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1 Theory and results

The coherent (ensemble average) transmitted and reflected fields from a particulate slab are most commonly computed by the effective wavenumber approach, see *e.g.* [5]. The effective wave number is obtained from the roots of a determinant relation. An alternative method, presented in [1], solves the coherent transmitted and reflected fields from a particulate slab by solving a system of integral equations in the depth variable. We let a plane wave impinge at normal incidence on the slab, $z \in [0, d]$, containing spherical dielectric particles of radius a. The scattered fields (ensemble average) on either side of the slab, $[z_1, z_2] = [a, d-a]$, are

$$\boldsymbol{E}_{\mathrm{s}}^{\pm}(\boldsymbol{r}) = \frac{3f}{2(ka)^3} \sum_n \mathrm{i}^{-l+\tau-1} \boldsymbol{A}_n(\pm\hat{\boldsymbol{z}}) k \int_{z_1}^{z_2} \mathrm{e}^{\pm \mathrm{i}kz'} f_n(z') \,\mathrm{d}z' \mathrm{e}^{\pm \mathrm{i}kz}, \quad \begin{cases} z > d \\ z < 0 \end{cases}$$

The summation is over the multi-index $n = \{\tau, \sigma, m, l\}$, $\tau = 1, 2$, $\sigma = \mathrm{e}, \mathrm{o}$, $m = 0, 1, 2, \ldots, l$, and $l = 1, 2, 3\ldots$, and $\boldsymbol{A}_n(\hat{\boldsymbol{k}}_{\mathrm{i}})$ are the vector spherical harmonics, see [3] for more details. The volume fraction of the spheres is denoted f. The coefficients $f_n(z)$ satisfies a system of linear, one-dimensional integral equations in z [1], *viz.*,

$$f_n(z) = \mathrm{e}^{\mathrm{i}kz} \sum_{n'} T_{nn'} a_{n'} + k \int_{z_1}^{z_2} \sum_{n'} K_{nn'}(z - z') f_{n'}(z') \,\mathrm{d}z', \quad z \in [z_1, z_2]$$

The entries of the kernel in this set of integral equations consist of rapidly oscillating integrals. Fortunately, for the hole correction (HC), these integrals have a closed form solution in terms of a series of spherical waves [2]. Without this analytic solution of the integrals, the integral equation approach offers challenging numerical integration. The particles are completely characterized by the transition matrix $T_{nn'}$, which for a spherical particle is diagonal in its (pairwise) indices. The expansion coefficients of the plane wave in terms of regular spherical vector waves are denoted a_n, see [3].

The hole correction — an adequate approximation for gases and other tenuous media – gives less accurate results for *e.g.*, liquids or other amorphous materials. In this paper, we analyse the effect of the Percus-Yevick (P-Y) approximation of the pair distribution function on the transmitted and reflected fields [4]. This P-Y approximation enlarges the scope of the integral equation approach considerably, and we compare the effect of the P-Y approximation on reflection and transmission from a particulate slab of finite thickness. The kernel entries now include integrals with rapidly oscillating integrands, but, due to the form of the P-Y approximation, the integration interval is accurately approximated by a finite interval, which makes numerical integration feasible.

References

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