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Modelling of a Gas Turbine with Modelica™

Antonio Alejandro Gómez Pérez

Department of Automatic Control
Lund Institute of Technology
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Department of Automatic Control
Lund Institute of Technology
Box 118
SE-221 00 Lund Sweden

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Author(s)
Antonio Alejandro Gómez Pérez

Supervisors
A. Rantzer H. Tummescheit

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Abstract

The report describes the development of a global model of a simple gas turbine in order to simulate the dynamic behaviour. The Modelica language is used for the creation of the model. The model is based on the evaporative gas turbine located in the Department of Heat and Power Engineering in Lund. This turbine was run as a conventional turbine before including the heat exchanger and the evaporative tower. The model can be split up in three main parts: the compressor, the combustion chamber and the expander. These three models are based on the equations obtained from thermodynamic literature. Furthermore, information provided by the manufacturer was used for the implementation of the compressor and the expander models. Hence, the thesis is focused first on creating a model of a simple gas turbine by using as many components of the ThermoFlow library as possible and second on extending the library with reusable models for turbines and compressors. Since the model involves mechanical parts, components of the rotational sub-library are used for the task.

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who supported bravely my decision
of coming to Sweden for all this
period. I hope we can enjoy for all
the time that we were separated

Nomenclature

Symbol	**Unit**	**Physical Meaning**
A	m^2	surface area
c	J/(kgK)	specific heat capacity
F	N	axial force
h	J/kg	specific enthalpy
I	kgm/s	axial momentum
J	-	Jacobean matrix
m	kg/s	mass flow rate
M	kg	mass
n	rpm	rotational speed
p	Pa	pressure
P	W	power
q	J/kg	heat flow
Q	J/s	heat flow
R	J/(molK)	universal gas constant
s	J/(kgK)	specific entropy
t	s	time
T	K	temperature
u	J/kg	specific internal energy
U	J	internal energy

v	m^3/kg	specific volume
V	m^3	volume
w	kg/mol	molecular weight
w	J/kg	specific work
γ	-	ratio of specific heats
η	-	efficiency
ρ	kg/m^3	density
τ	Nm	torque
ω	rad/s	rotational speed
U	J	internal energy

1. Introduction

1.1 Background

Due to the wide range of interactions between different engineering fields, systems are getting more complex and heterogeneous. When trying to create models of these systems, problems arise with the interaction of the different parts. Many times, it is possible to find simulation programs with graphical user interfaces for creating complex model. The main problem using these interfaces is that they are normally very specific, and are constrained to a concrete engineering discipline. Some examples of this kind of programs are *Spice* and *Saber* for electronics simulations, or *ASPEN Plus* and *SpeedUp* for simulation of chemical processes. Therefore, they are not appropriate when dealing with interoperatibily in heterogeneous problems.

Among the recent research results in modelling and simulation, two concepts have strong relevance to this problem:

- *Object oriented modelling languages* have demonstrated how object oriented concepts can be successfully employed to support hierarchical structuring, reuse and evolution of large and complex models, independent from the application domain and specialised graphical formalisms. By using these languages, it is possible to support modularity on multiple levels. It means that a model can have many submodels, which have submodels themselves.

- *Non-causal modeling.* The traditional approach for simulation based on input and output blocks, is replaced by another one where interaction is not defined with inputs or outputs. This generalisation provides both simpler models and more efficient simulations, while retaining the capability to include submodels with fixed input output relations.

During the last four years the object-oriented, multi-domain language Modelica has been developed by an international group of engineers and researchers. The goal of the Modelica design is to become a de-facto standard for physical modeling languages. Another activity of the group is to also develop basic, free model libraries for several applications domains. The library for thermo-hydraulic processes is currently being developed at the Department of Automatic Control, Lund.

A base library like the one in progress in Lund can only prove its usefulness in practical applications with industrial relevance. Hence, the thesis is focused first on creating a model of a simple gas turbine by using as many components of the ThermoFlow library as possible and second on extending the library with reusable models for turbines and compressors. Since the model involves mechanical parts, components of the rotational sub-library are used for the task. This project is performed in close cooperation with the Department of Heat and Power Engineering, which runs a pilot plant for an evaporative gas turbine at Lund University.

1.2 Objectives

The objective of this thesis is to develop a global model of a simple gas turbine in order to simulate the dynamic behaviour. The Modelica language is used for the creation of the model. The model is based on the evaporative gas turbine located in the Department of Heat and Power Engineering in Lund *(Lindquist, 1999)*. This turbine was run as a conventional turbine before including the heat exchanger and the evaporative tower. Due to the large quantity of parameters involved in the model, the goal is to reproduce the general dynamic behaviour of the turbine. It should be noted that the approximation in the result obtained the model and the results obtained with the experiments depend on many model parameters. The tuning of the parameters of the model is beyond the scope the project.

The model can be split up in three main parts: the compressor, the combustion chamber and the expander. These three models are based on the equations obtained from thermodynamic literature, mainly in *(Cohen, 1996)*, *(Cengel, 1994)* and *(Philips, 1999)*. Furthermore, information provided by the manufacturer was used for the implementation of the compressor and the expander models.

A model of a hydraulic brake or another kind of power sink needs to be implemented in order to extract power from the turbine. The models of compressor, expander and brake are mechanically coupled.

1.3 Why a dynamic model?

When designing a gas turbine for generating electrical power, the most important thing is to obtain a high efficiency of the system. Therefore, an operating point is chosen, and all calculations are based on this design point. Static models are normally used for the design of gas turbines. However, it is not possible to know the response of the plant during transients by using static models. A concrete case of a transient can be a change in the requested power. It is known that in a gas turbine the highest temperature in the cycle is reached at the end of the combustion chamber, before the turbine, which is approximately the same than at the inlet of the expander. The maximum temperature that the turbine blades can withstand limits the turbine inlet temperature (TIT). Therefore, the maximum pressure ratio that can be used in the cycle is also limited, because the TIT is associated to this pressure ratio. Increasing the turbine inlet temperature has been one of the main approaches to improve the gas turbine efficiency. The development of new materials and new cooling techniques has made possible the increase of the TIT. This development has also resulted in more expensive components and the necessity of controlling that these components are working in the right operating range. Consequently, it is necessary to evaluate the behaviour of the plant during different transients in order to keep the TIT in the right range. Dynamic models are used for addressing questions like this.

It should be noted that these models are more complicated than the static models, and it could yield problems for evaluating the equations of the model. Hence, the models have to be simple and at the same time accurate enough for the simulations. Dynamic models are used mainly in order to control the plant. These models can be used for simulating different parts of the plant and to plug them together, for analysing the behaviour of the whole plant. Then, new controllers can be tested improving the global behaviour of the turbine. Another important function of the dynamic models is the training of the personal in power plant. Dynamics models are created to reproduce the behaviour of the plant. In this way, the staff of the company can be prepared by using simulation based on these models.

1.4 Phases of the project

The project was divided in the following phases:

- Study of the features of the language Modelica
- Study of the structure and models in the ThermoFlow library

- Study of the governing equations and technical data for the main parts of the plant.
- Development and implementation of the different models of the plant
- Simulation of the global model of the turbine
- Discussion of the results obtained in the simulations

2. Gas Turbines

2.1 Basic description

It is known that nowadays, all the society in general is dependent on the electrical power. Most of generation of heat and power is dominated by the use of fossil fuels. Turbines are the main devices for generating electrical power by using these fuels. There are basically two kinds of turbines: steam and gas turbines. Steam turbines are able to generate larger power than gas turbines and with an overall efficiency over 40%, but they have the drawback that rather complicated installations for generating steam are needed. On the other hand, gas turbines are much more compact power plants than steam turbines, since steam does not need to be generated and hot gases are used directly to run the turbine. Therefore, gas turbines have very short start-up times.

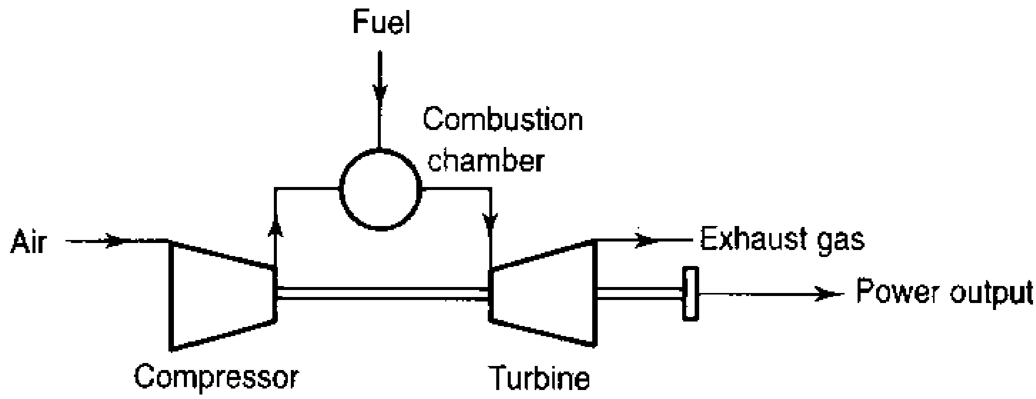

Figure 2.1 Simple cycle (Cohen,1996)

A gas turbine can be split up in three main parts: compressor, combustion chamber and expander. In the compressor, fresh air is taken from the atmosphere and is drawn into the compressor. The compressor uses mechanical energy to raise the pressure and the temperature of the air. This air is used as oxidizer in the combustion chamber, where fuel is burnt at constant pressure. The resulting high-temperature gases enter to the expander where they expand to the atmospheric pressure, producing power. Mechanical shaft power from the expander can be converted to electricity in a generator. Part of this power is used in the compressor. The exhausted gases are released into the atmosphere. Therefore, this cycle is classified as an open cycle

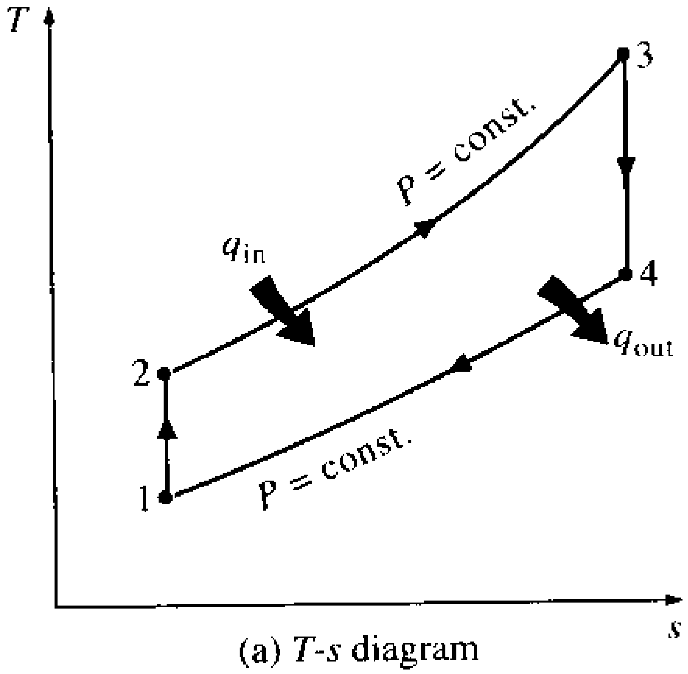

Figure 2.2 T-s diagram for the ideal Brayton cycle (Cengel,1994)

This open cycle can be modelled with a closed cycle, just using air in it. This cycle is essentially the same than the one explained before, but the combustion chamber is replaced by a heat exchanger and there is another heat exchanger which connects the outlet of the expander to the inlet of the compressor. The closed cycle is shown in figure 2.2. The ideal cycled obtained by closing the open cycle is the Brayton cycle. The Brayton cycle was first proposed by George Brayton in 1870. This cycle is made of four internally reversible processes:

- Isentropic compression
- Heat addition at constant pressure
- Isentropic expansion
- Heat rejection at constant pressure

There are two main factors, which affect to the performance of the gas turbine, component efficiencies and turbine working temperature. The higher values they take the better is the performance of the plant. Historically, these two factors have complicated the development of the gas turbine. For example in 1904, two

French engineers, Armegaud and Lemale, made a turbine, which could hardly turn itself. It was due to the low efficiency in the compressor, around 60%, and the limitation of the gas temperature, around 740 K. Nowadays, the efficiencies of the components are around 85-90% and the temperatures that the turbines can withstand exceed 1650 K.

2.2 Evaporative Gas Turbine

The evaporative gas turbine developed at the Department of Heat and Power Engineering in Lund is briefly described in this section.

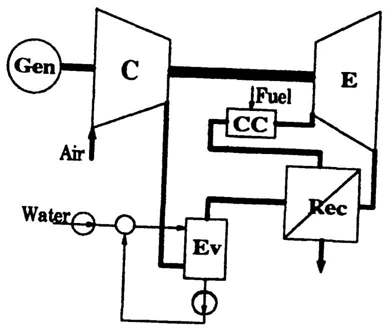

Figure 2.3 Evaporative Gas Turbine

The plant can be split up in the following components: compressor, combustion chamber, expander, recuperator, humidification tower and economiser. The first three parts where explained in the previous section. The scheme of the plant can be seen in the figure.

The air is compressed in the compressor and then flows to the humidification tower. There, the air is brought in contact with water in a counter flow, resulting in a temperature decrease and an elevated humid airflow. Therefore, the exit air temperature can always be kept low by means of the humidification tower, independently of the exit temperature of the compressed air in the compressor. This introduces the opportunity of using a recuperator after the humidification tower. In the recuperator, the air mass flow is heated by using the remaining energy in the exhausted gas mass flow. After the recuperator, the exhausted gases pass trough the economizer, where the water is heated before it enters in the humidification tower.

Tests carried out in the pilot plant showed that the efficiency increased from 22.27% in the simple cycle to 35% in the evaporative cycle. The NO_x emissions were reduced by 90% to under 10 ppm, and the UHC (uncombusted hydro carbons) and CO were not measurable when running the evaporative cycle at rated power output. Other advantages of the plant are that it is possible to reach full power output in less than five minutes and the investment costs for the evaporative cycle are much smaller than for other cycles with the same efficiency. More information about the EvGT can be found in (*Lindquist,1999)*.

3. Modelica language

3.1 Introduction

Modelica is an object-oriented language developed for creating large, complex and heterogeneous physical problems. General equations are used for modelling the physical phenomena. No particular variable needs to be solved for manually, since the Modelica tool will have enough information to do that automatically. Object-oriented and non-causal are important concepts in Modelica. Both concepts are sometimes confused. The concept of object-oriented refers to the structuring of the models whereas the concept of non-causal refers to the underlying description of the behaviour of the models. Nevertheless, these concepts are used together in constrast to the traditional concept based in block-oriented models, which comes more from concern with computational aspects than from user concerns. The Modelica language relies on these concepts of object-oriented and non-causal model. In this chapter some general ideas about these two notions are given.

3.2 Characteristics of object-oriented modeling

The principal point is that object-oriented modelling is able to study a system as a set of interacting objects. The total system is decomposed into simpler elements, which are easier to study. Each object encapsulates data, behaviour and structure. Once each object is defined, it is necessary to define the different connections for these objects and finally their behaviour. With these simple units it is possible to build models and submodels in an easy way. Models define facts and relations, rather that being procedures for computing data. In object-oriented modelling the models are treated as objects. These objects are described by a class, which can be seen as a blue-print of the model. A model representation must support modularity on multiple levels, which means that one model can have many submodels which have submodels themselves. A model is also studied as something abstract, which means that it can be used without knowing all the details about its definition. In an abstract model, it is possible to speak about an interface and internal description. The interface of the model describes how the variables that are internal to the model interact with the environment. The part of the model that has no interaction with the environment is the internal definition.

3.3 Non-Causal Modeling

In order to allow reuse of components models, the equations should be stated in a neutral form without consideration of computational order, i.e. non-causal modeling. Most of the general-purpose simulation softwares on the market assume that the systems have to be split up into block diagrams structures. Therefore, these models are expressed as an interconnection of submodels on explicit state-space form, ODE (Ordinary Differential Equation) :

$$\frac{dx}{dt} = f(x,u)$$

$$y = f(x,u)$$

where u is input, y is output and x is the state. Normally, equations of the models need to be manipulated in order to get this form. A great effort has to be spent in terms of analysis and analytical transformations. It requires a lot of engineering skills and manpower and it is an error-prone process. There is a fundamental limitation of block diagram modelling. The blocks have an unidirectional data flow from inputs to outputs. Therefore, the need of manual transformations implies that it is rather complicated to build physics based model libraries with a block diagram language. A general solution for this problem requires a shift of paradigm.

In Modelica it is possible to write the equations in their natural form, i.e. as a system of differential-algebraic equations, DAE:

$$f\left(x, \frac{dx}{dt}, y, u\right) = 0$$

where x is the vector of unknowns that appear differentiated in the equation and y is the vector of unknowns that do not appear differentiated. Modelica has been carefully designed in such a way that computer algebra can be used to achieve the same efficient simulation code than if the model would have been converted to ODE form manually.

4. ThermoFlow library

For creating the complete model of the gas turbine, the *ThermoFlow* library was used. The library is under development at the *Department of Automatic Control* at the *Lund University*. This chapter is dedicated to briefly describe the library, in order to give an idea of how it is used. Further information about the library can be obtained at *www.control.lth.se/~hubertus/ThermoFluid.*

4.1 Introduction

Since the range of different thermo-hydraulic applications is very wide, it is not feasible to try to create complete models for all these applications. When creating a library to use in these applications, emphasis has to be put in the construction of reusable models. Therefore, the *ThermoFlow* library is designed to provide extensibility of basic building blocks rather than for creating complete models for specific applications. In this way, the user of the library can combine several of these basic models to obtain a complete one in a certain application. The basic physics of flows, fluids and heat need to be covered by the library. Complete physical properties for different mediums are also needed. For this reason a great effort is put to develop basic flow models and control volumes. In the control volume is possible to choose the suitable property model for the desired application. The models in the library are designed for system level simulation, not for detailed simulation. The models are thus discretized in one dimension or even lump parameter approximations

The Thermo-Flow library follows basically the following guidelines:

- One unified library both for lumped and distributed models
- Separation of the medium submodels, which can be selected through class parameters
- Both bi and unidirectional flows are supported
- Assumptions can be selected through class parameter

The main idea of the library is to enable the user to create complex models based on the simple models provided.

4.2 Basic Design Ideas

A large number of engineering problems involve mass flow in and out of the system. In many books in thermodynamics these systems are modeled as control volumes. Inside the control volumes, energy and mass flow are set. In the *ThermoFlow* library, control volumes are the basic entity. But another model is necessary for calculating the mass flow and the convective energy associated to the mass flow. Because of this, flow models are introduced. A flow model is the result of a modeling abstraction, where the volume is neglected. These flow models contain either an algebraic equation that relates pressure drop and mass flow, or an expression for the dynamic momentum balance. The *ThermoFlow* library is based on an alternating sequence of control volumes and flow models. It can be said that storage of mass and energy are modeled in the control volume, whereas the flow of mass and energy are modeled in the flow models. Control volumes and flow models are connected through flow connectors. The flow connector for a single medium flow without dynamic momentum balance contains the following variables:

$$\{p, h, m, q_c, r, T, s, k\}$$

where the quantities are pressure, specific enthalpy, mass flow, convective heat flow, density, temperature, specific entropy and ratio of specific heat, respectively. All the information for the mass and energy balance is contained in the variables m and q_c, which are evaluated in the flow model. The rest of variables (p, h, r, T, s, k) are evaluated in the control volume.

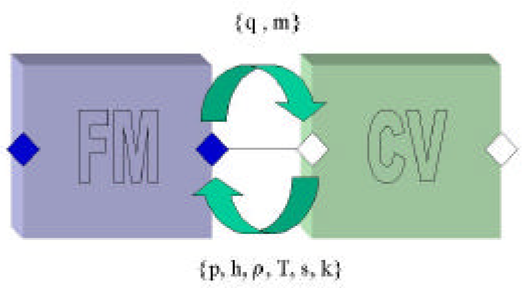

Figure 4.1 Interaction between CV and FM

In order to build up new models using the library it is very important to understand these two basic models, the control volume and the flow model.

Control Volumes

Control volumes are in fact one of the most important “pillars” of the *ThermoFlow* library. As it was said before, a control volume contains energy and mass balances. But it is also necessary to include a model for calculating all the thermophysical properties, which is called *medium model*. The user can choose the medium model depending on the application. Control volumes contain also connectors, which are the links between the control volume and the environment. Through the connectors, the control volume interacts with the rest of the system. There are two different kinds of connectors:

- Flow connectors, which were already explained above.
- Heat transfer connectors. In this connectors there is no mass flow. These connectors are used for modeling heat transfer between fluids and solid bodies.

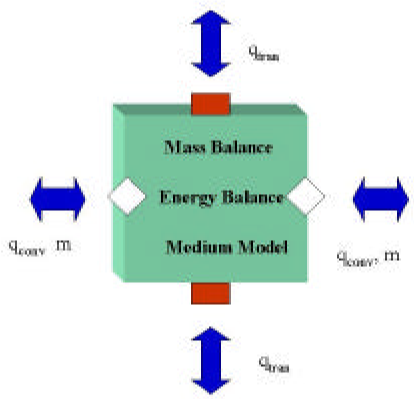

Figure 4.2 Control Volume

Once the connectors have been defined, mass and energy balances in the control volume can be written as:

$$\frac{d}{dt}(M) = \sum_{i}^{n} m_n \qquad (4.1)$$

$$\frac{d}{dt}(U) = \sum_{i}^{n} \dot{q}_{conv,i} + \sum_{i}^{l} \dot{q}_{transfer,j} \tag{4.2}$$

Where M is the total mass, U the total inner energy, *n* is the number of flow connections (associated to mass flow m and heat flow q_{conv}) and *l* the number of heat transfer connectors (associated to heat transfer q_{tran}). Positive sign is associated to flows into the control volume. For simplicity, pressure volume work and dissipative work have been neglected here.

Flow Models

Two volumes have to be connected with a flow model, which contains mainly the momentum balance. In the *ThermoFlow* library, two types of flow models are defined:

- Stationary pressure drop models.
- Dynamic momentum balances for pipes with constant cross-sectional area.

The user can choose the type of flow model to use depending on the model that he wants to create. It should be kept in mind that the dynamic momentum balance is only of interest when fast wave dynamics of the system are of interest. When the main interest is in the thermal behaviour, the stationary pressure drop should be used.

Figure 4.3 Flow Model

The momentum equation for a pipe with a constant cross-section area and with volume *V* is:

$$I = \int_V r \cdot w \cdot dV = \int_{\Delta z} \int_A r \cdot w \cdot dA \cdot dz = \dot{m} \cdot \Delta z \tag{4.3}$$

Where w is the velocity in z direction, A is the normal flow area and m is the mass flow. Using Newton's law with only pressure and friction forces acting on the CV :

$$\Delta z \frac{d\dot{m}}{dt} = \frac{dI}{dt} = \dot{I}_1 - \dot{I}_2 + (p_1 - p_2) \cdot A - F_{wall} \tag{4.4}$$

The equation can be simplified in order to obtain a stationary pressure drop model:

$$0 = (p_1 - p_2) \cdot A - F_{wall} \tag{4.5}$$

Where F_{wall} is the friction between the fluid and the wall. It depends on the flow characteristics. Different expressions for F_{wall} can be found in the literature.

When creating models like a compressor for example, there are many different relationships between mass flow rate, pressure ratio, angular speed, etc. Then the flow model can contain these expressions instead of the dynamic momentum equation.

Medium Models

It is important to have accurate medium models in order to make the library really reusable. On the other hand, for the purpose of dynamic simulations, it is also important to have fast medium models. At the moment, the following medium models are implemented:

- Pure Ideal gases
- Mixture of ideal gases
- CO_2
- Water

The models implemented in the library are all very accurate and are taken from some recommended formulations or standards like IAPWS/IF97 for water.

Medium models are necessary in control volumes. By using this medium models it is possible to compute all remaining variables of interest using the mass and energy balances.

State variable transformation

Mass and energy balances are implemented in the control volume model. Total mass and internal energy (M and U) are the states in these equations. For using the medium models provided in the library, these variables are not very suitable. Different variables are chosen as states depending on the choice of the medium model used. When working with ideal gases, which are used for the turbine model, p and T are chosen as states, mainly for efficiency reasons. In ideal gases, all medium properties depend on T. Hence, if h were chosen as a state, there would always be a non-linear system of equations for calculating T. Because of this there is a special class in the library called *StateTransformation*, which changes the states *M* and *U* to different states according to the desired model.

A differentiation of $M = rV$ and $U = uM$ for a constant volume yields:

$$V\frac{dr}{dt} = \frac{dM}{dt} \tag{4.6}$$

$$M\frac{du}{dt} = \frac{dU}{dt} - u\frac{dM}{dt} \tag{4.7}$$

So the energy and mass balances described above can be rewritten as:

$$\frac{d}{dt}\begin{pmatrix} M \\ U \end{pmatrix} = V\begin{pmatrix} 1 & 0 \\ u & r \end{pmatrix}\frac{d}{dt}\begin{pmatrix} r \\ u \end{pmatrix} \tag{4.8}$$

Where r is the density and u is the specific inner energy. These primary equations are then transformed into secondary forms to give differential equations in the states that are best suited for the medium model. For the case of perfect gases, p and T are chosen as states. For simplicity, the composition of the gas is assumed constant, so it is not considered in the transformation:

$$\frac{d}{dt}\begin{pmatrix} r \\ u \end{pmatrix} = \begin{pmatrix} \frac{dr}{dp}_T & \frac{dr}{dT}_p \\ \frac{du}{dp}_T & \frac{du}{dT}_p \end{pmatrix}\frac{d}{dT}\begin{pmatrix} p \\ T \end{pmatrix} \tag{4.9}$$

Where:

$$J = \begin{pmatrix} \left.\frac{dr}{dp}\right|_T & \left.\frac{dr}{dT}\right|_p \\ \left.\frac{du}{dp}\right|_T & \left.\frac{du}{dT}\right|_p \end{pmatrix} \tag{4.10}$$

And for ideal gases:

$$\left.\frac{dr}{dp}\right|_T = \frac{1}{R \cdot T} \tag{4.11}$$

$$\left.\frac{dr}{dT}\right|_p = -\frac{p}{R \cdot T^2} \tag{4.12}$$

$$\left.\frac{du}{dp}\right|_T = 0 \tag{4.13}$$

$$\left.\frac{du}{dT}\right|_p = C_v \tag{4.14}$$

To obtain differential equations for pressure and temperature, the following expression is used:

$$\frac{d}{dt}\begin{pmatrix} p \\ T \end{pmatrix} = J^{-1}\frac{d}{dt}\begin{pmatrix} r \\ u \end{pmatrix} \tag{4.15}$$

The inverse of the Jacobian is computed as follows:

$$J^{-1} = \frac{1}{\det J} \cdot \begin{pmatrix} \left.\frac{du}{dT}\right|_p & -\left.\frac{dr}{dT}\right|_p \\ -\left.\frac{du}{dp}\right|_T & \left.\frac{dr}{dp}\right|_T \end{pmatrix} \tag{4.16}$$

An the determinant is:

$$\det J = \left.\frac{du}{dT}\right|_p \cdot \left.\frac{dr}{dp}\right|_T - \left.\frac{dr}{dT}\right|_p \cdot \left.\frac{du}{dp}\right|_T = C_v \cdot \frac{1}{R \cdot T} - \left(\frac{-p}{R \cdot T^2}\right) \cdot 0 = \frac{C_v}{R \cdot T} \tag{4.17}$$

So the inverse of the Jacobian can be rewritten as:

$$J^{-1} = \frac{C_v}{R \cdot T} \cdot \begin{pmatrix} C_v & \frac{p}{R \cdot T^2} \\ 0 & \frac{1}{R \cdot T} \end{pmatrix} \quad (4.18)$$

Similar expressions are also implemented in the Library for other pairs of state variables, for example (*p,h*), (*r,T*), (*r,T,x)*,...where x refers to the composition of a mixture of gases.

4.3 Sequence of calculation in a dynamic simulation

Once control volumes have been presented, a brief explanation of the way of operating is given. Pressure and temperature are assumed to be the states. It should be noted that this sequence of calculation is determined automatically by the Dymola tool, without any help from the user

- First of all, initial values of the states pressure and temperature (p_0 and T_0) are needed. The user has to supply these values.

- Knowing the temperature, it is possible to evaluate other termodynamical variables (h, ρ, s, k) by using the medium model.

- The flow models use the value of these variables for evaluating the mass and energy flows. The flow model accesses this information through the connectors.

- All the mass and energy flows calculated in the flow models surrounding the control volume, are used in the mass and energy balances. Time derivatives of total mass and total inner energy are then calculated (dM, dU).

- The class *StateTransformation* is used to transform the time derivatives of total mass and inner energy (dM, dU) into the time derivatives of the chosen states (dp, dT).

- The time derivative of pressure and temperature are used for evaluating the new values of the states. The sequence is repeated again.

The sequence can be seen in the figure.

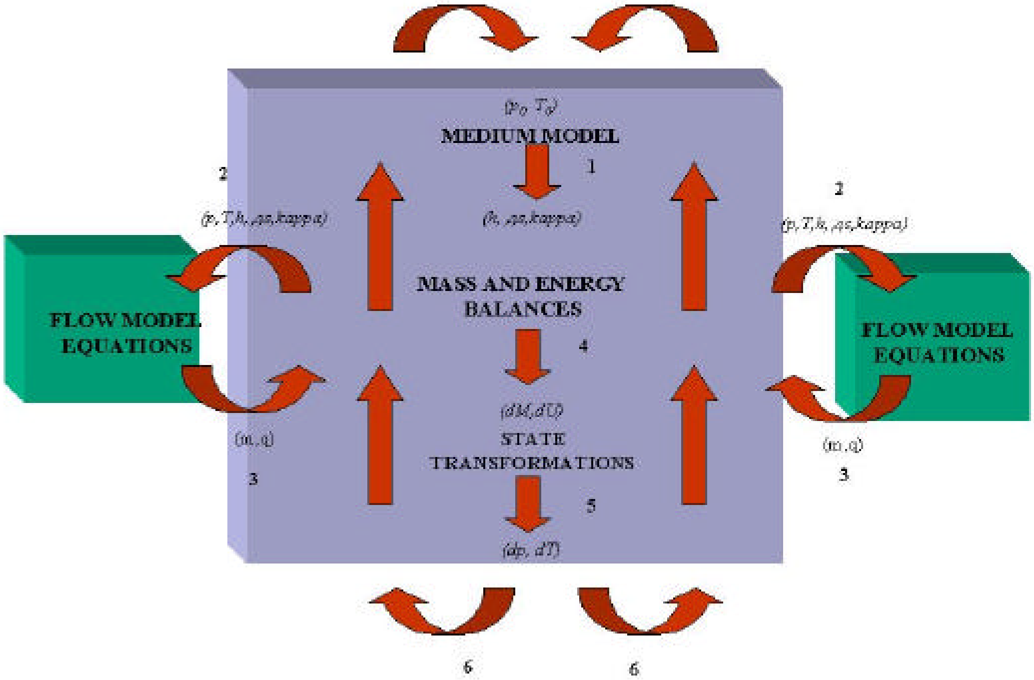

Figure 4.4 Sequence of calculation

5.Compressor

5.1 Introduction

The compressor for gases has the same function that the pump for liquids. Basically, mechanical energy is added to the gases and this is used to raise the pressure, hence the name compressor. This adding of energy causes the gases to flow from one unit operation to the next. A compressor is the same than a turbine in reverse, compressing rather than expanding the gases that pass through it.

The model created for the compressor is based on the general equation for steady state flow. In this way, faster dynamics are neglected, and replaced by static relationships. This approach is used when a model has some fast and some slow dynamics. Higher frequency transients can be neglected, since the low frequency transients dominate the response. When this assumption is used, it is possible to base a model on steady state data. A compressor map is used in the compressor model. By using this map, the polytropic efficiency and the mass flow can be calculated as functions of the pressure ratio and the rotor speed. The compressor model includes equation for the mechanical behaviour. Inside the model, mechanical and thermodynamic powers are connected in an equation.

During the first part of the chapter, the governing equations are explained. In the second part, the way of implementing the compressor map is shown. Finally, a short description about the Modelica models created for the compressor is given.

5.2 Governing equations

First of all, a steady state energy balance is used. This equation can be found in many books of thermodynamics *(Philip, 1999),(Cengel, 1996)*:

$$dq - dw - dh - cdc - gdz = 0 \tag{5.1}$$

where the *dq* is the specific external heating, *dw* is the specific work, *dh* is the enthalpy, *cdc* is the kinetic energy term and *gdz* the potential energy term. Normally a compressor can be considered as adiabatic, so the term *dq* can be neglected. The same consideration can be taken for the height difference, so that *dz=0*. There is a small difference between the inlet and the outlet velocities, but it can be also neglected, i.e. *cdc=0*, *(Philip, 1996)*. According to this, the equation obtained is:

$$-dw = dh \tag{5.2}$$

And integrating over the compressor section gives:

$$-w = h_2 - h_1 \tag{5.3}$$

Where the subscripts 1 and 2 refer to the input and the output sections of the compressor respectively, and *w* is the specific work done on the gas. From the definition of constant specific heat $c_p(T)$, for ideal gases, equation (5.3) can be rewritten as:

$$-w = \int_1^2 c_p(T)dT \tag{5.4}$$

where $c_p(T)$ is a function of the temperature. In the Figure (5.1) is possible to see the variation of $c_p(T)$ with the temperature for different gases. For the case of air, the variation of specific heat with the temperature is smooth, and may be approximated as linear over intervals of a few hundred degrees, *(Cengel, 1994)*. Then, the specific heat in equation (5.4) can be replaced by a constant average specific heat value. An approach is to take the average for the temperatures at compressor inlet an outlet.

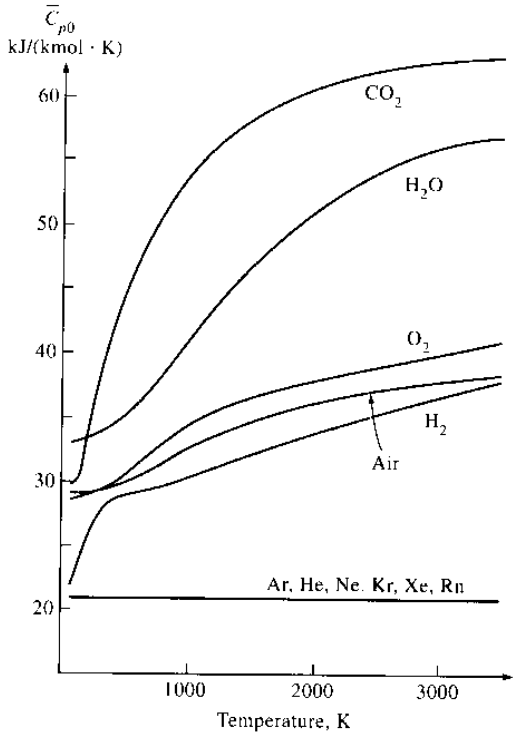

Figure 5.1 Ideal gas constant-pressure specific heats for some gases (Cohen,1996)

Denoting the average specific heat with c_p, the following equation is obtained:

$$-w = c_p \cdot (T_2 - T_1) = c_p \cdot T_1 \cdot \left(\frac{T_2}{T_1} - 1 \right) \tag{5.5}$$

For gases, it is possible to use the following equation for calculating the specific heat (*Phillip, 1999)*:

$$c_p = \frac{g}{g - 1} \cdot R \tag{5.6}$$

where R is the gas constant for the gas, and γ is the ratio of the specifics heats C_p and C_v:

$$g = \frac{C_p}{C_v} \tag{5.7}$$

The specific heat ratio also varies with the temperature, but this variation is very small. For the case of air, the value of the specific heat ratio is around 1.4 *(Cengel,1996)*. The equation (5.5) can be rewritten as:

$$-w = \frac{g}{g-1} \cdot R \cdot T_1 \cdot \left(\frac{T_2}{T_1} - 1 \right) \tag{5.8}$$

Since *R* and g are constants and T_1 is the ambient temperature, the only variable in equation (5.8) is T2. Therefore, the specific work is a minimum when the temperature T2 is a minimum. This occurs when the compression is isentropic (adiabatic and reversible). In an isentropic compression the relation between pressures and temperatures is the following:

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{(g-1)}{g}} \tag{5.9}$$

Where the subscript *s* is referred to the isentropic temperature. Substituting from equation (5.9) into equation (5.8), the isentropic specific work w_s is obtained:

$$-w_s = \frac{g}{g-1} \cdot R \cdot T_1 \cdot \left(\left(\frac{p_2}{p_1} \right)^{\frac{g-1}{g}} - 1 \right) \tag{5.10}$$

In order to calculate the actual specific work, the concept of isentropic efficiency is used. The isentropic efficiency is defined as the ratio between the real enthalpy difference and the theoretical enthalpy difference, when considering the process as isentropic:

$$h_s = \frac{h_{2s} - h_1}{h_2 - h_1} \tag{5.11}$$

Where h_{2s} is the specific enthalpy at the outlet of the compressor, when the process is isentropic. Measuring the pressures and temperature at the inlet and outlet sections of the compressor, an experimental determination of this isentropic efficiency can be obtained. T_{2s} can be calculated with the equation (5.9), and assuming the gas has a constant specific heat, the isentropic efficiency can be evaluated as:

$$h_s = \frac{c_p \cdot (T_{2s} - T_1)}{c_p \cdot (T_2 - T_1)} = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad (5.12)$$

The definition of isentropic efficency, is based on a ratio of specific work to actual specific work, across the complete section of the compressor. When the pressure ratio changes, an overall efficiency as obtained in equation (5.12) does not remain constant. In fact it is found that h_s tends to decrease when the pressure ratio raises, *(Cohen,1996)*. These considerations have led to the concept of polytropic efficiency, which is defined as the isentropic efficiency of an elemental stage in the process, arbitrarily defined in such a way that this efficiency is constant throughout the whole process:

$$h_p = \frac{dw_s}{dw} = \text{constant for the whole compressor} \qquad (5.13)$$

Assuming c_p constant, the equation (5.13) can be transformed into:

$$h_p = \frac{dh_s}{dh} = \frac{c_p \cdot dT_s}{c_p \cdot dT} = \frac{dT_s}{dT} \qquad (5.14)$$

Using the equation for an isentropic compression process:

$$\frac{T}{p^{\left(\frac{g-1}{g}\right)}} = \text{constant} \qquad (5.15)$$

Expressions (5.14) and (5.15) can be combined obtaining:

$$\frac{dT_s}{T} = \frac{g-1}{g} \cdot \frac{dp}{p} \qquad (5.16)$$

Using the definition of polytropic efficiency:

$$\frac{dT}{T} \cdot h_p = \frac{g-1}{g} \cdot \frac{dp}{p} \qquad (5.17)$$

If the expression above is integrated between inlet 1 and outlet 2 assuming h_p constant by definition, the following expression is stated:

$$h_p = \frac{\ln\left(\frac{p_2}{p_1}\right)^{\frac{g-1}{g}}}{\ln\left(\frac{T_2}{T_1}\right)} \qquad (5.18)$$

This expression allows the calculation of the value for h_p by using the values of p and T at the inlet and the outlet of the compressor. This expression can be rewritten in the following form:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(g-1)}{g \cdot h_p}} \qquad (5.19)$$

Then, it is possible to define a new coefficient:

$$m = \frac{h_p \cdot g}{1 - g \cdot (1 - h_p)} \qquad (5.20)$$

So that equation (5.19) can be rewriten as:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(m-1)}{m}} \qquad (5.21)$$

Equation (5.21) can be combined with equation (5.1) for evaluating the actual specific work:

$$-w = c_p \cdot (T_2 - T_1) = c_p \cdot T_1 \cdot \left(\frac{T_2}{T_1} - 1\right) = c_p \cdot T_1 \cdot \left(\left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}} - 1\right) \qquad (5.22)$$

And dividing the equation (5.10) for the isentropic specific work by the expression (5.22) for the actual specific work:

$$h_s = \frac{c_p \cdot T_1 \cdot \left(\left(\frac{p_2}{p_1}\right)^{\frac{(g-1)}{g}} - 1\right)}{c_p \cdot T_1 \cdot \left(\left(\frac{p_2}{p_1}\right)^{\frac{(m-1)}{m}} - 1\right)} \qquad (5.23)$$

By using this expression, the isentropic efficiency can be calculated as a function of the pressure ratio and the polytropic efficiency. Figure 5.2 shows the decrease in the isentropic efficiency when the pressure ratio raises.

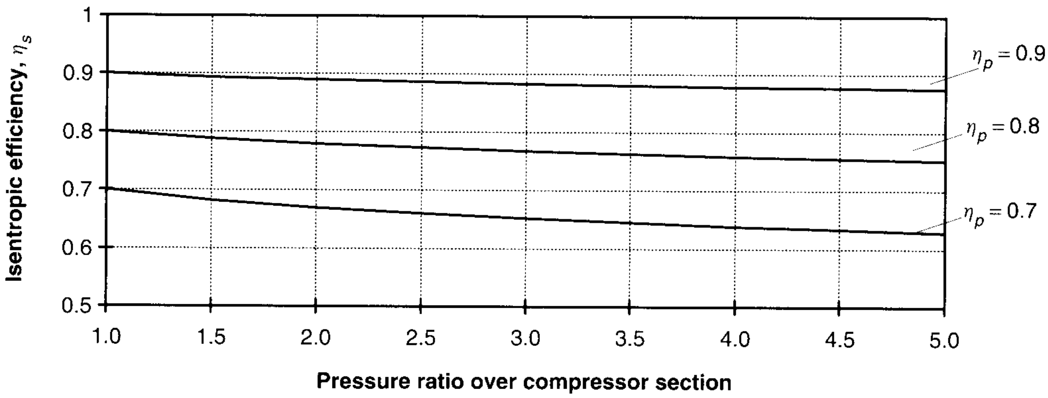

Figure 5.2 Isentropic efficiency against pressure ratio for different polytropic efficiencies (Philip,1999)

The actual power needed for the compression is the product of the actual specific work and the mass flow rate of gas:

$$P = -\dot{m} \cdot w \tag{5.24}$$

This expression can be rewritten using the isentropic efficiency as:

$$P_{comp} = \frac{-\dot{m} \cdot w_s}{h_s} \tag{5.25}$$

The shaft connected to the turbine provides this power, so it can also be expressed as:

$$P_{comp} \cdot h_{mec} = t_{comp} w_{comp} \tag{5.26}$$

Where t_{comp} is the torque, w_{comp} is the angular velocity and h_{mec} in the compressor. In this way, the mechanical and the thermodynamic behaviours are connected.

5.3 Use of the compressor map

The map of the manufacture was used for the implementation of the compressor model. In this map, the pressure ratio is plotted against the corrected mass flow for a range of corrected speed and polytropic efficiency curves. The corrected mass flow and the corrected compressor speed are used in the map to compensate for different environmental conditions under which the steady state experiments were carried out.

For the compressor map, the corrected mass flow is defined as:

$$m_{cor} = \frac{\dot{m} \cdot \sqrt{T_1}}{p_1} \qquad (5.26)$$

And the corrected rotational speed:

$$n_{cor} = \frac{n}{\sqrt{T_1}} \qquad (5.27)$$

It is important to point out that there are two critical regions that have to be taken into account, the surge and the stall lines. Along the surge line, the rotor speed contours become nearly horizontal. To the left of the surge line, the speed contours drop with respect to the pressure ratio. This may create an unstable phenomenon called surging, which can destroy the compressor *(Cohen, 1996)*. At each rotor speed, a pressure for which surge occurs can be identified. Along the stall line, the mass flow becomes choked. These regions have to be implemented in the model.

Figure 5.3 shows an example of compressor map. Surge line is indicated in the figure.

All the information given in the map must to be processed by the model. It is necessary to translate the information from the graphical form supplied by the manufacturer to the Modelica model. There are several methodologies for transforming this information.

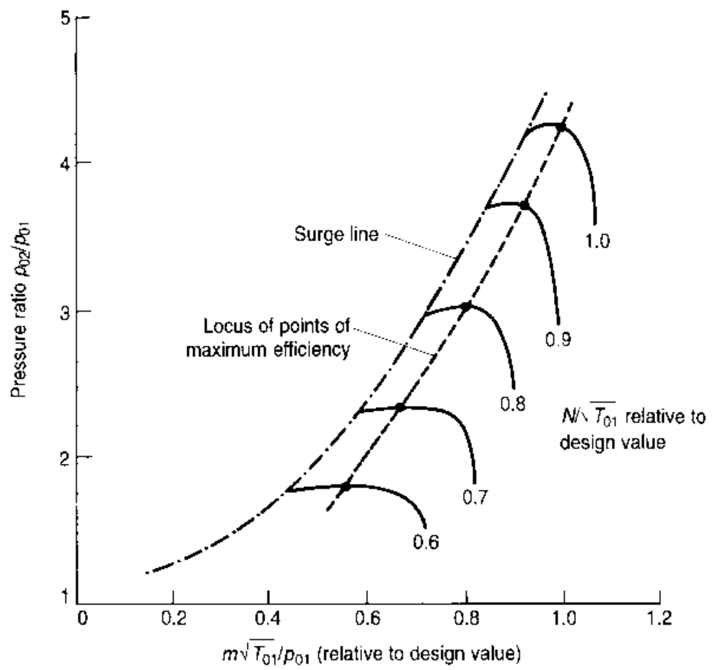

Figure 5.3 Map for a centrifugal compressor (Cohen,1999)

One of these approaches could be to place all the information provided in tables by choosing several points of the map and then to use these tables in a look-up manner *(Munns, 1996)*. But using the information, it can cause problems in the dynamic simulations. In these tables, data points for several pressure ratios and speed parameters are given. When a point for a different pressure ratio or speed parameter needs to be evaluated, linear interpolation has to be used. When a dynamic simulation moves over one of the given points, there is a discontinuous derivative caused by the linear interpolation, which yields in problems for the dynamic simulation. For this reason, the approach was rejected.

Another approach is to implement a couple of functions where pressure and speed parameter are given as inputs and the efficiency and the mass flow parameter are obtained as output for each one of the functions (*Gustafsson, 1998)*.

In order to get a function for evaluating the corrected mass flow, an ellipsoid equation is used:

$$\left(\frac{x}{a}\right)^z + \left(\frac{y}{b}\right)^z = c \qquad (5.28)$$

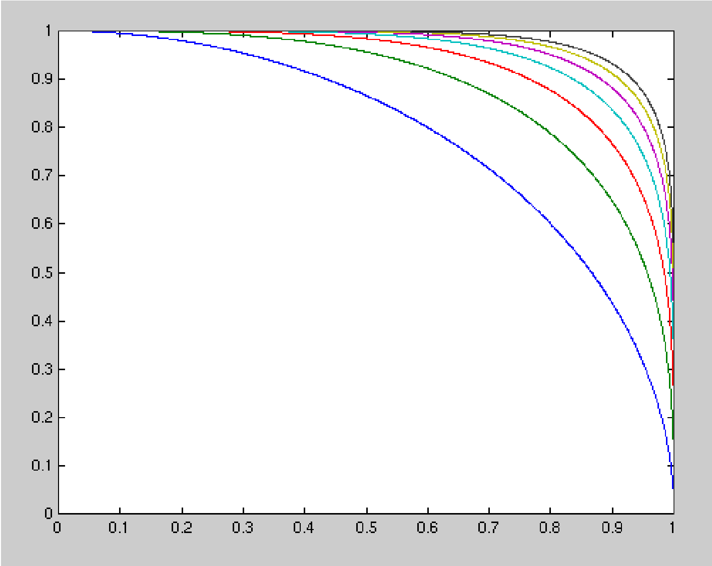

Figure 5.4 Ellipsoid curves for different values of z

In figure 5.4, a set of the curves is shown. These curves have been obtained using the ellipsoid equation for different values of the parameter z. The values of *a*, *b* and *c* are all equal to one.

The lines on the left are for lower values of z. When the value of z increases, the shape of the curves obtained is very similar to shape of the corrected speed curves in the map.

Now, it is going to be shown how to calculate the mass flow by using the ellipsoid equation. First, z and n_{cor} (corrected rotational speed) are assumed to be fixed. The variable n_{cor} is calculated from the angular velocity, which is a dynamic state, and can thus be regarded as known. The variable z is a function of n_{cor}.

For the value of the constant a in the ellipsoid equation, the value of the mass flow when the pressure ratio equals one is given. This value corresponds to the mass flow when the compressor is stalled. For calculating the value of the constant b, the ellipse equation is used. Therefore, once the constant *a* is known, x and y are given as inputs and b is obtained as an output. The value of x corresponds to the pressure ratio at the surge line, while y corresponds to the mass flow at the surge line. In this way, the equation takes the value of the map.

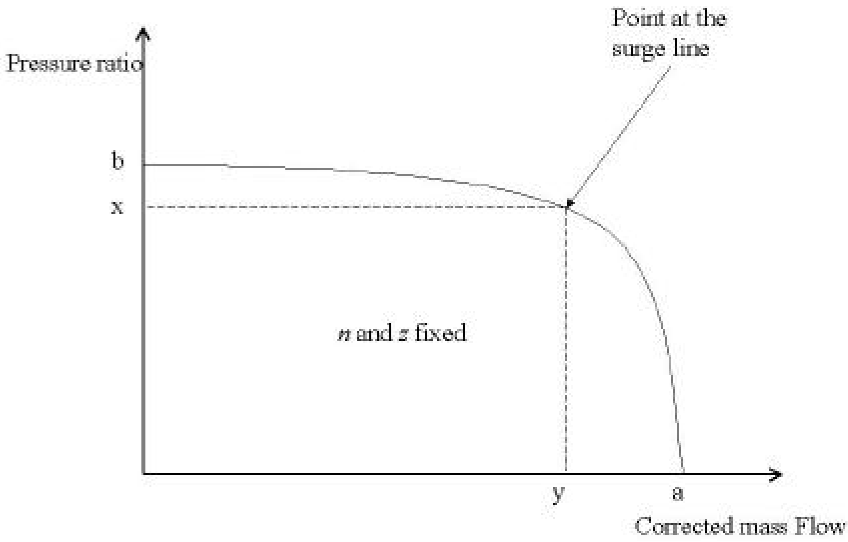

Figure 5.5 Estimation of the value of b

Once values a, b and z are known and x (pressure ratio) is given as an input, y (mass flow) can be obtained easily by using the equation (5.28).

In order to be able to use this equation over the whole range of speeds, it is necessary to have a, b, and z as functions depending on the corrected speed:

$$a = f(n_{cor})$$
$$b = f(n_{cor})$$
$$z = f(n_{cor})$$

For calculating a as a function of the speed, the points defined in the map for the eight constant corrected speeds were used. For each one of these speeds, the values of the corrected mass flow at the pressure ratio one are read. Then a relation between the mass flow at pressure one and the corrected speed is calculated by fitting a polynomial to the data points. Finally it is possible to calculate a as a function of the corrected speed:

$$a = a_4 \cdot n_{cor}^{\ 4} + a_3 \cdot n_{cor}^{\ 3} + a_2 \cdot n_{cor}^{\ 2} + a_1 \cdot n_{cor} + a_0 \tag{5.29}$$

where a_0, a_1, a_2, a_3 and a_4 are the coefficients of the polynomial fitting. The same approach is adopted to obtain a function of b depending on the corrected speed:

$$b = b_4 \cdot n_{cor}^{\ 4} + b_3 \cdot n_{cor}^{\ 3} + b_2 \cdot n_{cor}^{\ 2} + b_1 \cdot n_{cor} + b_0 \qquad (5.30)$$

where b_0, b_1, b_2, b_3 and b_4 are the coefficients of the polynomial fitting. The value of z is calculated as a linear function of the corrected speed. A value z_1 for the lowest speed and another z_2 for the highest speed are chosen, and in between a linear interpolation is used:

$$z = z_1 + n_{cor} \cdot \frac{(z_2 - z_1)}{(n_{\max} - n_{\min})} \qquad (5.31)$$

Once a, b and z are known, it is possible to use the ellipsoid equation for the whole map. The pressure ratio x is given as input and the mass flow y is obtained as output.

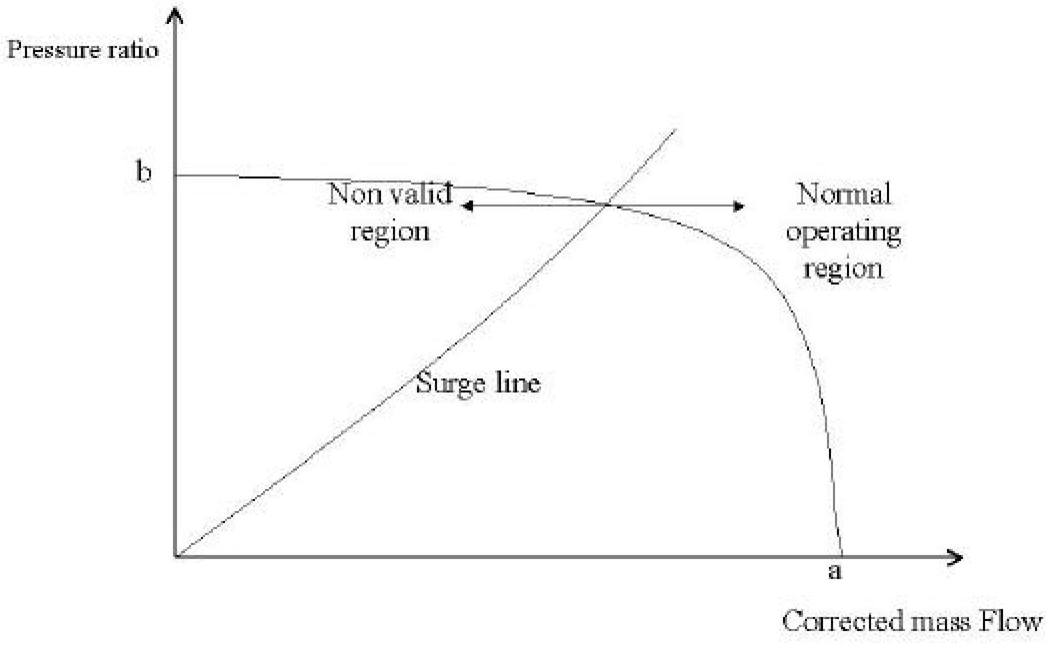

Figure 5.6 Valid region for calculation with ellipsoidal equation

The surge line of the map needs to be considered. If the compressor reaches the surge line, the corrected mass flow is not the one calculated with the ellipsoid equation (Figure 5.6).

Because of this, the corrected mass flow for a given pressure ratio has to be calculated and then it is necessary to check if the surge line has been reached. A polynomial relation can be fitted between the mass flow and the pressure ratio at the surge line by using the points obtained in the map. The approach is the same that was taken for calculating *a* and *b*:

$$m_{surge} = m_4 \cdot r^4 + m_3 \cdot r^3 + m_2 \cdot r^2 + m_1 \cdot r \quad + m_0 \tag{5.32}$$

Where m_{surge} is the mass flow at the surge line for a given value of the pressure ratio *r*.

In order to know if the surge line has been reached, the mass flow for a given speed and pressure ratio is calculated by using the ellipsoid equation. Then the mass flow at the surge line is evaluated with equation (5.32). Once both mass flows have been calculated, they are compared:

$$m_{ellipse} \leq m_{surge} \rightarrow surge$$

Another function for calculating the polytropic efficiency of the compressor was also needed, but getting a function for it was much more difficult, since the information provided by the map was not very accurate.

For this reason, it was decided to create a speed-dependent function for evaluating the value of the maximum efficiency. Then a function for the degradation of this efficiency was fitted.

So first of all, a polynomial relation can be fitted between the corrected speed and the value of the maximum efficiency:

$$h_{\max} = m_4 \cdot n_{cor}{}^4 + m_3 \cdot n_{cor}{}^3 + m_2 \cdot n_{cor}{}^2 + m_1 \cdot n_{cor} \quad + m_0 \tag{5.33}$$

where m_0, m_1, m_2, m_3 and m_4 are the constants for the polynomial fitting. The same can be done between the speed and the pressure ratio for maximum efficiency:

$$p_{\max\_eff} = p_4 \cdot n_{cor}{}^4 + p_3 \cdot n_{cor}{}^3 + p_2 \cdot n_{cor}{}^2 + p_1 \cdot n_{cor} \quad + p_0 \tag{5.34}$$

where p_0, p_1, p_2, p_3 and p_4 are the constants for the polynomial fitting. The maximum polytropic efficiency and the corresponding pressure ratio for the present corrected speed are calculated with equation (5.33) and (5.34). The actual pressure ratio supplied may not be the optimum one, and the actual efficiency can be lower. The difference in optimum pressure ratio can be expressed as a difference in optimum flow. Once this difference is known, a correction for the efficiency is made. This correction assumes a symmetrical degradation on both sides of the optimum flow. This degradation is based on a parabolic equation:

$$h = h_{\max} - c \cdot \left(m - m_{\max\_eff}\right)^2 \qquad (5.35)$$

Where c is a constant. For fitting this constant several points on the map were chosen.

5.4 Compressor model in Modelica

The compressor model belongs to the class of flow models. It uses the equations explained at the beginning of this chapter in order to evaluate the mass and energy flows. These flows provide the control volumes the information to evaluate the mass and energy balances, as it was explained in chapter [4], which was dedicated to the *ThermoFlow* library. An equation to link the thermal power to the mechanical power is also needed.

A brief explanation of the implementation of the models is given here. All the models used for the creation of the compressor are in a package called *NewCompressors*.

CorrectedMass1

It is a function used to calculate the corrected mass flow by using equation (5.28). Corrected speed and pressure ratio are the inputs of this function.

CorrectedMass2

Corrected mass flow at the surge line is calculated here by using the equation (5.32). Pressure ratio is needed as input.

P_maxeff

The value of pressure at the maximum efficiency for a given speed is calculated here by using equation (5.34).

Maxeff

The value of maximum efficiency for a given speed is calculated here by using equation (5.33).

Efficiency

The function with the degradation of the efficiency is given here (equation (5.34.)). Corrected speed and pressure ratio are the inputs and the polytropic efficiency is the output. Values obtained in *P_maxeff* and *maxeff* are used internally in this function.

CompressorMap

This class uses the functions *CorrectedMass1* and *CorrectedMass2* for evaluating the actual mass flow in the compressor. A Boolean variable called *Surge* is defined here. This variable is used to check if the compressor reaches the surge line. The difference between the corrected mass flow in the compressor and the corrected mass flow for the same pressure ratio at the surge line is defined in a variable. Hence, it is possible to know how close is the compressor to the surge line

IsentropicVariables

The class *IsentropicVariables* is a record. A record is a restricted form of class that may not have any equations. It is used for setting different variables not defined in the main model.

FlowModelBaseMD

The model called *FlowModelBaseMD* inherits the class *FlowVariablesMultiStatic*, which was already available, in the *ThermoFlow* library. This class is a shell model with two connectors for flow model. The connectors contains the following variables:

- *mass fraction for each component of the mixture*
- *pressure*
- *enthalpy*
- *mass flow for each component of the mixture*
- *energy flow*
- *density*
- *Ratio of specific heat capacities*
- *entropy*

The class connects the variables internal to the model, to the variables at the connector a and b.

PolytropicEfficiency

The class *PolytropicEfficiency* inheritances all the variables from the class *IsentropicVariables.* Polytropic efficiency is calculated In this class by using the functions *P_maxeff, Maxeff* and *Efficiency*. Once this is known, the isentropic efficiency is used to evaluate the specific work in the compressor.

Compressor

The class *Compressor* is the complete thermodynamic model. It inherits all the classes explained above.

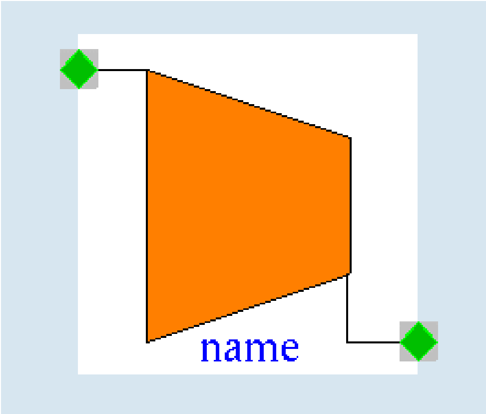

Figure 5.7 Icon of the class Compressor

CompressorMec

In the model *CompressorMec*, equation (5.26), which links the thermodynamical and mechanical behaviour, is added. A new connector for the mechanical information is also added. This connector contains the following variables:

- *Absolute rotation angle of flange*
- *Torque in the flange*

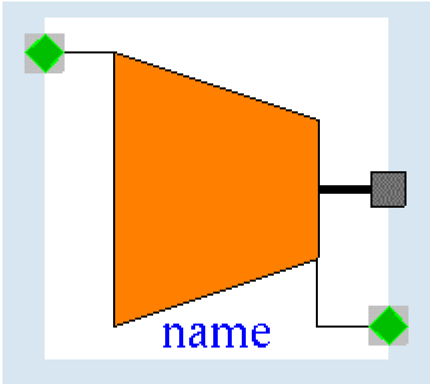

Figure 5.8 Icon of the class CompressorMec

CompleteCompressor

The last model in the package NewCompressor is the model CompleteCompressor. This model contains a parameter called *J*, which is the inertia of the compressor. The units of *J* are $Kg*m^2$. The user can modify this parameter.

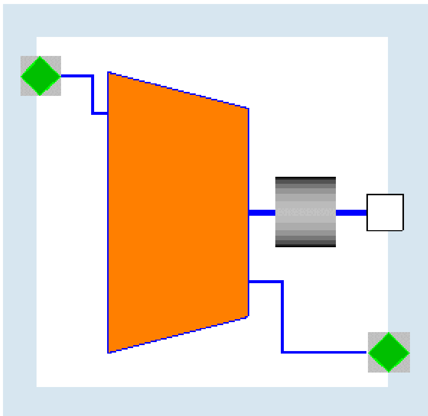

Figure 5.9 Icon of the class CompleteCompressor

6.Turbine

6.1 Governing equations

One form of the energy equation for general steady state flow is given by the following equation, *(Cohen,1996)*:

$$dq - dw - dh - cdc - gdz = 0 \qquad (6.1)$$

where the *dq* is the specific external heating, *dw* is the specific work, *dh* is the enthalpy, *cdc* is the kinetic energy term and *gdz* the potential energy term. The terms *dq* and *dz* can be neglected, because the flow is adiabatic and the height change is very small. Integrating between the 1 (inlet section of the turbine) and 2 (outlet section of the turbine), the following expression is obtained:

$$h_1 + \frac{1}{2} \cdot c_1^2 - h_2 + \frac{1}{2} \cdot c_2^2 - w = 0 \qquad (6.2)$$

Where *w* is the specific work obtained in the turbine and *c* and *h* are speed and enthalpy. The kinetic energy term at the inlet of the turbine can be neglected, because the air flow velocity at the turbine entrance is close to zero. For neglecting the value of the kinetic term at the output of the turbine, the following consideration can be taken. A value for the velocity in the output of 200 m/s has associated a specific kinetic energy of 20 kJ/kg. The specific enthalpy of the air from tables, at the temperature of 900°C is 1023.25 kJ/kg. It means that the kinetic term is 1.917 % of the enthalpy of the air. It gives us an idea about the size of both terms. After this consideration, it seems logical to neglect the kinetic terms for the dynamic model *(Philip,1999)*. Therefore, the next equation arises:

$$w = h_1 - h_2 \qquad (6.3)$$

A constant specific heat was also assumed for the turbine. A constant specific heat c_p for the model is calculated as an average for the temperatures at turbine inlet and outlet. Therefore, equation (6.3) can be rewritten using the temperatures:

$$w = c_p \cdot (T_1 - T_2) = c_p \cdot T_1 \cdot \left(1 - \frac{T_2}{T_1}\right) \qquad (6.4)$$

Using equation (5.6) of the chapter [5], it is possible to rewrite equation (6.4) as:

$$w = \frac{g}{g-1} \cdot R \cdot T_1 \cdot \left(1 - \frac{T_2}{T_1}\right) \tag{6.5}$$

If the expansion process occurs in isentropic conditions, the work obtained is the isentropic work:

$$w_s = \frac{g}{g-1} \cdot R \cdot T_1 \cdot \left(1 - \frac{T_{2s}}{T_1}\right) \tag{6.6}$$

where T_{2s} is the isentropic temperature. For an isentropic expansion, the relation between pressures and temperatures is:

$$\frac{T_1}{T_{2s}} = \left(\frac{p_1}{p_2}\right)^{\frac{(g-1)}{g}} \tag{6.7}$$

So expression (6.6) can be rewritten as:

$$w_s = \frac{g}{g-1} \cdot R \cdot T_1 \cdot \left(1 - \left(\frac{p_{2s}}{p_1}\right)^{\frac{g-1}{g}}\right) \tag{6.8}$$

The isentropic efficiency for a turbine is defined as the ratio between the actual enthalpy difference and the isentropic enthalpy difference:

$$h_s = \frac{h_2 - h_1}{h_{2s} - h_1} \tag{6.9}$$

Where h_{2s} is the enthalpy at the outlet when the process is isentropic. As it happens in the compressor, when pressure ratio changes, the isentropic efficiency does not remain constant. For the case of the turbine, h_S tends to increase when the pressure ratio grows, (*Cohen,* 1996). In order to take this into account, polytropic efficiency is introduced:

$$h_p = \frac{dw}{dw_s} = \text{constant for the whole turbine} \tag{6.10}$$

Following the same approach that was followed in chapter [5], it is possible to get a function for evaluating the isentropic efficiency as a function of the polytropic efficiency and the pressure ratio:

$$h_s = \frac{c_p \cdot T_1 \cdot \left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{(m-1)}{m}}\right)}{c_p \cdot T_1 \cdot \left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{(g-1)}{g}}\right)} \tag{6.11}$$

Where the coefficient *m* is:

$$m = \frac{g}{h_p \cdot (1-g) + g} \tag{6.12}$$

Expression (6.9) and (6.11) can be combined in order to evaluate the actual specific work in the turbine:

$$-w = h_s \cdot \frac{g}{g-1} \cdot R \cdot T_1 \cdot \left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{g-1}{g}}\right) \tag{6.13}$$

The actual power released in the expansion is the product of the actual specific work and the mass flow rate of gas:

$$P = \dot{m} \cdot w \tag{6.14}$$

For taking into account the mechanical losses, the mechanical efficiency is introduced:

$$P' = P - P_{ml} = P \cdot h_{mec} \tag{6.15}$$

where *P'* is the mechanical shaft power and P_{ml} is the term referred to mechanical losses. Then, mechanical and thermal power can be connected:

$$P' = t_{tur} w_{tur} = P \cdot h_{mec} \tag{6.17}$$

6.2 Turbine map

For the case of the compressor, the manufacturer provided a map for evaluating the mass flow and the efficiency. For turbines there are also maps but they have special characteristics. A typical map for a turbine can be seen in figure 6.1. The performance is expressed by plotting the polytropic efficiency h_p and the corrected mass flow against the pressure ratio for various values of the corrected speed.

For the turbine map, corrected flow mass is defined as:

$$m_{cor} = \frac{\dot{m} \cdot \sqrt{T_1}}{p_1} \tag{6.26}$$

And the corrected speed of rotation:

$$n_{cor} = \frac{n}{\sqrt{T_1}} \tag{6.27}$$

Where the subscript *1* is referred to the input of the turbine. The map shows the relative speed to the design value:

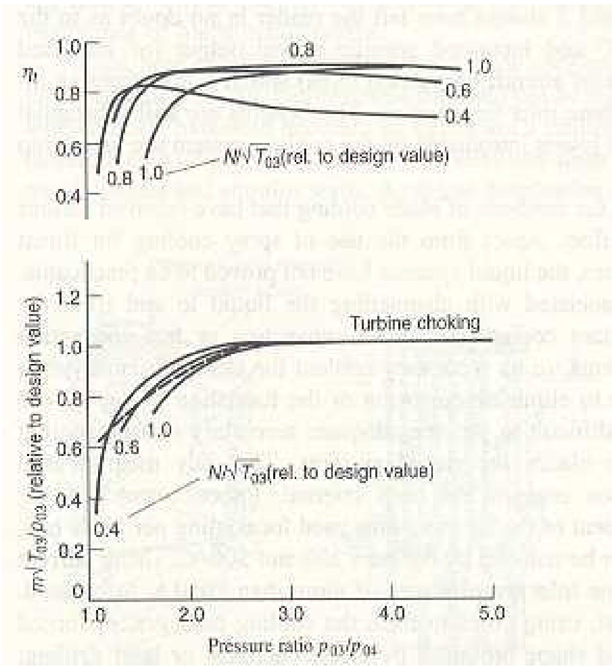

Figure 6.1 Map of a turbine (Cohen, 1996)

The efficiency plot shows that the efficiency remains constant over a wide range of corrected speeds and pressure ratios. For the case of the corrected mass flow, the maximum value of it is reached at a pressure ratio, which produces choking conditions at some point in the turbine. In this situation all the constant speed lines merge into a single horizontal line as indicated on the mass flow plot.

Taking into account that for the dynamic simulation the turbine works in choked conditions, a good approximation can be to assume that the corrected mass flow remains constant. In this way, the equation for the design of nozzles is used for evaluating the mass flow *(Cohen,1996)*:

$$\frac{m \cdot \sqrt{T_1}}{A_{thr} \cdot p_1} = \sqrt{\frac{g}{R} \cdot \left(\frac{2}{g+1}\right)^{\frac{(g+1)}{(g-1)}}} \tag{6.28}$$

Where A_{thr} is the equivalent nozzle throat area. This equation is used for dimensioning the nozzle area based on a known design point. Therefore, A_{thr} can be evaluated knowing the rest of variables at the design point. Then, this area is used to evaluate the mass flow.

The efficiency is assumed constant for all the range of speeds. The user can adjust the value of the efficiency.

6.3 Turbine model in Modelica

The turbine model implemented in Modelica is essentially a flow model. The equations described in this chapter are included in the model. The model is very similar to the compressor model, but the turbine model does not include a class for the implementation of the map. In the turbine model this map is replaced by equation (6.28), where choked conditions are assumed. The classes used for the creation of the turbine model are located in the package NewTurbines. Some of the classes in this package are described in the rest of the chapter.

FlowModelBaseDM

FlowModelBaseMD is a shell model with two connectors for flow model. The connectors contains the following variables:

- *mass fraction for each component of the mixture*
- *pressure*
- *enthalpy*
- *mass flow for each component of the mixture*
- *energy flow*
- *density*
- *Ratio of specific heat capacities*
- *entropy*

The class connects the convent variables internal to the model, to the variables at the connector a and b.

IsentropicVariables

The class *IsentropicVariables* is a record. In this record, different variables, which are necessary for the turbine model, are defined.

PolytropicEfficiency

This class includes equations for calculating the isentropic efficiency as a function of the polytropic efficiency and the pressure ratio. Isentropic efficiency then is used to evaluate the specific work obtained in the turbine.

Turbine

The *Turbine* class is the complete thermodynamic model. All the classes that were shown above are inherited from this class. Therefore, mass and energy flows are evaluated in this model.

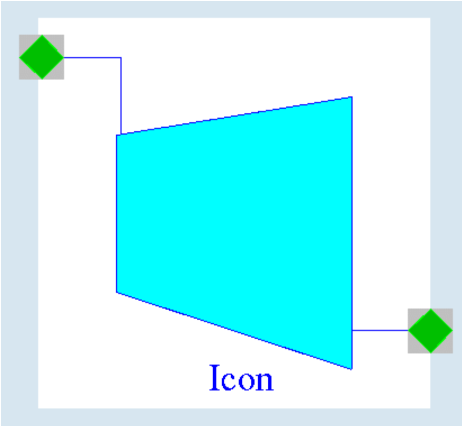

Figure 6.2 Icon of the class Turbine

TurbineMec

This class includes equation (6.17), which links the mechanical power to the thermal power. A mechanical connector is included. The mechanical connector contains the following variables:

- *Absolute rotation angle of flange*
- *Torque in the flange*

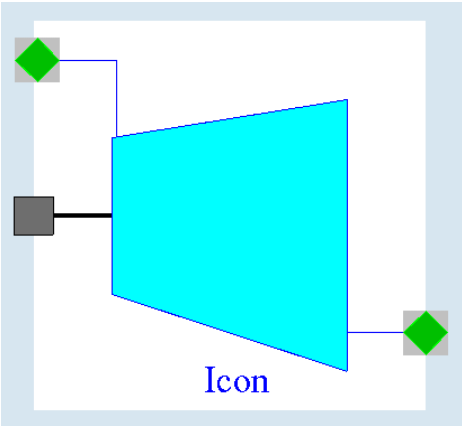

Figure 6.3 Icon of the class TurbineMec

CompleteTurbine

This model includes the model *Inertia*, which is inherited from the Rotational library of Modelica. This model is a rotational component with inertia and two rigidly connected flanges. The model includes the following equation:

$$J \cdot a = \sum_{i}^{n} t_i \tag{6.29}$$

where *J* is the moment of inertia, *a* is the angular acceleration, t is the torque and *n* is the number of mechanical connectors.

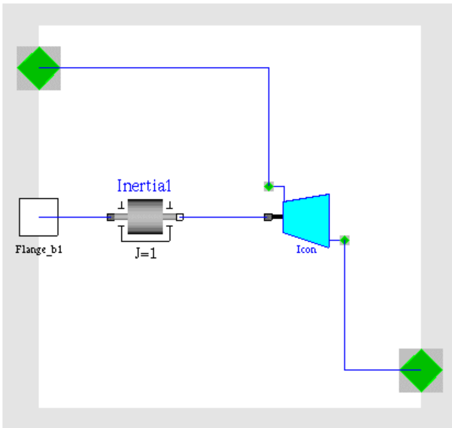

Figure 6.4 Interior of the class Complete Turbine

The moment of inertia is a parameter that can be supplied by the user.

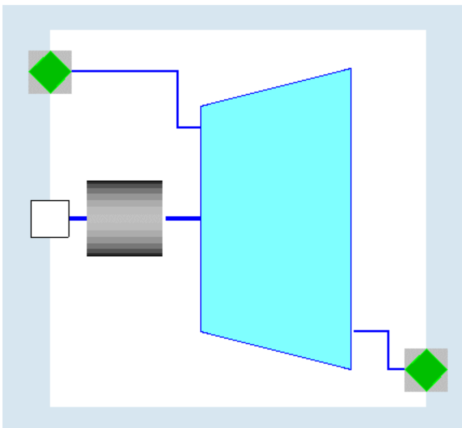

Figure 6.5 Icon of the class CompleteTurbine

7.Combustion Chamber

7.1 Introduction

Models presented in previous chapters were limited to be non-reacting thermodynamic systems. On the other hand, for the model of the combustion chamber, chemical reactions must be taken into account. In non-reacting systems just the notions of *sensible internal energy* (associated with temperature and pressure changes) and *latent internal energy* (associated to phase changes) were used. When dealing with reacting systems, it is necessary to consider the *chemical internal energy*, which is associated with the destruction and the formation of chemical bonds between the atoms.

In the combustion chamber, the chemical reaction involved is called combustion. In a combustion, some molecules are destroyed for the creation of new molecules with a release of a large amount of energy. Consequently, the energy and mass balances have to include the chemical equation of the combustion. Normally, energy and mass balances are treated in control volumes, but since chemical reactions are involved a new class had to be developed.

Some additional assumptions have to be made for the derivation of the model. These assumptions are:

- The combustion is going to be considered as instantaneous. This is very a logical consideration since the transients involved in the combustion are much faster than all other transients in the system.
- Kinetic and potential energy are going to be neglected in the energy balance.
- The efficiency of the combustion chamber is assumed as a constant parameter, i.e. the user can select the desired value before running the simulation.

7.2 Governing equations

Chemical equations govern the behaviour of the combustion chamber. In this section, equations for energy and mass balances are shown. The assumption of ideal gas properties for all components is used for calculating all the properties of the gas mixtures.

Mass balance

The air is considered as a mixture of CO_2, H_2O, N_2 and O_2. The fuel used for the turbine is natural gas. The composition of the natural gas is variable, depending on the provider. Natural gas is mainly composed of a mixture of hydrocarbon fuels, but other gases can also be found in it. For the model, natural gas is considered as a mixture of the following elements: CH_4, C_2H_6, C_3H_8, C_4H_{10}, N_2 and CO_2. From the chemical point of view, N_2, CO_2, which can be found either in the air or in the fuel, and H_2O, which can be found just in the air, are assumed to be inert. It means that they are not involved in any chemical reaction. Therefore, only hydrocarbon fuels (CH_4, C_2H_6, C_3H_8 and C_4H_{10}) react. The chemical equation for the combustion of a general hydrocarbon fuel assuming the stoichiometric amount of O_2 is:

$$C_mH_n + \left(m + \frac{n}{4}\right) \cdot O_2 \rightarrow m \cdot CO_2 + \left(\frac{n}{2}\right) \cdot H_2O \quad (7.1)$$

Where *m* and *n* depend on the kind of hydrocarbon, e.g for Methane m and n are equal to 1 and 4. Equation (7.1) implies that each kmol of C_mH_n reacts with (m+n/4) kmol of O_2, producing m kmol of CO2 and (n/2) kmol of H2O. According to this, the following expressions arise for CH_4, C_2H_6, C_3H_8 and C_4H_{10}:

$$CH_4 + 2 \cdot O_2 \rightarrow CO_2 + 2 \cdot H_2O \quad (7.2)$$

$$C_2H_6 + 3.5 \cdot O_2 \rightarrow 2 \cdot CO_2 + 3 \cdot H_2O \quad (7.3)$$

$$C_3H_8 + 5 \cdot O_2 \rightarrow 3 \cdot CO_2 + 4 \cdot H_2O \quad (7.4)$$

$$C_4H_{10} + 6.5 \cdot O_2 \rightarrow 4 \cdot CO_2 + 5 \cdot H_2O \quad (7.5)$$

Complete combustion is assumed, which means that there is not CO among the products and all hydrocarbon fuel is consumed in the reaction.

It is possible to transform expressions (7.2), (7.3), (7.4) and (7.5) into expressions for mass flow (kg/s) by using the molecular weight of the components involved in the reactions:

$$\text{molecular weight} \rightarrow w = \frac{\text{kg of component}}{\text{kmol of component}} \qquad (7.6)$$

In this way:

$$CH_4 + 2 \cdot \left(\frac{wO_2}{wCH_4} \right) \cdot O_2 \rightarrow \left(\frac{wCO_2}{wCH_4} \right) \cdot CO_2 + 2 \cdot \left(\frac{wH_2O}{wCH_4} \right) \cdot H_2O$$

(7.7)

$$C_2H_6 + 3.5 \cdot \left(\frac{wO_2}{wC_2H_6} \right) \cdot O_2 \rightarrow 2 \cdot \left(\frac{wCO_2}{wC_2H_6} \right) \cdot CO_2 + 3 \cdot \left(\frac{wH_2O}{wC_2H_6} \right) \cdot H_2O$$

(7.8)

$$C_3H_8 + 5 \cdot \left(\frac{wO_2}{wC_3H_8} \right) \cdot O_2 \rightarrow 3 \cdot \left(\frac{wCO_2}{wC_3H_8} \right) \cdot CO_2 + 4 \cdot \left(\frac{wH_2O}{wC_3H_8} \right) \cdot H_2O$$

(7.9)

$$C_4H_{10} + 6.5 \cdot \left(\frac{wO_2}{wC_4H_{10}} \right) \cdot O_2 \rightarrow 4 \cdot \left(\frac{wCO_2}{wC_4H_{10}} \right) \cdot CO_2 + 5 \cdot \left(\frac{wH_2O}{wC_4H_{10}} \right) \cdot H_2O$$

(7.10)

Where wCO_2, wCH_4, wC_2H_6, wC_3H_8, wC_4H_{10}, wH_2O and wO_2 are the molecular weighs of CO_2, CH_4, C_2H_6, C_3H_8, C_4H_{10}, H_2O and O_2 respectively.

Equations above used the soichiometric amount of O_2. If the amount of O_2 is bigger than the stoichiometric, there is O_2 in the exhausted gases. Gas turbines operate with excess of air, i.e. O_2 is not totally consumed in the reaction. Therefore, the composition of the exhausted gases for the gas turbine is a mixture of CO_2, H_2O, N_2 and O_2. Mass flows for each one of the components at the outlet of the compressor can be evaluated by using the expressions (7.7), (7.8), (7.9) and (7.10). Considering that m_{in} [CO_2], m_{in} [H_2O], m_{in} [N_2] and m_{in} [O_2], are the mass flows of CO_2, H_2O, N_2 and O_2.at the inlet of the combustion chamber, and m_{fuel} [CH_4], m_{fuel} [C_2H_6], m_{fuel} [C_3H_8], m_{fuel} [C_4H_{10}], m_{fuel} [N_2], m_{fuel} [CO_2] are the mass flows of CH_4, C_2H_6, C_3H_8, C_4H_{10}, N_2 and CO_2, the following relation can be written:

$$m_{out}[CO_2] = m_{in}[CO_2] + m_{fuel}[CH_4] \cdot \frac{wCO_2}{wCH_4} + \\ + 2 \cdot m_{fuel}[C_2H_6] \cdot \frac{wCO_2}{wC_2H_6} + 3 \cdot m_{fuel}[C_3H_8] \cdot \frac{wCO_2}{wC_3H_8} + \\ + 4 \cdot m_{fuel}[C_4H_{10}] \cdot \frac{wCO_2}{wC_4H_{10}} + m_{fuel}[CO_2] \tag{7.11}$$

$$m_{out}[H_2O] = m_{in}[H_2O] + 2 \cdot m_{fuel}[CH_4] \cdot \frac{wH_2O}{wCH_4} + \\ + 3 \cdot m_{fuel}[C_2H_6] \cdot \frac{wH_2O}{wC_2H_6} + 4 \cdot m_{fuel}[C_3H_8] \cdot \frac{wH_2O}{wC_3H_8} + \\ + 5 \cdot m_{fuel}[C_4H_{10}] \cdot \frac{wH_2O}{wC_4H_{10}} \tag{7.12}$$

$$m_{out}[N_2] = m_{in}[N_2] + m_{fuel}[N_2] \tag{7.13}$$

$$m_{out}[O_2] = m_{in}[O_2] - 2 \cdot m_{fuel}[CH_4] \cdot \frac{wO_2}{wCH_4} - \\ - 3.5 \cdot m_{fuel}[C_2H_6] \cdot \frac{wO_2}{wC_2H_6} - 5 \cdot m_{fuel}[C_3H_8] \cdot \frac{wO_2}{wC_3H_8} - \\ - 6.5 \cdot m_{fuel}[C_4H_{10}] \cdot \frac{wH_2O}{wC_4H_{10}} \tag{7.14}$$

Where m_{out} [CO_2], m_{out} [H_2O], m_{out} [N_2] and m_{out} [O_2], are the mass flows of CO_2, H_2O, N_2 and O_2.at the outlet of the combustion chamber.

Energy Balance

Once the mass balance is calculated, the next step is to evaluate the energy balance. Now it is necessary to use the concept of *enthalpy of formation*.

It is possible to establish a reference for enthalpy in the study of reactive systems by setting the value zero to the enthalpy of the *stable elements* in a state called *standard reference state* defined for T_{ref}=298.15 K and p_{ref}=1 atm. Stable elements are set to zero at the standard conditions. The term *stable* is used in the meaning chemically stable. This means that at the *standard state*, the stable forms of Nitrogen, Oxygen and Hydrogen are N_2, O_2 and H_2 and not N, O and H. Using this reference, it is possible to assign values for the enthalpy of formation of the components. The enthalpy of formation of a component at the standard reference state, is the value of the energy that is released or absorbed for the same when it is created from its primary forms (O_2, C, H_2,...), at T_{ref} and p_{ref}. If the enthalpy of formation is negative, energy is released during the creation of the component, and when it is positive, energy is absorbed during the process. If the conditions differ from the standard reference, the enthalpy of formation will decrease or increase. For example, if the temperature of the components is higher than T_{ref}, the released energy decreases because some energy is needed to raise the temperature.

Consequently, before writing the energy balance, it is necessary to express the enthalpy of a component in a form suitable for use in reacting systems. The enthalpies of formation of each of the components need to be taken into account for the energy balance in the combustion chamber. A new definition of enthalpy called total enthalpy is introduced. This enthalpy is the sum of the enthalpy of formation of the component at 25 C and 1 atm and the sensible enthalpy of the component of the component relative to the reference temperature:

$$Enthalpy = h_f^0 + \left(h - h_{\mathrm{Ref}}\right) \tag{7.15}$$

In *ThermoFlow*, zero Kelvin is chosen as the reference state for calculating the enthalpy. The expression implemented in *ThemoFlow* for calculating the sensible enthalpy is based on the NASA tables *(Gordon,1994)*.

With the enthalpy defined as above, including the enthalpy of formation, the energy balance can be evaluated. When the changes in kinetic and potential energies are negligible, the conservation of energy relation for a chemically reacting steady-flow system can be expressed in the following form *(Cengel, 1994)*:

$$\dot{Q} - \dot{W} = \sum \dot{m}_p \cdot \left(h_f^0 + \left(h - h_{\mathrm{ref}}\right)\right)_p - \sum \dot{m}_r \cdot \left(h_f^0 + \left(h - h_{\mathrm{ref}}\right)\right)_r \tag{7.16}$$

Where the subscript *p* is taken to refer products, while *r* refers to reactants. In the combustion chamber, the work term can be neglected. Then, the chemical energy released during a combustion process is either lost as heat to the surroundings or it is used internally to raise the temperature of the combustion products. In the limit case of no heat loss to the surroundings (Q=0), the temperature of the products will reach a maximum, which is called the *adiabatic flame or adiabatic combustion temperature*. In this case, the expression (16) can be rewritten as:

$$\sum \dot{m}_p \cdot \left(h_f^0 + \left(h - h_{\mathrm{ref}}\right)\right)_p = \sum \dot{m}_r \cdot \left(h_f^0 + \left(h - h_{\mathrm{ref}}\right)\right)_r \tag{7.17}$$

In gas turbines, the highest temperature to which the blades can be exposed is limited by metallurgical considerations. Therefore, the adiabatic temperature is an important factor in the design of gas turbines. The maximum temperatures which occurs in gas turbines are considerably lower than the adiabatic flame temperature due to the following reasons:

- When a combustion chamber operates with excess of air, which is the normal case, it serves as a coolant
- The combustion is usually incomplete
- Some heat loss takes place
- Some combustion gases dissociate at high temperatures

For taking into account the first three points, the combustion efficiency (η_{cc}) can be introduced, so expression (7.17) is rewritten as:

$$\sum \dot{m}_p \cdot \left(h_f^0 + \left(h - h_{\mathrm{ref}}\right)\right)_p = \eta_{cc} \cdot \left(\sum \dot{m}_r \cdot \left(h_f^0 + \left(h - h_{\mathrm{ref}}\right)\right)_r\right) \tag{7.18}$$

7.3 Combustion chamber model in Modelica

In the chapter [4], it was explained that the construction of model was based in an alternative sequence of flow models and control volumes. The mass and energy balances are calculated in the control volumes. But for the model of the combustion chamber, mass and energy balances are implemented in a flow model. For the case of the combustion chamber, equations explained above are used to evaluate the mass and energy balances. An equation for calculating the relation between the mass flow and the variation of the pressure is also needed. The combustion chamber model evaluates the mass and energy flows, which are then used in the control volume for evaluating the mass and energy balances. Temperature in the combustion chamber is obtained there.

All the models used for the creation of the combustion chamber are in the package called *CombustionChamber*.

Some of the classes included in this package are explained briefly.

MassBalance.

This class is used for evaluating the composition of each of the components of the gas mixture at the output of the combustion chamber. This class contains the equations (7.11), (7.12), (7.13) and (7.14)

Valve

This model was created for controlling the mass flow of fuel into the combustion chamber. The model *Valve* is a simple flow model, with two flow connectors and one connector with an input signal. This last connector is used to regulate the fuel flow. There is a parameter called *mdot_max*, which can be supplied by the user. This parameter is the maximum fuel flow in kg/s that the valve can supply. It is possible to regulate this fuel flow by using an input signal with a value between zero and one.

NaturalGasResS_pTX

The model simulates an infinitive reservoir of natural gas. It is a control volume in which pressure p and temperature T can be assigned. The model also includes a medium model for natural gas, which includes the components CH_4, C_2H_6, C_3H_8, C_4H_{10}, N_2 and CO_2. The user can select the natural gas fuel composition in fuel in % of mass fraction.

ThreePort

This class is essentially a three port flow model. These three connectors are used for the incoming air, fuel and exhausted gases, respectively. This class also inherits the record called *FlowVariablesSingleStatic,* which contains some necessary variables for a flow model. In this class, these variables are connected to the variables in the connectors.

CombustionChamber

This model is the real combustion model. It inherits the classes *ThreePort* and *MassBalance*. The model contains an equation that relates the mass flow *m* to the pressure drop in the combustion chamber. The equation is the following:

$$\frac{\dot{m}^2}{dp} = \frac{\dot{m}_0^{\,2}}{dp_0} \qquad (7.19)$$

Where *dp* is:

$$dp = p1 - p2 \qquad (7.20)$$

and m_0 and dp_0 are the mass flow *m* and pressure loss *dp* in the combustion chamber at the design point.

The energy balance of equation (7.18) is also included in this class.

8. Testing models of the turbine

The three main models, compressor, combustion chamber and expander, have been presented in previous chapters. However, other models are necessary for the creation of the complete dynamic model of the turbine. In this chapter, two different complete models of the turbine are presented. The first one corresponds to the turbine connected to a hydraulic brake without any controller. The second model includes a speed controller, which compares the rotational speed to a reference and regulates the fuel mass flow in order to maintain the speed constant. A description of these models, including the models that have not been presented before, is given in the chapter.

8.1 System model with hydraulic brake

The model is shown in Figure 1. It can be seen that the input for this model is the fuel mass flow. A description of the models not presented before is given in this section.

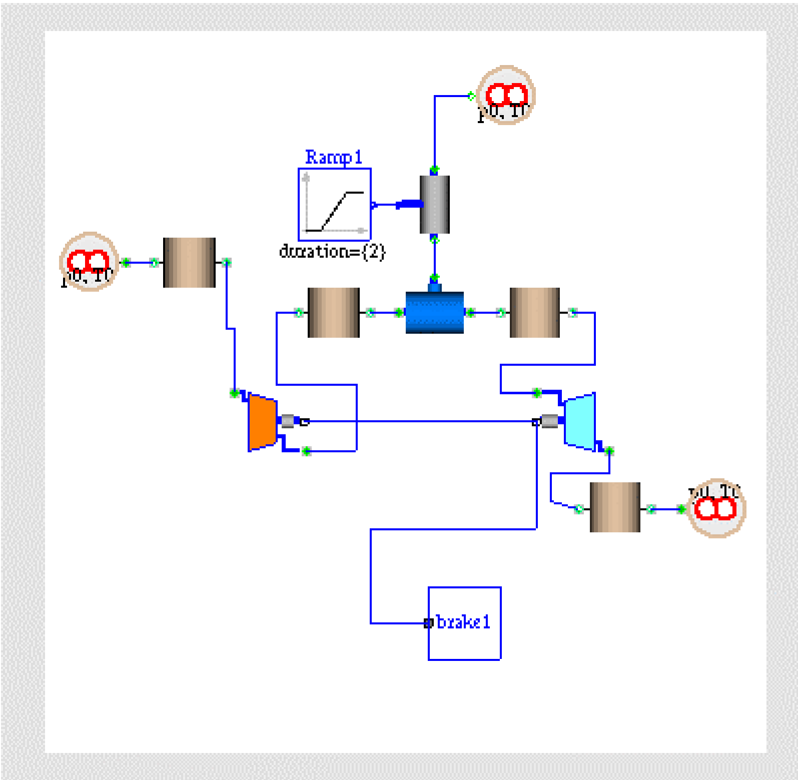

Figure 8.1 Complete model of the gas turbine in open loop

In order to define environmental conditions e.g. temperature, pressure, composition of the air, it was necessary to develop a model of the environment. In the ThermoFlow library, thermodynamic reservoirs are used for this purpose. A reservoir is considered as a control volume of infinite size, i.e. the values of the states do not change when energy and mass are introduced or extracted. In the turbine model, these states are temperature, pressure and composition of the mixture of the gases CO_2, H_2O, N_2 and O_2. These states are parameters, which can be modified by the user for each simulation. In this way, it is easy to choose different environmental conditions for the simulations. However, the compressor and the turbine are not connected directly to the environment. Pipes and filters cause a pressure drop between the atmospheric pressure and the pressures at the inlet of the compressor and at the outlet of the combustion chamber. In order to take this into account, a flow model with a simple equation for the pressure drop is added to the reservoir model. The complete model created by a reservoir and a pressure drop model is called source. Two air sources are used in the turbine model, one situated at the beginning of the cycle, i.e. before the compressor and another at the end of the cycle, i.e. after the expander.

Another reservoir is used for simulating the fuel tank. Nevertheless, the states for this reservoir are pressure, temperature, and composition of the mixture of the gases CH_4, C_2H_6, C_3H_8, C_4H_{10}, N_2 and CO_2. In this way, it is possible to set different compositions of natural gas for the experiments.

In order to extract the power obtained in the expander without having to control the process, a model of a brake is needed to dissipate the energy. A hydraulic brake model was implemented for the turbine system model. This model is based on a relation between the torque and the speed. The relation is parabolic:

$$t = c_1 + c_2 \cdot n^2$$

Where τ is the torque, n is the rotational speed and c_1 and c_2 are parameters, which can be modified by the user. In this way, the model can be simulated for different speeds by varying the values of c_1 and c_2.

The model also contains four control volumes. As it was explained in the ThermoFlow chapter, control volumes are used for evaluating the mass and energy balances. The area and length of the control volumes are parameters and have to be set by the user.

8.2 System model with speed controller

This second model can be used for reproducing situations which are closer to the actual use of the turbine in the pilot plant. This model is very similar to the model explained above, but the model of the brake is replaced by providing a torque as an input, which could e.g. come from a generator. When the value of the torque increases, the speed decreases. The value of the speed is compared to the reference speed and the controller increases the value of the fuel mass flows, in order to increase the speed until it reaches steady state. The model of the PI controller was taken from the Block sub-library. The complete model can be seen in the figure.

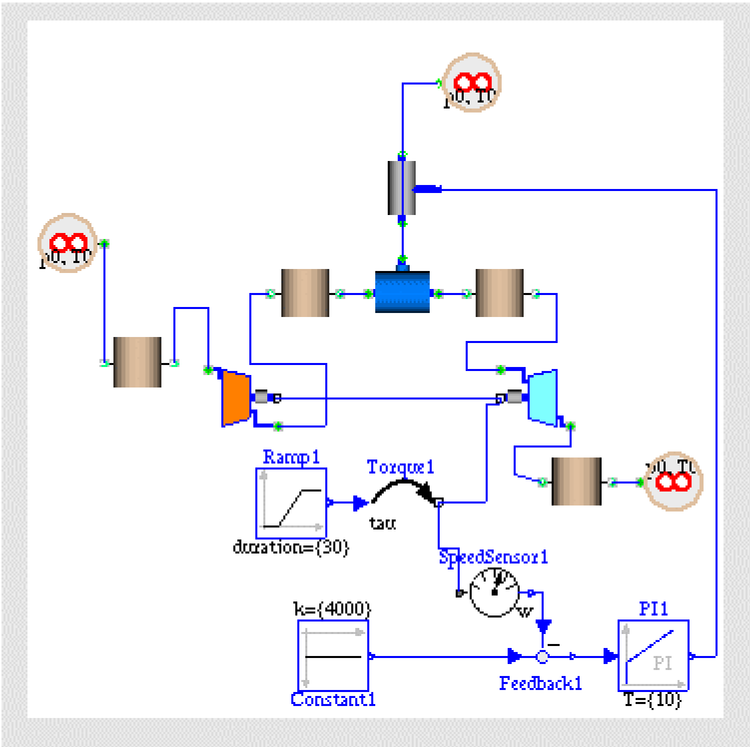

Figure 8.2 Complete model of the gas turbine with a speed controller

9.Simulation Results

The simulations were carried out for the two system models presented in the previous chapter. Results of different simulations are shown in this chapter.

9.1 Simulation in the model with hydraulic brake

This first simulation was carried out varying the fuel mass flow injected to the combustion chamber. An increase of the mass flow was simulated in the natural gas valve. Figure 9.1 shows the fuel mass flow trajectory that was given to the model in the first simulation.

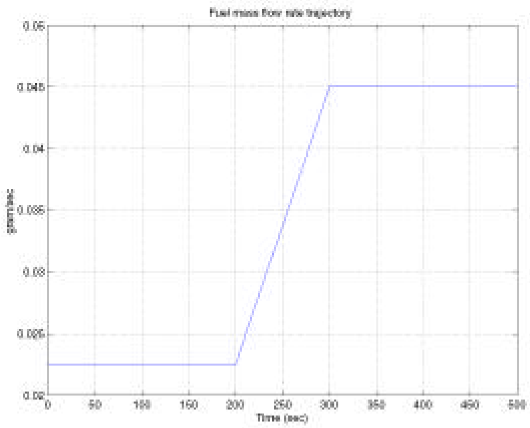

Figure 9.1 Fuel mass flow trajectory. Ramp 1

Figure 9.2 shows the polytropic and isentropic efficiencies in the compressor. During the transient, a maximum efficiency value is reached and then it decreases until it gets stable. Before the transient starts, the compressor is working in a point of the map located on the left of the line of maximum efficiency. When the mass flow increases, the pressure ratio and the mass flow increase as well, and the compressor goes to another point in the map to the right of the maximum efficiency line. Therefore, the maximum efficiency line is crossed during the transient. When it happens, the maximum value of the efficiency is reached. The isentropic efficiency is calculated as a function of the polytropic efficiency and the pressure ratio. When the pressure ratio increases, the isentropic efficiency decreases (*Cohen, 1996*). Therefore, the increase in the difference between both efficiencies can be is due to this increase in the compressor pressure ratio.

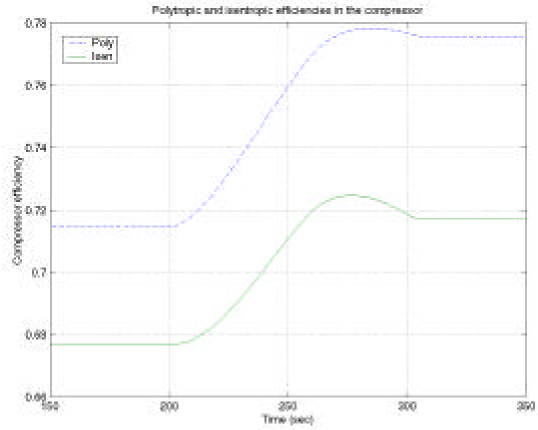

Figure 9.2 Polytropic and isentropic efficiencies in the compressor

It is important to know how close the compressor is to the surge line. In figure 9.3 it is possible to see this. The margin to surge is the difference between the corrected mass flow in the compressor and the corrected mass flow for the same pressure ratio at the surge line. Hence, figure 9.3 shows that the compressor was close to the surge line, and during the transient it moves away from the surge line.

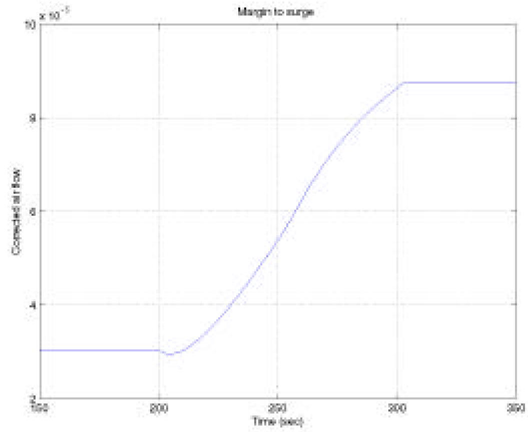

Figure 9.3 Margin to the surge line

The trajectories of the compressor mass flow and rotational speed can be seen in figure 9.4 and 9.5. Both them go up due to the increase of the fuel mass flow

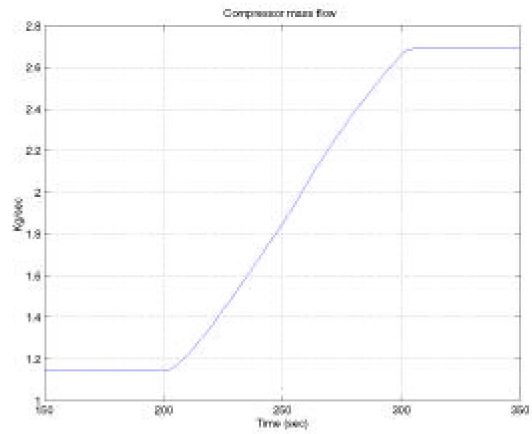

Figure 9.4 Compressor mass flow

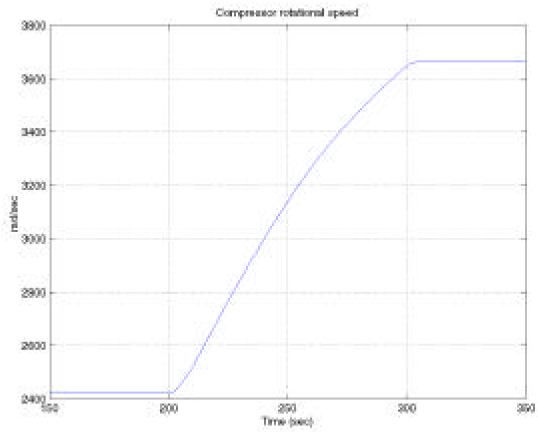

Figure 9.5 Compressor rotational speed

Figure 9.6 shows the increase of the pressure at the outlet of the compressor. The compressor receives more power from the turbine, and this is why the pressure at the outlet increases.

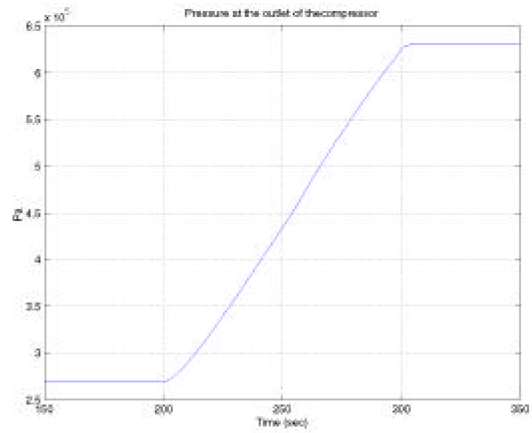

Figure 9.6 Pressure at the outlet of the compressor

The temperature at the outlet of the compressor goes up due to the increase of the pressure. It can be seen in figure 9.7. The increase of the power in the compressor is shown in figure 9.8.

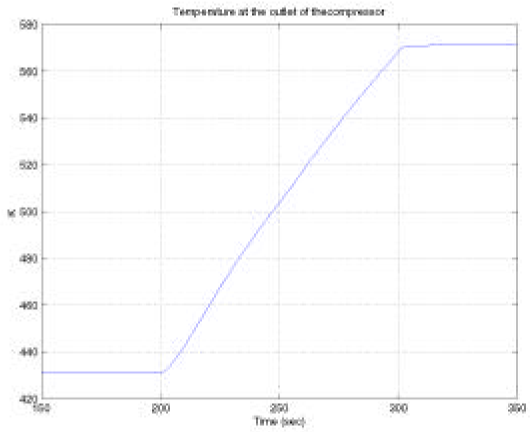

Figure 9.7 Compressor outlet temperature

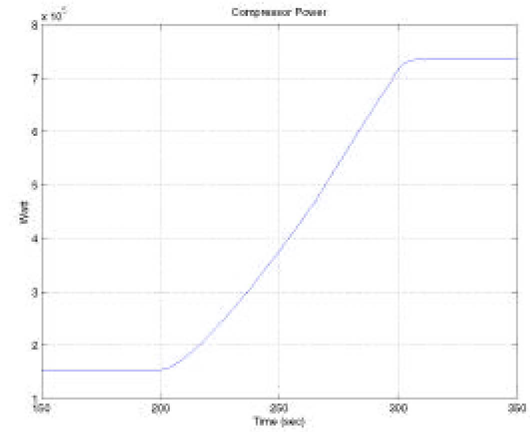

Figure 9.8 Compressor power

The air fuel ratio AF is defined as the ratio of the mass of air to the mass of fuel in the combustion chamber. It can be seen in figure 9.9. When the fuel mass flow increases, the AF in the combustion chamber decreases because of the time constants of the process. The mass flow through the compressor is delayed because of the acceleration of turbine and compressor and the dead time of the flow of hot air through the system.

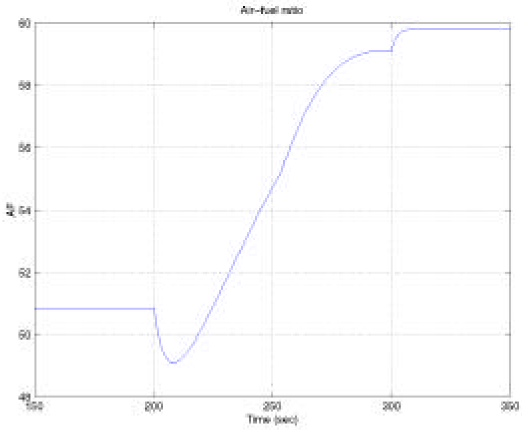

Figure 9.9 Air-fuel ratio AF

The temperature in the combustion chamber is shown in Figure 9.10. This temperature depends on the shape of the factor AF.

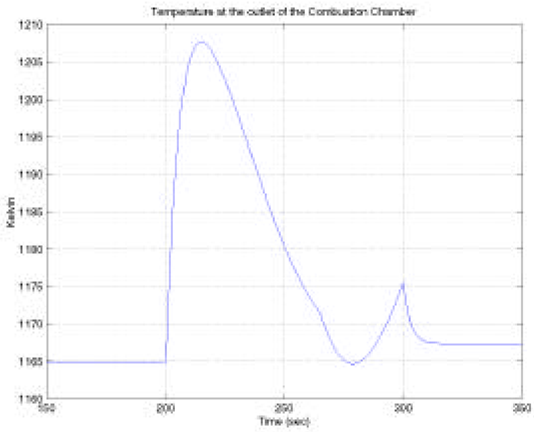

Figure 9.10 Combustion chamber temperature

Figures 9.11, 9.12, 9.13 and 9.14 show the mass flows of CO_2, H_2O, N_2 and O_2 at the inlet and at the outlet of the combustion chamber.

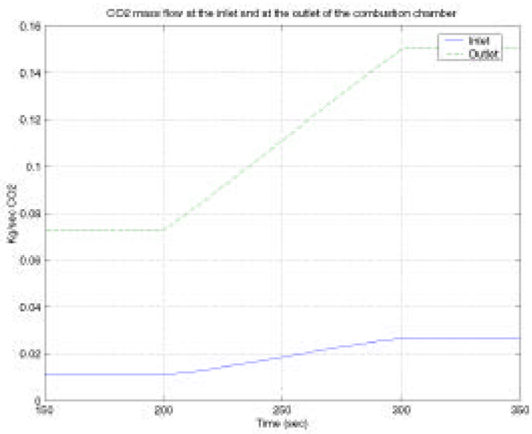

Figure 9.11 CO_2 mass flow

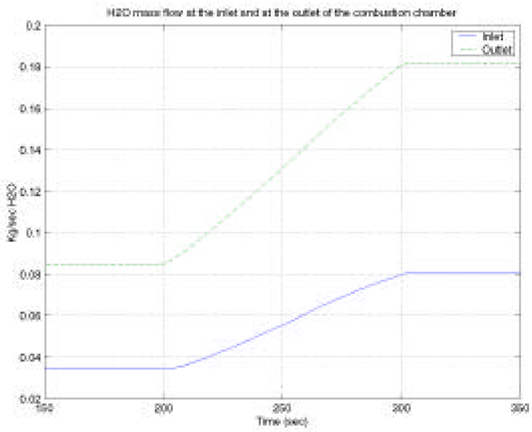

Figure 9.12 H_2O mass flow

The mass flows of CO_2 and H_2O increase at the outlet due to the combustion. Since the N_2 was assumed to be inert, the mass flow is the same at the inlet and at the outlet. However, the mass flow of O_2 decreases because it is consumed in the reaction.

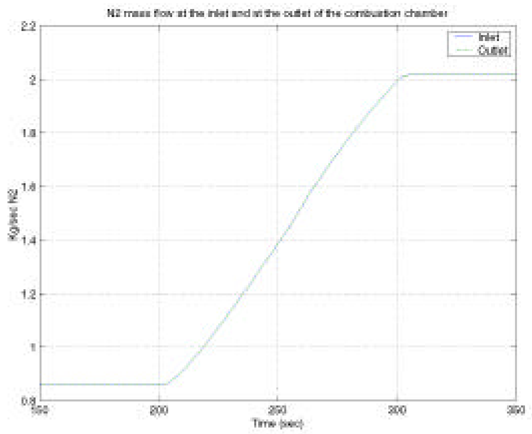

Figure 9.13 N_2 mass flow

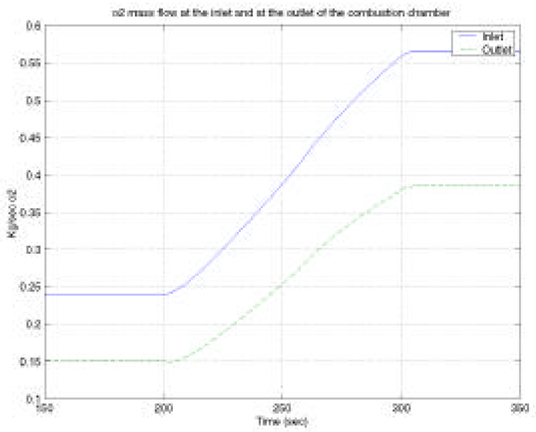

Figure 9.14 O_2 mass flow

It should be noted that at the beginning of the transient, the mass flow of O2 decreases. When the transient starts, the fuel mass flow increases and more O2 is needed for reacting with the fuel. After a while, the amount of air flowing through the compressor increases so the O2 at the outlet of the combustion chamber increases as well.

Figure 9.15 shows the temperatures at the inlet and the outlet of the turbine. The difference between both temperatures increases due to the increase in the pressure ratio and the increase of the efficiency in the turbine.

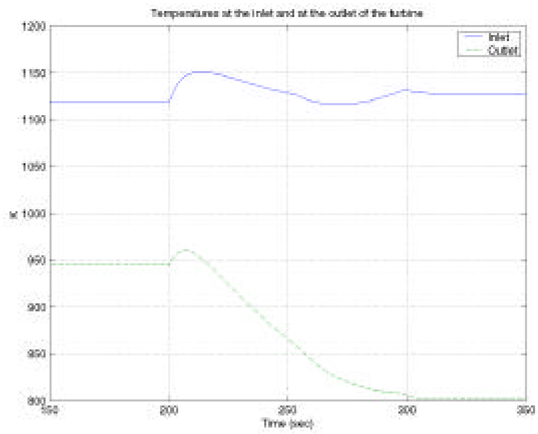

Figure 9.15 Temperature at the inlet and aoutlet of the turbine

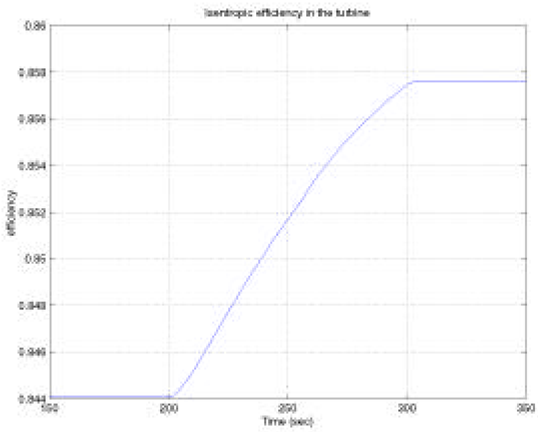

Figure 9.16 Isentropic efficiency in the turbine

Figure 9.16 shows the increase of the isentropic efficiency due to the increase of the pressure ratio *(Cohen,1996)*. Polytropic efficiency was assumed constant in the turbine.

The power of the turbine is shown in Figure 9.17. Comparing to the power of the compressor, it yields that around the 68% of the power goes to the compressor, and the rest goes to the brake.

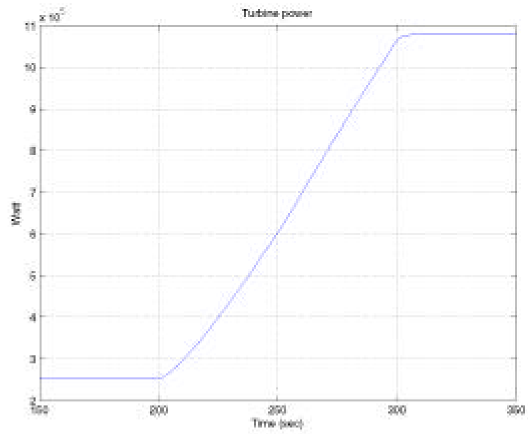

Figure 9.17 Turbine power

A different trajectory for the fuel gas was introduced to the system model in a different simulation. The values of the fuel mass flow at the beginning and at the end of the ramp were the same than in the first ramp but this ramp was faster than the previous. The duration of the ramp was 10 sec, so ten times shorter than the first one. The ramp can be observed in Figure 9.18.

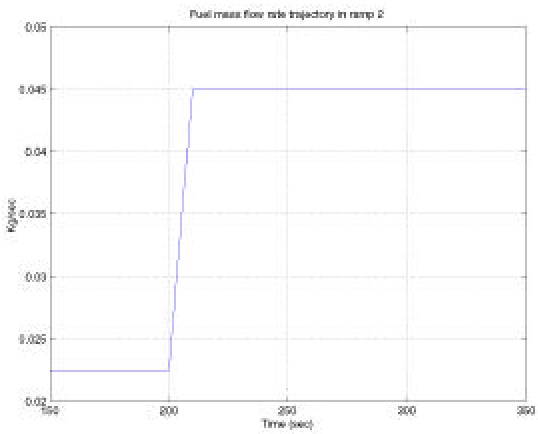

Figure 9.18 Fuel mass flow trajectory. Ramp2

Temperatures at the combustion chamber for ramp 1 and ramp 2 are compared in Figure 9.19. It is possible to see a higher increase in the temperature obtained with the faster ramp (ramp 2). After a while, both temperatures reach the same value. Therefore, the same increase in the fuel mass flow can yield different transient temperatures in the turbine. This can not be reproduced by using a static model.

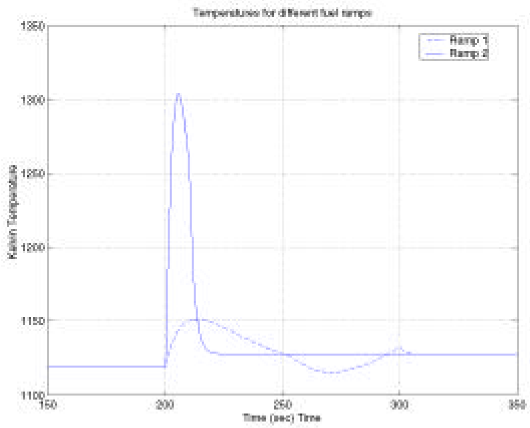

Figure 9.19 Comparation of temperature for Ramp 1 and ramp 2

9.2 Simulation of the Model with speed controller

A power ramp was simulated for the model with the speed controller, by using a torque as an input. Some results were compared to measurements carried out in the real plant in 1997. The results of these experiments can be found in (*Lindquist, 1999)*. Figure 9.20 shows the torque introduced to the model. It goes from zero, idle conditions, to the maximum torque, 119 Nm. Assuming that the rotational speed is constant and with a value of 4020 rpm, the maximum torque corresponds to a power output of 480 KW.

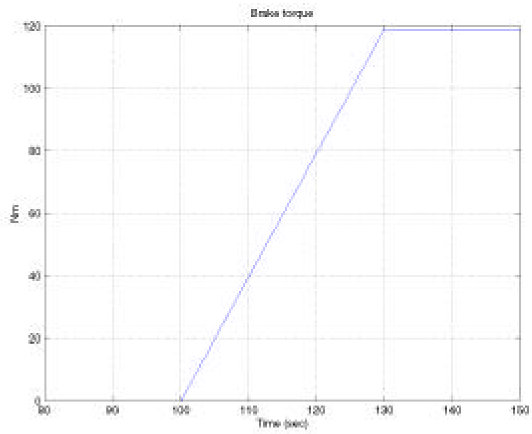

Figure 9.20 Torque absorbed by the brake

The air mass flow in the compressor is shown in Figure 9.21. In the simulation, the value of the mass flow is 3.436 kg/s at idle and 3.33 kg/s at 480 KW. The actual results in experiments gave a result of 3.45 kg/s at idle and 3.41 kg/s at 480, so the maximum error is 2.3%.

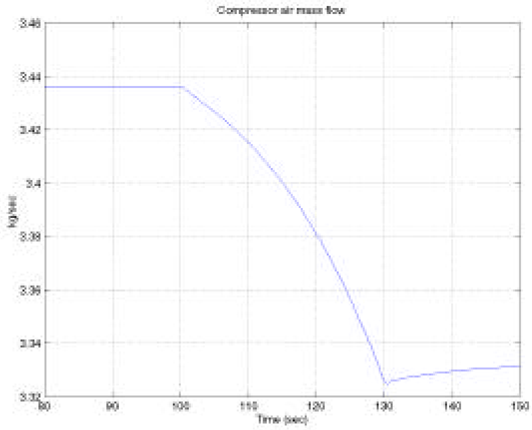

Figure 9.21 Compressor mass flow

Figure 9.22 shows the pressure at the outlet of the compressor. The results in the simulation are 7.35 Bar and 8.3 bar at idle and at 480 KW respectively. The actual results obtained in the experiments are 7.3 bar and 8.2 bar. The maximum error is 1.2%.

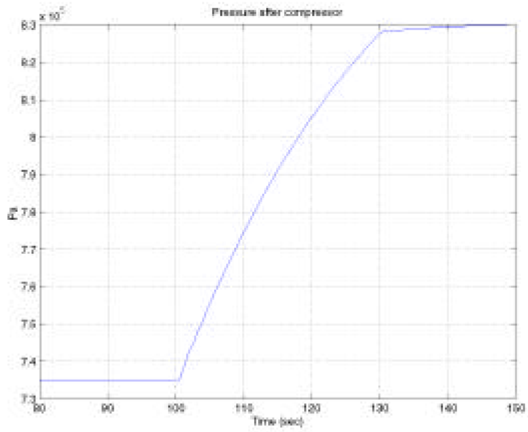

Figure 9.22 Pressure at the outlet of the compressor

Figures 9.23 shows the temperature at the outlet of the compressor. The actual temperature is around 600 K over the whole power output range. In the simulation, the value of the temperature varies slightly from 616 K at idle to 626 at 480 KW. The maximum error comparing to actual data is 4.3%.

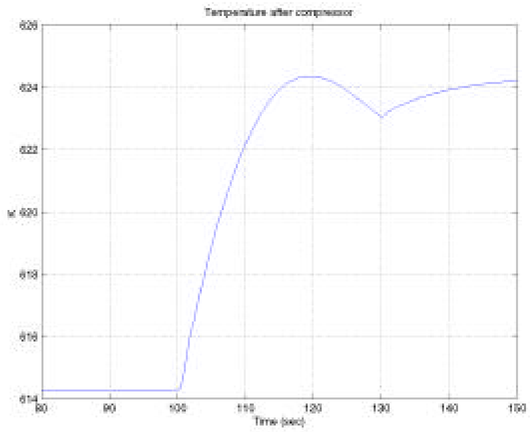

Figure 9.23 Temperature at the outlet of the compressor

The expander outlet temperature is shown in Figure 9.24. The results in the simulation are 626 K and 828 K at idle and at 480 KW respectively. The actual result goes from 623.5 K to 773 k. The maximum error is reached at 480 KW, and its value is 7.3%.

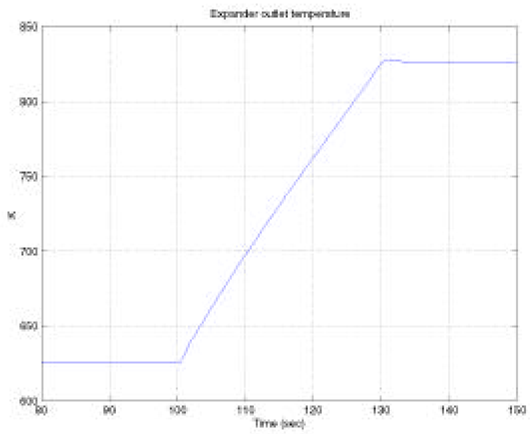

Figure 9.24 Temperature at the outlet of the turbine

10. Conclusions and future work

10.1 Conclusions

Some of the conclusions obtained at the end of the work are:

- The system turbine model reproduces well the dynamic behaviour of a real plant. The results obtained when the model reaches stationary conditions are very close to the results obtained in the experiments carried out in the real plant. On the other hand, there was no data for comparing to some results obtained simulating transients. However, the results were discussed with people of the Heat and Power Department, concluding that they results were close to reality
- The model can be reused for different plants by changing some of the classes. The model developed is general, with the exception of the compressor map and the equation for the expander.
- Some of the models can be used as new components of the ThermoFlow library, continuing in this way the development of it.

10.2 Future work

The model developed and the results obtained were very interesting, but improvements and extensions should be done. Better maps for compressor and turbines can replace the present ones in order to simulate different transients, e.g. a start-up of the plant. Models for the regenerative heat exchanger and evaporative tower can be developed and added to the model in order to simulate the complete plant.

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