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Reinforced Concrete Structures Subjected to Imposed Deformations: A Study of Cracking due to Shrinkage in Slab Foundations for Residential Houses

Betongkonstruktioner utsatta för tvångsdeformationer: En studie av sprickbeteende orsakad av krympning i villaplattor

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Abstract

The need for limiting crack widths in reinforced concrete structures is due to durability, tightness towards gaseous and liquid substances, and for aesthetic reasons. Although concrete cracks at relatively low tensile stresses and can not be avoided, strategies for crack control can be adopted to meet the requirements. For certain structures, tensile stresses and consequently cracking may arise due to imposed deformations. That is when a restrained structure is exposed to temperature variations or shrinkage strains. Eurocode 2 provides guidance for crack control, but the formulas are adopted by assuming stabilized cracking, which may be a too conservative assumption for restrained structures. The study presented in this master thesis aims to investigate the crack behavior of reinforced concrete structures exposed to imposed deformations, where the focus is on smaller slab foundations for residential houses.

The crack behaviour was analysed with nonlinear finite element analysis in Atena 2D. First, real experimental tie-rod tests were modelled with finite elements and analysed with different fracture energy models, bond models and mesh sizes. It was shown that nonlinear finite element analysis can predict cracks well for the intended purpose. The obtained results and observations were the foundation for a further study of cracking in slab foundations. For that purpose, the drying was simulated with respect to modern concrete properties, and a shrinkage profile could be obtained. A parametric study was performed by varying following parameters: stiffness of the sub-base, friction, slab length, slab height, bar diameter, reinforcement ratio, concrete class, type of strain distribution. The obtained results were compared with analytical methods in Eurocode 2 and a method from Chalmers University by Engström.

The parametric study showed that the number of cracks and the crack widths can be controlled by varying different parameters. The length of the slab and the stiffness in the sub-base were crucial parameters for the crack growth, but are, however, parameters that for a designer are difficult to control. Crack widths could on the other hand be limited by increasing the bar diameter or the reinforcement amount. Eurocode 2 guidelines for minimum reinforcement may be too conservative for smaller concrete slabs. For slabs in which the reinforcement ratio did not fulfil the requirement of minimum reinforcement, the crack widths obtained from the finite element analysis still turned out to be acceptable to some extent. The overall conclusion is that new guidelines for these types of structures is necessary.

Keywords: Reinforced concrete structure, nonlinear finite element analysis, crack width, imposed deformation, shrinkage, foundation slabs, fracture mechanics, Atena 2D.

Sammanfattning

Sprickbredder i armerade betongkonstruktioner bör begränsas med hänsyn till hållbarhet, täthet och estetik. Eftersom betong har en relativt låg draghållfasthet kan sprickor dock sällan undvikas, men kan med olika strategier minskas för att uppfylla de givna kraven. I betongkonstruktioner kan dragspänningar och följaktligen sprickor uppstå på grund av tvångsdeformationer, som kan uppstå på grund av temperaturvariationer och krympning. I Eurocode 2 erhålls metoder för att beräkna minsta tillåtna armeringsinnehåll och erforderlig sprickbredd för en betongkonstruktion. Dessa metoder utgår från antagandet att all last i ett sprucket tvärsnitt ska bäras av armeringen och att uppsprickningen är fullt utvecklad, vilket sällan är fallet för konstruktioner utsatta för tvång. Studien som presenteras i detta examensarbete har som syfte att undersöka sprickbeteendet för plattor på mark utsatta för tvångsdeformationer, samt att utreda om tillgängliga analytiska metoder använder sig av för konservativa antaganden.

Sprickutvecklingen undersöktes med hjälp av icke-linjär finita elementmetod i Atena 2D. Ett centrisk armerad betongprisma utsatt för ren dragbelastning modellerades och jämfördes med experimentella försök för att utvärdera olika materialmodeller för brottenergi och vidhäftningsförmåga. De materialmodeller som vid kalibreringen uppvisade ett sprickbeteende likt de utförda försöken användes i fortsatta analyser, i vilka plattor på mark studerades. I det syftet erhöles en krympprofil genom att simulera uttorkning med hänsyn till materialegenskaper i modern betong. En parameterstudie utfördes för att undersöka hur utformning och materialval påverkar sprickutvecklingen och den slutgiltiga sprickbredden, samt hur väl dessa beräkningar stämmer överens med analytiska metoder. Parametrar som ingick i studien var följande: styvhet i undergrund, friktion, plattlängd, plathöjd, armeringsdiameter, armeringsinnehåll, betongkvalitet och olika typer av krympvariation över tvärsnittet.

Studien visar att längden på plattorna och styvheten i undergrunden är av stor betydelse för sprickbeteendet, men är parametrar som är svåra att påverka. Sprickbredden kan dock begränsas genom val gällande armeringsinnehåll och armeringsdiameter. I fall med ett lägre armeringsinnehåll än vad Eurocode 2 rekommenderar, uppvisar de numeriska beräkningarna ett liknande sprickbeteende som uppfyller dessa krav och sprickbredder som till ett visst mått kan anses acceptabla. Detta indikerar att Eurocode 2 är för konservativ när sprickor studeras i villaplattor utsatta för tvång. För att minska den erforderliga armeringsmängden i plattor på mark bör därför inverkan av återhållande element beaktas vid beräkning av minsta armeringsmängd.

Nyckelord: Armerade betongkonstruktioner, icke-linjär finita element analys, sprickbredd, tvångsdeformationer, krympning, platta på mark, brottmekanik, Atena 2D.

Preface

This master thesis was carried out at the Division of Structural Engineering at Lund University and Skanska Teknik in Malmö during the spring of 2020.

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List of Symbols

Abbreviations

COD	Crack Opening Displacement
EC2	Eurocode 2
FEA	Finite Element Analysis
FPZ	Fracture Process Zone
MC10	Model Code 2010
MC90	Model Code 1990
PPB	ProduktionsPlanering Betong
RH	Relative Humidity
SLS	Service Limit State
ULS	Ultimate Limit State

Greek letters

$\beta_{as}(t)$	Coefficient considering variation in time
β_{RH}	Ambient relative humidity factor
ϕ	Bar diameter
ρ	Density
ρ_r	Reinforcement ratio
σ	Stress
σ_c	Concrete stress
σ_s	Steel stress
σ_{ct}	Concrete tensile stress
τ_b	Bond stress
ε	Strain

ε_c	Concrete strain
ε_{cd}	Concrete drying shrinkage strain
ε_{cs}	Concrete shrinkage strain
ε_{cs}	Total shrinkage strain
$\varepsilon_{cs}(\infty)$	Final shrinkage strain
$\varphi(t, t_0)$	Creep coefficient

Latin letters

A	Total cross sectional area
A_c	Concrete cross sectional area
A_{eff}	Total cross sectional area
b	Width of cross section
c	Concrete cover
d	Effective height of cross section
E	Modulus of elasticity
E_s	Modulus of elasticity for steel
E_{cm}	Mean modulus of elasticity for concrete
E_c	Effective modulus of elasticity for concrete
E_c	Modulus of elasticity for concrete
F	Force
f_{cm}	Mean concrete compressive strength
$f_{ctk,0.05}$	Lower characteristic of concrete tensile strength
$f_{ctk,0.95}$	Higher characteristic of concrete tensile strength
f_{ctk}	Characteristic concrete tensile strength
f_{ctm}	Mean concrete tensile strength
f_t	Tensile strength
G_f	Concrete fracture energy
h	Height of cross section
h_0	Notional height
$J(t, t')$	Creep compliance function

l	Length
l_t	Transfer length
$l_{t,max}$	Maximum transfer length
N	Axial normal force
N_{cr}	Cracking load
p	Pressure
R	Restraint degree
s	Slip
$s_{r,max}$	Maximum crack spacing
$s_{r,min}$	Minimum crack spacing
s_{rm}	Mean crack spacing
T	Temperature
t	Time
u	Displacement
w	Crack width
w_k	Characteristic crack width
w_m	Mean crack width
n_{cr}	Number of cracks

Chapter 1

Introduction

1.1 Background

Limitation of crack widths in reinforced concrete structures is important to maintain durability. As cracks open up, harmful substances causing corrosion to the steel can ingress and negatively affect the structure. For certain structures, crack widths are limited for aesthetic reasons. However, concrete cracks at relatively low tensile stresses and can rarely be avoided. Control of cracking is therefore important to fulfil requirements. Studying cracking in concrete requires knowledge in the constitutive behaviour of concrete and steel, and the bond between them. When cracking occurs due to imposed deformations in a restrained structure, however, the global stiffness decreases and thereby the stresses as well. A proper reinforcement amount can limit crack widths by distributing the tensile forces and create several narrow cracks instead of few wider cracks [16].

The effect of imposed deformation is a matter of time and age; at an early age, imposed deformation is due to the hydration process causing plastic shrinkage and temperature gradients; at mature age, imposed deformation is due to drying shrinkage and ambience temperature changes. Imposed deformations will result in cracking when the structure is externally or/and internally restrained. Internal restraints may be caused by the bond between the concrete and the reinforcement or by strain gradients. External restraints are related to boundary conditions, i.e. the connection of a member to other parts of the structure.

Both analytical and numerical approaches have been proven accurate to predict cracks. In practical design, simplified models are often used which can result in an overestimation of reinforcement amount for certain types of structures. A common method for crack control is to require a minimum amount of reinforcement, such that the stress in the reinforcement upon cracking does not exceed the yield stress. Minimum reinforcement is, however, not sufficient for restrained structures.

In the field of structural engineering, there is no consensus on how to consider imposed deformation when estimating crack widths for slab foundations. Different approaches

can be problematic when designers have to review each other calculations. A common perception is that the reinforcement amount, using standards such as Eurocode, becomes too large.

In recent years, as a response to the raised awareness on climate change, new, more eco-friendly, types of concrete mixtures have been introduced to the market. Usually, parts of the cement are replaced with waste material from the steel and coal industry. These types of concrete may have different properties regarding crack initiation and propagation than ordinary concrete. A common drawback is the longer drying time and the denser material structure. In Eurocode, the maximum crack width is directly dependent on the concrete strength class. When the production requires shorter drying times, the designer usually increases the concrete strength class. A higher concrete strength class provides more reinforcement to meet the crack width requirements. As a result, the environmental benefits of using these types of concrete become less significant.

In this thesis, cracking in concrete foundation slabs for residential houses subjected to imposed deformation is investigated, i.e. smaller slabs with a thickness usually around 100 mm. Such a slab will be exposed to the following problems: After casting, the slab will start to shrink; stresses will develop due to external and internal restraints, and cracking may occur. The shrinkage strain is simulated for concrete containing silica, which can be categorized as green concrete.

1.2 Aim

This thesis aims to investigate the cracking behaviour of concrete slab foundations for residential houses subjected to imposed deformations. Crack width calculations from analytical methods found in Eurocode 2 and a method developed at Chalmers University are compared with nonlinear finite element calculations in order to compare crack widths obtained from numerical and analytical methods. A parametric study is performed to evaluate how, and to what degree, different variables affect the crack propagation and the final crack width.

1.3 Method

To understand the described problem and the crack formation process in concrete, a theoretical basis regarding material behaviour, and restraining effects, was carried out. Also, some existing analytical approaches of how to estimate crack width were investigated.

Tie rod tests carried out at the University of New South Wales [63], were modelled in Atena 2D in order to evaluate material models and parameters. The obtained results became the basis for further analyses in the parametric study.

The shrinkage load was determined by simulating the relative humidity in the software

”ProduktionsPlanering Betong”. Thereby, the shrinkage strain could be determined as a gradient over the elements cross section.

1.4 Limitations

The effect of temperature loads, relaxation and creep on crack behaviour was investigated but not considered in the analysis. Only dead-load from the slab and the shrinkage was considered. The concrete was assumed to have a 28-days strength when load was applied.

The FE-analysis was performed in 2D and a plane stress condition was used. All material properties were time independent. The influence of ground material was not considered except for the insulation material, which was modelled as a linear elastic material.

1.5 Outline of the Thesis

Chapter 2 covers the properties and constitutive modelling for concrete. It is followed by **Chapter 3** in which the shrinkage and creep are introduced. **Chapter 4** concerns boundary conditions, i.e. restraints causing tensile compressive or flexural stresses. In **Chapter 5**, cracking in concrete is presented, including the crack behaviour, the causes of cracking and concepts of fracture mechanics. **Chapter 6** covers the finite element formulation used in the numerical analysis. **Chapter 7** presents the analytical methods including EC2 and the method by Engström. In this chapter, two examples are highlighted to show how the calculations were made. In **Chapter 8** different material models is compared with experimental tests. Finally, in **Chapter 9**, the results from the parametric study are presented which follows by a discussion part in **Chapter 10**. Conclusions are, together with suggested further research, presented in **Chapter 11**.

Chapter 2

Reinforced Concrete Structures

Even though the Romans were not the first to build with concrete¹, it would be unfortunate not to mention them. Through empirical studies, they were very successful in their development of concrete; after 2000 years there are still buildings standing tall with Roman concrete². After the fall of the Roman Empire, the use of concrete became rare until the mid 18th century. The Portland Cement was developed in 1824 and 15 years later, the reinforcement was invented. Today concrete is the most widely used construction material in the world [6], which is likely due to its durability, sustainability and its low cost. This chapter covers the components in modern concrete and its fundamental constitutive models.

2.1 Properties and Constitutive Modelling of Concrete

Concrete consists of aggregate, cement, water and possibly additives. When the constituent materials in the concrete are mixed, the water and the cement react with each other and form a binding cement-paste which keeps all materials together. As shown in Fig. 2.1, the water-cement ratio, w/c , will determine the properties of the material, such as the strength of the material [38]. An equivalent w/c -ratio can also be defined as the quota between water content, W , and the amount of binding cement, C , and equivalent additives, $k \cdot D$:

$$w/c_{eqv} = \frac{W}{C + k \cdot D} \quad (2.1)$$

Since concrete is a mixture of different materials, where the ingoing materials have a heterogeneous structure, the finished concrete will have a non homogeneous structure.

¹Burning gypsum dated to 9000 BC was found in Asia, and lime as part of a floor dated back to 7500 BC was found in Europe [47]. The invention of concrete is however a subject of speculations.

²Roman concrete *opus cementicium* was a mixture of volcanic ash and lime.

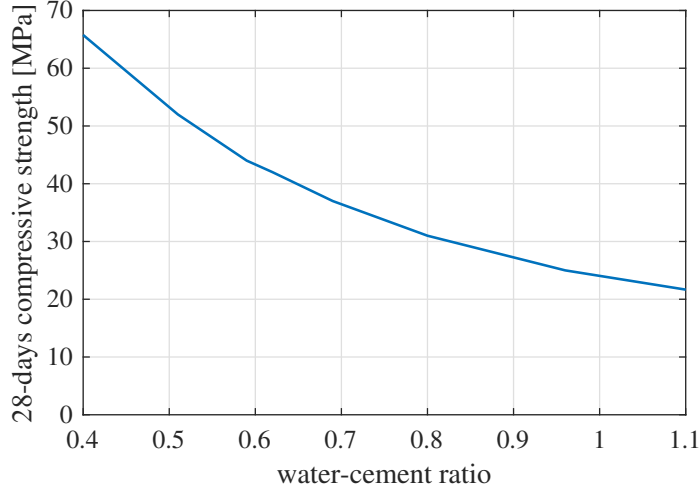


Fig. 2.1: Relationship between w/c and compressive strength for concrete at age 28 days.

Hydrated cement paste will contain numerous discontinuities such as pores, micro-cracks and voids [22]. The non-homogeneity will lead to weak zones in the material, creating local flaws in the bond between the cement paste and the aggregate. Under tensile load, micro-cracks will form in the weak areas of the material and allow cracks to propagate in the structure. Thus, the compressive strength of concrete is about ten times higher than its tensile strength. The tensile strength is usually not considered in the Ultimate Limit State (ULS) design but plays a fundamental role in the field of fracture mechanics, e.g. for determining crack widths. The compressive strength is however considered to be the most important mechanical property of the material. A typical uniaxial stress-strain diagram for a concrete material is shown in Fig. 2.2.

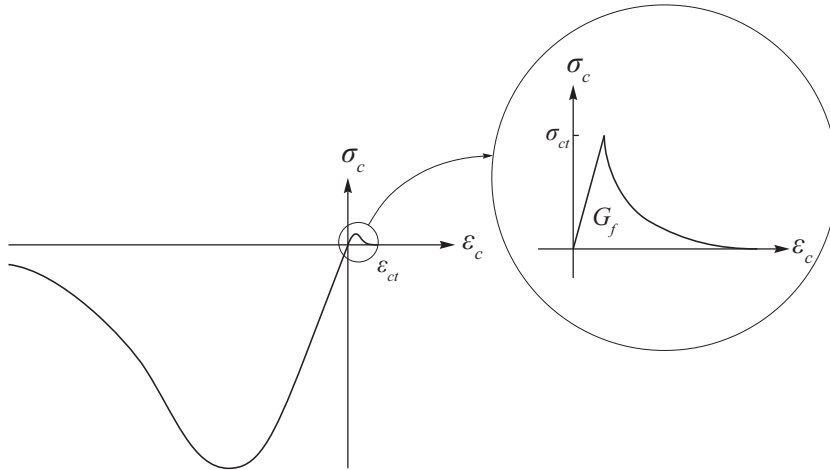


Fig. 2.2: Idealized stress-strain relationship for concrete. G_f is the fracture energy.

In standards, the concrete is usually classified on the basis of its characteristic cylinder and cubic compressive strength, which is measured 28 days after casting. For instance, a concrete class denoted as C35/45 has a characteristic cylinder compressive strength of 35 MPa, and a characteristic cubic compressive strength of 45 MPa. Often when producing a concrete batch, random cubic tests are cast from the delivered batch and

later tested to ensure that the desired strength is achieved.

When testing concrete specimens, cast from the same batch, the measured strength will have a natural scatter. By evaluating a large number of tests, a frequency distribution can show how the material strength properties vary within the same concrete class. Fig. 2.3 illustrates an idealized normal distribution curve, which is symmetric around the mean value. The lower characteristic strength is defined as the 0.05-fractile of the frequency distribution, meaning that a maximum of 5% of the tests is allowed to have lower strength than the characteristic strength value. Depending on what is evaluated, the mean strength value or the high characteristic value might be of use. Often when designing in ULS, an unfavourable situation should be considered and, therefore the lower strength value should be used.

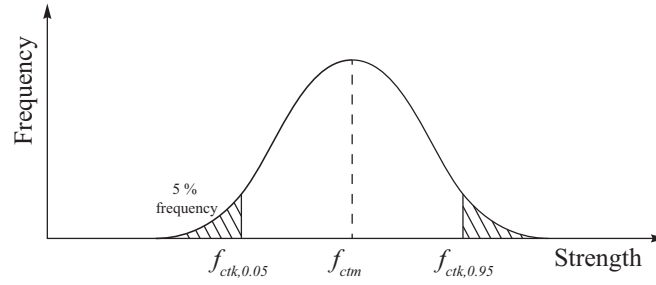


Fig. 2.3: Principle frequency curve for tensile strength of concrete. f_{ctm} is the mean tensile strength and f_{ctk} is the characteristic tensile strength.

For practical reasons, concrete is often tested in compression. The relationship between tensile strength and compressive strength are not proportional to each other, particularly not for higher strength concrete. For higher strength grades, an increase of the compressive strength only leads to a minor increase of the tensile strength [19]. In Eurocode, the relationship between the compressive strength and the mean tensile strength is defined as

$$f_{ctm} = 0.30 \cdot (f_{ck})^{2/3} \leq C50/60 \quad (2.2)$$

$$f_{ctm} = 2.12 \cdot \ln\left(1 + \frac{f_{cm}}{10}\right) > C50/60 \quad (2.3)$$

Concrete subjected to a sustained strain will lose strength with time due to a reduction in aggregate interlock and dowel action. To account for a decrease in strength, a reduction of the short term characteristic tensile strength is needed to get a more realistic response for long term loading. For this reason, Eq. (2.4), suggested by [19], can be used as a reduction of the tensile strength for sustained loading situations.

$$f_{ctk,sus} = \alpha \cdot f_{ctk} \quad (2.4)$$

where f_{ctk} is the short term tensile strength of the concrete and

$$\alpha = 0.6 \text{ for normal strength concrete}$$

$\alpha = 0.75$ for high strength concrete

2.2 Steel Reinforcement

Reinforcement is usually made out of steel, forming bars, although other types of materials can be used such as polymers, glass fibres and carbon. The main purpose of adding reinforcement to a concrete structure is to resist tensile stresses after cracking and to make the composition less brittle, i.e. to enable the structure to deform before failure. For a cracked section, the reinforcement will carry the majority of the tensile forces and limit the crack growth. Therefore, adding reinforcement can also be a way to reduce crack width when designing in Serviceability Limit State (SLS).

The reinforcement bars are classified on different characteristics such as surface geometry, yield strength and ductility. The interaction between the concrete and the reinforcement will strongly depend on the surface geometry of the bar [37]. Thus, the bars are in most cases manufactured with ribs, lugs or indentations to increase the bond between the materials and to reduce the risk of slippage.

Unlike concrete, the reinforcement steel has the same material behaviour in both tension and compression, although the contribution from reinforcement in compression is usually not taken into account when studying a concrete section. In addition, steel is a ductile material which means that it will deform before failure. The reinforcement steel can be idealized as a bilinear plastic material with a yield strength around 500 MPa. When the stress in the steel reaches the yield stress, additional loading will solely result in deformation. A typical stress-strain diagram for a reinforcement bar is shown in the figure below.

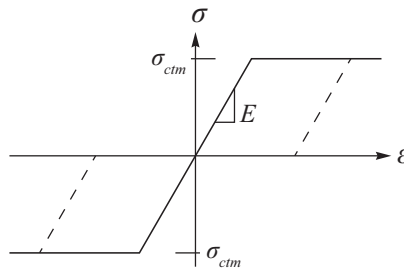


Fig. 2.4: *Bilinear stress-strain diagram for steel.*

It is important to protect the reinforced steel from chlorides to avoid corrosion. The resulting rust from corrosion can cause expansion, which in turn leads to tensile stresses and eventually cracking, delamination and spalling [37]. Another effect of corrosion is that the steel area decreases, leading to reduced strength in the reinforcement. In concrete structures, a proper cover of the reinforcement steel can prevent harmful substances to penetrate the concrete. In Eurocode, the minimum concrete cover is determined with regards to the exposure class, the w/c-ratio and the intended service life of the structure.

2.3 Early Age Concrete

There is no established definition of early age concrete, but it can be characterized by the rapid development of its properties due to the hydration. The early age period is usually considered to be the first few days or the first week after casting [39]. Hardening of the concrete will take place during the hydration period. The strength in the concrete will increase very fast during this period but the strength growth will gradually decrease with time. Betonghandboken [37] divides the hardening process into four phases. In the first phase, "fresh concrete", some hardening takes place but the concrete will remain workable. In the next phase, "young age concrete", the actual hardening begins and material properties change rapidly. In the third and fourth phase, "almost hardened concrete" and "hardened concrete", the properties are almost fully established.

In a normal concrete mixture, the cement particles are often very small, in the range of a couple of micrometres in diameter [37]. The particles are assumed to be spherical which gives a large surface area per weight unit. When adding water to the cement mixture, these particles will be suspended freely and will have a large surface area in direct contact with water. The contact with water immediately starts a chemical reaction, namely the hydration of the cement, where the cement binds water to its own molecular structure. The hydration of the cement transforms the clinker particles to a gel-like structure of small crystallised grains. As the hydration proceeds, pores arise between the forming gel particles.

The development of the gel starts as a thin coat around the cement particles which gradually thickens in the area between unreacted cement and the outer surface of the cement gel, see Fig. 2.5. The rate of hydration is dependent on how much water the cement particles have access to. While the gel thickens, the water reaching the cement decreases, leading to a reduction in hydration-rate [37]. When a cement particle hydrates and the gel is formed, the volume of the initial grain increases. After some time, the cement gel from different cement particles reaches each other. At this stage, strength starts to develop in the concrete. Therefore, the strength development in a concrete mixture is dependent on the distance between cement particles, i.e. the w/c-ratio.

In the hydration of the cement, an exothermic process will take place, called the heat of hydration, which will allow large amounts of heat to develop. The heat of hydration is most affected by the type of clinker material, but also by the water-cement ratio, the particle size and the curing temperature. In large concrete structures, the produced heat may be higher than the structure can dissipate, i.e. adiabatic conditions are present. The high temperature is more likely to occur in central parts of a structure, which may cause undesirable stresses as the concrete cools to the ambient temperature. One way to control the heat development is to adjust the proportion of clinker material, e.g. C_3S and C_3A which have the highest rate of heat evolution [12]. In most situations, adiabatic conditions are unlikely due to the fact that the temperature in the concrete structure will be affected by the surrounding temperature. For a concrete slab, the dissipation of heat from the surface may cause temperature gradients and result in thermal cracking.

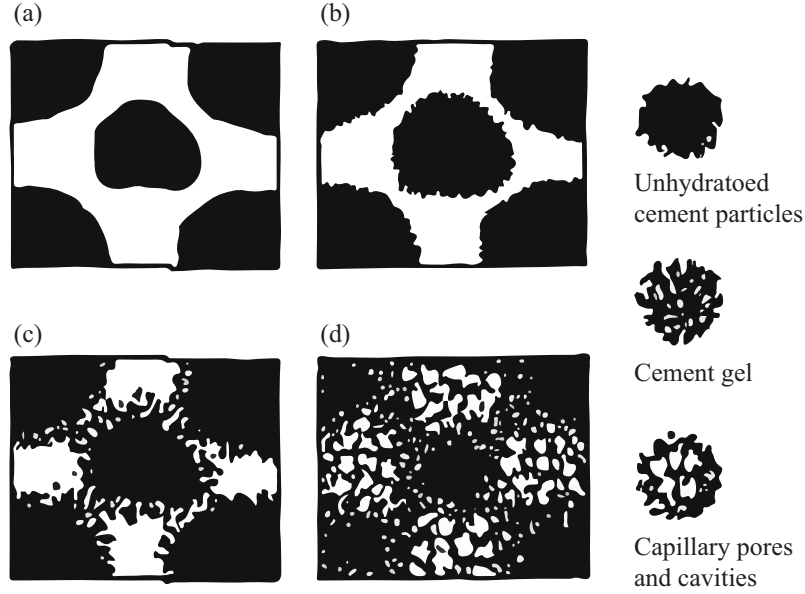


Fig. 2.5: *Structure of cement paste or mix (a) immediately after mixing. (b) after a few minutes. (c) upon setting. (d) after some month. Figure after [37].*

In the early age concrete a volumetric change in the concrete, in combination with external and internal restraints, induces stresses in the structure which can lead to cracks in the hardening process. For modern concrete with a low water-cement ratio and additional microsilica, the risk of cracking tend to increase. This can be explained by the increase of autogenous deformation, a higher rate of heat evolution and increased brittleness [41].

To consider the development of the tensile strength of concrete during the hydration is important when predicting cracks. Experimental results based on uniaxial tension tests are few for early age concrete because this type of tests is difficult to conduct. Instead, alternative methods have been developed where a concrete specimen can be loaded by compression to find the corresponding tensile strength. Examples of such tests are the so called Brazilian tests and three point bending tests [41].

2.4 Green Concrete

The manufacturing of cement contributes to around eight percent of the world's total CO₂ emissions [35]. Various efforts from the concrete industry and researchers have been conducted to find less energy-intensive concrete mixtures. In the period 1998-2002, a number of leading companies joined forces to bring knowledge of more environmental friendly types of concrete, which resulted in a 4-year long project named Green Concrete³ [59]. For today, green concrete means environmentally friendly concrete, although no general definition is established. According to [19], green concrete is characterized by "having a significantly improved sustainability compared to ordi-

³The full title name for the project was 'Centre for Resource Saving Concrete Structures', or 'Danish Centre for Green Concrete'.

nary structural concrete”. In practice, green concrete types are usually produced with fine materials replacing the cement, a reduction of the total binder content or with recycled aggregates. Fig. 2.6 illustrates the interest in green concrete in research.

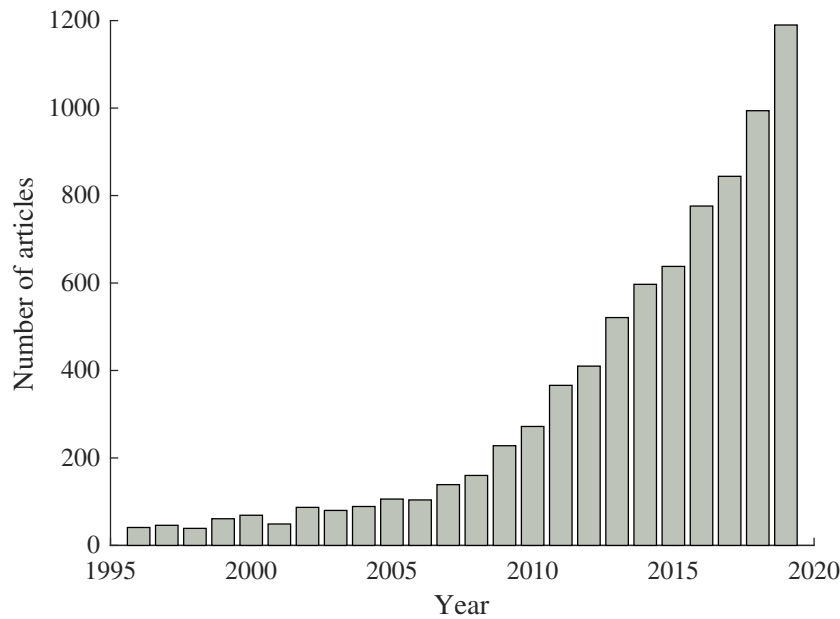


Fig. 2.6: *The figure shows the number of articles containing the phrase "green concrete". The data is provided by Google Scholar.*

Common alternatives to the traditional Portland cement are byproducts from the steel and coal industry, such as fly ash, blast furnace slag and microsilica. Since these materials are byproducts, it is considered to be emission free when used in concrete. All cement in a concrete mix can not however be substituted, but it can significantly reduce the emission of greenhouse gases. [59].

In earlier studies [54], a comparison of strength properties between traditional portland cement based concrete and three types of green concrete were conducted. Concrete specimens were tested after 7 and 28 days, in both in compression and tension. The studies showed a slight reduction of both compressive and tensile strength as well as a reduction in elasticity for the green concrete specimens. However, the difference in strength for the traditional concrete and the green concrete was significantly higher after 7 days than after 28 days, which indicates a delay in strength growth for green concrete mixes. The reason for the delay in strength development is the higher content of finer particles in green concrete, which decrease the water diffusion in the material. In addition, the green concrete specimens contained a lower chloride permeability than the traditional concrete samples. This indicates that green concrete has good durability properties compared to normal concrete.

2.5 Stresses and Strains

In a reinforced concrete structure, the stress and the associated strain are not constant, but will rather change with time as creep and shrinkage develop [21]. It is

therefore important to apply time-functions for both strain and stress when analysing a concrete structure over a longer period. Likewise, different type of strains has to be distinguished. According to [16], strains can either be stress-dependent or stress-independent. When a structure is subjected to a load, the strains that occur in the material are stress-dependent. Stress-dependent strains are also developed when a restrained structure has a need for deformation. It is common in practice to assume that stress-dependent strains for an uncracked concrete structure with regards to a short-term response can be described with Hooke's law as follows:

$$\varepsilon_c(t_0) = \frac{\sigma_c(t_0)}{E_c(t_0)} \quad (2.5)$$

where $\sigma_c(t_0)$ is the concrete stress and $E_c(t_0)$ is the modulus of elasticity of concrete at age t_0 . Under sustained stress, creep will increase the strain with time and the total strain can be obtained as:

$$\varepsilon_c(t) = \frac{\sigma_c(t_0)}{E_c(t_0)}[1 + \varphi(t, t_0)] \quad (2.6)$$

where $\varphi(t, t_0)$ is the creep factor which depend on the age of loading, t_0 , and the age, t , for which the strain is calculated. Creep is further investigated in Chapter 3.

Concrete subjected to shrinkage or thermal variations develops stresses if the structure is restrained. For free shrinkage, the strain that develops between the time t_s , which is the time when shrinkage starts to develop, and the later instant t , can be expressed as

$$\varepsilon_{cs}(t, t_s) = \varepsilon_{cs0}\beta_s(t - t_s) \quad (2.7)$$

where ε_{cs0} is the final value for shrinkage and $\beta_s(t - t_s)$ is a function adopted from [18]. If the concrete structure is restrained, stresses will develop. Thermal variations can also cause strains that are independent of stresses, where the thermal concrete strain ε_{cT} is a function of the change of temperature, ΔT , and the coefficient of thermal expansion for concrete, α_{cT} . However, this thesis will not look into stresses developed from temperature variations.

Reinforced concrete is a composite structure which utilizes the high compressive strength of the concrete and the tensile strength of the reinforcement steel. The interaction between the two materials is therefore fundamental and determines how well the structure will function. The interaction between the two materials affects how stresses are carried and distributed within the structure, consequently determining how the structure will crack and deform.

In an uncracked reinforced concrete structure, concrete and steel reinforcement can be assumed to fully interact with each other [16]. This means that the strain in the concrete and in the steel are equal and increase at the same rate:

$$\Delta\varepsilon_c = \Delta\varepsilon_t \quad (2.8)$$

Since the ultimate tensile strain for the concrete is significantly lower than for the steel, the concrete will fail before the steel is fully utilized. In an ordinary reinforced structure, the steel will only be utilized up to about 5% before failure occurs in the concrete, and cracks develop. This means that a large amount of reinforcement can not prevent the occurrence of cracks. To allow some cracks to develop in the structure, the steel strain can be increased and utilized more efficiently. Therefore, the purpose of reinforcement should be to distribute cracks instead of preventing them, and thus limit the width of each crack [16].

2.6 Slab Foundation

Slab foundation⁴ is a common type of concrete foundation system used for all types of buildings, both residential and industrial. Usually, the foundation type does not need any major excavation work, but it is more or less built directly on the existing ground. The thickness of the slab can vary from 100 mm for smaller slabs, where no significant loads are applied, up to a few hundred millimetres for larger industrial floors. There are no specific rules regarding the minimum thickness of smaller slabs, but the requirements for sufficient concrete cover must be fulfilled and will often determine the minimum thickness. According to [5], foundation slabs that are thinner than 130 mm and without edge beams may have a risk of curling due to an uneven strain profile. An effective way to increase the stiffness, and thereby decrease the risk of curling, is to make the slab thicker. Nevertheless, some curling effect will be present for all slabs thinner than 400 mm. Slabs usually have locally thickened areas, called hauches, to be able to resist large local loads. The hauches may be located at an edge of the slab, as edge beams, to resist loads from the exterior walls, but they may also be located where supporting walls or columns are connected to the foundation.

In regions with a colder climate, it is normal to use a layer of insulation to fulfil the energy requirements. The insulation assures that the relative humidity (RH) in the slab is kept to an acceptable level. It is common in Sweden to place a 300 mm insulation layer underneath the slab but it can in some local areas be thinner, for example under hauches, see Fig. 2.7. Firm mineral wool or foam insulation are commonly used materials to isolate foundations. The edge beams are often cast in prefabricated moulds with pre-installed insulation, an approach used to avoid thermal bridges. Although the insulation can be situated directly on the soil, it is common to have a drained capillary breaking layer of at least 150 mm granular material, for example macadam. A rule of thumb is that the insulation should never be exposed to capillary water [37]. To further ensure that no moisture diffusion from the ground reaches the slab, the isolation joints can be displaced with respect to each other for different layers.

⁴Slab-on-grade may be an alternative term for the type of slab foundations which is investigated in this report.

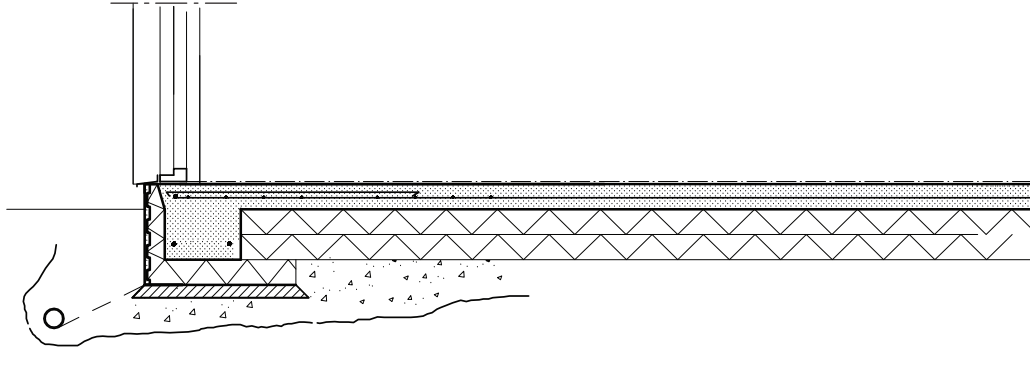


Fig. 2.7: *Typical section for a concrete slab foundation for a residential house. The concrete is usually resting on polystyrene material.*

The cast sequence for a slab foundation depends on both the slab geometry and the intended use of the building. For example, for larger foundations, used for industrial buildings, it is common to cast in stages, connected with joints. Slabs used in residential houses are, however, often smaller and can be cast at once. A conventional approach is to cast thicker parts, such as haunches, before casting the relatively thin slab. For areas that require a certain slope, such as bathrooms, the slab is usually lowered by a couple of millimetres to allow for levelling work later on.

The production time for residential houses is often very short and together with many floor manufacturers demand of a low RH in the concrete, the drying of the concrete slab is a critical activity in this kind of projects. In order to shorten the drying time, a higher quality concrete class is usually used. The cost of a higher concrete quality is relatively small compared to the profit from reduced production time. In contrast, higher concrete classes increase the shrinkage strain which results in a higher reinforcement amount. In addition, the amount of cement in the concrete mixture increase, which in turn contributes to higher CO₂ emissions.

Chapter 3

Shrinkage and Creep of Concrete

Two physical properties in concrete, which evolve over time, are shrinkage and creep. To some extent, these properties depend on the concrete's material structure i.e. the concrete class, cement type, water content and even to some extent on the geometry of the structure. However, external parameters in the surrounding environment such as the temperature and the relative humidity have an effect on how fast, and to what extent, these phenomena will develop. Creep is the deformation exhibited from a sustained load, while shrinkage is a process which will lead to stress-independent deformations. This chapter will cover different types of shrinkage, creep and how they evolve over time.

3.1 Shrinkage

Concrete goes through processes where the concrete mix both loses and absorbs moisture. The change of moisture content will result in a volume change in the concrete. A loss of moisture is called shrinkage and will cause the concrete to contract. An absorption of moisture is called swelling and will make the concrete expand. A distinction can be made between different types of shrinkage, depending on when the loss of moisture occurs and in which stage of the hardening process that it is in.

3.1.1 Plastic shrinkage

Plastic shrinkage takes place within six hours after placing due to an immediate loss of water through evaporation from the concrete surface before it has hardened. When the rate of evaporation, from the concrete surface, exceeds the rate in which the water can be transported, namely bleeding, the concrete at the surface layer will contract more than the layers underneath. Since the surface layer is restrained by the wet underlying concrete, the contraction induces tensile stresses which exceed the capacity of the plastic concrete and form cracks [43].

The rate in which water evaporates from the surface of a concrete structure depends on

several factors such as humidity, temperature and the wind speed in the surrounding environment. The bleeding rate inside the concrete strongly depends on the w/c-ratio, where a higher ratio will increase the bleeding and vice versa. Another material property that affects the bleeding is the size of the particle in the cement paste. Finer particles make the pore system narrower which decreases the bleeding rate. Additional factors, such as the depth of the structure, may also affect the rate of bleeding [52].

Furthermore, a slow bleeding rate combined with a fast rate of drying at the surface can lead to other problems such as delamination. When the surface of the concrete structure hydrates and densifies faster than the underlying, still plastic, concrete there is a risk of water and air getting trapped under the surface layer from the continued bleeding [31]. These areas where air and water voids exist become weak zones where the concrete tend to crack.

In practice, plastic shrinkage can be prevented, or limited, by covering the surface with plastic sheeting to decrease the evaporation from the concrete surface. At the time of casting, the concrete can be vibrated to further lower the risk of cracking. The vibration increases the water transported to the surface, which prevents it from drying out.

3.1.2 Autogenous shrinkage

Autogenous shrinkage takes place during the hydration phase of the cement paste, that usually starts within a couple of hours after mixing [51]. The concrete loses moisture through chemical reactions where the cement clinker binds water without any transfer of moisture to the surrounding environment, which results in an external volume change. The rate of the autogenous shrinkage is at its highest during the curing of the concrete but the main component of autogenous shrinkage gradually decreases within the first days of hardening.

The mechanisms of autogenous shrinkage can be explained with different approaches. A common way to describe the driving forces behind the shrinkage is based on the capillary tension theory [64]. In a concrete mix a pore system form when water is consumed in the hydration process. In the continued hydration process, more water bind to the cement and build up an internal pressure in the pore system which result in a volume reduction. The magnitude of the pore pressure is influenced by the distance between particles in a concrete mix, where a more dense mix gives an increase of pressure [28]. The size of the pore system may also influence the amount of potential water being stored in the concrete. Smaller pore sizes reduce the internal RH in the mixture that causes an increased shrinkage deformation.

According to Eurocode [17], autogenous shrinkage is linearly dependent on the concrete strength, for a specific type of concrete. The absolute value of the autogenous shrinkage, ε_{cs} , can be estimated as

$$\varepsilon_{cs}(\infty) = 2.5(f_{ck} - 10) \cdot 10^{-6} \quad (3.1)$$

A time dependent coefficient can be multiplied to calculate the autogenous shrinkage at a specific time according to:

$$\varepsilon_{cs}(t) = \beta_{as}(t) \cdot \varepsilon_{cs} \quad (3.2)$$

where $\beta_{as}(t)$ is a time function and $\varepsilon_{ca}(\infty)$ is the final autogenous shrinkage. $\beta_{as}(t)$ can be determined as

$$\beta_{as}(t) = 1 - e^{-0.2 \cdot t^{0.5}} \quad (3.3)$$

where t is the number of days from the casting of the concrete.

3.1.3 Drying shrinkage

The moisture loss which takes place after the concrete has hardened, causing an additional contraction of the material, is called drying shrinkage. Water, stored in the concrete's pore system, is transferred to the surrounding environment, which usually has lower relative humidity than the concrete itself. The contraction caused by the drying shrinkage is to some degree reversible, meaning that the concrete absorbs moisture and expands when it is subjected to a higher level of relative humidity. The reversible part of the drying shrinkage is between 40 - 70 %, depending on when the drying occurs and on the degree of the hydration. The irreversible part of the shrinkage can be explained by the additional physical and chemical reactions which take place when adsorbed water is removed. The reactions form bonds and close off parts of the pore system, which allow less water to enter the body on re-wetting [43]. The pores in the concrete have a wide range of sizes and will, in turn, evolve with time due to hydration and ageing. To a large extent, the driving mechanism of the water transportation inside the concrete is dependent on the size of the pore system. This makes the estimation of drying a complex problem, since the pore system changes over time.

The drying of the concrete is a slow process, one which occurs over a long period of time, usually over dozens of years. A high rate of drying takes place in the initial time period after hardening, and will gradually decrease while the concrete is ageing. The rate in which shrinkage develops is strongly dependent on the relative humidity and the temperature in the surrounding environment, as well as the water content in the concrete [37]. After several years, in a constant climate, the RH in the concrete will be in equilibrium with its surrounding environment and the shrinkage reaches a limit value.

In addition, the size of the structural member with respect to its surface area, effective thickness (h_0), will affect the rate of drying. For larger members, the internal water has a longer path before reaching a surface exposed to drying and will therefore dry slower [43]. The effective thickness can be expressed according to [17] as

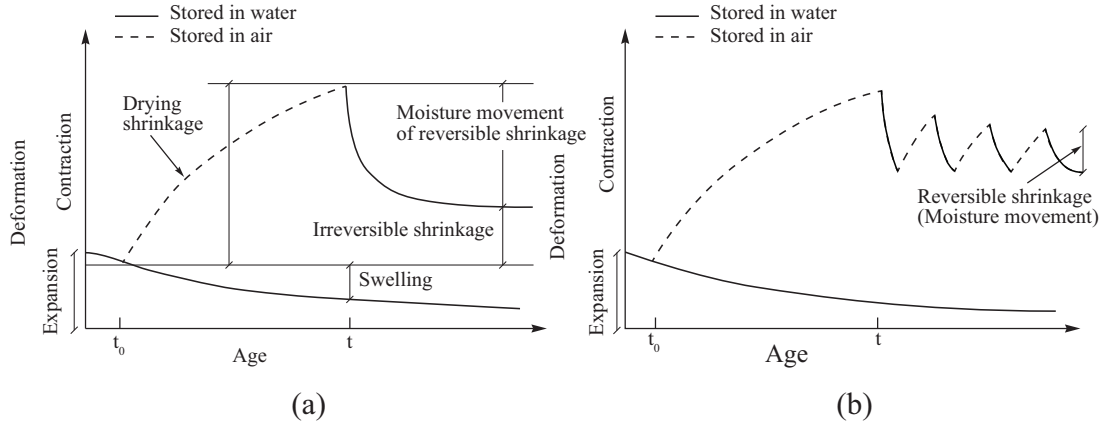


Fig. 3.1: *Moisture movement and deformation in concrete. In (a) the concrete is dried from t_0 to t and then resaturated. In (b) the concrete is dried from t_0 to t and then subjected to cycles of drying and wetting.*

$$h_0 = \frac{2 \cdot A_c}{u} \quad (3.4)$$

where A_c is the cross section area of the concrete and u is the circumference of the cross section exposed to drying.

Cross section variation

The moisture content in a concrete member will have a nonlinear distribution over the cross section, resulting in a different need for movement. Depending on how the structure is able to dry, the distribution of the strains can be either symmetric or non-symmetric. For a symmetric distribution, when the moisture condition is similar at the top and the bottom, the assumption of a uniform strain distribution is valid. However, if the drying conditions at top and bottom of a concrete member is significantly different, the assumption of a uniformly strain distribution over the entire cross section is no longer valid. In this case, a skewed distribution of the shrinkage strain will be a better representation of how the actual strain distribution looks like. In addition, for thick structures with equal drying conditions at top and bottom, a nonlinear strain distribution can be of interest.

Externally unrestrained shrinkage

Unrestrained shrinkage can be defined as the shrinkage deformation a body would get due to a change of moisture content without any moisture gradient within the body [37]. However, even unrestrained drying shrinkage gives a risk of cracking. When the cement paste contracts, the aggregate in the cement does not change in volume and acts as a “ridged” point for the cement paste. This may result in tensile forces acting on the cement paste and corresponding compression forces acting on the aggregates,

see Fig. 3.2. As a result, micro-cracking starts to occur in the cement paste. When studying and estimating the effect of shrinkage in a concrete structure it is therefore convenient to study the concrete as a homogeneous material, and not accounting for the material being composed by different materials.

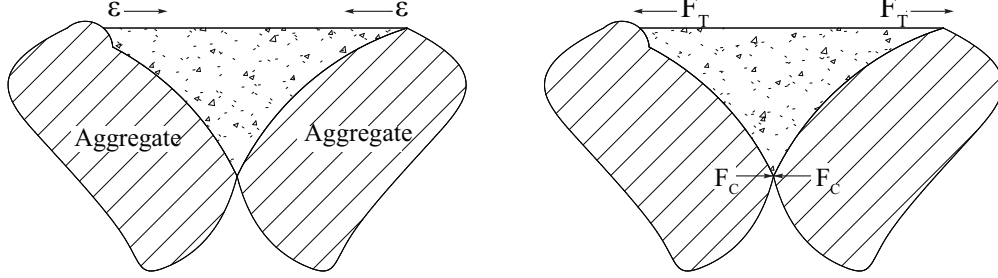


Fig. 3.2: *Forces acting on the aggregate.*

The free, external shrinkage of a body can be measured as the external movement of an unloaded structure. The measurable shrinkage of a body will be unique for the specific situation as long as a moisture gradient is present. After a long time, no moisture gradient will exist in the body and the internal stresses will be very small. Theoretically, some stresses due to nonlinear effects, such as uneven hydration and the formation of micro-cracks, will always be found in the body. In design practice, though, it is convenient to use a reference shrinkage, representing the ultimate shrinkage strain for a concrete body in a standardized environment. According to [37], the reference shrinkage can be expressed as

$$\varepsilon_{s0} = \varepsilon_{\infty}(t_s = 28 \text{ d, RH} = 50\%, T = 20^{\circ} \text{ C}) \quad (3.5)$$

3.2 Creep

Concrete subjected to sustained load will have additional deformation with time, compared with the elastic deformation, referred to as creep. It can be defined as the difference in deformation between a loaded specimen and an unloaded specimen, both geometrically equal and with the same environment history. This phenomenon was discovered by Hatt 1907 [23], and has been a complex problem ever since. Internal chemical reactions changing material properties, moisture diffusion through the material and other nonlinear effects makes it difficult to devise creep tests [4].

Creep can be separated into two superposed strains, namely basic creep deformation and drying creep deformation. The former is the deformation occurring without drying, and the latter is the additional deformation in the case of drying. The inner moisture state is of paramount importance in the delayed behaviour of concrete. The lower relative humidity, the lower the creep strains, which has been shown by experimental tests by [60]. Other important factors affecting the creep are the following: duration of loading, temperature level and age of the concrete at loading. In addition, internal factors as the water-cement ratio, type of cement and type of aggregate can influence the creep.

Creep is linearly dependent on the stress for loads less than 40 %¹ of concrete strength [4], and at high stresses, higher than 40 %, the creep-stress relation appears to be nonlinear, see Fig. 3.3. The creep effect is highly influenced by the degree of hydration when the load is applied and for how long the structure is subjected to a constant level of loading. Furthermore, the history of water content and temperature in a concrete member will affect the creep but are neglected in design practice as a simplification. However, for the assumption that the stresses remain less than 40% of the strength, the creep is assumed to be linear, and may thus be characterized by the compliance function $J(t, t')$. $J(t, t')$ depends on both t and t' separately, and as t' increase, the creep value for fixed $(t-t')$ diminishes. For sequenced loading and unloading, the principle of superposition, introduced by Boltsmanns, can be implied as long as the creep deformation is proportional to the stresses. An integral type formulation can be introduced as

$$\varepsilon(t) - \varepsilon^0(t) = \int_{t_1}^t J(t, t') d\sigma(t') \quad (3.6)$$

in which t is the time measured from casting of concrete, t' is the time from casting when the load is applied, σ is the linearized stress, ε is the linearized stress strain and ε_o is the given stress-independent inelastic strain comprising shrinkage and thermal dilatation.

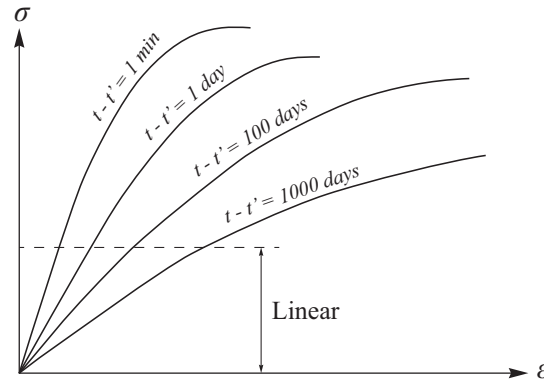


Fig. 3.3: The figure shows creep curves for loading at different ages.

Creep strains are partially reversible, meaning that if the load is removed the strain will recover. This phenomena is illustrated in Fig. 3.4, which shows that the recovery response will vary with time.

In Eurocode, creep has to be considered in SLS, and in ULS if second-order effects are of importance. Usually in design practice, creep is considered by a reduced Young's modulus, $E_{c,eff}$, calculated as

$$E_{c,eff} = \frac{E_c}{1 + \varphi(t, t_0)} \quad (3.7)$$

¹Eurocode states 45 %

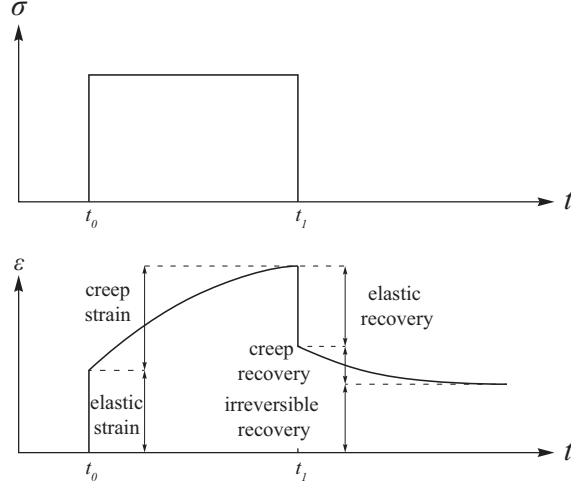


Fig. 3.4: *Creep and recovery for a specimen loaded at time t_0 and unloaded at time t_1 .*

where E_c is the initial Young's modulus and $\varphi(t, t_0)$ is the creep coefficient. According to Eurocode, the creep coefficient can be determined as

$$\varphi(t, t_0) = \beta(t, t_0)\varphi_0 \quad (3.8)$$

where $\beta(t, t_0)$ is the time function. φ_0 is the notional creep coefficient and can be estimated as

$$\varphi_0 = \varphi_{RH} \quad (3.9)$$

where

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \quad (3.10)$$

$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \left[\frac{35}{f_{cm}} \right]^{0.7} \right] \left[\frac{35}{f_{cm}} \right]^{0.2} \quad (3.11)$$

$$\varepsilon(t)_{c, creep} = \varphi(t, t_0) \frac{\sigma_c}{E_c} \quad (3.12)$$

where t is the actual time considered and t_0 is the concrete age when load was applied.

Chapter 4

Restraints in Reinforced Concrete Structures

A concrete element subjected to a volume change will always, to a certain degree, be restrained [15]. The restraints are provided either by the supports or by different parts of the material, namely external and internal restraints, respectively. This chapter concerns restraints causing tensile, compressive or flexural stresses in the concrete due to volume changes.

4.1 Internal Restraint

Internal restraints, or eigenstresses, occurs due to differential volume change in a member. This is the case when nonlinear distribution of shrinkage and temperature are present, or due to constituents with different material properties. Eigenstress can be defined as the stress given by self-equilibrated internal stresses caused by internal restraints without a need for external forces as follows:

$$\sum_A \sigma_i \cdot dA = 0 \quad (4.1)$$

At early age, as autogeneous shrinkage and thermal expansion arise, internal restraints will generate stresses between the cement paste and aggregates and initial cracking may occur. At mature age, however, the total shrinkage is a more concern. In a reinforced concrete structure subjected to shrinkage, the reinforcement will counteract the desired contraction of the concrete, and the restricted movement will consequently develop stresses. These stresses will be concentrated at the interface between the concrete and the reinforcement and decrease further away. The magnitude of the stresses depends not solely on the degree of shrinkage, but also on the stiffness ratio between the two materials as well as the structural geometry.

Shrinkage may not only be a concern due to different material properties, but due to

differences in relative humidity causing uneven shrinkage within the structure. The uneven contraction of the structure can be seen as different layers in the concrete restraining each other, which will contribute to bending stresses developing in the member [24]. In the case of a concrete slab foundation, the relative humidity is usually higher at the top than the bottom causing a nonlinear strain distribution. In design, the nonlinear distribution is usually simplified to a linear or a uniform distribution as seen in the figure below.

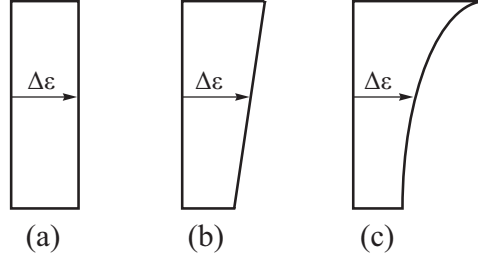


Fig. 4.1: Strain distribution due to shrinkage: (a) uniform (b) linear (c) nonlinear

4.2 External restraint

External restraints are due to friction or adjoining members preventing the structure to move freely. In reality, structural connections are often complex which can make it difficult to estimate the degree of restraint. Usually in design, two simple cases of external restraints are considered: continuous edge restraint and short end restraint, see Fig. 4.2. In a continuous edge restrained structure, the adjacent material is preventing the concrete from obtaining its desired volume change along with one of its boundaries while the rest of the structure is allowed to move freely. The degree of restraint along the restrained boundary will depend on the geometry as well as the strength and elasticity relation between the two materials [15]. Throughout the structure, the degree of restraint will decrease as the distance to the boundary gets larger. For an end restrained structure, the degree of restraint in the horizontal direction will be more or less uniform.

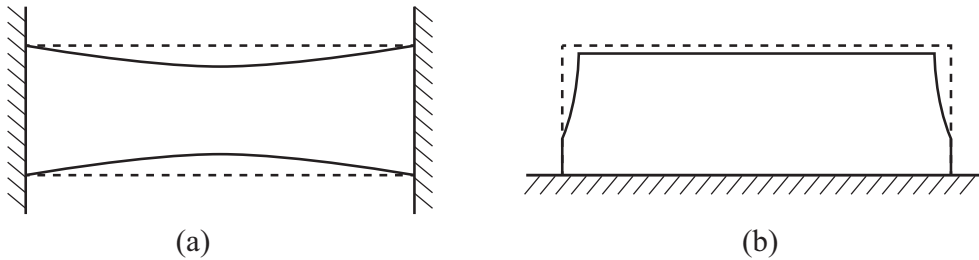


Fig. 4.2: Illustration of (a) an end restrained bar. (b) a continuous edge restrained wall.

The restraint stress, σ_R , can be found by utilizing Hooke's law as

$$\sigma_R = \varepsilon_R E \quad (4.2)$$

where ε_R is the restrained strain and E is Young's Modulus.

4.3 Degree of restraint

The degree of restraint, R , is the ratio of how effectively a restraint prevents the need for free movements. It can be defined as follows [16]:

$$R = \frac{\text{actual stress}}{\text{stress when completely restrained}} \quad (4.3)$$

A restraint degree equal to 0 indicate no restraint, i.e. the structure can move freely, and a restraint degree equal to 1 indicates full restraint, i.e. the movement is completely prevented. In the case of a short ended structure, the restraint degree can be determined according to Eq. (4.4). The restraint degree between the embedded reinforcement and concrete can be expressed according to Eq. (4.5).

$$R = \frac{\frac{\sigma_c}{E_c}}{\sigma_c \left(\frac{1}{E_c} + \frac{A_c}{S \cdot l} \right)} \quad (4.4)$$

$$R = \frac{1}{1 + \frac{A_{M1} E_{M1}}{A_{M2} E_{M2}}} \quad (4.5)$$

Since the restraint degree depends on the stiffness of both the concrete and the adjacent material, the restraint force will depend on the combined structural response to a certain strain. A change of stiffness, for example due to cracking and creep, will therefore decrease the restraint force. In practice, however, the need of movement is neither fully restrained nor free. To account for flexibility of the boundaries, a structure can be studied as partially restrained.

4.4 Friction

For concrete slab foundations, external restraints may be caused by frictional forces, haunches or piles. In order to reduce the external restraints, joints may be arranged to allow movements in the slab, which is a common method for larger foundation slabs.

Friction are usually divided into three categories: dry friction, fluid friction and internal friction. Fluid friction occurs when adjacent layers in a fluid are moving at different velocities. Dry friction occurs when surfaces of two solids are in contact, and internal friction is the force resisting the movement of two particles in one material. For a concrete slab foundation, both dry friction and internal friction may have an impact on the structure [45].

For dry friction, also called Coulomb friction, the friction force will act in the opposite direction to the movement of the slab [37]. The magnitude of the friction force between two materials can be described with the frictional coefficient, μ , defined as

$$\mu = \frac{F_f}{F_N} \quad (4.6)$$

where F_f is the maximal frictional force in the tangential direction, F_N is the normal force acting perpendicular to the interface surface and $\mu = \mu_s$ (static) or $\mu = \mu_k$ (kinetic). The frictional force can be assumed proportional to the normal force as long as the body is in equilibrium. When the friction force exceeds the maximum static frictional force, the body is no longer in equilibrium and it will start to slide. The kinetic friction is often lower than the maximum static friction but the displacement occurring before sliding starts is negligible. For structures resting on ground this means that the friction-displacement diagram can be simplified as a bilinear curve with maximum friction force corresponding to the static frictional force [45].

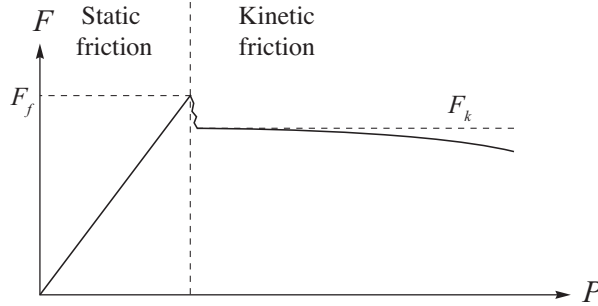


Fig. 4.3: *Illustration of a friction curve for dry friction. After [45].*

Several studies have been conducted to estimate the friction relations for concrete cast on ground, e.g. [45] conducted laboratory test for 0.2 m thick concrete slabs cast on sand and crushed aggregate. Between the 1920's and 1950's, American friction tests were performed which are summarized in [58]. Real full-scale tests in Denmark can be found in [20]. However, studies on the interlayer friction characteristics of insulation material appear to be rare. In [55], tests were conducted for the interface strength between cast in place concrete and insulation material (geofoam). High residual shear stresses were exhibited which was assumed to be due to the roughness of the concrete. The frictional coefficient from the performed tests was estimated between 0.6 to 1.2. Suggestions from [37] on the friction coefficient for common sub-grade materials, including insulation material, are presented in Tab. 4.1.

It was concluded that friction will cause external restraint and thus stresses when the structure is subjected to e.g. shrinkage. Thus, the friction may be seen as unfavourable for the structure, at least to a certain point. When cracking occurs, the frictional forces will function in analogy with the reinforcement and redistribute the stresses along the length, and thereby reduce the crack widths [44].

Sub-grade material	Static friction coefficient (μ_s)
Crushed aggregate	2.0
Crushed aggregate	1.5
Insulation material	1.0
Sand layer (even). Plastic sheeting	0.75

Tab. 4.1: *Friction coefficients for common sub-grade materials. From [36].*

Chapter 5

Cracking in Concrete

Concrete cracks when the tensile stress exceeds the tensile strength of the material. Since the tensile strength of concrete is comparably low, cracks can hardly be avoided and are therefore considered in design practice. After cracking occurs in a reinforced structure, the tensile force at the cracked section will be resisted only by the reinforcement, and the stiffness of the whole structure will be reduced. Concrete in tension can be considered to be a linear elastic material before cracking. Once the tensile capacity is reached and a crack is initiated, a nonlinear behaviour has to be considered. Cracks occur due to internal settlements, early-age plastic shrinkage, early drying out, temperature development and inappropriate heat curing. Cracks can also be caused due to shrinkage, creep, external movements, environmental influence and fire [16]. This chapter explains the crack behaviour, the causes of cracking and briefly the concepts of fracture mechanics for concrete structures.

5.1 Plastic Shrinkage Cracks

Cracks due to plastic shrinkage will appear one to three hours after casting. The cracks are usually oriented parallel in relation to one another but in some cases in a random pattern. The length of the cracks vary from dozens of millimetres to 1-2 metres [37]. The crack propagates at the surface but can extend throughout the entire cross section of a structure, usually with a very narrow crack-width [52].

To reduce the risk of plastic shrinkage cracks, the most effective approach is to prevent evaporation. Such an approach can be achieved by covering the concrete with polyethylene sheets or by using fog spray during the most critical time.

5.2 Thermal Cracks

Thermal cracking occurs due to thermal differences in the structure, caused by the hydration process in the concrete. Temperature variations in the surrounding envi-

ronment can also cause thermal cracks, but will in practice, for slab foundations for residential houses, only be the case for fire damages. Thermal cracks appear as a random pattern on the surface of a structure, usually 0.01-0.1 mm wide and less than 50 mm deep. In plates thinner than 0.7 m, thermal cracks are rare [37].

5.3 Drying Shrinkage Cracks

A concrete member exposed to shrinkage will crack if the member is restrained. External restrains will cause cracks to propagate perpendicular to the direction of the desired movement. The crack-width depends on the number of cracks and the total movement of the member, which in turn depends on the design and amount of the reinforcement. To reduce shrinkage cracks, without reducing the magnitude and the velocity of the shrinkage, a proper reinforcement design giving an increased amount of cracks and thus smaller cracks is advantageous. For bigger slabs, joints can be placed at certain distances to reduce the relative movement for each section.

5.4 Crack Width Limitations

Crack widths should be limited to satisfy requirements concerning durability, functionality and appearance. Reinforcement may be subjected to moisture and oxygen due to cracks, which in turn will affect the structural behaviour. However, in many cases, surface-crack widths will be more of an aesthetic concern [15]. This is usually the case for slab foundations for residential houses.

Surface-cracks are easy to measure and are therefore often the focus in limitations. According to Eurocode, the crack width limits depends on the exposure class of the structure, see Tab. 5.1.

Exposure class	Corrosion sensitivity			Little corrosion sensitivity		
	L 100	L 50	L 50	L 100	L 50	L 50
XC0	-	-	-	-	-	-
XC1	0.40	0.45	-	0.45	-	-
XC2	0.30	0.40	0.45	0.40	0.45	-
XC2, XC4	0.20	0.30	0.40	0.30	0.40	-
XS1, XS2, XD1, XD2	0.15	0.20	0.30	0.20	0.30	0.40
XS3, XD3	0.10	0.15	0.20	0.15	0.20	0.30

Tab. 5.1: Permitted crack width, w_k , in mm, according to the Swedish National Annex, EKS 2 [9].

In concrete slabs for residential houses, radon leakage may also be a problem if the crack width is too large. The gas will pass through the concrete due to air pressure

gradients, but since the gas flow through an uncracked structure is very small, this can often be accepted. However, if cracks appear in the structure, the flow of gas through the structure will drastically increase. The gas penetration can be calculated as a function of the crack width according to Eq. (5.1). The equation states that if the crack width, w , doubles, gas penetration will increase by eight times.

$$q = \frac{\Delta p l w^3}{12 \eta t} \quad (5.1)$$

According to Betonghandboken [37], crack width should be limited to 0.2 mm with regards to gas penetration.

5.5 Crack Control in Reinforced Concrete

To keep crack widths below certain values, a strategy for crack control should be applied in the design [16]. Assume a prismatic reinforced concrete bar as shown in Fig. 5.1, subjected to an axial tension F . The figure shows the difference of the crack formation due to imposed deformation and external load. When a member is subjected to a continuously increasing axial tensile load, the deformation will increase instantly when a crack is formed. Next crack will appear for a small additional load if the concrete strength is assumed to be rather uniform. In the case of a displacement-loaded structure, i.e. when a member is subjected to temperature variation or shrinkage, the internal forces within the member will instantly reduce due to the decreased stiffness when a crack is formed. These two cases can be distinguished by force-induced and displacement-induced cracking, respectively.

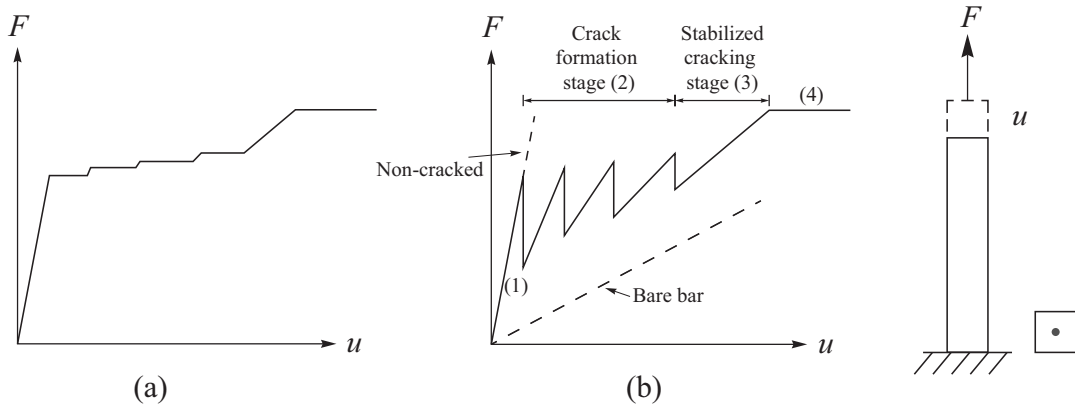


Fig. 5.1: The figure shows the relation between displacement and force for a reinforced bar caused by: (a) force controlled loading. (b) displacement controlled loading.

In Fig. 5.1, four stages of cracking can be distinguished: the uncracked stage (1), the crack formation stage (2), the stabilized stage (3) and the steel yielding stage (4). The tensile stress in the reinforcement is transferred along the transmission length, l_t , to the uncracked concrete sections. Since the tensile stress is gradually transferred to the

concrete, the stress will not reach a critical value within the transmission zone, allowing no new cracks to appear. Hence, only a certain number of cracks can be formed for a reinforced concrete bar. When the maximum amount of cracks are reached, the bar is in its stabilized crack pattern.

5.6 Reinforcement Bond Models

Shear stresses will appear at the interface between the concrete and the reinforcement if there is a different need for deformation. These stresses are associated with bond stresses and are usually expressed as a bond-slip relationship. It defines the bond strength, τ_b , as a function of the current slip, s , between the reinforcement and the surrounding concrete.

A bond between reinforcement and concrete can be attributed to three mechanisms: chemical adhesion, friction and mechanical interlocking. The contribution from adhesion is, however, small and negligible as soon as the slip between the reinforcement and the concrete starts [53]. When the slip starts, bond stresses develop between the reinforcement and the concrete due to the relative displacement, $s = u_s - u_x$, between the two materials. Along with a certain distance beside the crack, the reinforcement steel will transfer the tensile stress to the concrete by the bond. This distance is called the transmission length, l_t , and is illustrated in Fig. 5.2. The relationship between bond stress and bond slip is dependent on the rib geometry but also on the concrete strength, the position and orientation of the bar during casting, the state of stress, the boundary conditions and the concrete cover [19]. However, for a reinforced member with a single crack subjected to a tensile force F , the bond stresses will be at a maximum close to the cracked section and decreases along the transmission length.

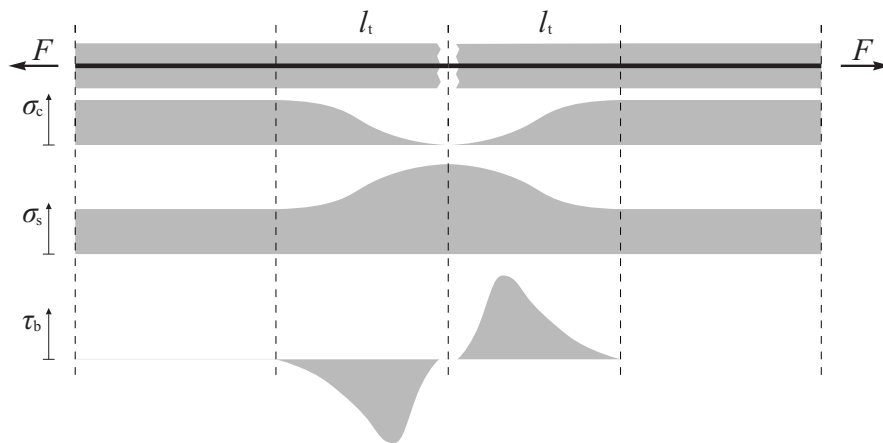


Fig. 5.2: *The figure shows the principal variation of stresses along a reinforced concrete bar. Figure after [16].*

A general relationship between the relative displacement between concrete and reinforcement $s = u_s - u_x$ and bond stress, τ_b , is shown in Fig. 5.3

Model Code 2010 [19] proposes a relationship between the bond stress and bond slip as follows:

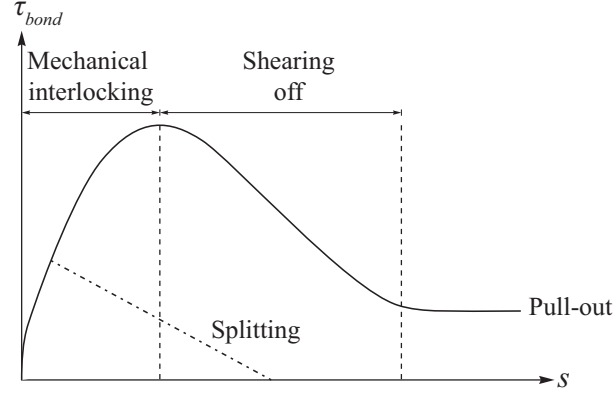


Fig. 5.3: General bond stress-slip relation for a pull-out test.

$$\tau_b = \tau_{bmax} (s/s_1)^\alpha \quad \text{for } 0 \leq s \leq s_1 \quad (5.2)$$

$$\tau_b = \tau_{bmax} \quad \text{for } s_1 \leq s \leq s_2 \quad (5.3)$$

$$\tau_b = \tau_{bmax} - (\tau_{bmax} - \tau_{bf})(s - s_2)/(s_3 - s_2) \quad \text{for } s_2 \leq s \leq s_3 \quad (5.4)$$

$$\tau = \tau_{bf} \quad \text{for } s_3 < s \quad (5.5)$$

The parameters in Eqs. (5.2)-(5.5) are defined in Model Code 2010 [19].

A bond model was presented by Bigaj in [7] which accounts for the bond quality, concrete cubic compressive strength and the reinforcement bar radius. The stress-strain relationship is defined with four points (1-4), forming a triple-linear curve.

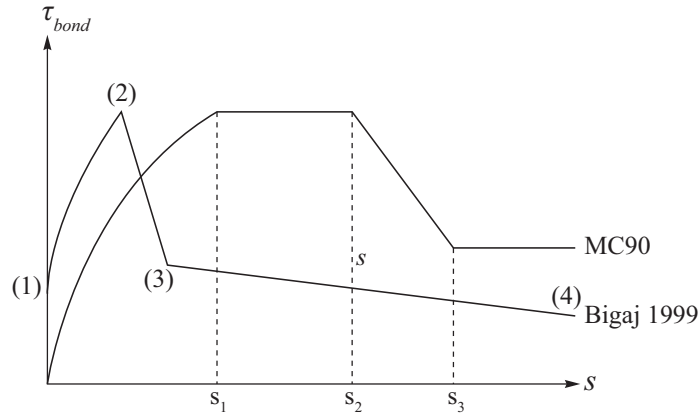


Fig. 5.4: Two bond-slip models: CEB-FIP Model Code 1990 and Bigaj 1999. Both models are available in Atena 2D.

Jaccoud in [30] proposed an expression for the ascending part of the bond stress-slip relation as

$$\tau_b(s) = 0.22 \cdot f_{cm} \cdot s^{0.21} \quad (5.6)$$

5.7 Crack Propagation and Fracture Energy

Concrete can be studied as a linear-elastic material, both in tension and in compression, before cracking occurs. On the other hand, when evaluating concrete at and after the state of cracking, a nonlinear material model is necessary in order to describe the structural behaviour. To evaluate concrete at the state of cracking and to account for the nonlinear failure mode, a model of fracture mechanics is usually adopted.

There are three basic types of failure modes, describing how two different parts of a material is displaced with respect to each other on opposite sides of a crack, see Fig. 5.5. Mode I is the most frequently used model for simulating the crack propagation of concrete [8]. This mode considers two part of a material being pulled apart from each other by tension acting perpendicular to the crack direction. Mode I indicates that the crack will grow in the x -axis direction and the stress contributing to crack growth is oriented in the y -direction. Mode II and Mode III describe a situation where two parts are sliding in the opposite direction with respect to each other in the xy -plane respectively perpendicular to the xy -plane. However, for concrete materials, crack propagation usually occur according to mode I.

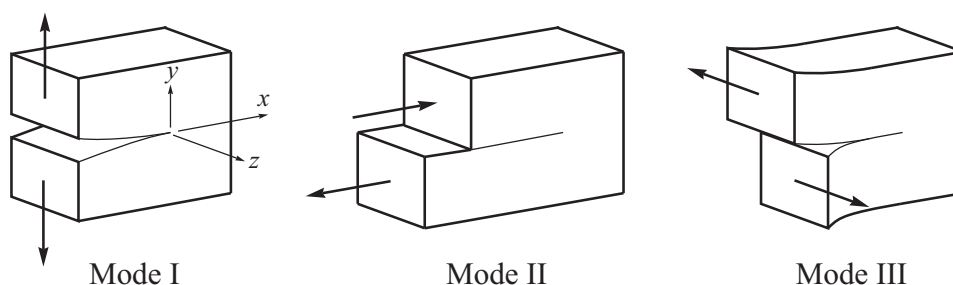


Fig. 5.5: *The figure shows the fracture modes: Opening mode (mode I), tearing mode (mode II) and sliding mode (mode III).*

When a brittle material, such as concrete, is loaded in pure tension with controlled deformation, the tensile stress will gradually increase with the deformation. Before the material reaches its maximum tensile capacity, the amount of microcracks occurring in the material is relatively small and evenly distributed within the specimen. At the same stress level as the material tensile capacity, the microcracking will be concentrated to a limited area, a so called Fracture Process Zone (FPZ), in which the cracking will propagate. This zone starts to develop in weak parts of the material and where local stress concentration is present, often due to the curing of the concrete. At this point, all additional microcracking is limited to the FPZ only. In the concentrated area, microcracks will start to merge and form continuous, visual cracks. Fully developed cracks will increase the deformation of the material consequently reducing the stresses, leading to a softening behaviour of the material [8].

Moreover, crack propagation of a pre-existing crack is developed with the same principle as discussed above. Worth mentioning is that further crack growth will propagate from the existing cracks tip, where the FPZ will continue to develop. When studying crack growth with a linear elastic approach, the stresses near the crack tip will theoretically be very large, which is not the case in reality. Thus, a fracture model which

accounts for the nonlinear zone ahead of the crack tip can be introduced. Several models have been developed for this purpose, including the fictitious crack model¹, the crack band model, the two-parameter fracture model, among others [66]. With a nonlinear approach, the closure stress will be zero at the tip of the crack, where the crack-opening displacement (COD) has reached a certain width, w_c . The closure stress will increase over the FPZ until it reaches the critical tensile strength of the material [56].

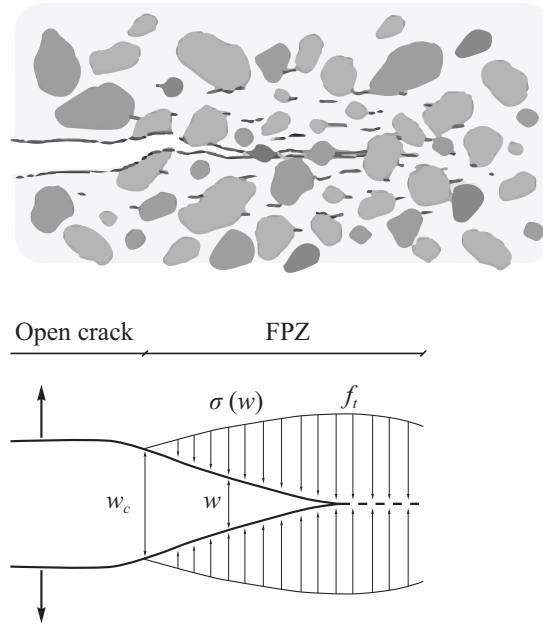


Fig. 5.6: *Concept of fracture process zone (FPZ) and tension-softening in concrete. After [56].*

To describe the behaviour after the material has reached its maximum capacity , $\varepsilon > \varepsilon_{crack}$, a fracture model according to [25] can be adopted, where the described tension softening process is included. In this model the behaviour of the FPZ can be described with a stress - crack-opening relation. The crack-opening, w , is an absolute deformation that describes the deformation of the fractured zone. To increase the deformation, energy per unit area is consumed ($\sigma \cdot dw$). The amount of energy consumed to develop a unit area of crack is defined as the fracture energy, G_f , of the material [25], see Eq. (5.7). A fully developed crack is here defined as a crack where no stress can be transferred over.

$$G_f = \int_0^{w_c} \sigma \cdot dw \quad (5.7)$$

The fracture energy depends primarily on the water-cement ratio, the maximum aggregate size and the age of concrete [19]. For a material such as concrete the shape of the softening curve, hence the fracture energy does not strongly depend on the

¹Arne Hillerborg, professor in Building Materials at Lund Institute of Technology 1973-1988, has played an important role in the development of fracture mechanics. In one of his most significant work [26], the fictitious crack model was introduced.

specimen type and size. Therefore, the fracture energy for concrete can be seen as a material parameter and may only depend on the concrete strength [25]

According to Model Code 2010 [19], the fracture energy should be determined by testing, but in the absence of real tests it may, for ordinary normal strength concrete, be estimated as

$$G_f = 74 \cdot f_{cm}^{0.18} \quad (5.8)$$

Model Code 1990 [18] presents an alternative method to calculate the fracture energy. This method takes into account that different aggregate sizes, d_{max} , affects the amount of energy released upon cracking. The fracture energy is calculated as follows

$$G_f = G_{f0} \left(\frac{f_{cm}}{f_{cm0}} \right)^{0.7} \quad (5.9)$$

in which the coefficient G_{f0} is estimated as Tab. 5.2.

d_{max}	8	16	32
G_f	25	30	58

Tab. 5.2: *Coefficients for determination of the fracture energy.*

5.8 Transverse Reinforcement Effect on Cracking Behaviour

Concrete slabs structures are generally reinforced in two directions which may cause locations in the structure where the reinforcement has to be overlapped. Most of the experimental work on cracking behaviour for tension members is concentrated on structures reinforced in one direction. In [49], the effect of transverse reinforcement for concrete members subjected to pure tension was tested. It was found that stress concentrations around the transverse reinforcement induced cracks at their location. Furthermore, the crack spacing was influenced by the transverse reinforcement spacing. The first cracks appeared at the transverse reinforcement location. No consideration is taken in Eurocode [17] due to the effects of transversal reinforcement.

Chapter 6

Finite Element Formulation

Finite Element Analysis (FEA) is a powerful tool for solving partial differential equations for structures that are too complex for regular analytical solutions. In the field of civil engineering, FEA is often used to study structural behaviour for large, composite structures exposed to different loading conditions. For the vast majority of structural problems, a linear analysis provides an acceptable precision, at least for parts of a structure. However, in cases where nonlinear behaviour may affect the result, this may have to be accounted for in the model. Nonlinear analysis can be classified into mainly three types of nonlinearities: geometrical nonlinearity for large deformations, which mostly includes stability problems, material nonlinearity, and contact nonlinearity [14]. This chapter describes the fundamentals of Finite Element Analysis using a Lagrangian formulation, the constitutive model SBETA used in Atena 2D and solving methods for nonlinear problems.

6.1 General Problem Formulation

Studying nonlinear problems with the finite element method, give rise to the need for solving a set of nonlinear equations. The equations describe the static conditions for each node in the analyzed body, i.e. the acceleration is set aside. That is to say, the solution to the equations is found when the external forces are in equilibrium with the internal forces in each node [42]. The equilibrium equations can also be referred to as the global equations since the equilibrium should hold for the entire body. By disregarding the acceleration, the equations of equilibrium can be formulated as:

$$\psi = \mathbf{f}_{int} - \mathbf{f}_{ext} \quad (6.1)$$

$$\psi = 0 \quad (6.2)$$

The response of a material due to a known load, $\mathbf{f}_{ext}$, will depend on how the structure has been loaded previously to the current loading occasion, i.e. the loading history.

Therefore, the equations can not simply be solved, applying all external load at once. By gradually increasing the external load, with an incremental procedure called load-stepping-schemes, the nonlinear behaviour of the material can be captured.

For structural analysis, a Lagrangian formulation is the most common approach to solve the equations of equilibrium resulting from the finite element discretization [14]. Two forms of the Lagrangian formulations can be adopted: total Lagrangian or updated Lagrangian, where the former uses the undeformed configuration, i.e. $t = 0$, as the computational domain, whereas the latter uses the most recent deformed configuration at time t . In a Lagrange formulation, the behaviour of infinitesimal particles dV is of interest. The governing equations for a total Lagrangian formulation can be established from the expression of virtual work:

$$\int_{v^o} \rho^o \delta \mathbf{u} \cdot \ddot{\mathbf{u}} dv^o + \int_{v^o} \delta \omega dv^o - \int_{s^o} \delta \mathbf{u} \cdot \mathbf{t}^o ds^o - \int_{v^o} \rho^o \delta \mathbf{u} \cdot \mathbf{b} dv^o = 0 \quad (6.3)$$

Due to the principle of virtual work, following holds for a static loading situation:

$$V(\mathbf{u}, \delta \mathbf{u}) = V_{int} - V_{ext} = 0 \quad (6.4)$$

where

$$V_{int} = \int_{v^o} \mathbf{tr}(\delta \mathbf{E} \cdot \mathbf{S}) dv^o \quad (6.5)$$

$$V_{ext} = \int_{s^o} \delta \mathbf{u} \cdot \mathbf{t}^o ds^o - \int_v \rho^o \delta \mathbf{u} \cdot \mathbf{b} dv^o \quad (6.6)$$

for the finite element formulation.

In continuum mechanics, the 2nd Piola-Kirchoff stress tensor and the Green-Lagrange's strain tensor are usually used to express the forces and strains, respectively. For a plane-stress situation, Green-Lagrange's strain tensor, $\delta \underline{\mathbf{E}}$, and the 2nd Piola-Kirchhoff stress tensor can be expressed in column matrix format as

$$\underline{\mathbf{S}} = \begin{bmatrix} S_{xx} \\ S_{yy} \\ S_{xy} \end{bmatrix} \quad \delta \underline{\mathbf{E}} = \begin{bmatrix} \delta E_{xx} \\ \delta E_{yy} \\ 2\delta E_{xy} \end{bmatrix} \quad (6.7)$$

$$\delta \underline{\mathbf{E}} = \begin{bmatrix} \frac{\partial \delta u_x}{\partial x^o} \\ \frac{\partial \delta u_y}{\partial y^o} \\ \frac{\partial \delta u_x}{\partial y^o} + \frac{\partial \delta u_y}{\partial x^o} \end{bmatrix} + \begin{bmatrix} \frac{\partial u_x}{\partial x^o} \frac{\partial \delta u_x}{\partial x^o} + \frac{\partial u_y}{\partial x^o} \frac{\partial \delta u_y}{\partial x^o} \\ \frac{\partial u_x}{\partial y^o} \frac{\partial \delta u_x}{\partial y^o} + \frac{\partial u_y}{\partial y^o} \frac{\partial \delta u_y}{\partial y^o} \\ \frac{\partial u_x}{\partial x^o} \frac{\partial \delta u_x}{\partial y^o} + \frac{\partial u_x}{\partial x^o} \frac{\partial \delta u_y}{\partial y^o} + \frac{\partial u_y}{\partial x^o} \frac{\partial \delta u_x}{\partial x^o} + \frac{\partial u_y}{\partial x^o} \frac{\partial \delta u_y}{\partial x^o} \end{bmatrix} \quad (6.8)$$

Eq. (6.8) can be expressed in a more compatible form by introducing two linear operators and a matrix depending upon displacement

$$\tilde{\nabla}_o = \begin{bmatrix} \frac{\partial}{\partial x^o} & 0 \\ 0 & \frac{\partial}{\partial y^o} \\ \frac{\partial}{\partial y^o} & \frac{\partial}{\partial x^o} \end{bmatrix} \quad \bar{\nabla}_o = \begin{bmatrix} \frac{\partial}{\partial x^o} & 0 \\ 0 & \frac{\partial}{\partial y^o} \\ 0 & \frac{\partial}{\partial x^o} \\ \frac{\partial}{\partial y^o} & 0 \end{bmatrix} \quad \mathbf{A}(\mathbf{u}) = \begin{bmatrix} \frac{\partial u_x}{\partial x^o} & 0 & \frac{\partial u_y}{\partial x^o} & 0 \\ 0 & \frac{\partial u_x}{\partial y^o} & 0 & \frac{\partial u_y}{\partial y^o} \\ \frac{\partial u_x}{\partial y^o} & \frac{\partial u_x}{\partial x^o} & \frac{\partial u_y}{\partial y^o} & \frac{\partial u_y}{\partial x^o} \end{bmatrix} \quad (6.9)$$

The variation of Green-Lagrange's strain can now be written as

$$\delta \underline{\mathbf{E}} = \nabla_o \delta \mathbf{u} + \mathbf{A}(\mathbf{u}) \bar{\nabla}_o \delta \mathbf{u} \quad (6.10)$$

For the second non-linear term in Eq. (6.10), following property holds:

$$\mathbf{A}(\mathbf{u}) \bar{\nabla}_o \delta \mathbf{u} = \mathbf{A}(\delta \mathbf{u}) \bar{\nabla}_o \mathbf{u} \quad (6.11)$$

The matrices for the displacements, $\mathbf{u}$, the arbitrary velocities, $\delta \mathbf{u}$, the surface tractions, $\mathbf{t}^o$, and the body forces, $\mathbf{b}$, are defined as

$$\mathbf{u} = \begin{bmatrix} u_x \\ u_y \end{bmatrix} \quad \delta \mathbf{u} = \begin{bmatrix} \delta u_x \\ \delta u_y \end{bmatrix} \quad \mathbf{t}^o = \begin{bmatrix} t_x^o \\ t_y^o \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_x \\ b_y \end{bmatrix} \quad (6.12)$$

By using the virtual work expression given in Eq. (6.3), which is valid for large deformations, and by using the matrices defined in Eq. (6.12), following is obtained

$$\int_{v^o} \delta \underline{\mathbf{E}}^T \underline{\mathbf{S}} dv^o - \int_{s^o} \delta \mathbf{u}^T \mathbf{t}^o ds^o - \int_{v^o} \rho^o \delta \mathbf{u}^T \mathbf{b} dv^o = 0 \quad (6.13)$$

To obtain the FE-formulation in matrix format, the displacement field $\mathbf{u}$ is approximated as

$$\mathbf{u}(\mathbf{x}^o) = \mathbf{N}(\mathbf{x}^o) \mathbf{a} \quad (6.14)$$

where $\mathbf{N}$ is the global shape functions and $\mathbf{a}$ is the nodal displacement vector. With Galerkins method it is possible to derive Eqs. (6.15)-(6.16). For the full derivation, the reader is encourage to read [14].

$$\mathbf{f}_{int} = \int_v \mathbf{B}^T \boldsymbol{\sigma} dv \quad (6.15)$$

$$\mathbf{f}_{ext} = \int_s \mathbf{N}^T \mathbf{t} ds + \int_v \rho \mathbf{N}^T \mathbf{b} dv \quad (6.16)$$

Different iteration schemes have different strategies to solve the nonlinear systems. Two commonly used iteration methods for non-linear problems are the Newton-Raphson and Arc-length method.

6.2 Solving Equilibrium Equations

The incremental scheme for Newton-Raphson is based on using the tangent for a known point, where the equilibrium is fulfilled, to estimate the displacement for the next load step by extrapolation of the tangent.

Consider a point in equilibrium n , in which the external force $f_{ext} = f_n$ and the displacement a_n^1 is known. The load is increased by one load step, $f_{ext} = f_n + \Delta f = f_{n+1}$, and the new displacement a_n^2 is estimated by extrapolating the tangent from point n until the external force $f_{ext} = f_n$ is reached. The difference between the estimated solution and the true solution, drift, can be calculated as a residual. If the residual is smaller than the equilibrium criteria set, the estimated solution is accepted as the displacement for the next point, $n + 1$.

$$a_n^2 = a_{n+1}^1$$

However, if the residual is larger than the equilibrium criteria, the calculated internal forces are corrected by the residual and a new tangent is calculated for point (a_n^2) and once more extrapolated to find the displacement corresponding to the next load step. This procedure is repeated until the equilibrium requirements are reached and the estimated displacement can be accepted as the next load step. A Newton-Raphson scheme is illustrated in Fig. 6.1

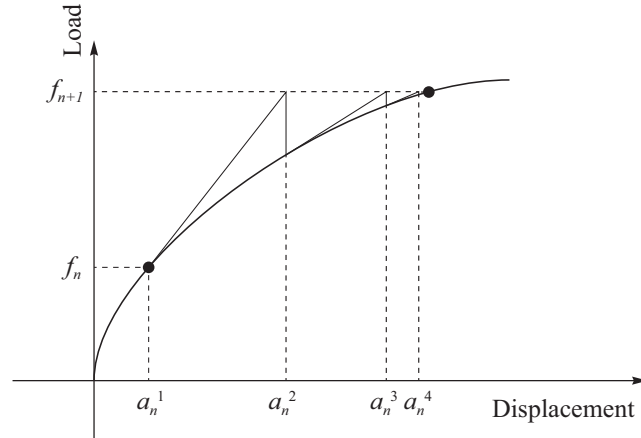


Fig. 6.1: *Illustration of the Newton-Raphson method.*

A Newton-Raphson scheme works both for loading and unloading and has a fast convergence rate [42]. In contrary, every iteration is costly in terms of time and memory since the tangent needs to be calculated and inverted for each iteration. For situations with several degrees of freedom it may be advantages to use a modified Newton-Raphson method, in which the same tangent stiffness is used in the iteration. However, for each load step in a Newton-Raphson scheme, the load increase is kept constant during the iterations to find the next point of equilibrium. This makes it difficult to trace a corresponding displacement when a material reaches its maximum capacity. For an external load above the maximum capacity of a material (peak load), the iterations to find a displacement of equilibrium will simply continue forever.

When modelling a material with a brittle or ductile response, it is often of interest to examine the post-peak behaviour. When reaching a local maximum or minimum the load pattern need to be adjusted in order to stay on the path of equilibrium. This can not be done with a load-controlled Newton-Raphson scheme since a so called snap-through behaviour is obtained [61], see Fig. 6.2. Therefore, examining material behaviour such as buckling or softening with this type of iteration method can cause converging errors.

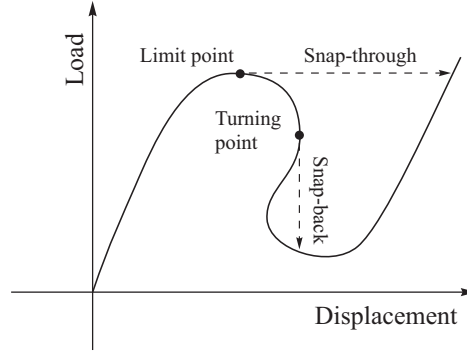


Fig. 6.2: *Snap-through and snap-back behaviour for a one-dimensional problem.*

When a nonlinear system of equations exhibits one or more critical points, traditional simple load or displacement controlled algorithms, such as the Newton-Raphson method, will fail to find a solution. To be able to solve this type of problems, path-following methods need to be introduced.

The arc-length method, introduced by [48] and [62], is a well established method for solving nonlinear systems of equations where critical points are present. The method can solve instability-problems as well as materially nonlinear problems, without claiming efficiency nor robustness. The main idea is to vary both the displacement and the load vector coefficient simultaneously, rather than to keep the load vector coefficient constant and iteratively solve for the displacement.

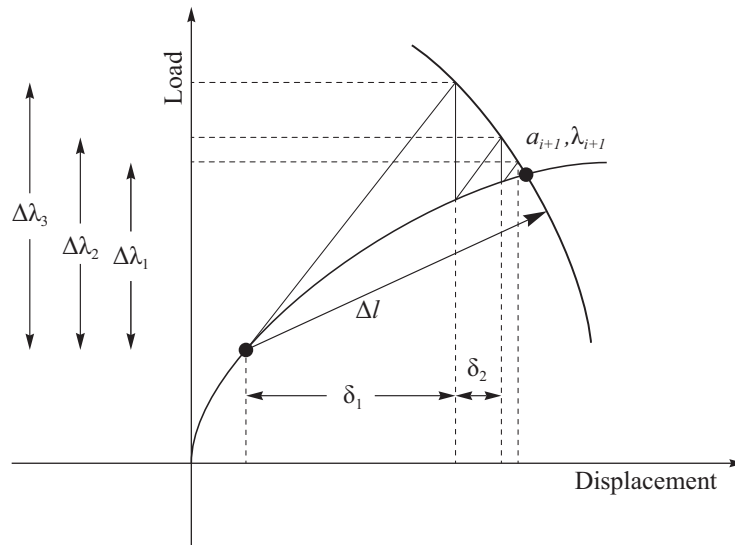


Fig. 6.3: *Illustration of the arc-length method.*

To be able to control the loading, a loading parameter, λ , is introduced. After derivations, the arc length equation can be expressed as

$$(\Delta u + \delta u)^T (\Delta u + \delta u) + \psi^2 (\Delta \lambda + \delta \lambda)^2 (q^T q) = \Delta l^2 \quad (6.17)$$

where ψ is a scaling parameter and Δl can be defined as the radius of the arc, determining how far to search for the next equilibrium [61].

There are several variations of the arc-length method, suited for different purposes. For crack analysis in concrete, the Crisfield method is to prefer.

6.3 Constitutive Model

In Atena 2D, the constitutive material model used for concrete is SBETA. The constitutive relation accounts for the nonlinear behaviour in compression and uses concepts of fracture mechanics in tension. The latter is important in order to evaluate the number of cracks and crack widths. Material properties in the model are based on a smeared approach for cracks or distributed reinforcement [14]. A smeared crack model is preferred over a discrete crack model. The latter describes the crack as a geometrical discontinuity, whereas the former as a continuum. In finite element analysis, a discrete crack is modelled by separating element edges, which can be problematic since a remeshing is necessary [50]. However, in a smeared crack model, a cracked solid is imagined to be a continuum and the model does not require a new mesh formulation. In addition, the orientation of the crack planes is not restricted.

For the smeared crack modelling, either a fixed crack model or a rotated crack model can be used. For both models, cracks will appear when the tensile capacity is reached. However, for the fixed crack model, cracks will propagate in the same direction as the principal stress and the crack direction will be fixed for further loading. In the rotated crack model, cracks will form in the direction of the principal strain. The crack direction will in this case not be fixed but change direction as the principal strain rotates.

The stress-strain diagram for concrete due to monotonic loading is shown in Fig. 6.4. The ascending branch of the compression is a second degree parabola, adopted from [18]. When the peak stress is reached, i.e. $\sigma_y^{ef} = f_c^{ef}$, a linear softening law is introduced. In tension, the concrete is assumed to be linear elastic until the peak stress, f_t^{ef} , is reached and the concrete starts to crack.

For the crack opening, two types of formulations can be adopted: a fictitious crack model or a stress-strain relation in a material point. The former is based on a crack-opening law and fracture energy in combination with crack band theory. The latter is only used in special cases. Furthermore, five softening models are available in the SBETA-model, where the most accurate model for describing the softening behaviour is, according to [33], the exponential softening model. This crack opening law was proposed by [29] and is based on a comprehensive analysis of experimental results. It

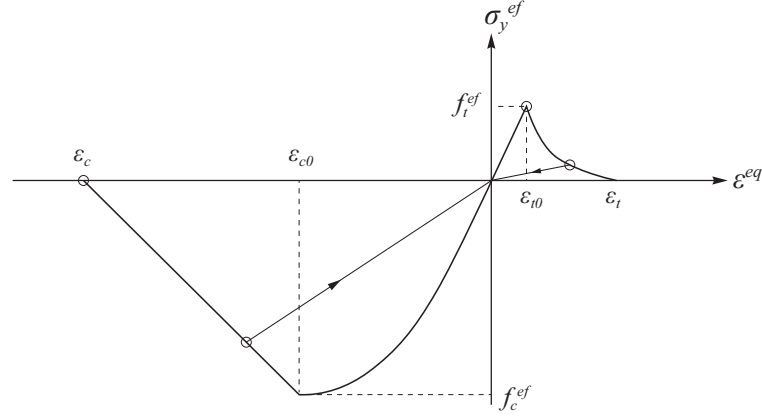


Fig. 6.4: *Uniaxial stress-strain law for concrete in Atena.*

can be formulated as

$$\sigma = f^c(\epsilon^c) = f_t \left(\left[1 + \left(\frac{c_1 \epsilon^c}{\epsilon_f} \right)^3 \right] \exp \left(-\frac{c_2 \epsilon^c}{\epsilon_f} \right) - e^{-c_2} (1 + c_1^3) \frac{\epsilon^c}{\epsilon_f} \right) \quad (6.18)$$

where f_t is the uniaxial tensile strength, ϵ_f is the strain at which the crack becomes stress-free, and c_1 and c_2 are dimensionless material parameters [27]. Recommended values are $c_1 = 3$ and $c_2 = 6.93$. The corresponding softening curve is illustrated in Fig. 6.5.

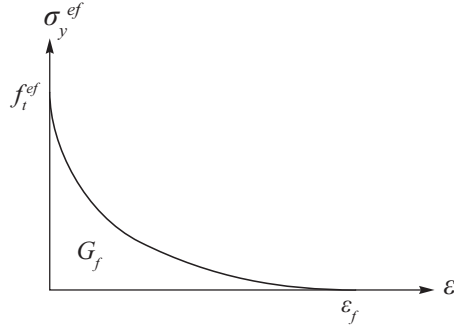


Fig. 6.5: *Exponential crack opening law.*

Crack band theory is used in order to localize the cracks. A crack is modelled by changing the isotropic elastic moduli matrix to an orthotropic one. The material stiffness is reduced in the direction normal to the cracks. The method is effective since no remeshing is necessary. In Atena, so-called localization limiter represents the crack band for discrete failure, see Fig. 6.6.

The choice of element size is important in order to achieve accurate results. In theory, the approximate solution for a finite element model should converge toward the exact solution when decreasing the element size. However, the cost of a finer mesh is an increase in CPU-time. A convergence study can be performed to find a suitable mesh size, such that a decrease in element size will not significantly affect the result.

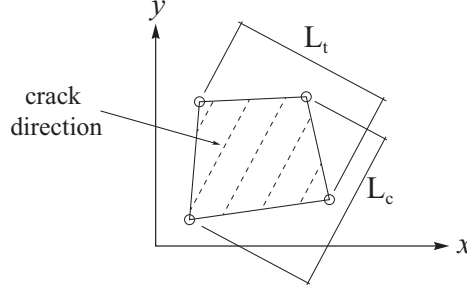


Fig. 6.6: *Definition of localization bands for a 4-noded element.*

6.3.1 Interface material

Interaction between the concrete and the underground material, in this case insulation, can be considered with contact mechanics. Normal contact problems are often treated with either penalty formulation or the Lagrange multiplier methods. In penalty formulation, the bodies are allowed to penetrate each other and an increase in depth of penetration will increase the contact force and vice versa [65].

In Atena, a default interface material model is used to model the interface between the concrete slab and foundation material. The model is based on Mohr-Coulumb criterion with tension cut off [14]. The constitutive relation is given as

$$\begin{Bmatrix} \tau_1 \\ \sigma \end{Bmatrix} = \begin{bmatrix} K_{tt} & 0 \\ 0 & K_{nn} \end{bmatrix} \begin{Bmatrix} \Delta v \\ \Delta u \end{Bmatrix} \quad (6.19)$$

in which Δu and Δv is the sliding and opening displacements of the interface, respectively.

Input parameters are initial normal and tangential stiffness, K_{nn} and K_{tt} , tensile strength, f_t , cohesion, c and coefficient of friction at the interface ϕ . In addition, two parameters denoted as K_{nn}^{min} and K_{tt}^{min} are provided in order to preserve the positive definiteness of the global system of equations after failure occurs. However, guidelines for the coefficients are provided by Atena. The interface tensile strength should be kept to approximately half of the cohesion. The material parameters can not be defined as arbitrary but should be chosen according to Eq. (6.20). The parameters c , f_t and ϕ should be greater than zero.

$$\begin{aligned} f_t &< \frac{c}{\phi} \\ f_t &< c \end{aligned} \quad (6.20)$$

To estimate the initial normal and tangential stiffness, following can be obtained

$$K_{nn} = \frac{E}{t}, K_{tt} = \frac{G}{t} \quad (6.21)$$

in which E is the minimal elastic modulus, G is the minimal shear modulus and t is the width of the interface zone.

Chapter 7

Analytical Methods

In this chapter, design approaches of how to determine shrinkage strain, creep and crack width according to EC2 are presented. In addition, an alternative method for crack width calculation by Engström is introduced. The crack width, calculated using the presented analytical approaches, will later be used as a reference in the numerical parametric study.

7.1 Shrinkage

The total shrinkage strain, ε_{cs} , of a concrete structure is calculated as the sum of the drying shrinkage strain, ε_{cd} , and the autogenous shrinkage strain, ε_{ca} , as follows:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} \quad (7.1)$$

Although autogenous shrinkage is time-dependent, the main component of the strain is developed within a few days after casting. Therefore, the final shrinkage value, $\varepsilon_{cs}(\infty)$, calculated according to Eq. (7.2), is usually used in design calculations. However, if the shrinkage after a specific time has to be studied, it can be accounted for using Eqs. (7.3-7.4).

$$\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10) \cdot 10^{-6} \quad (7.2)$$

$$\beta_{as}(t) = 1 - e^{-0.2 \cdot t^{0.5}} \quad (7.3)$$

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{cs} \quad (7.4)$$

Factor	Description	Comment
$\varepsilon_{ca}(\infty)$	Autogenous shrinkage strain after infinite time	
f_{ck}	Characteristic compressive cylinder strength	After 28 days
$\varepsilon_{ca}(t)$	Autogenous shrinkage strain at a certain time	
$\beta_{as}(t)$	Coefficient considering variation in time	
t	Age of concrete at the considered time	In days

Tab. 7.1: *Description of parameters used when calculating the autogenous shrinkage according to Eqs. (7.2-7.4).*

To calculate the drying shrinkage strain, a basic drying shrinkage strain, $\varepsilon_{cd,0}$, is calculated with regards to the cement class, concrete quality and the surrounding RH as

$$\varepsilon_{cd,0} = 0.85 \left[(220 + 100 \cdot \alpha_{ds1}) \cdot \exp \left(-\alpha_{ds2} \cdot \frac{f_{cm}}{f_{cmo}} \right) \right] \cdot 10^{-6} \cdot \beta_{RH} \quad (7.5)$$

where

$$\beta_{RH} = 1.55 \left[1 - \left(\frac{RH}{RH_0} \right)^3 \right] \quad (7.6)$$

Factor	Description	Comment
$\varepsilon_{cd,0}$	Basic drying shrinkage strain	
α_{ds1}	Coefficient depending on cement type	= 3 for Class S = 4 for Class N = 6 for Class R
α_{ds2}	Coefficient depending on cement type	= 0.13 for Class S = 0.12 for Class N = 0.11 for Class R
f_{cm}	Mean compressive strength of concrete	
f_{cmo}	A constant	= 10 MPa
β_{RH}	Coefficient considering the relative humidity in the surrounding environment	
RH	Ambient relative humidity	
RH_0	Reference value	= 100 %

Tab. 7.2: *Description of parameters used when calculated the basic shrinkage strain according to Eqs. (7.5-7.6).*

Since the drying shrinkage strain develops slowly as moisture migrates from the hardened concrete, an adjustment with regards to time is often required in accordance with Eqs. (7.7-7.8). If the final drying shrinkage strain is studied, the time-coefficient can be taken as $\beta_{ds}(t, t_s) = 1$.

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} \quad (7.7)$$

$$\beta_{ds}(t, t_s) = \frac{(t - t_s)}{(t - t_s) + 0.04\sqrt{h_0^3}} \quad (7.8)$$

$$h_0 = \frac{2 \cdot A_c}{u} \quad (7.9)$$

Factor	Description	Comment
$\varepsilon_{cd}(t)$	Drying shrinkage strain at a specific time	
$\beta_{ds}(t, t_s)$	Coefficient considering variation in time	
k_h	Coefficient depending on the notional size	See Tab. (7.4)
$\varepsilon_{cd,0}$	Basic drying shrinkage strain	See Eq. (7.5)
t	Age of the concrete at the moment considered	In days
t_s	Age of the concrete in days, at the beginning of drying shrinkage	Normally at the end of curing
h_0	Notional size of cross section	
A_c	Cross sectional area	
u	Perimeter of the cross section which is exposed to drying	

Tab. 7.3: *Description of parameters used when calculated the drying shrinkage strain at a specific time according to Eqs. (7.7-7.9).*

h_0	k_h
100	1.0
200	0.85
300	0.75
≥ 500	0.7

Tab. 7.4: *Values on k_h depending on the notional size h_0 .*

7.2 Creep

Eurocode provides two methods for estimating the creep. With the simplified method, the final creep may be extrapolated from graphs. If the development over time has to be considered, a more detailed calculation according to Annex B in Eurocode 2 can be used, which is presented below.

According to Eurocode 2, the compliance function is defined using the creep coefficient as follows:

$$J(t, t') = \frac{1}{E(t')} + \frac{\varphi(t, t')}{1.05E_{cm}} \quad (7.10)$$

Following implementation supports only linear creep, i.e. for stresses below 0.45 of the characteristic compressive strength. The creep coefficient is given as

$$\varphi(t, t') = \varphi_{RH} \frac{16.8}{f_{cm}} \frac{1}{0.1 + t'^{0.2}} \left(\frac{t - t'}{\beta_H + t - t'_T} \right)^{0.3} \quad (7.11)$$

with

$$\varphi_{RH} = \left(1 + \alpha_1 \frac{1 - RH}{0.1(h_0)^{1/3}} \right) \cdot \alpha_2 \quad (7.12)$$

$$\beta_H = 1.5 \left[1 + \left(1.2 \frac{RH}{100} \right)^{0.18} \right] h_0 + 250 \cdot \alpha_3 \leq 1500 \quad (7.13)$$

where $\alpha_{1,2,3} = 1$ for $f_{cm} \leq 35$ MPa, else

$$\alpha_1 = \left(\frac{35}{f_{cm}} \right)^{0.7} \quad \alpha_2 = \left(\frac{35}{f_{cm}} \right)^{0.2} \quad \alpha_3 = \left(\frac{35}{f_{cm}} \right)^{0.5} \quad (7.14)$$

The effect of cement type to the creep coefficient may be taken into account by modifying the age at loading, t_0 , as follows:

$$t_0 = t_{0,T} = \left(\frac{9}{2 + t_{0,T}^{1.2}} + 1 \right) \geq 0.5 \quad (7.15)$$

$$t_T = \sum_{i=1}^n e^{-(4000/[273+T(\Delta t_i)]-13.65)} \cdot \Delta t_i \quad (7.16)$$

Factor	Description	Comment
β_H	Coefficient depending on the relative humidity and notional member size.	RH in %. $h_0 = 2A_c/u$
t	Age of concrete in days at the moment considered	In days
t_0	Age of concrete at loading	In days
$t - t_0$	Non-adjusted duration of loading	In days
$\alpha_{1,2,3}$	Coefficient considering the influence of the concrete strength	
$t_{0,T}$	Temperature adjusted age of concrete at loading	See Eq. (7.16)
α	Power which depends of type of cement	= -1 for cement Class S = 0 for cement Class N = 1 for cement Class R

Tab. 7.5: Description of material parameters for creep calculation according to Eurocode.

For situations when the compressive stress exceeds 0.45 of the compressive strength a nonlinear notional creep coefficient should be obtained. Eurocode provides an exponential expression for those situations, but will not be presented here.

7.3 Minimum Reinforcement

According to EN - 1992-1-1, crack control in a concrete member subjected to tensile stresses require a minimum amount of reinforcement to ensure proper bond. The minimum reinforcement amount can be estimated according to Eq. (7.17) with factors defined in Tab. 7.6. The calculation is based on an equilibrium between steel and the concrete just before cracking occurs, where the allowed steel stress is equal to the yield stress. Note that the factor k may also be taken according to EKS10 [11] as $k = 0.9$ for h_{flange} or b_{web} less than 200 mm.

$$A_{s,\min}\sigma_s = k_c k f_{ct,\text{eff}} A_{ct} \quad (7.17)$$

Factor	Description	Comment
$A_{s,\min}$	Minimum reinforcement area within the tensile zone	
σ_s	Maximum allowed tensile stress in reinforcement immediately after formation of the crack	May be taken as the characteristic strength of the reinforcement f_{yk}
k_c	Coefficient considering the stress distribution over the cross section prior to cracking and of the change of the lever arm	= 1 for pure tension
k	Coefficient considering the effect of non-uniform self-equilibrating stresses	In Eurocode, $k = 1.0$ for webs with $h \leq 300$ mm or flanges with widths less than 300 mm
$f_{ct,\text{eff}}$	Mean value of the tensile strength of the concrete	= f_{ctm} or = $f_{ctm}(t)$ if cracking is expected earlier than 28 days
A_{ct}	Area of concrete within the tensile zone	

Tab. 7.6: *Description of factors used in Eq. (7.17).*

7.4 Crack Width

7.4.1 EN 1992-1-1

Calculation of crack widths according to section 7.3.4 in EN 1992-1-1 (EC2-1), is presented in Eqs. (7.18)-(7.20), and the factors are defined in Table 7.7.

$$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) \quad (7.18)$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{p,eff}} (1 + \alpha_e \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \quad (7.19)$$

$$s_{r,max} = k_3 c + k_1 k_2 k_4 \phi / \rho_{p,eff} \quad (7.20)$$

Factor	Description	Comment
w_k	Characteristic crack width at the surface	
$s_{r,max}$	Maximum crack spacing	
ε_{sm}	Mean strain in the reinforcement along the distance $s_{r,max}$	
ε_{cm}	Mean strain in the concrete between cracks	
k_1	Coefficient considering bond properties in the reinforcement	= 0.8 for high bond bars
k_2	Coefficient considering distribution of strain	= 0.5 for bending = 1.0 for pure tension
k_3		can be taken as $7\phi/c$ according to EKS9 [10]
k_4		= 0.425
$\rho_{p,eff}$	Reinforcement ratio	= $A_s/A_{c,eff}$ for a non-prestressed member
σ_s	Stress in reinforcement for a cracked section	
k_t	Factor considering duration of the load	= 0.6 for short term load loading = 0.4 for long term loading
$f_{ct,eff}$	Mean value of the tensile strength of the concrete	Depend on the concrete quality
α_e	Ratio between reinforcement and concrete	= E_s/E_{cm}
E_s	Young's modulus for reinforcement	

Tab. 7.7: Description of factors used in Eq. (7.18-7.20)

7.4.2 EN 1992-3

An alternative method to design reinforcement in order to control cracking in a restrained structure can be found in EN 1992-3 (EC2-3), where particular two conditions

of restraint are considered: end restraint and continuous edge restraint, see Fig. 7.1. The two restrain conditions, are assumed to behave similarly before cracking, i.e. the first crack occurs when the restrained strain exceeds the tensile strain capacity. However, after cracks have occurred, the the two cases deals with the redistribution of stresses differently [3].

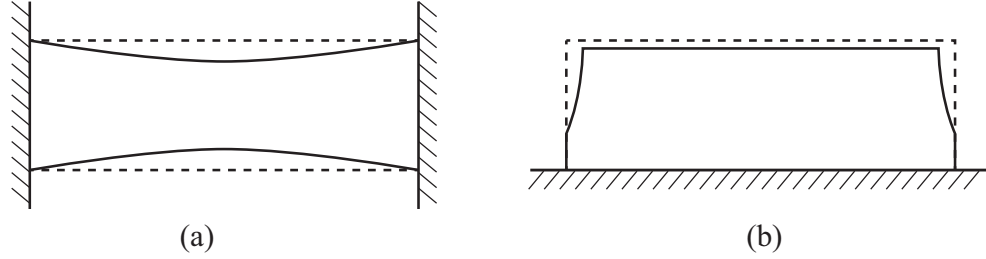


Fig. 7.1: Illustration of (a) end restrained bar. (b) continuous edge restrained wall.

EN 1992-3 refers to Eq. (7.18), from EN 1992-1-1, for the calculation of the characteristic crack width. The equation is then applied for both end restrained and continuous edge restrained structures, but $(\varepsilon_{sm} - \varepsilon_{cm})$ is calculated differently for the two types of restraints. For an end restrained member, $(\varepsilon_{sm} - \varepsilon_{cm})$ is given as Eq. (7.21) and for a continuous edge restrained member as Eq. (7.22).

$$(\varepsilon_{sm} - \varepsilon_{cm}) = 0.5\alpha_c k_c k f_{ct,eff} (1 + 1/(\alpha_e \rho)) / E_s \quad (7.21)$$

$$(\varepsilon_{sm} - \varepsilon_{cm}) = R_{ax} \varepsilon_{free} \quad (7.22)$$

Factor	Description	Comment
R_{ax}	Degree of external axial restraint	Can be found in Annex L in EN 1992-3
ε_{free}	Free contraction	
$f_{ct,eff}$	Effective tensile strength of concrete at the time of cracking	
ρ	Reinforcement ratio	$= A_s / A_{ct}$
α_e	Modular ratio	
E_s	Modulus of elasticity of reinforcement	
k	Coefficients for stress distribution and the effect of self-equilibrating stresses (see EN1992-1-1, 7.3.2 Minimum reinforcement areas)	

Tab. 7.8: Description of factors used in Eqs. (7.21-7.22).

The two approaches consider the effect of restraints differently. For a continuous edge restraint member, no account is taken for the bond stress between the concrete and the steel, despite the expression for the minimum area of reinforcement where the bond between the concrete and the reinforcement is the only consideration.

Eurocode states that for a situation with several cracks, the crack widths and the crack spacing, $s_{r,max}$, will be of equal size. On the contrary, studies by Kheder [34] shows

that the crack spacing and crack widths is not consistent. The secondary cracks will probably not be of equal size as the primary cracks, and likewise, the spacing of the secondary cracks will differ from the spacing of the primary cracks.

An example is given in order to illustrate the calculation procedure of crack width according to EN 1992-1-1 and EN 1992-3. The same procedure is used for the further analysis of a slab foundation.

Example 1

A 10 metre concrete slab on ground, of concrete quality C35/45, is subjected to shrinkage strain and is assumed to be restrained along its length. The height of the slab is set to 100 mm. Material properties and cross sectional data is given below.

Factor	Description	Value
f_{cm}	Concrete compression strength	43 MPa
f_{ct}	Concrete tensile strength	3.2 MPa
f_{ck}	Concrete characteristic strength	35 MPa
E_{cm}	Young modulus of concrete	34 GPa
E_s	Young modulus of concrete	200 GPa
A_c	Cross sectional area of slab (1 m strip)	10 000 mm ²
ϕ	Reinforcement diameter	10 mm
f_{yk}	Yield strength of reinforcement	500 MPa

Tab. 7.9: *Material and geometric data used in example.*

1. Determining the shrinkage strain

Assume $RH = 50\%$ and cement class N $\Rightarrow \alpha_{ds1} = 4.0$ and $\alpha_{ds2} = 0.12$.

Surface exposed to drying, $u = 1000$ mm. The notional size of the cross section can be calculated as:

$$h_0 = 2 \cdot u / A_c = 2 \cdot 1000 / 10000 = 200 \text{ mm}$$

β_{RH} is determined according to Eq. (7.6)

$$\beta_{RH} = 1.55 \left[1 - \left(\frac{50}{100} \right)^3 \right] = 1.36$$

Basic drying shrinkage strain $\varepsilon_{cd,0}$ is calculated according to Eq. (7.5).

$$\varepsilon_{cd,0} = 0.85 \left[(220 + 100 \cdot 4) \cdot \exp \left(-0.12 \cdot \frac{43}{10} \right) \right] \cdot 10^{-6} \cdot 1.36 = 4.54 \cdot 10^{-4}$$

According to Tab. 7.4, the coefficient of notional size can be determined to $k_h = 0.85$. Final value for drying shrinkage strain can now be determined according to Eq. (7.7), where $\beta_{ds}(t, t_s) = 1$

$$\varepsilon_{cd,\infty} = k_h \cdot \varepsilon_{cd,0} = 3.86 \cdot 10^{-4}$$

Autogenous shrinkage is calculated according to Eq. (7.2)

$$\varepsilon_{ca,\infty} = 2.5(35 - 10) \cdot 10^{-6} = 0.625 \cdot 10^{-4}$$

The total shrinkage is the sum of drying shrinkage and autogenous shrinkage, according to Eq. (7.1)

$$\varepsilon_{cs,\infty} = 0.624 \cdot 10^{-4} + 3.86 \cdot 10^{-4} = 4.49 \cdot 10^{-4}$$

2. Determining minimum cross sectional area of reinforcement

Considering pure tension, gives $k_c = 1$ and a height less than 200 mm entails $k = 0.9$ according to Tab. (7.6).

The minimum reinforcement amount is calculated according to Eq. (7.17)

$$A_{s,min} = (1 \cdot 0.9 \cdot 3.2 \cdot 100000) / 500 = 576 \text{ mm}^2$$

Choose a distance between bars as $s = 100$ mm, which gives a total area of reinforcement of 785 mm^2 . Reinforcement ratio is equal to $7.85 \cdot 10^{-3}$.

3. Calculate crack width

First, the distance between cracks is determined from Eq. (7.20). Following values for k-coefficient, by considering good bond and pure tension, is given according to Tab.(7.7).

$$k_1 = 0.8$$

$$k_2 = 1$$

$$k_3 = 7 \cdot 10 / 45 = 1.6$$

$$k_4 = 0.425$$

$$s_{r,max} = \frac{1.6 \cdot 45 + 0.8 \cdot 1.0 \cdot 0.425 \cdot 10}{7.85 \cdot 10^{-3}} = 503 \text{ mm}$$

At this point, two options are possible in Eurocode. Crack width calculation according to En 1992-1-1 or EN 1992-3, appendix M.

According to EN 1992-1-1

$k_t = 0.4$, considering long term loading, see Tab. (7.7). The ratio between reinforcement and concrete, α_e , is calculated according to Tab. (7.7).

$$\alpha_e = 200/34 = 5.88$$

Difference in strain between concrete and steel is calculated according to Eq. (7.19), where $\sigma_s = E_s \varepsilon_{cs}$:

$$\varepsilon_{sm} - \varepsilon_{cm} = \max \left\{ \begin{array}{l} \frac{\frac{4.49 \cdot 10^{-4}}{200 \cdot 10^9} - 0.4 \frac{3.2 \cdot 10^6}{7.85 \cdot 10^{-3}} (1 + 5.88 \cdot 7.85 \cdot 10^{-3})}{200 \cdot 10^9} = -4.04 \cdot 10^{-4} \\ 0.6 \frac{4.49 \cdot 10^{-4}}{200 \cdot 10^9} = 2.69 \cdot 10^{-4} \end{array} \right.$$

The characteristic crack width can now be determined with Eq. (7.18)

$$w_k = 503 \cdot 2.69 \cdot 10^{-4} = 0.14 \text{ mm}$$

According to EN 1992-3

In Annex L, the restraint factor (R) can be taken as, $R = 0.5$. The crack width is calculated by Eq. (7.18) considering the strain from Eq(7.22)

$$\varepsilon_{sm} - \varepsilon_{cm} = 0.5 \cdot 4.48 \cdot 10^{-4} = 2.24 \cdot 10^{-4}$$

$$w_k = 505 \cdot 2.24 \cdot 10^{-4} = 0.11 \text{ mm}$$

The calculated crack widths are 0.11 mm and 0.14 mm.

As shown in calculation the use of EN 1992-3, Annex M and L, gives a crack width approximately 20 % smaller than EN 1992-1-1.

7.4.3 Model Provided by Engström

Usually in structural design codes, the tools for calculating the crack width due to restraint cracking is insufficient and the designer has to deal with minimum reinforcement amount or models developed for load-induced cracking by assuming stabilized cracking case, which is rarely the case. A crack model has been developed by Engström [16], where the bond characteristics of the embedded reinforcement is taken into account. The main concept of the model is that cracking due to restraint can be analyzed by assuming that the cracks act as nonlinear springs. The model takes its form from the ascending part of the curve for the bond-slip relationship found in Eq. (5.6). From this equation, an expression for the mean crack width and transmission length was derived by Jaccoud [30] and later reformulated by Engström [16]. The formulas are defined in Eqs. (7.23-7.26), in which both the transmission length, l_t , and the crack width, w , depend on the steel stress, σ_s . The reinforcement diameter, ϕ , is taken in millimetre.

$$w_m(\sigma_s) = 0.420 \left(\frac{\phi \sigma_s^2}{0.22 f_{cm} E_s (1 + \frac{E_s A_s}{E_c A_{ef}})} \right)^{0.826} + \frac{\sigma_s}{E_s} 4\phi \quad (7.23)$$

$$w_{net}(\sigma_s) = 0.420 \left(\frac{\phi \sigma_s^2}{0.22 f_{cm} E_s (1 + \frac{E_s A_s}{E_c A_{ef}})} \right)^{0.826} \quad (7.24)$$

$$w_k(\sigma_s) = 1.3 \cdot w_m(\sigma_s) \quad (7.25)$$

$$l_t(\sigma_s) = 0.443 \frac{\phi \sigma_s}{0.22 f_{cm} w_{net}^{0.21} (1 + \frac{E_s A_s}{E_c A_{ef}})} + 2\phi \quad (7.26)$$

For members subjected to imposed deformations, equilibrium condition can be defined as

$$\varepsilon_{ce} l + \varepsilon_{cc} l + n_{cr} w + \varepsilon_c l + u = 0 \quad (7.27)$$

where l is the length of the member, ε_{ce} is the elastic deformation in the concrete, ε_c creep deformation, n_{cr} number of cracks, w crack width and u is the movements of the supports. The restraint degree is calculated as

$$R = \frac{1}{1 + \frac{E_c A_c}{S \cdot l}} \quad (7.28)$$

The restraint degree between the reinforcement and concrete when the reinforcement is considered as an external restraint in relation to the net concrete section can be found as

$$R = \frac{1}{1 + \frac{E_c A_{net}}{E_s A_s}} \quad (7.29)$$

In Eq. (7.28), the stiffness of the support, S , is given as

$$S = \frac{N}{u} \quad (7.30)$$

Finally, the deformation compatability condition can be described by using the equation below

$$\frac{N(\sigma_s)}{E_c A_I} (1 + \varphi_{ef}) + n_{cr} \cdot w(\sigma_s) - R \cdot \varepsilon_c \cdot l = 0 \quad (7.31)$$

where $N(\sigma_s) = \sigma_s A_s$ is the axial force acting on the uncracked parts of the element, l is the total length of the element, n_{cr} is the number of cracks, ε_{cs} is the final shrinkage strain and R is the degree of restraint.

The first term in Eq. (7.31) represents the elongation of the uncracked parts, the second parts accounts for accumulated deformation due to crack opening, and the last term represents a possible displacement of the support. By combining Eq. (7.31) with Eq. (7.23), the only unknown terms will be the steel stress, σ_s , and the number of cracks, n_{cr} . The equation can then be solved by an iterative process in which the model is initiated by assuming one crack, $n_{cr} = 1$. If the tensile stress in the member is enough to initiate a new crack, i.e. $N(\sigma_s) > N_{cr}$, the equation has to be solved for $n_{cr} = n_{cr} + 1$ number of cracks. The required force to initiate a new crack can be calculated according to Eq. (7.32). The procedure is repeated until the tensile force is not enough to form a new crack, or if the maximum number of cracks is reached, where the maximum number of cracks is found as $n_{cr,max} = l/(2 \cdot l_t) + 1$.

$$N_{cr} = f_{ctm} \cdot (A_{ef} + (\alpha_{ef} - 1)A_s) \quad (7.32)$$

The method provided by Engström is illustrated the example below.

Example with Engström method

Consider the slab presented in the previous example, but as short-end restrained.

The effective cross sectional area of the uncracked part of the slab, where the width of the slab is the distance between bars, i.e. 100 mm, can be calculated as:

$$A_{I,eff} = A_c + (E_s/E_c - 1)A_s = 0.1 \cdot 0.1 + (200/34 - 1) \cdot \frac{0.01^2 \pi}{4} = 0.0104 \text{ m}^2$$

1. Determining the restraint stress

In Example 1, the shrinkage strain is calculated to $\varepsilon_{cs} = 4.49 \cdot 10^{-4}$ and the degree of restraint is set to $R = 0.5$.

The restrained strain can now be calculated as

$$\varepsilon_R = R \cdot \varepsilon_{cs} = 0.5 \cdot 4.49 \cdot 10^{-4} = 2.25 \cdot 10^{-4}$$

2. Determine if the slab will crack or not

The stress is calculated assuming that no cracks occur and compared with the tensile strength of the concrete.

$$\sigma_c = \varepsilon_R E_c = 2.25 \cdot 10^{-4} \cdot 34 \cdot 10^9 = 7.65 \text{ MPa} > f_{ctm}$$

The slab will crack.

3. Determine how many times the slab will crack

The restrained elongation can be calculated as:

$$\Delta L = \varepsilon_R L = 2.25 \cdot 10^{-3}$$

Assume the number of cracks, $n_{cr} = 1$, that use that the restrained elongation should be equal to the elastic deformation of the concrete and the the crack width, to determine the force, F , in the reinforcement.

$$2.25 \cdot 10^{-3} = \frac{FL}{E_C A_{I,eff}} + 1 \cdot \left[0.42 \left(\frac{\phi \cdot (F_s/A_s)^2}{0.22 f_{cm} E_s \left(1 + \frac{E_s A_s}{E_c A_{I,eff}} \right)} \right)^{0.826} + 4\phi \frac{F}{A_s E_s} \right] / 1000$$

Note that the bar diameter should be inserted in mm.

Assume the force is constant along the bar, $F = F_s$, gives

$$\Rightarrow F = 49 \text{ kN}$$

Corresponding to a stress in the concrete:

$$\sigma_c = 49 \cdot 10^3 / 0.0104 = 4.72 \text{ MPa} > f_{ctm}$$

I.e. more than 1 crack will appear.

Increase the number of cracks and repeat step 3 until $\sigma_c < f_{ctm}$.

When 3 cracks are assumed, the corresponding stress in the concrete will be below the tensile strength of concrete and no further cracking can there force occur. The stress in the concrete and the force in the reinforcement at this stage is calculated to:

$$\sigma_c = 3.09 \text{ MPa} < f_{ctm}$$

$$F_s = 32.1 \text{ kN}$$

4. Checking the maximum number of cracks that can appear

Calculating w_{net} according to Eq. (7.24), where the force in the bar is N_{cr} according to Eq. (7.32).

$$N_{cr} = 3.2 \cdot 10^6 \cdot 0.0104 = 33.2 \text{ kN}$$

$$w_{net}(\sigma_s) = 0.420 \left(\frac{10 \cdot \left(\frac{33.2 \cdot 10^3}{\frac{0.012 \cdot \pi}{4}} \right)^2}{0.22 \cdot 43 \cdot 10^6 \cdot 200 \cdot 10^9 \left(1 + \frac{200}{34} \frac{0.012 \cdot \pi}{4 \cdot 0.0104} \right)} \right)^{0.826} = 0.387 \text{ mm}$$

Now the transmission length, l_t , is calculated according to Eq. (7.26):

$$l_t(\sigma_s) = 0.443 \frac{10 \cdot \frac{33.2 \cdot 10^3}{\frac{0.01^2 \cdot \pi}{4}}}{0.22 \cdot 43 \cdot 10^6 \cdot 0.365^{0.21} \left(1 + \frac{200}{34} \frac{\frac{0.01^2 \cdot \pi}{4}}{0.0104} \right)} + 2 \cdot 10 = 252 \text{ mm}$$

The total number of cracks allowed are

$$n_{cr,max} = \frac{10000}{2 \cdot 252} + 1 = 20.9$$

The assumption of 3 cracks are therefore valid.

5. Determining crack width

The calculated force is inserted into Eq. (7.23):

$$\begin{aligned} w(\sigma_s) &= 0.420 \left(\frac{10 \cdot \left(\frac{32.1 \cdot 10^3}{\frac{0.01^2 \cdot \pi}{4}} \right)^2}{0.22 \cdot 43 \cdot 10^6 \cdot 200 \cdot 10^9 \left(1 + \frac{200}{34} \frac{\frac{0.01^2 \cdot \pi}{4}}{0.0104} \right)} \right)^{0.826} + \frac{4 \cdot \left(\frac{32.1 \cdot 10^3}{\frac{0.01^2 \cdot \pi}{4}} \right) \cdot 10}{200 \cdot 10^9} \\ &= 0.447 \text{ mm} \end{aligned}$$

The characteristic crack width is calculated with Eq. (7.25)

$$w_k = 1.3 \cdot 0.447 = 0.581$$

Calculated crack width is approximately 0.58 mm.

As can be seen in Fig. 7.2, which illustrates the force- and crack width pattern width from Example 2, the stabilized crack pattern has not been reached. The number of cracks that appear is three, which can be compared with the maximum allowed number of cracks that is 20. In Example 1, the crack width was calculated to 0.11 and 0.14 mm according to EC 2, to compare with 0.58 mm from Example 2 in which the crack width was calculated according to Engström.

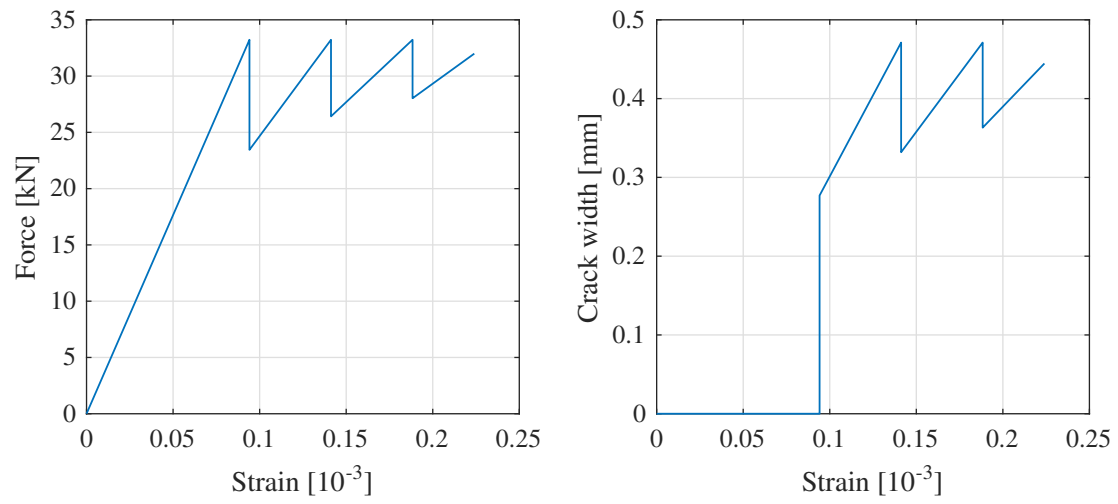


Fig. 7.2: *Restraint force and crack width as a function of the shrinkage strain from Example with Engström method.*

Chapter 8

Calibration and Shrinkage Estimation

In this chapter, a comparison of material models and material parameters, used when analysing the slab foundations, are performed. Two fracture energy models, as well as two bond models, are investigated and compared against experimental tie-rod tests. In addition, a procedure for determining the shrinkage load, via simulated RH, is presented as well as the calculated shrinkage strain for the studied slab foundations.

8.1 Calibration of Material Models

Experimental tie-rod tests carried out at the University of New South Wales (UNSW) [63], and numerical results were compared to verify the FE-model and evaluate which material models that are most suitable for crack analysis in further studies. Proposed fracture energy models and bond models have been studied. In addition, a convergence study for different mesh sizes was carried out to achieve a functional model with sufficient precision. Good correlation between numerical and experimental results proves that the finite element program is capable of capturing the crack behaviour in reinforced concrete structures.

8.1.1 Experimental Tie-rod Tests

The specimens, that were carried out at the UNSW, where 1100 mm long concrete prisms with a square cross section of 100x100 mm². Each specimen contained one reinforcement bar with a bar diameter of either 12 mm or 16 mm, located at the centroid of the cross section. The reinforcement bar was extended on each side of the concrete prism, where the axial tensile load was applied. See Fig. 8.1 for details of the specimens.

The specimens were loaded until 3 % elongation of the specimen were reached, using

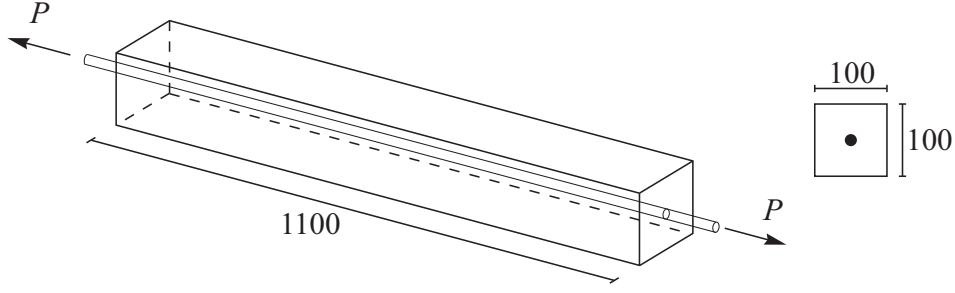


Fig. 8.1: Dimensions of the specimen used in the experiment at UNSW [63].

displacement control. To limit the effect of shrinkage, the concrete bodies were cured under wet burlap for three weeks and tested immediately after curing. No significant shrinkage was observed in the specimens prior to testing. The tests were conducted in a tensile testing machine and the the applied load, elongation and crack locations were registered.

Companion standard 150 by 150 mm² sized cylinders were casted from the same batch of concrete as the tie rod specimens. Material properties, such as compressive strength, f_c , and the modulus of elasticity, E_c , were measured and determined from pure compression tests. The tensile strength, f_{ct} , were estimated using the Brazil test. The tests, determining the material properties were performed at the time of loading for the tie rod test specimens. The reinforcement had a module of elasticity of $E_s = 200$ GPa. The measured material data are presented in Tab. 8.1.

Specimen name	f_c [MPa]	f_{ct} [MPa]	E_c [MPa]
STN12	21.56	2.04	22400
STN16	21.56	2.04	22400

Tab. 8.1: Measured short term material properties for the test specimens.

The results from the performed tests are presented in Figs. 8.2 and 8.3 as a strain-force diagram, showing the tension stiffening process of the test specimens. The number of cracks, mean and maximum crack width are presented in Tab. 8.2.

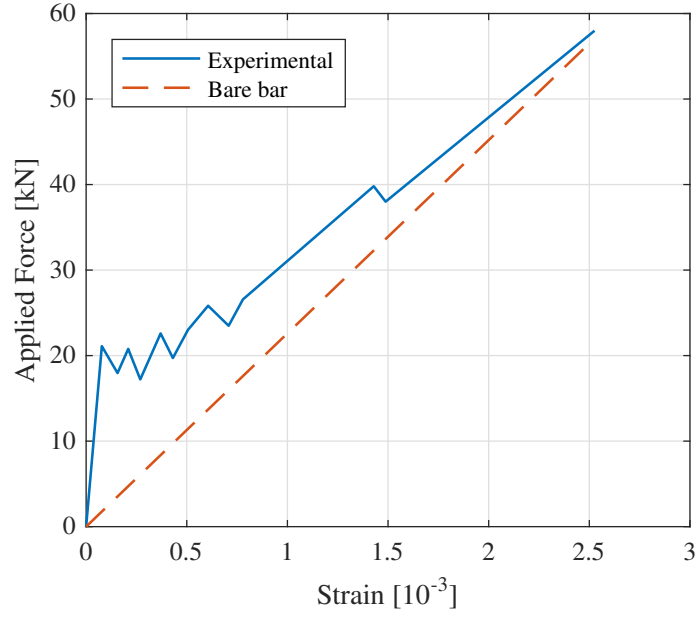


Fig. 8.2: *Strain versus load for STN12, $\phi = 12$ mm.*

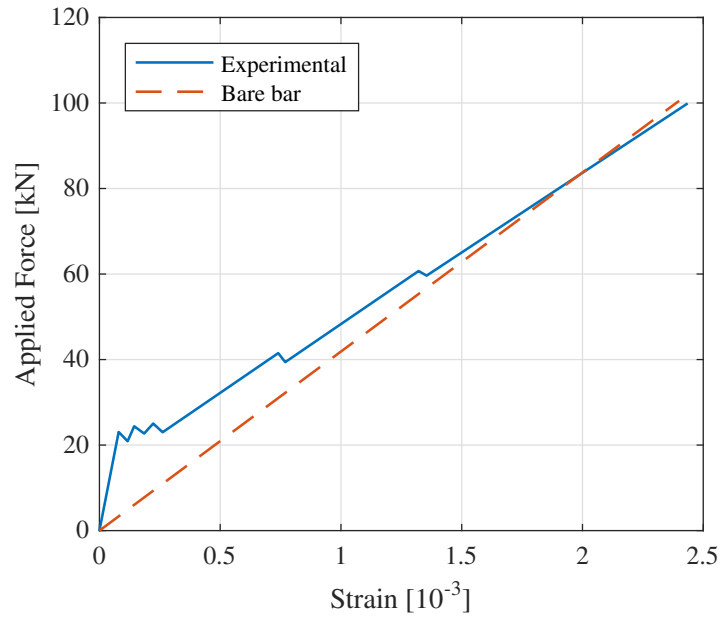


Fig. 8.3: *Strain versus load for STN16, $\phi = 16$ mm.*

Studying the tension-stiffening curves, shown in Fig. 8.2 and Fig. 8.3, some initial remarks can be made. For both tests the specimens remain uncracked until the applied force reach approximately 21 kN. When a crack appears, the force will decrease while the strain increases. In the plot of the force-strain diagram this is represented by a discontinuity of the curve.

By utilizing Hooke's law it can be shown that the inclination of the curve corresponds to the effective stiffness of each specimen:

$$\begin{cases} \sigma = E \cdot \varepsilon = E \cdot A_{eff} \cdot \varepsilon \\ F = k \cdot \varepsilon \end{cases} \Rightarrow k = E \cdot A_{eff} \quad (8.1)$$

When a new crack appear, the effective area of the specimen decrease which result in a lower stiffness. In the force-strain diagram, this is represented as a decrease in inclination. Studying the slope after final cracking, i.e. when no more cracks can appear, it can be concluded that the stiffness of the whole specimen is equal or close to the stiffness of a bare reinforcement bar.

Specimen name	Load stage	Mean crack width[mm]	Maximum crack width[mm]
STN12	$P = 40$ kN	0.185	0.300
STN12	$P = 50$ kN	0.200	0.375
STN16	$P = 75$ kN	0.215	0.325
STN16	$P = 90$ kN	0.245	0.375
STN16	$P = 100$ kN	0.3	0.5

Tab. 8.2: Measured crack width for the two test specimens at different load stages.

8.1.2 FE-analysis of the Tie-rod Test

Topology, boundary conditions and load

Symmetry conditions in the vertical direction (y-direction) were used to reduce the number of elements and consequently reduce the processing time. Thus, only half the height of the specimen was modelled. Since the reinforcement was located in the middle of the prisms, where symmetry conditions were applied, the cross sectional area of the reinforcement bar was reduced by half, see Fig. 8.4. To account for the vertical symmetry conditions in the bond model, only half the circumference of the reinforcement was used. Note that this symmetry is only in the y - direction, hence, the real thickness of the specimen was modelled.

The number of elements was further reduced by using symmetry condition in the horizontal direction (x-direction) as well. To ensure that the symmetry assumptions would not affect the results, a comparison between a full-length model and a symmetry-model was conducted. It was detected that in the symmetry model, close to the symmetry boundary, no vertical through-cracks could appear. It may therefore be a risk of not detecting cracks forming in the middle of the test specimen. In the end, the two models gave similar results and symmetry was used in the further analyzes.

In the real experiment, the load was applied to the protruding reinforcement. In a FE-model, however, applying load directly to a reinforcement bar "in air" can be problematic since all nodes must be included in the structure. In order overcome this problem, the reinforcement bar was extended from the test prism and the free bar end was wrapped inside a small elastic cube with a high young modulus, see Fig. 8.5.

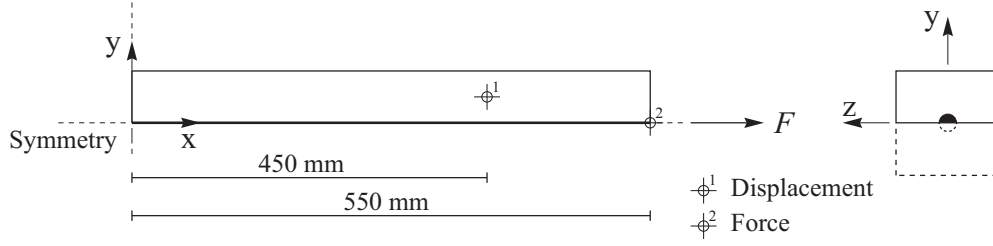


Fig. 8.4: *Dimensions and symmetry lines in the FE-model.*

By disregarding slip at the bar end/beginning, a perfect connection between the bar and the cube was achieved [46]. When load was applied to the elastic cube, all load could be transferred to the reinforcement due to the perfect bond, which resulted in a loading situation that corresponds well with the real experiment.

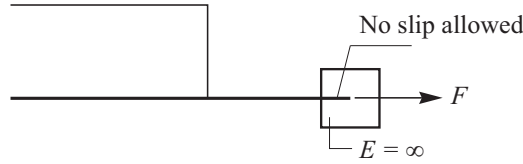


Fig. 8.5: *Reinforcement bar enclosed in an elastic cube*

Method

The force applied to the structure was measured as an external force at a point according to Fig. 8.4. However, since symmetry was used and only half the prism was modelled, the force was multiplied by a factor of 2 to get the acting force for the entire body.

To evaluate the cracking and post-cracking behavior of the test specimen, the tension-stiffening curve, i.e. the force-strain diagram, was plotted against the real test results. Moreover, the crack width from the analysis was measured at the surface of the concrete specimen, the top boundary. Measurements were taken at two different load levels, for the 12 mm bar diameter at 40 kN, 50 kN and for the 16 mm bar diameter at 75 kN respectively 90 kN. The mean and maximum crack width was compared with the performed tests. The crack pattern was rendered for crack widths greater than 0.05 mm, to illustrate where cracks in the specimen appeared.

For a small area in the concrete specimen, close to where the reinforcement extends beyond the concrete body, inclined cracks appeared at relatively low load levels. This was due to the anchorage which gave rise to inclined tensile stresses in the same direction as the principal strains, see Fig. 8.6. The resulting cracks may locally reduce the bond capacity but will not affect the bond behavior outside of this area. The inclined crack may however result in a locally reduced stiffness, consequently leading to an unrealistic displacement for this region. To account for this problem the displacement was measured 450 mm from the left symmetry boundary, see Fig. 8.4.

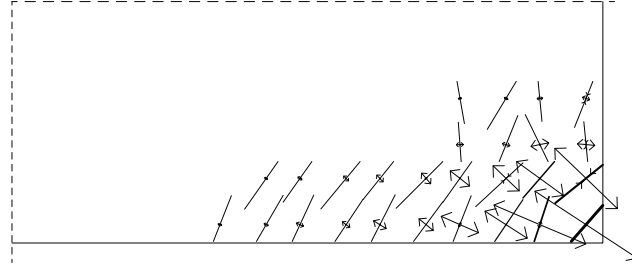


Fig. 8.6: *Direction of principal strains at the anchorage zone.*

Concrete material model

The compressive strength, presented in Table 8.2, correspond with a concrete class between C20/25 and C25/30. The cubic compressive strength can be calculated as:

$$f_{cu} = 1.25 \cdot f_c = 1.25 \cdot 21.56 = 26.95 MPa \quad (8.2)$$

A comparison of fracture energy model was performed between FIB1990 and FIB2010. The amount of fracture energy are, for simplicity, calculated by assuming a concrete class of C20/25. For the FIB 1990 fracture energy model, an aggregate size of 8 mm was assumed. Results are presented in Figs. 8.7 and 8.8.

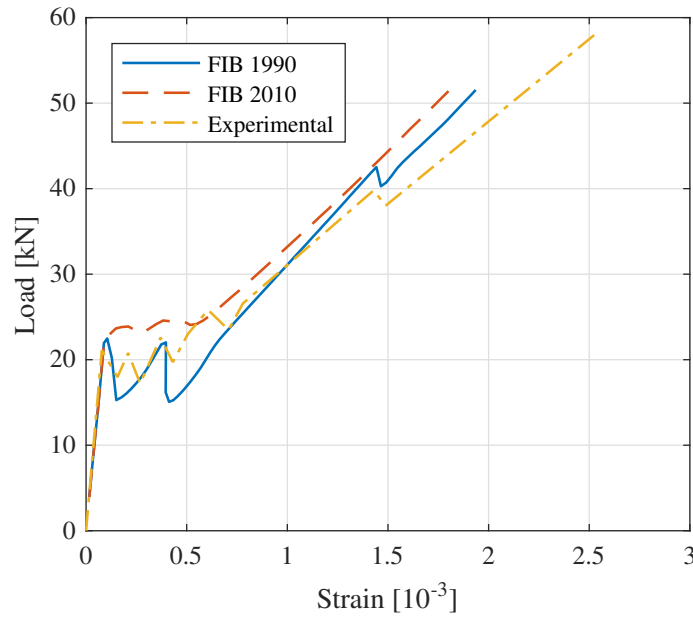


Fig. 8.7: *Comparison between fracture energy-models for STN12.*

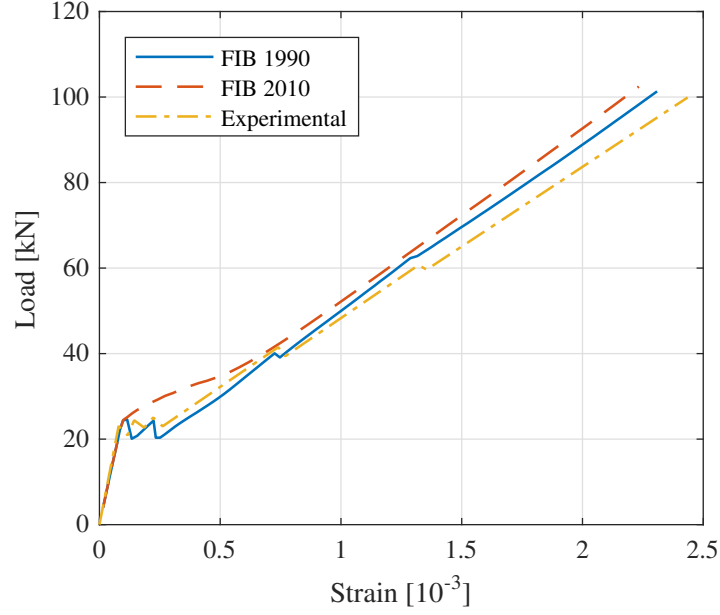


Fig. 8.8: Comparison between fracture energy-models for STN16.

Reinforcement and bond model

The reinforcement was modelled as a discrete bar, only taking load in the axial direction, meaning no bending or shear could be carried by the reinforcement. The material was assumed to follow a bilinear ideal elastic-plastic stress-strain law where the modulus of elasticity was set to $E_s = 200$ GPa and the yield stress to $\sigma_y = 500$ MPa. In this analysis the weight of the reinforcement was neglected.

A comparison between Bigaj slip law and slip law according to Model Code 2010 [19] was performed. The bond-slip relation for Bigaj bond model are presented in Tab. 8.3 for the bar diameters 12 mm and 16 mm. Note that the bond strength, and not the bond stress, are presented for the correlating slip.

Slip $\phi 12$ [mm]	Slip $\phi 16$ [mm]	$f_{strength}$ [MPa]
0	0	2.32
0.36	0.48	9.29
0.56	0.75	3.2
5.96	7.68	0

Tab. 8.3: Bond strength-slip relation

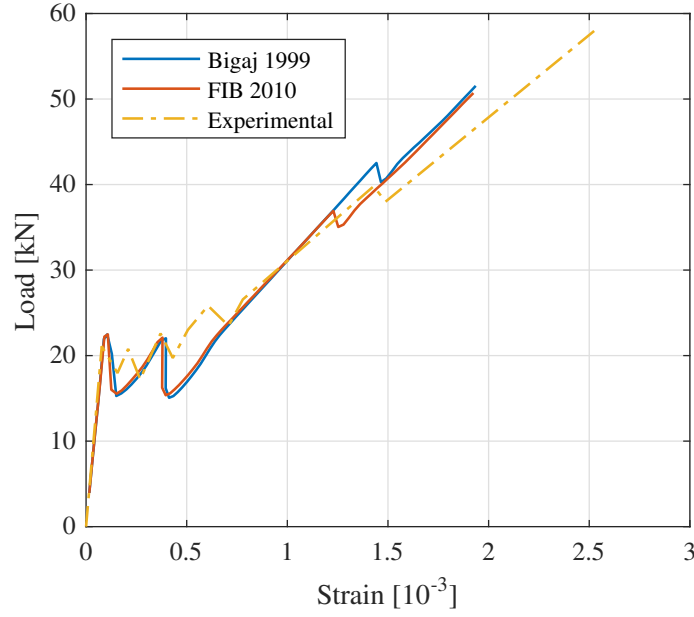


Fig. 8.9: Comparison between Bigaj 1999 and Model Code 2010 bond-slip models for $\phi = 12$ mm reinforcement.

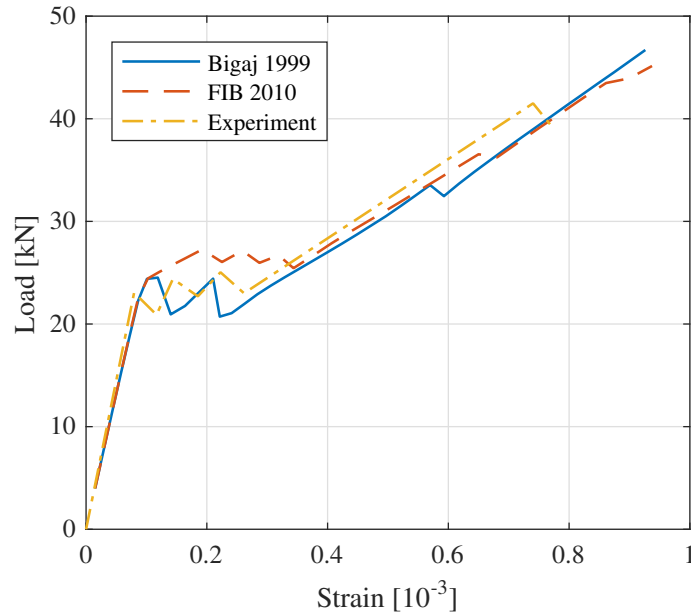


Fig. 8.10: Force-strain relation for Bigaj 1999 and Model Code 2010 bond-slip models with $\phi = 16$ mm reinforcement.

Mesh size evaluation

The choice of element size is important in order to achieve accurate results. In theory, the approximated solution for a finite element model should converge toward the exact solution when decreasing the element size. However, the cost of a finer mesh is an increase of CPU-time. A convergence study can be performed to find a suitable mesh size, such that the element size will not significantly affect the result.

A convergence study was performed with six different element sizes. The number of cracks, maximum crack width and mean crack width, presented in Tab. 8.4, are similar in magnitude.

Mesh size	Max crack width		Mean crack width		Number of cracks	
	$F = 40$	$F = 50$	$F = 40$	$F = 50$	$F = 40$	$F = 50$
25	0.33	0.39	0.30	0.28	2	3
12.5	0.34	0.40	0.28	0.22	2	4
10	0.32	0.40	0.29	0.27	2	3
50/7	0.31	0.38	0.23	0.29	3	3
5.0	0.31	0.38	0.29	0.28	2	3
2.5	0.31	0.40	0.21	0.27	3	3

Tab. 8.4: Crack widths for different mesh sizes at different loading stage

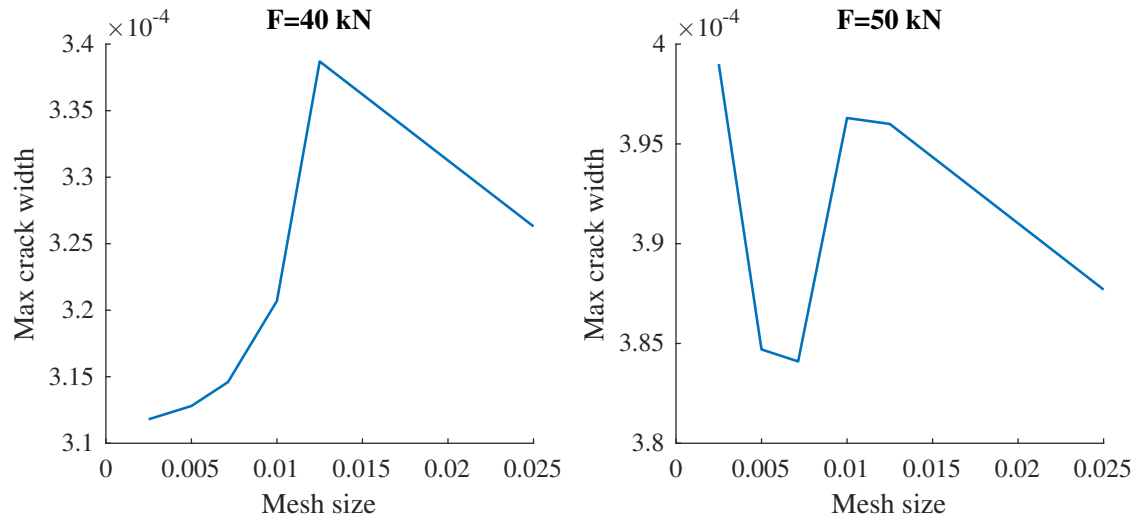


Fig. 8.11: Relation between maximum crack width and element size for $F = 40$ kN and $F = 50$ kN.

8.1.3 Comparasion with Method by Engström

The method by Engström appears to correspond well with the experimental results for STN12, as can be seen in Fig. 8.12. The total number of cracks amounted to four and the transmission length can be calculated according to Eq. 7.26 to 181 mm. In the experiment, the total number of cracks amounted to five and the average crack spacing to 191 mm.

The cracking behaviour with the method by Engström correspond well with the results from the analysis for STN16, as seen in Fig. 8.13. The total number of cracks amounted to four to compare with five in the experiment.

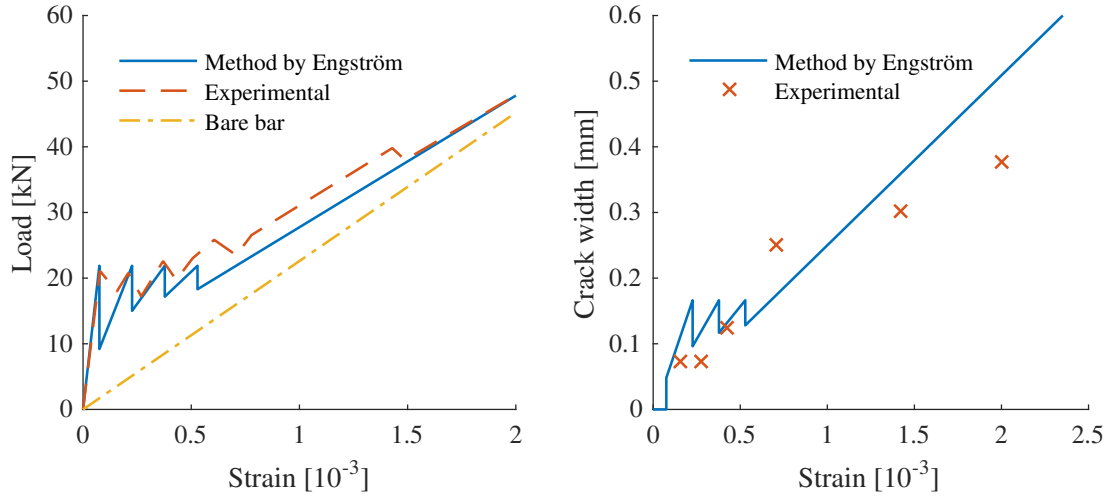


Fig. 8.12: Load and crack width versus strain for STN12.

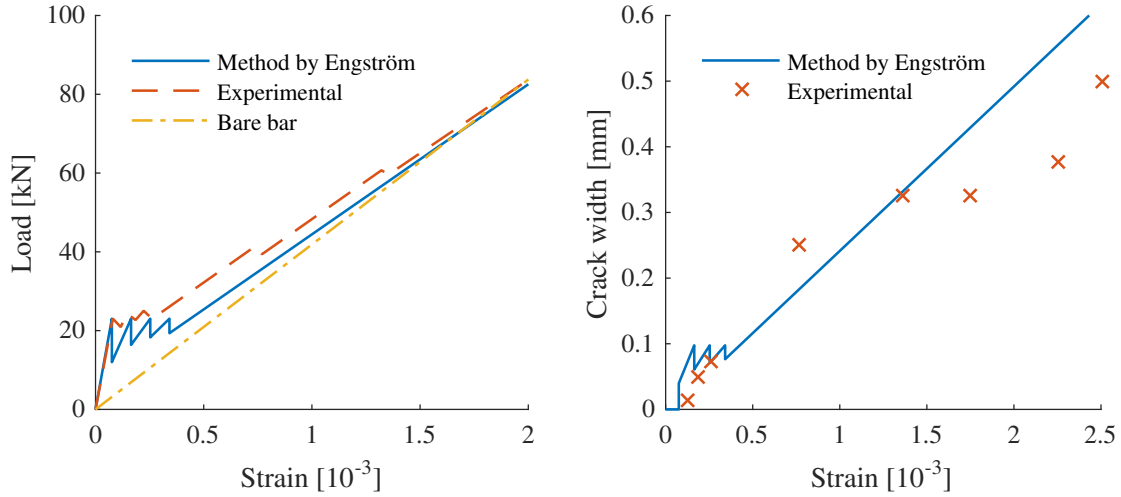


Fig. 8.13: Load and crack width versus strain for STN16.

8.1.4 Concluding Remarks

From the calibration, following conclusions has been made:

- The fracture energy model according to Model Code 1990 [18] correlates well with real experiment regarding crack behaviour.
- Symmetry can be used in the crack analysis without any major impact on the crack behaviour.
- The bond model according to Bigaj shows good correlation with the experimental results.
- The mesh size will have an impact on the crack behaviour; however, when the mesh size is smaller than 10 mm, the impact of the mesh size is very small.
- The experimental results correlates well with the method by Engström.

These conclusions has been adopted in the parametric study.

8.2 Estimation of Drying Shrinkage Strain

The volume change due to drying shrinkage is highly influenced by the relative humidity of the surrounding environment. In Eurocode, the mean drying shrinkage is taken as uniform for the entire cross section of a structure, which is rarely the case in reality, and especially not for a concrete slab on ground. In [32], a procedure is proposed on how to calculate the shrinkage gradient. A variant of this procedure is adapted to estimate the cross section variation of the shrinkage strain.

The method to estimate a linear cross section variation of drying shrinkage strain is as follows:

1. Simulate RH at a certain depth of the slab (in this thesis at 80 % depth of the slab) and calculate the drying shrinkage strain at the point according to Eurocode.
2. Calculate the drying shrinkage at the top of the slab with the assumed RH in the surrounding environment.
3. Extrapolate a strain over the cross section.

The software used to simulate RH in this study was "Produktionsplanering Betong 2.0" (PPB). The software accounts for material properties of modern cement types containing silica, which makes the concrete more dense than ordinary Portland cement. This is typical for green concrete. The program use data from the Swedish Meteorological and Hydrological Institute (SMHI) to simulate weather conditions such as temperature, precipitation and wind speed. Moreover, the program simulates the construction phase and can be customized to a specific construction schedule, taking into account when waterproof structure is estimated to be completed and when specific drying conditions are applied [57]. The program is limited to 10 000 days simulation period, which corresponds to approximately 27 years. However, when the drying shrinkage was calculated after 27 years and compared with the theoretical long term shrinkage, only a difference of 1 % was obtained. Therefore, in further analysis, the 27-years shrinkage value was used as the long term shrinkage, $\varepsilon_{cs,\infty}$.

The drying is highly dependent on the conditions that exists during the first period after casting and the choice of casting date will affect the long term shrinkage strain [57]. To obtain a representative shrinkage value, the date of casting should be chosen such that the shrinkage will represent a mean value. Fig. 8.14 shows how the mean shrinkage strain vary with respect to the casting date. The plotted data is obtained for a simulation time of 5.5 years.

It can be seen in Fig. 8.14 that a representative casting date is either during the spring month, March - May or in the fall, October - December. In further simulations the casting date was set to 1st of March.

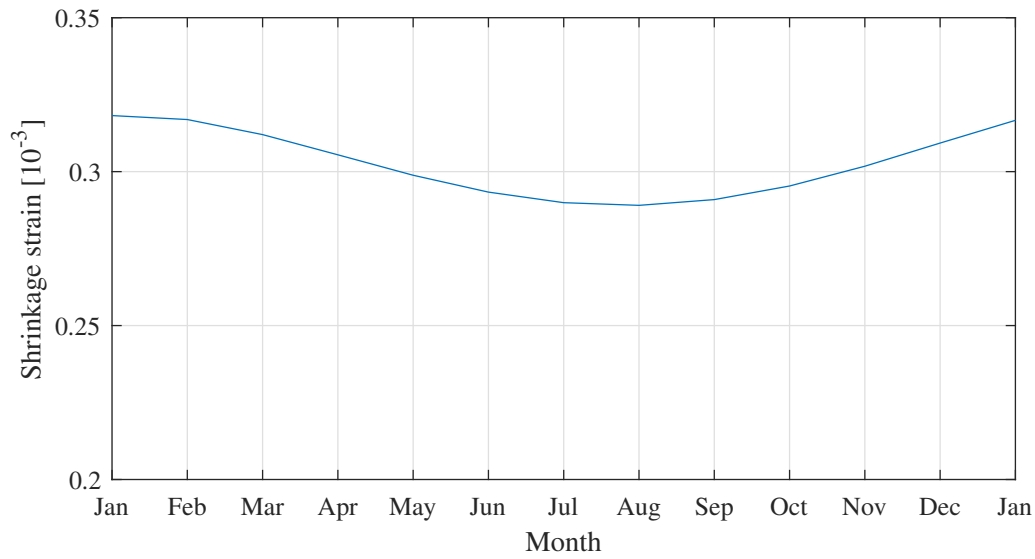


Fig. 8.14: *The variation of drying shrinkage depending on casting date.*

Input parameters for long term RH simulations are presented in Tab. 8.5.

Parameter	Value
Simulation time:	10 000 days
Outdoor climate:	Simulated, based on climate data for Malmö
WaterProof House:	After 55 days
Drying conditions:	After 56 days (RH = 50 %, Temp = 20 C)
Coverage after casting:	Tarpaulin
Coverage time:	336 hours after casting
Heating:	None
Insulation material:	Foam insulation
Insulation thickness:	300 mm
Underground material:	Moraine/Gravel
Underground temperature:	Simulated

Tab. 8.5: *Input parameters to PPB for analysis of RH in slab.*

The RH is simulated for the different concrete qualities as well as for the different slab heights used in the crack analysis. Results from preformed simulations are presented in Tab. 8.6, where the RH and the shrinkage strain at top/ bottom is showed. For calculation procedure, see Appendix C. Note that the total shrinkage are presented, i.e. drying and autogenous shrinkage.

Concrete quality	Slab height	Simulated RH	Strain at top	Strain at bottom
	[mm]	[%]	[10 ⁻³]	[10 ⁻³]
C28/35	100	78.2	0.465	0.253
C32/40	100	78.3	0.455	0.252
C35/45	100	77	0.449	0.266
C45/55	100	72.2	0.430	0.307
C50/60	100	69.9	0.422	0.323
C35/45	80	74.2	0.476	0.308
C35/45	120	79.0	0.430	0.237
C35/45	140	80.5	0.412	0.214

Tab. 8.6: *Results obtained from the shrinkage simulation for different concrete qualities.*

Chapter 9

Study of Cracking in Concrete Slab Foundations: Analytical and Numerical Results

9.1 Parametric Study of Slab Foundation

A parametric study of a slab foundation was performed with nonlinear finite element analysis in Atena 2D. The crack behaviour was recorded as the shrinkage load was applied stepwise. Creep was not considered in the analysis. A mesh size of 10 mm was used for the concrete part, since it gave sufficient precision for crack width estimation in the calibration. The sub-grade was modelled as an elastic isotropic material with a mesh size of 50 mm but refined at the boundary to the concrete, as seen in Fig. 9.1. To account for friction, the contact between the sub-base and the slab was modelled with an interface material. The fracture energy was calculated according to Model Code 1990, assuming an 8 mm aggregate size, which was proven to be accurate in the calibration. Furthermore, a bond model by Bigaj 1999 [7] was adopted. The SBETA material model was implemented to the concrete and a plane stress condition was assumed. Reinforcement was centrally placed in the concrete slab and slip was disabled at the bar ends. Finally, the shrinkage load was applied as a linear strain gradient with a load stepping procedure. The final load was reached after 20 load steps but another ten steps were recorded, which in the diagrams are indicated with a grey area. The self-weight of the concrete was applied before the shrinkage, in a separate load step.

The parameters considered in the parametric study were:

- Boundary conditions
- Stiffness of the sub-base
- Friction coefficient
- Slab length

- Slab height
- Bar diameter
- Reinforcement ratio
- Concrete Class
- Type of strain distribution (uniform, linear and nonlinear)

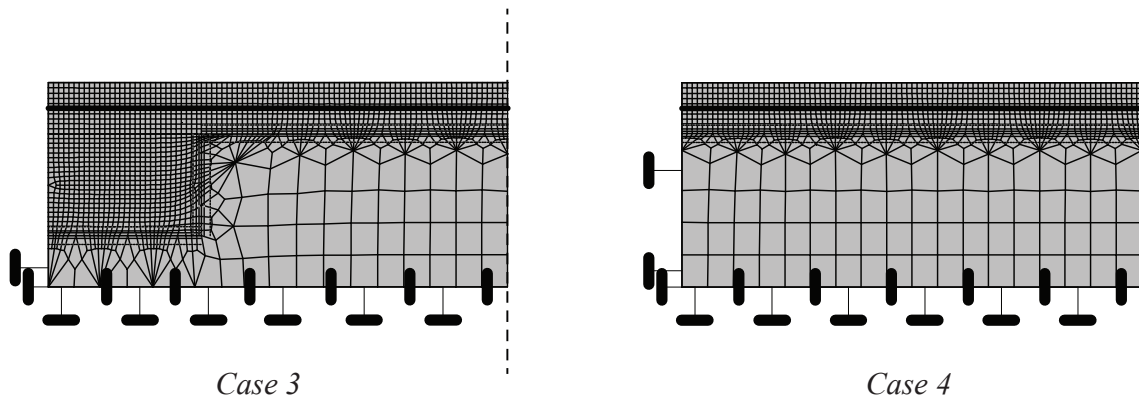


Fig. 9.1: The figure shows the mesh for the FE-models Case 3 and Case 4. The mesh size of the elastic sub-base material was set to 50 mm, and 10 mm for the concrete. The mesh was refined at the interface between the concrete and the sub-base.

For each analysis, an illustration of the crack pattern is presented in Appendix A. Only cracks with a width greater than 0.01 mm are shown.

9.1.1 Boundary Conditions

Four different geometric FE-models are investigated, see Fig. 9.2. Case 1 and 2 represent the theoretical models used in the analytical methods, i.e. EC2 and the method by Engström. Case 3 and 4 take into account the friction and the stiffness of the insulation material, which better correspond to the real conditions of a slab foundation. Material properties for the concrete are presented in Tab. 9.1.

Concrete class	G_f [N/m]	f_c [MPa]	f_{ct} [MPa]	E_c [MPa]	h [mm]	φ [%]
C35/45	55.5	43	1.92	34000	100	0.79

Tab. 9.1: Material properties used in the analysis, comparing different geometric FE-models.

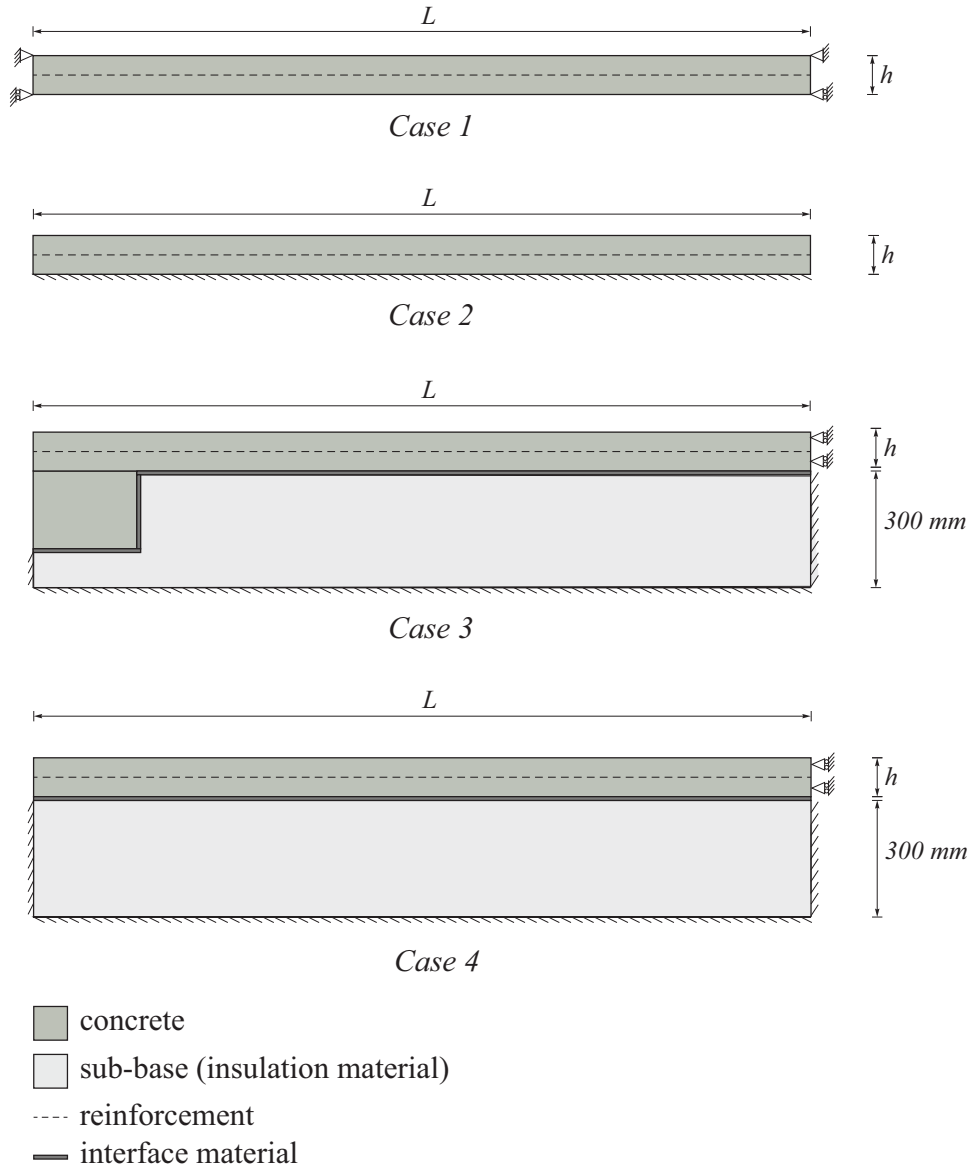


Fig. 9.2: Investigated models in the FE-analysis. Case 1 and 2 represents a continuous edge restraint and a end restrained slab. Case 3 and 4 represent models which correspond to reality, with friction and stiffness of the sub-base taken into account.

The four models in Fig. 9.2 are loaded with the same shrinkage strain. Fig. 9.3 shows the maximum crack width as a function of the shrinkage strain for the FE-analysis and Fig. 9.5 shows the crack width obtained with the method by Engström.

The numerical analysis gives for Case 1 a maximum crack width at about 0.25 mm. The cracks propagate at a relatively low strain and the width of the cracks increase quite fast as the strain gets higher. The maximum crack width decrease to some extent when the strain is further increased. This is due to the members ability to deform in the y-direction, where the deformation keeps the existing cracks together. The method by Engström gives a maximum crack width of 0.21 mm when the slab is fully restrained, which is of the same magnitude as the numerical results. In the

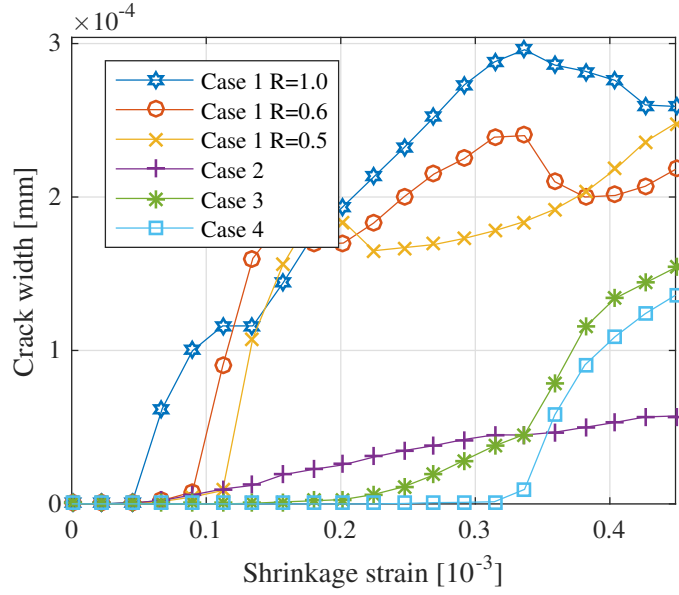


Fig. 9.3: Crack width propagation for the various cases. In Case 1, the restrained factor is set to $R = 0.5/0.6/1.0$.

method by Engström, the degree of restraint increases the amount of cracks, but does not however influence the crack width. The FE-analysis, on the other hand, shows greater crack widths for a higher degree of restraint.

For Case 2, the maximum crack width is 0.6 mm, which is the lowest measured crack width of the four cases. Fig. 9.3 shows that cracks are also initiated at a low strain. In contrary to Case 1, the crack growth is almost constant and cracks are distributed more evenly along the length of the slab, see Fig. 9.4.

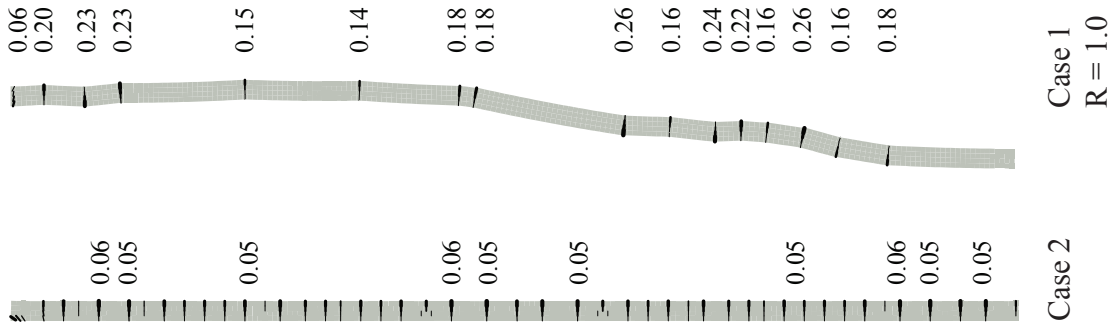


Fig. 9.4: Crack pattern and crack widths for Case 1 ($R=1.0$) and Case 2.

Case 3 shows slightly larger cracks than Case 4, although Case 3 is restrained from the footing. The maximum crack width is about 0.15 mm for the two cases and they show similar crack behaviour, where the cracks are initiated at a slightly lower strain for Case 3.

In the method by Engström, the maximum crack width is reached after a relatively low strain and an increase of strain will only lead to more cracks of equal size. The method gives similar results as the numerical analysis for Case 1. However, in comparison to the

realistic models, the method by Engström does not show a realistic crack behaviour. In further parametric studies, only Case 3 and Case 4 are evaluated and compared to EC2. Results obtained with the method by Engström is presented in Appendix B, but is not included further in this chapter. For a more profound analysis of this method, the reader is encouraged to read previous master's thesis carried out at Chalmers University: [40], [1] and [2].

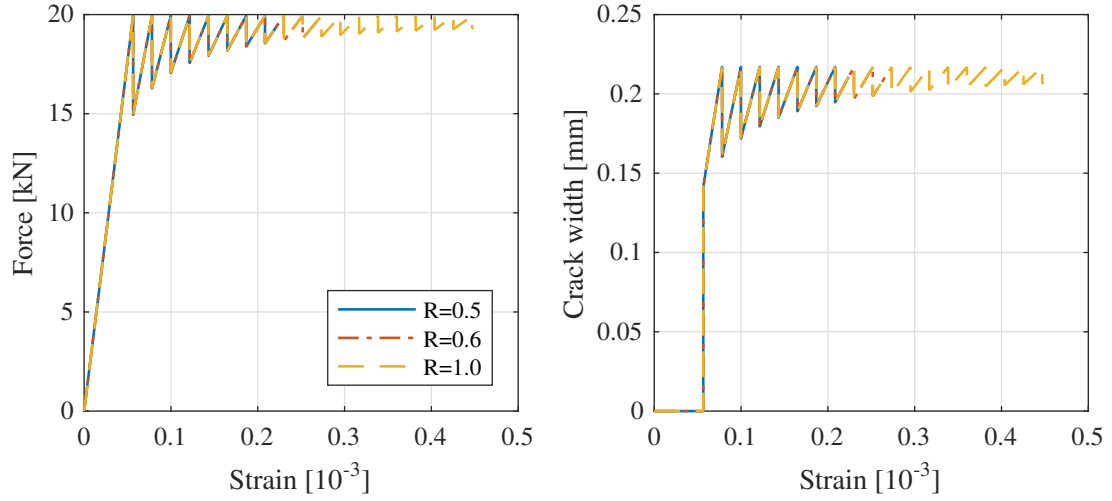


Fig. 9.5: Force and crack width versus strain with method by Engström.

9.1.2 Influence of Stiffness of the Sub-base

It was stated that the E-modulus for insulation material is approximately 15 MPa for short-term loading and 5 MPa for long-term loading. In order to investigate how the crack width is affected by the E-modulus of the insulation, several analyses were carried out with different stiffnesses of the sub-base, see Tab. 9.2.

Concrete class	E-modulus [MPa]	Cases
C35/45	5/15/30/60/120/10000	3/4

Tab. 9.2: *Parameters used in the performed analysis.*

The result shows a wide variety of the maximum crack width for the different E-modulus. With increased stiffness, the external restraint will also increase, and thus also the crack width. Case 3 and 4 shows a similar behaviour, as can be seen in Fig. 9.6, even though the slab in Case 4 was not restricted by the insulation material in the x-direction.

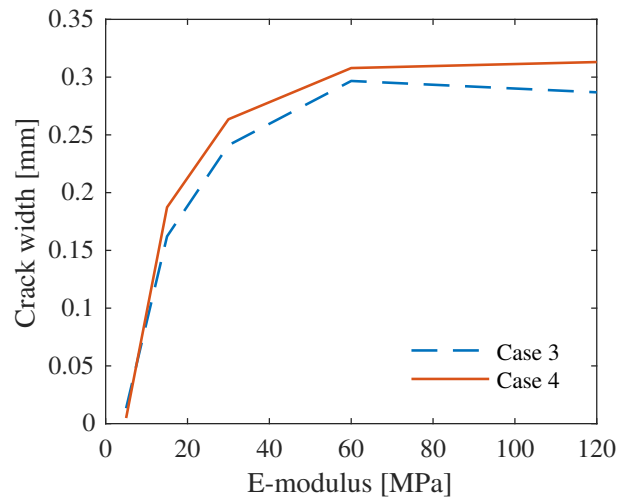


Fig. 9.6: *Crack width for various stiffness of the sub-base.*

Studying Fig. 9.7, the initiating cracking starts later as the stiffness decreases. Once the crack has initiated, however, the crack growth is much faster. It is not only the crack width which increases for a higher stiffness, but also the amount of cracks. For $E = 5$ MPa, only one crack occur, and for $E = 120$ MPa, seven cracks are calculated.

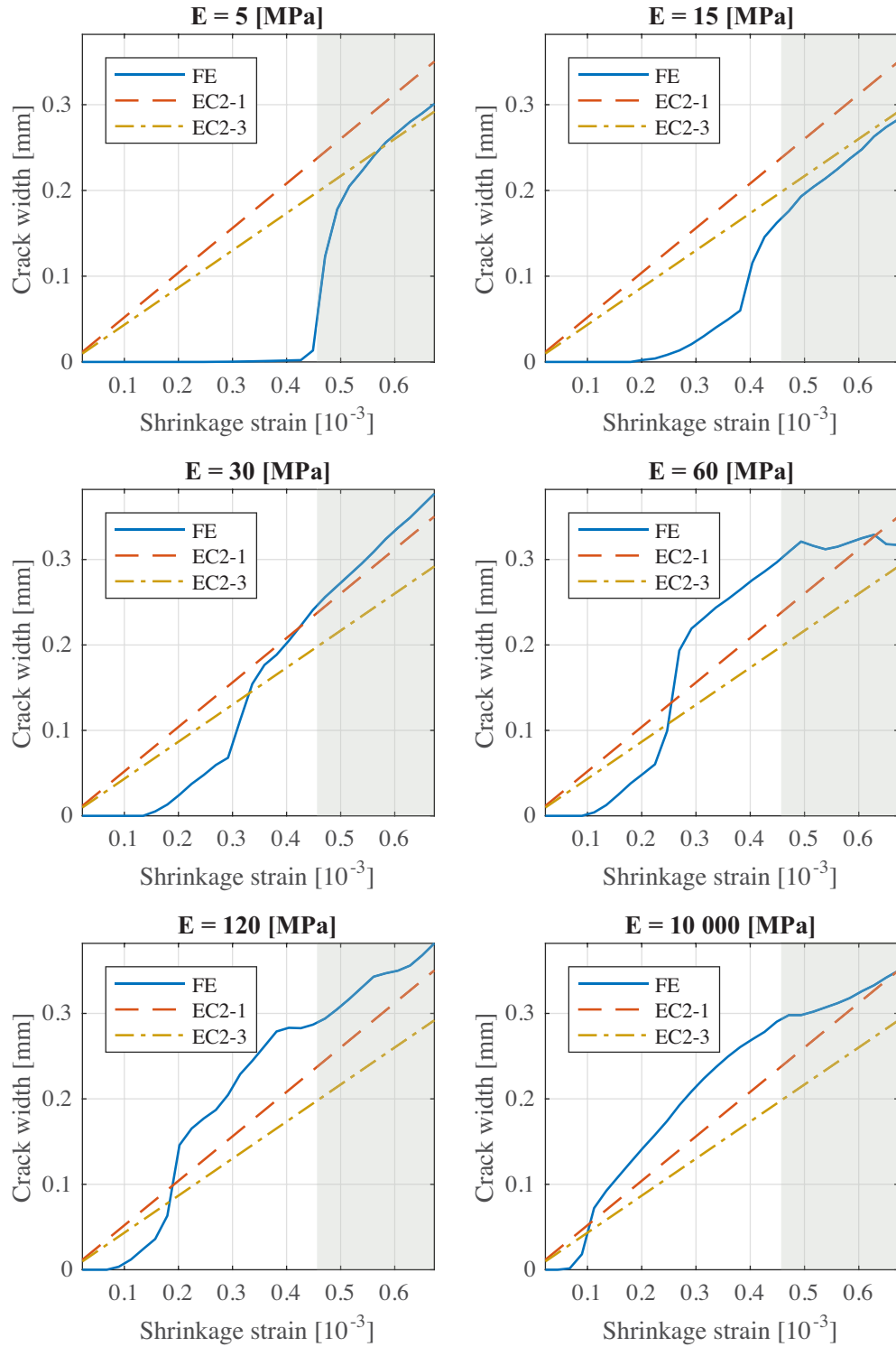


Fig. 9.7: Crack propagation for different insulation stiffness. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.

9.1.3 Influence of Friction Coefficient

FE-analyses were performed for various friction coefficients and with different E-modulus for the sub-grade material.

Concrete class	Friction coefficient [μ]	E-modulus [MPa]	Case
C35/45	0/0.5/0.75/1.5	30/10000	3

Tab. 9.3: *Parameters used in the performed analysis.*

For a resilient sub-base material with $E = 30$ MPa, the maximum crack width varies noncontinuous and with a slight variation. A further analysis was performed for a rigid sub-base material with $E = 10000$ MPa. The result from this analysis shows a greater variation, as seen in Fig. 9.8. It is also shown that the crack width decreases with increased friction.

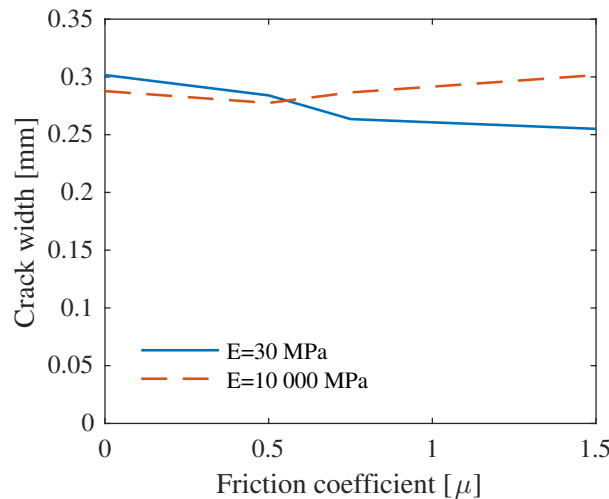


Fig. 9.8: *Crack width for various friction coefficients [μ].*

Studying Fig. A.8, the number of cracks for the rigid sub-base material is more than double than for the resilient sub-base material. The number of cracks does not appear to vary for different coefficients of friction.

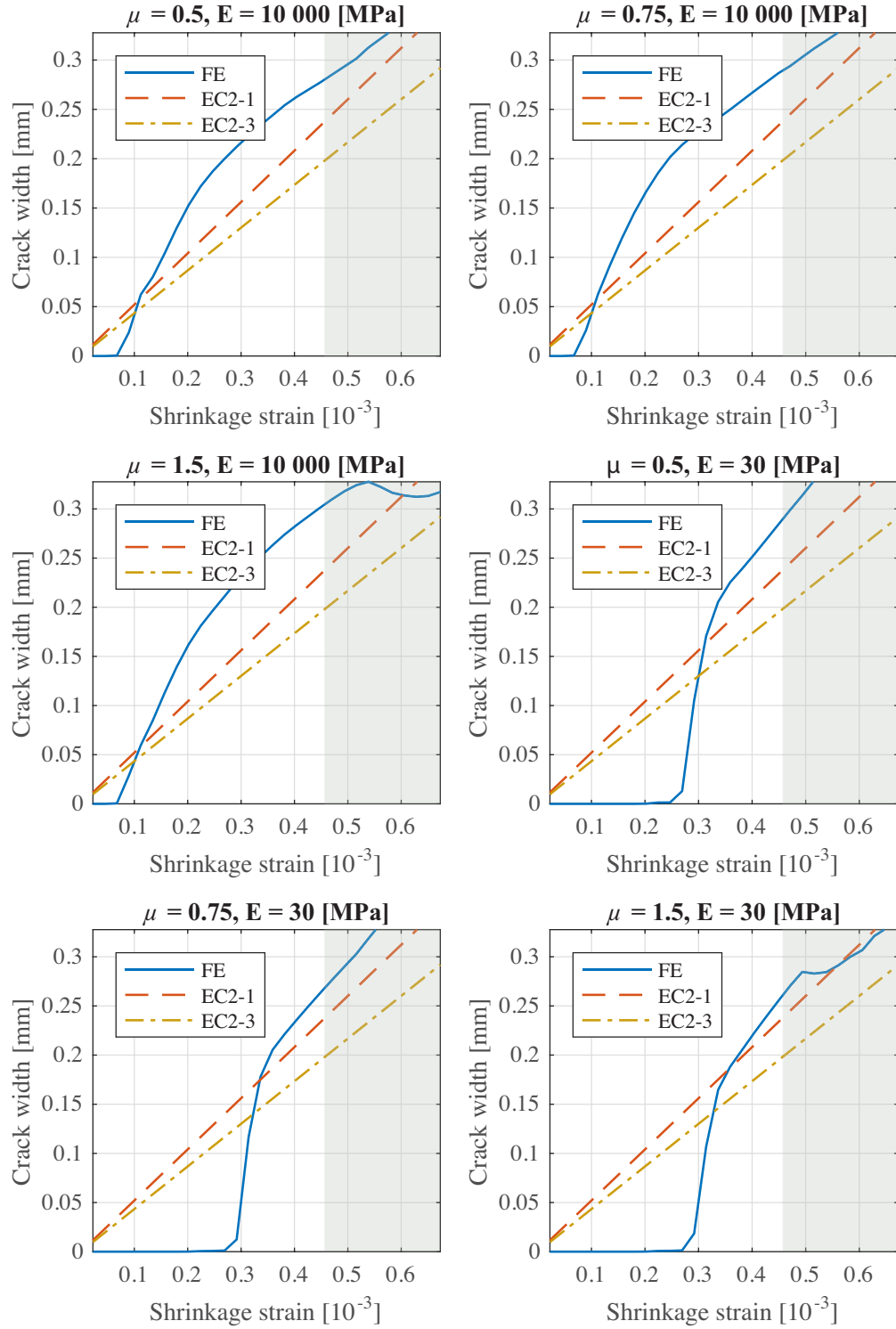


Fig. 9.9: Crack propagation for different friction coefficient and insulation stiffness. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.

9.1.4 Influence of Slab Length

FE-analyses were performed for various slab lengths, presented in Tab. 9.4. No parameters other than the slab lengths were varied in this analysis.

Concrete class	Slab length [m]	Bar diameter [mm]	Reinforcement ratio [%]	Case
C35/45	5/7.5/10/12.5/15	10	0.79	3

Tab. 9.4: *Parameters in the performed analysis.*

As can be seen in Fig. 9.10, the crack widths increases as the length of the slab increases. For a 5 and 7.5 meter long slab, the crack widths are not larger than 0.12 mm. For a 15 m long slab, the maximum crack width obtained is about 0.24 mm.

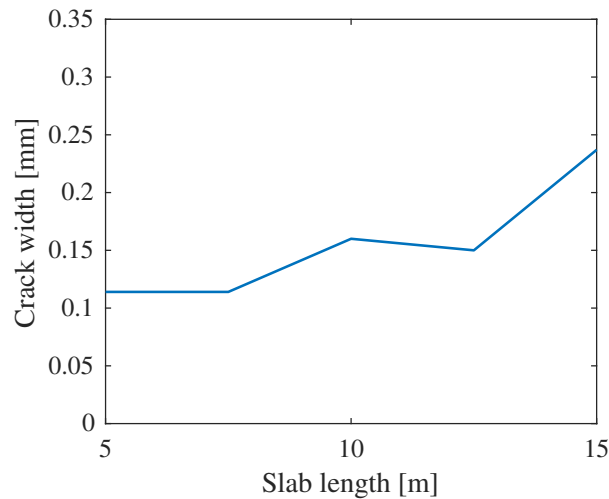


Fig. 9.10: *Maximum crack widths obtained with different slab lengths.*

The crack behaviour presented in Fig. 9.11 shows a wide diversity of the crack width for the different slab length. At the calculated shrinkage strain, the 10 m long slab seems to provide a larger crack width than the 12.5 m long slab, but for further strain the 12.5 m slab grows significantly faster.

A total of seven cracks can be obtained for the 15 m long slab, and three for the 5 m long slab.

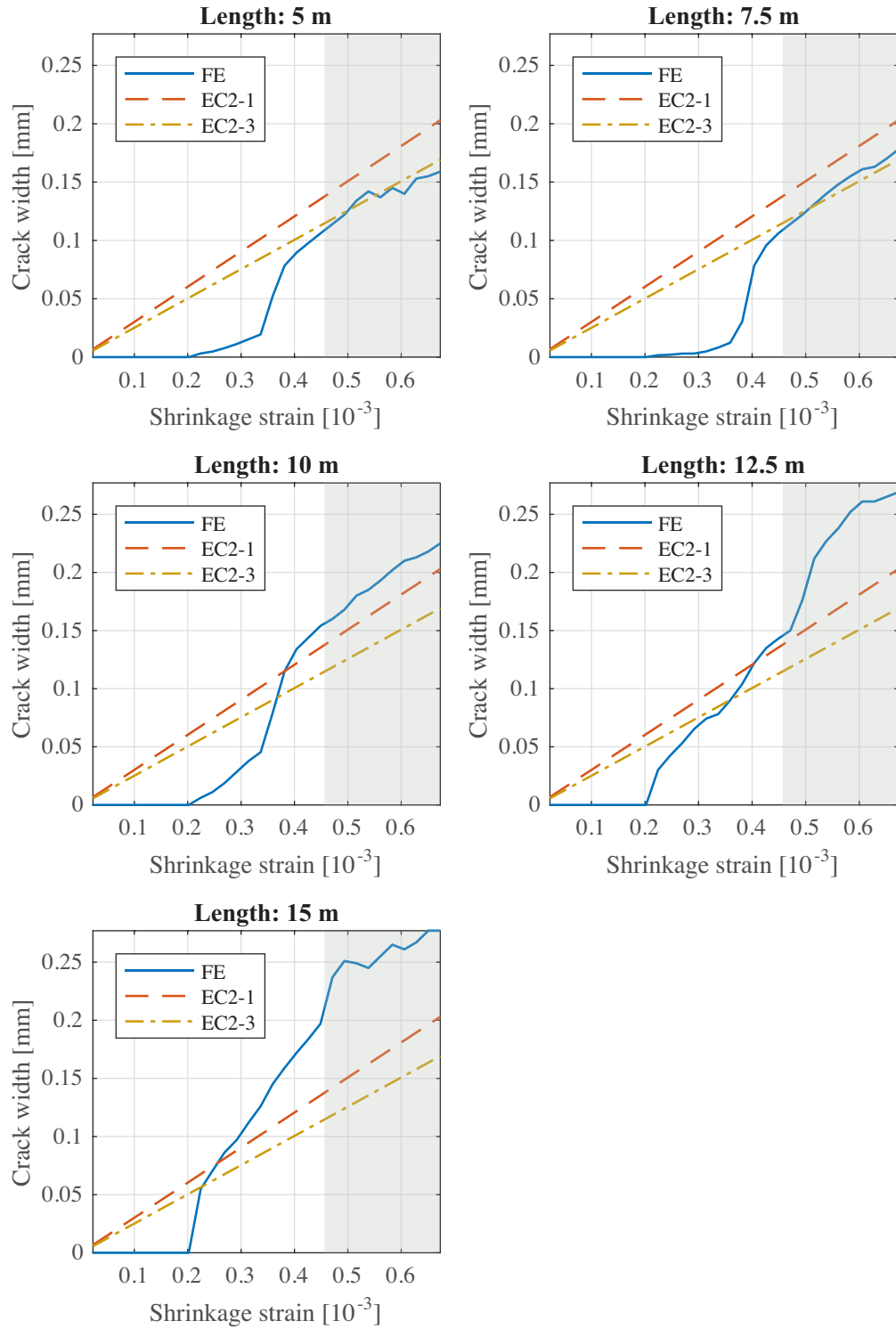


Fig. 9.11: Crack propagation for different slab lengths. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.

9.1.5 Influence of Slab Height

In order to investigate how the geometry of the slab will affect the crack behaviour, several analyses were carried out on different slab heights. The final shrinkage strain was calculated for each slab height, see Tab. 8.6.

Concrete class	Slab height [mm]	Bar diameter [mm]	Case
C35/45	80/100/120/140	10	3

Tab. 9.5: *Parameters in the performed analysis.*

Fig. 9.12 shows that as the slab height increases the maximum crack width will also increase, except for the case with 80 mm slab height. The maximum crack obtained for 140 mm height is approximately 30 % wider than for the 100 mm height.

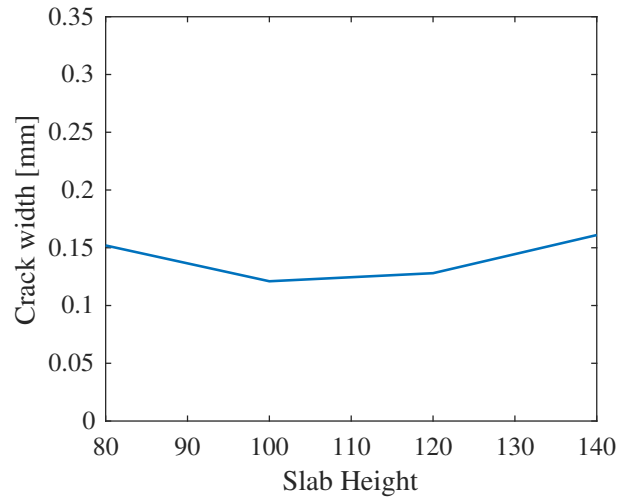


Fig. 9.12: *Maximum crack widths obtained with different slab heights.*

The FE-results shows similar crack widths as EC2, as seen in Fig. 9.13. It should be noted that the 140 mm height slab does not reach the minimum reinforcement requirement.

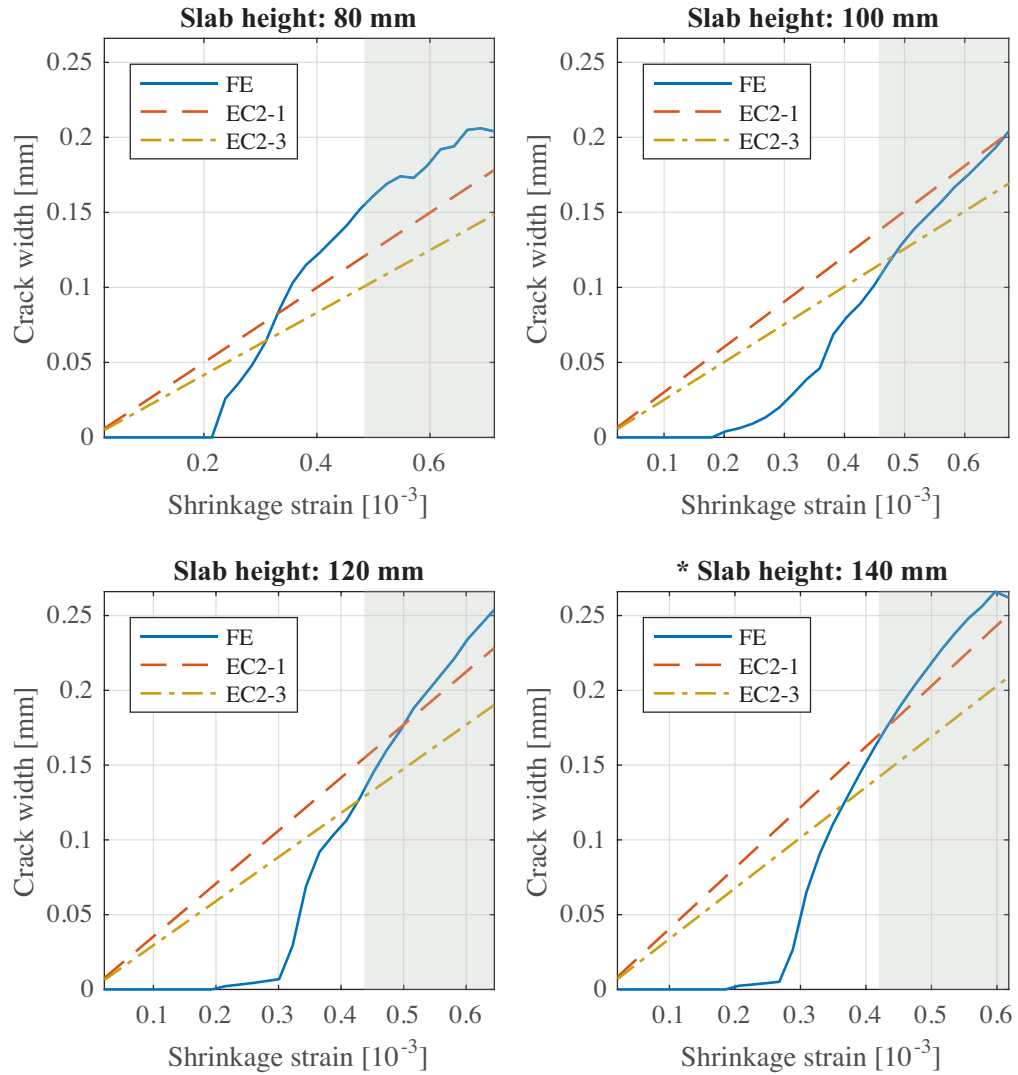


Fig. 9.13: Crack propagation for different slab heights. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.
 * - Requirement regarding minimum reinforcement according to EC2 was not fulfilled.

9.1.6 Influence of Bar Diameter

The influence of how the bar diameter affect the crack width was examined by varying the bar diameter but keeping the reinforcement ratio constant. The investigated cases in the analysis are presented in Tab. 9.6. In the analysis, a new bond was calculated for each bar diameter according to the bond model from Bigaj [7]. The reinforcement ratio was chosen to the minimum reinforcement criteria according to EC2.

Concrete class	Bar diameter [mm]	Reinforcement ratio [%]	Case
C35/45	6/8/10/12	0.58	3

Tab. 9.6: *Parameters in the performed analysis.*

As can be seen in Fig. 9.14, the maximum crack width increase with increasing bar diameter. The parameters which influence the result are the stress-strain relation for the bond, and the contact area between the concrete and the reinforcement which increases for smaller bar diameters.

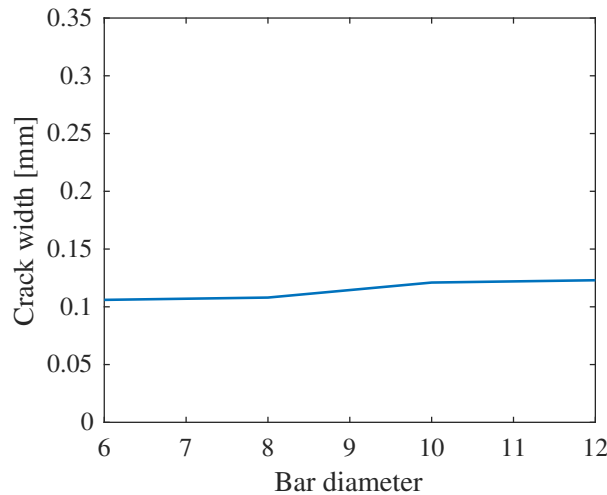


Fig. 9.14: *Various bar diameter with the same reinforcement ratio.*

The crack behaviour for the four cases follows roughly the same pattern.

According to Fig. 9.15 on the next page, EC2 provides smaller crack widths only for the case with $\phi = 6$ mm bar reinforcement. The amount of cracks for the different bar diameters are six or seven.

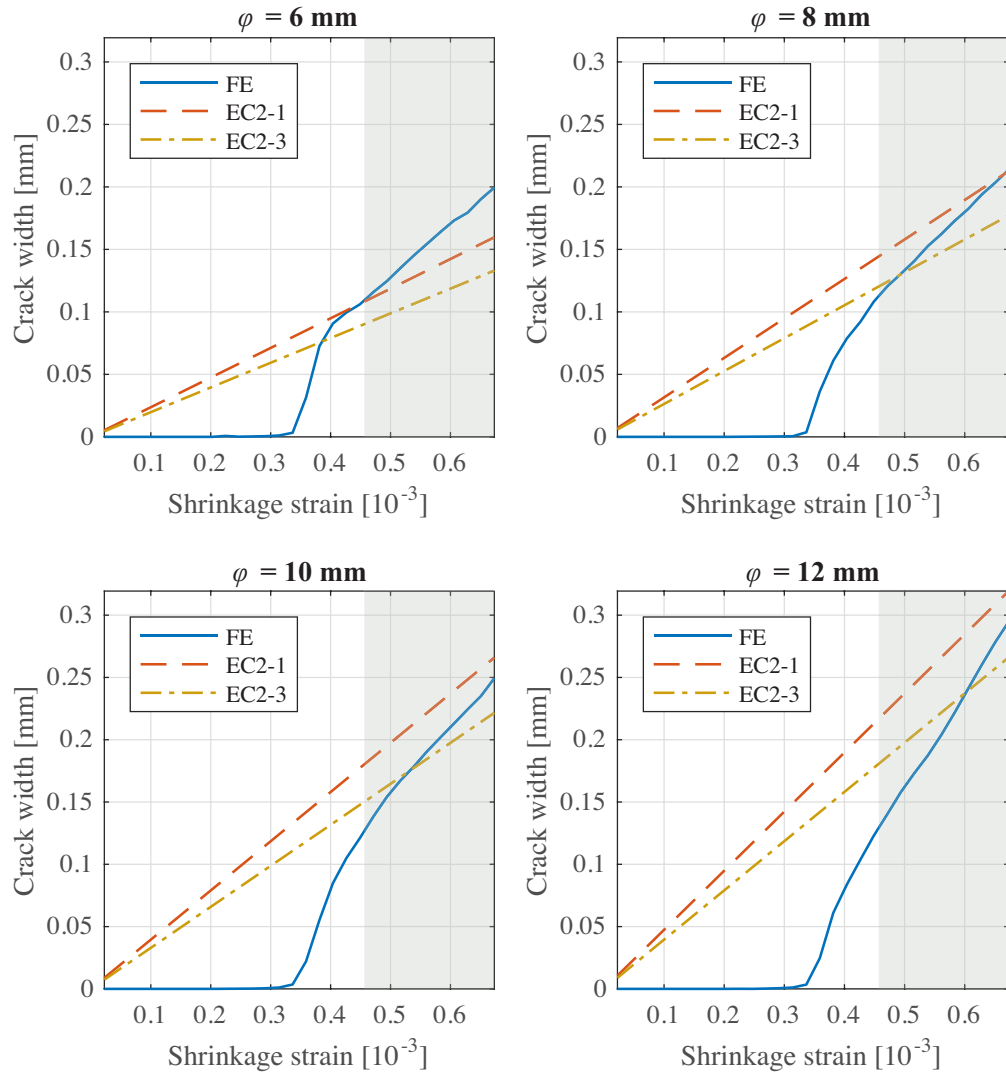


Fig. 9.15: Crack propagation for the different bar diameters. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.

9.1.7 Influence of Reinforcement Ratio

The influence of reinforcement ratio was examined. The reinforcement ratio was calculated for different c/c -distances: 50/75/100/150/200/250 mm. The resulting reinforcement ratio and parameters used in the analysis are presented in Tab. 9.16.

Concrete class	Bar diameter [mm]	Reinforcement ratio [%]	Case
C35/45	10	1.01/0.67/0.50/0.34/0.25/0.20	3

Tab. 9.7: *Parameters in the performed analysis.*

The result clearly shows that an increased amount of reinforcement results in smaller cracks. Note that if the reinforcement ratio is increased from 0.2 % to 1.0 %, which corresponds to an increase of 500 %, the crack width decreases with approximately 50 %.

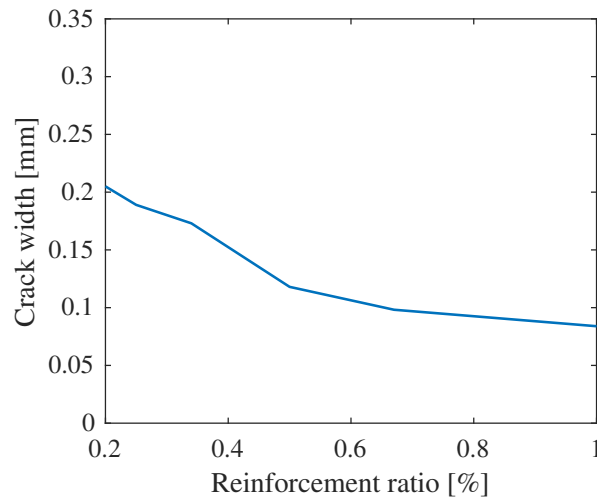


Fig. 9.16: *Various reinforcement ratio with the same bar diameter.*

As denoted in Fig. 9.17 with an asterisk (*), the minimum reinforcement is not fulfilled for four of the cases. As has been stated, the crack width calculation according to EC2 requires that the minimum reinforcement is fulfilled. Apart from that, when the reinforcement ratio decreases, the difference between the numerical results and the FE-results seems to increase.

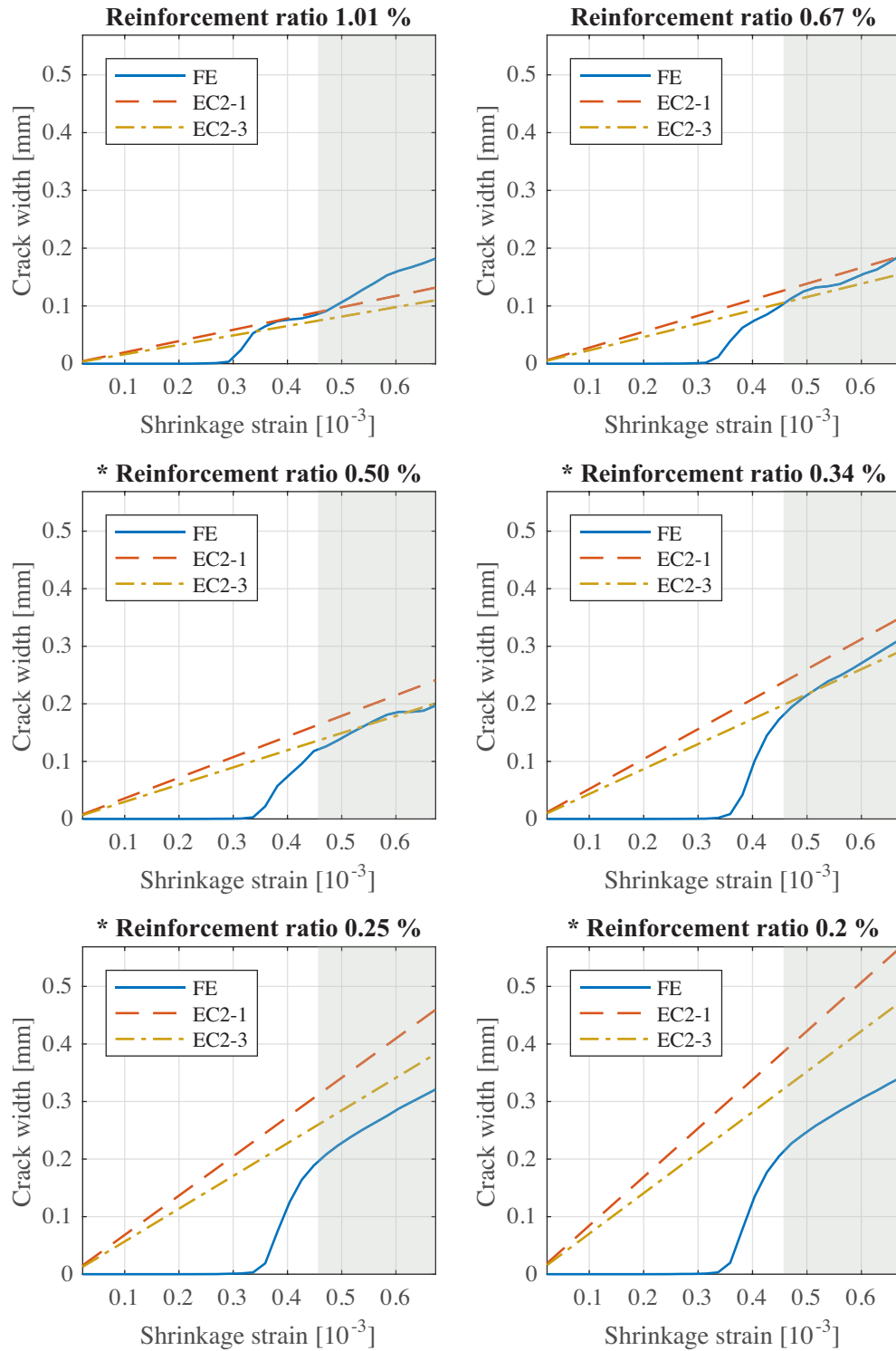


Fig. 9.17: Crack propagation for different reinforcement ratios. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain. * - Requirement regarding minimum reinforcement according to EC2 was not fulfilled.

9.1.8 Influence of Strain Distribution

Three different strain distribution was evaluated according to Fig. 9.18. In order to model a nonlinear strain distribution, the FE-model was divided into three layers, of which each layer was distributed a unique linear shrinkage. For all cases, the strain was set to the same value at the top- Thus, the total strain was greater for the uniform distribution, followed by the linear distribution and the nonlinear distribution, in that order.

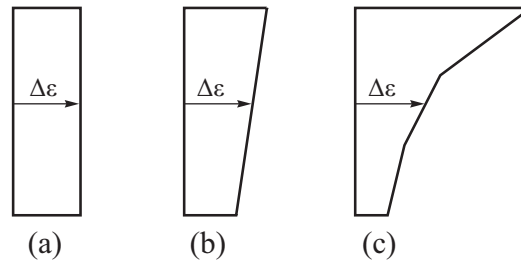


Fig. 9.18: Investigated strain distributions: (a) uniform, (b) linear, (c) nonlinear.

As can be seen in Fig. 9.19, the maximum crack width for a linear strain distribution is 50 % greater for a linear strain distribution, after final shrinkage is reached. The uniform shrinkage differ from the other two and exhibits small crack widths. As can be seen the crack propagation curves, Fig. 9.20, the maximum crack width increases drastically after the final simulated shrinkage value is reached.

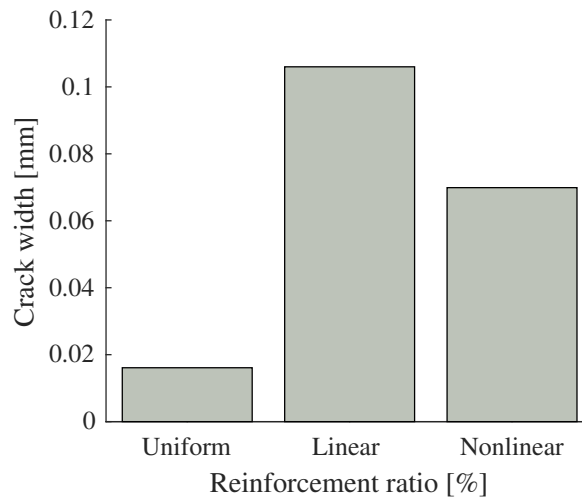


Fig. 9.19: Maximum crack width after final shrinkage.

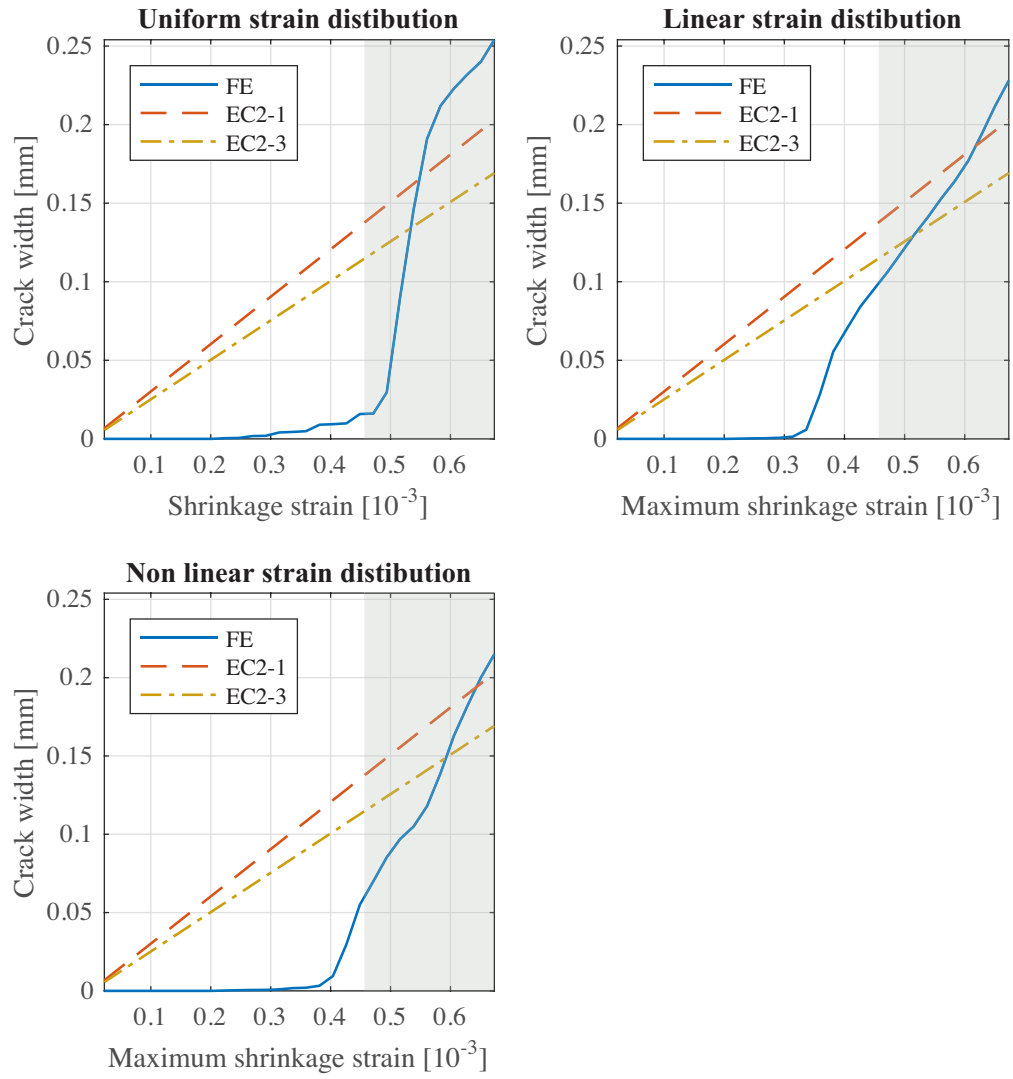


Fig. 9.20: Crack propagation for different strain distributions. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.

9.1.9 Influence of Concrete Class

To investigate how the concrete class will affect the crack width, the FE-model were analysed with different concrete types. The material properties used in the analysis are presented in Tab. 9.8. For every concrete class, the total shrinkage strain was calculated according to EC2 by calculating the relative humidity with PPB. With the concrete class, it follows that the fracture energy decrease or increase with the concrete compressive strength, as well as the bond. The interface material was kept the same for all concrete classes. The bar diameter was set to 10 mm with a c/c-distance of 100 mm.

Concrete class	f_{ck} [MPa]	f_{ctm} [MPa]	f_{cm} [MPa]	E_{cm} [GPa]	G_f [N/m]
C28/35	28	2.8	36	32	49
C32/40	32	3	40	33.4	52.8
C35/45	35	3.2	43	34	55.5
C45/55	45	3.8	53	36	64.3
C50/60	50	4.1	58	37	68.5

Tab. 9.8: *Material properties used in the FE-analysis.*

The analysis performed shows that a higher Concrete Class results in increased maximum cracks, as can be seen in Fig. 9.21.

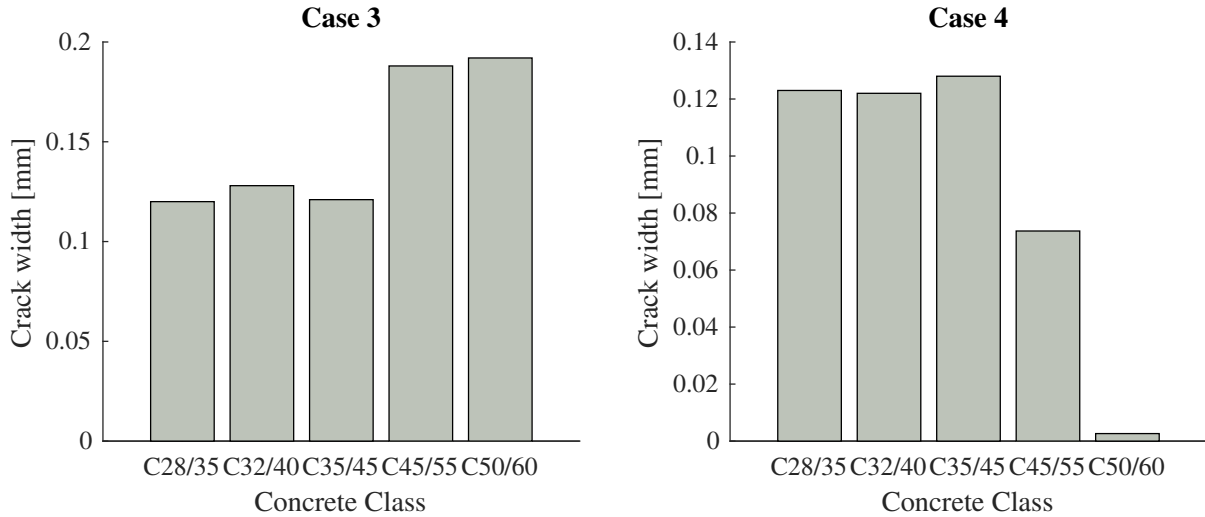


Fig. 9.21: *Maximum crack width after final shrinkage.*

The crack propagation for the two cases are presented in Fig. 9.22 and Fig. 9.23. When studying the propagation for Case 3, a higher concrete quality does not seem to affect at which strain cracks are initiated. The crack pattern in Appendix A.9, reveals that a crack in the area between the slab and the footing which for the case of a high concrete class get very large. The propagation for Case 4, shows a delay of crack initiation as the concrete class increases but an increase of the rate in which the cracks grows.

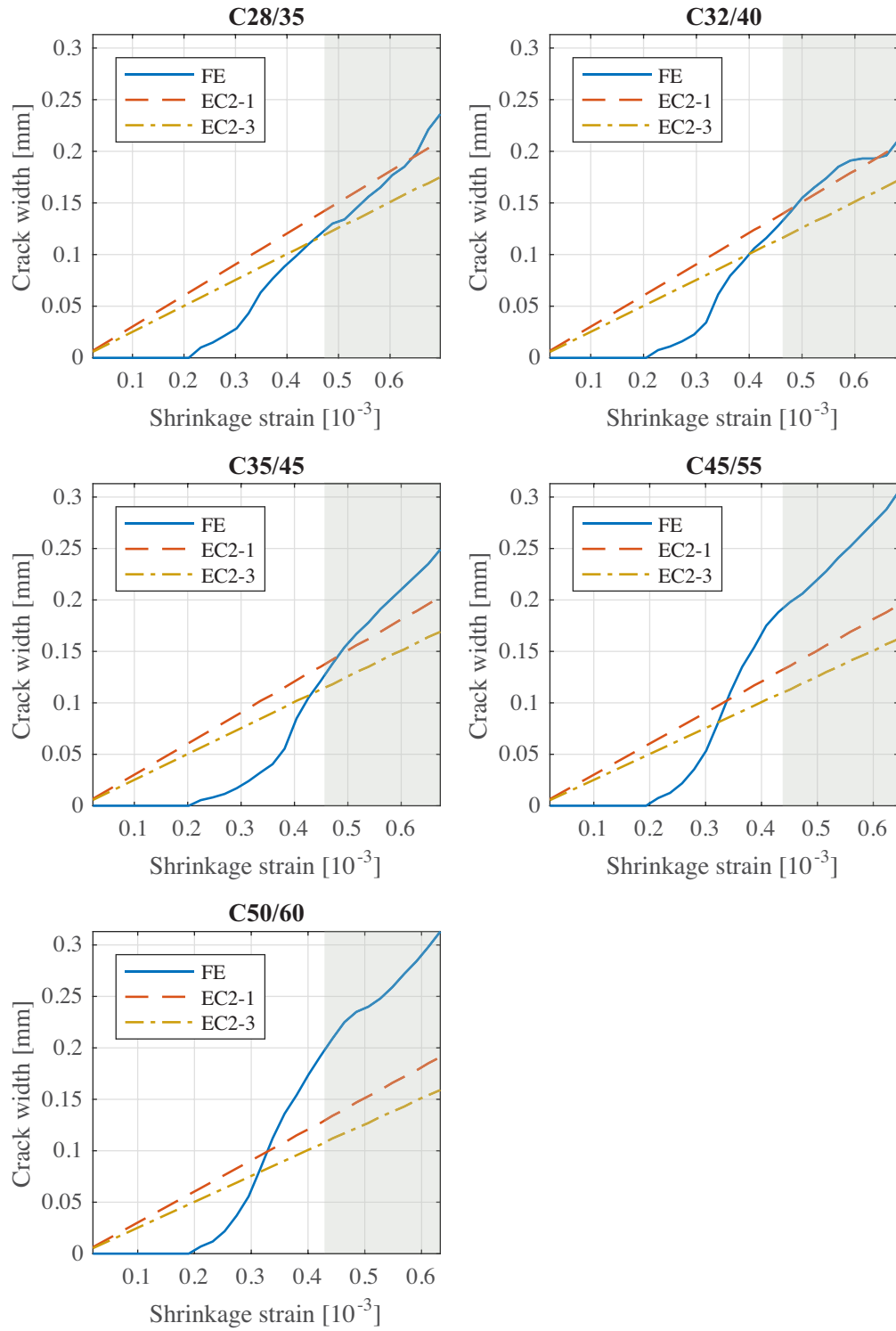


Fig. 9.22: Crack propagation for different concrete qualities, Case 3. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.

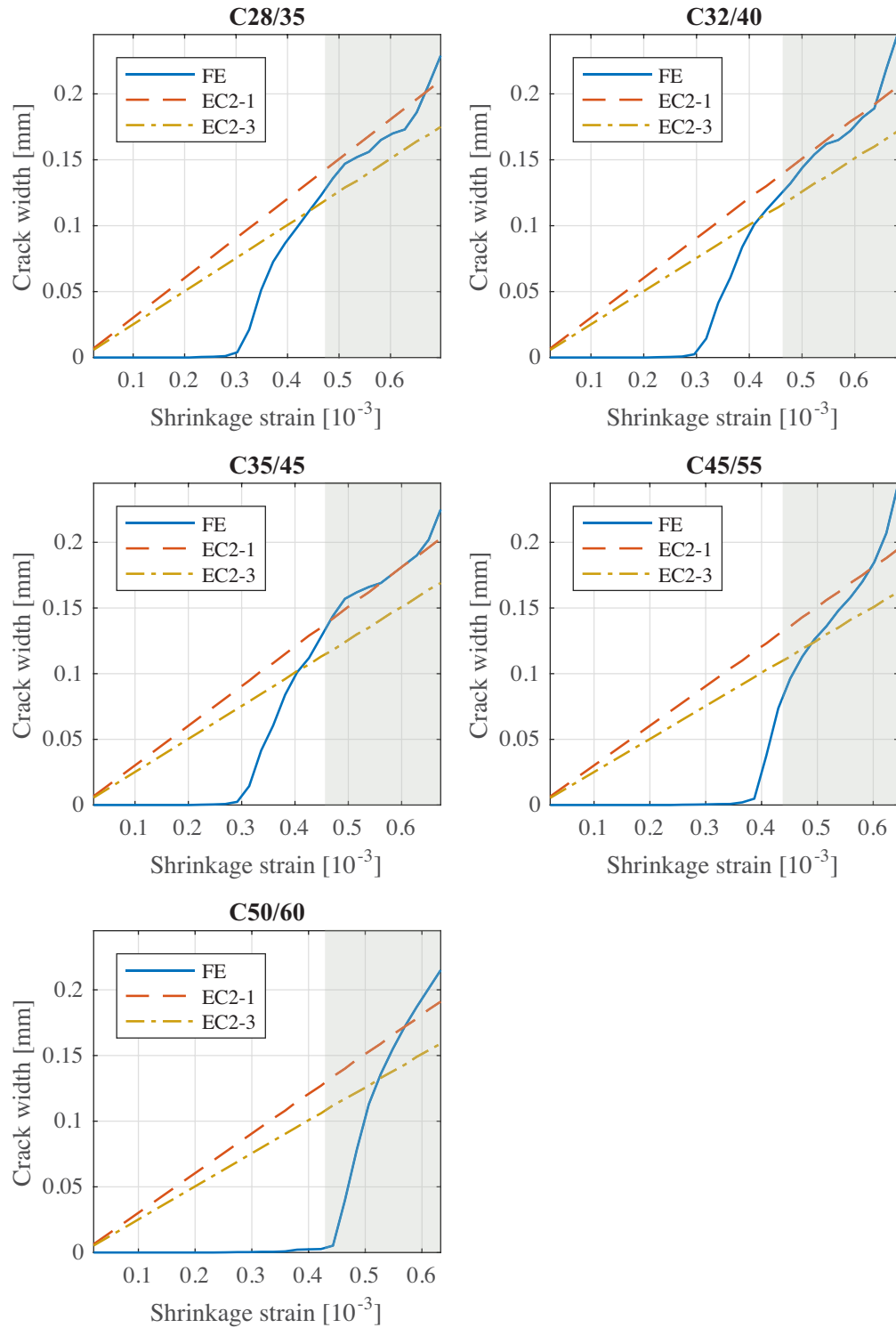


Fig. 9.23: Crack propagation for different concrete qualities, Case 4. Comparison between FE-analysis and Eurocode calculation. Grey area indicates a fictive strain.

Chapter 10

Discussion

One of the main issues, which to a large extent initiated this work, is how to interpret and implement the calculation procedures in EC2 regarding crack width and minimum reinforcement for smaller concrete slabs. After discussions with experienced structural engineers, it was understood that the introduction of EC2 has led to an increased amount of reinforcement for small foundation slabs. The increase of reinforcement does not, however, solely depend on the conventional formulations in standards, but also on the fact that new concrete types, with longer drying time, are used nowadays. When a short production time is required in a project, the designer usually has to assign a higher concrete class to shorten the drying time, which in turn provides more reinforcement. In the light of the aforementioned problem, nonlinear finite element analyses have been carried out on smaller concrete slabs. A parametric study was performed and compared with analytical methods. This chapter discusses the obtained results and observations.

10.1 Material Models and Shrinkage Load

The crack behaviour is directly dependent on the fracture energy. A larger fracture energy gives fewer but wider cracks, and vice versa. The fracture energy depends on the w/c-ratio, the maximum aggregate size and the age of concrete. In the calibration, the fracture energy was determined from both Model Code 2010 and Model Code 1990, and compared to each other. Model Code 1990 showed a slightly improved correlation with the experiment and was thus used in the parametric study. One main advantage with the Model Code 1990 model is that it accounts for the aggregate size. Usually for smaller slabs for residential houses, the aggregate size is comparably small. The aggregate size was assumed to be 8 mm, which gives a relatively low value of the fracture energy.

The linear shrinkage gradient was calculated using a simulated RH in the slab. Since the estimation of the RH in a concrete structure is dependent on the ambient climate, there are some uncertainties about the calculated shrinkage gradient. Parameters such as casting date, drying climate, weather protection, to mention a few, are specific for

each project and should therefore be adjusted for the specific conditions. However, the situation in this thesis is assumed to represent a case where a fast production time is required, which in turn gives a larger shrinkage strain.

10.2 Parametric Study

In the parametric study, the influence of boundary conditions was examined by varying the friction coefficient and the stiffness of the sub-base. The influence of friction on crack formation turned out to be comparatively small. In the case of a stiff sub-base material, $E = 10000$ MPa, it is difficult to draw any conclusions at all. For a low stiffness in the underlying material, $E = 30$ MPa, on the other hand, the crack width decreased as the friction coefficient increased, which was an expected behaviour. Friction is a function of the normal force, and in the analysis the only external force accounted for was the self-weight of the slab. In reality, additional dead loads from the structure and live loads may affect the force of friction acting on the foundation slab. In addition, an upward bending of the edges can be detected, separating the concrete slab from the interface material. This separation is more distinct for a high stiffness. Therefore, it might be imprudent to say to what extent the friction will affect the crack formation.

The results from various sub-base stiffness show that at low stiffness, $E = 5$ MPa, almost no cracks are formed. As the stiffness increases, so does the cracking. Most surprising is the difference between the two analyzed cases, which is almost insignificant. In Case 3, the slab is resisted in the horizontal direction and should theoretically be more restrained than in Case 4. The similarity in crack width for the both cases is presumed to arise due to two things: In case 3, the slab may curl more than in reality, and the resistance in the x-direction from the insulation thus becomes less important. The second part is that the vertical stiffness of the insulation has a major impact on the crack formation. If the slab is more free to move in the vertical direction, the stresses can be redistributed.

The influence of the geometry was examined by varying the slab length and the slab height. From the analysis of different slab lengths it is shown that the crack widths becomes wider as the length increases. The same pattern is observed for various slab heights, as the height increases so does the crack width. An exception is observed for the case of a slab height of 80 mm, where large cracks can be found at the footing, see Fig. 10.1. The crack in the footing propagates from the 90° corner, where the footing and the slab are connected. In the sharp corner, the calculated stresses are unproportionately large due to a singularity in the model. In reality, however, the stresses will probably be within reasonable levels and cracks will not grow as large in this area. The curling of the slab is believed to be the underlying phenomenon that allows the crack to grow. A more slender slab is curling more due to a reduced section modulus, which makes the cracks in the connection larger. However, it is debatable if the footing is allowed to curl this much in reality. The footing is often stiffer due to additional reinforcement, and the loads acting on the footing will counteract the curling as well. This indicates that the crack in the footing might be overestimated in

the numerical analysis.

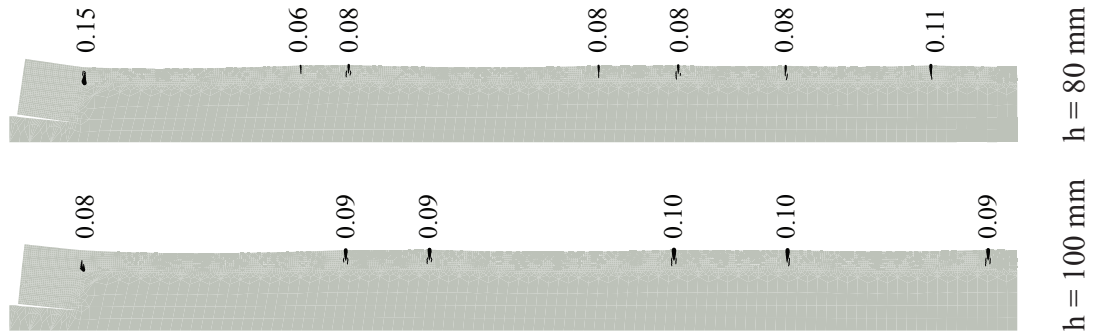


Fig. 10.1: Crack width for an 80 mm high slab and a 100 mm high slab.

Reinforcement has a major impact on the crack behaviour. In the parametric study, the influence of bar diameter and reinforcement ratio were examined. Studying Fig. 9.14, the crack width becomes wider for larger bar diameters, even though the amount of reinforcement remains constant. This verifies the initial thought that a larger reinforcement bar is restraining the structure more, consequently inducing additional forces to the structure, resulting in wider cracks. When the bar diameter is increased from 6 mm to 12 mm, the crack width becomes about 15 % wider. Note that the amount of reinforcement remains constant in the comparison. A smaller bar diameter, densely arranged, may therefore be advantageous. As suspected, the crack width decrease with an increase of reinforcement amount. The amount of reinforcement was varied between 0.2 % to 1 %, which led to a decrease in crack width with about 50 %.

In Figs. 9.22 and 9.23, the crack propagation for different concrete classes are compared. By studying the results, a few observations can be made. As the concrete class is increased, the tensile capacity of the material is heightened accordingly, delaying the initiation of cracks to a greater strain. For the lower concrete classes, the crack grows gradually as the applied strain is increased. However, for higher concrete classes, the rate of crack growth increases i.e. the cracks gets wider for a certain strain increment. The reason for this behaviour may be explained by the difference in fracture energy, where a higher concrete classes is assigned a higher value of fracture energy.

A comparison between different shrinkage profiles is presented in section 9.1.8, where a uniform, a linear gradient and a nonlinear strain variation are evaluated. In calculations according to EC2 a constant strain profile is assumed over the entire cross section. However, this does not correspond with the real conditions. In BBK04 a suggestion can be found on how to account for a linear strain profile in a slab where the maximum shrinkage strain can be estimated as $1.25 \cdot \varepsilon_{cs}$, and the corresponding minimum shrinkage as $0.75 \cdot \varepsilon_{cs}$. Results from the analysis show a difference in crack propagation depending on which type of strain profile that is applied. In the case of nonuniform strain profile, the slab seem to crack at smaller strains than for the uniform strain profiles. The behavior can be explained by the fact that a nonuniform strain will induce bending stresses in the slab, making the slab curl upwards. The weight of the the structure is, however, counteracting the curling movement which induces additional stresses in the top of the structure. Depending on the final shrinkage strain,

the impact of different strain profiles will vary.

10.3 Comparison between Analytical and Numerical Results

When calculating the crack width according to EC2-1, the crack width will be dependent on the crack spacing and the strain difference between the reinforcement steel and the concrete. In situations where shrinkage is the governing load, inserting the shrinkage strain directly into Eq. 7.19 implies that the structure is seen as fully restrained [13]. This means that all load is to be carried by the reinforcement in a cracked section, which may lead to excessive reinforcement in order to control the crack width. By integrating EC-3 into EC-1, the effects of restraints can be taken into account when estimating the crack width.

EC2 assume all cracks to be independent of each other. When a crack occurs, no strain relief is assumed beyond $s_{r,max}$, which could lead to a change in crack width. This assumption may be acceptable for a completely restrained structure, but in practice only partially restrained structures exist. The existing cracks, as well as the subsequent cracking is influenced by the strain relief. This can explain the behavior observed in the analysis for different lengths, where the crack width decreases for a greater strain. Magnification of the observed behaviour is shown in Fig. 10.2.

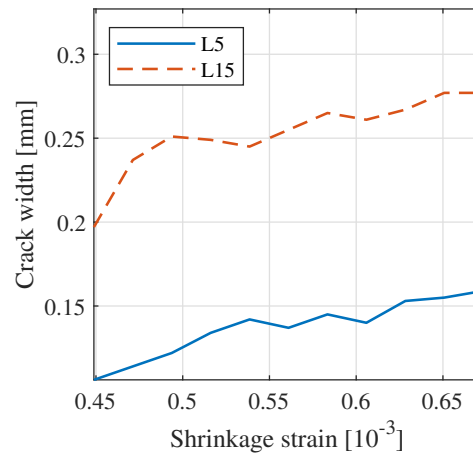


Fig. 10.2: *Magnification of crack behaviour observed in the analysis for different lengths*

When comparing the numerical results of how the bar diameter and the reinforcement ratio affects the crack width with EC2 a few remarks can be made. As shown in Fig. 10.3 the estimated crack width increases much faster for the analytical methods according to EC2 than for the FE results, when the reinforcement ratio decreases. For smaller ratios, there is a risk of overestimating the crack width when using EC2, leading to a conservative design. In addition, the influence of bar diameter on the final crack width is much higher using EC2 then the FE results shows, see Fig. 10.3.

However, for a smaller bar diameter, 6 mm, EC2 crack width estimation corresponds well with the FE-results.

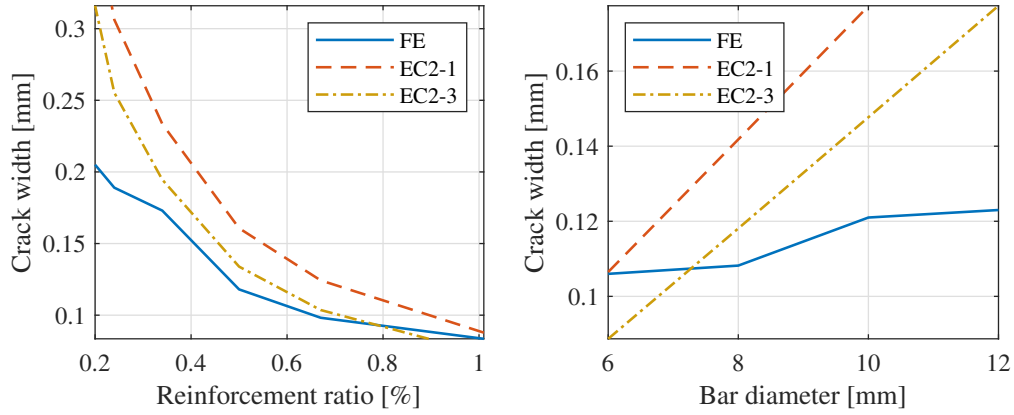


Fig. 10.3: *The relation between reinforcement ratio and crack width as well as bar diameter and crack width, respectively*

Studying Fig. 9.17, the minimum amount of reinforcement according to EC2 is not fulfilled for four of the cases. However, the numerical analysis does not show a change in crack behaviour for the "under-reinforced" specimens. The minimum amount of reinforcement is calculated on the basis that all load is carried by the reinforcement in a cracked section. In reality, some of the load is transferred and carried by the restraining elements, leading to a reduction of steel stress. Therefore, a refined approach of calculating the minimum amount of reinforcement is suggested by [3], taking the degree of restraint into account. The proposed method is for an edge restrained element as follows

$$A_{s,min} = (1 - R_{Edge})k_c k A_{ct} f_{ct,eff} / f_{yk}$$

in which the notations are described in Tab. 7.17.

When varying the length of the foundation slab, the numerical results show a dependence between slab length and crack width. The crack width gets larger as the slab length increases. However, in EC2 no dependence between slab length and crack width is presented. For smaller slab lengths, 5 m - 7.5 m, the EC2 estimation of the crack widths are larger than the FE-results, but for larger slabs, 12.5 m - 15 m, the results indicates that EC2 may underestimate the actual crack width.

10.4 Creep

Creep can be considered by a reduction of Young's modulus. The result from the method by Engström showed that a higher creep coefficient requires a higher strain to initiate a new crack. Consequently, for a higher creep coefficient fewer but wider cracks will form. Creep was not considered in the numerical analysis; however, in the

case of imposed deformations, the stresses in the slab will decrease with time due to creep. That is, when creep is considered for restraint structures, one may not only account for a reduction in the modulus of elasticity, but also for the loss of stresses.

10.5 Economic and Environmental Benefits

It was stated in the introduction of this chapter that one way for a designer to meet drying out requirements, is to increase the concrete class. With a higher concrete strength the w/c-ratio decreases, and so does the drying time, which in turn provides more reinforcement to meet the crack width requirements. From an economic point of view, an increase of reinforcement for a slab foundation is not necessarily a major financial loss if one only considers the material costs. However, with heavier reinforcement meshes, a two-man job may become a task requiring lifting machines. Usually for residential houses, the financial capacity may be small and additional costs can be devastating. This type of problem was a recurring theme during discussions with construction engineers.

The same type of problem can be applied to the use of green concrete. As green concrete usually has higher w/c-ratio, the drying time increases, and to meet the drying out requirements, a higher concrete class is used. It follows that not only the reinforcement amount increases, but so does the amount of cement. The environmental benefit of using green concrete may therefore be significantly less or none at all.

Chapter 11

Final Remarks

In this master thesis, a study on cracking in slab foundations was performed. Nonlinear finite element analyzes were made in Atena 2D with the constitutive model SBETA. Different cases representing both theoretical models and real situations were modelled, and the crack widths were studied for gradient shrinkage. The shrinkage gradient load was calculated using PPB to account for modern concrete types drying properties. A theoretical basis was given regarding cracking mechanisms in a concrete structure as well as for the structural behaviour for concrete as a material. This chapter summarizes the key findings and the main points from the study.

11.1 Conclusions

From the study on crack formation for slab foundations, the following conclusions can be drawn:

- The stiffness of the sub-base has a major impact on the crack formation. For a low stiffness, the cracks are fewer and more narrow, and vice versa. This parameter can however be hard to account for in analytical methods.
- The diameter of the reinforcement bar has an effect on the crack width. This is likely due to the internal restraint. The bond between the concrete and the reinforcement depends on the contact area. With a constant reinforcement ratio, the total contact area will increase when decreasing the bar diameter. Reinforcement bars with a small diameter, densely arranged, might therefore be advantageous for reducing the final crack width.
- The influence of bar diameter on the crack width is higher for calculations according to EC2 compared to the results from the FE-analyses. In the FE-analyses a bond-slip model according to [7] was adopted, but several other bond-slip models can be used.
- By integrating EN 1992-3 into EN 1992-1-1, restraints may be taken into account when calculating crack width according to Eurocode.

- Minimum reinforcement according to standards may be too conservative for restrained structures, especially for smaller slabs. One may take into account the load-bearing capacity of the restraining element to get a better estimation of the minimum amount of reinforcement.
- For slabs in which the reinforcement amount did not achieve Eurocodes recommendation on minimum reinforcement, the finite element analysis showed similar crack behaviour as for the slabs with sufficient reinforcement. Therefore, for lower reinforcement ratios, EC2 may overestimate the crack widths.
- The method by Engström showed great correlation to experimental tie-rod tests carried out at UNSW [63]. However, in the study of the slab foundation, the method by Engström did not correlate well with the numerical results, nor with the EC2, and may be an insufficient tool for crack width calculations for smaller slab foundations.
- The crack formation varies for different types of strain profiles. If a shrinkage gradient is considered, cracks will mainly form at the top of the slab.

11.2 Further Research

Since the stiffness of the sub-base material has a major effect on the crack formation, it is of interest to determine the module of elasticity for different insulation materials. Data from manufacturers are in some cases not presented, and the data available have a wide scatter. Another interesting property of the insulation material is the friction coefficient between concrete and insulation. Even though the FE-analysis shows that the friction does not have a huge impact on the final maximum crack width, there is reason to better determine this parameter.

The degree of restraint depends on the structural geometry of the slab such as length, slab height, stiffness ratio between different materials and the reinforcement amount. To determine and investigate how different design choices affect the degree of restraint is therefore of interest.

The crack width was evaluated with a finite element model in this thesis. Furthermore, the finite element model showed good agreement to experimental tie-rod tests. However, the results in the study on slab foundations have not been verified against real values. A field study on crack widths of smaller slabs would therefore be interesting.

The shrinkage was calculated as the final shrinkage, even though it develops over time. Other time-dependent material properties are creep and tensile strength. However, time dependence was not taken into account as it would require much more complicated models. A study that accounts for the time-dependency would therefore be interesting and could make the calculations more precise.

In the numerical analyses, only single, centrally placed reinforcement was considered. An investigation of how placement of the reinforcement affects the crack width is of interest. Since more cracks developed at the top of the slab, a higher location of the

reinforcement bar might be advantageous. Additional reinforcement, locally at the footing, might reduce the width of the cracks developing in this area. By eliminating these cracks, a thinner slab design might be possible.

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Appendices

Appendix A

FEM Analysis

A parametric study of the FE-model is here presented.

A.1 Boundary Conditions

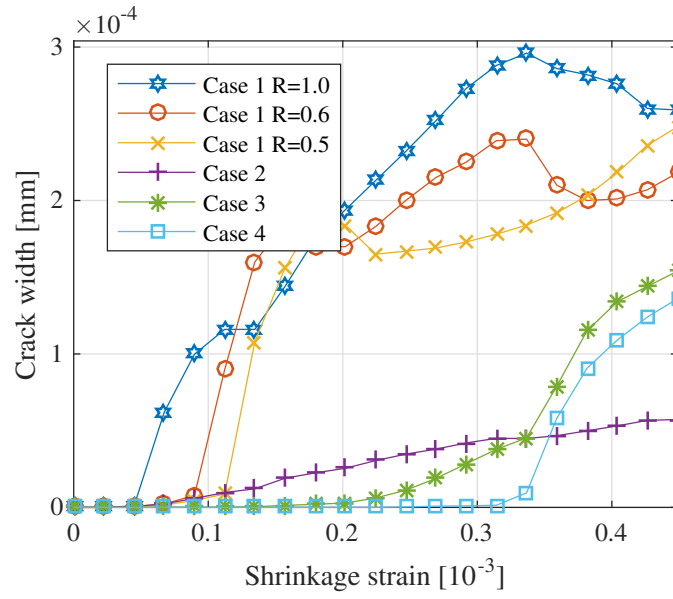


Fig. A.1: *Maximum crack width versus shrinkage strain for different boundary conditions.*

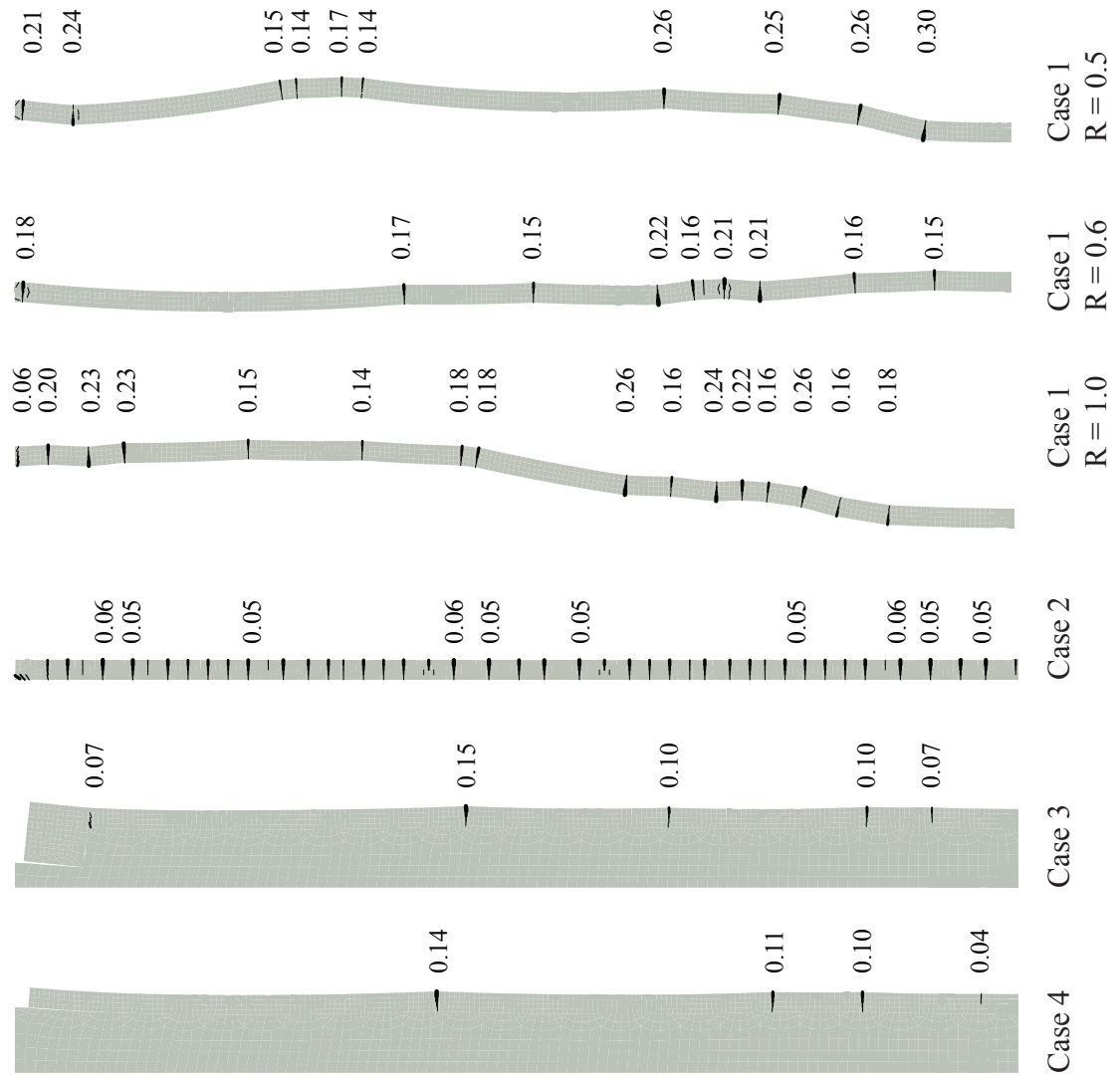


Fig. A.2: Crack pattern and crack widths [mm] at final shrinkage for different boundary conditions.

A.2 Stiffness of the Sub-base

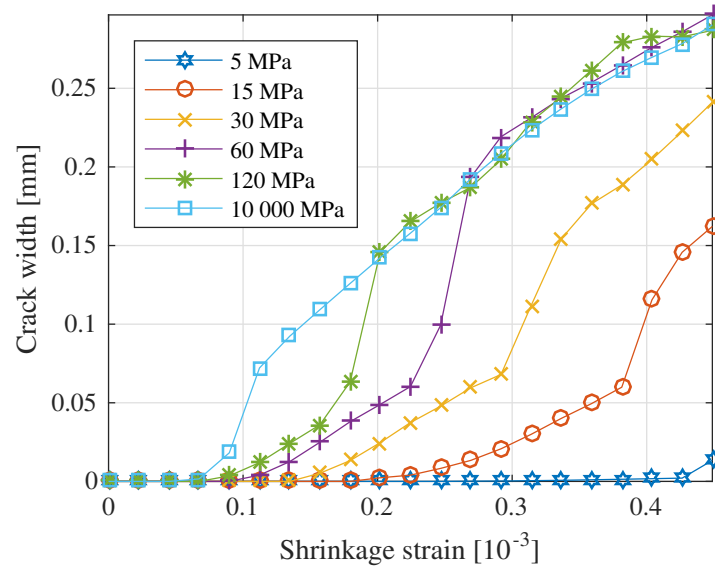


Fig. A.3: Growth of maximum crack width for various stiffness for Case 3.

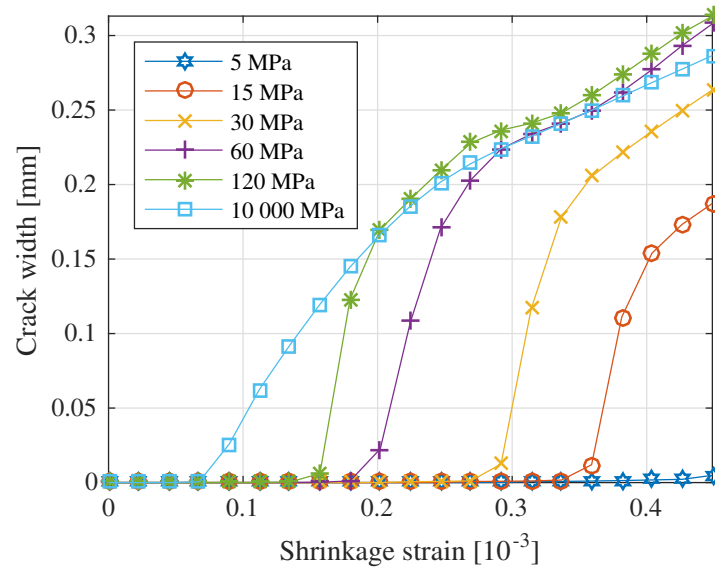


Fig. A.4: Growth of maximum crack width for various stiffness for Case 4.

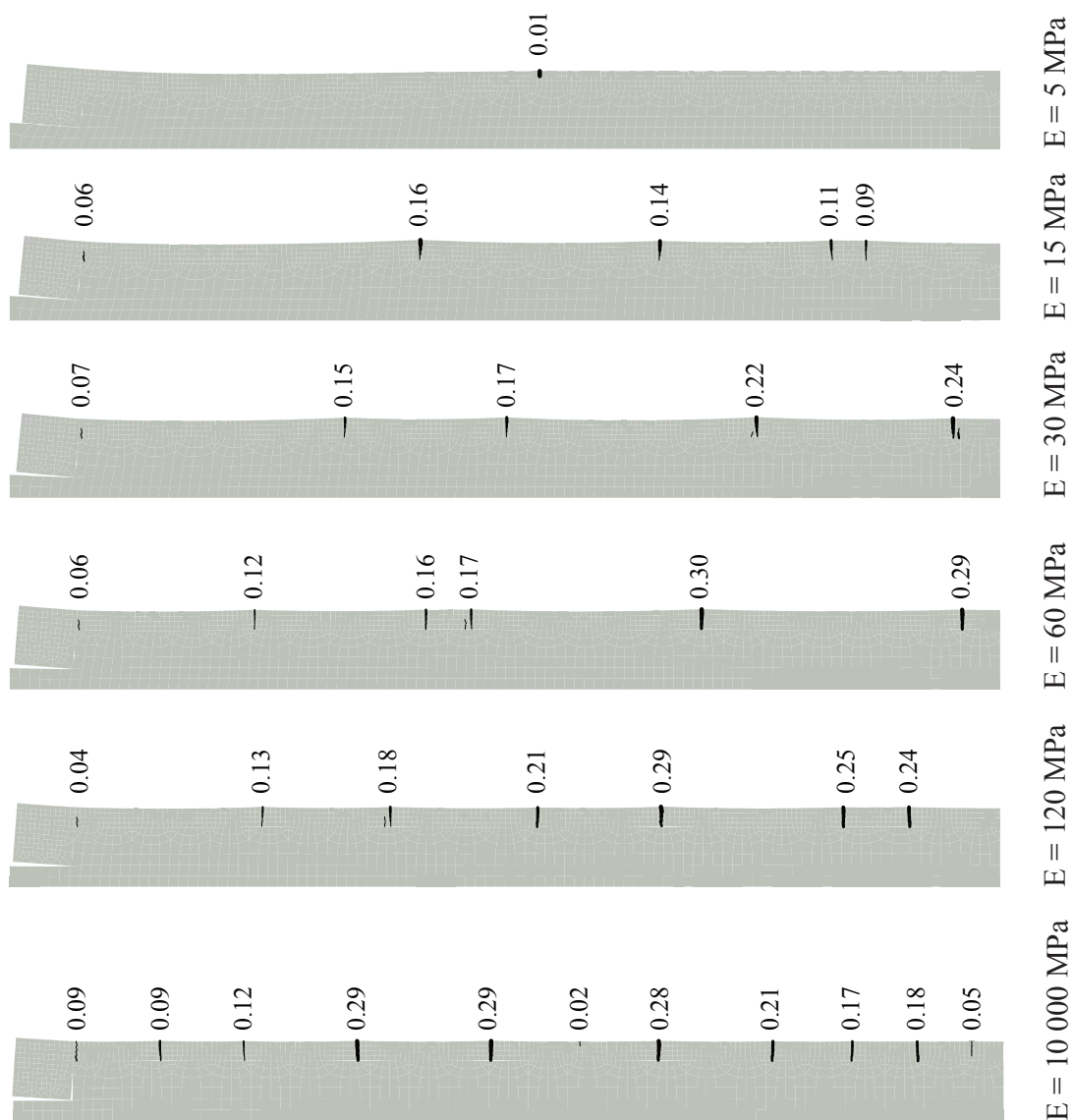


Fig. A.5: Crack pattern and crack widths [mm] at final shrinkage for stiffness for the sub-base.

A.3 Friction Coefficient

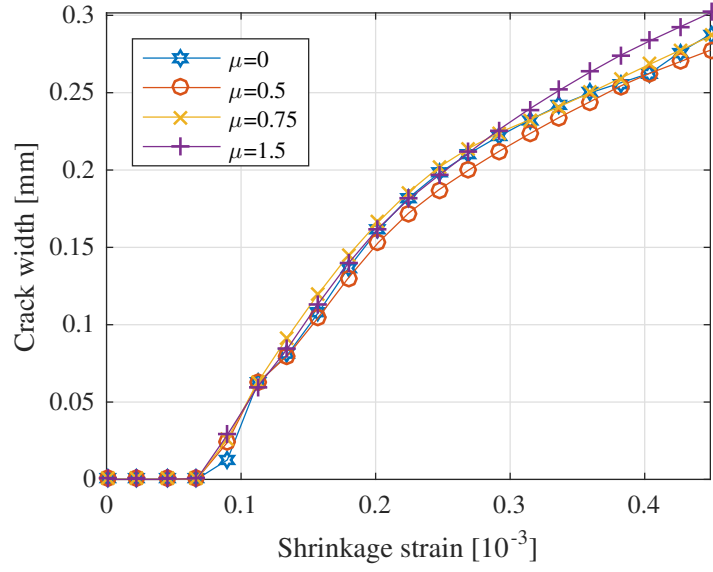


Fig. A.6: *Maximum crack width versus shrinkage strain for different frictions coefficients, with sub-grade $E = 10000$ MPa.*

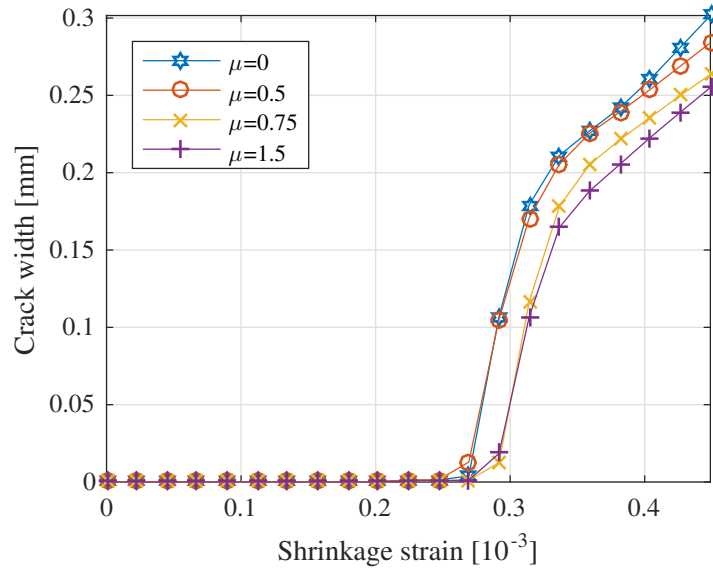


Fig. A.7: *Maximum crack width versus shrinkage strain for different frictions coefficients, with sub-grade $E = 30$ MPa.*

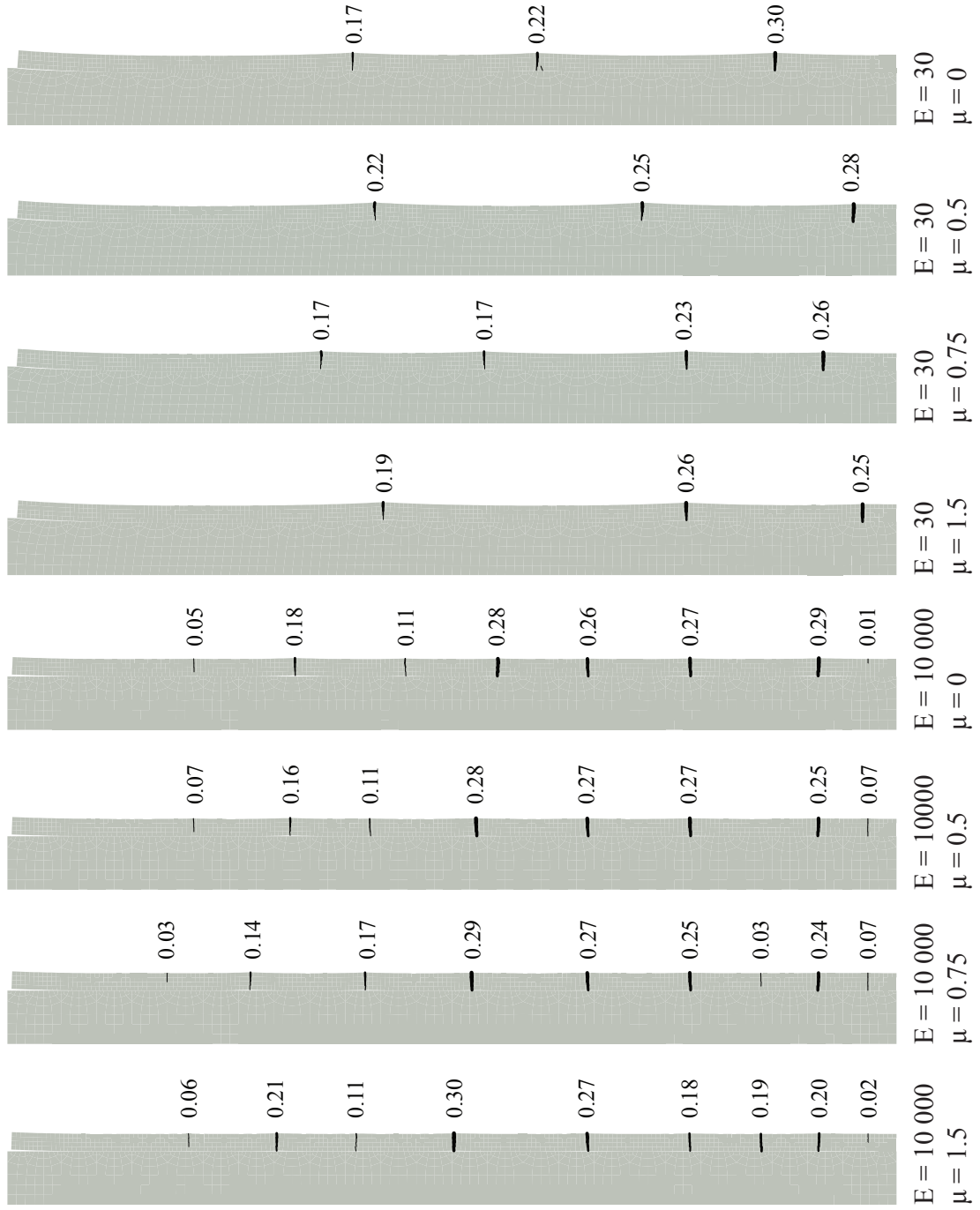


Fig. A.8: Crack pattern and crack widths [mm] at final shrinkage for different friction coefficients, μ .

A.4 Slab Length

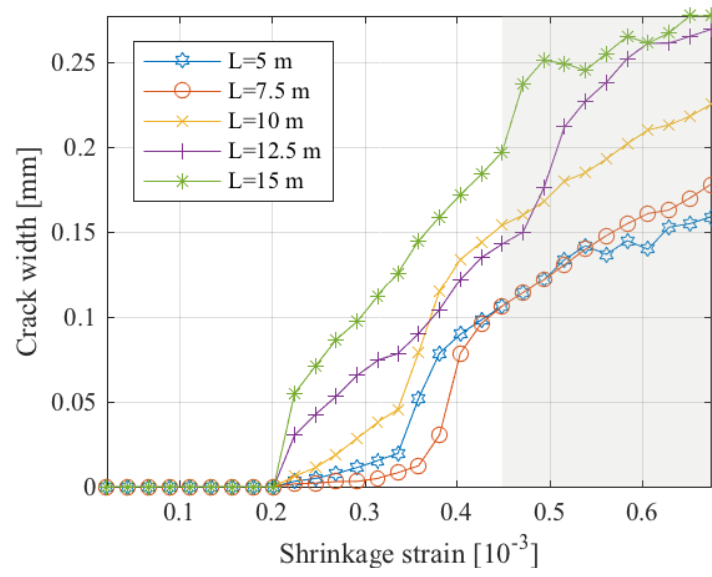


Fig. A.9: Growth of maximum crack width for various slab length for case 3.

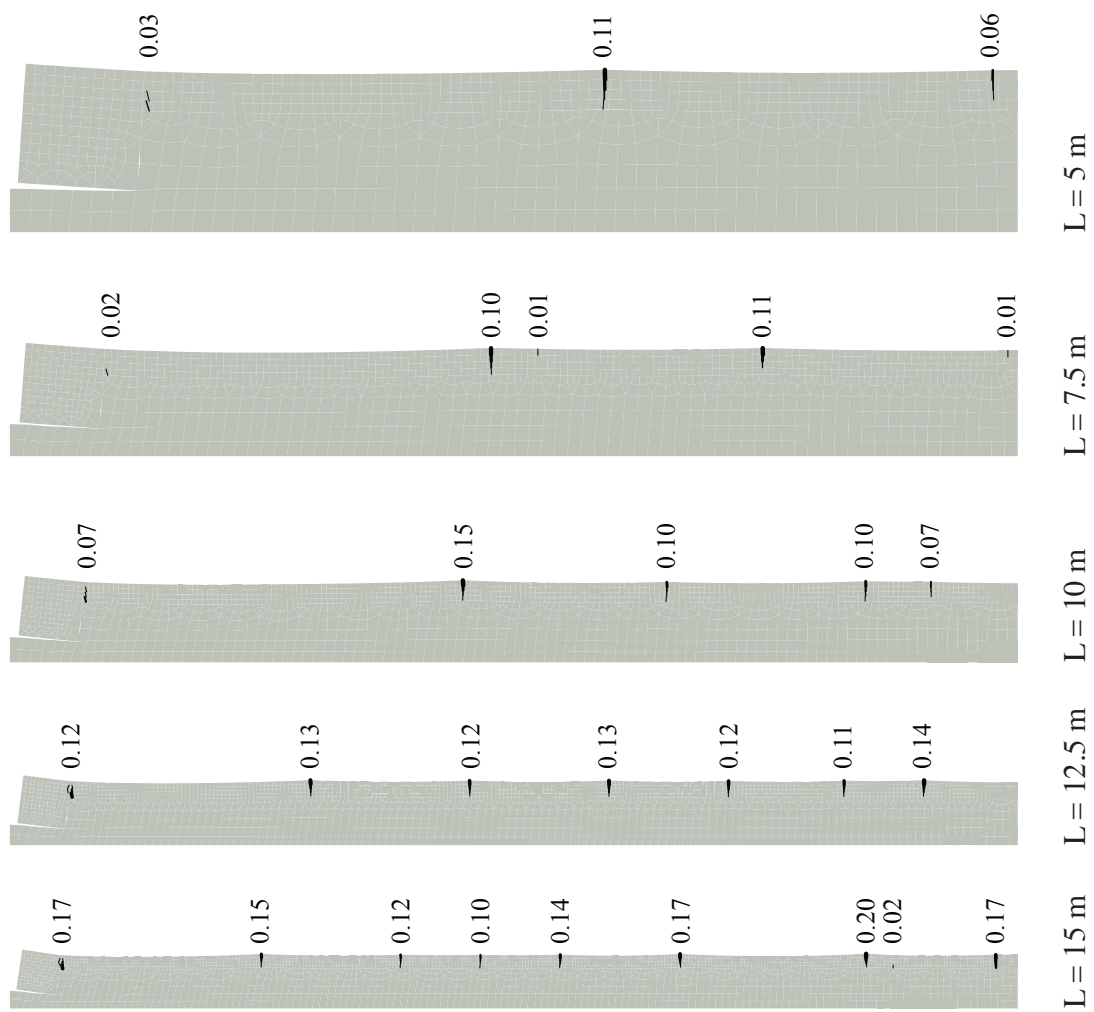


Fig. A.10: Crack pattern and crack widths [mm] at final shrinkage for different slab lengths, L .

A.5 Bar Diameter

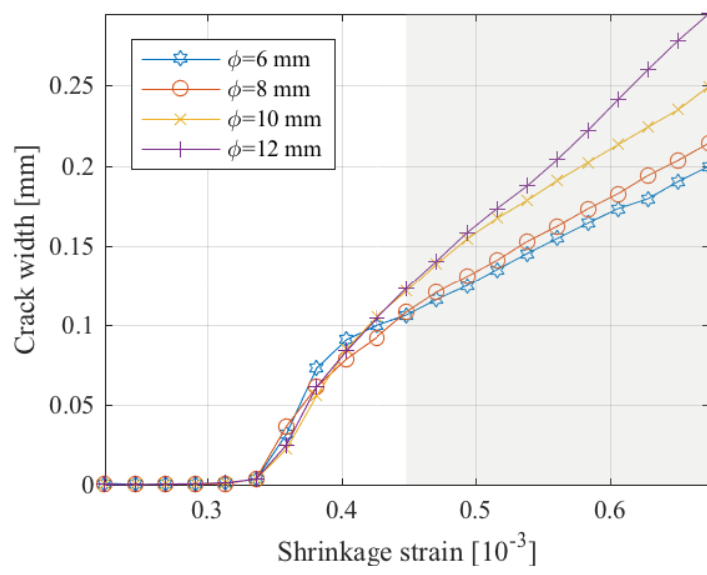


Fig. A.11: Growth of maximum crack width for various bar diameters for case 3.

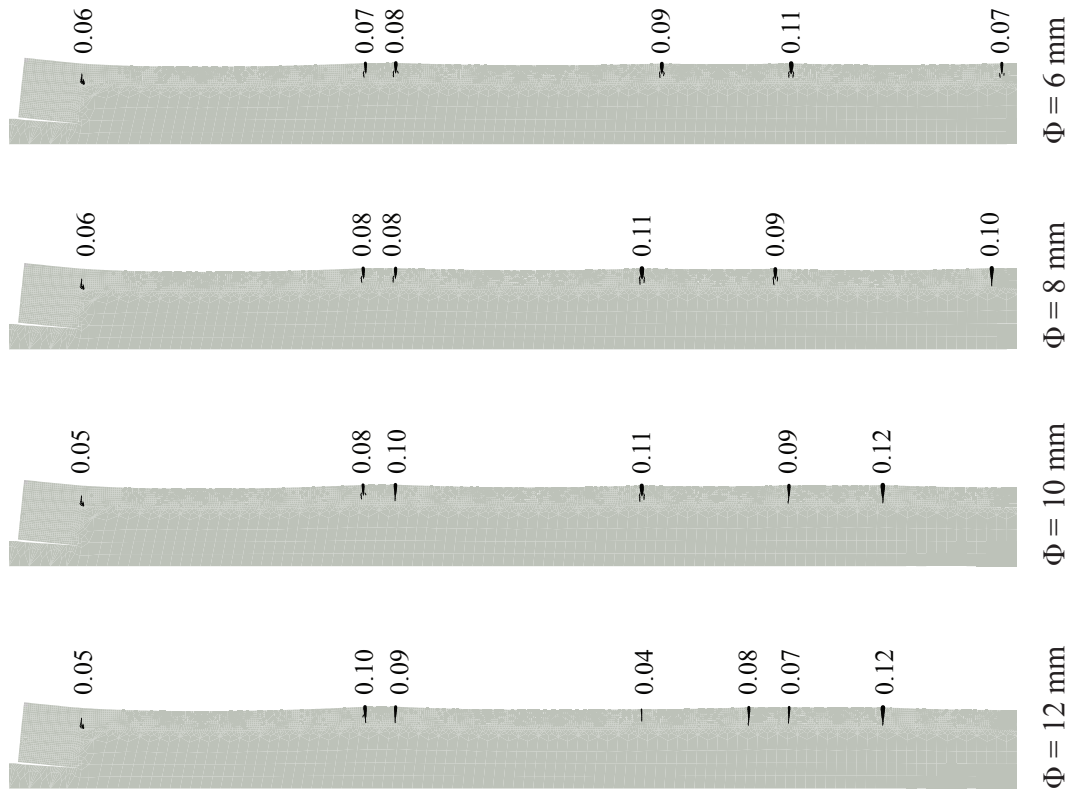


Fig. A.12: Crack pattern and crack width [mm] for different bar diameter, ϕ .

A.6 Reinforcement Ratio

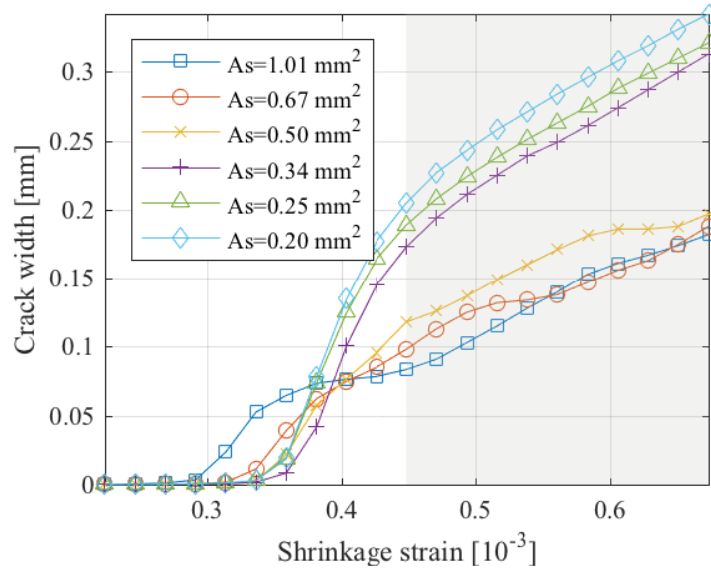


Fig. A.13: Growth of maximum crack width for various reinforcement ratios for case 3.

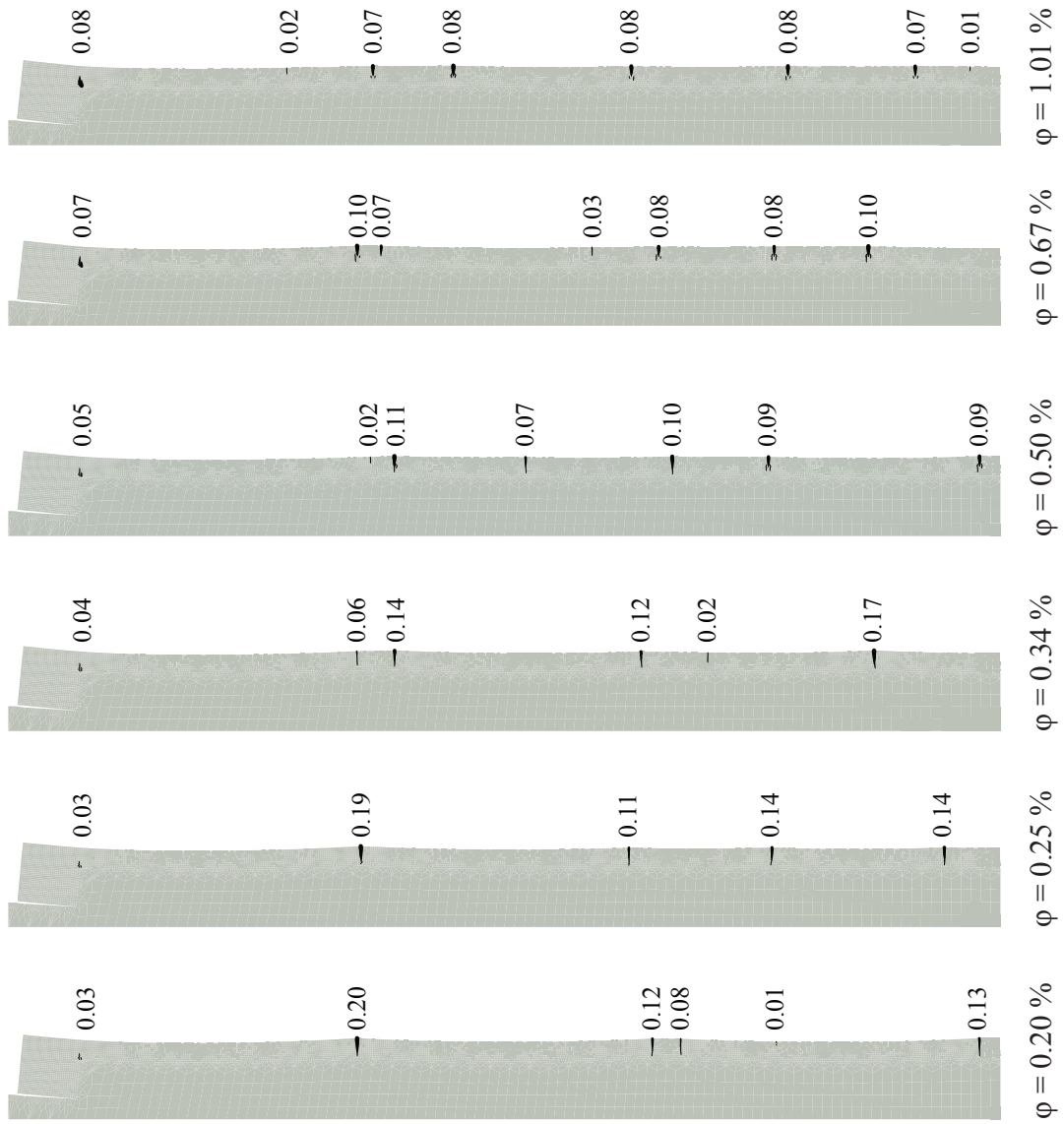


Fig. A.14: Crack pattern and crack widths [mm] at final shrinkage for different reinforcement ratio, ϕ .

A.7 Slab Height

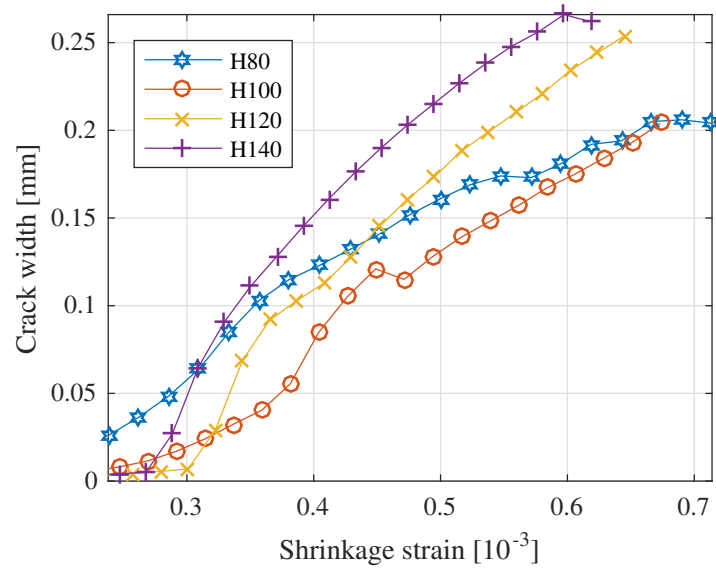


Fig. A.15: Growth of maximum crack width for various slab height for case 3.

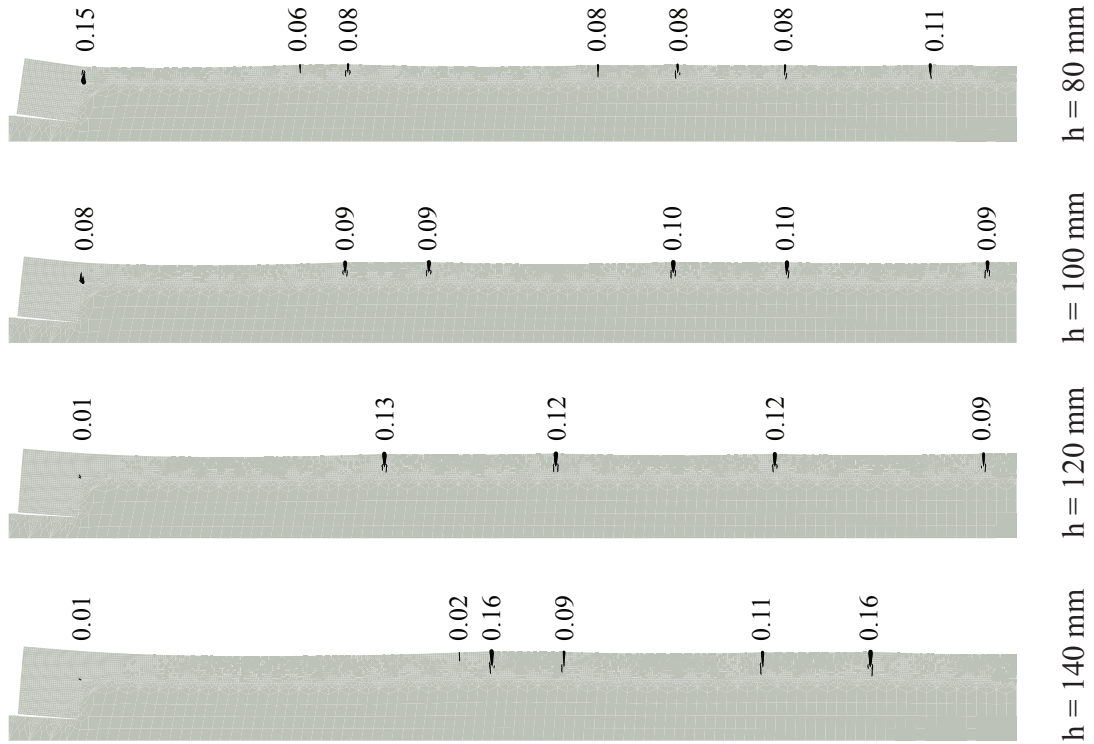


Fig. A.16: Crack pattern and crack width [mm] for different slab heights, h .

A.8 Concrete Class, Case 3

Concrete class	f_{ck} [MPa]	f_{ctm} [MPa]	f_{cm} [MPa]	E_{cm} [GPa]	G_f [N/m]
C28/35	28	2.8	36	32	49
C32/40	32	3	40	33.4	52.8
C35/45	35	3.2	43	34	55.5
C45/55	45	3.8	53	36	64.3
C50/60	50	4.1	58	37	68.5

Tab. A.1: *Material properties*

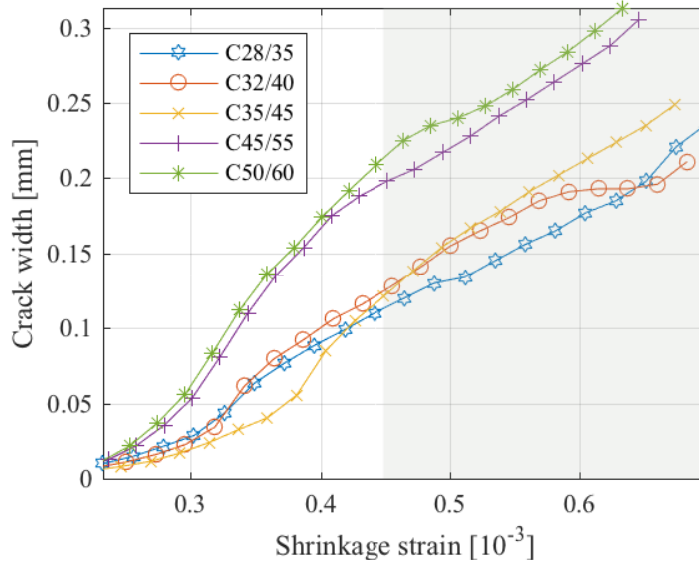


Fig. A.17: *Crack width with different concrete classes for Case 3.*

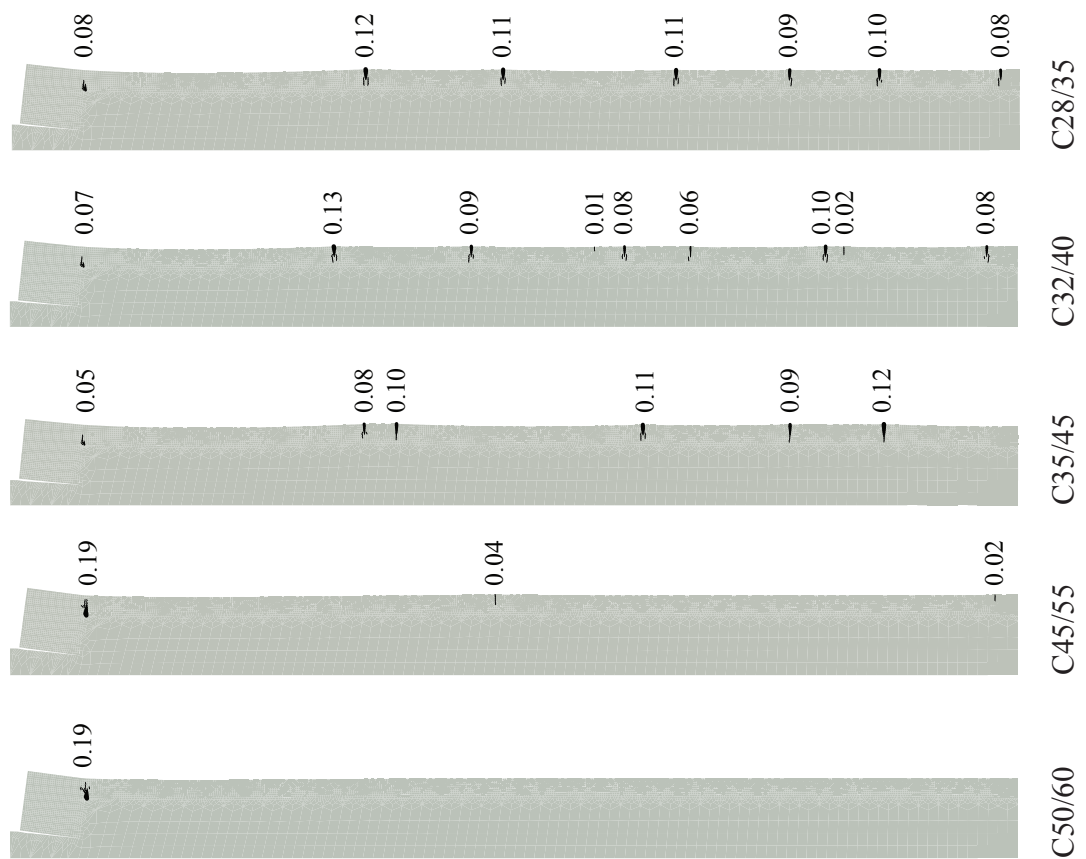


Fig. A.18: Crack pattern and crack widths [mm] at final shrinkage for concrete classes, Case 3.

A.9 Concrete Class, Case 4

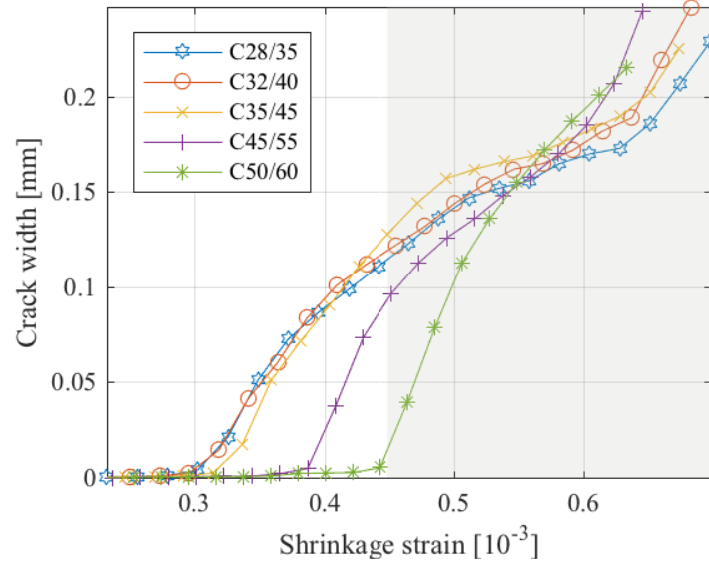


Fig. A.19: Crack width with different concrete classes for Case 4.

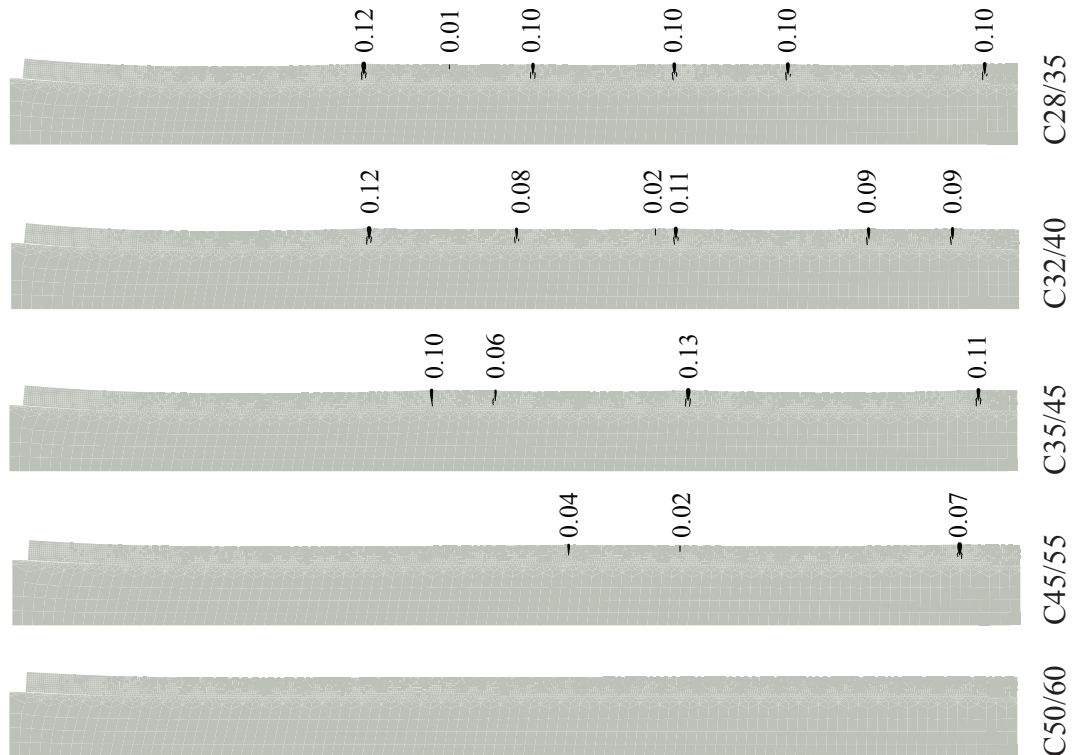


Fig. A.20: Crack pattern and crack widths [mm] at final shrinkage for concrete classes, Case 4.

A.10 Strain Distribution

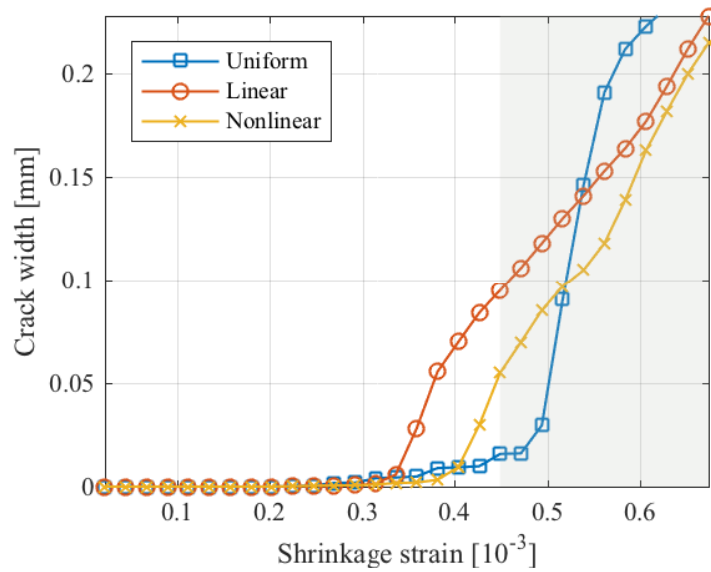


Fig. A.21: Crack width with different strain distribution for Case 3.

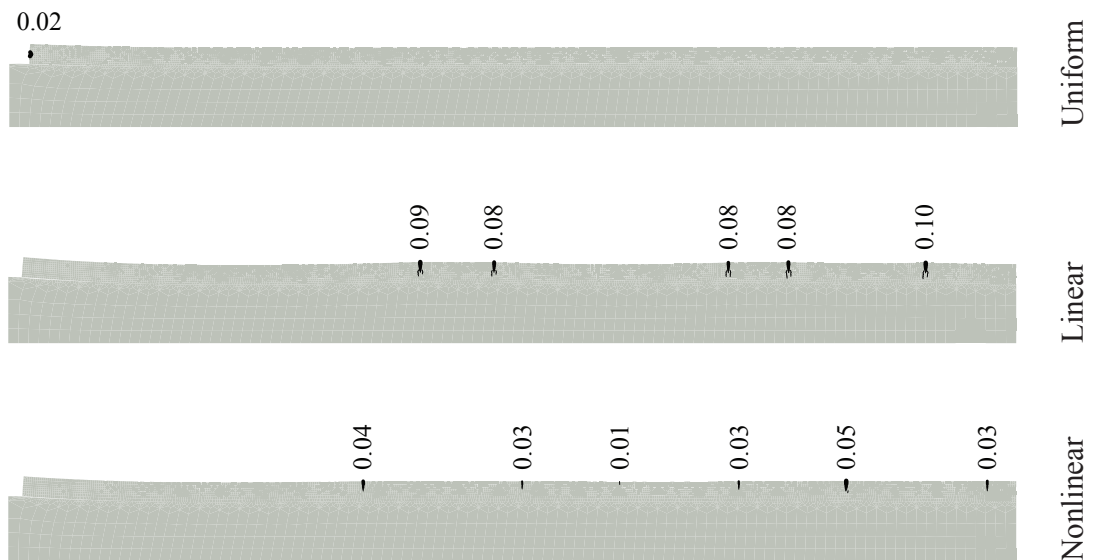


Fig. A.22: Crack pattern and crack widths [mm] at final shrinkage for different shrinkage gradients.

Appendix B

Method by Engström

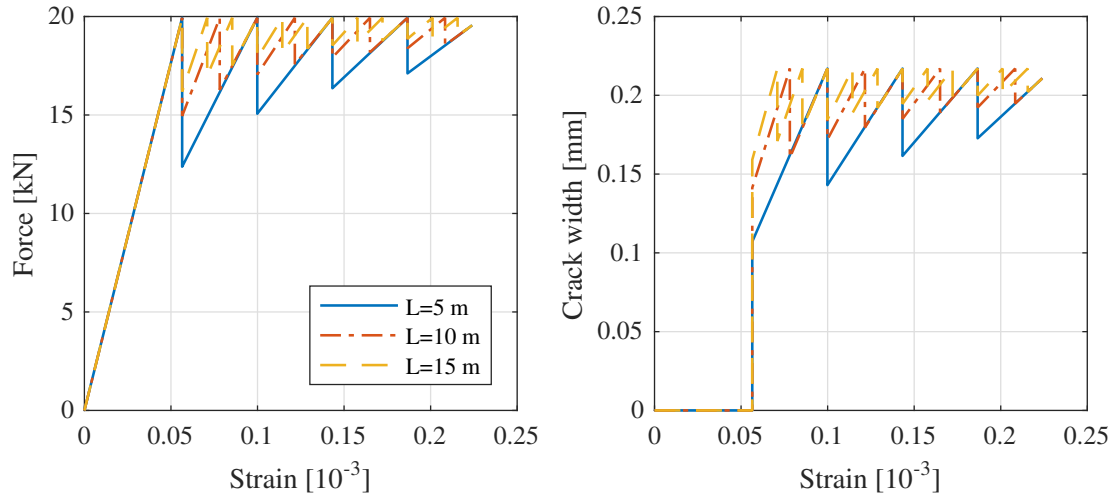
Calculations were performed in Matlab and the function-file can be found in Appendix D. Parameters used in the analysis are presented in Tab. B.1.

CC	length [m]	height [mm]	ϕ [mm]	width [mm]	creep coeff.
C35/45	5/10/15	80/100/120/140	6/8/10/12	50/100/150/250	0/1/2/3

Tab. B.1: *Parameters in the method by Engström analysis.*

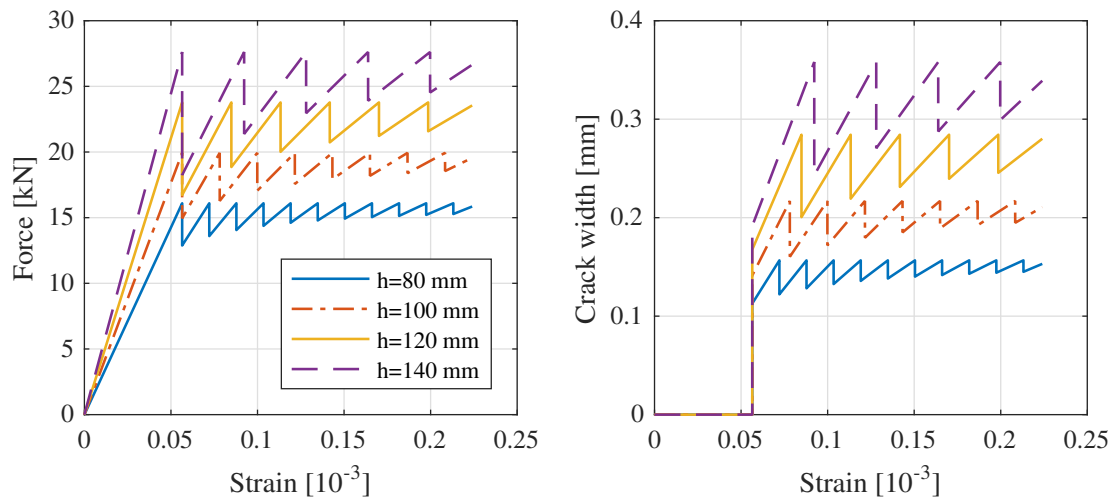
B.1 Influence of Length

CC	length [m]	height [mm]	ϕ [mm]	width [mm]	creep coeff.
C35/45	5/10/15	80/100/120/140	6/8/10/12	50/100/150/250	0/1/2/3



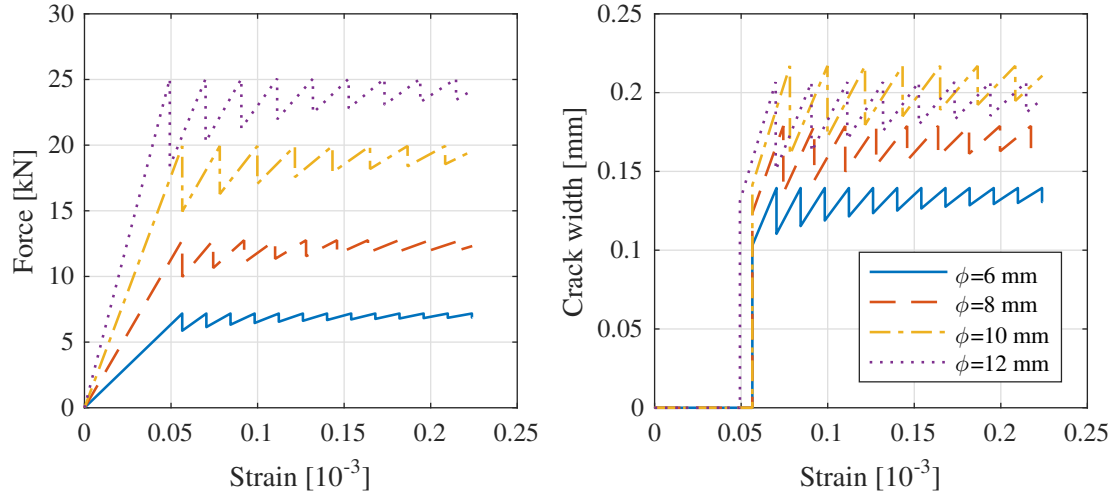
B.2 Influence of Height

CC	length [m]	height [mm]	ϕ [mm]	width [mm]	creep coeff.
C35/45	5/10/15	80/100/120/140	6/8/10/12	50/100/150/250	0/1/2/3



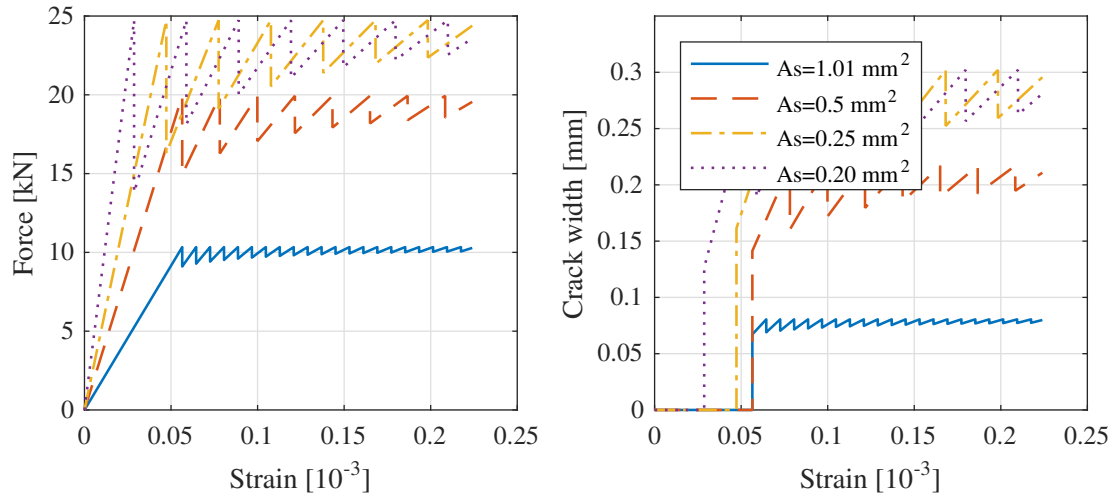
B.3 Influence of Bar Diameter

CC	length [m]	height [mm]	ϕ [mm]	width [mm]	creep coeff.
C35/45	5/10/15	80/100/120/140	6/8/10/12	50/100/150/250	0/1/2/3



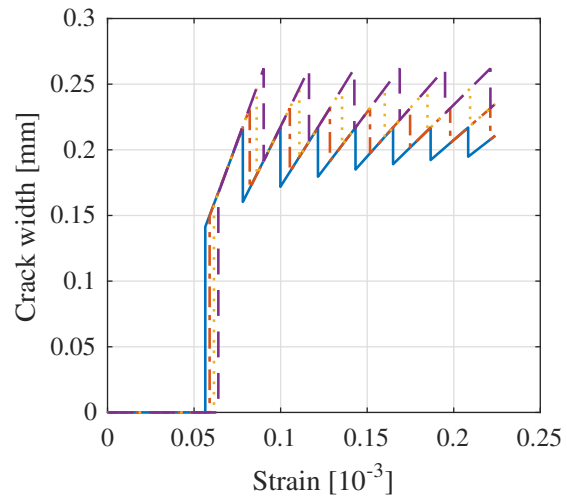
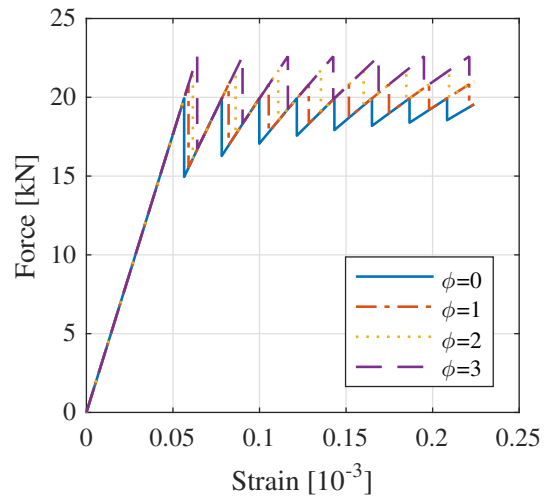
B.4 Influence of Reinforcement Amount

CC	length [m]	height [mm]	ϕ [mm]	width [mm]	creep coeff.
C35/45	5/10/15	80/100/120/140	6/8/10/12	50/100/150/250	0/1/2/3



B.5 Influence of Creep

CC	length [m]	height [mm]	ϕ [mm]	width [mm]	creep coeff.
C35/45	5/10/15	80/100/120/140	6/8/10/12	50/100/150/250	0/1/2/3



Appendix C

Shrinkage Calculation

Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

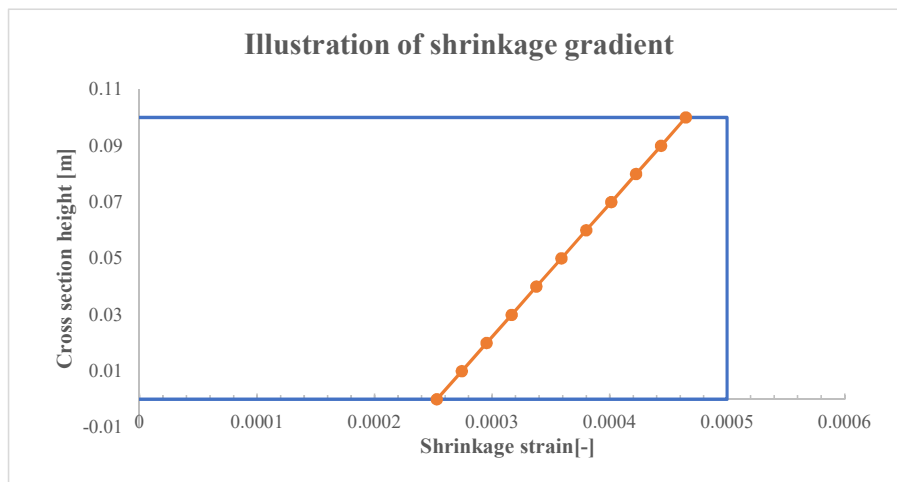
Concrete class	C28/35	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 100
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	$A_c [\text{mm}^2] = 100000$	Compressive strength	$f_{ck} [\text{MPa}] = 28$
Notional size of cross-section	$h_0 [\text{mm}] = 200$	Mean compressive strength	$f_{cm} [\text{MPa}] = 36$
Coeff. - relative humidity	$\beta_{RH} = 1.36$	α -coeff.	$\alpha_{ds1} = 4$
Basic drying shrinkage strain	$\epsilon_{cd,0} = 3.64\text{E-}04$		$\alpha_{ds2} = 0.12$
Coeff. Depending on h_0	$k_h = 0.85$	Drying shrinkage	$\epsilon_{cd,\infty} = 4.20\text{E-}04$
		Autogenous shrinkage	$\epsilon_{ca,\infty} = 4.50\text{E-}05$
Total shrinkage at top	$\epsilon_{cs,\infty} = 4.65\text{E-}04$		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	$h_{sim} [\%] = 80$		
Simulated RH at h_{sim}	$RH_{sim} [\%] = 78.2$		
Coeff. - relative humidity	$\beta_{RH} = 0.81$		
Basic drying shrinkage strain	$\epsilon_{cd,0} = 3.64\text{E-}04$	Drying shrinkage	$\epsilon_{cd,\infty} = 2.50\text{E-}04$
Coeff. Depending on h_0	$k_h = 0.85$	Autogenous shrinkage	$\epsilon_{ca,\infty} = 4.50\text{E-}05$
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty} = 2.95\text{E-}04$		

Determining shrinkage gradient

Shrinkage at bottom boundary	$k_{bottom} [-] = 2.53\text{E-}04$
Shrinkage increase with height	$k_y [1/\text{m}] = 2.12\text{E-}03$
Shrinkage at top boundary	$k_{top} [-] = 4.65\text{E-}04$



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

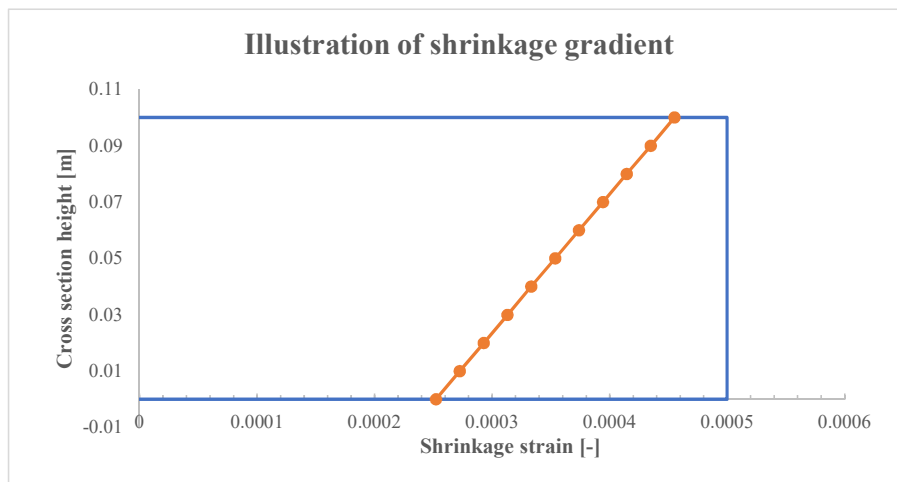
Concrete class	C32/40	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 100
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 100000	Compressive strength	f_{ck} [MPa] = 32
Notional size of cross-section	h_0 [mm] = 200	Mean compressive strength	f_{cm} [MPa] = 40
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.47E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.85	Drying shrinkage	$\epsilon_{cd,\infty}$ = 4.00E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 5.50E-05
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.55E-04		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 80		
Simulated RH at h_{sim}	RH_{sim} [%] = 78.3		
Coeff. - relative humidity	β_{RH} = 0.81		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.47E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.38E-04
Coeff. Depending on h_0	k_h = 0.85	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 5.50E-05
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty}$ = 2.93E-04		

Determining shrinkage gradient

Shrinkage at bottom boundary	k_{bottom} [-] = 2.52E-04
Shrinkage increase with height	k_y [1/m] = 2.03E-03
Shrinkage at top boundary	k_{top} [-] = 4.55E-04



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

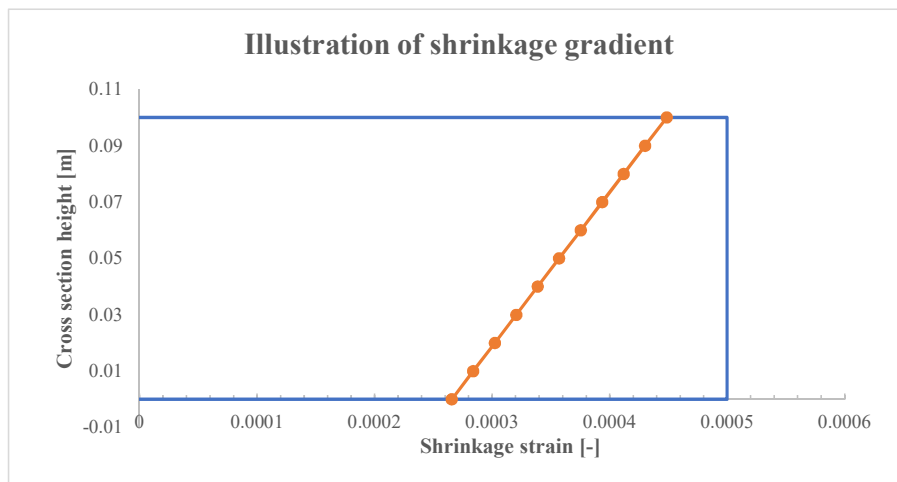
Concrete class	C35/45	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 100
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 100000	Compressive strength	f_{ck} [MPa] = 35
Notional size of cross-section	h_0 [mm] = 200	Mean compressive strength	f_{cm} [MPa] = 43
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.85	Drying shrinkage	$\epsilon_{cd,\infty}$ = 3.86E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.49E-04		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 80		
Simulated RH at h_{sim}	RH_{sim} [%] = 77		
Coeff. - relative humidity	β_{RH} = 0.84		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.40E-04
Coeff. Depending on h_0	k_h = 0.85	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty}$ = 3.02E-04		

Determining shrinkage gradient

Shrinkage at bottom boundary	k_{bottom} [-] = 2.66E-04
Shrinkage increase with height	k_y [1/m] = 1.83E-03
Shrinkage at top boundary	k_{top} [-] = 4.49E-04



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

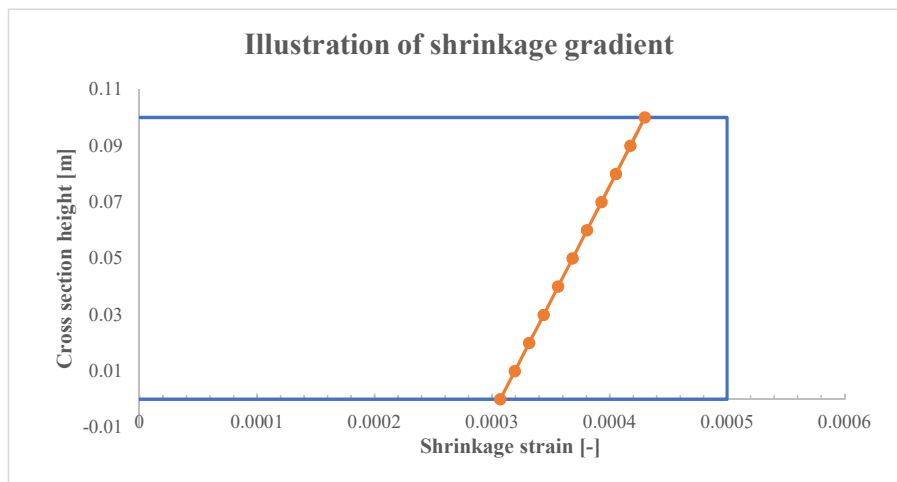
Concrete class	C45/55	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 100
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 100000	Compressive strength	f_{ck} [MPa] = 45
Notional size of cross-section	h_0 [mm] = 200	Mean compressive strength	f_{cm} [MPa] = 53
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 2.97E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.85	Drying shrinkage	$\epsilon_{cd,\infty}$ = 3.42E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 8.75E-05
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.30E-04		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 80		
Simulated RH at h_{sim}	RH_{sim} [%] = 72.2		
Coeff. - relative humidity	β_{RH} = 0.97		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 2.97E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.44E-04
Coeff. Depending on h_0	k_h = 0.85	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 8.75E-05
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty}$ = 3.32E-04		

Determining shrinkage gradient

Shrinkage at bottom boundary	k_{bottom} [-] = 3.07E-04
Shrinkage increase with height	k_y [1/m] = 1.23E-03
Shrinkage at top boundary	k_{top} [-] = 4.30E-04



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

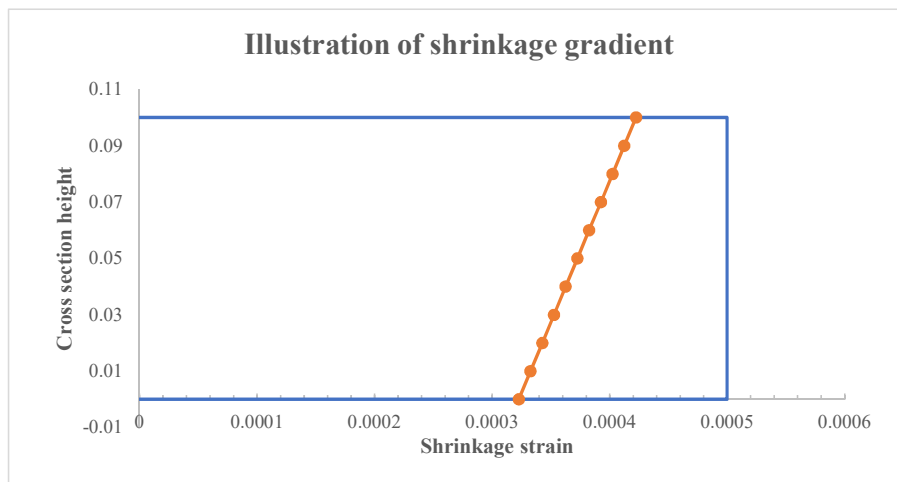
Concrete class	C50/60	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 100
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 100000	Compressive strength	f_{ck} [MPa] = 50
Notional size of cross-section	h_0 [mm] = 200	Mean compressive strength	f_{cm} [MPa] = 58
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 2.80E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.85	Drying shrinkage	$\epsilon_{cd,\infty}$ = 3.22E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 1.00E-04
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.22E-04		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 80		
Simulated RH at h_{sim}	RH_{sim} [%] = 69.9		
Coeff. - relative humidity	β_{RH} = 1.02		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 2.80E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.43E-04
Coeff. Depending on h_0	k_h = 0.85	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 1.00E-04
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty}$ = 3.43E-04		

Determining shrinkage gradient

Shrinkage at bottom boundary	k_{bottom} [-] = 3.23E-04
Shrinkage increase with height	k_y [1/m] = 9.97E-04
Shrinkage at top boundary	k_{top} [-] = 4.22E-04



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

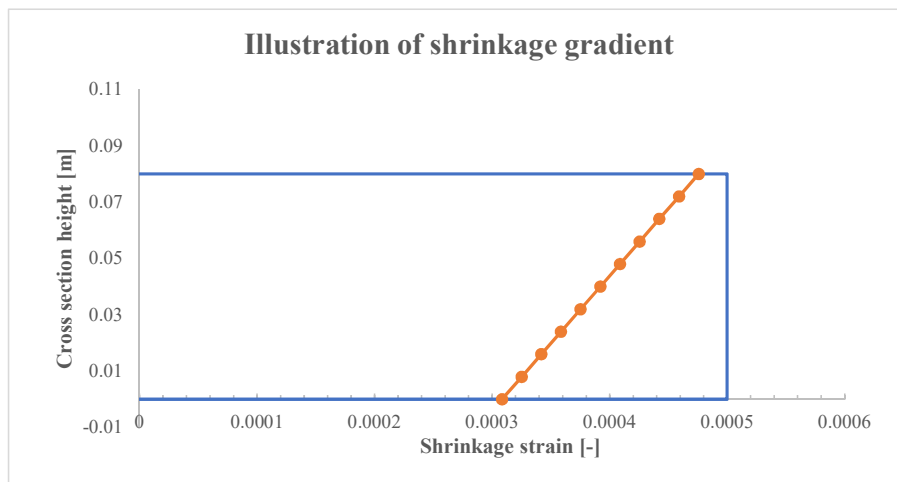
Concrete class	C35/45	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 80
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 80000	Compressive strength	f_{ck} [MPa] = 35
Notional size of cross-section	h_0 [mm] = 160	Mean compressive strength	f_{cm} [MPa] = 43
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.91	Drying shrinkage	$\epsilon_{cd,\infty}$ = 4.13E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.76E-04		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 80		
Simulated RH at h_{sim}	RH_{sim} [%] = 74.2		
Coeff. - relative humidity	β_{RH} = 0.92		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.79E-04
Coeff. Depending on h_0	k_h = 0.91	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty}$ = 3.42E-04		

Determining shrinkage gradient

Shrinkage at bottom boundary	k_{bottom} [-] = 3.08E-04
Shrinkage increase with height	k_y [1/m] = 2.09E-03
Shrinkage at top boundary	k_{top} [-] = 4.76E-04



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

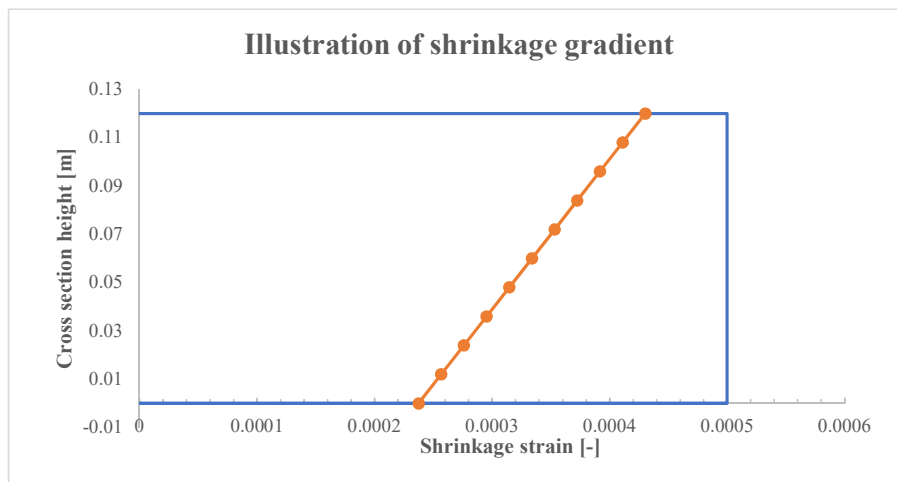
Concrete class	C35/45	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 120
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 120000	Compressive strength	f_{ck} [MPa] = 35
Notional size of cross-section	h_0 [mm] = 240	Mean compressive strength	f_{cm} [MPa] = 43
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.81	Drying shrinkage	$\epsilon_{cd,\infty}$ = 3.68E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.30E-04		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 80		
Simulated RH at h_{sim}	RH_{sim} [%] = 78.95		
Coeff. - relative humidity	β_{RH} = 0.79		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.14E-04
Coeff. Depending on h_0	k_h = 0.81	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty}$ = 2.76E-04		

Determining shrinkage gradient

Shrinkage at bottom boundary	k_{bottom} [-] = 2.37E-04
Shrinkage increase with height	k_y [1/m] = 1.61E-03
Shrinkage at top boundary	k_{top} [-] = 4.30E-04



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

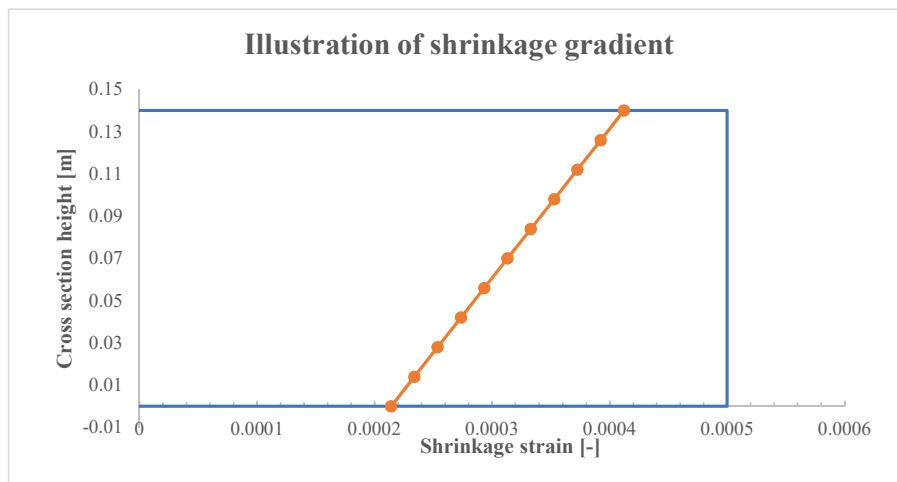
Concrete class	C35/45	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 140
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 140000	Compressive strength	f_{ck} [MPa] = 35
Notional size of cross-section	h_0 [mm] = 280	Mean compressive strength	f_{cm} [MPa] = 43
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.77	Drying shrinkage	$\epsilon_{cd,\infty}$ = 3.50E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.12E-04		

Determining shrinkage at 80 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 80		
Simulated RH at h_{sim}	RH_{sim} [%] = 80.5		
Coeff. - relative humidity	β_{RH} = 0.74		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 1.91E-04
Coeff. Depending on h_0	k_h = 0.77	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at 80 % depth	$\epsilon_{cs,\infty}$ = 2.54E-04		

Determining shrinkage gradient

Shrinkage at bottom boundary	k_{bottom} [-] = 2.14E-04
Shrinkage increase with height	k_y [1/m] = 1.42E-03
Shrinkage at top boundary	k_{top} [-] = 4.12E-04



Input =

Determining shrinkage at top boundary according to EN 1993-1-1 App.B

Concrete class	C35/45	Width of strip	b [mm] = 1000
Cement class	2: N	Height of slab	h [mm] = 100
RH in surrounding environment	RH [%] = 50	Perimeter exposed to drying	u [mm] = 1000
Concrete cross-section	A_c [mm ²] = 100000	Compressive strength	f_{ck} [MPa] = 35
Notional size of cross-section	h_0 [mm] = 200	Mean compressive strength	f_{cm} [MPa] = 43
Coeff. - relative humidity	β_{RH} = 1.36	α -coeff.	α_{ds1} = 4
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04		α_{ds2} = 0.12
Coeff. Depending on h_0	k_h = 0.85	Drying shrinkage	$\epsilon_{cd,\infty}$ = 3.86E-04
		Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at top	$\epsilon_{cs,\infty}$ = 4.49E-04		

Determining shrinkage at 30 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 30		
Simulated RH at h_{sim}	RH_{sim} [%] = 69.8		
Coeff. - relative humidity	β_{RH} = 1.02		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.91E-04
Coeff. Depending on h_0	k_h = 0.85	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at 30 % depth	$\epsilon_{cs,\infty}$ = 3.54E-04		

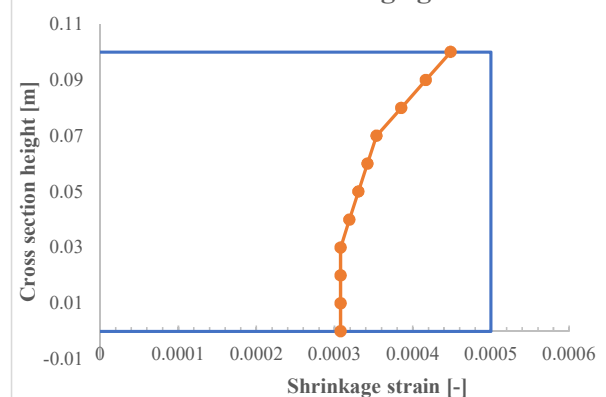
Determining shrinkage at 70 % depth according to EN 1993-1-1 App.B

RH simulated at depth	h_{sim} [%] = 70		
Simulated RH at h_{sim}	RH_{sim} [%] = 76.3		
Coeff. - relative humidity	β_{RH} = 0.86		
Basic drying shrinkage strain	$\epsilon_{cd,0}$ = 3.35E-04	Drying shrinkage	$\epsilon_{cd,\infty}$ = 2.45E-04
Coeff. Depending on h_0	k_h = 0.85	Autogenous shrinkage	$\epsilon_{ca,\infty}$ = 6.25E-05
Total shrinkage at 70 % depth	$\epsilon_{cs,\infty}$ = 3.08E-04		

Determining shrinkage gradient

Shrinkage at top boundary	k_{top} [-] = 4.49E-04
Shrinkage at 30 % depth	k_{30} [-] = 3.54E-04
Shrinkage increase with height	$k_{y,0-30}$ [1/m] = 3.16E-03
Trance of shrinkage at 30 % depth	$k_{bottom,30}$ [-] = 1.32E-04
Shrinkage at 70 % depth boundary	k_{70} [-] = 3.08E-04
Shrinkage increase with height	$k_{y,30-70}$ [1/m] = 1.15E-03
Trance of shrinkage at 70 % depth	$k_{bottom,70}$ [-] = 2.73E-04
Shrinkage at bottom boundary	k_{bottom} [-] = 3.08E-04
Shrinkage increase with height	$k_{y,70-100}$ [1/m] = 0.00E+00

Illustration of shrinkage gradient



Appendix D

Function-file Matlab, Method by Engström

```
1  function [str_i ,fykAfter ,wy_i]=engstromCracks(diaBar ,width ,  
    height ,creep ,l ,restDeg)  
2  
3  %——Calculation of crack widths according to Bjorn Engstrom  
    ——%  
4  
5  %%——Input——%  
6  %—————%  
7  %——Geometry [m]——%  
8  l          = 1;  
9  c          = (height-diaBar)/2;  
10 restDeg = restDeg;  
11 nrBars   = 1;  
12  
13 %——Material Parameters [Pa]——%  
14 Es       = 200e9;  
15 Ec       = 34e9;  
16 fctm     = 1.92e6;  
17 fcm      = 43e6;  
18 fy       = 500e6;  
19 creep    = creep;  
20 alphaEff = Es/Ec*(1+creep);  
21  
22 %——strain——%  
23 shrink   = 0.448*1e-3;  
24 %shrink  = 1e-3;  
25 shrink   = restDeg*shrink;  
26  
27 %——Calculations [m]——%  
28 Ac       = width*height;  
29 As       = nrBars*diaBar^2*pi/4;
```

```

30 h_eff = 2*(c+diaBar/2)*10^-3;
31 Aeff = h_eff*width;
32 Aeff = min(Ac,( height*2.5*(c+diaBar/2)));
33 AIeff = Ac+(Es/Ec-1)*As;
34 dL = shrink*l;
35
36 %——Maximum allowed force——%
37 N1 = fctm*( Aeff+(alphaEff-1)*As)
38 fyk = N1/As;
39
40 %——Maximum Crack Width——%
41 wy = (((0.420*((( diaBar*1000*fyk^2)/(0.22*fcm*Es*(1+Es/
    Ec*As/Aeff))))^0.826))))...
42      +(4*diaBar*1000*fyk/Es))/1000;
43
44 %——For plots——%
45 str_i = [0];
46 fykAfter = [0];
47 wy_i = [0 0];
48
49 n_c = 0;
50 N_i = N1;
51
52 %Transfer length
53 wnet = 0.42*(( diaBar*1000*fyk^2)/...
54      (0.22*fcm*Es*(1+Es/Ec*As/AIeff)))^0.826
55 lt = (0.443*( diaBar*1000*fyk)/(0.22*fcm*(wnet^0.21)*(1+Es
    /Ec*As/AIeff)))...
56      +2*diaBar*1000)/1000
57
58 %Maximum allowed number of cracks
59 n_cr_max = 1/(2*lt)+1
60
61 %——Deformation conditions——%
62 check = 0;
63 iter = 0;
64 while (N_i >= N1 && check == 0)
65     iter = iter + 1;
66
67     %Determine force for n_cr cracks
68     equF = @(sigmaS) 1*sigmaS*As/(Ec*AIeff)...
69         +n_cr*(0.42*(( diaBar*1000*sigmaS^2)/...
70             (0.22*fcm*Es*(1+Es/Ec*As/Aeff)))^0.826...
71             +(4*diaBar*1000*sigmaS/Es))/1000-shrink*l;
72     Fs = fzero(equF,1e9);
73     N_i = Fs*As;
74
75     if N_i > N1

```

```

76      %Determine strain when n_cr appear
77      equStrain = @(shrink_cr) 1*fyk*As/(Ec*Aleff) ...
78      +n_cr*(0.42*((diaBar*1000*fyk^2)/...
79      (0.22*fcm*Es*(1+Es/Ec*As/Aleff)))^0.826...
80      +(4*diaBar*1000*fyk/Es))/1000-shrink_cr*1;
81
82      str = fzero(equStrain,0.0001);
83      str_i = [str_i str str];
84
85      n_cr = n_cr+1;
86
87      %Determine force directly after n_cr cracks appear
88      equForce = @(fykTest) 1*fykTest*As/(Ec*Aleff) ...
89      +n_cr*(0.42*((diaBar*1000*fykTest^2)/...
90      (0.22*fcm*Es*(1+Es/Ec*As/Aleff)))^0.826...
91      +(4*diaBar*1000*fykTest/Es))/1000-str_i(2*iter)*1;
92
93      F_s = max(fzero(equForce,15000));
94      fykAfter = [fykAfter fyk F_s];
95
96      %Crack width directly after n_cr cracks appear
97      wy_n = (((0.420*(((diaBar*1000*F_s^2)/(0.22*fcm*Es
98      *(1+Es/Ec*As/Aleff)))^0.826))) ...
99      +(4*diaBar*1000*F_s/Es)))/1000;
100     wy_i = [wy_i wy_n wy];
101
102     end
103
104     if n_cr+1 > n_cr_max
105         check = 1;
106         iter = iter + 1;
107     end
108     N_i = F_s*As;
109
110     end
111     equForce = @(fykTest) 1*fykTest*As/(Ec*Aleff) ...
112     +n_cr*(0.42*((diaBar*1000*fykTest^2)/...
113     (0.22*fcm*Es*(1+Es/Ec*As/Aleff)))^0.826...
114     +(4*diaBar*1000*fykTest/Es))/1000-shrink*1;
115
116     str_i(2*iter) = shrink;
117     fykAfter(2*iter) = max(fzero(equForce,1e9));
118
119     wy_i(length(wy_i)) = (((0.420*(((diaBar*1000*fykAfter(2*iter)
120     ^2) ...
121     /(0.22*fcm*Es*(1+Es/Ec*As/Aleff)))^0.826))) ...
122     +(4*diaBar*1000*fykAfter(2*iter)/Es)))/1000;
123 end

```