Investigating the performance of fundamentally-weighted portfolios

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Abstract

Market indices based on market capitalization have been argued to be the most mean-variance efficient for a long time. In recent years, this argument has been questioned and other methods of weighting portfolios have been suggested. One of these is the weighting by fundamental indexation. This thesis investigates whether fundamentally-weighted portfolios outperform portfolios weighted by market capitalization. For this purpose, four different fundamentally-weighted portfolios are constructed and compared to a composite portfolio, an equally weighted portfolio and a portfolio based on market capitalization. These portfolios consist of the 100 largest stocks from the Swedish stock market and their performance is studied over a period of eight years. The results show that the Equal Weight portfolio outperforms the fundamentally-weighted portfolios and the Market Capitalization portfolio. Thereafter, the Sharpe ratio is compared to different performance measures to see if they suggest a different ranking of the portfolios. Moreover, the Atkinson index, which is originally a measure of income inequality, is introduced. The Atkinson index serves as a measure of financial risk and is in turn also converted into a performance measure. Despite that it was possible to show that the portfolio returns do not follow a normal distribution, the comparisons of the different performance measures did not lead to any substantial difference in the ranking of the portfolios.

Contents

1 Introduction

This section provides an introduction to the thesis and to this specific research area in finance. The background of different indexation methods and performance measures will be introduced. Then the aim and purpose of this thesis will be presented. Finally, the disposition of the study is highlighted.

1.1 Background

Investing in market indices is common and major stock indices, such as the OMXS30 and S&P 500, are weighted according to market capitalization. This kind of market indices are argued to be the most mean-variance efficient by the capital asset pricing model (CAPM) (Bodie, Kane & Marcus, 2014, p. 291-299). This model shows that every investor should invest in a market portfolio, which consists of all the existing assets in the market, and is defined as a portfolio where each asset appears in proportion equal to the ratio of its market value to the total market value of all assets combined (Danthine & Donaldson, 2015, p. 211). Since the model shows that it is rather difficult to include every existing asset in a portfolio, a market index such as the ones mentioned above, is said to be a good approximation of a market portfolio (Bodie et al., 2014, p. 293).

However, in recent years studies have shown that the portfolio constructed with weights according to market capitalization is not the most mean-variance efficient investment. It has been demonstrated that portfolios based on different weights perform better. Arnott, Hsu and Moore (2005) suggest an approach of constructing portfolios according to fundamental indexation. This method involves assigning weights to stocks according to fundamental values of companies. They show that the method of fundamental indexation is more mean-variance efficient and results in significantly better performance compared to the market capitalization method. Moreover, it has been argued that weighting stocks according to market capitalization results in investing more in overvalued stocks and simultaneously, less in undervalued stocks (Hsu, 2006). This has been explained to be due to the price inefficiency in the market (Hsu, 2006).

It is therefore of interest to investigate if portfolios based on fundamental weights outperform portfolios which are constructed according to market capitalization and this thesis will apply this investigation to the Swedish stock market.

1.2 Aim and purpose

This thesis has two main objectives which are:

1. To see if portfolios created according to the method of fundamental indexation outperform the portfolios which are created according to the method of market capitalization. This is done according to the methodology that Arnott, Hsu and Moore (2005) have laid out.
2. To show whether the choice of performance measure results in a different ranking of portfolios. This is done by comparing standard performance measures to more advanced ones. A particularly interesting aspect is that a new performance measure developed by Fischer and Lundtofte (2018) will be included in this evaluation.

1.3 Disposition

The disposition of this thesis is as follows. First, a literature review of previous research will be presented in section 2. In order to be able to understand and discuss different methods, section 3 will serve as a theoretical background of the subject. Then the research design, including the collection of data and the construction of the different portfolios, will be presented in section 4. The results will then be analyzed and discussed in section 5, together with comparisons with previous studies. Finally, section 6 provides a conclusion based on the results from this study. This last section also includes a short explanation of some limitations of this study and furthermore a suggestion for further research.

2 Literature review

This section will present the previous research regarding the method of constructing portfolios by weighting according to market capitalization, fundamental indexation and equal weighting.

Portfolio construction based on market capitalization has been the main strategy for a very long time in the financial world and stock indices such as S&P 500 and the OMXS30 are weighted according to this method. This method of weighting stocks has been regarded to be the most efficient from a mean-variance perspective (Bodie et al., 2014, p. 291-299). Additionally a passive strategy, such as this, has the benefit of not requiring any active rebalancing of weights and thus has low transaction costs (Hsu, 2006). However, studies have shown that there are other possible methods to construct market indices that are in fact more mean-variance efficient.

Hsu (2006) argues that market capitalization weighted indices are sub-optimal due to price inefficiency. The author explains that giving stocks a weight which is proportional to their market capitalization, tends to underestimate the value of stocks which have a low market capitalization. Hsu (2006) furthermore shows that this leads to an underperformance of the portfolios created by this method. This has also been demonstrated in an article by Hsu and Campollo (2006), where they show that one of the major flaws of market capitalization indices is that the overvalued companies are given extra weight when constructing portfolios, in comparison to the companies that are undervalued.

Arnott et al. (2005) provide an alternative approach and show that it is possible to construct market indices that are more mean-variance efficient than those which are based on market capitalization. According to the fundamental indexation proposed by Arnott et al. (2005), it is beneficial to create market indices where different measures of company size is used in order to construct the portfolio weights. The authors show that this method of choosing weights to assign to the stocks, which are included in the portfolio, results in being more mean-variance efficient than the market capitalization weighting method. They do this by weighting market indices by fundamental metrics such as book value, cash flow, gross dividends, revenues, sales and total employment of companies. Their findings show that the indices created by fundamental indexation results in significantly better performance. Additionally, it is presented that the fundamental indices have on average 2.15 % higher annual returns as compared to the market capitalization index. Tamura and Shimizu (2005) have also studied the performance of fundamental indices and constructed these according to the method by Arnott et al. (2005). Instead of studying the market in the United States, they chose to apply their research to the world market. In accordance with the study by Arnott et al. (2005), their results showed that the fundamental indices outperform the indices based on the market capitalization for all the countries included in their study.

Although several studies have shown that portfolios created with the help of fundamental indexation outperform the market capitalization portfolio, other find the theory flawed. Kaplan (2008) demonstrates that the conditions that need to hold in order for the fundamental indexation to outperform the market capitalization index are not always fulfilled, and concludes that the better performance it not always shown. The suggestion that weighting the stocks by the companies market capitalization results in investing more in overvalued stocks and less in undervalued stocks is called the noisy market hypothesis (Perold, 2007). The author demonstrates that noisy market prices do not lead to an performance drag, as it is being argued by Hsu (2006).

Another approach for creating indices and portfolios, which can be more efficient than the market capitalization portfolio, is the so called equal weighting strategy[1]. It has

[1]Equal weighting is also known as naive diversification.

been shown that equal weighted portfolios perform better than the market capitalization portfolios (Plyakha, Uppal & Vilkov, 2012). In their article, the authors showed that an equal weighted portfolio yielded both a higher Sharpe ratio, and a higher certainty equivalent return. They also showed that this portfolio in fact had a higher standard deviation and a higher kurtosis, characteristics which are seen as less favorable. Interestingly, the equal weighted portfolio was still able to perform better.

While previous research has shown that equal weighted portfolios perform better than the market capitalization portfolios, there exist arguments that this is not the case and that the opposite is true (Kritzman, Page & Turkington, 2010). The authors explain that the construction of an equal weighted portfolio implies that there is no investment knowledge present. In their article, they show that equal weighted portfolios do not have superior performance over market capitalization portfolios.

In conclusion of this literature review, it is obvious that different ideas and arguments about the different methods of indexation exist. Studies have shown that portfolios based on market capitalization can be outperformed by portfolios based on fundamental metrics and portfolios based on the simple method of equal weighting. On the other hand, these results have also been contradicted. This motivates for further research in the subject.

3 Theoretical background

This section consists of explanations behind the theories that will be used in this study. First, the capital asset pricing model will be described. Standard performance characteristics, such as the mean return and standard deviation, will be presented together with the Sharpe ratio, which is the most commonly used performance measure for financial assets. This is followed by the theory of a normal distribution, which will be shown to be a quite important aspect to consider in this study. Then other performance measures will be explained and finally, the Atkinson index will be introduced together with the theory of certainty equivalent rate of return, utility levels and risk aversion.

3.1 Capital asset pricing model

The capital asset pricing model (CAPM) is based on the theory of equilibrium[2] in a market and is a central model in financial economics (Danthine & Donaldson, 2015, p. 209-210). The model was published by William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) and builds on the foundation of the modern portfolio theory developed by Harry Markowitz (Bodie et al., 2014, p. 291). As it is recognized to be difficult to construct models without making simplifying assumptions, it is needless to say that the CAPM too makes some assumptions. The model assumes that every investor has the same expectations about future returns (Danthine & Donaldson, 2015, p. 210). A second assumption is that there exists a risk-free asset on the market, in which investors can invest and receive a safe return (Danthine & Donaldson, 2015, p. 210). Moreover, it is

[2]The state of equilibrium in a market requires that demand equals supply and that this takes place at specific equilibrium price and quantity. The state of equilibrium in this theory, thus, makes the assumption that the supply and demand of existing assets are equal and, in turn, that the observed prices of assets are the equilibrium prices (Danthine & Donaldson, 2015, p. 210).

assumed that investors can borrow and lend this risk-free asset without any limitation[3] (Danthine & Donaldson, 2015, p. 210).

The model consists of a set of predictions about the relationship between the risk of an asset and its expected return in equilibrium. In his original article, Sharpe (1964) argues that the systematic risk is the only risk component which is relevant when predicting this relationship. The explanation to this is that the non-systematic risk can be diversified away and thereby avoided. As a result, it will not have any relevant influence on the assets price. It is thus shown that the investor is only rewarded with a risk premium for the systematic risk in this model.

The model shows that every investor is holding the same risky portfolio which in turn is shown to be equivalent to the market portfolio[4] (Bodie et al., 2014, p. 291-293). These results are derived from the assumptions previously mentioned. It is demonstrated that the risky portfolio includes all the risky assets in the market[5] and that the weight of every asset equals the ratio of the market value of that asset to the market value of all assets combined. Therefore, it is easy to see that this risky portfolio is equivalent to the market portfolio, which is defined as a portfolio consisting of all assets, where each asset appears in proportion equal to the ratio of its market value to the total market capitalization.

Moreover, it is shown that the market portfolio is efficient because it is located on the efficient frontier and as a result, is defined as a efficient mean-variance portfolio (Bodie et al., 2014, p. 292).

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[3]In a state of equilibrium, these two will cancel each other out (Bodie et al., 2014, p. 294).

[4]The assumptions made in the model imply that the mean-variance efficient frontier is the same for every investor and that every investor has the same optimal portfolio which consists of the risk-free asset and the tangency portfolio (Bodie et al., 2014, p. 292).

[5]This has to do with the definition of equilibrium. If there for instance would be an asset in the market that is not included in the risky portfolio, the interpretation would be that there is no demand for it. However, since the asset is on the market, and thus in positive supply, this would contradict the implication of market equilibrium. This does therefore not hold true, by the assumption of the model (Danthine & Donaldson, 2015, p. 211).

For efficient portfolios, the linear relationship in equation (1) holds.

$$E[r_p] = r_f + \frac{E[r_M] - r_f}{\sigma_M}\sigma_p \tag{1}$$

Where $(E[r_M] - r_f)/\sigma_M$ is the slope of the capital market line and denotes the reward of risk taking (Bodie et al., 2014, p. 292). This can be used to determine the expected return for any individual asset i as shown by equation (2).

$$E[r_i] = r_f + (E[r_M] - r_f)\frac{\sigma_{iM}}{\sigma_M^2} \tag{2}$$

Where σ_{iM}/σ_M^2 is defined as the parameter β_i in CAPM[6]. This ratio measures the covariance between the return on asset i and the returns on the market portfolio divided by the variance of the market returns. This expression can be rearranged in the following way.

$$E[r_i] = r_f + \left(\frac{E[r_M] - r_f}{\sigma_M}\right)\beta_i\sigma_M = r_f + \left(\frac{E[r_M] - r_f}{\sigma_M}\right)\rho_{iM}\sigma_i \tag{3}$$

What is interesting to note in equation (3), is that it is only the fraction of ρ_{iM} of the total risk of an asset, σ_i, which is rewarded. This is in line with the suggestion made by Sharpe (1964) that a part[7] of the risk is diversified away when including asset i into the market portfolio. The intuitive reasoning behind this is that, since all investors hold the market portfolio with a risk of σ_M, they are solely interested in how the risk of asset i influences the risk of the market portfolio, σ_M. This is measured by $\rho_{iM}\sigma_i$[8]. Note that

[6]It should be noted that this definition of β means that it is calculated as a weighted average where the weights correspond to the asset weights in the specific portfolio (Bodie et al., 2014, p. 302).

[7]More precisely, $(1 - \rho_{iM}\sigma_i)$ is the part which is diversified away.

[8]This is equivalent to $\beta_i\sigma_M$.

this is referred to as the marginal contribution of risk by asset i to σ_M, while the β_i of the same asset is referred to as the assets systematic risk measure. By definition, the efficient portfolio will have $\rho_{iM} = 1$ since the total risk and the systematic risk is equivalent for this portfolio [9].

Finally, equation (3) can be rearranged as follows.

$$E[r_i] = r_f + \beta_i(E[r_M] - r_f) \Longrightarrow E[r_i] - r_f = \beta_i(E[r_M] - r_f) \tag{4}$$

This is the expected-return beta relationship. As this relationship is said to hold for any individual asset it also holds for any combination of assets and can therefore be used to calculate the expected return for the overall portfolio (Bodie et al., 2014, p. 297). Accordingly, the portfolio beta can be calculated using equation (5).

$$E[r_p] = r_f + \beta_p(E[r_M] - r_f) \tag{5}$$

3.2 Evaluation of portfolio returns in the mean-variance framework

One of the most basic ways of evaluating the performance of a portfolio is to compute its mean return. There are two different definitions of mean return, namely the geometric ($E_g[r]$) and arithmetic mean ($E_a[r]$), see equations (6) and (7) in which t represents a time period and r_t denotes the asset return in that period.

$$E_g[r] = \sqrt[T]{\prod_{t=1}^{T}(1 + r_t)} - 1 \tag{6}$$

[9]The market portfolio includes all assets and can therefore not be diversified any further.

$$E_a[r] = \frac{1}{T}\sum_{t=1}^{T} r_t \tag{7}$$

The geometric mean takes into account that the return of an asset is a multiplicative process and thus that compounding occurs between periods (Bodie et al., 2014, p. 130-132). Consequently, it calculates the average compound rate over the period. The arithmetic mean on the other hand is simply an average of the returns. It is commonly used to estimate the expected future return of assets but does not consider the compounding between the periods (Bodie et al., 2014, p. 130-132).

The variance of asset returns measures how much the return deviates from the mean return. The arithmetic variance of returns is defined in equation (8). In order to represent the variance with the same units as the expected returns the standard deviation is instead often used which is equal to the square root of the variance as $\sigma_a = \sqrt{\sigma_a^2}$.

$$\sigma_a^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - E_a[r])^2 \tag{8}$$

The geometric variance of returns can be derived analogously to the geometric mean. First equation (6) can be rearranged as.

$$E_g[r] + 1 = \sqrt[T]{\prod_{t=1}^{T}(1 + r_t)} \tag{9}$$

Then taking the natural logarithm of both sides yields.

$$\ln\{E_g[r]+1\} = \ln\left\{\sqrt[T]{\prod_{t=1}^{T}(1+r_t)}\right\} = \ln\left\{[(r_1+1)\cdot(r_2+1)\cdots(r_T+1)]^{\frac{1}{T}}\right\} \qquad (10)$$

But because $\ln(x_1 x_2)^{x_3} = x_3(\ln x_1 + \ln x_2)$, the right side of equation (10) can be simplified as.

$$\ln\{E_g[r]+1\} = \frac{1}{T}[\ln(1+r_1)+\cdots+\ln(1+r_T)] = \frac{1}{T}\sum_{t=1}^{T}\ln\{r_t+1\} \qquad (11)$$

It can be seen that the right side of this equation is the arithmetic mean of the sample: $\ln(1+r_1)+\ln(1+r_2)+\cdots+\ln(1+r_n)$. Therefore the arithmetic variance can be used as.

$$\ln\{\sigma_g^2+1\} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\ln\{r_t+1\} - \ln\left\{E_g[r]+1\right\}\right)^2 \qquad (12)$$

And it follows that the geometric variance can be calculated as.

$$\sigma_g^2 = \exp\left\{\frac{1}{T-1}\sum_{t=1}^{T}\left(\ln\{r_t+1\} - \ln\left\{E_g[r]+1\right\}\right)^2\right\} - 1 \qquad (13)$$

The Sharpe ratio is commonly used to evaluate the performance of financial assets and can in this study be applied to measure portfolio performance (Bodie et al., 2014, p. 134). The Sharpe ratio divides average portfolio excess return over the sample period by the standard deviation of returns over that same period (Bodie et al., 2014, p. 840). Hence, the measure presents the trade-off between the reward of holding a certain portfolio and the portfolios total volatility, which denotes the standard deviation of the portfolio. The Sharpe ratio is given by equations (14) and (15) where the difference is in whether the geometric or arithmetic expected return is used.

$$\mathrm{SR}_a = \frac{\left(E_a[r] - E_a[r_f]\right)}{\sigma_a} \tag{14}$$

$$\mathrm{SR}_g = \frac{\left(E_g[r] - E_g[r_f]\right)}{\sigma_g} \tag{15}$$

The numerator in these equations, see equation (16), is called the expected excess return ($E[R]$) and yields the expected return compared to the risk free rate r_f, where the risk free rate is defined in table 1.

$$E[R] = E[r] - E[r_f] \tag{16}$$

The distribution of the Sharpe ratio can be estimated, and is done so to see if the ratios for the different portfolios are statistically distinguished from each other (Fischer & Lundtofte, 2018). The 95 % confidence interval ($\mathrm{CI}_{\mathrm{SR}}$) can be determined according to equation (17) where SE is the standard error according to equation (18).

$$\mathrm{CI}_{\mathrm{SR}} = \mathrm{SR} \pm 1.96\mathrm{SE} \tag{17}$$

$$\mathrm{SE} = \sqrt{\frac{1}{T}\left[1 + \frac{1}{2}\mathrm{SR}^2 - \mathrm{Skewness} \cdot \mathrm{SR} + \frac{1}{4}\left(\mathrm{Kurtosis} - 3\right)\mathrm{SR}^2\right]} \tag{18}$$

While the calculation of the Sharpe ratio can seem quite simple and easy to interpret as the computation results in one single number, the ratio has got some limitations. The performance measure is based on the mean-variance model, which assumes that the mean and standard deviation of returns are adequate when evaluating portfolio performance (Sharpe, 1994). In order words, it is assumed that the higher moments are not needed to be taken into account. Furthermore, under the assumption that the portfolio returns are normally distributed, the Sharpe ratio is considered to be an accurate measure of the performance of a portfolio (Bodie et al., 2014, p. 137). In the following section, the normal distribution will be explained and portfolio characteristics such as skewness and kurtosis will be presented.

3.3 Deviations from normality

The normal distribution ($\mathrm{nd}(r_t \mid E_a[r], \sigma_a^2)$) of returns is described by the mean return ($E_a[r]$) and the variance (σ_a^2). It is defined in equation (19) and it can be seen that the normal distribution is symmetric around its mean, implying that the probability and extent of deviation from the mean is equal in both directions (Bodie et al., 2014, p. 136).

$$\mathrm{nd}(r_t \mid E_a[r], \sigma_a^2) = \frac{1}{2\pi\sigma_a^2} e^{\frac{(r_t - E_a[r])^2}{2\sigma_a^2}} \tag{19}$$

As a result, it is possible to measure risk simply as the standard deviation of the returns (Bodie et al., 2014, p. 136). It furthermore holds that when assets which have normally distributed returns are included in a portfolio, the portfolio return is also normally distributed (Bodie et al., 2014, p. 136). It can therefore be understood that the normal distribution simplifies portfolio selection to a great extent.

However, in an article about the statistical properties of asset returns, Cont (2001) shows that asset returns do not follow a normal distribution. It is presented, with the help of empirical evidence, that it is common for asset returns to contain heavy tails and sharp peaks (Cont, 2001). The finding of these properties mean negative skewness and excess kurtosis of asset returns, see equations (20) and (21). As a result, asset returns are shown to deviate from the normal distribution mentioned above.

$$\text{Skewness} = \frac{E_a\left[(r_t - E_a[r])^3\right]}{\sigma_a^3} = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_t - E_a[r])^3}{\left[\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t - E_a[r])^2}\right]^3} \tag{20}$$

$$\text{Excess Kurtosis} = \frac{E_a\left[(r_t - E_a[r])^4\right]}{\sigma_a^4} - 3 = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_t - E_a[r])^4}{\left[\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_t - E_a[r])^2}\right]^4} - 3 \tag{21}$$

The skewness of a normal distribution has by definition got the value of zero (Bodie et al., 2014, p. 138). A property such as negative skewness would therefore imply asymmetry and more importantly, result in an underestimation of risk (Bodie et al., 2014, p. 138). The kurtosis, on the other hand, measures the degree of fat tails and has got a value of three for a normal distribution (Bodie et al., 2014, p. 139). In order to detect if the portfolios in this study deviates from normality in this manner, excess kurtosis can be calculated according to equation (21). By definition, the excess kurtosis of a normal distribution has got a value of zero. Any excess kurtosis results in a distribution that has got fatter tails as well as it has the property of a higher peak compared to the normal distribution.

3.4 Performance evaluation of non-normally distributed returns

While the most common performance measure is the Sharpe ratio, see equation (15) (Sharpe, 1994), it is not always an adequate measure when it comes to the evaluation of portfolio performance. It is shown that the Sharpe ratio is not consistent with the maximization of expected utility in the case when the portfolio returns are not normally distributed (Fischer & Lundtofte, 2018). As mentioned, the distribution of asset returns often deviates from a normal distribution. Additional performance measures are therefore introduced in this section.

Jensen's alpha (α) is a measure of how much higher the return of the portfolio is from the return predicted by the CAPM, given the beta and the market return (Bodie et al., 2014, p. 840). The measure is risk-adjusted and based on the CAPM explained in section 3.1 (Jensen, 1968). A positive value of alpha implies that the portfolio has earned a return that is above the expected return, while a negative alpha suggests the opposite. The measure is defined in equation (22) where r_p is the return of the portfolio, r_f is the risk free rate, β is the beta of the portfolio and r_M is the market return.

$$\alpha = r_p - r_f - \beta \left(r_M - r_f \right) \tag{22}$$

The Treynor measure is similar to the Sharpe ratio as it gives the excess return $E_a[R]$ per unit of risk (Bodie et al., 2014, p. 840). The Treynor measure is given by equation (23).

$$\text{Treynor} = \frac{E_a[R]}{\beta} \tag{23}$$

However, the measure differs in that it only considers the systematic risk, which is denoted by β (Bodie et al., 2014, p. 840). This is the part of the risk that cannot be diversified away, as explained in section 3.1.

The tracking error (TE) of an portfolio shows how the performance of the investment deviates from the performance of its benchmark (Reilly & Brown, 2012, p. 553). The measure is calculated according to equation (24).

$$\mathrm{TE} = \frac{1}{T-1}\sqrt{\sum_{t=1}^{T}(r_{t,p} - r_{t,M})^2} \tag{24}$$

It is measured by taking the standard deviation of the differences between the returns on a certain portfolio r_p and the returns of the market portfolio r_M, which serves as the benchmark (Reilly & Brown, 2012, p. 553).

The Information ratio (IR) divides the α of the portfolio by the portfolios tracking error (Bodie et al., 2014, p. 840). The definition is given by equation (25).

$$\mathrm{IR} = \frac{\alpha}{\mathrm{TE}} \tag{25}$$

The measure defines the excess return of holding a portfolio compared to the market portfolio, per unit of volatility of those returns (Goodwin, 1998). By definition, the Information ratio for the market portfolio is zero.

So far we have compared different performance measures to the standard Sharpe ratio, which is considered to be an adequate measure only if the assumption of normal returns holds. It has already been shown that the asset returns commonly don't follow a normal distribution (Cont, 2001). Due to this, moments higher than the mean and variance of returns are needed in order to study the performance of portfolios (Bodie et al., 2014, p. 137-139). It is important to mention that when the standard deviation of returns is used as a measurement of risk, both the upside and downside risk is included in the risk measure (Fischer & Lundtofte, 2018). With this reason, it has been argued that the downside risk should be of more importance when considering the risk of an investment (Harlow, 1991). Therefore, three different performance measures which focus on downside risk of an investment will now be presented. The following measures are related to the lower partial moment, which is defined as equation (26).

$$\mathrm{LPM}_n(\bar{r}) = \int_{-\infty}^{\bar{r}} (\bar{r} - r)^n f(r) dr \tag{26}$$

Where $\bar{r}$ represents a chosen threshold value and n is the order of moment. The lower partial moment is an asymmetric measure which focuses only on the returns which fall below a specific level of return, also called threshold return (Harlow, 1991). The performance measures are said to focus on the downside risk of returns due to that the focus is set on the left tail of the distribution (Harlow, 1991).

The following measures, which deal with the downside risk, have been derived in a more generalized form by Kaplan and Knowles (2004). This was done with the help of using lower partial moment as a risk measure and equations (27) to (29) will be presented in this form. The Omega ratio was developed by Keating and Shadwick (2002) and is defined by equation (27).

$$\text{Omega} = \frac{E_a[R]}{LPM_1 \left[E_a[r_f]\right]} + 1 \tag{27}$$

The measure places focus on the downside risk of a portfolio while it also captures the higher moments of the returns (Keating & Shadwich, 2002). The authors explain that while the Sharpe ratio tries to minimize losses of portfolio returns, the performance measure also minimizes the potential gains of an investment. This has to do with that the Sharpe ratio considers both the returns above and below the mean to be risky to the same extent. In order to create a performance measure which gets around this problem, they based the Omega ratio around a threshold return. This return differs for individuals and in this study, the risk-free rate will be used. This performance measure looks at the returns which are considered as a loss separately from the ones which are considered to be gains[10]. Consequently, the Omega ratio is a ratio which presents the gains to losses of a portfolio return. Hence, it is therefore quite intuitive to understand that a higher value of the Omega ratio is preferable.

The Sortino ratio was created by Sortino and van der Meer (1991) and is defined in equation (28).

$$\text{Sortino} = \frac{E_a[R]}{\sqrt{LPM_2\left[E_a[r_f]\right]}} \tag{28}$$

As it can be seen, the performance measure is similar to the Sharpe ratio based on arithmetic mean of returns. This has both to do with that it is a ratio of the excess return over a threshold return, which in this case is the risk-free rate, and that the measure considers the second moment of returns (Kaplan & Knowles, 2004). The performance measure was initially developed because of the argument that standard deviation and beta are not completely adequate risk measures (Sortino & van der Meer, 1991). The authors wanted to find a risk measure that did not punish upside potential and thus, suggested that downside variance is to be considered a better measure of risk.

[10]This will depend on the value of the threshold return which can be seen as the minimum accepted return.

Finally, the Kappa3 ratio (Kaplan & Knowles, 2004) is defined in equation (29).

$$\text{Kappa3} = \frac{E_a[R]}{\sqrt[3]{LPM_3\left[E_a[r_f]\right]}} \tag{29}$$

It can be seen that this measure includes the third lower partial moment and that there exists a resemblance between this particular measure and the two measures which are previously presented. Kaplan and Knowles (2004) suggest that the Omega and Sortino ratio can be defined as two different versions of the Kappa measure, where the order of moments changes [11] and therefore argue that the Kappa measure is a generalized form of the two [12].

A different way to incorporate risk into a performance measure can be done by the absolute deviation of returns. The mean absolute deviation (MAD) is defined in equation (30).

$$\text{MAD} = \frac{E_a[R]}{E_a\left[|r - E_a[r]|\right]} \tag{30}$$

The performance measure swaps the standard deviation for the absolute deviation of returns, $|r - E_a[r]|$, and thus uses this as a measure of variability (Bodie et al., 2014, p. 251).

The Calmar ratio uses the maximum drawdown as a measure of risk (Fischer & Lundtofte, 2018). The maximum drawdown (MD) is defined as the worst possible return of an asset.

$$\text{Calmar} = \frac{E_a[R]}{-\text{MD}} \tag{31}$$

[11] It is shown that equation (29) is also simply a version of the more general Kappa measure.

[12] For the derivation of equation (28), see Kaplan & Knowles (2004).

Value at risk (VaR) is often used when considering tail events of return distributions (Bodie et al., 2014, p. 139). The 5 % VaR represents the best rate of an asset return out of 5 % worst case scenarios (Bodie et al., 2014, p. 139). Studying the value of the 5% VaR therefore implies that the rest of the returns, 95 % of them, will exceed this value (Bodie et al., 2014, p. 139). The VaR with an quantile of α is defined in equation (32) where z_α denotes the α-quantile of the standard normal distribution.

$$\mathrm{VaR}_p^\alpha = -(E[r_p] + \sigma_p z_\alpha) \tag{32}$$

It is shown that VaR can be used as a measure of risk in the following way (Dowd, 2000). The Dowd ratio is defined in equation (33).

$$\mathrm{Dowd}^\alpha = \frac{E_a[R]}{VaR^\alpha} \tag{33}$$

Even though the measure shows similarities to the standard Sharpe ratio, it differs in that it measures returns of an asset against the assets extreme risk, instead of the total risk of the returns, which is represented by the standard deviation of returns in the Sharpe ratio.

3.5 Utility theory and the Atkinson index

When comparing the performance of different portfolios, it is also of significance to take an investors degree of risk aversion into account[13]. An individual's level of utility can be presented in the form of a utility function, which describes preferences, and the first and second derivative can reveal if the individual is risk averse. The level of utility can

[13]An individual who is risk averse is described to dislike risk more than an individual who is neutral to taking risk or a person who is described as being a risk lover.

also be defined as the certainty equivalent rate of return which is the rate that a risk-free investment would need in order to yield the same utility as an investment in the risky portfolio (Bodie et al., 2014, p. 170-171).

There exists several different kinds of utility functions. The utility function described in equation (34) is a quadratic utility function, where U stands for the level of utility, $E[r]$ is the expected return, A is a measure of risk aversion and σ^2 is the variance of the returns (Bodie et al., 2014, p. 170).

$$\mathrm{U} = E[r] - \frac{1}{2}A\sigma^2 \tag{34}$$

The following can be noted from the case of a quadratic utility function. The utility level receives a higher value when the portfolio has got higher expected returns and on the other hand, receives a lower value if the level of risk increases. Thus, an outcome of quadratic utility is that only the mean ($E[r]$) and variance (σ^2) of returns are considered important. This kind of utility function is commonly used in finance and is more precisely, often assumed in the mean-variance framework (Huang & Litzenberger, 1993, p. 25-26). However, a quadratic utility function implies a satiation point and increasing absolute risk aversion (Huang & Litzenberger, 1993, p. 25-26). A satiation point suggests that as an individual's wealth increases beyond a threshold value, the individual's utility begins to decrease in value, thus the marginal utility turns negative. It has been demonstrated that this characteristic of the utility function is quite unrealistic since most individuals will always prefer more to less, regardless if it is income or wealth which is being considered (Danthine & Donaldson, 2015, p. 166-168). Moreover, an increasing absolute risk aversion implies that an individual will invest less in, for instance risky portfolios, as the individual receives more wealth (Danthine & Donaldson, 2015, p. 121). This has in turn also been demonstrated not to be a reasonable assumption (Danthine & Donaldson, 2015,

p. 121).

Another classic variety of a utility function has got the standard form of constant relative risk aversion (CRRA).

$$U(c) = \frac{c^{1-\rho} - 1}{1 - \rho} \tag{35}$$

Where ρ represents a coefficient of relative risk aversion and c denotes consumption. The CRRA utility function is one of the most commonly used forms of utility functions (Danthine & Donaldson, 2015, p. 137). A beneficial characteristic of the CRRA utility function is that it, in contrast to the quadratic utility function, takes higher moments into account. As it turns out, this form of utility function will be important in this study as it is the underlying utility function in the performance measure which will be presented in the following section.

The Atkinson index is initially a measure of income inequality, where the inequality is measured by studying the distribution of income (Atkinson, 1970). Measuring inequality can intuitively be explained to be related to measuring risk as the former measure involves the calculation of the distribution of income while the latter looks at the distribution of returns. This is suggested in the article by Atkinson (1970) and shown in an article by Fischer and Lundtofte (2018)[14]. The authors have shown this by applying the Atkinson index to asset returns and by doing so, creating a measure of financial risk.

Atkinson originally begins to derive his measure of inequality by introducing the concept of the equally distributed equivalent level of income, which is further demonstrated to be

[14]The idea of applying a measure of inequality to financial returns in order to create a measure of risk is not new and was first done by Yitzhaki (1982) when he used the Gini coefficient to do this (Fischer & Lundtofte, 2018).

closely related to the certainty equivalent rate of return (Atkinson, 1970)[15]. The Atkinson index, A, is defined according to equation (36) (Fischer & Lundtofte, 2018)[16].

$$A = 1 - \frac{CE}{E[r]} \tag{36}$$

Where CE represents the certainty equivalent and $E[r]$ is the mean of returns. The Atkinson index can only take a value between zero and one, where zero indicates complete equality and one indicates complete inequality (Atkinson, 1970). In financial economics, the value of zero would indicate that there is no financial risk, as the value of A can be compared to the standard deviation of returns.

When solving for the Atkinson index in the case of constant relative risk aversion, we have equation (37). This particular expression has been derived by Fischer and Lundtofte (2018). The reason behind this is that it has been demonstrated that the underlying utility function of the Atkinson index is of CRRA form (Atkinson, 1970).

$$\mathrm{A}(\rho) = \begin{cases} 1 - \dfrac{1}{E_a[r]}(E_a[r^{1-\rho}])^{\frac{1}{1-\rho}} & \rho > 0,\ \rho \neq 1, \\ 1 - \dfrac{1}{E_a[r]}e^{E_a[\ln(r)]} & \rho = 1 \end{cases} \tag{37}$$

Where ρ represents the relative risk aversion. Additionally, in order to make comparisons with other performance measures, certainty equivalent returns can be defined by rearranging equation (36) and solving for CE.

[15]The original measure of inequality was defined as $I = 1 - \frac{y_{EDE}}{\mu}$, where y_{EDE} represents the equally distributed equivalent level of income.

[16]Note that this A differs from the one seen in equation (34).

$$CE = E[r](1 - A) \tag{38}$$

Where the value of A has been calculated according to equation (37). It can be observed that as A increases, which can be compared to the standard deviation and hence a level of risk, the certainty equivalent rate will decrease. This implies that the utility level of an individual will decrease with an increasing level or risk.

It is shown that the certainty equivalent return can be used as a performance measure (Fischer & Lundtofte, 2018). Their performance measure takes the higher moments such as kurtosis and skewness into account, as opposed to the standard Sharpe ratio. This performance measure is also shown to be consistent with the maximization of expected utility. This is in contrast to the Sharpe ratio which is only consistent with maximization of expected utility if the returns are normally distributed or if the utility function is quadratic. These assumptions have been explained to be unrealistic and moreover, it will be shown that the portfolio returns in this study do not follow a normal distribution. The performance measure developed by Fischer and Lundtofte (2018) depends on individual preferences. However, the authors demonstrate that the measure can be made more general by varying the coefficient of relative risk aversion within a reasonable range (Fischer & Lundtofte, 2018). Additionally, the new performance measure is applied on hedge funds returns, resulting in a different ranking of the funds when compared to the other performance measures. In contrast to their findings, Eling and Schuhmacher (2007) have done a similar study where they calculated various performance measures of hedge funds and concluded that the ranking does not change substantially for different performance measures. It is therefore interesting to see if this measure suggests a different ranking of the portfolios created in this study compared to the other performance measures which have been introduced.

4 Research design

This section consists of a description of the data sample used, how the returns are computed and what risk free rate is used. It moreover consists of a detailed explanation of the method used to construct the different portfolios.

4.1 Data sample

The Orbis database, which provides company information, was used in order to create the portfolios. To begin with, a list of companies which have OMX Stockholm as their main market was created and was used throughout the study. The companies included in this study were selected by the criteria that they were one of the 100 largest companies ranked by current market capitalization in 2010 [17]. They furthermore had to exist during the whole period in order to be selected. These 100 largest companies were chosen to create the Market Capitalization portfolio which served as a benchmark, in accordance with the article written by Arnott et al. (2005).

The companies from the same original list were also ranked by every fundamental metric chosen and included in the respective fundamental portfolio. Additionally, a Composite portfolio was created by combining equal proportions of the weights of each company from the fundamental portfolios. This procedure is explained more in detail in section 4.2.

Finally, an Equal Weight portfolio was constructed by giving equal weights to the stocks of the companies from the list of the 100 largest companies by market capitalization as of the year 2010.

[17]This is the first year included in the Orbis database.

The sample period was set to be between January 2010 and December 2017. The market capitalization and fundamental metrics data (i.e. employees, cash flow, revenue, dividends per share) was taken on an annual basis and used to first create the portfolios at year 2010 and then rebalance them every year.

The stock data needed for calculating the returns of the different portfolios was obtained from Nasdaq. The data consisted of daily closing prices, adjusted for dividends, from the period of January 2010 to December 2017. This time period is rather short which could potentially lead to a difficulty in finding statistical significance when comparing the portfolios. However, as the portfolios consisted of daily stock closing prices, the total amount of observations was in fact 2005. The daily return at day (t) for all the stocks was calculated according to equation (39).

$$r_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \tag{39}$$

Where $r_{i,t}$ is the return of stock i at day t, $P_{i,t}$ is the price of stock i at day t and $P_{i,t-1}$ is the price of stock i from the previous day.

The data for the risk-free rate was collected from the Swedish Central Bank and is presented in table 1. In principal, Treasury bills are often used in order to represent the risk-free rate and therefore this study will use the 1-month Swedish T-bill as a substitute for the risk-free rate (Bodie et al., 2014, p. 129).

Table 1: Annualized risk free rate based on 1-month Swedish T-bill.

Year	2010	2011	2012	2013	2014	2015	2016	2017
Rate	0.5	1.9	1.5	1.0	0.5	-0.3	-0.6	-0.7

4.2 Portfolio construction

The construction of the different portfolios was performed in accordance with the procedure outlined in the article by Arnott et al. (2005). All the companies listed on the Stockholm stock exchange that had it as the main market were ranked by their current market capitalization as of January 2010. The 100 companies with highest rank were selected and used throughout the study. Hence, the same 100 companies selected in the beginning of the study were used for the construction of the seven different portfolios: Market Capitalization portfolio, Employees portfolio, Cash Flow portfolio, Operating Revenue portfolio, Dividends per Share portfolio, Composite portfolio, Equal Weight portfolio. These portfolios differed in the manner that the assets had different weights assigned to them and were therefore considered to be of different importance.

The weights were in turn calculated as follows. For the Market Capitalization portfolio the weights were calculated as shown in equation (40).

$$w_{i,t}^{\text{Market Cap.}} = \frac{M_{i,t}}{\sum\limits_{j=1}^{100} M_{j,t}} \tag{40}$$

In this equation $w_{i,t}^{\text{Market Cap.}}$ denotes the weight assigned to asset i at time t, $M_{i,t}$ denotes the market capitalization for company i at time t and $\sum_{j=1}^{N=100} M_{j,t}$ is the total market capitalization for all the companies combined.

For the fundamental metric portfolios (i.e. Employees portfolio, Cash Flow portfolio, Operating Revenue portfolio and Dividends per Share portfolio) equation (41) was used to determine the weights $w_{i,t}^{n}$. Where $F_{i,t}^{n}$ denotes the fundamental size of respective metric n for company i at time t (e.g. the number of employees in the case of the Employees portfolio). $F_{j,t}^{n}$ is the total fundamental size of metric n for all the companies combined.

$$w_{i,t}^{n} = \frac{F_{i,t}^{n}}{\sum_{j=1}^{100} F_{j,t}^{n}} \tag{41}$$

For the Composite portfolio the weights were calculated using equation (42).

$$w_{i,t}^{\text{Composite}} = \sum_{n=1}^{4} \frac{w_{i,t}^{n}}{4} \tag{42}$$

The weights for the Equal Weight portfolio $w_{i,t}^{\text{Equal}}$ were all set to the same weight as stated in equation (43).

$$w_{i,t}^{\text{Equal}} = \frac{1}{100} \tag{43}$$

Finally it should be noted that by using equations (40) to (43), then equation (44) holds.

$$\sum_{i=1}^{100} w_{i,t} = 1 \tag{44}$$

The portfolios were rebalanced as of January 1st each year. This method of rebalancing is in line with the research done by Arnott et al. (2005). The authors argue that the advantage of rebalancing the portfolios once a year is being able to maintain low transaction costs. In their paper, it is also shown that rebalancing the portfolios on a more frequent basis, such as monthly or quarterly, increases the portfolio turnover but does not result in any noticeable additional return of the portfolios.

5 Results and Analysis

In the following section, the results will be presented and an analysis of them will be given. First, the mean of returns, variance and standard deviation of the portfolio returns will be presented. Additionally, the cumulative increase for all the portfolios will be illustrated in a figure. Then, the different performance measures will be computed in order to evaluate and compare the performance of the portfolios.

5.1 Portfolio returns

The seven portfolios were constructed as outlined in section 4.2. The geometric and arithmetic mean of returns for every portfolio were calculated as depicted in equations (6) and (7). The arithmetic and geometric mean, variance and standard deviation of returns for all portfolios are presented in table 2. These values have been calculated based on the whole data set, however, they have been converted to represent the daily rate on an annual basis according to equation (45) where n is the number of trading days in a year[18]. Furthermore, the cumulative increase of every portfolio is shown in figure 1.

$$
\begin{aligned}
E_a[r]_{\text{annual}} &= nE_a[r]_{\text{daily}} \\
E_g[r]_{\text{annual}} &= \left(\prod_{t=1}^{T}(1 + r_t)\right)^{n/T} - 1
\end{aligned}
\tag{45}
$$

It can be observed that the arithmetic mean for all the portfolio returns have higher

[18]Calculations besides these have not been annualized. This will however not affect the ranking of the portfolios.

Table 2: Annualized values for arithmetic and geometric mean of returns, arithmetic and geometric variance of returns and arithmetic and geometric standard deviation of returns.

Portfolio	$E_a[r]$ (%)	$E_g[r]$ (%)	σ_a^2 (%)	σ_g^2 (%)	σ_a (%)	σ_g (%)
Employees	10.24 [3]	8.95 [5]	3.34 [6]	3.41 [6]	18.28 [6]	20.10 [6]
Cash Flow	10.55 [1]	9.31 [2]	3.29 [5]	3.36 [5]	18.13 [5]	19.92 [5]
Dividends	9.86 [7]	9.01 [4]	2.46 [2]	2.50 [2]	15.67 [2]	17.01 [2]
Revenue	9.98 [6]	8.61 [7]	3.42 [7]	3.50 [7]	18.50 [7]	20.37 [7]
Composite	10.16 [4]	9.02 [3]	3.04 [3]	3.10 [3]	17.43 [3]	19.08 [3]
Equal Weight	10.49 [2]	9.72 [1]	2.41 [1]	2.46 [1]	15.54 [1]	16.86 [1]
Market Cap.	10.14 [5]	8.94 [6]	3.15 [4]	3.21 [4]	17.74 [4]	19.44 [4]

values than the geometric returns of the returns[19]. This is not surprising and is due to the multiplicative nature of the return process, as mentioned earlier. The Cash Flow portfolio has the highest arithmetic mean of returns, while the Dividends per Share portfolio is the one which has got the lowest. Furthermore, it can be seen that both the Composite portfolio and the Equal Weight portfolio have got higher mean of returns than the Market Capitalization portfolio. When it comes to the geometric mean of returns, it is the Equal Weight portfolio that has the highest value while the Operating Revenue portfolio has the lowest. It can furthermore be noted that the difference between the two averages increases with volatility, see for instance the Operating Revenue portfolio which has the highest variance of returns and the Equal Weighted portfolio, which has got the lowest variance of returns. Although these differences can be observed, a paired t-test has been calculated which shows that there is no statistically significant difference between the returns of the portfolios compared to the Market Capitalization portfolio.

By having the return distributions of the portfolios summarized by geometric mean ($E_g[r]$) and geometric variance (σ_g^2) as in table 2, it becomes possible to rank the portfolios according to the mean-variance criterion (Danthine & Donaldson, 2015, p. 56-58)[20]. It

[19]For normally distributed returns, it holds that $E_g[r] = E_a[r] - 1/2\sigma^2$ (Bodie et al., 2014, p. 170).

[20]The mean-variance criterion is defined as follows. For portfolio A to dominate portfolio B, the following must hold: $E[r_A] \geq E[r_B]$ and $\sigma_A \leq \sigma_B$ and at least one of these inequalities must hold in order to rule out indifference between the portfolios (Bodie et al., 2014, p. 172-173).

can be concluded that the Equal Weight portfolio mean-variance dominates as it has got both the highest geometric mean of returns and the lowest geometric variance of returns. Values within the brackets show the ranking of the portfolios, with [1] indicating the best performance. Note that this will generate that the value [1] will be assigned to the highest mean of returns while it will be assigned to the lowest variance of returns. The portfolio based on Operating Revenue is shown to have the lowest mean return while it simultaneously has the highest variance of returns. These characteristics suggest that a rational investor, according to the mean-variance criterion, would rank this portfolio the lowest. However, when comparing the rest of the portfolios, it is not as clear to see which dominates the others in terms of the mean-variance criterion. When there is no clear dominance between the different portfolios, the investments are explained to be preference dependent (Danthine & Donaldson, 2015, p. 56-58). This implies that each investor's preferences regarding the trade-off between expected return and risk needs to be taken into account before drawing conclusions. Furthermore, since individual preferences vary between every individual, the ranking of these portfolios will not be same for all investors.

From figure 1 it can be observed that the cumulative increase for the Equal Weight portfolio is the highest. In fact, this portfolio surpassed the Market Capitalization portfolio during the year of 2016. Nevertheless, it is easy to see that the portfolio returns follow each other closely throughout the whole period. This can be further verified by table 3, where the correlation of the portfolio returns are presented. There is a high correlation between all the portfolios but it can be noticed that the Equal Weight portfolio has got the lowest correlation with the Market Capitalization portfolio.

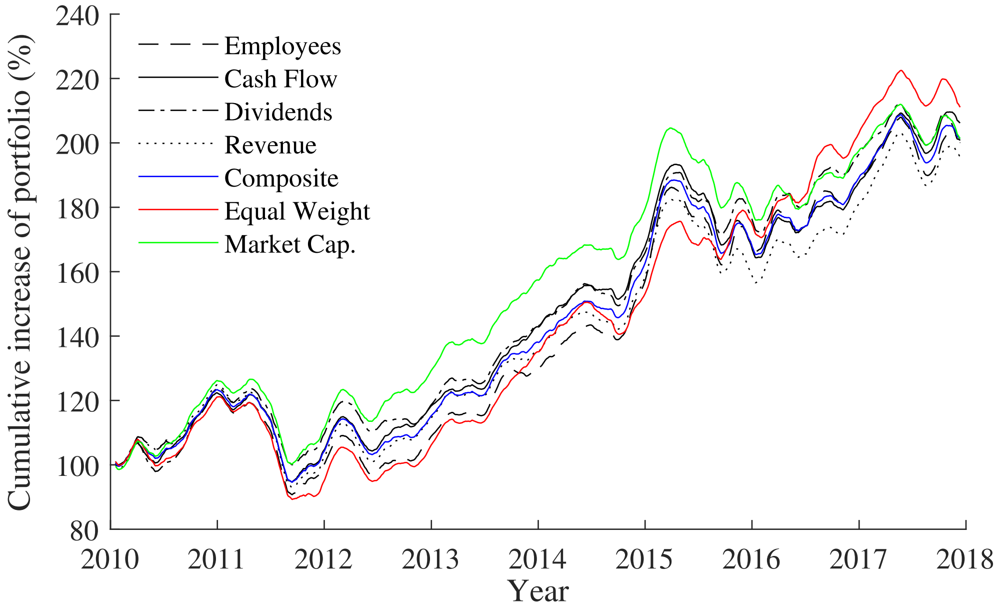

Figure 1: Cumulative increase for all portfolios.

Table 3: Correlation of portfolio returns.

Portfolio	Employees	Cash Flow	Dividends	Revenue	Composite	Equal Weight	Market Cap.
Employees	1.00						
Cash Flow	0.97	1.00					
Dividends	0.95	0.96	1.00				
Revenue	0.98	0.99	0.96	1.00			
Composite	0.99	0.99	0.98	0.99	1.00		
Equal Weight	0.94	0.95	0.97	0.95	0.96	1.00	
Market Cap.	0.97	0.99	0.96	0.99	0.99	0.95	1.00

5.2 Portfolio performance

The Sharpe ratio and excess returns, based on both arithmetic and geometric expected returns, are presented in table 4. The standard error of each Sharpe ratio, together with the corresponding confidence intervals, have also been included. They have been calculated in order to see if there is a statistical significant difference between them, with an accuracy of 95 %. In order for a Sharpe ratio to be significantly different than another, the confidence intervals are not allowed to overlap. This implies that the only portfolio which

Table 4: Annualized values for the Sharpe ratio and excess returns based on both geometric and arithmetic mean of returns, see equations (14) to (16) and (45). The standard error (SE(SR)) and confidence interval ($CI_{SR,1}$, $CI_{SR,2}$) are determined using equations (17) and (18).

Portfolio	SR_a	SR_g	$E_a[R]$ (%)	$E_g[R]$ (%)	SE(SR)	$CI_{SR_a,1}$	$CI_{SR_a,2}$
Employees	0.53 [6]	0.42 [6]	9.76	8.46	0.028	0.48	0.59
Cash Flow	0.56 [3]	0.44 [4]	10.07	8.83	0.029	0.50	0.61
Dividends	0.60 [2]	0.50 [2]	9.37	8.53	0.030	0.54	0.66
Revenue	0.51 [7]	0.40 [7]	9.49	8.13	0.028	0.46	0.57
Composite	0.55 [4]	0.45 [3]	9.67	8.53	0.029	0.50	0.61
Equal Weight	0.64 [1]	0.55 [1]	10.00	9.24	0.032	0.58	0.71
Market Cap.	0.54 [5]	0.43 [5]	9.65	8.45	0.028	0.49	0.60

fulfills the requirements is the Equal Weight portfolio against the Operating Revenue portfolio.

It is evident that the Equal Weight portfolio has got the highest Sharpe ratio, which suggests that it is a favorable investment. This is true for both the Sharpe ratio which is based on the arithmetic and the geometric mean of returns. It is also worth noting that, while the Sharpe ratio based on geometric mean of returns is smaller, there is a relatively big difference between the values of the Equal Weight portfolio and the Market Capitalization portfolio. The Operating Revenue portfolio has in turn got the lowest Sharpe ratio, suggesting a less favorable investment. Values within the brackets represent the ranking of the portfolios, with [1] indicating the best performance. Nearly all the portfolios have the same ranking based on both the arithmetic and geometric Sharpe ratio. The only difference is in the ranking of the Cash Flow portfolio and the Composite portfolio where the two portfolios switch places. This can possibly be due to that the calculations of the excess return based on arithmetic and geometric mean of returns differ.

When comparing the Sharpe ratio of the portfolios based on fundamental metrics and the Market Capitalization portfolio, the following can be stated. The Cash Flow portfolio and the Dividends per Share portfolio have got higher values than the Market Capitalization portfolio. On the other hand, the two other fundamental portfolios, Number of Employees portfolio and Operating Revenue portfolio, have got lower values. However, it still turns out that the Composite portfolio, which consists of a combination of all the fundamental portfolios, performs better than the Market Capitalization portfolio. In summary, four out of six portfolios perform better than the Market Capitalization portfolio according to the Sharpe ratio, where the Equal Weight portfolio performs the best. Due to this, the answer to whether the portfolios based on fundamental metrics are more mean-variance efficient than the portfolio based on market capitalization seems to be ambiguous.

The values for skewness and excess kurtosis for the portfolios in this study are presented in table 5 and shown in figure 2. Additionally, the black solid line shows a normal distribution with a mean which is equal to the mean of the portfolio. It can be observed that all the portfolios have got negative skewness, of which the Equal Weight portfolio has got the highest value in absolute terms. This same portfolio has got an excess kurtosis of 4.44, which is the highest of all the portfolios in this study. This can be illustrated in figure 2, where the daily stock returns of the Equal Weight portfolio are shown to have fatter tails and a higher peak at the same time as it is more skewed to the left. This indicates that the Equal Weight portfolio has higher probability of extreme events, due to the fat tails, and moreover, a higher probability of negative returns.

Table 5: Annualized values for the arithmetic expected returns ($E_a[r]$), arithmetic variance of returns σ_a^2, skewness and excess kurtosis.

Portfolio	$E_a[r]$ (%)	σ_a^2 (%)	Skewness	Excess kurtosis
Employees	10.24	3.34	-0.33	3.64
Cash Flow	10.55	3.29	-0.38	4.13
Dividends	9.86	2.46	-0.43	3.86
Revenue	9.98	3.42	-0.39	4.26
Composite	10.16	3.04	-0.39	4.09
Equal Weight	10.49	2.41	-0.52	4.44
Market Cap.	10.14	3.15	-0.30	4.17

It is interesting to note that this portfolio has got the lowest variance and a slightly lower arithmetic mean than the Cash Flow portfolio which has got the highest. The Market Capitalization portfolio has got the least negative skewness of all the portfolios and a lower value of excess kurtosis compared to the Equal Weight portfolio. Additionally, tests for normality have been computed and are presented in table 12. The results show that all the portfolios failed these tests at the 5 % significance level and thus further supports the fact that the portfolio returns in this study are not normally distributed.

Since it is shown that the normality assumption does not hold in this study, it is of importance to include other performance measures when evaluating the portfolios. First, the CAPM parameters, alpha and beta, are calculated. The values of α and β for the portfolios are shown in table 6. Additionally, the confidence levels of 95 % were calculated and presented in table 6.

Table 6: CAPM parameters: Jensen's α and β.

Portfolio	$CI_{\alpha 1}$	α (% / year)	$CI_{\alpha 2}$	$CI_{\beta 1}$	β	$CI_{\beta 2}$
Employees	-2.98	0.11	3.19	0.989	1.00	1.011
Cash Flow	-1.79	0.35	2.49	1.000	1.01	1.015
Dividends	-1.93	1.21	4.34	0.835	0.85	0.857
Revenue	-2.57	-0.43	1.71	1.020	1.03	1.036
Composite	-1.60	0.31	2.21	0.964	0.97	0.977
Equal Weight	-1.39	1.97	5.32	0.821	0.83	0.844
Market Cap.	0.00	0.00	0.00	1.00	1.00	1.000

By definition, the values of α and β for the Market Capitalization portfolio are 0.00 and 1.00 respectively, since it serves as the market portfolio in this study. We can observe that the Equal Weight portfolio has got the highest alpha while the Operating Revenue portfolio has got the lowest alpha. The values of alpha do not show a difference which is statistically significant, at the 5 % level. The Equal Weight portfolio has earned a 1.97 % return above what is needed given the portfolio market risk, according to CAPM. On the other hand, the Operating Revenue portfolio has got a negative alpha, implying underperformance in relation to the Market Capitalization portfolio. As can be seen, the Operating Revenue portfolio has got a β value of 1.03, which is the highest β of these portfolios. The beta is a measure of the systematic risk in the CAPM, as explained in section 3.1. This means that the Operating Revenue portfolio has got 3 % more systematic risk than the Market Capitalization portfolio and implies that this portfolio could be expected to earn more due to the higher risk. The confidence intervals for beta show that there is a statistical significant difference for the Equal Weight portfolio, Composite portfolio and the Dividends per Share portfolio, compared to the Market Capitalization portfolio, at the 5 % level. Interestingly, the beta of the Equal Weight portfolio has got the lowest value while it also has been shown to have the highest mean of geometric returns. A beta lower than one suggests that the portfolio is less volatile than the market portfolio. This can also be verified with that the Equal Weight portfolio has got the lowest variance and standard deviation of returns, while the Market Capitalization portfolio has got the fourth highest.

The Treynor ratio for all the portfolios is presented in table 7. It is evident that the Equal Weight portfolio has got the highest value, which indicates that this is the portfolio with the highest excess return per unit of systematic risk, which is defined as β. This is not particularly surprising as this portfolio has got the lowest beta, and furthermore, also has the second highest value of excess returns. The opposite is true for the Operating Revenue portfolio which has got the lowest value of the Treynor ratio.

The Information ratio has been computed for all the portfolios and is presented in table 7. It is shown that the portfolio based on equal weights has got the highest Information ratio, indicating that this portfolio has performed better than the portfolio based on market capitalization. On the contrary, the Operating Revenue portfolio has got a negative value.

Moreover, the tracking error of all the portfolios have been calculated. The tracking error of the Market Capitalization portfolio is zero by definition. The portfolio that has got the highest tracking error is the portfolio based on equal weights. This is also the portfolio that has got the highest Sharpe ratio, the highest Treynor ratio and the highest Information ratio. The ranking of the portfolios according to the previous explanation is also presented in table 7.

Table 7: Treynor measure (TR), Information ratio (IR), Tracking error (TE) and Sharpe ratio (SR).

Portfolio	TR	IR	TE	SR_a
Employees	0.085 [6]	0.136 [5]	0.008 [3]	0.53 [6]
Cash Flow	0.088 [4]	0.627 [3]	0.006 [5]	0.56 [3]
Dividends	0.101 [2]	1.273 [2]	0.009 [2]	0.60 [2]
Revenue	0.079 [7]	-0.770 [7]	0.006 [4]	0.51 [7]
Composite	0.088 [3]	0.613 [4]	0.005 [6]	0.55 [4]
Equal Weight	0.111 [1]	1.927 [1]	0.010 [1]	0.64 [1]
Market Cap.	0.085 [5]	0.000 [6]	0.000 [7]	0.54 [5]

The Equal Weight portfolio is shown to perform better than the Market Capitalization portfolio and the fundamentally-weighted portfolios, according to every performance measure.

Additionally, the Omega ratio, the Sortino ratio and the Kappa3 measure have been computed for all portfolios and are presented in table 8. It can be observed that the ranking of the portfolios correspond to the ranking of the portfolios according to the Sharpe ratio which is based on the arithmetic mean of returns. This is in contrast to what was demonstrated in the article by Kaplan and Knowles (2004), where it was shown that the ranking of the hedge funds was affected by changing the order of moments.

The 5 % value at risk (VaR) has been calculated for all the portfolios and is presented in table 9. Since it is a measure of financial risk and losses have a negative impact on the performance of a portfolio, lower values are preferred to higher ones. The ranking of the portfolios according to VaR is therefore done by assigning the value one [1], to the lowest value of VaR. For the Equal Weight portfolio, it can be observed that the portfolio returns

Table 8: Omega, Sortino, Kappa3.

Portfolio	Omega	Sortino	Kappa3
Employees	1.0978 [6]	0.0475 [6]	0.0318 [6]
Cash Flow	1.1035 [3]	0.0492 [3]	0.0326 [3]
Dividends	1.1107 [2]	0.0527 [2]	0.0350 [2]
Revenue	1.0950 [7]	0.0453 [7]	0.0300 [7]
Composite	1.1031 [4]	0.0491 [4]	0.0325 [4]
Equal Weight	1.1213 [1]	0.0562 [1]	0.0369 [1]
Market Cap.	1.1014 [5]	0.0484 [5]	0.0322 [5]

are suggested to, with a 95 % significance, exceed a VaR of 1.57 %. This is a lower value compared to the portfolios based on fundamental metrics and the market portfolio, which has got a VaR of 1.80 %. The Dowd ratio, which uses the VaR as a measure of risk, has been calculated and is shown in table 9.

The mean absolute deviation (MAD) and the Calmar ratio are also calculated and presented in table 9. There is a slightly different ranking shown when comparing the Calmar ratio with the other two performance measures. The difference in the definition of this measure, as compared to the MAD and Dowd, is that it includes the worst possible returns of an portfolio. This results in that the Employees portfolio receives a higher ranking, which might suggest that it has a less negative worst return relative to its absolute deviation and value at risk.

Table 9: Value at risk, MAD, Calmar, Dowd.

Portfolio	VaR (%)	MAD	Calmar	Dowd
Employees	1.86 [6]	0.0467 [6]	0.0029 [3]	0.0209 [6]
Cash Flow	1.84 [5]	0.0493 [3]	0.0028 [5]	0.0218 [3]
Dividends	1.59 [2]	0.0526 [2]	0.0030 [2]	0.0235 [2]
Revenue	1.88 [7]	0.0454 [7]	0.0025 [7]	0.0201 [7]
Composite	1.77 [3]	0.0491 [4]	0.0028 [4]	0.0218 [4]
Equal Weight	1.57 [1]	0.0574 [1]	0.0032 [1]	0.0254 [1]
Market Cap.	1.80 [4]	0.0483 [5]	0.0026 [6]	0.0214 [5]

Finally, the new measure of financial risk, defined in equation (37) has been calculated for all the portfolios. The parameter ρ, which represents the relative risk aversion, was varied between one and ten. These values of ρ were chosen since they are considered to be reasonable (Fischer & Lundtofte, 2018). The results are presented in table 10. The three different values of ρ have been included in table 10 to show how the Atkinson index changes with different values of ρ. A ranking of the different portfolios has been included. Since this is a measure of financial risk, a lower value will give a higher ranking.

Table 10: Atkinson Index for $\rho = 1$, $\rho = 5$ and $\rho = 10$.

Portfolio	$\rho = 1$	$\rho = 5$	$\rho = 10$
Employees	0.017 [6]	0.084 [6]	0.170 [6]
Cash Flow	0.016 [5]	0.083 [5]	0.168 [5]
Dividends	0.012 [2]	0.062 [2]	0.125 [2]
Revenue	0.017 [7]	0.086 [7]	0.175 [7]
Composite	0.015 [3]	0.077 [3]	0.155 [3]
Equal Weight	0.012 [1]	0.061 [1]	0.124 [1]
Market Cap.	0.016 [4]	0.079 [4]	0.160 [4]

It can be observed that the ranking of the portfolios does not depend on the relative risk aversion. Furthermore, it might be worth mentioning that the index has been computed for when ρ takes the values of one to ten (see table 13). However, it is shown that the portfolios still have the same ranking. Furthermore, it can be observed that, while having the same ranking, the difference between the values of the Atkinson index increases with an increasing value of ρ. This implies that, the higher relative risk aversion a person has, the bigger becomes the difference in the Atkinson index for the different portfolios. A comparison of the portfolio ranking according to this index and the standard deviation of the portfolio returns (see table 2) reveals that a ranking of the standard deviation, both based on arithmetic and geometric, suggests the exact same ranking. This is an interesting comparison to make since both of these measures are measures of financial risk.

As it has been demonstrated that the certainty equivalent works as a performance measure, it was calculated according to equation (38). The results are presented in table 11 and, as in the previous case, a more detailed presentation is shown in table 14.

Table 11: Certainty equivalent for $\rho = 1$, $\rho = 5$ and $\rho = 10$

Portfolio	$\rho = 1$	$\rho = 5$	$\rho = 10$
Employees	0.984 [6]	0.916 [6]	0.830 [6]
Cash Flow	0.984 [5]	0.917 [5]	0.832 [5]
Dividends	0.988 [2]	0.938 [2]	0.875 [2]
Revenue	0.983 [7]	0.914 [7]	0.825 [7]
Composite	0.985 [3]	0.924 [3]	0.845 [3]
Equal Weight	0.988 [1]	0.939 [1]	0.877 [1]
Market Cap.	0.985 [4]	0.921 [4]	0.840 [4]

Overall, the main characteristics which were found regarding the Atkinson index hold for these values as well. The ranking of the portfolios are the same and are thus shown to not depend on the value of ρ. It is also observed that the differences between the values increase with an increasing value of ρ. First of all, this implies that the level of utility is the highest when holding the Equal Weight portfolio but it is also observed that the difference in the level of utility is only slightly lower when investing in the Dividends portfolio, which is considered second best, when the value of ρ equals one. Even if an individual would invest in another one of the portfolios, the utility level would not dramatically drop as the range of the certainty equivalent for all the portfolios is between 0.988 and 0.983. However, when making the same illustration of the certainty equivalent values when ρ is ten, already when investing in the portfolio which has the third highest value as compared to the portfolio which is considered to be best, there is a relatively bigger difference in the level of utility. This supports the somewhat intuitive reasoning that individuals with high risk aversion will receive higher utility when holding a portfolio which has got a relatively lower risk.

Given these findings, it is not possible to state that the portfolios based on fundamental indexation result in significantly better portfolio performance. This is in contrast to what has been argued by Arnott et al. (2005). On the other hand it is in line with the results demonstrated by Perold (2007) and Kaplan (2008). The findings in this study have shown that the Equal Weight portfolio performs better than the portfolio based on market capitalization. This is in line with the findings shown in the article by Plyakha et al. (2012). This motivates the use of the strategy with equal weights when constructing portfolios.

Moreover, rankings of all the portfolios according to the different performance has been presented and show no substantial difference. Thus, examining whether the choice of performance measure results in a different ranking does not result in any difference. This is in contrast to what Fischer and Lundtofte (2018) found when applying their performance measure to hedge fund returns. On the other hand, the result in this study is consistent with the results shown by Eling and Schuhmacher (2007).

6 Conclusion

This section includes the main findings from the results which are previously shown in section 5. There is also a short explanation of some limitations in this study. Furthermore, recommendations for further research in this area of financial economics are presented.

6.1 Main findings

The first objective of this thesis was to investigate if fundamentally-weighted portfolios perform better than portfolios based on market capitalization. The method of investigating this was done by constructing four different fundamental portfolios according to the method by Arnott et al. (2005). These fundamental portfolios were also combined into a Composite portfolio. And furthermore, an Equal Weight portfolio was constructed. The second objective was to investigate if the choice of different performance measures would suggest a different ranking of the portfolios. This part of the thesis included a new performance measure, which originally was developed to be used as a measure of income inequality.

The results presented in section section 5 show that two of the fundamental portfolios perform better than the Market Capitalization portfolio, while the other two are outperformed by the Market Capitalization portfolio. These results imply that we are not able to draw conclusions on whether portfolios weighted according to fundamental indexation are more mean-variance efficient than the portfolio based on market capitalization. Although these results are ambiguous, the Equal Weight portfolio was shown to have the highest geometric mean of returns and the lowest standard deviation of returns. It has also got the highest excess return and is ranked highest according to all the performance measures. Hence, the results show that the Equal Weight portfolio included in this study performed better than both the fundamentally-weighted portfolios and the portfolio based on market capitalization. As the study finds that the Equal Weight portfolio has got the best performance, the results are more align with the research made by Plyakha et al. (2012) than that of Arnott et al. (2005) and Tamura and Shimizu (2005).

Secondly, the ranking of the portfolios is shown to not change substantially when comparing between different performance measures. As a result, it can be stated that it does not matter if portfolio returns are evaluated according to standard Sharpe ratio or a more complicated performance measure. Eling and Schuhmacher made similar findings in their study (2007). Applying the new performance measure (Fischer & Lundtofte, 2018) on the portfolio returns, did not result in a different ranking of the portfolios, not even when the coefficient of relative risk aversion was varied within a range of one to ten. This would suggest, surprisingly enough, that regardless on the degree of risk aversion, an investor should choose to invest in the Equal Weight portfolio.

6.2 Limitations

The limitations are the following. Firstly, limited data has prevented studying the performance of the portfolios over a longer time period. It would have been interesting to see what the results would show over a longer time period since more data points would then have been included in the study. Secondly, the stocks included in the portfolios are taken from the Swedish stock market. The results could have been made more general, for instance, by including stocks from different countries. Additionally, almost all of the portfolio rankings according to the different performance measures are identical. As this thesis does not show any evidence to why this is the case, I can only make speculations. Additionally, there is no statistically significant difference between the portfolio returns. This makes it difficult to draw any clear conclusions in this thesis.

6.3 Recommendations for further research

The aim of this thesis was to investigate if portfolios constructed according to fundamental indexation outperforms portfolios which are constructed by market capitalization. Furthermore, it also aimed to show whether applying different performance measures would suggest a different ranking of the portfolios. The results suggest that the fundamental portfolios not always perform better than the portfolio based on market capitalization and that there is no substantial difference in the ranking of the portfolios.

However, recognizing the limitations in this thesis leads us to the following recommendations for future research. As the time period studied is rather short, one possible extension could be to extend the time periods. Moreover, this thesis is focused on constructing portfolios with stocks from the Swedish stock market. A suggestion for further research could therefore be to expand this environment and include stocks from several different countries, and possibly even increase the number of stocks in every portfolio. Including other kinds of assets in the portfolios is also a suggestion. Another limitation is the fact that the 100 largest stocks, according to market capitalization, were selected in the first year of the study and then used throughout the study[21]. Therefore, a different kind of method when selecting, and rebalancing, the stocks included in the portfolios could be suggested for further research.

[21]This results in that the Equal Weight portfolio has been constructed by the same stocks, and also the same weights of the stocks, every year.

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Appendices

Table 12: Jarque-Bera (JB), Anderson-Darling (AD), Kolmogorov-Smirnov (KS) and Lilliefors (LF) normality tests of portfolio returns. The significance level was 5 %.

Portfolio	JB	AD	KS	LF
Employees	No	No	No	No
Cash Flow	No	No	No	No
Dividends	No	No	No	No
Revenue	No	No	No	No
Composite	No	No	No	No
Equal Weight	No	No	No	No
Market Cap.	No	No	No	No

Table 13: Atkinson index.

Portfolio	$\rho = 1$	$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 6$	$\rho = 7$	$\rho = 8$	$\rho = 9$	$\rho = 10$
Employees	0.017 [6]	0.034 [6]	0.050 [6]	0.067 [6]	0.084 [6]	0.101 [6]	0.118 [6]	0.136 [6]	0.153 [6]	0.170 [6]
Cash Flow	0.016 [5]	0.033 [5]	0.050 [5]	0.066 [5]	0.083 [5]	0.100 [5]	0.117 [5]	0.134 [5]	0.151 [5]	0.168 [5]
Dividends	0.012 [2]	0.025 [2]	0.037 [2]	0.049 [2]	0.062 [2]	0.075 [2]	0.087 [2]	0.100 [2]	0.112 [2]	0.125 [2]
Revenue	0.017 [7]	0.034 [7]	0.052 [7]	0.069 [7]	0.086 [7]	0.104 [7]	0.122 [7]	0.139 [7]	0.157 [7]	0.175 [7]
Composite	0.015 [3]	0.030 [3]	0.046 [3]	0.061 [3]	0.077 [3]	0.092 [3]	0.108 [3]	0.123 [3]	0.139 [3]	0.155 [3]
Equal Weight	0.012 [1]	0.024 [1]	0.036 [1]	0.049 [1]	0.061 [1]	0.073 [1]	0.086 [1]	0.098 [1]	0.111 [1]	0.124 [1]
Market Cap.	0.016 [4]	0.032 [4]	0.047 [4]	0.063 [4]	0.079 [4]	0.095 [4]	0.111 [4]	0.128 [4]	0.144 [4]	0.160 [4]

Table 14: Certainty equivalent.

Portfolio	$\rho = 1$	$\rho = 2$	$\rho = 3$	$\rho = 4$	$\rho = 5$	$\rho = 6$	$\rho = 7$	$\rho = 8$	$\rho = 9$	$\rho = 10$
Employees	0.984 [6]	0.967 [6]	0.950 [6]	0.933 [6]	0.916 [6]	0.899 [6]	0.882 [6]	0.865 [6]	0.847 [6]	0.830 [6]
Cash Flow	0.984 [5]	0.967 [5]	0.951 [5]	0.934 [5]	0.917 [5]	0.901 [5]	0.884 [5]	0.867 [5]	0.850 [5]	0.832 [5]
Dividends	0.988 [2]	0.976 [2]	0.963 [2]	0.951 [2]	0.938 [2]	0.926 [2]	0.913 [2]	0.901 [2]	0.888 [2]	0.875 [2]
Revenue	0.983 [7]	0.966 [7]	0.949 [7]	0.931 [7]	0.914 [7]	0.896 [7]	0.879 [7]	0.861 [7]	0.843 [7]	0.825 [7]
Composite	0.985 [3]	0.970 [3]	0.955 [3]	0.939 [3]	0.924 [3]	0.908 [3]	0.893 [3]	0.877 [3]	0.861 [3]	0.845 [3]
Equal Weight	0.988 [1]	0.976 [1]	0.964 [1]	0.952 [1]	0.939 [1]	0.927 [1]	0.915 [1]	0.902 [1]	0.889 [1]	0.877 [1]
Market Cap.	0.985 [4]	0.969 [4]	0.953 [4]	0.937 [4]	0.921 [4]	0.905 [4]	0.889 [4]	0.873 [4]	0.857 [4]	0.840 [4]

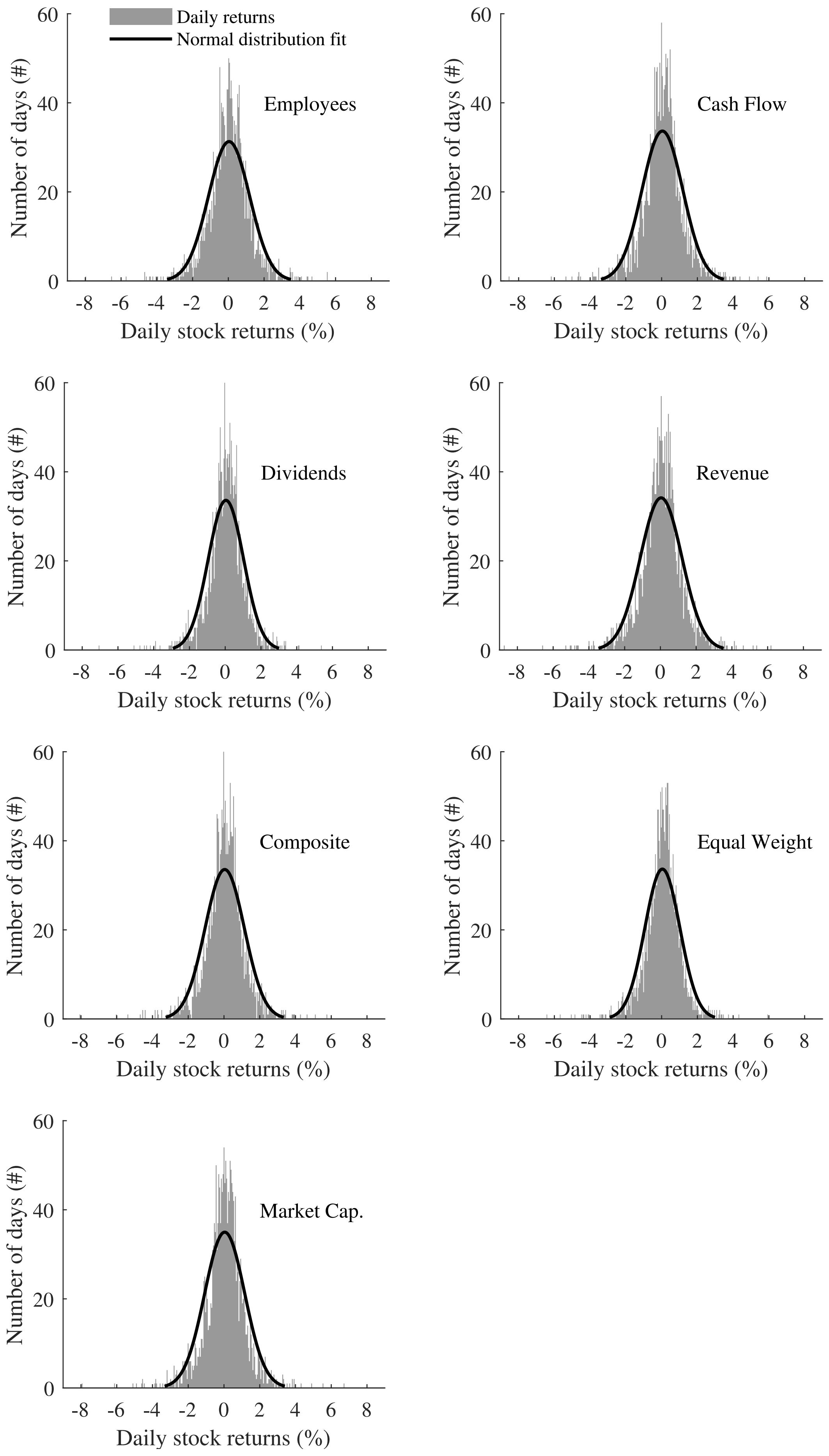

Figure 2: Distribution of daily stock returns for all portfolios. Additionally, the black solid line shows a normal distribution fit of the data.