\alpha h_\beta{}^\beta \\ & + \; a_2\,\kappa\, h_{\mu\nu}h_\gamma{}^\gamma\,\partial^\mu h^{\nu\alpha}\partial_\alpha h_\beta{}^\beta \;+\; \cdots, \end{aligned} \tag{2.49}$$

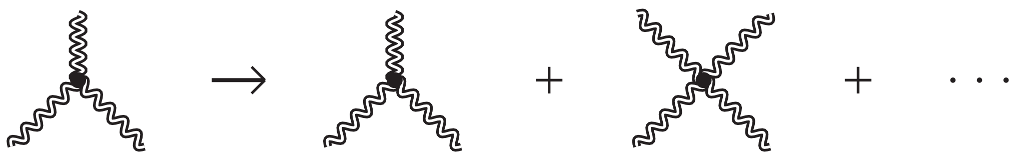


Figure 3: Field redefinition for the triple graviton vertex.

Thus, the field redefinition generates an expansion of the triple graviton vertex as shown in Fig. 3, giving two new contributions for the quadruple graviton vertex with the two parameters (a_1, a_2). So, by choosing a proper value for these parameters, we can cancel some of the contributions to the quadruple graviton vertex in the standard Lagrangian.

For our fields, we use the most general parameterized expansions which are relevant to the lowest order vertices. This means that we write all possible parameterized combinations up to four h for the gravitational field $h_{\mu\nu}$ as

$$\begin{aligned} h'_{\mu\nu} = h_{\mu\nu} + \kappa\Big[&c_1 h_{\mu\alpha}h_\nu{}^\alpha + c_2 h_{\mu\nu}h_\alpha{}^\alpha\Big] \\ &+ \kappa^2\Big[c_3 h_{\mu\nu}h_\alpha{}^\alpha h_\beta{}^\beta + c_4 h_{\mu\nu}h^{\alpha\beta}h_{\alpha\beta} + c_5 h_{\mu\alpha}h_\nu{}^\alpha h_\beta{}^\beta + c_6 h_{\mu\alpha}h_{\nu\beta}h^{\alpha\beta}\Big] \\ &+ \kappa^3\Big[c_7 h_{\mu\nu}h_\alpha{}^\alpha h_\beta{}^\beta h_\gamma{}^\gamma + c_8 h_{\mu\nu}h_\alpha{}^\alpha h^{\beta\gamma}h_{\beta\gamma} + c_9 h_{\mu\nu}h^{\alpha\beta}h_\beta{}^\gamma h_{\gamma\alpha} \\ &\quad + c_{10} h_{\mu\alpha}h_\nu{}^\alpha h_\beta{}^\beta h_\gamma{}^\gamma + c_{11} h_{\mu\alpha}h_\nu{}^\alpha h_{\beta\gamma}h^{\beta\gamma} + c_{12} h_{\mu\alpha}h_{\nu\beta}h^{\alpha\beta}h_\gamma{}^\gamma \\ &\quad + c_{13} h_{\mu\alpha}h_\nu{}^\beta h^{\alpha\gamma}h_{\beta\gamma}\Big] + \cdots, \end{aligned} \tag{2.50}$$

up to three powers of h for the scalar field ϕ as

$$\begin{aligned}\phi' = \phi + \kappa\Big[e_1 h_\alpha{}^\alpha \phi\Big] \\ + \kappa^2\Big[e_2 h_\alpha{}^\alpha h_\beta{}^\beta \phi + e_3 h_{\alpha\beta} h^{\alpha\beta}\phi\Big] \\ + \kappa^3\Big[e_4 h_\alpha{}^\alpha h_\beta{}^\beta h_\gamma{}^\gamma \phi + e_5 h_{\alpha\beta} h^{\alpha\beta} h_\gamma{}^\gamma \phi + e_6 h^{\alpha\beta} h_\beta{}^\gamma h_{\gamma\alpha}\phi\Big] + \cdots ,\end{aligned} \tag{2.51}$$

up to two powers of h for the ghost field χ_μ as

$$\begin{aligned}\chi'_\mu = \chi_\mu + \kappa\Big[g_1 h_\alpha{}^\alpha \chi_\mu + g_2 h_{\alpha\mu}\chi^\alpha\Big] \\ + \kappa^2\Big[g_3 h_\alpha{}^\alpha h_\beta{}^\beta \chi_\mu + g_4 h^{\alpha\beta} h_{\alpha\beta}\chi_\mu + g_5 h_\alpha{}^\alpha h_{\beta\mu}\chi_\beta + g_6 h^{\alpha\beta} h_{\alpha\mu}\chi_\beta\Big] + \cdots ,\end{aligned} \tag{2.52}$$

and up to two powers of h for the anti-ghost field $\bar{\chi}_\mu$ as

$$\begin{aligned}\bar{\chi}'_\mu = \bar{\chi}_\mu + \kappa\Big[f_1 h_\alpha{}^\alpha \bar{\chi}_\mu + f_2 h_{\alpha\mu}\bar{\chi}^\alpha\Big] \\ + \kappa^2\Big[f_3 h_\alpha{}^\alpha h_\beta{}^\beta \bar{\chi}_\mu + f_4 h^{\alpha\beta} h_{\alpha\beta}\bar{\chi}_\mu + f_5 h_\alpha{}^\alpha h_{\beta\mu}\bar{\chi}^\beta + f_6 h^{\alpha\beta} h_{\alpha\mu}\bar{\chi}_\beta\Big] + \cdots ,\end{aligned} \tag{2.53}$$

where c_i, e_i, g_i, f_i are parameters that will be chosen later to simplify the Feynman rules.

3 Feynman Rules

In this section, we calculate the standard Feynman rules [6]. After that, we explain our strategies to simplify the Feynman rules for gravity, and then we show the resulting simplified Feynman rules.

To perform these calculations, we use the FORM program which is a symbolic manipulation system that can manipulate symbolic expressions and do mathematical operations, then return symbolic results [14, 15]. In addition, some short pieces of code that are relevant to our calculations are shown in App. F.

3.1 The Standard Calculations

First, we calculate the Lagrangian for matter up to three powers of h and the Lagrangian for gravity up to four powers of h from Eqs. (2.14, 2.15) respectively using the expansion Eq. (2.13) and the definitions (2.6–2.10). Second, we calculate the Lagrangian for fixing the gauge up to four h and the Lagrangian for ghosts up to two h from Eqs. (2.35, 2.34) using the de Donder (harmonic) gauge condition Eq. (2.36) and the gauge transformation Eq. (2.26). Thus, we get the total Lagrangian as

$$\mathcal{L}_{\text{Total}}(h,\phi,\chi,\bar{\chi}) = \mathcal{L}_{\text{Gravity}}(h) + \mathcal{L}_{\text{Matter}}(h,\phi) + \mathcal{L}_{\text{FG}}(h) + \mathcal{L}_{\text{Ghost}}(\chi,\bar{\chi},h)\,. \tag{3.1}$$

From this total Lagrangian, we get the standard Feynman rules which are listed in App. B. We have also verified that they agree with the Feynman rules in [6].

3.2 The Simplified Calculations

Again, we calculate the Lagrangian for matter up to three powers of h and the Lagrangian for gravity up to four powers of h from Eqs. (2.14, 2.15) respectively using the expansion Eq. (2.13) and the definitions (2.6−2.10). Second, we calculate the Lagrangian for fixing gauge up to four h and the Lagrangian for ghost up to two h from Eqs. (2.34, 2.35) using the general parameterized gauge condition Eq. (2.37) and the gauge transformation Eq. (2.26). Third, we add the total derivative Lagrangians Eqs. (2.43−2.45). Fourth, we put the previous Lagrangians together to obtain the total Lagrangian as

$$\begin{aligned}\mathcal{L}_{\text{Total}}(h,\phi,\chi,\bar{\chi}) = &\ \mathcal{L}_{\text{Gravity}}(h) + \mathcal{L}_{\text{Matter}}(h,\phi) + \mathcal{L}_{\text{FG}}(h) + \mathcal{L}_{\text{Ghost}}(\chi,\bar{\chi},h) \\ &+ \mathcal{L}_{\text{TD}}(h) + \mathcal{L}_{\text{TD}}(\phi,h) + \mathcal{L}_{\text{TD}}(\chi,\bar{\chi},h)\,.\end{aligned} \tag{3.2}$$

After that, we redefine all the fields in the general parameterized way as given in Eqs. (2.50−2.53). Thus, we get the total Lagrangian Eq. (3.2) with eight sets of parameters:

	h	ϕ	$\chi,\bar{\chi}$
$\mathcal{L}_{\text{TD}}$	$(a_1,\cdots,a_{50})$	$(d_1,\cdots,d_{25})$	$(h_1,\cdots,h_{36})$
Field Redefinition	$(c_1,\cdots,c_{13})$	$(e_1,\cdots,e_6)$	$(f_1,\cdots,f_6),(g_1,\cdots,g_6)$
$\mathcal{C}_\mu(h)$	$(b_1,\cdots,b_{22})$		

Finding the proper values of these parameters by inspection is what took the most time in this thesis. In order to find suitable values for the parameters we used the following strategies:

1. Ensuring the same propagators as in the standard calculations.
2. Minimizing the number of terms as much as possible, especially for the triple and quadruple graviton vertices.
3. Cancelling all terms that have second order derivative of our fields as $\partial\partial h$. Because when we integrate by parts, the second order derivative of our fields as $h\partial\partial h$ gives two terms that have first order derivative of our fields as $\partial h\partial h$. Some of these terms can cancel terms in the standard Lagrangian.
4. Trying to keep terms that have the same indices for the partial derivatives, such as $\partial_\mu h_{\nu\alpha}\partial^\mu h^{\nu\alpha}$ and not $\partial_\mu h_{\nu\alpha}\partial^\nu h^{\mu\alpha}$, in order to get simpler expressions in momentum space.

Thus, we choose the parameters as given in App. A, where we list each set of the parameters separately and we also show the parameters that contribute to each Feynman rule. These parameters are the main results of our work.

To illustrate the simplified calculations, let us take an example of the triple graviton vertex from our work. The FORM code for the total Lagrangian of this vertex is shown below, where only parameters from the following sets $(c_1, c_2, \cdots, c_{13})$, $(a_1, a_2, \cdots, a_{50})$, $(b_1, b_2, \cdots, b_{22})$ can appear:

Result

```
LagT3 =
 + H(mu,mu)*H(nu,nu)*H(al,al,be,be)*Fact(1 + 4*a9,4)
 + H(mu,mu)*H(nu,nu)*H(al,be,al,be)*Fact( - 1 + 4*a13,4)
 + H(mu,mu)*H(nu,nu,al)*H(al,be,be)*Fact( - 1 - b5 + 2*b3 + a19 + 2*a13,1)
 + H(mu,mu)*H(nu,nu,al)*H(be,al,be)*Fact(1 + 2*b5 + a17,1)
 + H(mu,mu)*H(nu,nu,al,be)*H(al,be)*Fact( - 1 + a11,1)
 + H(mu,mu)*H(nu,al)*H(nu,al,be,be)*Fact( - 1 + a19,1)
 + H(mu,mu)*H(nu,al)*H(nu,be,al,be)*Fact(2 + a17 + a15,2)
 + H(mu,mu)*H(nu,al)*H(al,be,nu,be)*Fact(2 + a17 + a15,2)
 + H(mu,mu)*H(nu,al,al)*H(nu,be,be)*Fact(1 - 4*c2 - 4*b3 + 8*a9,4)
 + H(mu,mu)*H(nu,al,be)^2*Fact( - 3 + 4*c2 + 4*a11,4)
 + H(mu,mu)*H(nu,al,be)*H(al,nu,be)*Fact(1 + 2*a15,2)
 + H(mu,mu,nu)*H(nu,al)*H(al,be,be)*Fact(2 + 2*b8 - b7 + a21 + a17,2)
 + H(mu,mu,nu)*H(nu,al,be)*H(al,be)*Fact(2 + 2*b4 + a20 + 2*a14,2)
 + H(mu,mu,nu)*H(al,nu,be)*H(al,be)*Fact( - 4 + 2*b6 + a22 + a18,2)
 + H(mu,mu,nu,nu)*H(al,be)^2*Fact( - 1 + 2*a10,2)
 + H(mu,mu,nu,al)*H(nu,be)*H(al,be)*Fact(2 + a12,1)
 + H(mu,nu)^2*H(al,be,al,be)*Fact(1 + 2*a14,2)
 + H(mu,nu)*H(mu,nu,al)*H(al,be,be)*Fact(2 - b6 + a21 + a15,2)
 + H(mu,nu)*H(mu,nu,al)*H(be,al,be)*Fact( - 4 + 2*b6 + a22 + a18,4)
 + H(mu,nu)*H(mu,nu,al,be)*H(al,be)*Fact(2 + a20,1)
 + H(mu,nu)*H(mu,al)*H(nu,al,be,be)*Fact(2 + a21,4)
 + H(mu,nu)*H(mu,al)*H(nu,be,al,be)*Fact( - 4 + a18 + a16,1)
 + H(mu,nu)*H(mu,al,nu,be)*H(al,be)*Fact( - 2 + a22,2)
 + H(mu,nu)*H(mu,al,al)*H(nu,be,be)*Fact( - 1 - 2*b8 + 2*a19,2)
 + H(mu,nu)*H(mu,al,al)*H(be,nu,be)*Fact(2 + 2*b8 - b7 + a21 + a17,4)
 + H(mu,nu)*H(mu,al,be)*H(nu,al,be)*Fact(3 + 2*a20,2)
 + H(mu,nu)*H(mu,al,be)*H(al,nu,be)*Fact( - 2 + a22 + a16,1)
 + H(mu,nu)*H(mu,al,be,be)*H(nu,al)*Fact(6 + 3*a21,4)
 + H(mu,nu)*H(nu,mu,al)*H(al,be,be)*Fact(2 - b6 + a21 + a15,2)
 + H(mu,nu)*H(nu,mu,al)*H(be,al,be)*Fact( - 4 + 2*b6 + a22 + a18,4)
 + H(mu,nu)*H(nu,al,mu,be)*H(al,be)*Fact( - 2 + a22,2)
 + H(mu,nu)*H(nu,al,al)*H(be,mu,be)*Fact(2 + 2*b8 - b7 + a21 + a17,4)
 + H(mu,nu)*H(al,mu,nu)*H(al,be,be)*Fact( - 1 + c2 - c1 - b4 + a11 + 2*a10,1)
 + H(mu,nu)*H(al,mu,nu)*H(be,al,be)*Fact(2 + 2*b4 + a20 + 2*a14,2)
 + H(mu,nu)*H(al,mu,al)*H(be,nu,be)*Fact( - 2 + 2*b7 + a18,1)
 + H(mu,nu)*H(al,mu,be)*H(al,nu,be)*Fact(3 + 2*c1 + 2*a12,1)
 + H(mu,nu)*H(al,mu,be)*H(be,nu,al)*Fact( - 1 + a16,1)
 ;
```

Result

Here Fact(x,y)= $\frac{x}{y}$, H(mu,nu)= $h_{\mu\nu}$, H(al,mu,nu)= $\partial_\alpha h_{\mu\nu}$, H(al,be,mu,nu)= $\partial_\alpha\partial_\beta h_{\mu\nu}$, and all the indices are contracted in the proper way.

Now, if we plug our choice of the parameters as shown in App. A, we get the following simplified expression for this vertex:

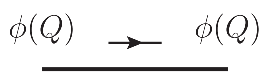
Result
```
Time =        5.66 sec    Generated terms =          4
             LagT3        Terms in output =          4
                          Bytes used      =        444

LagT3 =
      + H(mu,mu)*H(nu,al,al)*H(nu,be,be)*Fact(1,8)
      + H(mu,nu)*H(mu,al,be)*H(nu,al,be)*Fact(-1,2)
      + H(mu,nu)*H(mu,al,be)*H(al,nu,be)*Fact(1,1)
      + H(mu,nu)*H(al,mu,nu)*H(al,be,be)*Fact(-1,4)
     ;
```
Result

As a result, we efficiently reduce the triple graviton vertex from 40 to 4 terms. In the next subsection we will show all the simplified Feynman rules that we obtain.

3.3 The Simplified Feynman Rules

Here we present the simplified Feynman rules that we obtain from our choice of parameters, in App. A, together with some comparisons with the standard Feynman rules that are given in App. B. As already mentioned, our choice of parameters ensures the same propagators as in the standard Feynman rules. For completeness, we give below the Lagrangian and the corresponding propagator in momentum space for the scalar field $\phi(Q)$

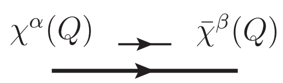


$$\mathcal{L}_{\phi\phi} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2\,, \tag{3.3}$$

in momentum space

$$S_{\{\phi\phi\}}(Q,m) = \frac{i}{Q^2 - m^2 + i\epsilon}\,,$$

for the ghost field $\chi(Q)$

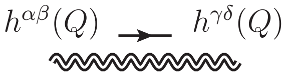


$$\mathcal{L}_{\{\bar{\chi}\chi\}} = -\eta_{\mu\nu}\partial_\lambda\bar{\chi}^\mu\partial^\lambda\chi^\nu\,, \tag{3.4}$$

in momentum space

$$S^{\alpha\beta}_{\{\bar{\chi}\chi\}}(Q) = -\frac{i}{Q^2}\eta^{\alpha\beta}\,,$$

and for the gravitational field $h^{\alpha\beta}(Q)$



$$\mathcal{L}_{hh} = \frac{1}{2}\partial_\mu h_{\nu\lambda}\partial^\mu h^{\nu\lambda} - \frac{1}{4}\partial_\mu h_\nu{}^\nu \partial^\mu h_\lambda{}^\lambda\,, \tag{3.5}$$

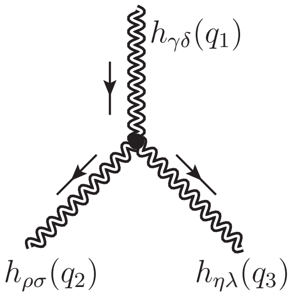
in momentum space

$$S^{\alpha\beta\gamma\delta}_{\{hh\}}(Q) = \frac{i}{Q^2}P^{\alpha\beta\gamma\delta}\,,$$

where $P^{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\alpha\delta}\eta^{\beta\gamma} - \eta^{\alpha\beta}\eta^{\gamma\delta})$.

Turning now to the vertices, our choice of parameters successfully simplifies most of them, comparing with the standard ones as listed in App. B. Here we discuss each vertex separately. First, our main effort is to get the simplest form of the triple graviton vertex $V^{\{hhh\}}_{\gamma\delta\rho\sigma\eta\lambda}(q_1, q_2)$ since it can appear in many one-loop diagrams, as shown in Sec. 5, and its standard expression has 40 terms, as shown in Eq. (B.6). This can lead to messy calculations when this vertex appears twice or more in a diagram. For example, a diagram with three triple graviton vertices such as Fig. 12i can give about 64 000 terms. However, using our choice of parameters, we successfully reduce it to just four terms as follows

$$\mathcal{L}_{hhh} = \frac{\kappa}{2}\Big(\frac{1}{4}h_\mu{}^\mu \partial_\nu h_\alpha{}^\alpha \partial^\nu h_\beta{}^\beta - h^{\mu\nu}\partial_\mu h^{\alpha\beta}\partial_\nu h_{\alpha\beta} + 2\,h^{\mu\nu}\partial_\mu h^{\alpha\beta}\partial_\alpha h_{\nu\beta} - \frac{1}{2}h^{\mu\nu}\partial_\alpha h_{\mu\nu}\partial^\alpha h_\beta{}^\beta\Big)\,, \tag{3.6}$$

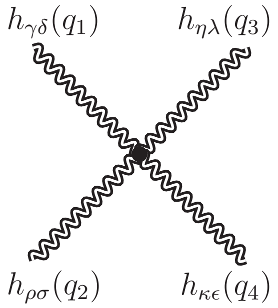


$\Big(q_3 = q_1 - q_2\Big)$

in momentum space

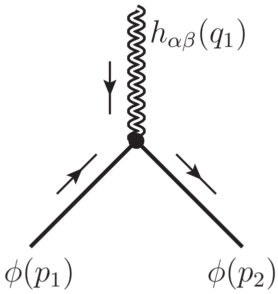
$$\begin{aligned} V^{\{hhh\}}_{\gamma\delta\rho\sigma\eta\lambda}(q_1, q_2) = i\frac{\kappa}{2}\Big[\ &\frac{1}{2}q_1{\cdot}q_2(\eta_{\rho\sigma}\eta_{\gamma\delta}\eta_{\eta\lambda} - \eta_{\rho\sigma}\eta_{\gamma\eta}\eta_{\delta\lambda} - \eta_{\rho\eta}\eta_{\sigma\lambda}\eta_{\gamma\delta}) \\ &+ \frac{1}{2}q_1{\cdot}q_3(\eta_{\rho\sigma}\eta_{\gamma\delta}\eta_{\eta\lambda} - \eta_{\rho\gamma}\eta_{\sigma\delta}\eta_{\eta\lambda} - \eta_{\rho\eta}\eta_{\sigma\lambda}\eta_{\gamma\delta}) \\ &+ \frac{1}{2}q_2{\cdot}q_3(-\eta_{\rho\sigma}\eta_{\gamma\delta}\eta_{\eta\lambda} + \eta_{\rho\sigma}\eta_{\gamma\eta}\eta_{\delta\lambda} + \eta_{\rho\gamma}\eta_{\sigma\delta}\eta_{\eta\lambda}) \\ &- 2\,\eta_{\rho\gamma}\eta_{\sigma\delta}q_{1\eta}q_{2\lambda} + \eta_{\rho\gamma}\eta_{\sigma\eta}(q_{1\lambda}q_{2\delta} - q_{2\lambda}q_{3\delta}) \\ &+ 2\,\eta_{\rho\delta}\eta_{\gamma\eta}(q_{1\sigma}q_{2\lambda} + q_{1\lambda}q_{3\sigma}) + \eta_{\rho\eta}\eta_{\sigma\lambda}q_{2\gamma}q_{3\delta} \\ &+ 2\,\eta_{\rho\eta}\eta_{\gamma\lambda}(q_{1\sigma}q_{3\delta} - q_{2\delta}q_{3\sigma}) - \eta_{\gamma\eta}\eta_{\delta\lambda}q_{1\rho}q_{3\sigma}\Big]\,. \end{aligned}$$

Second, in addition to the triple graviton vertex, our main goal is to get the simplest form of the quadruple graviton vertex $V^{\{hhhh\}}_{\gamma\delta\rho\sigma\eta\lambda\kappa\epsilon}(q_1, q_2, q_3)$. This vertex has a very complicated standard form, 113 terms, as shown in Eq. (B.7). In our method, the parameters in App. A successfully reduces the number of terms to just 12 as follows



$$\begin{aligned}\mathcal{L}_{hhhh} = \frac{\kappa^2}{4}\Big(&- \frac{5}{16} h_\mu{}^\mu h_\nu{}^\nu \partial_\alpha h_\beta{}^\beta \partial^\alpha h_\tau{}^\tau + \frac{1}{2} h_\mu{}^\mu h^{\nu\alpha} \partial_\nu h_{\beta\tau} \partial_\alpha h^{\beta\tau} \\ &- h_\mu{}^\mu h^{\nu\alpha} \partial_\nu h^{\beta\tau} \partial_\beta h_{\alpha\tau} + h_\mu{}^\mu h^{\nu\alpha} \partial_\beta h_{\nu\tau} \partial^\beta h_\alpha{}^\tau \\ &- \frac{1}{8} h_{\mu\nu} h^{\mu\nu} \partial_\alpha h_\beta{}^\beta \partial^\alpha h_\tau{}^\tau + h^{\mu\nu} \partial_\mu h_{\nu\alpha} \partial^\beta h^{\alpha\tau} h_{\beta\tau} \\ &+ \frac{1}{4} h^{\mu\nu} \partial_\mu h_\alpha{}^\alpha h_{\nu\beta} \partial^\beta h_\tau{}^\tau - 2\, h^{\mu\nu} \partial_\mu h^{\alpha\beta} h_{\nu\alpha} \partial^\tau h_{\beta\tau} \\ &+ h^{\mu\nu} \partial_\mu h_{\alpha\beta} h_{\nu\tau} \partial^\tau h^{\alpha\beta} - 2\, h^{\mu\nu} \partial_\mu h_{\alpha\beta} h^{\alpha\tau} \partial_\tau h_\nu{}^\beta \\ &+ h^{\mu\nu} h_{\nu\alpha} \partial_\beta h_{\mu\tau} \partial^\beta h^{\alpha\tau} + 2\, h^{\mu\nu} \partial_\nu h_{\alpha\beta} h^{\alpha\beta} \partial^\tau h_{\mu\tau} \Big) \,. \end{aligned} \tag{3.7}$$

Third, the scalar-scalar-graviton vertex $V^{\{\phi\phi h\}}_{\alpha\beta}(p_1, p_2, m)$, which is the only vertex that has the same expression as in the standard rules Eq. (B.2), is given by

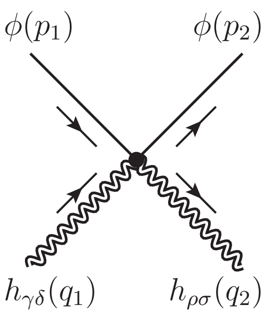


$$\mathcal{L}_{\phi\phi h} = \frac{\kappa}{2}\Big(- \frac{1}{2} h_\mu{}^\mu \phi^2 m^2 + \frac{1}{2} h_\mu{}^\mu \partial_\nu \phi \partial^\nu \phi - h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \Big) \,, \tag{3.8}$$

in momentum space

$$V^{\{\phi\phi h\}}_{\alpha\beta}(p_1, p_2, m) = i\frac{\kappa}{2}\Big[(p_{1\alpha} p_{2\beta} + p_{2\alpha} p_{1\beta}) - \eta_{\alpha\beta}(p_1.p_2 - m^2) \Big] \,.$$

Fourth, as mentioned before, the parameters are chosen in order to get the triple graviton and quadruple graviton vertices as simple as possible, but at the same time the scalar-scalar-graviton-graviton vertex $V^{\{\phi\phi hh\}}_{\gamma\delta\rho\sigma}(p_1, p_2)$ is surprisingly reduced from six terms, as in Eq. (B.3), to just two terms. Moreover, this vertex is now independent of the scalar mass m as follows

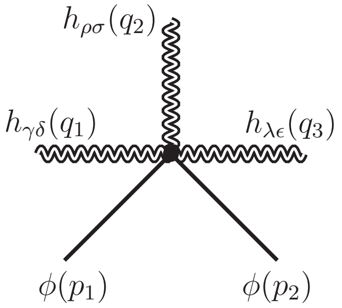


$$\mathcal{L}_{\phi\phi hh} = \frac{\kappa^2}{4}\Big(h^{\mu\nu} h_\nu{}^\alpha \partial_\mu \phi \partial_\alpha \phi - \frac{1}{2} h_\mu{}^\mu h^{\nu\alpha} \partial_\nu \phi \partial_\alpha \phi \Big) \,, \tag{3.9}$$

in momentum space

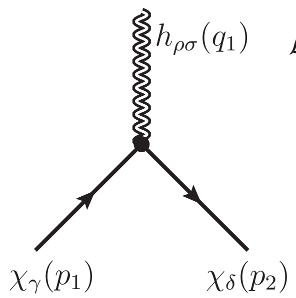
$$\begin{aligned} V^{\{\phi\phi hh\}}_{\gamma\delta\rho\sigma}(p_1, p_2) = i\frac{\kappa^2}{8}\Big[&- \eta_{\gamma\delta}(p_{1\rho} p_{2\sigma} + p_{1\sigma} p_{2\rho}) + \eta_{\gamma\rho}(p_{1\delta} p_{2\sigma} + p_{1\sigma} p_{2\delta}) \\ &+ \eta_{\gamma\sigma}(p_{1\delta} p_{2\rho} + p_{1\rho} p_{2\delta}) + \eta_{\delta\rho}(p_{1\gamma} p_{2\sigma} + p_{1\sigma} p_{2\gamma}) \\ &+ \eta_{\delta\sigma}(p_{1\gamma} p_{2\rho} + p_{1\rho} p_{2\gamma}) - \eta_{\rho\sigma}(p_{1\gamma} p_{2\delta} + p_{1\delta} p_{2\gamma}) \Big] \,. \end{aligned}$$

Fifth, the scalar-scalar-graviton-graviton-graviton vertex $V^{\{\phi\phi hhh\}}_{\gamma\delta\rho\sigma\lambda\epsilon}(p_1, p_2, q_1, q_2)$ is reduced from 10 terms, as in Eq. (B.4), to seven terms in the simplified method, where this reduction is due to the choice of the redefinition parameters for scalar field e_4, e_6 as shown in Tab. 8, as follows



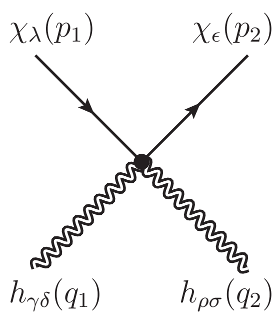
$$\begin{aligned}\mathcal{L}_{\phi\phi hhh} = \frac{\kappa^3}{8}\Big[&-\frac{1}{4}m^2\phi^2 h_\mu{}^\mu h^{\nu\alpha}h_{\nu\alpha} - \frac{1}{16}\phi\,\partial_\mu\phi\partial^\mu h_\nu{}^\nu h_\alpha{}^\alpha h_\beta{}^\beta \\ &-\frac{1}{2}\phi\,\partial_\mu\phi\partial^\mu h^{\nu\alpha}h_\nu{}^\beta h_{\alpha\beta} + \frac{1}{4}\partial_\mu\phi\partial^\mu\phi h_\nu{}^\nu h^{\alpha\beta}h_{\alpha\beta} \\ &+\frac{1}{8}\partial_\mu\phi\partial_\nu\phi h^{\mu\nu}h_\alpha{}^\alpha h_\beta{}^\beta - \frac{1}{2}\partial_\mu\phi\partial^\nu\phi h^{\mu\alpha}h_{\nu\alpha}h_\beta{}^\beta \\ &-\partial^\mu\phi\partial^\nu\phi h_{\mu\alpha}h_{\nu\beta}h^{\alpha\beta}\Big]\,. \end{aligned} \tag{3.10}$$

Sixth, the ghost-ghost-graviton vertex $V^{\{\bar{\chi}\chi h\}}_{\rho\sigma\gamma\delta}(p_1,p_2)$ also appears in our one-loop diagrams as shown in Sec. 5. It has 11 terms, compared to 8 in the standard vertex, and it takes the form



$$\begin{aligned}\mathcal{L}_{\bar{\chi}\chi h} = \frac{\kappa}{2}\Big(&-\bar{\chi}^\mu\chi^\nu\partial_\mu\partial_\nu h_\alpha{}^\alpha + \bar{\chi}_\mu\partial^\mu\chi_\nu\partial_\alpha h^{\nu\alpha} + 2\,\bar{\chi}_\mu\chi^\nu\partial_\nu\partial_\alpha h^{\mu\alpha} \\ &-\frac{1}{2}\bar{\chi}_\mu\partial_\nu\chi^\nu\partial^\mu h_\alpha{}^\alpha - \bar{\chi}_\mu\partial_\nu\chi_\alpha\partial^\mu h^{\nu\alpha} + \bar{\chi}_\mu\partial_\nu\chi_\alpha\partial^\alpha h^{\mu\nu} \\ &-\partial^\mu\bar{\chi}_\mu\partial_\nu\chi_\alpha h^{\nu\alpha} - \partial^\mu\bar{\chi}^\nu\partial_\mu\chi_\nu h_\alpha{}^\alpha - \partial_\mu\bar{\chi}_\nu\partial^\mu\chi_\alpha h^{\nu\alpha} \\ &+\partial_\mu\bar{\chi}_\nu\partial_\alpha\chi^\nu h^{\mu\alpha} - \partial_\mu\bar{\chi}_\nu\partial_\alpha\chi^\alpha h^{\mu\nu}\Big)\,. \end{aligned} \tag{3.11}$$

Seventh, the ghost-ghost-graviton-graviton vertex $V^{\{\bar{\chi}\chi hh\}}_{\gamma\delta\rho\sigma\lambda\epsilon}(p_1,p_2,q_1)$ is the last vertex that is needed in the calculations of our one-loop diagrams, where it appears in Fig. 12e. It has 29 terms while it vanishes in the standard rules. It can be written as



$$\begin{aligned}\mathcal{L}_{\bar{\chi}\chi hh} = \frac{\kappa^2}{8}\Big(&-\bar{\chi}^\mu\partial^\nu\chi_\nu h_{\mu\alpha}\partial_\beta h^{\alpha\beta} + \bar{\chi}_\mu\partial^\mu\chi^\nu h_{\nu\alpha}\partial_\beta h^{\alpha\beta} + 2\,\bar{\chi}_\mu\partial^\mu\chi^\nu\partial_\nu h^{\alpha\beta}h_{\alpha\beta} \\ &+\bar{\chi}_\mu\partial^\mu\chi^\nu\partial^\alpha h_{\nu\alpha}h_\beta{}^\beta + 2\,\bar{\chi}_\mu\chi^\nu h^{\mu\alpha}\partial_\nu\partial_\alpha h_\beta{}^\beta + \bar{\chi}^\mu\chi^\nu\partial_\mu h_\alpha{}^\alpha\partial_\nu h_\beta{}^\beta \\ &+2\,\bar{\chi}^\mu\chi^\nu\partial_\nu h_{\mu\alpha}\partial^\alpha h_\beta{}^\beta + 2\,\bar{\chi}^\mu\chi^\nu\partial_\nu\partial^\alpha h_{\mu\alpha}h_\beta{}^\beta + \bar{\chi}^\mu\partial^\nu\chi_\mu h_{\nu\alpha}\partial_\beta h^{\alpha\beta} \\ &+\bar{\chi}_\mu\partial^\mu\chi_\nu h^{\nu\alpha}\partial_\alpha h_\beta{}^\beta - \bar{\chi}^\mu\partial^\nu\chi_\nu\partial_\alpha h_{\mu\beta}h^{\alpha\beta} + \bar{\chi}^\mu\partial^\nu\chi^\alpha h_{\mu\nu}\partial_\alpha h_\beta{}^\beta \\ &+\bar{\chi}_\mu\partial_\nu\chi^\alpha h^{\mu\nu}\partial^\beta h_{\alpha\beta} - \bar{\chi}^\mu\partial^\nu\chi^\alpha\partial_\mu h_{\nu\alpha}h_\beta{}^\beta - \bar{\chi}^\mu\partial^\nu\chi^\alpha h_{\mu\alpha}\partial_\nu h_\beta{}^\beta \\ &+\bar{\chi}^\mu\partial^\nu\chi^\alpha h_{\mu\alpha}\partial^\beta h_{\nu\beta} - \bar{\chi}^\mu\partial^\nu\chi^\alpha\partial_\mu h_\beta{}^\beta h_{\nu\alpha} - \bar{\chi}^\mu\partial^\nu\chi^\alpha\partial_\nu h_{\mu\alpha}h_\beta{}^\beta \\ &-4\,\bar{\chi}^\mu\partial^\nu\chi^\alpha h_{\nu\alpha}\partial^\beta h_{\mu\beta} + \bar{\chi}^\mu\partial^\nu\chi^\alpha\partial_\alpha h_{\mu\nu}h_\beta{}^\beta - \partial^\mu\bar{\chi}_\mu\partial^\nu\chi^\alpha h_{\nu\alpha}h_\beta{}^\beta \\ &-\partial^\mu\bar{\chi}^\nu\partial_\mu\chi_\nu h^{\alpha\beta}h_{\alpha\beta} - 2\,\partial^\mu\bar{\chi}^\nu\partial_\mu\chi^\alpha h_{\nu\alpha}h_\beta{}^\beta + \partial^\mu\bar{\chi}^\nu\partial^\alpha\chi_\nu h_{\mu\alpha}h_\beta{}^\beta \\ &-\partial^\mu\bar{\chi}^\nu\partial^\alpha\chi_\alpha h_{\mu\nu}h_\beta{}^\beta - \partial^\mu\bar{\chi}_\nu\partial^\alpha\chi_\alpha h_{\mu\beta}h^{\nu\beta} - 4\,\partial^\mu\bar{\chi}^\nu\partial^\alpha\chi^\beta h_{\mu\nu}h_{\alpha\beta} \\ &+2\,\partial^\mu\bar{\chi}^\nu\partial^\alpha\chi^\beta h_{\mu\alpha}h_{\nu\beta} + 2\,\partial^\mu\bar{\chi}^\nu\partial^\alpha\chi^\beta h_{\mu\beta}h_{\nu\alpha}\Big)\,. \end{aligned} \tag{3.12}$$

Our ghost vertices above are more complicated than the standard ghost vertices because our general parameterized gauge Eq. (2.37) is more complicated than the de Donder gauge

Eq. (2.36). However, the ghost vertices just appear in few diagrams in scalar-graviton scattering as shown in Sec. 5, so they do not affect our calculations so much if they have slightly more complicated form.

Finally, after deriving the simplified Feynman rules, it is time to apply these rules on scattering processes as shown in the next section.

4 Tree Level Scattering

Now all the ingredients are in place to start the calculations, but before diving into one-loop diagrams, we calculate the amplitudes of scalar-graviton scattering and graviton-graviton scattering at tree level. So, we first show how we use the helicity formalism [11] to express the resulting amplitudes at tree level. After that, we calculate these amplitudes by using the simplified Feynman rules, as shown in the previous section, then repeat the calculations for the same amplitudes but using the standard Feynman rules, as shown in App. B. Finally, we compare the results obtained in the two ways and check against published results where available.

4.1 Helicity Amplitudes

A helicity amplitude is an amplitude $\mathcal{M}$ that is evaluated for fixed helicity of the external particles, where the helicity of a particle is the projection of its spin on its momentum. Namely, if we consider the process: $a_1 + a_2 \to a_3 + a_4$, then the total amplitude $\mathcal{M}$ will be decomposed into helicity amplitudes $\mathcal{M}_{(\lambda_1,\lambda_2;\lambda_3,\lambda_4)}$ each one representing the amplitude of transition from a particular helicity state λ_1, λ_2 of the incoming particles a_1, a_2 to a particular helicity state λ_3, λ_4 of the outgoing particles a_3, a_4.

In our case, the graviton has a polarization tensor $\epsilon^{\pm 2}_{\mu\nu}(p)$ for helicity ± 2 which can be written in terms of the polarization vector $\epsilon^{\pm 1}_{\mu}(p)$ for helicity ± 1 as

$$\epsilon^{\pm 2}_{\mu\nu}(p) = \epsilon^{\pm 1}_{\mu}(p)\epsilon^{\pm 1}_{\nu}(p)\,.$$

In addition, the graviton is a spin-2 massless particle of a symmetric gravitational field $h_{\mu\nu}$. This implies that its polarization tensor $\epsilon_{\mu\nu}$ is transverse, traceless and symmetric [11]:

$$p^{\mu}\epsilon^{\pm 2}_{\mu\nu}(p) = p^{\nu}\epsilon^{\pm 2}_{\mu\nu}(p) = 0\,, \tag{4.1}$$
$$\eta^{\mu\nu}\epsilon^{\pm 2}_{\mu\nu}(p) = \epsilon^{\pm 2}{}_{\nu}{}^{\nu}(p) = 0\,, \tag{4.2}$$
$$\epsilon^{\pm 2}_{\mu\nu}(p) = \epsilon^{\pm 2}_{\nu\mu}(p)\,, \tag{4.3}$$

where our choice of polarization vectors and four momenta in the Center-of-Mass frame (CM) are given in App. C together with other useful kinematic relations.

Finally, helicity amplitudes have some useful properties. For example, in general the results are given by simple expressions and the total amplitude can be squared directly without having to use the completeness relations. Above all, only some of the helicity amplitudes are actually independent. The others can be calculated by using various symmetries [7], such as parity

$$\mathcal{M}_{(\lambda_3,\lambda_4;\lambda_1,\lambda_2)} = (-1)^{m-n}\mathcal{M}_{(-\lambda_3,-\lambda_4;-\lambda_1,-\lambda_2)}\,, \tag{4.4}$$

time-reversal

$$\mathcal{M}_{(\lambda_3,\lambda_4;\lambda_1,\lambda_2)} = (-1)^{m-n}\mathcal{M}_{(\lambda_1,\lambda_2;\lambda_3,\lambda_4)}\,, \tag{4.5}$$

charge conjugation

$$\mathcal{M}_{(\lambda_3,\lambda_4;\lambda_1,\lambda_2)} = (-1)^{m-n}\mathcal{M}_{(\lambda_4,\lambda_3;\lambda_2,\lambda_1)}\,, \tag{4.6}$$

and exchanging bosons

$$\mathcal{M}_{(\lambda_3,\lambda_4;\lambda_1,\lambda_2)}(s,t,u) = (-1)^{m-2s_1}\mathcal{M}_{(\lambda_3,\lambda_4;\lambda_2,\lambda_1)}(s,u,t)\,, \tag{4.7}$$

where $m=\lambda_1-\lambda_2,\;\; n=\lambda_3-\lambda_4$ and s_1,s_2,s_3,s_4 are the spin of the particles.

4.2 Scalar-Graviton Scattering

For this process, there are four possible diagrams at tree level as shown in Fig. 4. We use the helicity amplitudes formalism, as discussed before, to write down the amplitudes. For this process we have two independent helicity amplitudes $\mathcal{M}_{(0,+2;0,+2)}$, $\mathcal{M}_{(0,+2;0,-2)}$ and the others can be obtained by applying the symmetries that are given in the previous subsection.

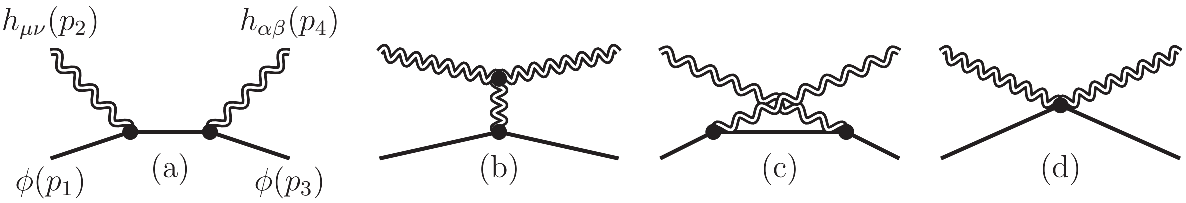


Figure 4: Scalar-Graviton scattering at tree level, where (a), (b) and (c) represent s, t and u-channels respectively, while (d) is just a simple scalar-scalar-graviton-graviton vertex.

The amplitudes of the respective diagrams in Fig. 4 can then be written as:

$$\begin{aligned}
\mathcal{M}_{(a)} &= \epsilon^{\mu\nu}(p_2)\;\; V^{\{\phi\phi h\}}_{\mu\nu}(p_1,Q_1,m)\;\; S(Q_1,m)\;\; V^{\{\phi\phi h\}}_{\alpha\beta}(p_3,Q_1,m)\;\; \epsilon^{*\alpha\beta}(p_4)\,,\\
\mathcal{M}_{(b)} &= \epsilon^{\mu\nu}(p_2)\;\; V^{\{\phi\phi h\}}_{\gamma\delta}(p_1,p_3,m)\;\; S^{\gamma\delta\rho\sigma}_{\{hh\}}(Q_2)\;\; V^{\{hhh\}}_{\mu\nu\alpha\beta\rho\sigma}(p_2,p_4)\;\; \epsilon^{*\alpha\beta}(p_4)\,,\\
\mathcal{M}_{(c)} &= \epsilon^{\mu\nu}(p_2)\;\; V^{\{\phi\phi h\}}_{\mu\nu}(p_1,Q_3,m)\;\; S(Q_3,m)\;\; V^{\{\phi\phi h\}}_{\alpha\beta}(p_3,Q_3,m)\;\; \epsilon^{*\alpha\beta}(p_4)\,,\\
\mathcal{M}_{(d)} &= \epsilon^{\mu\nu}(p_2)\;\; V^{\{\phi\phi hh\}}_{\mu\nu\alpha\beta}(p_1,p_3)\;\; \epsilon^{*\alpha\beta}(p_4)\,,
\end{aligned} \tag{4.8}$$

where $Q_1 = p_1 + p_2$, $Q_2 = p_1 - p_3$ and $Q_3 = p_1 - p_4$.
Adding these amplitudes together, we get the total amplitude as

$$\mathcal{M}_{(\text{Total})} = \mathcal{M}_{(\text{a})} + \mathcal{M}_{(\text{b})} + \mathcal{M}_{(\text{c})} + \mathcal{M}_{(\text{d})} \,. \tag{4.9}$$

In the helicity formalism, this total amplitude can be written as

$$\mathcal{M}_{(\text{Total})} = \mathcal{M}_{(0,+2;0,+2)} + \mathcal{M}_{(0,-2;0,-2)} + \mathcal{M}_{(0,+2;0,-2)} + \mathcal{M}_{(0,-2;0,+2)} \,. \tag{4.10}$$

Using the simplified Feynman rules that we derived in the previous section and applying kinematics in the CM frame according to the choice of momenta and polarization vectors as shown in App. C, the two independent helicity amplitudes are given by

$$\begin{aligned}
\mathcal{M}_{(0,+2;0,+2)} &= \kappa^2 \frac{k^4}{(s-m^2)(u-m^2)t}\big(1+\cos(\theta)\big)^2\Big[m^4 + 4km^2E + 8k^2m^2 + 8k^3E + 8k^4\Big]\,,\\
\mathcal{M}_{(0,+2;0,-2)} &= \kappa^2 \frac{k^4\, m^4}{(s-m^2)(u-m^2)t}\big(1-\cos(\theta)\big)^2\,,
\end{aligned}$$

where θ is the scattering angle and k is the momentum of the incoming particles in the CM frame.
However, in the CM frame, we have:

$$\begin{aligned}
s &= m^2 + 2k^2 + 2kE\,,\\
E^2 &= m^2 + k^2\,.
\end{aligned}$$

Then, the two independent helicity amplitudes can be written as:

$$\begin{aligned}
\mathcal{M}_{(0,+2;0,+2)} &= \kappa^2 \frac{k^4\, s^2}{(s-m^2)(u-m^2)t}\big(1+\cos(\theta)\big)^2\,,\\
\mathcal{M}_{(0,+2;0,-2)} &= \kappa^2 \frac{k^4\, m^4}{(s-m^2)(u-m^2)t}\big(1-\cos(\theta)\big)^2\,.
\end{aligned} \tag{4.11}$$

After obtaining the results with our simplified Feynman rules, we can start the process of comparison and checking the results. First, we have verified that the standard Feynman rules in App. B give the same results for the independent helicity amplitudes Eq. (4.11). In addition, our results for the scalar-graviton scattering helicity amplitudes also agree with the results of M. T. Grisaru and P. Van Nieuwenhuizen and C. C. Wu in [7].

4.3 Graviton-Graviton Scattering

For this process, there are also four possible diagrams at tree level as shown in Fig. 5, and the helicity amplitude formalism is again used to express the final results. This process has four independent helicity amplitudes $\mathcal{M}_{(+2,+2;+2,+2)}$, $\mathcal{M}_{(+2,-2;+2,-2)}$, $\mathcal{M}_{(+2,+2;+2,-2)}$, $\mathcal{M}_{(+2,+2;-2,-2)}$.

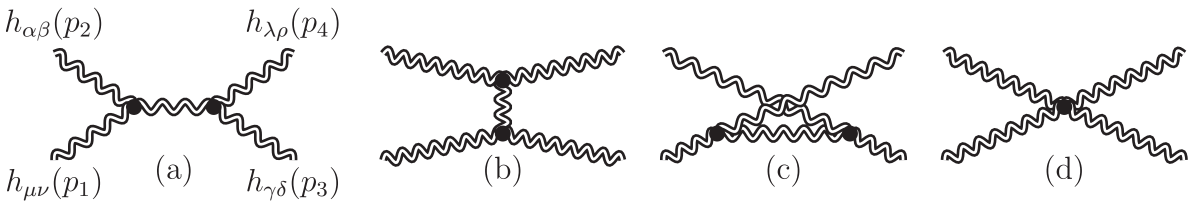


Figure 5: Graviton-Graviton scattering at tree level, where (a), (b) and (c) represent s, t and u-channels respectively, while (d) is just a simple quadruple graviton vertex.

The amplitudes of the respective diagrams in Fig. 5 can then be written as:

$$\mathcal{M}_{(\mathrm{a})} = \epsilon^{\mu\nu}(p_1)\epsilon^{\alpha\beta}(p_2)\ \ V^{\{hhh\}}_{\mu\nu\alpha\beta\eta\sigma}(p_1,p_2)\ \ S^{\eta\sigma\epsilon\kappa}_{\{hh\}}(Q_1)\ \ V^{\{hhh\}}_{\gamma\delta\lambda\rho\epsilon\kappa}(p_3,p_4)\ \ \epsilon^{*\gamma\delta}(p_3)\epsilon^{*\lambda\rho}(p_4)\,,$$

$$\mathcal{M}_{(\mathrm{b})} = \epsilon^{\mu\nu}(p_1)\epsilon^{\alpha\beta}(p_2)\ \ V^{\{hhh\}}_{\mu\nu\alpha\beta\eta\sigma}(p_1,p_3)\ \ S^{\eta\sigma\epsilon\kappa}_{\{hh\}}(Q_2)\ \ V^{\{hhh\}}_{\gamma\delta\lambda\rho\epsilon\kappa}(p_2,p_4)\ \ \epsilon^{*\gamma\delta}(p_3)\epsilon^{\lambda\rho}(p_4)\,,$$

$$\mathcal{M}_{(\mathrm{c})} = \epsilon^{\mu\nu}(p_1)\epsilon^{\alpha\beta}(p_2)\ \ V^{\{hhh\}}_{\mu\nu\alpha\beta\eta\sigma}(p_1,p_4)\ \ S^{\eta\sigma\epsilon\kappa}_{\{hh\}}(Q_3)\ \ V^{\{hhh\}}_{\gamma\delta\lambda\rho\epsilon\kappa}(p_2,p_3)\ \ \epsilon^{*\gamma\delta}(p_3)\epsilon^{*\lambda\rho}(p_4)\,,$$

$$\mathcal{M}_{(\mathrm{d})} = \epsilon^{\mu\nu}(p_1)\epsilon^{\alpha\beta}(p_2)\ \ V^{\{hhhh\}}_{\mu\nu\alpha\beta\gamma\delta\lambda\rho}(p_1,p_2,p_3)\ \ \epsilon^{*\gamma\delta}(p_3)\epsilon^{*\lambda\rho}(p_4)\,, \tag{4.12}$$

where $Q_1 = p_1 + p_2$, $Q_2 = p_1 - p_3$ and $Q_3 = p_1 - p_4$.
Adding these amplitudes together, we get the total amplitude as

$$\mathcal{M}_{(\mathrm{Total})} = \mathcal{M}_{(\mathrm{a})} + \mathcal{M}_{(\mathrm{b})} + \mathcal{M}_{(\mathrm{c})} + \mathcal{M}_{(\mathrm{d})}\,. \tag{4.13}$$

In the helicity formalism, this total amplitude can be written as

$$\begin{aligned}\mathcal{M}_{(\mathrm{Total})} = \;& \mathcal{M}_{(+2,+2;+2,+2)} + \mathcal{M}_{(-2,-2;-2,-2)} + \mathcal{M}_{(+2,-2;+2,-2)} + \mathcal{M}_{(-2,+2;-2,+2)} + \mathcal{M}_{(+2,-2;-2,+2)} \\ &+ \mathcal{M}_{(-2,+2;+2,-2)} + \mathcal{M}_{(+2,+2;+2,-2)} + \mathcal{M}_{(-2,-2;-2,+2)} + \mathcal{M}_{(+2,-2;+2,+2)} + \mathcal{M}_{(-2,+2;-2,-2)} \\ &+ \mathcal{M}_{(+2,+2;-2,+2)} + \mathcal{M}_{(-2,-2;+2,-2)} + \mathcal{M}_{(-2,+2;+2,+2)} + \mathcal{M}_{(+2,-2;-2,-2)} + \mathcal{M}_{(+2,+2;-2,-2)} \\ &+ \mathcal{M}_{(-2,-2;+2,+2)}\,.\end{aligned}$$

Again, using our simplified Feynman rules and applying kinematics in the CM frame gives the four independent helicity amplitudes:

$$\begin{aligned}\mathcal{M}_{(+2,+2;+2,+2)} &= \kappa^2\,\frac{1}{4}\,\frac{s^3}{t\,u}\,, \\ \mathcal{M}_{(+2,-2;+2,-2)} &= \kappa^2\,\frac{1}{4}\,\frac{u^3}{s\,t}\,, \\ \mathcal{M}_{(+2,+2;+2,-2)} &= 0\,, \\ \mathcal{M}_{(+2,+2;-2,-2)} &= 0\,.\end{aligned} \tag{4.14}$$

Applying the symmetry relations Eqs. (4.4–4.7) gives all helicity amplitudes for graviton-graviton scattering:

$$\begin{aligned}
&\mathcal{M}_{(+2,+2;+2,+2)} = \mathcal{M}_{(-2,-2;-2,-2)} = \kappa^2 \frac{1}{4} \frac{s^3}{t\,u} \,, \\
&\mathcal{M}_{(+2,-2;+2,-2)} = \mathcal{M}_{(-2,+2;-2,+2)} = \kappa^2 \frac{1}{4} \frac{u^3}{s\,t} \,, \\
&\mathcal{M}_{(+2,-2;-2,+2)} = \mathcal{M}_{(-2,+2;+2,-2)} = \kappa^2 \frac{1}{4} \frac{t^3}{s\,u} \,, \\
&\mathcal{M}_{(+2,+2;+2,-2)} = \mathcal{M}_{(-2,-2;-2,+2)} = \mathcal{M}_{(+2,-2;+2,+2)} = \mathcal{M}_{(-2,+2;-2,-2)} = 0 \,, \\
&\mathcal{M}_{(+2,+2;-2,+2)} = \mathcal{M}_{(-2,-2;+2,-2)} = \mathcal{M}_{(-2,+2;+2,+2)} = \mathcal{M}_{(+2,-2;-2,-2)} = 0 \,, \\
&\mathcal{M}_{(+2,+2;-2,-2)} = \mathcal{M}_{(-2,-2;+2,+2)} = 0 \,.
\end{aligned} \tag{4.15}$$

Once more, comparing with the standard Feynman rules, the same results are reached. Our results for these helicity amplitudes also agree with the results of J. F. Donoghue and T. Torma in their paper [8] and M. T. Grisaru, P. Van Nieuwenhuizen and C. C. Wu in [7].

5 One-Loop Correction

While the goal in the previous section was to verify the simplified Feynman rules, the goal in this section is to show the usefulness of these rules at loop level. So, we first show how to treat the loop integrals in the loop calculations by using dimensional regularization and the Passarino-Veltman method [1, 12, 13]. After that, we calculate some one-loop diagrams for scalar-graviton scattering using the simplified rules, then repeat the calculations using the standard rules, and finally compare the results obtained in the two ways.

5.1 Loop Integral

To calculate our loop integrals, we use dimensional regularization with Passarino-Veltman reduction since it preserves gauge and Lorentz invariance [1, 13].

5.1.1 Dimensional Regularization

Dimensional regularization is widely used to regularize loop integrals and separate out the UV divergences [1]. The main idea is to change the dimensionality of the loop integral from the dimension where it diverges to a lower dimension d where the integral converges.

In our case, the loop integrals are in four-dimensional Minkowski space. So, we move them to dimension $d = 4 - 2\epsilon$, where ϵ is a parameter and the limit $\epsilon \to 0$ will be taken at the end of the calculation. Then, we perform the integrals and go back to the original dimension by doing analytic continuation. Moreover, there are some considerations when using $d \neq 4$ dimensions that have to be taken into account: the metric tensor becomes

$$g_4^{\mu\nu} \to g_d^{\mu\nu}\,, \qquad \Rightarrow \qquad g^{\mu\nu} g_{\mu\nu} = \delta^\mu_\mu = d = 4 - 2\epsilon\,,$$

and the measures become

$$\int \frac{d^4 p}{(2\pi)^4} \qquad \to \qquad \int \frac{(\mu)^{2\epsilon}\, d^d p}{(2\pi)^d}\,,$$

where μ is a regulator parameter with dimension $[\mu] = M$.

Briefly, the standard procedure of this regularization is [1]: transfer to Euclidean space, do the Wick rotation, apply Feynman parameters, shift the integration variable, perform the integral, go back to Minkowski space. These steps will be explained in more detail and applied in the next section.

5.1.2 Scalar Integrals

The general form of a scalar one-loop integral for a N-point function with external momenta $p_1, \cdots, p_{N-1}$ (with p_N from momentum conservation) as shown in Fig. 6 is given by

$$I^N(p_1, \cdots, p_{N-1}, m_0, \cdots, m_{N-1}) \sim \tag{5.1}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m_0^2 + i\epsilon)((k+q_1)^2 - m_1^2 + i\epsilon) \cdots ((k+q_{N-1})^2 - m_{N-1}^2 + i\epsilon)}\,,$$

where k is the undefined momentum in the loop that will be integrated over, and $q_1, \cdots, q_{N-1}$ are the internal momenta that are related to the external momenta by $q_i = \sum_{k=1}^{i} p_k$ as shown in Fig. 6, and $m_0, \cdots, m_{N-1}$ are the masses of the propagators involved in the loop.

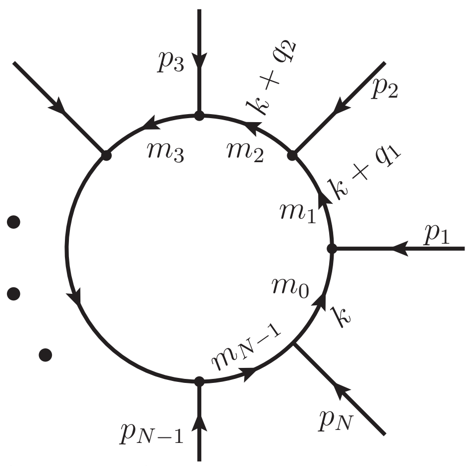


Figure 6: Generic diagram of N-point function with N external momenta at one-loop.

In our case of scalar-graviton scattering to one-loop, only the scalar field propagator has a mass m, so the masses $m_0, \cdots, m_{N-1}$ are either zero or m. In the following, we will only need one-loop diagrams up to the box diagrams with four propagators so we limit ourselves to the following scalar integrals:

$$A_0(m_0) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m_0^2 + i\epsilon)} ,$$

$$B_0(p_1, m_0, m_1) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m_0^2 + i\epsilon)((k+q_1)^2 - m_1^2 + i\epsilon)} , \tag{5.2}$$

$$C_0(p_1, p_2, m_0, m_1, m_2) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m_0^2 + i\epsilon)((k+q_1)^2 - m_1^2 + i\epsilon)((k+q_2)^2 - m_2^2 + i\epsilon)} ,$$

$$D_0(p_1, p_2, p_3, m_0, m_1, m_2, m_3) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m_0^2 + i\epsilon)((k+q_1)^2 - m_1^2 + i\epsilon)((k+q_2)^2 - m_2^2 + i\epsilon)((k+q_3)^2 - m_3^2 + i\epsilon)} ,$$

where the loop integrals are denoted with respect to the number of propagators involved in the loop: (A_0) for one propagator, (B_0) for two propagators, (C_0) for three propagators and (D_0) for four propagators.

Before continuing we note that as a result of the standard procedure of dimensional regularization, all massless tad-pole diagrams, as illustrated in Fig. 7, vanish.

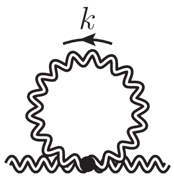


Figure 7: One-loop massless tad-pole diagram.

More specifically these diagrams correspond to the massless scalar integral A_0 given by

$$A_0(0) = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 + i\epsilon)} . \tag{5.3}$$

As can be seen in App. D, this integral vanishes for all d since it is proportional to $(M^2)^{d/2-2}$ where in this case $M^2 = 0$. Thus, the whole integral is zero. The general form of this result can be written as

$$\int \frac{d^dk}{(k^2)^\alpha} = 0 \qquad \text{for} \quad \forall\, \alpha,\, d \in \mathbb{C} , \tag{5.4}$$

which is known as Veltman's formula [12].

5.1.3 Tensor Integrals

The general form of a tensor one-loop integral of rank-M for a N-point function with external momenta $p_1, \cdots, p_{N-1}$ (with p_N from momentum conservation) as shown in Fig. 6 is given by

$$I^N_{\mu_1,\cdots,\mu_M}(p_1,\cdots,p_{N-1},m_0,\cdots,m_{N-1}) \sim \int \frac{d^dk}{(2\pi)^d} \frac{k_{\mu_1}\cdots k_{\mu_M}}{(k^2-m_0^2+i\epsilon)((k+q_1)^2-m_1^2+i\epsilon)\cdots((k+q_{N-1})^2-m_{N-1}^2+i\epsilon)}, \tag{5.5}$$

where $\mu_1, \cdots, \mu_M$ are indices that represent the rank of the integral, and the rest is the same as in Eq. (5.1).

In our case of scalar-graviton scattering to one-loop, the maximum number of indices that can appear in the numerator for A and D integrals is four, for B is five, and C is six. To calculate these tensor integrals, we will follow the Passarino-Veltman method, which will be discussed in the next section.

5.1.4 Passarino-Veltman Reduction

The idea of the Passarino-Veltman method [13] is to write the tensor integrals in terms of scalar integrals, Eq. (5.2), with the help of Passarino-Veltman reduction formula which takes the general form

$$k\cdot p_i = \frac{1}{2}[((k+q_i)^2-m_i^2)-((k+q_{i-1})^2-m_{i-1}^2)+m_i^2-m_{i-1}^2-q_i^2+q_{i-1}^2]\,. \tag{5.6}$$

To illustrate this idea, let us take an example of a rank-one tensor integral, a vector integral, for a two-point function which is given by

$$I^2_\mu(p_1,m_0,m_1) = B_\mu(p_1,m_0,m_1) = \int \frac{d^dk}{(2\pi)^d}\frac{k_\mu}{(k^2-m_0^2)((k+q_1)^2-m_1^2)}, \tag{5.7}$$

where $q_1 = p_1$. Since p_1 is the only four vector in this integral which can carry the index μ in the result, it follows that this integral can be written as $p_{1\mu}$ multiplied by a scalar function $B_1(p_1, m_0, m_1)$

$$\int \frac{d^dk}{(2\pi)^d}\frac{k_\mu}{(k^2-m_0^2)((k+p_1)^2-m_1^2)} = p_{1\mu}B_1(p_1,m_0,m_1)\,. \tag{5.8}$$

Multiplying by p_1^μ from both sides gives

$$\int \frac{d^dk}{(2\pi)^d}\frac{k\cdot p_1}{(k^2-m_0^2)((k+p_1)^2-m_1^2)} = p_1\cdot p_1\; B_1(p_1,m_0,m_1)\,. \tag{5.9}$$

Now we can use the Passarino-Veltman reduction formula Eq. (5.6) which takes the following form in this example,

$$k\cdot p_1 = \frac{1}{2}[((k+p_1)^2-m_1^2)-(k^2-m_0^2)+m_1^2-m_0^2-p_1^2]\,. \tag{5.10}$$

Inserting Eq. (5.10) into Eq. (5.9) gives

$$\begin{aligned} p_1^2 B_1(p_1, m_0, m_1) = &\ \frac{1}{2}\Bigg[\int \frac{d^d k}{(2\pi)^d}\frac{1}{(k^2-m_0^2)} - \int \frac{d^d k}{(2\pi)^d}\frac{1}{((k+p_1)^2-m_1^2)} \\ &+ (m_1^2 - m_0^2 - p_1^2)\int \frac{d^d k}{(2\pi)^d}\frac{1}{(k^2-m_0^2)((k+p_1)^2-m_1^2)}\Bigg] \\ = &\ \frac{1}{2}\Big[A_0(m_0) - A_0(m_1) - (-m_1^2 + m_0^2 + p_1^2)B_0(p_1, m_0, m_1)\Big]\,. \end{aligned}$$

From this it follows that

$$\begin{aligned} B_\mu(p_1, m_0, m_1) &= p_{1\mu} B_1(p_1, m_0, m_1) \\ &= \frac{p_{1\mu}}{2p_1^2}\Big[A_0(m_0) - A_0(m_1) - (-m_1^2 + m_0^2 + p_1^2)B_0(p_1, m_0, m_1)\Big]\,. \end{aligned} \tag{5.11}$$

As a result, the tensor integral $B_\mu(p_1, m_0, m_1)$ can be written in terms of the scalar integrals $A_0(m_0)$, $A_0(m_1)$ and $B_0(p_1, m_0, m_1)$. This procedure can be generalized for any tensor integral, and for completeness we give all relevant tensor integrals in App. E.

5.2 Scalar-Graviton Scattering to One-Loop Order

In order to study scalar-graviton scattering to one-loop order, we first need to draw all the diagrams that can contribute to this process up to one-loop. We start with the tree level diagrams as shown in Fig. 8. Then, we insert all possible one-loop corrections, as shown in the next sections, into the tree level diagrams to get all possible one-loop diagrams for scalar-graviton scattering. More specifically, in this case we need to insert the propagator corrections, the triple graviton vertex corrections, the scalar-scalar-graviton vertex corrections and the scalar-scalar-graviton-graviton vertex corrections.

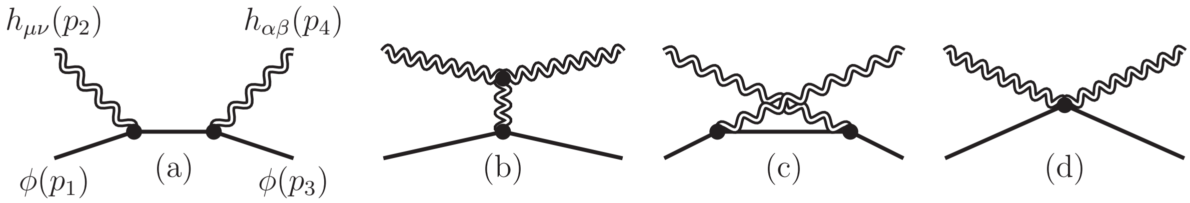


Figure 8: scalar-graviton scattering at tree level.

Moreover, we need to take into account that there are permutations of external graviton legs whenever it is possible. For example, the loop corrections to the scalar-scalar-graviton-graviton vertex has a permutation in external graviton legs as shown in Fig. 9. In addition, when dealing with loops, it is important to remember the minus sign that arises from fermion loops.

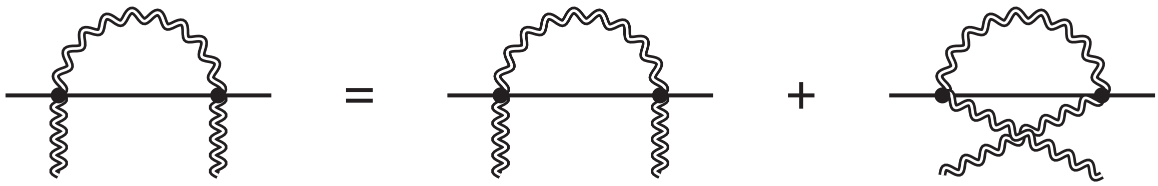


Figure 9: Permutation of external graviton legs.

The last point to describe before starting the calculations is how the graviton propagator behaves in dimensional regularization at loop level. As starting point, we recall the Lagrangian for the graviton propagator, Eq. (3.5), which can be written as

$$\mathcal{L}_{hh} = \frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} - \frac{1}{4}\partial_\lambda h_\nu{}^\nu \partial^\lambda h_\mu{}^\mu = \frac{1}{2}h_{\mu\nu}\partial^\lambda\partial_\lambda\Big(I^{\mu\nu\alpha\beta} - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}\Big)h_{\alpha\beta}\,, \tag{5.12}$$

where $I^{\mu\nu\alpha\beta} = \frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha})$ is the identity tensor.
Then, we solve the Green's function equation in d dimensions

$$\Big(I^{\mu\nu\alpha\beta} - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}\Big)\partial^\lambda\partial_\lambda S_{\alpha\beta\gamma\delta}(x-y) = -I^{\mu\nu}{}_{\gamma\delta}\,\delta^{(d)}(x-y)\,, \tag{5.13}$$

to find the propagator. To solve this equation, we perform a Fourier transform which gives

$$\Big(I^{\mu\nu\alpha\beta} - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}\Big)(-k^2)S_{\alpha\beta\gamma\delta}(k) = -I^{\mu\nu}{}_{\gamma\delta}\,. \tag{5.14}$$

To solve this equation, we use the following ansatz

$$S_{\alpha\beta\gamma\delta}(k) = \frac{1}{k^2}\Big(aI_{\alpha\beta\gamma\delta} + b\eta_{\alpha\beta}\eta_{\gamma\delta}\Big)\,, \tag{5.15}$$

which gives

$$a = 1\,, \qquad\qquad b = -\frac{1}{d-2}\,, \tag{5.16}$$

where $\eta_{\mu\nu}\eta^{\mu\nu} = \delta^\mu_\mu = d = 4 - 2\epsilon$.
Thus, the graviton propagator in momentum space in d dimensions is

$$S_{\mu\nu\alpha\beta}(k) = \frac{i}{k^2}\Big(\frac{1}{2}I_{\mu\nu\alpha\beta} - \frac{1}{d-2}\eta_{\mu\nu}\eta_{\alpha\beta}\Big) = \frac{i}{k^2}P_{\mu\nu\alpha\beta}\,, \tag{5.17}$$

where $P_{\mu\nu\alpha\beta} = \frac{1}{2}I_{\mu\nu\alpha\beta} - \frac{1}{d-2}\eta_{\mu\nu}\eta_{\alpha\beta}$.

5.2.1 Self-Energy Corrections

There are two types of propagators, scalar and graviton, in the tree level diagrams shown in Fig. 8, and each one has different loop corrections. So, let us start with the graviton propagator which has six one-loop diagrams as shown in Fig. 10.

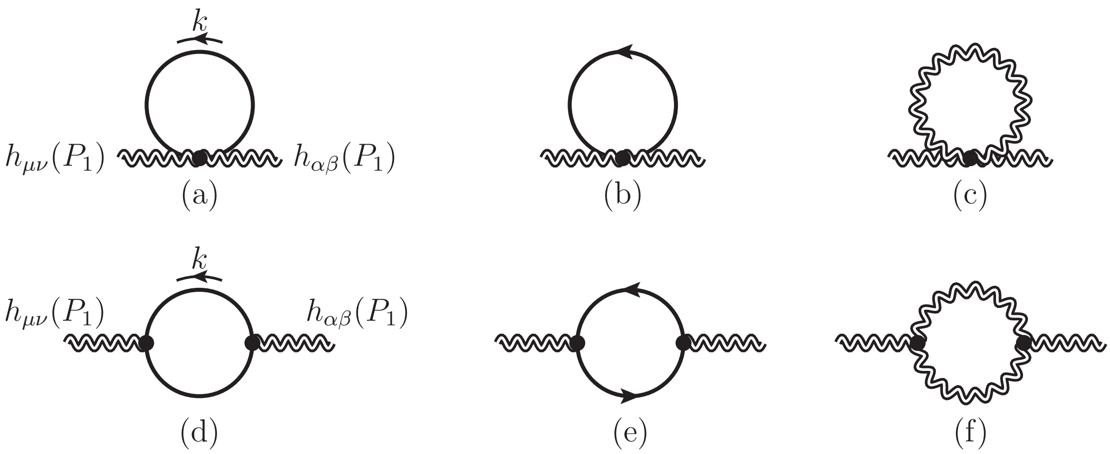


Figure 10: The graviton propagator at one-loop level with: one (a-c), two (d-f) propagators.

In terms of the Feynman rules, the diagrams with one propagator in Fig. 10a-c can be written as follows:

$$\mathcal{M}^{(\mathrm{a})}_{\mu\nu\alpha\beta} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} V^{\{\phi\phi hh\}}_{\mu\nu\alpha\beta}(k) \qquad S_{\{\phi\phi\}}(k,m)\,, \tag{5.18}$$

$$\mathcal{M}^{(\mathrm{b})}_{\mu\nu\alpha\beta} = \int \frac{d^4k}{(2\pi)^4} V^{\{\bar{\chi}\chi hh\}}_{\mu\nu\alpha\beta\rho\sigma}(P_1,k) \qquad S^{\rho\sigma}_{\{\bar{\chi}\chi\}}(k) \qquad = 0\,, \tag{5.19}$$

$$\mathcal{M}^{(\mathrm{c})}_{\mu\nu\alpha\beta} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} V^{\{hhhh\}}_{\mu\nu\alpha\beta\rho\sigma\gamma\delta}(P_1,k) \; S^{\rho\sigma\gamma\delta}_{\{hh\}}(k) \qquad = 0\,, \tag{5.20}$$

where $\frac{1}{2}$ is the symmetry factor. $\mathcal{M}^{(\mathrm{b})}_{\mu\nu\alpha\beta}$ and $\mathcal{M}^{(\mathrm{c})}_{\mu\nu\alpha\beta}$ vanish according to the relation Eq. (5.4), which means that all massless tad-pole diagrams vanish. Using the simplified rules, we get

$$\mathcal{M}^{(\mathrm{a})}_{\mu\nu\alpha\beta} = \frac{\kappa^2}{8}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2-m^2}\left[\eta_{\mu\nu}k_\alpha k_\beta - \eta_{\mu\alpha}k_\nu k_\beta - \eta_{\mu\beta}k_\nu k_\alpha - \eta_{\nu\alpha}k_\mu k_\beta - \eta_{\nu\beta}k_\mu k_\alpha + \eta_{\alpha\beta}k_\mu k_\nu\right],$$

which can be written in terms of scalar integrals as

$$\mathcal{M}^{(\mathrm{a})}_{\mu\nu\alpha\beta} = \kappa^2\,\frac{m^2}{4d}\,A_0(m)\,\left[\eta_{\mu\nu}\eta_{\alpha\beta} - \eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\alpha\nu}\right]. \tag{5.21}$$

On the other hand, if we use the standard Feynman rules to calculate the amplitude $\mathcal{M}^{(\mathrm{a})}_{\mu\nu\alpha\beta}$, then the calculation is slightly more complicated because the scalar-scalar-graviton-graviton vertex $V^{\{\phi\phi hh\}}_{\mu\alpha\beta}$ has six terms in the standard rules Eq. (B.2), compared to two terms in the simplified rules Eq. (3.9).
Next we consider the diagrams with two propagators in the loop as in Fig. 10d-f, which can be written as:

$$\mathcal{M}^{(\mathrm{d})}_{\mu\nu\alpha\beta} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} V^{\{\phi\phi h\}}_{\mu\nu}(k,Q_1,m) \;\; S_{\{\phi\phi\}}(k,m) \;\; S_{\{\phi\phi\}}(Q_1,m) \;\; V^{\{\phi\phi h\}}_{\alpha\beta}(k,Q_1,m)\,,$$

$$\mathcal{M}^{(\mathrm{e})}_{\mu\nu\alpha\beta} = \int \frac{d^4k}{(2\pi)^4} V^{\{\bar\chi\chi h\}}_{\mu\nu\rho\gamma}(k,Q_1) \qquad S^{\rho\sigma}_{\{\bar\chi\chi\}}(k) \qquad S^{\gamma\delta}_{\{\bar\chi\chi\}}(Q_1) \qquad V^{\{\bar\chi\chi h\}}_{\sigma\delta\alpha\beta}(k,Q_1)\,,$$

$$\mathcal{M}^{(\mathrm{f})}_{\mu\nu\alpha\beta} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} V^{\{hhh\}}_{\mu\nu\rho\sigma\eta\lambda}(k,Q_1) \qquad S^{\rho\sigma\gamma\delta}_{\{hh\}}(k) \qquad S^{\eta\lambda\epsilon\kappa}_{\{hh\}}(Q_1) \qquad V^{\{hhh\}}_{\gamma\delta\epsilon\kappa\alpha\beta}(k,Q_1)\,,$$

where $\frac{1}{2}$ is the symmetry factor and $Q_1 = k + P_1$.
Plugging in the Feynman rules will give lengthy expressions. Therefore, we only show $\mathcal{M}^{(\mathrm{f})}_{\mu\nu\alpha\beta}$, which has two triple graviton vertices, while the other amplitudes $\mathcal{M}^{(\mathrm{d})}_{\mu\nu\alpha\beta}, \mathcal{M}^{(\mathrm{e})}_{\mu\nu\alpha\beta}$ are approximately the same in the standard and simplified rules. Using the simplified rules and doing the Passarino-Veltman reduction, the amplitude $\mathcal{M}^{(\mathrm{f})}_{\mu\nu\alpha\beta}$ can be written in terms of the scalar integral $B_0(0,0,P_1)$ as

$$\begin{aligned}
\mathcal{M}^{(\mathrm{f})}_{\mu\nu\alpha\beta} =& \frac{\kappa^2\, B_0(0,0,P_1)}{64\,d^4 - 256\,d^3 + 192\,d^2 + 256\,d - 256}\Big[\\
&+ P_{1\mu}P_{1\nu}P_{1\alpha}P_{1\beta}\big(\,d^6 - 2\,d^4 - 116\,d^3 + 312\,d^2 + 144\,d - 256\big) \\
&+ \eta_{\mu\nu}\eta_{\alpha\beta}P_1^3\big(\,d^6 - 5\,d^5 + 31\,d^3 + 6\,d^2 - 36\,d - 8\big) \\
&+ P_1^2\big(\eta_{\mu\nu}P_{1\alpha}P_{1\beta} + \eta_{\alpha\beta}P_{1\mu}P_{1\nu}\big)\big(64 - \,d^6 + 3\,d^5 + 13\,d^4 - 34\,d^3 - 76\,d^2 + 40\,d\big)\Big] \\
&+ \frac{\kappa^2\, B_0(0,0,P_1)}{64\,d^3 - 128\,d^2 - 64\,d + 128}\Big[\\
&+ P_1^3\big(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}\big)\big(3\,d^3 - 24\,d^2 - 8\,d + 16\big) \\
&+ P_1^2\big(\eta_{\mu\alpha}P_{1\nu}P_{1\beta} + \eta_{\mu\beta}P_{1\nu}P_{1\alpha} + \eta_{\nu\alpha}P_{1\mu}P_{1\beta} + \eta_{\nu\beta}P_{1\mu}P_{1\alpha}\big)\big(32\,d^2 - 7\,d^3 + 20\,d - 16\big)\Big]\,.
\end{aligned}$$

Now, if we instead use the standard Feynman rules to calculate the amplitude $\mathcal{M}^{(\mathrm{f})}_{\mu\nu\alpha\beta}$, then the calculation is more complicated because the triple graviton vertex $V^{\{hhh\}}_{\mu\nu\rho\sigma\eta\lambda}$ has 40 terms in the standard rules Eq. (B.6), compared to only four terms in the simplified rules Eq. (3.6). In addition, this amplitude has two triple graviton vertices, so the number of Lagrangian terms involved from the vertices in the standard rules is 1600, compared to 16 terms in the simplified rules. Since these calculations are very lengthy, we only show the comparison between the running time in the FORM program, after doing Passarino-Veltman reduction and writing the amplitude $\mathcal{M}^{(\mathrm{f})}_{\mu\nu\alpha\beta}$ in terms of scalar integrals:

simplified way

```
WTime =       0.58 sec   Generated terms =        10
            HH2a         Terms in output =        10
                         Bytes used      =      4024
```

simplified way

standard way

```
WTime =      26.38 sec   Generated terms =        10
            HH2b         Terms in output =        10
                         Bytes used      =      3960
```

standard way

As shown above, there is a large difference in the running time. In addition, the running time will increase considerably for more complicated diagrams as we will show in the next sections.

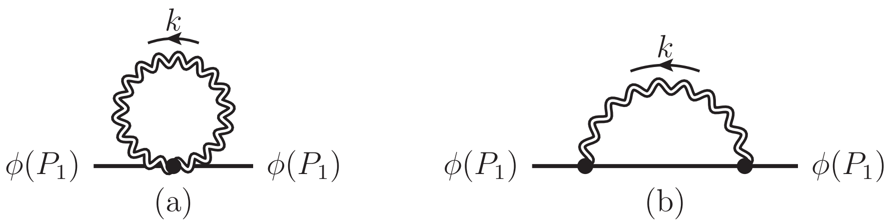


Figure 11: The scalar propagator at one-loop level with: one (a), two (b) propagators.

Finally, we consider the scalar propagator which has just two diagrams that can contribute as shown in Fig. 11. Again, the massless tad-pole in Fig. 11a vanishes. So, the diagrams can be written as:

$$\mathcal{M}^{(a)} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} V^{\{\phi\phi hh\}}_{\mu\nu\alpha\beta}(P_1) \qquad S^{\mu\nu\alpha\beta}_{\{hh\}}(k) \quad = 0\,, \tag{5.22}$$

$$\mathcal{M}^{(b)} = \int \frac{d^4k}{(2\pi)^4} V^{\{\phi\phi h\}}_{\mu\nu}(P_1, Q_1, m) \;\; S^{\mu\nu\alpha\beta}_{\{hh\}}(k) \;\; S_{\{\phi\phi\}}(Q_1) \;\; V^{\{\phi\phi h\}}_{\alpha\beta}(P_1, Q_1, m)\,,$$

where $\frac{1}{2}$ is the symmetry factor and $Q_1 = k + P_1$.
Plugging the Feynman rules in $\mathcal{M}^{(b)}$ gives

$$\begin{aligned}
\mathcal{M}^{(b)} = {} & \kappa^2 \left[P_1^2 + m^2\right] \int \frac{d^dk}{(2\pi)^d} \frac{k\cdot P_1}{k^2((k+P_1)^2 - m^2)} \\
& + \kappa^2 \frac{P_1^2}{2} \int \frac{d^dk}{(2\pi)^d} \frac{k^2}{k^2((k+P_1)^2 - m^2)} \\
& + \kappa^2 \left[\frac{-2P_1^4 - 4m^2P_1^2 + d\,P_1^4 + 2d\,m^2P_1^2 - d\,m^4}{2(d-2)}\right] \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2((k+P_1)^2 - m^2)}\,,
\end{aligned} \tag{5.23}$$

which can be written in terms of scalar integrals as

$$\mathcal{M}^{(b)} = -\kappa^2 \frac{m^2}{2} A_0(m) + \kappa^2 \frac{m^2}{d-2} \left[d\, P_1^2 - 2P_1^2 - m^2\right] B_0(0, m, P_1)\,. \tag{5.24}$$

Both the simplified and the standard rules give the same results, since the scalar-scalar-graviton vertex $V_{\mu\nu}^{\{\phi\phi h\}}$ and the propagators are the same in both.

5.2.2 Triple Graviton Vertex Corrections

At tree level, the triple graviton vertex appears in the t-channel diagram as shown in Fig. 8b, and it has nine one-loop contributions: three of them are tad-pole diagrams as in Fig. 12a-c, three are bubble diagrams as in Fig. 12d-f and three are triangle diagrams as in Fig. 12g-i. However, in this section we only discuss the bubble diagram with two graviton propagators, as shown in Fig. 13f in detail, while we only give overall comparisons for the other diagrams.

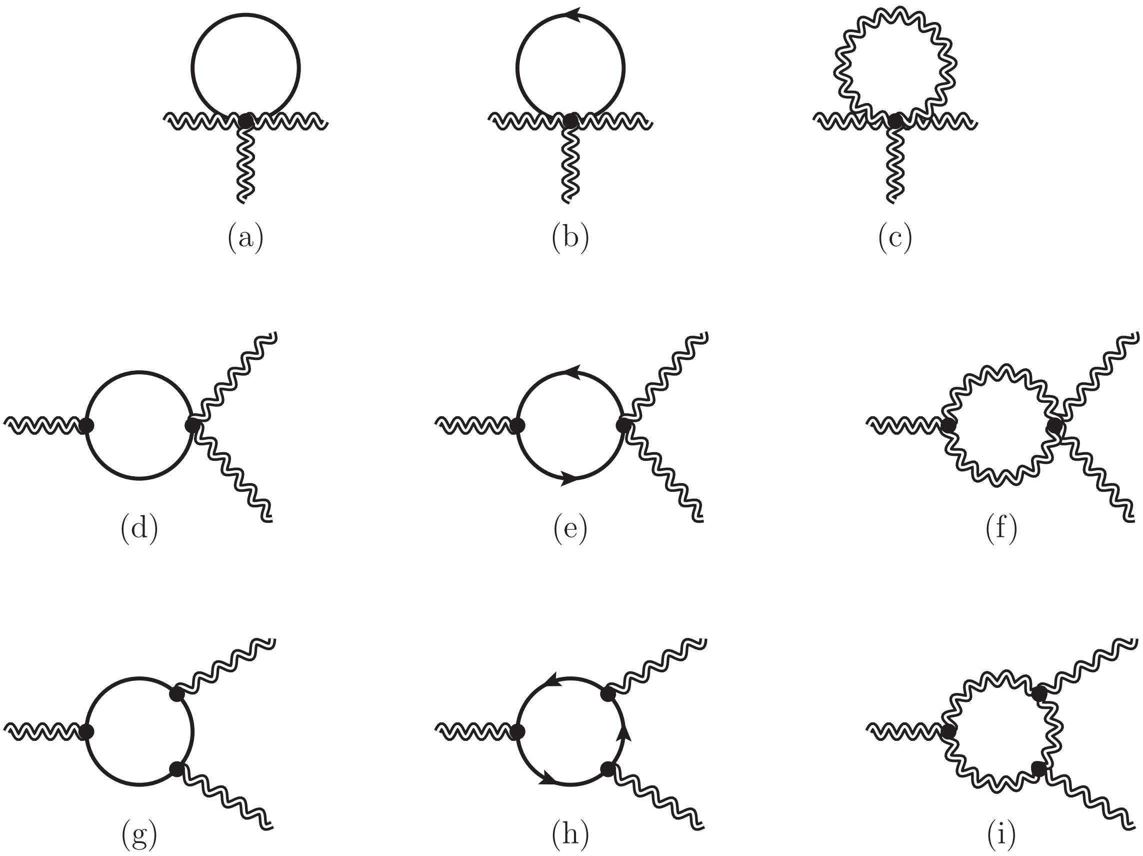


Figure 12: Triple graviton vertex at one-loop level with: one (a-c), two (d-e), three (g-i) propagators.

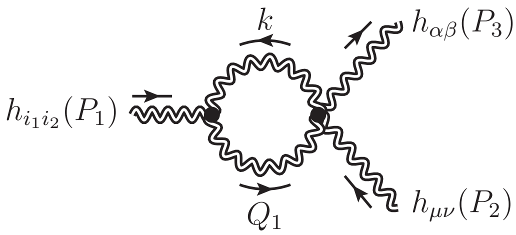


Figure 13: Triple graviton vertex at one-loop level with two graviton propagators.

Using the momentum assignments in Fig. 13, the amplitude of this diagram can be written as

$$\mathcal{M}_{\mu\nu\alpha\beta\, i_1 i_2} = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} V^{\{hhh\}}_{\mu\nu\rho\sigma\eta\lambda}(k, Q_1)\;\; S^{\rho\sigma\gamma\delta}_{\{hh\}}(k)\;\; S^{\eta\lambda\epsilon\kappa}_{\{hh\}}(Q_1)\;\; V^{\{hhhh\}}_{\gamma\delta\epsilon\kappa\alpha\beta\, i_1 i_2}(k, Q_1, P_2)\,,$$

where $\frac{1}{2}$ is the symmetry factor and $Q_1 = k + P_1$.

In this case, we note that the standard triple graviton vertex $V^{\{hhh\}}_{\mu\nu\rho\sigma\eta\lambda}$ Eq. (B.6) has 40 terms and the standard quadruple graviton vertex $V^{\{hhhh\}}_{\gamma\delta\epsilon\kappa\alpha\beta\, i_1 i_2}$ Eq. (B.7) has 113 terms whereas the simplified ones Eqs. (3.6, 3.7) have only 4 and 12 terms respectively. Thus, for this diagram, the number of Lagrangian terms involved from the vertices in the standard way is 4520, compared to 48 terms in the simplified way. To compare the two, we again consider the running time in FORM, after doing Passarino-Veltman reduction and writing the amplitude in terms of scalar integrals:

simplified way

```
WTime =       20.15 sec    Generated terms =         99
          V3HH2a           Terms in output =         99
                           Bytes used      =      32776
```

simplified way

standard way

```
WTime =      803.35 sec    Generated terms =         99
          V3HH2b           Terms in output =         99
                           Bytes used      =      34184
```

standard way

which shows that the running time in the standard way is about 40 times the running time in the simplified way. Similarly, for the diagram in Fig 12i the running time is about seven minutes in the simplified way while in the standard way it is more than two hours.

Finally, we list, in Tab. 1, the number of Lagrangian terms for all one-loop corrections to the triple graviton vertex in the standard and simplified ways.

Table 1: The number of Lagrangian terms when using the standard rules and the simplified ones for calculating the one-loop corrections to the triple graviton vertex as shown in Fig. 12.

Diagram	The standard way[1]	The simplified way[1]
(a)	10	7
(b)	The amplitude vanishes[2]	The amplitude vanishes[2]
(c)	The amplitude vanishes[2]	The amplitude vanishes[2]
(d)	18	6
(e)	The amplitude vanishes	319
(f)	4 520	48
(g)	27	27
(h)	512	1 331
(i)	64 000	64

[1] Since the propagators are the same in the standard and simplified rules, we only consider the terms of the vertices in all our comparisons of the numbers of the Lagrangian terms.
[2] all massless tad-pole diagrams vanish according to the relation Eq. (5.4).

5.2.3 Scalar-Scalar-Graviton Vertex Corrections

At tree level, the scalar-scalar-graviton vertex appears twice in the s and u-channel diagrams and once in the t-channel as shown in Fig. 8, and it has six one-loop diagrams as shown in Fig. 14: the tad-pole diagram in Fig. 14a, the three bubble diagrams in Fig. 14b-d and the two triangle diagrams in Fig. 14e-f.

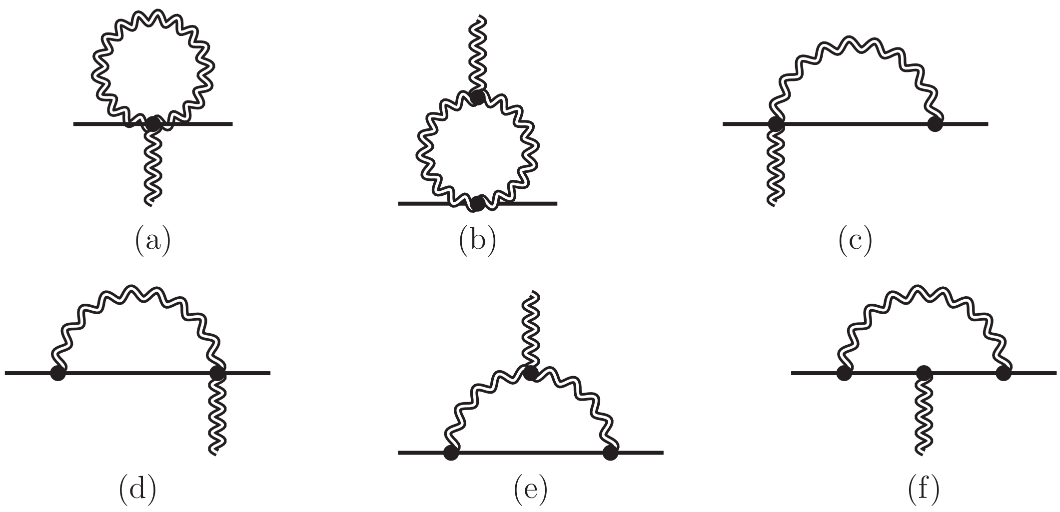


Figure 14: Scalar-Scalar-Graviton vertex at one-loop level with: one (a), two (b-d), three (e-f) propagators.

Again, we list in Tab. 2 the comparison of the number of Lagrangian terms using the standard and simplified rules to show the usefulness of the simplified rules and how the number of Lagrangian terms can be reduced by a factor 30 for some of the diagrams in Fig. 14.

Table 2: The number of Lagrangian terms when using the standard rules and the simplified ones for calculating the one-loop corrections to the scalar-scalar-graviton vertex as shown in Fig. 14.

Diagram	The standard way	The simplified way
(a)	The amplitude vanishes[1]	The amplitude vanishes[1]
(b)	240	8
(c)	18	6
(d)	18	6
(e)	360	36
(f)	27	27

[1] all massless tad-pole diagrams vanish according to the relation Eq. (5.4).

5.2.4 Scalar-Scalar-Graviton-Graviton Vertex Corrections

The scalar-scalar-graviton-graviton vertex also appears once at tree level in the last diagram in Fig. 8. However, it has 16 one-loop diagrams as shown in Fig. 15: the tad-pole diagram in Fig. 15a, the five bubble diagrams in Fig. 15b-f, the seven triangle diagrams in Fig. 15g-m and the three box diagrams in Fig. 15n-p.

Again, since the aim of this section is to show the usefulness of the simplified rules, we only discuss the statistics of the results for the diagram in Fig. 15h. For this diagram, the number of Lagrangian terms involved from the vertices in the standard way is 9 600, compared to 32 terms in the simplified way. To compare the two, we again consider the running time in FORM, after doing Passarino-Veltman reduction and writing the amplitude in terms of scalar integrals:

simplified way
```
WTime =        91.16 sec   Generated terms =      21830
       V2phi2H3a           Terms in output =        116
                           Bytes used      =    1786232
```
simplified way

standard way
```
WTime =      3940.32 sec   Generated terms =      34572
       V2phi2H3b           Terms in output =        116
                           Bytes used      =    1823344
```
standard way

which shows that the running time in the simplified way is less than two minutes while in the standard way it is more than one hour.

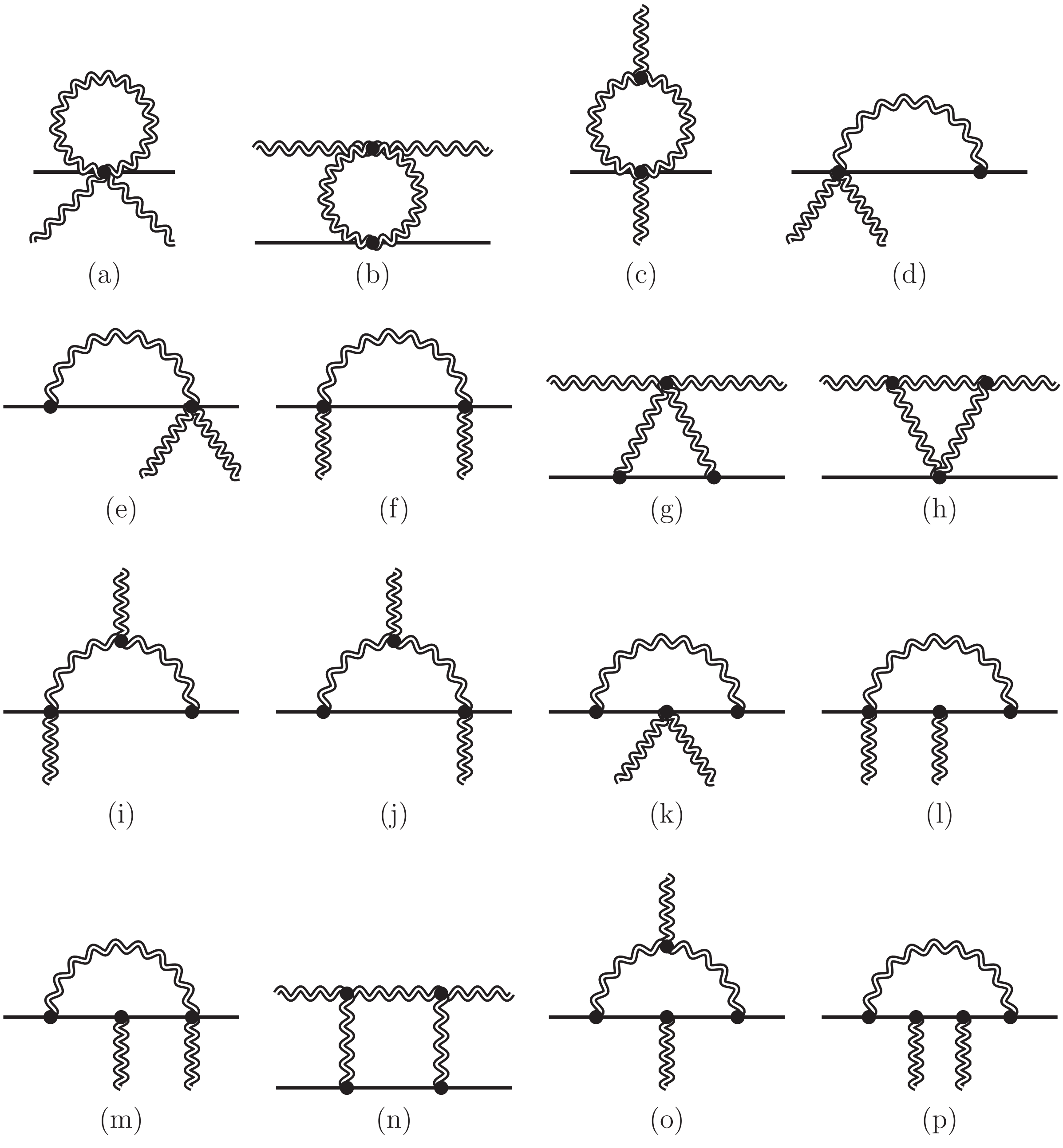


Figure 15: Scalar-Scalar-Graviton-Graviton vertex at one-loop level with: one (a), two (b-f), three (g-m), four (n-p) propagators.

Finally, we list, in Tab. 3, the number of Lagrangian terms for all one-loop corrections to the triple graviton vertex in the standard and simplified ways.

Table 3: The number of Lagrangian terms when using the standard rules and the simplified ones for calculating the one-loop corrections to the scalar-scalar-graviton-graviton vertex as shown in Fig. 15.

Diagram	The standard way	The simplified way
(a)	The amplitude vanishes[1]	The amplitude vanishes[1]
(b)	678	24
(c)	400	28
(d)	30	21
(e)	30	21
(f)	36	4
(g)	1 017	108
(h)	9 600	32
(i)	720	24
(j)	720	24
(k)	54	18
(l)	54	18
(m)	54	18
(n)	14 400	144
(o)	1 080	108
(p)	81	81

[1] all massless tad-pole diagrams vanish according to the relation Eq. (5.4).

6 Conclusions

In this work we have shown how it is possible to simplify the Feynman rules for gravity by using the freedom of choosing the gauge, adding total derivative terms and redefining the fields, which can change the form of Lagrangian without changing the information that it contains. In particular, the triple graviton and quadruple graviton vertices were reduced from 40 to 4 terms and from 113 to 12 terms respectively.

In order to check our simplified Feynman rules, we have compared the resulting amplitudes from the simplified rules with the resulting amplitudes from the standard rules for scalar-graviton and graviton-graviton scattering at tree level, and we indeed found that the resulting amplitudes are in agreement. In addition, our results at tree level also agree with the results of M. T. Grisaru, P. Van Nieuwenhuizen and C. C. Wu [7] as well as J. F. Donoghue and T. Torma [8].

Besides tree level amplitudes, we have also calculated some one-loop diagrams for scalar-graviton scattering, and we have shown how the calculations become simpler by using the simplified Feynman rules. In particular, for those diagrams that have triple or quadruple graviton vertices. Moreover, we have shown how the running time in the FORM program can be considerably reduced, up to 40 times faster for some diagrams, by using the simplified Feynman rules to calculate the amplitudes.

These simplified rules can also be used to simplify the calculations of more complicated diagrams such as the quadruple graviton vertex at one-loop level in graviton-graviton scattering with four propagators. In the latter example, the number of Lagrangian terms involved from the vertices in the standard way is 2 560 000 terms, compared to 256 terms in the simplified way. In addition, our aim was to simplify the lowest order vertices. However, this technique can also be used to simplify higher order vertices. Finally, this thesis may open the door to finding more freedoms and tools to further manipulate the Lagrangian in order to reach even simpler Feynman rules for gravity.

A Parameters

Table 4-10 in this appendix shows the values of the parameters that were chosen to obtain the simplified Feynman rules as shown in Sec. 3.

Table 4: The parameters of the total derivative Lagrangian for the gravitational field Eq. (2.43) that remove all second order derivative terms for the propagator S_{hh} and the vertices V_{hhh}, V_{hhhh}.

Propagator/Vertex	Parameter	Value	Parameter	Value	Parameter	Value
	a_1	-2	a_2	2		
S_{hh}	a_3	-1	a_4	2	a_5	1
	a_6	-1	a_7	-3	a_8	2
V_{hhh}	a_9	-1/4	a_{10}	1/2	a_{11}	1
	a_{12}	-2	a_{13}	1/4	a_{14}	-1/2
	a_{15}	-1/2	a_{16}	1	a_{17}	-3/2
	a_{18}	3	a_{19}	1	a_{20}	-2
	a_{21}	-2	a_{22}	2		
V_{hhhh}	a_{23}	-1/24	a_{24}	1/4	a_{25}	-1/3
	a_{26}	1/24	a_{27}	-1/4	a_{28}	1/3
	a_{29}	1/4	a_{30}	-1/2	a_{31}	-1/8
	a_{32}	1/4	a_{33}	1/4	a_{34}	-1/2
	a_{35}	-1	a_{36}	2	a_{37}	-3/8
	a_{38}	3/4	a_{39}	1	a_{40}	-2
	a_{41}	-1	a_{42}	2	a_{43}	3/2
	a_{44}	-2	a_{45}	-3	a_{46}	2
	a_{47}	1/2	a_{48}	-1	a_{49}	-1
	a_{50}	2				

Table 5: The parameters of the gravitational field redefinition Eq. (2.50) that reduce the number of Lagrangian terms for the vertices V_{hhh} and V_{hhhh}.

Vertex	Parameter	Value	Parameter	Value	Parameter	Value
V_{hhh}	c_1	1/2	c_2	-1/4		
V_{hhhh}	c_3	3/32	c_4	0	c_5	-1/8
	c_6	1/4				

Table 6: The parameters of the gauge condition Eq. (2.37), where the values of b_1, b_2 ensure the same de Donder propagator S_{hh} Eq. (B.5) as in the standard gauge Eq. (2.36), and the other b's parameters reduce the number of Largrangian terms for the vertices V_{hhh} and V_{hhhh}.

Propagator/Vertex	Parameter	Value	Parameter	Value	Parameter	Value
S_{hh}	b_1	1	b_2	-1/2		
V_{hhh}	b_3	-1/8	b_4	1/2	b_5	1/4
	b_6	-1/2	b_7	-1/2	b_8	1/2
V_{hhhh}	b_9	-1/64	b_{10}	1/16	b_{11}	1/8
	b_{12}	-1/2	b_{13}	1/32	b_{14}	-1/8
	b_{15}	-1/8	b_{16}	1/4	b_{17}	-1/8
	b_{18}	1/4	b_{19}	3/8	b_{20}	-1/4
	b_{21}	1/8	b_{22}	-1/4		

Table 7: The parameters of the total derivative Lagrangian for the scalar field Eq. (2.44) that remove all second order derivative terms for the propagator $S_{\phi\phi}$ and the vertices $V_{\phi\phi h}$, $V_{\phi\phi hh}$, $V_{\phi\phi hhh}$.

Propagator/Vertex	Parameter	Value
$S_{\phi\phi}$	d_1	0
$V_{\phi\phi h}$	$d_2, \cdots, d_5$	0
$V_{\phi\phi hh}$	$d_6, \cdots, d_{14}$	0
$V_{\phi\phi hhh}$	$d_{15}, \cdots, d_{22}$	0

Table 8: The parameters of the scalar field redefinition Eq. (2.51) that reduce the number of Lagrangian terms for the vertices $V_{\phi\phi h}$, $V_{\phi\phi hh}$ and $V_{\phi\phi hhh}$.

Vertex	Parameter	Value	Parameter	Value	Parameter	Value
$V_{\phi\phi h}$	e_1	0				
$V_{\phi\phi hh}$	e_2	0	e_3	0		
$V_{\phi\phi hhh}$	e_4	-1/384	e_5	0	e_6	-1/48

Table 9: The parameters of the total derivative Lagrangian for the ghost and antighost fields Eq. (2.45) that remove all second order derivative terms for the propagator $S_{\bar{\chi}\chi}$ and the vertices $V_{\bar{\chi}\chi h}$, $V_{\bar{\chi}\chi hh}$, $V_{\bar{\chi}\chi hhh}$.

Propagator/Vertex	Parameter	Value	Parameter	Value	Parameter	Value
$S_{\bar{\chi}\chi}$	h_1	-1				
$V_{\bar{\chi}\chi h}$	$h_2, \cdots, h_{10}$	0	h_{11}	1/2	h_{12}	-1/2
	h_{13}	-1/2	h_{14}	-1/4	h_{15}	-1/2
$V_{\bar{\chi}\chi hh}$	h_{20}	0	h_{21}	-1/8	h_{22}	1/4
	h_{23}	-1/32	h_{24}	1/8	h_{25}	-1/8
	h_{26}	1/8	h_{27}	1/8	h_{28}	-1/4
	h_{29}	0	h_{30}	0	h_{31}	-1/8
	h_{32}	1/8	h_{33}	-1/2	h_{34}	1/4
	h_{35}	1/4	h_{36}	0		

Table 10: The parameters of the ghost and antighost field redefinition Eqs. (2.52, 2.53) that reduce the number of Lagrangian terms for the vertices $V_{\bar{\chi}\chi h}$ and $V_{\bar{\chi}\chi hh}$.

Vertex	Parameter	Value	Parameter	Value	Parameter	Value
$V_{\bar{\chi}\chi h}$	f_1	1/4	f_2	0	g_1, g_2	0
$V_{\bar{\chi}\chi hh}$	f_3	-1/32	f_4	1/8	f_5	1/8
	f_6	-1/8	$g_3, \cdots, g_6$	0		

B Standard Feynman Rules

For completeness we give here the standard Feynman rules that we derived using the weak gravitational field expansion Eq. (2.10) and de Donder gauge Eq. (2.36):

- The scalar propagator:

$$\mathcal{L}_{\phi\phi} = \frac{1}{2}\Big(\partial^\mu\phi\partial_\mu\phi - \phi^2 m^2\Big)\,. \tag{B.1}$$

- The scalar-scalar-graviton vertex:

$$\mathcal{L}_{\phi\phi h} = \frac{\kappa}{4}\Big(- \phi^2 h_\mu{}^\mu m^2 + \partial^\mu\phi\partial_\mu\phi h_\nu{}^\nu - 2\,\partial_\mu\phi\partial_\nu\phi h^{\mu\nu}\Big)\,. \tag{B.2}$$

- The scalar-scalar-graviton-graviton vertex:

$$\begin{aligned}\mathcal{L}_{\phi\phi hh} = \frac{\kappa^2}{8}\Big(&- \frac{1}{2}\phi^2 h_\mu{}^\mu h_\nu{}^\nu m^2 + \phi^2 h^{\mu\nu}h_{\mu\nu}m^2 + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi h_\nu{}^\nu h_\alpha{}^\alpha - \partial^\mu\phi\partial_\mu\phi h^{\nu\alpha}h_{\nu\alpha}\\ &- 2\,\partial_\mu\phi\partial_\nu\phi h^{\mu\nu}h_\alpha{}^\alpha + 4\,\partial^\mu\phi\partial_\nu\phi h_{\mu\alpha}h^{\nu\alpha}\Big)\,.\end{aligned} \tag{B.3}$$

- The scalar-scalar-graviton-graviton-graviton vertex:

$$\begin{aligned}\mathcal{L}_{\phi\phi hhh} = \frac{\kappa^3}{16}\Big(&- \frac{1}{6}h_\mu{}^\mu h_\nu{}^\nu h_\alpha{}^\alpha \phi^2 m^2 + \frac{1}{6}h_\mu{}^\mu h_\nu{}^\nu h_\alpha{}^\alpha \partial^\beta\phi\partial_\beta\phi - h_\mu{}^\mu h_\nu{}^\nu h^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\\ &+ h_\mu{}^\mu h^{\nu\alpha}h_{\nu\alpha}\phi^2 m^2 - h_\mu{}^\mu h^{\nu\alpha}h_{\nu\alpha}\partial^\beta\phi\partial_\beta\phi + 4\,h_\mu{}^\mu h^{\nu\alpha}h_{\alpha\beta}\partial_\nu\phi\partial^\beta\phi\\ &+ 2\,h^{\mu\nu}h_{\mu\nu}h^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \frac{4}{3}h^{\mu\nu}h_\mu{}^\alpha h_{\nu\alpha}\phi^2 m^2 + \frac{4}{3}h^{\mu\nu}h_\mu{}^\alpha h_{\nu\alpha}\partial^\beta\phi\partial_\beta\phi\\ &- 8\,h^{\mu\nu}h_{\nu\alpha}h^{\alpha\beta}\partial_\mu\phi\partial_\beta\phi\Big)\,.\end{aligned} \tag{B.4}$$

- The graviton propagator:

$$\mathcal{L}_{hh} = \frac{1}{2}\Big(- \frac{1}{2}\partial^\mu h_\nu{}^\nu \partial_\mu h_\alpha{}^\alpha + \partial^\mu h^{\nu\alpha}\partial_\mu h_{\nu\alpha}\Big)\,. \tag{B.5}$$

- The triple graviton vertex:

$$\begin{aligned}\mathcal{L}_{hhh} = \kappa\Big(&- \frac{1}{4}h_\mu{}^\mu h_\nu{}^\nu \partial^\alpha\partial^\beta h_{\alpha\beta} + \frac{1}{4}h_\mu{}^\mu h_\nu{}^\nu \partial^\alpha\partial_\alpha h_\beta{}^\beta - h_\mu{}^\mu \partial^\nu h_{\nu\alpha}\partial^\alpha h_\beta{}^\beta + h_\mu{}^\mu \partial_\nu h^{\nu\alpha}\partial^\beta h_{\alpha\beta}\\ &- h_\mu{}^\mu \partial^\nu\partial_\nu h^{\alpha\beta}h_{\alpha\beta} - h_\mu{}^\mu h^{\nu\alpha}\partial_\nu\partial_\alpha h_\beta{}^\beta + h_\mu{}^\mu h^{\nu\alpha}\partial_\nu\partial^\beta h_{\alpha\beta} + h_\mu{}^\mu h^{\nu\alpha}\partial_\alpha\partial^\beta h_{\nu\beta}\\ &+ \frac{1}{4}h_\mu{}^\mu \partial^\nu h_\alpha{}^\alpha \partial_\nu h_\beta{}^\beta - \frac{3}{4}h_\mu{}^\mu \partial^\nu h^{\alpha\beta}\partial_\nu h_{\alpha\beta} + \frac{1}{2}h_\mu{}^\mu \partial^\nu h^{\alpha\beta}\partial_\alpha h_{\nu\beta} + \partial^\mu h_{\mu\nu}h^{\nu\alpha}\partial_\alpha h_\beta{}^\beta\end{aligned}$$

$$
\begin{aligned}
&-\frac{3}{2}\partial_\mu h^{\mu\nu}h_{\nu\alpha}\partial_\beta h^{\alpha\beta}+\partial_\mu h^{\mu\nu}\partial_\nu h_{\alpha\beta}h^{\alpha\beta}-2\partial_\mu h^{\mu\nu}\partial_\alpha h_{\nu\beta}h^{\alpha\beta}-\frac{1}{2}\partial^\mu\partial_\mu h_\nu{}^\nu h^{\alpha\beta}h_{\alpha\beta}\\
&+2\partial^\mu\partial_\mu h^{\nu\alpha}h_{\nu\beta}h_\alpha{}^\beta+\frac{1}{2}h^{\mu\nu}h_{\mu\nu}\partial_\alpha\partial_\beta h^{\alpha\beta}+h^{\mu\nu}\partial_\mu h_{\nu\alpha}\partial^\alpha h_\beta{}^\beta-h^{\mu\nu}\partial_\mu h_{\nu\alpha}\partial_\beta h^{\alpha\beta}\\
&+2h^{\mu\nu}\partial_\mu\partial_\nu h^{\alpha\beta}h_{\alpha\beta}+\frac{1}{2}h^{\mu\nu}h_{\mu\alpha}\partial_\nu\partial^\alpha h_\beta{}^\beta-h^{\mu\nu}h_{\mu\alpha}\partial_\nu\partial_\beta h^{\alpha\beta}-h^{\mu\nu}\partial_\mu\partial_\alpha h_{\nu\beta}h^{\alpha\beta}\\
&-\frac{1}{2}h^{\mu\nu}\partial_\mu h_\alpha{}^\alpha\partial_\nu h_\beta{}^\beta+\frac{1}{2}h^{\mu\nu}\partial_\mu h_\alpha{}^\alpha\partial^\beta h_{\nu\beta}-2h^{\mu\nu}\partial_\mu\partial_\alpha h^{\alpha\beta}h_{\nu\beta}+\frac{3}{2}h^{\mu\nu}\partial_\mu h^{\alpha\beta}\partial_\nu h_{\alpha\beta}\\
&-2h^{\mu\nu}\partial_\mu h^{\alpha\beta}\partial_\alpha h_{\nu\beta}+\frac{3}{2}h^{\mu\nu}\partial_\mu\partial^\alpha h_\beta{}^\beta h_{\nu\alpha}+h^{\mu\nu}\partial_\nu h_{\mu\alpha}\partial^\alpha h_\beta{}^\beta-h^{\mu\nu}\partial_\nu h_{\mu\alpha}\partial_\beta h^{\alpha\beta}\\
&-h^{\mu\nu}h_{\nu\alpha}\partial^\alpha\partial^\beta h_{\mu\beta}-h^{\mu\nu}\partial_\nu\partial_\alpha h_{\mu\beta}h^{\alpha\beta}+\frac{1}{2}h^{\mu\nu}\partial_\nu h_\alpha{}^\alpha\partial^\beta h_{\mu\beta}-h^{\mu\nu}\partial^\alpha h_{\mu\nu}\partial_\alpha h_\beta{}^\beta\\
&+h^{\mu\nu}\partial_\alpha h_{\mu\nu}\partial_\beta h^{\alpha\beta}-\frac{1}{2}h^{\mu\nu}\partial^\alpha h_{\mu\alpha}\partial^\beta h_{\nu\beta}+3h^{\mu\nu}\partial^\alpha h_\mu{}^\beta\partial_\alpha h_{\nu\beta}-h^{\mu\nu}\partial^\alpha h_{\mu\beta}\partial^\beta h_{\nu\alpha}\Big)\,.
\end{aligned}
\tag{B.6}
$$

- The quadruple graviton vertex:

$$
\begin{aligned}
\mathcal{L}_{hhhh} = \kappa^2\Big(&-2\,h_{\mu\mu}h_{\nu\nu}h_{\alpha\alpha}\partial_\beta\partial_\gamma h_{\beta\gamma}+2\,h_{\mu\mu}h_{\nu\nu}h_{\alpha\alpha}\partial_\beta\partial_\beta h_{\gamma\gamma}-\frac{1}{4}h_{\mu\mu}h_{\nu\nu}h_{\alpha\beta}\partial_\alpha\partial_\beta h_{\gamma\gamma}\\
&+\frac{1}{4}h_{\mu\mu}h_{\nu\nu}h_{\alpha\beta}\partial_\alpha\partial_\gamma h_{\beta\gamma}+\frac{1}{4}h_{\mu\mu}h_{\nu\nu}h_{\alpha\beta}\partial_\beta\partial_\gamma h_{\alpha\gamma}-\frac{1}{4}h_{\mu\mu}h_{\nu\nu}\partial_\alpha h_{\alpha\beta}\partial_\beta h_{\gamma\gamma}\\
&+\frac{1}{4}h_{\mu\mu}h_{\nu\nu}\partial_\alpha h_{\alpha\beta}\partial_\gamma h_{\beta\gamma}+\frac{1}{16}h_{\mu\mu}h_{\nu\nu}\partial_\alpha h_{\beta\beta}\partial_\alpha h_{\gamma\gamma}-\frac{3}{16}h_{\mu\mu}h_{\nu\nu}\partial_\alpha h_{\beta\gamma}\partial_\alpha h_{\beta\gamma}\\
&+\frac{1}{8}h_{\mu\mu}h_{\nu\nu}\partial_\alpha h_{\beta\gamma}\partial_\beta h_{\alpha\gamma}-\frac{1}{4}h_{\mu\mu}h_{\nu\nu}\partial_\alpha\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}+\frac{1}{4}h_{\mu\mu}h_{\nu\alpha}h_{\nu\alpha}\partial_\beta\partial_\gamma h_{\beta\gamma}\\
&+\frac{1}{4}h_{\mu\mu}h_{\nu\alpha}h_{\nu\beta}\partial_\alpha\partial_\beta h_{\gamma\gamma}-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}h_{\nu\beta}\partial_\alpha\partial_\gamma h_{\beta\gamma}-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}h_{\alpha\beta}\partial_\beta\partial_\gamma h_{\nu\gamma}\\
&+\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\nu h_{\alpha\beta}\partial_\beta h_{\gamma\gamma}-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\nu h_{\alpha\beta}\partial_\gamma h_{\beta\gamma}-\frac{1}{4}h_{\mu\mu}h_{\nu\alpha}\partial_\nu h_{\beta\beta}\partial_\alpha h_{\gamma\gamma}\\
&+\frac{1}{4}h_{\mu\mu}h_{\nu\alpha}\partial_\nu h_{\beta\beta}\partial_\gamma h_{\alpha\gamma}+\frac{3}{4}h_{\mu\mu}h_{\nu\alpha}\partial_\nu h_{\beta\gamma}\partial_\alpha h_{\beta\gamma}-h_{\mu\mu}h_{\nu\alpha}\partial_\nu h_{\beta\gamma}\partial_\beta h_{\alpha\gamma}\\
&+h_{\mu\mu}h_{\nu\alpha}\partial_\nu\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\nu\partial_\beta h_{\alpha\gamma}h_{\beta\gamma}-h_{\mu\mu}h_{\nu\alpha}\partial_\nu\partial_\beta h_{\beta\gamma}h_{\alpha\gamma}\\
&+\frac{3}{4}h_{\mu\mu}h_{\nu\alpha}\partial_\nu\partial_\beta h_{\gamma\gamma}h_{\alpha\beta}+\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\alpha h_{\nu\beta}\partial_\beta h_{\gamma\gamma}-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\alpha h_{\nu\beta}\partial_\gamma h_{\beta\gamma}\\
&+\frac{1}{4}h_{\mu\mu}h_{\nu\alpha}\partial_\alpha h_{\beta\beta}\partial_\gamma h_{\nu\gamma}-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\alpha\partial_\beta h_{\nu\gamma}h_{\beta\gamma}-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\beta h_{\nu\alpha}\partial_\beta h_{\gamma\gamma}\\
&+\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\beta h_{\nu\alpha}\partial_\gamma h_{\beta\gamma}-\frac{1}{4}h_{\mu\mu}h_{\nu\alpha}\partial_\beta h_{\nu\beta}\partial_\gamma h_{\alpha\gamma}+\frac{3}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\beta h_{\nu\gamma}\partial_\beta h_{\alpha\gamma}\\
&-\frac{1}{2}h_{\mu\mu}h_{\nu\alpha}\partial_\beta h_{\nu\gamma}\partial_\gamma h_{\alpha\beta}+\frac{1}{2}h_{\mu\mu}\partial_\nu h_{\nu\alpha}h_{\alpha\beta}\partial_\beta h_{\gamma\gamma}-\frac{3}{4}h_{\mu\mu}\partial_\nu h_{\nu\alpha}h_{\alpha\beta}\partial_\gamma h_{\beta\gamma}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{2}h_{\mu\mu}\partial_\nu h_{\nu\alpha}\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}-h_{\mu\mu}\partial_\nu h_{\nu\alpha}\partial_\beta h_{\alpha\gamma}h_{\beta\gamma}-\frac{1}{4}h_{\mu\mu}\partial_\nu\partial_\nu h_{\alpha\alpha}h_{\beta\gamma}h_{\beta\gamma}\\
&+h_{\mu\mu}\partial_\nu\partial_\nu h_{\alpha\beta}h_{\alpha\gamma}h_{\beta\gamma}+\frac{1}{2}h_{\mu\nu}h_{\mu\nu}h_{\alpha\beta}\partial_\alpha\partial_\beta h_{\gamma\gamma}-\frac{1}{2}h_{\mu\nu}h_{\mu\nu}h_{\alpha\beta}\partial_\alpha\partial_\gamma h_{\beta\gamma}\\
&-\frac{1}{2}h_{\mu\nu}h_{\mu\nu}h_{\alpha\beta}\partial_\beta\partial_\gamma h_{\alpha\gamma}-\frac{1}{8}h_{\mu\nu}h_{\mu\nu}\partial_\alpha h_{\beta\beta}\partial_\alpha h_{\gamma\gamma}+\frac{3}{8}h_{\mu\nu}h_{\mu\nu}\partial_\alpha h_{\beta\gamma}\partial_\alpha h_{\beta\gamma}\\
&-\frac{1}{4}h_{\mu\nu}h_{\mu\nu}\partial_\alpha h_{\beta\gamma}\partial_\beta h_{\alpha\gamma}-\frac{1}{3}h_{\mu\nu}h_{\mu\alpha}h_{\nu\alpha}\partial_\beta\partial_\gamma h_{\beta\gamma}-h_{\mu\nu}h_{\mu\alpha}h_{\nu\beta}\partial_\alpha\partial_\beta h_{\gamma\gamma}\\
&+\frac{3}{4}h_{\mu\nu}h_{\mu\alpha}h_{\nu\beta}\partial_\alpha\partial_\gamma h_{\beta\gamma}+\frac{1}{4}h_{\mu\nu}h_{\mu\alpha}h_{\nu\beta}\partial_\alpha\partial_\gamma h_{\gamma\beta}+h_{\mu\nu}h_{\mu\alpha}h_{\nu\beta}\partial_\beta\partial_\gamma h_{\alpha\gamma}\\
&-\frac{1}{2}h_{\mu\nu}h_{\mu\alpha}\partial_\nu h_{\alpha\beta}\partial_\beta h_{\gamma\gamma}+\frac{1}{8}h_{\mu\nu}h_{\mu\alpha}\partial_\nu h_{\beta\beta}\partial_\alpha h_{\gamma\gamma}-\frac{3}{2}h_{\mu\nu}h_{\mu\alpha}\partial_\nu h_{\beta\gamma}\partial_\alpha h_{\beta\gamma}\\
&+\frac{1}{4}h_{\mu\nu}h_{\mu\alpha}\partial_\nu h_{\beta\gamma}\partial_\beta h_{\alpha\gamma}+\frac{1}{4}h_{\mu\nu}h_{\mu\alpha}\partial_\nu h_{\beta\gamma}\partial_\gamma h_{\alpha\beta}-\frac{1}{2}h_{\mu\nu}h_{\mu\alpha}\partial_\nu\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}\\
&+h_{\mu\nu}h_{\mu\alpha}\partial_\nu\partial_\beta h_{\alpha\gamma}h_{\beta\gamma}+\frac{1}{4}h_{\mu\nu}h_{\mu\alpha}\partial_\beta h_{\nu\alpha}\partial_\beta h_{\gamma\gamma}+\frac{1}{8}h_{\mu\nu}h_{\mu\alpha}\partial_\beta h_{\nu\beta}\partial_\gamma h_{\alpha\gamma}\\
&-3h_{\mu\nu}h_{\mu\alpha}\partial_\beta h_{\nu\gamma}\partial_\beta h_{\alpha\gamma}+\frac{1}{4}h_{\mu\nu}h_{\mu\alpha}\partial_\beta h_{\nu\gamma}\partial_\gamma h_{\alpha\beta}+\frac{3}{4}h_{\mu\nu}h_{\nu\alpha}h_{\alpha\beta}\partial_\beta\partial_\gamma h_{\mu\gamma}\\
&-\frac{1}{2}h_{\mu\nu}h_{\nu\alpha}\partial_\alpha h_{\mu\beta}\partial_\beta h_{\gamma\gamma}+\frac{1}{4}h_{\mu\nu}h_{\nu\alpha}\partial_\alpha h_{\beta\gamma}\partial_\beta h_{\mu\gamma}+\frac{1}{4}h_{\mu\nu}h_{\nu\alpha}\partial_\alpha h_{\beta\gamma}\partial_\gamma h_{\mu\beta}\\
&+\frac{3}{4}h_{\mu\nu}h_{\nu\alpha}\partial_\alpha\partial_\beta h_{\mu\gamma}h_{\beta\gamma}+\frac{3}{4}h_{\mu\nu}h_{\nu\alpha}\partial_\beta h_{\mu\alpha}\partial_\beta h_{\gamma\gamma}+\frac{3}{8}h_{\mu\nu}h_{\nu\alpha}\partial_\beta h_{\mu\beta}\partial_\gamma h_{\alpha\gamma}\\
&+\frac{3}{4}h_{\mu\nu}h_{\nu\alpha}\partial_\beta h_{\mu\gamma}\partial_\gamma h_{\alpha\beta}-\frac{5}{4}h_{\mu\nu}\partial_\mu h_{\nu\alpha}h_{\alpha\beta}\partial_\beta h_{\gamma\gamma}+h_{\mu\nu}\partial_\mu h_{\nu\alpha}h_{\alpha\beta}\partial_\gamma h_{\beta\gamma}\\
&-h_{\mu\nu}\partial_\mu h_{\nu\alpha}\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}+2h_{\mu\nu}\partial_\mu h_{\nu\alpha}\partial_\beta h_{\alpha\gamma}h_{\beta\gamma}+\frac{3}{8}h_{\mu\nu}\partial_\mu h_{\alpha\alpha}h_{\nu\beta}\partial_\beta h_{\gamma\gamma}\\
&+\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\alpha}\partial_\nu h_{\beta\gamma}h_{\beta\gamma}-\frac{1}{4}h_{\mu\nu}\partial_\mu h_{\alpha\alpha}\partial_\beta h_{\nu\gamma}h_{\beta\gamma}-\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}h_{\nu\alpha}\partial_\beta h_{\gamma\gamma}\\
&-\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}h_{\nu\beta}\partial_\alpha h_{\gamma\gamma}+\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}h_{\nu\gamma}\partial_\alpha h_{\beta\gamma}+\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}h_{\nu\gamma}\partial_\beta h_{\alpha\gamma}\\
&+\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}h_{\alpha\gamma}\partial_\beta h_{\nu\gamma}+h_{\mu\nu}\partial_\mu h_{\alpha\beta}h_{\alpha\gamma}\partial_\gamma h_{\nu\beta}-\frac{3}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}\partial_\nu h_{\alpha\gamma}h_{\beta\gamma}\\
&-\frac{3}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}\partial_\nu h_{\beta\gamma}h_{\alpha\gamma}+\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}\partial_\nu h_{\gamma\gamma}h_{\alpha\beta}+\frac{1}{2}h_{\mu\nu}\partial_\mu h_{\alpha\beta}\partial_\alpha h_{\nu\gamma}h_{\beta\gamma}\\
&-2h_{\mu\nu}\partial_\mu\partial_\nu h_{\alpha\beta}h_{\alpha\gamma}h_{\beta\gamma}+\frac{3}{4}h_{\mu\nu}\partial_\mu\partial_\alpha h_{\nu\beta}h_{\alpha\gamma}h_{\beta\gamma}+\frac{5}{4}h_{\mu\nu}\partial_\mu\partial_\alpha h_{\alpha\beta}h_{\nu\gamma}h_{\beta\gamma}\\
&-h_{\mu\nu}\partial_\mu\partial_\alpha h_{\beta\beta}h_{\nu\gamma}h_{\alpha\gamma}-\frac{3}{2}h_{\mu\nu}\partial_\mu\partial_\alpha h_{\beta\gamma}h_{\nu\alpha}h_{\beta\gamma}+\frac{5}{8}h_{\mu\nu}\partial_\mu\partial_\alpha h_{\beta\gamma}h_{\nu\beta}h_{\alpha\gamma}\\
&+\frac{5}{8}h_{\mu\nu}\partial_\mu\partial_\alpha h_{\beta\gamma}h_{\nu\gamma}h_{\alpha\beta}-\frac{1}{2}h_{\mu\nu}\partial_\nu h_{\mu\alpha}h_{\alpha\beta}\partial_\beta h_{\gamma\gamma}+h_{\mu\nu}\partial_\nu h_{\mu\alpha}h_{\alpha\beta}\partial_\gamma h_{\beta\gamma}\\
&-h_{\mu\nu}\partial_\nu h_{\mu\alpha}\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}+\frac{1}{2}h_{\mu\nu}\partial_\nu h_{\alpha\beta}h_{\alpha\gamma}\partial_\beta h_{\mu\gamma}+\frac{1}{2}h_{\mu\nu}\partial_\nu h_{\alpha\beta}\partial_\alpha h_{\mu\gamma}h_{\beta\gamma}\\
&+\frac{1}{4}h_{\mu\nu}\partial_\nu\partial_\alpha h_{\mu\beta}h_{\alpha\gamma}h_{\beta\gamma}+\frac{1}{2}h_{\mu\nu}\partial_\alpha h_{\mu\nu}\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}-\frac{3}{2}h_{\mu\nu}\partial_\alpha h_{\mu\beta}\partial_\alpha h_{\nu\gamma}h_{\beta\gamma}
\end{aligned}
$$

$$
\begin{aligned}
&-2\partial_\mu h_{\mu\nu}h_{\nu\alpha}h_{\alpha\beta}\partial_\beta h_{\gamma\gamma}+\frac{3}{2}\partial_\mu h_{\mu\nu}h_{\nu\alpha}h_{\alpha\beta}\partial_\gamma h_{\beta\gamma}-2\partial_\mu h_{\mu\nu}h_{\nu\alpha}\partial_\alpha h_{\beta\gamma}h_{\beta\gamma}\\
&+2\partial_\mu h_{\mu\nu}h_{\nu\alpha}\partial_\beta h_{\alpha\gamma}h_{\beta\gamma}+\frac{1}{2}\partial_\mu h_{\mu\nu}\partial_\nu h_{\alpha\alpha}h_{\beta\gamma}h_{\beta\gamma}-2\partial_\mu h_{\mu\nu}\partial_\nu h_{\alpha\beta}h_{\alpha\gamma}h_{\beta\gamma}\\
&-\frac{1}{2}\partial_\mu h_{\mu\nu}\partial_\alpha h_{\nu\alpha}h_{\beta\gamma}h_{\beta\gamma}+4\partial_\mu h_{\mu\nu}\partial_\alpha h_{\nu\beta}h_{\alpha\gamma}h_{\beta\gamma}+\frac{1}{3}\partial_\mu\partial_\mu h_{\nu\nu}h_{\alpha\beta}h_{\alpha\gamma}h_{\beta\gamma}\\
&+\frac{1}{2}\partial_\mu\partial_\mu h_{\nu\alpha}h_{\nu\alpha}h_{\beta\gamma}h_{\beta\gamma}-2\partial_\mu\partial_\mu h_{\nu\alpha}h_{\nu\beta}h_{\alpha\gamma}h_{\beta\gamma}\Big)\,. \qquad \text{(B.7)}
\end{aligned}
$$

- The ghost propagator:

$$
\mathcal{L}_{\bar\chi\chi}=\bar\chi^\mu\partial^\nu\partial_\nu\chi_\mu\,. \qquad \text{(B.8)}
$$

- The ghost-ghost-graviton vertex:

$$
\begin{aligned}
\mathcal{L}_{\bar\chi\chi h}=\ &\kappa\Big(-\frac{1}{2}\bar\chi^\mu\partial_\mu\chi^\nu\partial_\nu h_\alpha{}^\alpha+\bar\chi^\mu\partial_\mu\chi^\nu\partial^\alpha h_{\nu\alpha}-\frac{1}{2}\bar\chi^\mu\chi^\nu\partial_\mu\partial_\nu h_\alpha{}^\alpha+\bar\chi^\mu\chi^\nu\partial_\nu\partial^\alpha h_{\mu\alpha}\\
&+\bar\chi^\mu\partial^\nu\partial_\nu\chi^\alpha h_{\mu\alpha}-\bar\chi^\mu\partial^\nu\chi^\alpha\partial_\mu h_{\nu\alpha}+\bar\chi^\mu\partial^\nu\chi^\alpha\partial_\nu h_{\mu\alpha}+\bar\chi^\mu\partial^\nu\chi^\alpha\partial_\alpha h_{\mu\nu}\Big)\,.
\end{aligned}
\qquad \text{(B.9)}
$$

- The ghost-ghost-graviton-graviton vertex vanishes.

C Kinematics

C.1 Scalar-Graviton Scattering ($\phi(p_1)h_{\mu\nu}(p_2) \to \phi(p_3)h_{\alpha\beta}(p_4)$):

There are four diagrams for this process as shown in Fig. 4, three of them (a, b, c) representing s, t and u-channels respectively. According to the conventions that we use in this thesis, the Mandelstam variables are given by

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \,, \\ t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \,, \\ u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 \,, \\ s + t + u &= \sum_i M_i^2 = 2m^2 \,, \end{aligned} \tag{C.1}$$

where m is the mass of scalar field ϕ.

In the CM frame, with the incoming particles along the z-axis, the momenta and the polarization vector can be chosen as:

$$\begin{aligned} p_1 &= (E, 0, 0, -k) \,, \\ p_2 &= (k, 0, 0, k) \,, \\ \epsilon_\mu^{\pm 1}(p_2) &= \left(0, \tfrac{1}{\sqrt{2}}, \pm\tfrac{i}{\sqrt{2}}, 0\right) \,. \end{aligned} \tag{C.2}$$

Choosing the outgoing particles to be in the yz-plane, the momenta and the complex conjugate polarization vector are:

$$\begin{aligned} p_3 &= (E, 0, -k\sin(\theta), -k\cos(\theta)) \,, \\ p_4 &= (k, 0, k\sin(\theta), k\cos(\theta)) \,, \\ \epsilon_\alpha^{*\pm 1}(p_4) &= \left(0, \frac{1}{\sqrt{2}}, \frac{\mp i\cos(\theta)}{\sqrt{2}}, \frac{\pm i\sin(\theta)}{\sqrt{2}}\right) \,, \end{aligned} \tag{C.3}$$

where θ is the scattering angle and $\epsilon_\alpha^{\pm 1}(p_4) = R_\alpha{}^\mu(\hat{p}_4)\, \epsilon_\mu^{\pm 1}(p_2)$ where $R_\alpha{}^\mu(\hat{p}_4)$ is a rotation. It is also possible to write the graviton polarization directly as a tensor:

$$\epsilon_{\mu\nu}^{\pm 2}(p_2) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \pm\frac{i}{2} & 0 \\ 0 & \pm\frac{i}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \,, \tag{C.4}$$

$$\epsilon_{\alpha\beta}^{*\pm 2}(p_4) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \mp\frac{i}{2}\cos(\theta) & \pm\frac{i}{2}\sin(\theta) \\ 0 & \mp\frac{i}{2}\cos(\theta) & -\frac{1}{2}\cos^2(\theta) & \frac{1}{2}\sin(\theta)\cos(\theta) \\ 0 & \pm\frac{i}{2}\sin(\theta) & \frac{1}{2}\sin(\theta)\cos(\theta) & -\frac{1}{2}\sin^2(\theta) \end{pmatrix} \,, \tag{C.5}$$

where $\epsilon^{\pm 2}_{\alpha\beta}(p_4) = R_\alpha{}^\mu(\hat{p}_4)\, R_\beta{}^\nu(\hat{p}_4)\, \epsilon^{\pm 2}_{\mu\nu}(p_2)$ where again $R_\alpha{}^\mu(\hat{p}_4)$, $R_\beta{}^\nu(\hat{p}_4)$ are rotation matrices. Finally, the following useful relations are valid in the CM frame:

$$p_1^\mu \epsilon_\mu^{\pm 1}(p_2) = 0\,, \tag{C.6}$$
$$p_3^\mu \epsilon_\mu^{\pm 1}(p_4) = 0\,. \tag{C.7}$$

C.2 Graviton-Graviton Scattering ($h^{\mu\nu}(p_1)h^{\alpha\beta}(p_2) \to h^{\gamma\delta}(p_3)h^{\lambda\rho}(p_4)$):

There are also four diagrams for this process as shown in Fig. 5, but the relation between Mandelstam variables for this process is now

$$s + t + u = 0\,. \tag{C.8}$$

In the CM frame, with the incoming particles, along the z-axis, the momenta and the polarization vectors can be chosen as:

$$\begin{aligned}
p_1 &= (k, 0, 0, k)\,,\\
p_2 &= (k, 0, 0, -k)\,,\\
\epsilon_\mu^{\pm 1}(p_1) &= \left(0, \tfrac{1}{\sqrt{2}}, \pm\tfrac{i}{\sqrt{2}}, 0\right)\,,\\
\epsilon_\alpha^{\pm 1}(p_2) &= \left(0, \tfrac{1}{\sqrt{2}}, \mp\tfrac{i}{\sqrt{2}}, 0\right)\,.
\end{aligned} \tag{C.9}$$

Choosing the outgoing particles to be in the yz-plane, the momenta and the complex conjugate polarization vectors are:

$$\begin{aligned}
p_3 &= (k, 0, k\sin(\theta), k\cos(\theta))\,,\\
p_4 &= (k, 0, -k\sin(\theta), -k\cos(\theta))\,,\\
\epsilon_\gamma^{*\pm 1}(p_3) &= \left(0, \frac{1}{\sqrt{2}}, \frac{\mp i\cos(\theta)}{\sqrt{2}}, \frac{\pm i\sin(\theta)}{\sqrt{2}}\right)\,,\\
\epsilon_\lambda^{*\pm 1}(p_4) &= \left(0, \frac{1}{\sqrt{2}}, \frac{\pm i\cos(\theta)}{\sqrt{2}}, \frac{\mp i\sin(\theta)}{\sqrt{2}}\right)\,,
\end{aligned} \tag{C.10}$$

where θ is the scattering angle.
Finally, the following useful relations are valid in the CM frame:

$$\begin{aligned}
&p_1^\mu \epsilon_\mu^{\pm 1}(p_2) = 0\,,\\
&p_3^\mu \epsilon_\mu^{\pm 1}(p_4) = 0\,,\\
p_4^\mu \epsilon_\mu^{\pm 1}(p_1) = -p_3^\mu \epsilon_\mu^{\pm 1}(p_1) \quad &, \quad p_4^\mu \epsilon_\mu^{\pm 1}(p_2) = -p_3^\mu \epsilon_\mu^{\pm 1}(p_2)\,,\\
p_2^\mu \epsilon_\mu^{\pm 1}(p_3) = -p_1^\mu \epsilon_\mu^{\pm 1}(p_3) \quad &, \quad p_2^\mu \epsilon_\mu^{\pm 1}(p_4) = -p_1^\mu \epsilon_\mu^{\pm 1}(p_4)\,.
\end{aligned} \tag{C.11}$$

D Dimensional Regularization of Scalar Integrals

To illustrate how the scalar integrals are calculated in dimensional regularization, let us take as an example the scalar integral B_0 Eq. (5.2) and then follow the standard procedure of dimensional regularization [1]. First, we move the loop integral from Minkowski space to Euclidean space by replacing the zeroth component of momentum in Minkowski space k_0 by the imaginary fourth component in Euclidean space ik_4. As a result of this replacement $k_0 \to ik_4$:

$$k^2 = k_0^2 - \vec{k}^2 \quad \to \quad -k_E^2 = -k_4^2 - \vec{k}^2\,, \tag{D.1}$$

$$d^4k \quad \to \quad id^4k_E\,. \tag{D.2}$$

Then, the scalar integral B_0 for $m_0 = m_1 = m$ and $q_1 = p_1$ becomes

$$\begin{aligned} B_0(p_1, m, m) &= \int \frac{id^dk_E}{(2\pi)^d} \frac{1}{(-k_E^2 - m^2 + i\epsilon)(-(k+p_1)_E^2 - m^2 + i\epsilon)} \\ &= \int \frac{id^dk_E}{(2\pi)^d} \frac{1}{(k_E^2 + m^2 - i\epsilon)((k+p_1)_E^2 + m^2 - i\epsilon)}\,. \end{aligned} \tag{D.3}$$

Second, since the integration over the fourth component of the momentum goes along the imaginary axis ik_4, we need to perform a Wick rotation to move to the integration along the real axis, making sure that the contour does not cross the poles of $\frac{1}{(k_E^2+m^2-i\epsilon)((k+p_1)_E^2+m^2-i\epsilon)}$. Third, in order to transform the product of several brackets in the denominator into a single bracket, we use the Feynman parameterization

$$\frac{1}{A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}} = \frac{\Gamma(\alpha_1 + \alpha_2 + \cdots + \alpha_n)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\cdots\Gamma(\alpha_n)} \tag{D.4}$$

$$\int dx_1 dx_2 \cdots dx_n \frac{\delta(1 - x_1 - x_2 - \cdots - x_n) x_1^{\alpha_1-1} x_2^{\alpha_2-1} \cdots x_n^{\alpha_n-1}}{[A_1x_1 + A_2x_2 + \cdots + A_nx_n]^{\alpha_1+\alpha_2+\cdots+\alpha_n}}\,,$$

where Γ is the gamma function.
In our case, $\alpha_1 = \alpha_2 = 1, n = 2$, and the integral becomes

$$\begin{aligned} B_0 &= \frac{i}{(2\pi)^d} \int d^dk_E\, \frac{1}{k_E^2 + m^2 - i\epsilon}\, \frac{1}{(k+p_1)_E^2 + m^2 - i\epsilon} \\ &= \frac{i}{(2\pi)^d} \int d^dk_E\, \frac{\Gamma(2)}{\Gamma(1)\Gamma(1)} \int_0^1 \frac{dx_1 dx_2\, \delta(1 - x_1 - x_2)}{[[k_E^2 + m^2 - i\epsilon]x_1 + [(k+p_1)_E^2 + m^2 - i\epsilon]x_2]^2} \\ &= \frac{i}{(2\pi)^d} \int d^dk_E\, \int_0^1 \frac{dx}{[k_E^2 + 2(k\cdot p_1)_E\, x + p_{1E}^2\, x + m^2 - i\epsilon]^2}\,, \end{aligned} \tag{D.5}$$

where $\Gamma(1) = \Gamma(2) = 1$, and from the second line to the third we used the delta function to do the integration over x_1 and finally renamed x_2 to x.

Fourth, to complete the square of k_E in the denominator, we need to shift the integration variable $k_E \to k_E - p_{1E}\,x$ to absorb the term $(k \cdot p_1)_E$ and obtain

$$B_0 = \frac{i}{(2\pi)^d} \int_0^1 dx \int \frac{d^d k_E}{[k_E^2 + p_{1E}^2\,x(1-x) + m^2 - i\epsilon]^2} = \frac{i}{(2\pi)^d} \int_0^1 dx \int \frac{d^d k_E}{[k_E^2 + M^2 - i\epsilon]^2}\,,$$

where $M^2 = p_{1E}^2\,x(1-x) + m^2$.
Fifth, we perform the momentum integral in Euclidean space, using spherical coordinates as follows

$$\begin{aligned} B_0 &= \frac{i}{(2\pi)^d} \int_0^1 dx\ \Omega_d \int_0^\infty dk_E\ k_E\ \frac{(k_E^2)^{d/2-1}}{[k_E^2 + M^2]^2} \\ &= \frac{i}{(2\pi)^d} \int_0^1 dx\ \Omega_d \int_0^\infty \frac{dk_E^2}{2}\ \frac{(k_E^2)^{d/2-1}}{[k_E^2 + M^2]^2} \\ &= \frac{i}{(2\pi)^d} \int_0^1 dx\ \frac{\Omega_d}{2}\ (M^2)^{d/2-2} \int_0^\infty \frac{dx\ x^{d/2-1}}{[x+1]^2} \\ &= \frac{i}{(4\pi)^{d/2}} \int_0^1 dx\ (M^2)^{d/2-2}\ \Gamma(2-d/2)\,, \end{aligned} \tag{D.6}$$

where $\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$ follows from the angular integration, and from the second line to the third $k_E^2 \to k_E^2\,M^2$ is used, and then $k_E^2 \to x$. However, we note that the gamma function $\Gamma(2-d/2)$ still diverges in the last result if we are in 4-dimensions.
Sixth, we transform back into Minkowski space by replacing again $d = 4 - 2\epsilon$ and doing an expansion in ϵ using the following relation [1]:

$$\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} \left(\frac{1}{M^2}\right)^{2-d/2} = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} - \gamma + \log(4\pi) - \log\!\big(M^2\big) + \mathcal{O}(\epsilon)\right). \tag{D.7}$$

Finally, we get

$$B_0(p_1, m, m) = \frac{i}{16\pi^2}\left(\frac{1}{\epsilon} - \gamma + \log(4\pi) - \int_0^1 dx \log[m^2 - p_1^2 x(1-x)]\right), \tag{D.8}$$

where $M^2 = p_{1E}^2\,x(1-x) + m^2 = -p_1^2\,x(1-x) + m^2$. The integral still blows up for $\epsilon \to 0$ but now the divergent part is separated from the finite terms. Similarly, we can calculate the other scalar integrals.

E Passarino-Veltman Reduction of Tensor Integrals

In the Passarino-Veltman method, the tensorial integrals can be written in terms of scalar functions as listed below. However, we only include the cases that are relevant to the scalar-graviton scattering to one-loop order (i.e., A and D integrals up to four indices, B integrals up to five indices and C integrals up to six indices in the numerator). In addition, all scalar functions C, D are symmetric under i, j, k, l, m, n indices in our notation below (e.g., $C_{21} = C_{12}$), and $i\epsilon$ was dropped for simplicity:

$$A = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m_0^2)} = A_0(m_0)\,, \tag{E.1}$$

$$A^\mu = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu}{(k^2 - m_0^2)} = 0\,, \tag{E.2}$$

$$A^{\mu\nu} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - m_0^2)} = \eta^{\mu\nu} A_{00}(m_0)\,, \tag{E.3}$$

$$A^{\mu\nu\alpha} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu k^\alpha}{(k^2 - m_0^2)} = 0\,, \tag{E.4}$$

$$A^{\mu\nu\alpha\beta} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu k^\alpha k^\beta}{(k^2 - m_0^2)} = \left[\eta^{\mu\nu}\eta^{\alpha\beta} + \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}\right] A_{0000}(m_0)\,, \tag{E.5}$$

$$B = \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2 - m_0^2)((k+q_1)^2 - m_1^2)} = B_0(p_1, m_0, m_1)\,, \tag{E.6}$$

$$B^\mu = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu}{(k^2 - m_0^2)((k+q_1)^2 - m_1^2)} = p_1^\mu\, B_1(p_1, m_0, m_1)\,, \tag{E.7}$$

$$\begin{aligned} B^{\mu\nu} &= \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2 - m_0^2)((k+q_1)^2 - m_1^2)} \\ &= \left[p_1^\mu\, p_1^\nu\right] B_{11}(p_1, m_0, m_1) + \eta^{\mu\nu} B_{00}(p_1, m_0, m_1)\,, \end{aligned} \tag{E.8}$$

$$\begin{aligned} B^{\mu\nu\alpha} &= \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu k^\alpha}{(k^2 - m_0^2)((k+q_1)^2 - m_1^2)} \\ &= \left[p_1^\mu\, p_1^\nu\, p_1^\alpha\right] B_{111}(p_1, m_0, m_1) + \left[\eta^{\mu\nu} p_1^\alpha + \eta^{\mu\alpha} p_1^\nu + \eta^{\nu\alpha} p_1^\mu\right] B_{001}(p_1, m_0, m_1)\,, \end{aligned} \tag{E.9}$$

$$B^{\mu\nu\alpha\beta} = \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu k^\alpha k^\beta}{(k^2 - m_0^2)((k+q_1)^2 - m_1^2)} \tag{E.10}$$

$$= \left[p_1^\mu p_1^\nu p_1^\alpha p_1^\beta\right] B_{1111}(p_1,m_0,m_1) + \left[\eta^{\mu\nu}\eta^{\alpha\beta} + \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}\right] B_{0000}(p_1,m_0,m_1)$$
$$+ \left[\eta^{\mu\nu} p_1^\alpha p_1^\beta + \eta^{\mu\alpha} p_1^\nu p_1^\beta + \eta^{\nu\alpha} p_1^\mu p_1^\beta + \eta^{\beta\nu} p_1^\alpha p_1^\mu + \eta^{\beta\alpha} p_1^\nu p_1^\mu + \eta^{\mu\beta} p_1^\nu p_1^\alpha\right] B_{0011}(p_1,m_0,m_1)\,,$$

$$\begin{aligned} B^{\mu\nu\alpha\beta\rho} &= \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu k^\alpha k^\beta k^\rho}{(k^2-m_0^2)((k+q_1)^2-m_1^2)} \\ &= p_1^\mu p_1^\nu p_1^\alpha p_1^\beta p_1^\rho\, B_{11111}(p_1,m_0,m_1) \\ &\quad + \Big[\eta^{\mu\nu} p_1^\alpha p_1^\beta p_1^\rho + \eta^{\mu\alpha} p_1^\nu p_1^\beta p_1^\rho + \eta^{\nu\alpha} p_1^\mu p_1^\beta p_1^\rho + \eta^{\beta\nu} p_1^\alpha p_1^\mu p_1^\rho + \eta^{\beta\alpha} p_1^\nu p_1^\mu p_1^\rho + \eta^{\mu\beta} p_1^\nu p_1^\alpha p_1^\rho \\ &\qquad + \eta^{\rho\mu} p_1^\alpha p_1^\beta p_1^\nu + \eta^{\rho\nu} p_1^\alpha p_1^\beta p_1^\mu + \eta^{\rho\alpha} p_1^\nu p_1^\beta p_1^\mu + \eta^{\rho\beta} p_1^\nu p_1^\alpha p_1^\mu\Big] B_{00111}(p_1,m_0,m_1) \\ &\quad + \Big[\eta^{\mu\nu}\eta^{\alpha\beta} p_1^\rho + \eta^{\mu\alpha}\eta^{\nu\beta} p_1^\rho + \eta^{\mu\beta}\eta^{\nu\alpha} p_1^\rho + \eta^{\rho\nu}\eta^{\alpha\beta} p_1^\mu + \eta^{\rho\alpha}\eta^{\nu\beta} p_1^\mu + \eta^{\rho\beta}\eta^{\nu\alpha} p_1^\mu \\ &\qquad + \eta^{\mu\rho}\eta^{\alpha\beta} p_1^\nu + \eta^{\mu\alpha}\eta^{\rho\beta} p_1^\nu + \eta^{\mu\beta}\eta^{\rho\alpha} p_1^\nu + \eta^{\mu\nu}\eta^{\rho\beta} p_1^\alpha + \eta^{\mu\rho}\eta^{\nu\beta} p_1^\alpha + \eta^{\mu\beta}\eta^{\nu\rho} p_1^\alpha \\ &\qquad + \eta^{\mu\nu}\eta^{\alpha\rho} p_1^\beta + \eta^{\mu\alpha}\eta^{\nu\rho} p_1^\beta + \eta^{\mu\rho}\eta^{\nu\alpha} p_1^\beta\Big] B_{00001}(p_1,m_0,m_1)\,, \end{aligned} \tag{E.11}$$

$$\begin{aligned} C &= \int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-m_0^2)((k+q_1)^2-m_1^2)((k+q_2)^2-m_2^2)} \\ &= C_0(p_1,p_2,m_0,m_1,m_2)\,, \end{aligned} \tag{E.12}$$

$$\begin{aligned} \mathcal{C}^\mu &= \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu}{(k^2-m_0^2)((k+q_1)^2-m_1^2)((k+q_2)^2-m_2^2)} \\ &= \sum_{i=1}^{2} p_i^\mu\, C_i(p_1,p_2,m_0,m_1,m_2)\,, \end{aligned} \tag{E.13}$$

$$\begin{aligned} C^{\mu\nu} &= \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu}{(k^2-m_0^2)((k+q_1)^2-m_1^2)((k+q_2)^2-m_2^2)} \\ &= \sum_{i,j=1}^{2} p_i^\mu\, p_j^\nu\, C_{ij}(p_1,p_2,m_0,m_1,m_2) + \eta^{\mu\nu} C_{00}(p_1,p_2,m_0,m_1,m_2)\,, \end{aligned} \tag{E.14}$$

$$\begin{aligned} C^{\mu\nu\alpha} &= \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu k^\alpha}{(k^2-m_0^2)((k+q_1)^2-m_1^2)((k+q_2)^2-m_2^2)} \\ &= \sum_{i,j,k=1}^{2} p_i^\mu\, p_j^\nu\, p_k^\alpha\, C_{ijk}(p_1,p_2,m_0,m_1,m_2) \\ &\quad + \sum_{i=1}^{2}\left[\eta^{\mu\nu} p_i^\alpha + \eta^{\mu\alpha} p_i^\nu + \eta^{\nu\alpha} p_i^\mu\right] C_{00i}(p_1,p_2,m_0,m_1,m_2)\,, \end{aligned} \tag{E.15}$$

$$\begin{aligned}
C^{\mu\nu\alpha\beta} &= \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu k^\alpha k^\beta}{(k^2-m_0^2)((k+q_1)^2-m_1^2)((k+q_2)^2-m_2^2)} \\
&= \sum_{i,j,k,l=1}^{2} p_i^\mu p_j^\nu p_k^\alpha p_l^\beta \, C_{ijkl}(p_1,p_2,m_0,m_1,m_2) \\
&\quad + \sum_{i,j=1}^{2} \Big[\eta^{\mu\nu} p_i^\alpha p_j^\beta + \eta^{\mu\alpha} p_i^\nu p_j^\beta + \eta^{\nu\alpha} p_i^\mu p_j^\beta + \eta^{\beta\nu} p_i^\alpha p_j^\mu + \eta^{\beta\alpha} p_i^\nu p_j^\mu \\
&\qquad\qquad + \eta^{\mu\beta} p_i^\nu p_j^\alpha\Big] C_{00ij}(p_1,p_2,m_0,m_1,m_2) \\
&\quad + \Big[\eta^{\mu\nu}\eta^{\alpha\beta} + \eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha}\Big] C_{0000}(p_1,p_2,m_0,m_1,m_2)\,,
\end{aligned} \tag{E.16}$$

$$\begin{aligned}
C^{\mu\nu\alpha\beta\rho} &= \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu k^\alpha k^\beta k^\rho}{(k^2-m_0^2)((k+q_1)^2-m_1^2)((k+q_2)^2-m_2^2)} \\
&= \sum_{i,j,k,l,m=1}^{2} p_i^\mu p_j^\nu p_k^\alpha p_l^\beta p_m^\rho \, C_{ijklm}(p_1,p_2,m_0,m_1,m_2) \\
&\quad + \sum_{i,j,k=1}^{2} \Big[\eta^{\mu\nu} p_i^\alpha p_j^\beta p_k^\rho + \eta^{\mu\alpha} p_i^\nu p_j^\beta p_k^\rho + \eta^{\nu\alpha} p_i^\mu p_j^\beta p_k^\rho + \eta^{\beta\nu} p_i^\alpha p_j^\mu p_k^\rho + \eta^{\beta\alpha} p_i^\nu p_j^\mu p_k^\rho + \eta^{\mu\beta} p_i^\nu p_j^\alpha p_k^\rho \\
&\qquad + \eta^{\rho\mu} p_i^\alpha p_j^\beta p_k^\nu + \eta^{\rho\nu} p_i^\alpha p_j^\beta p_k^\mu + \eta^{\rho\alpha} p_i^\nu p_j^\beta p_k^\mu + \eta^{\rho\beta} p_i^\nu p_j^\alpha p_k^\mu\Big] C_{00ijk}(p_1,p_2,m_0,m_1,m_2) \\
&\quad + \sum_{i=1}^{2} \Big[\eta^{\mu\nu}\eta^{\alpha\beta} p_i^\rho + \eta^{\mu\alpha}\eta^{\nu\beta} p_i^\rho + \eta^{\mu\beta}\eta^{\nu\alpha} p_i^\rho + \eta^{\rho\nu}\eta^{\alpha\beta} p_i^\mu + \eta^{\rho\alpha}\eta^{\nu\beta} p_i^\mu + \eta^{\rho\beta}\eta^{\nu\alpha} p_i^\mu \\
&\qquad + \eta^{\mu\rho}\eta^{\alpha\beta} p_i^\nu + \eta^{\mu\alpha}\eta^{\rho\beta} p_i^\nu + \eta^{\mu\beta}\eta^{\rho\alpha} p_i^\nu + \eta^{\mu\nu}\eta^{\rho\beta} p_i^\alpha + \eta^{\mu\rho}\eta^{\nu\beta} p_i^\alpha + \eta^{\mu\beta}\eta^{\nu\rho} p_i^\alpha \\
&\qquad + \eta^{\mu\nu}\eta^{\alpha\rho} p_i^\beta + \eta^{\mu\alpha}\eta^{\nu\rho} p_i^\beta + \eta^{\mu\rho}\eta^{\nu\alpha} p_i^\beta\Big] C_{0000i}(p_1,p_2,m_0,m_1,m_2)\,,
\end{aligned}$$

$$\begin{aligned}
C^{\mu\nu\alpha\beta\rho\sigma} &= \int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu k^\alpha k^\beta k^\rho k^\sigma}{(k^2-m_0^2)((k+q_1)^2-m_1^2)((k+q_2)^2-m_2^2)} \\
&= \sum_{i,j,k,l,m,n=1}^{2} p_i^\mu p_j^\nu p_k^\alpha p_l^\beta p_m^\rho p_n^\sigma \, C_{ijklmn}(p_1,p_2,m_0,m_1,m_2) \\
&\quad + \sum_{i,j,k,l=1}^{2} \Big[\eta^{\mu\nu} p_i^\alpha p_j^\beta p_k^\rho p_l^\sigma + \eta^{\mu\alpha} p_i^\nu p_j^\beta p_k^\rho p_l^\sigma + \eta^{\nu\alpha} p_i^\mu p_j^\beta p_k^\rho p_l^\sigma + \eta^{\beta\nu} p_i^\alpha p_j^\mu p_k^\rho p_l^\sigma + \eta^{\beta\alpha} p_i^\nu p_j^\mu p_k^\rho p_l^\sigma \\
&\qquad + \eta^{\mu\beta} p_i^\nu p_j^\alpha p_k^\rho p_l^\sigma + \eta^{\rho\mu} p_i^\alpha p_j^\beta p_k^\nu p_l^\sigma + \eta^{\rho\nu} p_i^\alpha p_j^\beta p_k^\mu p_l^\sigma + \eta^{\rho\alpha} p_i^\nu p_j^\beta p_k^\mu p_l^\sigma + \eta^{\rho\beta} p_i^\nu p_j^\alpha p_k^\mu p_l^\sigma \\
&\qquad + \eta^{\sigma\mu} p_i^\alpha p_j^\beta p_k^\rho p_l^\nu + \eta^{\sigma\nu} p_i^\alpha p_j^\beta p_k^\rho p_l^\mu + \eta^{\sigma\alpha} p_i^\nu p_j^\beta p_k^\rho p_l^\mu + \eta^{\sigma\beta} p_i^\alpha p_j^\nu p_k^\rho p_l^\mu \\
&\qquad + \eta^{\sigma\rho} p_i^\alpha p_j^\beta p_k^\nu p_l^\mu\Big] C_{00ijkl}(p_1,p_2,m_0,m_1,m_2)
\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i,j=1}^{2}\Big[\eta^{\mu\nu}\eta^{\alpha\beta}p_i^{\rho}p_j^{\sigma}+\eta^{\mu\alpha}\eta^{\nu\beta}p_i^{\rho}p_j^{\sigma}+\eta^{\mu\beta}\eta^{\nu\alpha}p_i^{\rho}p_j^{\sigma}+\eta^{\rho\nu}\eta^{\alpha\beta}p_i^{\mu}p_j^{\sigma}+\eta^{\rho\alpha}\eta^{\nu\beta}p_i^{\mu}p_j^{\sigma}\\
&\qquad+\eta^{\rho\beta}\eta^{\nu\alpha}p_i^{\mu}p_j^{\sigma}+\eta^{\mu\rho}\eta^{\alpha\beta}p_i^{\nu}p_j^{\sigma}+\eta^{\mu\alpha}\eta^{\rho\beta}p_i^{\nu}p_j^{\sigma}+\eta^{\mu\beta}\eta^{\rho\alpha}p_i^{\nu}p_j^{\sigma}+\eta^{\mu\nu}\eta^{\rho\beta}p_i^{\alpha}p_j^{\sigma}\\
&\qquad+\eta^{\mu\rho}\eta^{\nu\beta}p_i^{\alpha}p_j^{\sigma}+\eta^{\mu\beta}\eta^{\nu\rho}p_i^{\alpha}p_j^{\sigma}+\eta^{\mu\nu}\eta^{\alpha\rho}p_i^{\beta}p_j^{\sigma}+\eta^{\mu\alpha}\eta^{\nu\rho}p_i^{\beta}p_j^{\sigma}+\eta^{\mu\rho}\eta^{\nu\alpha}p_i^{\beta}p_j^{\sigma}\\
&\qquad+\eta^{\sigma\nu}\eta^{\alpha\beta}p_i^{\rho}p_j^{\mu}+\eta^{\sigma\alpha}\eta^{\nu\beta}p_i^{\rho}p_j^{\mu}+\eta^{\sigma\beta}\eta^{\nu\alpha}p_i^{\rho}p_j^{\mu}+\eta^{\mu\sigma}\eta^{\alpha\beta}p_i^{\rho}p_j^{\nu}+\eta^{\mu\alpha}\eta^{\sigma\beta}p_i^{\rho}p_j^{\nu}\\
&\qquad+\eta^{\mu\beta}\eta^{\sigma\alpha}p_i^{\rho}p_j^{\nu}+\eta^{\mu\nu}\eta^{\sigma\beta}p_i^{\rho}p_j^{\alpha}+\eta^{\mu\sigma}\eta^{\nu\beta}p_i^{\rho}p_j^{\alpha}+\eta^{\mu\beta}\eta^{\nu\sigma}p_i^{\rho}p_j^{\alpha}+\eta^{\mu\nu}\eta^{\alpha\sigma}p_i^{\rho}p_j^{\beta}\\
&\qquad+\eta^{\mu\alpha}\eta^{\nu\sigma}p_i^{\rho}p_j^{\beta}+\eta^{\mu\sigma}\eta^{\nu\alpha}p_i^{\rho}p_j^{\beta}+\eta^{\sigma\nu}\eta^{\alpha\rho}p_i^{\beta}p_j^{\mu}+\eta^{\sigma\alpha}\eta^{\nu\rho}p_i^{\beta}p_j^{\mu}+\eta^{\sigma\rho}\eta^{\nu\alpha}p_i^{\beta}p_j^{\mu}\\
&\qquad+\eta^{\mu\sigma}\eta^{\alpha\rho}p_i^{\beta}p_j^{\nu}+\eta^{\mu\alpha}\eta^{\sigma\rho}p_i^{\beta}p_j^{\nu}+\eta^{\mu\rho}\eta^{\sigma\alpha}p_i^{\beta}p_j^{\nu}+\eta^{\mu\nu}\eta^{\sigma\rho}p_i^{\beta}p_j^{\alpha}+\eta^{\mu\sigma}\eta^{\nu\rho}p_i^{\beta}p_j^{\alpha}\\
&\qquad+\eta^{\mu\rho}\eta^{\nu\sigma}p_i^{\beta}p_j^{\alpha}+\eta^{\sigma\nu}\eta^{\beta\rho}p_i^{\alpha}p_j^{\mu}+\eta^{\sigma\beta}\eta^{\nu\rho}p_i^{\alpha}p_j^{\mu}+\eta^{\sigma\rho}\eta^{\nu\beta}p_i^{\alpha}p_j^{\mu}+\eta^{\mu\sigma}\eta^{\beta\rho}p_i^{\alpha}p_j^{\nu}\\
&\qquad+\eta^{\mu\beta}\eta^{\sigma\rho}p_i^{\alpha}p_j^{\nu}+\eta^{\mu\rho}\eta^{\sigma\beta}p_i^{\alpha}p_j^{\nu}+\eta^{\sigma\alpha}\eta^{\beta\rho}p_i^{\nu}p_j^{\mu}+\eta^{\sigma\beta}\eta^{\alpha\rho}p_i^{\nu}p_j^{\mu}\\
&\qquad+\eta^{\sigma\rho}\eta^{\alpha\beta}p_i^{\nu}p_j^{\mu}\Big]C_{0000ij}(p_1,p_2,m_0,m_1,m_2)\\
&+\Big[\eta^{\mu\nu}\eta^{\alpha\beta}\eta^{\rho\sigma}+\eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\rho\sigma}+\eta^{\mu\beta}\eta^{\nu\alpha}\eta^{\rho\sigma}+\eta^{\sigma\nu}\eta^{\alpha\beta}\eta^{\rho\mu}+\eta^{\sigma\alpha}\eta^{\nu\beta}\eta^{\rho\mu}+\eta^{\sigma\beta}\eta^{\nu\alpha}\eta^{\rho\mu}\\
&\qquad+\eta^{\mu\sigma}\eta^{\alpha\beta}\eta^{\rho\nu}+\eta^{\mu\alpha}\eta^{\sigma\beta}\eta^{\rho\nu}+\eta^{\mu\beta}\eta^{\sigma\alpha}\eta^{\rho\nu}+\eta^{\mu\nu}\eta^{\sigma\beta}\eta^{\rho\alpha}+\eta^{\mu\sigma}\eta^{\nu\beta}\eta^{\rho\alpha}+\eta^{\mu\beta}\eta^{\nu\sigma}\eta^{\rho\alpha}\\
&\qquad+\eta^{\mu\nu}\eta^{\alpha\sigma}\eta^{\rho\beta}+\eta^{\mu\alpha}\eta^{\nu\sigma}\eta^{\rho\beta}+\eta^{\mu\sigma}\eta^{\nu\alpha}\eta^{\rho\beta}\Big]C_{000000}(p_1,p_2,m_0,m_1,m_2)\,,
\end{aligned}
$$

$$
\begin{aligned}
D &= \int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2-m_0^2)\big((k+q_1)^2-m_1^2\big)\big((k+q_2)^2-m_2^2\big)\big((k+q_3)^2-m_3^2\big)} \\
&= D_0(p_1,p_2,p_3,m_0,m_1,m_2,m_3)\,,
\end{aligned}
\tag{E.17}
$$

$$
\begin{aligned}
D^{\mu} &= \int\frac{d^dk}{(2\pi)^d}\frac{k^{\mu}}{(k^2-m_0^2)\big((k+q_1)^2-m_1^2\big)\big((k+q_2)^2-m_2^2\big)\big((k+q_3)^2-m_3^2\big)} \\
&= \sum_{i=1}^{3}p_i^{\mu}\;D_i(p_1,p_2,p_3,m_0,m_1,m_2,m_3)\,,
\end{aligned}
\tag{E.18}
$$

$$
\begin{aligned}
D^{\mu\nu} &= \int\frac{d^dk}{(2\pi)^d}\frac{k^{\mu}k^{\nu}}{(k^2-m_0^2)\big((k+q_1)^2-m_1^2\big)\big((k+q_2)^2-m_2^2\big)\big((k+q_3)^2-m_3^2\big)} \\
&= \eta^{\mu\nu}D_{00}(p_1,p_2,p_3,m_0,m_1,m_2,m_3)+\sum_{i,j=1}^{3}p_i^{\mu}p_j^{\nu}\;D_{ij}(p_1,p_2,p_3,m_0,m_1,m_2,m_3)\,,
\end{aligned}
\tag{E.19}
$$

$$
\begin{aligned}
D^{\mu\nu\alpha} &= \int\frac{d^dk}{(2\pi)^d}\frac{k^{\mu}k^{\nu}k^{\alpha}}{(k^2-m_0^2)\big((k+q_1)^2-m_1^2\big)\big((k+q_2)^2-m_2^2\big)\big((k+q_3)^2-m_3^2\big)} \\
&= \quad\sum_{i=1}^{3}\Big[\eta^{\mu\nu}p_i^{\alpha}+\eta^{\mu\alpha}p_i^{\nu}+\eta^{\nu\alpha}p_i^{\mu}\Big]D_{00i}(p_1,p_2,p_3,m_0,m_1,m_2,m_3)
\end{aligned}
\tag{E.20}
$$

$$+\sum_{i,j,k=1}^{3} p_i^\mu p_j^\nu p_j^\alpha \; D_{ijk}(p_1,p_2,p_3,m_0,m_1,m_2,m_3)\,,$$

$$\begin{aligned}
D^{\mu\nu\alpha\beta} &= \int \frac{d^dk}{(2\pi)^d}\frac{k^\mu k^\nu k^\alpha k^\beta}{(k^2-m_0^2)\big((k+q_1)^2-m_1^2\big)\big((k+q_2)^2-m_2^2\big)\big((k+q_3)^2-m_3^2\big)} \\
&= \Big[\eta^{\mu\nu}\eta^{\alpha\beta}+\eta^{\mu\alpha}\eta^{\nu\beta}+\eta^{\mu\beta}\eta^{\alpha\nu}\Big] D_{0000}(p_1,p_2,p_3,m_0,m_1,m_2,m_3) \\
&\quad+\sum_{i,j=1}^{3}\Big[\eta^{\mu\nu}p_i^\alpha p_j^\beta+\eta^{\mu\alpha}p_i^\nu p_j^\beta+\eta^{\nu\alpha}p_i^\mu p_j^\beta+\eta^{\mu\beta}p_i^\alpha p_j^\nu+\eta^{\nu\beta}p_i^\alpha p_j^\mu \\
&\qquad\qquad+\eta^{\alpha\beta}p_i^\mu p_j^\nu\Big] D_{00ij}(p_1,p_2,p_3,m_0,m_1,m_2,m_3) \\
&\quad+\sum_{i,j,k,l=1}^{3} p_i^\mu p_j^\nu p_k^\alpha p_l^\beta \; D_{ijkl}(p_1,p_2,p_3,m_0,m_1,m_2,m_3)\,.
\end{aligned} \tag{E.21}$$

Also, we use the following reduction formulas

$$k.p_i = \frac{1}{2}\Big[((k+q_i)^2-m_i^2)-((k+q_{i-1})^2-m_{i-1}^2)+m_i^2-m_{i-1}^2-q_i^2+q_{i-1}^2\Big]\,, \tag{E.22}$$

$$p_i.Q_i = \frac{1}{2}\Big[(Q_i^2-m_i^2)-(Q_{i-1}^2-m_{i-1}^2)+m_i^2-m_{i-1}^2+p_i^2\Big]\,, \tag{E.23}$$

$$p_i.Q_{i-1} = \frac{1}{2}\Big[(Q_i^2-m_i^2)-(Q_{i-1}^2-m_{i-1}^2)+m_i^2-m_{i-1}^2-p_i^2\Big]\,, \tag{E.24}$$

$$p_i.Q_{i-2} = \frac{1}{2}\Big[(Q_i^2-m_i^2)-(Q_{i-1}^2-m_{i-1}^2)+m_i^2-m_{i-1}^2-p_i^2-2p_{i-1}.p_i\Big]\,, \tag{E.25}$$

$$p_i.k = \frac{1}{2}\Big[(Q_i^2-m_i^2)-(Q_{i-1}^2-m_{i-1}^2)+m_i^2-m_{i-1}^2-2q_i.p_i+p_i^2\Big]\,, \tag{E.26}$$

$$p_i.k = \frac{1}{2}\Big[(Q_i''^2-m_i^2)-(Q_{i-1}'^2-m_{i-1}^2)+m_i^2-m_{i-1}^2-p_i^2-2p_i.p_{i-1}\Big]\,, \tag{E.27}$$

$$p_i.k = \frac{1}{2}\Big[(Q_i'^2-m_i^2)-(k^2-m_0^2)+m_i^2-m_0^2-p_i^2\Big]\,, \tag{E.28}$$

where the relations between the momenta are:

$$\begin{aligned}
Q_i &= k+p_1+\cdots+p_i = k+q_i\,,\\
q_i &= p_1+\cdots+p_i\,,\\
Q_i' &= k+p_i\,,\\
Q_i'' &= k+p_{i-1}+p_i\,.
\end{aligned}$$

Finally, we follow the Passarino-Veltman method as explained in Sec. 5.1.4 using the reduction formulas Eqs. (E.22–E.28) and other integral tools such as change of variables. As a result, we write all above scalar functions $A_{00}, A_{0000}, B_1, B_{00}, \ldots$ in terms of the scalar functions A_0, B_0, C_0 and D_0.

F The FORM Program

In this appendix, we show short pieces of our code to illustrate as much as possible how we perform the calculations in the FORM program. At the same time, we ensure that all the indices are contracted in a proper way in the code. However, let us start by introducing some notations in Tab. 11.

Table 11: Some notations that we use in our code.

$h_{\mu\nu}$ = `H(mu,nu)`	$\partial_\alpha h_{\mu\nu}$ = `H(al,mu,nu)`	$\partial_\alpha\partial_\beta h_{\mu\nu}$ = `H(al,be,mu,nu)`
$g_{\mu\nu}$ = `GL(mu,nu)`	$\partial_\alpha g_{\mu\nu}$ = `GL(al,mu,nu)`	$\partial_\alpha\partial_\beta g_{\mu\nu}$ = `GL(al,be,mu,nu)`
$g^{\mu\nu}$ = `GU(mu,nu)`	$\partial_\alpha g^{\mu\nu}$ = `GU(al,mu,nu)`	$\partial_\alpha\partial_\beta g^{\mu\nu}$ = `GU(al,be,mu,nu)`
ϕ = `phi`	$\partial_\alpha \phi$ = `phi(al)`	$\partial_\alpha\partial_\beta \phi$ = `phi(al,be)`
$\partial_\alpha \Gamma^\mu_{\nu\beta}$ = `Gamma(al,mu,nu,be)`	$\delta_{\mu\nu}$ = `d_(mu,nu)`	`MAXH`[1] = 4

[1] Since our calculations are up to the quadruple graviton vertex.

As an example, the square root of the determinant of metric $\sqrt{-g}$ = `SQRTMG` can be calculated from the relation Eq. (2.13) as:

Code
```
Local SQRTMG = 1+sum_(n,1,‘MAXH’,invfac_(n)/2^n*trlng^n);
#do i=1,‘MAXH’
    id,once trlng = -sum_(n,1,‘MAXH’
                    ,sign_(n)/n*epsh^n*kappa^n*HH(n,i‘i’x1,i‘i’x1));
    id epsh^{‘MAXH’+1} = 0;
#enddo
#do i=1,‘MAXH’
#do j=2,‘MAXH’
    id,once HH(n?,mu?,nu?) = H(mu,i‘i’x‘j’)*HH(n-1,i‘i’x‘j’,nu);
    id HH(0,mu?,nu?) = d_(mu,nu);
#enddo
#enddo
```
Code

In addition, the field redefinition can be done by replacing each field and its derivatives with the proper expansion. The code below shows the gravitational field redefinition Eq. (2.50), where the other fields were treated in the same way:

Code
```
#do i=1,‘MAXH’
multiply aa;
id,once aa*H(?a,mu?,nu?) =  a1*Der(?a,H(mu,nu))
 +kappa*epsh*(a3*Der(?a,H(mu,i{‘i’+1}x1),
   H(i{‘i’+1}x1,nu))+a2*Der(?a,H(mu,nu),H(i{‘i’+1}x1,i{‘i’+1}x1)))
 +kappa^2*epsh^2*(a4*Der(?a,H(mu,nu),H(i{‘i’+1}x1,i{‘i’+1}x1),
```

```
  H(i{`i'+1}x2,i{`i'+1}x2))+a5*Der(?a,H(mu,nu),H(i{`i'+1}x1,i{`i'+1}x2),
  H(i{`i'+1}x1,i{`i'+1}x2))+a6*Der(?a,H(mu,i{`i'+1}x1),H(nu,i{`i'+1}x1),
  H(i{`i'+1}x2,i{`i'+1}x2))+a7*Der(?a,H(mu,i{`i'+1}x1),H(nu,i{`i'+1}x2),
  H(i{`i'+1}x1,i{`i'+1}x2)))
 +kappa^3*epsh^3*(a8*Der(?a,H(mu,nu),H(i{`i'+1}x1,i{`i'+1}x1),
  H(i{`i'+1}x2,i{`i'+1}x2),H(i{`i'+1}x3,i{`i'+1}x3))+a9*Der(?a,H(mu,nu),
  H(i{`i'+1}x1,i{`i'+1}x1),H(i{`i'+1}x2,i{`i'+1}x3),
  H(i{`i'+1}x2,i{`i'+1}x3))+a10*Der(?a,H(mu,nu),H(i{`i'+1}x1,i{`i'+1}x2),
  H(i{`i'+1}x2,i{`i'+1}x3),H(i{`i'+1}x3,i{`i'+1}x1))+a11*Der(?a,
  H(mu,i{`i'+1}x1),H(nu,i{`i'+1}x1),H(i{`i'+1}x2,i{`i'+1}x2),
  H(i{`i'+1}x3,i{`i'+1}x3))+a12*Der(?a,H(mu,i{`i'+1}x1),H(nu,i{`i'+1}x1),
  H(i{`i'+1}x2,i{`i'+1}x3),H(i{`i'+1}x2,i{`i'+1}x3))+a13*Der(?a,
  H(mu,i{`i'+1}x1),H(nu,i{`i'+1}x2),H(i{`i'+1}x1,i{`i'+1}x2),
  H(i{`i'+1}x3,i{`i'+1}x3))+a14*Der(?a,H(mu,i{`i'+1}x1),H(nu,i{`i'+1}x2),
  H(i{`i'+1}x1,i{`i'+1}x3),H(i{`i'+1}x2,i{`i'+1}x3)));
id aa = 1;
id epsh^`MAXH' = 0;
.sort
#enddo
```

Code

Taking a derivative in our notation can also be achieved in FORM with the following procedure:

Code

```
repeat;
  id Deriv(H?fields(?a),?b) = H(?a)*Deriv(?b);
  id Deriv(mu?,H?fields(?a),?b) = H(mu,?a)*Deriv(?b)
                                 +H(?a)*Deriv(mu,?b);
  id Deriv(mu?,nu?,H?fields(?a),?b) = H(mu,nu,?a)*Deriv(?b)
                                     +H(mu,?a)*Deriv(nu,?b)
                                     +H(nu,?a)*Deriv(mu,?b)
                                     +H(?a)*Deriv(mu,nu,?b);
  id Deriv(mu?) = 0;
  id Deriv(mu?,nu?) = 0;
  id Deriv = 1;
endrepeat;
```

Code

Finally, the amplitude of a Feynman diagram can be calculated in FORM as shown below for the s-channel diagram in scalar-graviton scattering:

Code

```
* S-channel in scalar-graviton scattering:
*
* p2-> x             x p4->
```

```
4  * mu,nu x           x al,be
5  *        x          x
6  *  aaaaaaaXaaaaaaaYaaaaaaaa
7  *   p1->    P1->     p3->
8  *
9  *   X=p1x,i1x Y=p2x,i2x
10 
11 #include vertexprocedures.hf
12 #include symbols.hf
13 #include setexternal.hf
14 .sort
15 L Vleft =
16 #call vertex2phi1H`VERTEXTYPE'
17  ;
18 #call setmom(p1x,3)
19 #call takederiv
20 print +s;
21 .sort
22 skip;
23 L Vright =
24 #call vertex2phi1H`VERTEXTYPE'
25   ;
26 #call relabelindexij(1,2)
27 #call setmom(p2x,3)
28 #call takederiv
29 print +s;
30 .sort
31 #call pickoutphiin(p1,p1x,p1xp1x,2,Vleft)
32 #call pickoutphiout(p3,p2x,p2xp2x,2,Vright)
33 #call pickoutHin(mu,nu,p2,p1x,p1xp1x,2,Vleft)
34 #call pickoutHout(al,be,p4,p2x,p2xp2x,1,Vright)
35 .sort
36 drop Vleft,Vright;
37 G PPHH1`VERTEXTYPE' = i_^3*Vleft*Vright;
38 #call connectphi(P1,p1x,p1xp1x,1,p2x,p2xp2x,1,PPHH1`VERTEXTYPE')
39 print +s;
40 .store
41 save save/PPHH1`VERTEXTYPE'.sav PPHH1`VERTEXTYPE';
42 .end
```

Code

Where we start the code by defining the vertices and importing their corresponding expressions. In the example above, the left vertex defined in lines 15 and 16 is a scalar-scalar-graviton vertex, with its corresponding expression given by Eq. (3.8). Then in lines

18, 19, 31 and 33, we convert this vertex to momentum space. After that in line 37, we put all together and multiply by (i) for each vertex and propagator. Finally in line 38, we connect the two vertices by calling the propagator procedure, which also contains the `pickoutphiin` and `pickoutphiout` procedures.

Acronyms

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