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# An Automatic Start-Up Procedure for Multivariable Control Systems

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*Abstract*

This report proposes a start-up procedure for multivariable control systems. The procedure consists of three main parts: operator inputs, step-response experiments, and interaction analysis. An application to a paper machine model is shown.

In the appendix, eleven textbook process examples are presented.

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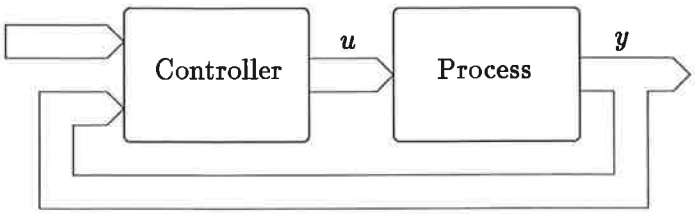


**Figure 1.** The multi-input multi-output control system.

# 1. Introduction

Automatic start-up procedures for single-input single-output (SISO) control systems are included in many industrial controllers. In this report, the multi-input multi-output (MIMO) problem is considered. It is highly motivated since MIMO processes are very common in practice, for instance in chemical process control systems. Today much engineering knowledge is used to design these control systems; knowledge that probably to some extent could be implemented in an automatic procedure. A collection of ideas is given below on how an automatic start-up procedure for multivariable control systems could be planned.

The goal of the start-up procedure described in this report is to, under secure conditions, estimate a process model good enough to be used for initiation of further experiments or controller design. A block diagram of the control system is shown in Figure 1. The thick lines illustrate the fact that there are several reference, control, and measured signals in the system. There are $m$ control signals collected in $u = (u_1, \ldots, u_m)$ and $p$ measured signals $y = (y_1, \ldots, y_p)$. In the following, the control signals are often called the inputs (to the process) and the measured signals the outputs (from the process). It is required that as little prior information as possible about the process should have to be added in the start-up procedure. The considered MIMO processes are assumed to be well approximated by a linear system and stable; for instance, no integrating processes are allowed. The number of inputs and outputs are assumed to be between five and fifteen. To avoid problems with initial states, the processes are assumed to be in steady-state at the beginning of the start-up procedure.

The MIMO start-up procedure must be far more complex than a SISO procedure. Possible cross couplings in the MIMO process cause one or more control signals to affect several outputs. These cross couplings have to be determined by the start-up procedure since they will highly influence the choice of controller structure. Also, the cross couplings may affect the identification experiments. An example of problems that can arise if SISO autotuning is generalized in one particular way is showed in [Johansson, 1993].

We have chosen to divide the start-up procedure into three main steps. They are

- Operator inputs,
- Step response experiments, and
- Interaction analysis.

These steps should then be followed by controller design or further modeling. The schedule is summarized in Figure 2. The **operator inputs** are assumed to be entered by a process engineer. Prior knowledge about the process can be used to enhance the quality of the modeling by, for instance, suggesting a relevant sample interval. Steps are used as excitation signals and through

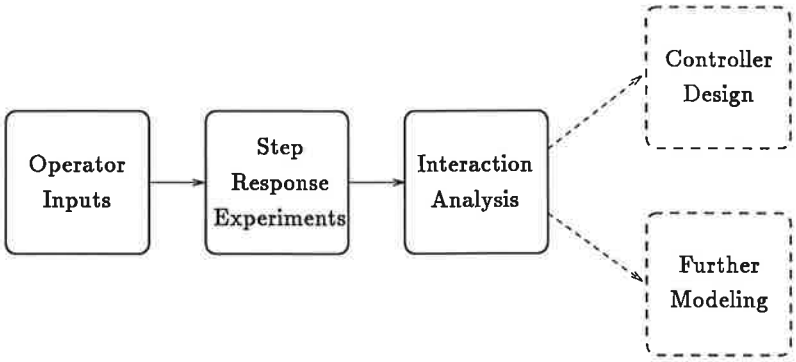


**Figure 2.** A schematic view of the start-up procedure.

the **step-response experiments** data are collected, which are used in the **interaction analysis** to produce a simple model of the process plant. Out of this model, possible control structures are suggested.

Section 2, Section 3, and Section 4 are devoted to the three steps in the MIMO start-up procedure, respectively. In Section 5, the procedure is applied to a paper machine process. A short summary is given in Section 6, together with ideas about future developments. Finally a number of MIMO processes, useful as simple test examples, are listed in the appendix.

## 2. Operator Inputs

The reason for having an input phase in the start-up procedure is to allow the user to include process knowledge in the procedure. This might, for instance, shorten the experiment time or give experiment results with a higher accuracy. But since all inputs have "intelligent" default values, it is also possible for the operator to simply confirm the given values.

The operator inputs are

1. Sample interval,
2. Step amplitude,
3. Step time,
4. Maximum output deviation,
5. Degree of input and output criticality, and
6. Known dynamics.

In Figure 3, an example is shown on an operator interface implemented in Matlab [MathWorks, 1992]. This interface originates from the paper machine example in Section 5.

In the main window "Operator Interface", there is a block illustrating the paper machine with its five inputs and five outputs. Above the block diagram, the operator can confirm or reject the suggested *sample interval.*

Connected to process input three, an "Edit Input Characteristics" window is opened. In this example, $u_3$ happened to be the slice opening control signal. A *step amplitude* for the following step response experiment is suggested. The amplitude is given in percent, which here is an absolute value. It is assumed that the actuators signals are tuned such that the steady-state control signal is in the approximate interval 10–90%. A good estimate of the required step amplitude can be found by applying consecutively higher and higher steps to

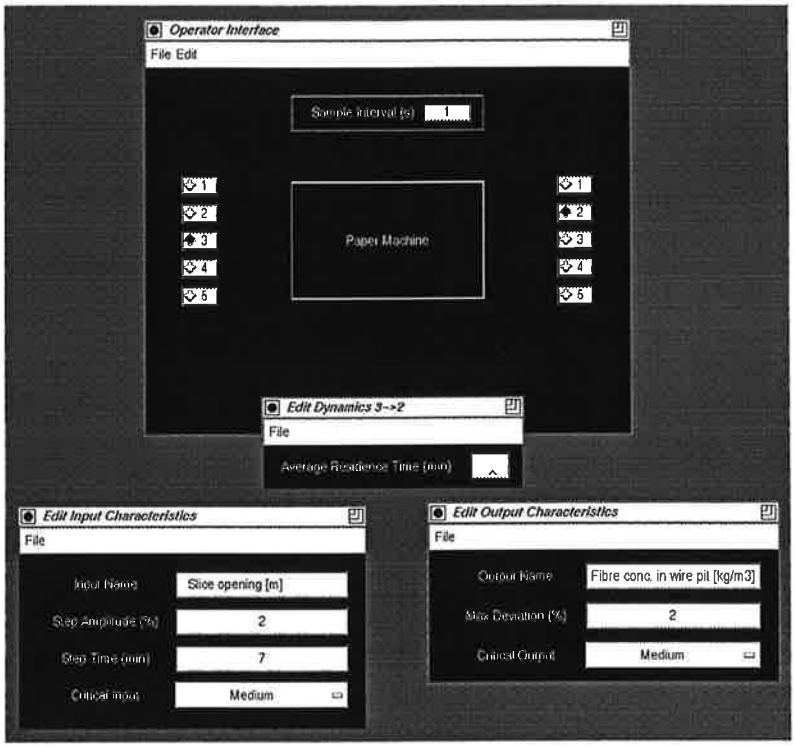


**Figure 3.** Operator interface used for the paper machine example in Section 5.

the input and stop when a sufficient change in the measured outputs is noticed. The length of the step is given by the *step time* parameter. A too short step may not let the step experiment reveal the dynamics of the coupling, and a too long step gives a start-up procedure which takes unnecessarily long time. If an input is specified to be *critical*, it can be used to suggest an overall sample interval relevant for the dynamics connected to that input.

Connected to process output two an "Edit Output Characteristics" window is opened. In this example, the measured signal is the fibre concentration in the wire pit. A *maximum deviation* of the output signal is suggested. It is given in percent in the same way as the input step amplitude. It can also be specified, how *critical* an output is.

Related to each input–output pair is an "Edit Dynamics" window. If an estimate of the dynamics is known, it can be entered here. It is specified as the *average residence time*, which is a rough measure of the time it takes for the step response to settle [Åström and Hägglund, 1994]. For instance, the known dynamics can be used to improve the suggestion of step time. The static gain is also important for the experiment set-up, thus it would be reasonable to include it as a parameter to be specified if it is known.

## 3. Step-Response Experiments

To identify the model of the plant, it has to be affected by test signals. In our approach, double step signals are consecutively applied to all inputs. This gives the following scenario. A double step of the shape shown in Figure 4 is applied to input 1 ($u_1$). The time length of the positive part of the step ($t_1$) is equal to the length of the negative part, as shown in the figure. After all measured outputs are settled, a double step of length $t_2$ is applied to $u_2$, etc.

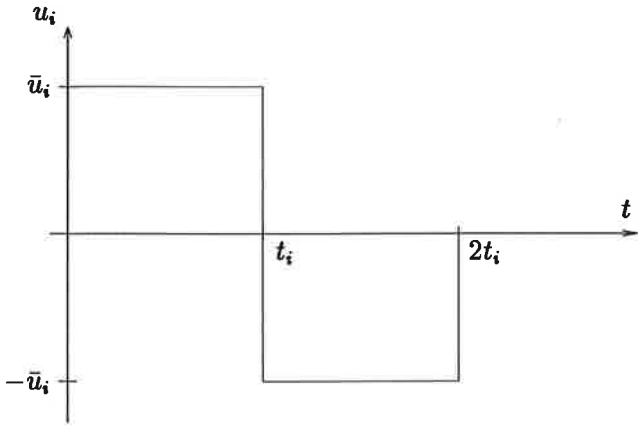


**Figure 4.** The input signal for the identification experiments is a double step.

In this way there will be $m$ double steps, equal to the number of inputs. The amplitude $\bar{u}_i$ of each step is determined in the way described in Section 2.

While doing step-experiments on a real process, it is important that the process states do not move far away from the operating point. Being close to the operating point is necessary because of safety reasons, but also since the linear model to be estimated is only a good approximation in a neighborhood of that point. There are often certain limits the measured outputs are not allowed to exceed. In our start-up procedure, the specification of these limits was described in Section 2. Upcrossings of the limits are avoided by a simple linear prediction. In the case of no predicted upcrossings, the length of the positive part of the step ($t_i$ in Figure 4) is equal to the step time specified in the operator input phase. However, this time is shortened if an upcrossing is predicted in one of the outputs. The prediction is done in the following way. Consider one of the outputs $y_j$, see Figure 5. At time $t_i$ it has reached the level marked with an asterisk. If we approximate $y_j$ at $t_i$ with a straight line, it follows the equation

$$y_j(t) \approx v \cdot (t - t_i) + y_j(t_i) \tag{1}$$

where $v$ is the slope of the line, that is, $v = \dot{y}_j(t_i)$. Assume the negative part of the double step is applied at $t_i$ as shown in Figure 4. Since the process is linear and the step response, without the negative step applied, follows (1), it is possible to derive an approximate value of $y_j(2t_i)$. This gives an equation for the second straight line in the figure:

$$y_j(t) \approx vt_i + y_j(t_i) - 2y_j(t_i) - v \cdot (t - 2t_i) = -vt + 3vt_i - y_j(t_i) \tag{2}$$

The intersection of the lines (1) and (2) is, for a broad class of systems, above the true maximum for $y_j(t), t \in (t_i, 2t_i)$.[1] Hence, the intersection can be treated as an estimate for the maximum of $y_j$. The estimate is given by

$$\hat{y}_j = vt_i$$

The upcrossing prediction described above is done in real-time. Thus, whenever $\hat{y}_j$ exceeds its maximum deviation limit (described in Section 2), the

1 Typically this holds for processes with step responses like the one in Figure 5. It is also easy to handle non-minimum phase processes just by switching off the upcrossing prediction for a short time interval in the beginning of the step experiment.

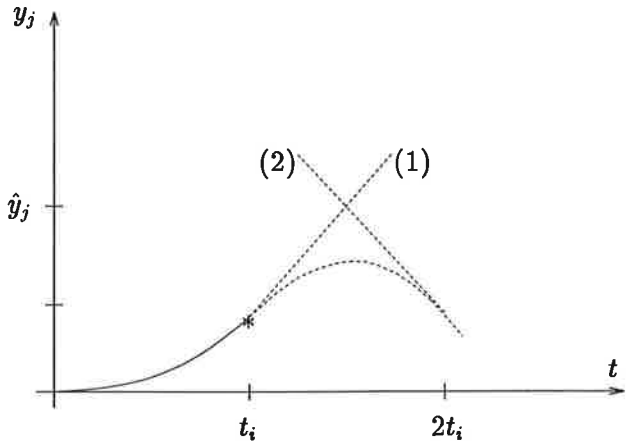


**Figure 5.** Upcrossing is avoided by linear prediction. The positive step is interrupted if $\hat{y}_j$ exceeds the maximum deviation limit.

negative step is applied. Also, since the process is MIMO, the first upcrossing predicted in *any* output $j \in \{1, \ldots, p\}$ acts as a trigger. Data from the $m$ step-response experiments from the $p$ outputs are logged. They are used in the modeling described in the next section.

Notice that after a double step experiment, the process will ideally remain in its initial state. This is the main reason for using double steps instead of single steps. Also, notice that the upcrossing prediction described above is quite a crude approximation. It is possible not only to use the derivative of $y_j$ at each time instant $t_i$ to estimate an upcrossing, but also the whole range of data collected from $t \in (0, t_i)$. These data can be used to identify a more complex model in real-time and thus, give a better prediction of upcrossings.

# 4. Interaction Analysis

The interaction analysis is divided into two parts: derivation of a process model and analysis of the model. The chosen class of process models is characterized by transfer function matrices $G$ with elements consisting of first-order transfer functions with time delays, that is,

$$G(s) = \begin{pmatrix} G_{11}(s) & \ldots & G_{1m}(s) \\ \vdots & & \vdots \\ G_{p1}(s) & \ldots & G_{pm}(s) \end{pmatrix}$$

where

$$G_{ji}(s) = \frac{K_{ji}}{1 + sT_{ji}} e^{-sL_{ji}} \tag{3}$$

The estimation and analysis of this model are described below.

## Modeling

The parameters of each transfer function $G_{ji}$ are determined from the data collected in the way described in Section 3. The so called *method of weighted moments* is used to estimate $G_{ji}$. It was suggested in [Bernhardsson and Persson, 1990], but is also thoroughly discussed in [Åström and Hägglund, 1994]. The ideas are recalled here. Let $Y_j$ and $U_i$ be the Laplace transforms of $y_j$ and $u_i$, respectively. Then the relation $Y_j(s) = G_{ji}(s)U_i(s)$ holds. By taking the

derivative of $Y_j$ and $U_i$, at a certain real valued point $s = \alpha$, we get from the definition of the Laplace transform

$$Y_j^{(n)}(\alpha) = (-1)^n \int_0^\infty e^{-\alpha t} t^n y_j(t) dt \tag{4}$$

$$U_i^{(n)}(\alpha) = (-1)^n \int_0^\infty e^{-\alpha t} t^n u_i(t) dt \tag{5}$$

Hence, the signals $y_j$ and $u_i$ can be seen as *weighted* by the function $e^{-\alpha t} t^n$. In this way, it is possible to derive an estimate of $G_{ji}(\alpha)$ and its derivatives $G'_{ji}(\alpha)$ and $G''_{ji}(\alpha)$. We also derive these three quantities from (3), and thus get three equations in $K_{ji}$, $L_{ji}$, and $T_{ji}$. The solution to these are

$$T_{ji} = \frac{a_{ji}}{1 - \alpha a_{ji}}$$

$$L_{ji} = -\frac{G'_{ji}(\alpha)}{G_{ji}(\alpha)} - a_{ji}$$

$$K_{ji} = (1 + \alpha T_{ji}) G_{ji}(\alpha) e^{\alpha L_{ji}}$$

$$a_{ji} = \sqrt{\frac{G''_{ji}(\alpha)}{G_{ji}(\alpha)} - \left(\frac{G'_{ji}(\alpha)}{G_{ji}(\alpha)}\right)^2}$$

which give the model estimate $G_{ji}$.

In practice, the infinite integral in (4) is approximated by a finite sum. A nice feature is that the method is not sensitive to mean zero noise in $y_j$. Since the shape of $u_i$ is known, the expressions given by (5) can be calculated in advance.

The choice of the parameter $\alpha$ is important for the behavior of the method of weighted moments. The initial part of the response should be weighted heavily to reveal the dynamic coupling between $u_i$ and $y_j$. It is reasonable to choose $\alpha$ as a guess from $1/(L_{ji} + T_{ji})$, that is, the reciprocal of the average residence time. If no guesses are available, we choose $\alpha = 2\ln(20)/t_i$.

### Analysis

The matrices

$$K = \begin{pmatrix} K_{11} & \dots & K_{1m} \\ \vdots & & \vdots \\ K_{p1} & \dots & K_{pm} \end{pmatrix} \quad L = \begin{pmatrix} L_{11} & \dots & L_{1m} \\ \vdots & & \vdots \\ L_{p1} & \dots & L_{pm} \end{pmatrix} \quad T = \begin{pmatrix} T_{11} & \dots & T_{1m} \\ \vdots & & \vdots \\ T_{p1} & \dots & T_{pm} \end{pmatrix}$$

estimated above can be used to determine the interaction in the process and to give a suggestion of control structure. It is interesting to know whether the process could be divided into a number of subprocesses, which can be controlled independently. The extreme, but often desirable, case is to control an $m \times m$ process with $m$ SISO controllers, for example PID controllers.

An obvious way to try to reveal the process structure is to compare the sizes of the matrix elements in $K$, $L$, and $T$, respectively. In simple cases, it is possible to permute the matrices so that independent subprocesses appear. Triangular structures may arise. These can typically be treated by feedforward together with ordinary control. We illustrate this in a simple example.

EXAMPLE 1—Feedforward
Assume the process has two inputs and two outputs, and the estimated model has the form

$$G(s) = \begin{pmatrix} G_{11}(s) & G_{12}(s) \\ 0 & G_{22}(s) \end{pmatrix}$$

Then, it is often sufficient to use two PID controllers and one feedforward: one PID controller in the $u_1$–$y_1$ loop, the other in the $u_2$–$y_2$ loop, and a feedforward from $u_2$ to $u_1$ removing the second loop's influence on the first one. □

Another standard case consists of processes which can be controlled by a cascade control structure:

EXAMPLE 2—Cascade Control
Assume the process has two inputs and one output, and the estimated model can be factorized as

$$G(s) = \begin{pmatrix} G_1(s) \\ G_2(s)G_1(s) \end{pmatrix}$$

Then, cascade control is often preferable. $u$ and $y_1$ can be connected by one PID controller which has the control signal from a second PID controller connected to $y_2$ as reference signal. □

Some of the searches for process structure described above can easily be automated, while others present severe difficulties, since all structures are not shown directly in the matrix elements. Next, we discuss another way of finding a good control structure.

The pairing problem is to connect $m$ SISO controllers to an $m \times m$ process in a best possible way. The problem is in many cases solved by deriving the relative gain array (RGA) [McAvoy, 1983]

$$\Lambda(0) = K .* K^{-T}$$

where $.*$ denotes the element-by-element (Schur) product and $K^{-T}$ is the transpose of the inverse of $K$. Permutation of $\Lambda(0)$ following the criteria in [McAvoy, 1983] gives a pairing suggestion. Basically, it tells us to put elements of $\Lambda(0)$ as close to unity as possible on the diagonal. RGA can also be used for finding diagonal structures in the process where the size of the diagonal blocks is larger than one. Then MIMO controllers should be used for these blocks and SISO controllers for the size one blocks.

Since only the steady-state gains are present in the RGA, no adjustment to *dynamics* in the process is done. One way to check, if the dynamics influence the control structure, is to derive a dynamic RGA

$$\Lambda(i\omega) = G(i\omega) .* G^{-T}(i\omega)$$

at a number of frequencies $\omega$, and check whether the structure of $\Lambda$ changes. If the pairing suggestions are different for different frequencies, it implies that the process may not be controlled by only SISO controllers.

Notice that the matrices $K$, $L$, and $T$ can be used for tasks other than determining the control *structure*. It is, of course, possible to *design* controllers out of them, for instance, a first controller can be designed and used while more sophisticated modeling is being done. In state-space MIMO identification, the time delays are crucial for the accuracy of the model, in particular, they heavily affect the model order. Then, the matrix $L$ can be useful for initial guesses regarding the true delays between the inputs and the outputs.

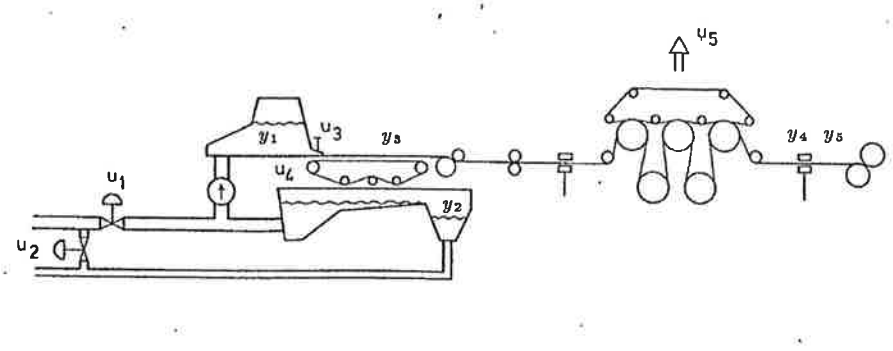


**Figure 6.** The paper machine process with five inputs and five outputs.

## 5. Paper Machine Example

The MIMO start-up procedure is now applied to an example of a paper machine. The procedure has been implemented in Matlab [MathWorks, 1992]. The considered process is a linear model of a paper machine and was presented in [Åström, 1973]. A sketch of the process is shown in Figure 6. Very briefly, it can be described as follows: the stock flow comes in from the left, enters the head box, and poors out on the wire. Some of the flow returns via the wire box. The rest will become paper, and after pressuring and drying, paper leaves the paper machine to the right. The process has five inputs and five outputs:

| | |
|---|---|
| $u_1$ | thick stock flow [$m^3/s$] |
| $u_2$ | thick stock fibre concentration [$kg/m^3$] |
| $u_3$ | slice opening [m] |
| $u_4$ | wire speed [m/s] |
| $u_5$ | water removal rate [kg/s] |
| $y_1$ | fibre concentration in headbox [$kg/m^3$] |
| $y_2$ | fibre concentration in wire pit [$kg/m^3$] |
| $y_3$ | wet line position [m] |
| $y_4$ | fibre weight [$kg/m^2$] |
| $y_5$ | water to fibre ratio |

The process has two states and is given in the state-space form

$$\dot{x} = \begin{pmatrix} -0.1250 & 0.1160 \\ 0.0002 & -0.0116 \end{pmatrix} x + \begin{pmatrix} 0.3004 & 0.3004 & -0.2958 & -0.2955 & 0 \\ 0.0005 & 0 & -0.0059 & 0 & 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} -2.7350 & 0 \\ 0 & -2.1036 \\ 0.2154 & 0 \\ 0.3965 & 0 \\ 0.1349 & 0 \end{pmatrix} x + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0788 & 0 & 0 \\ 0 & 0 & 1.1449 & 0 & 0 \\ 0 & 0 & 0.3898 & 0.3303 & 0.3292 \end{pmatrix} u$$

The time constants are 8 and 88 seconds. The inputs and outputs are normalized to be 80% at steady-state.

### Operator Inputs

In this example, we make a quite arbitrary choice of input parameters. Some of them are shown in Figure 3. The input parameters are

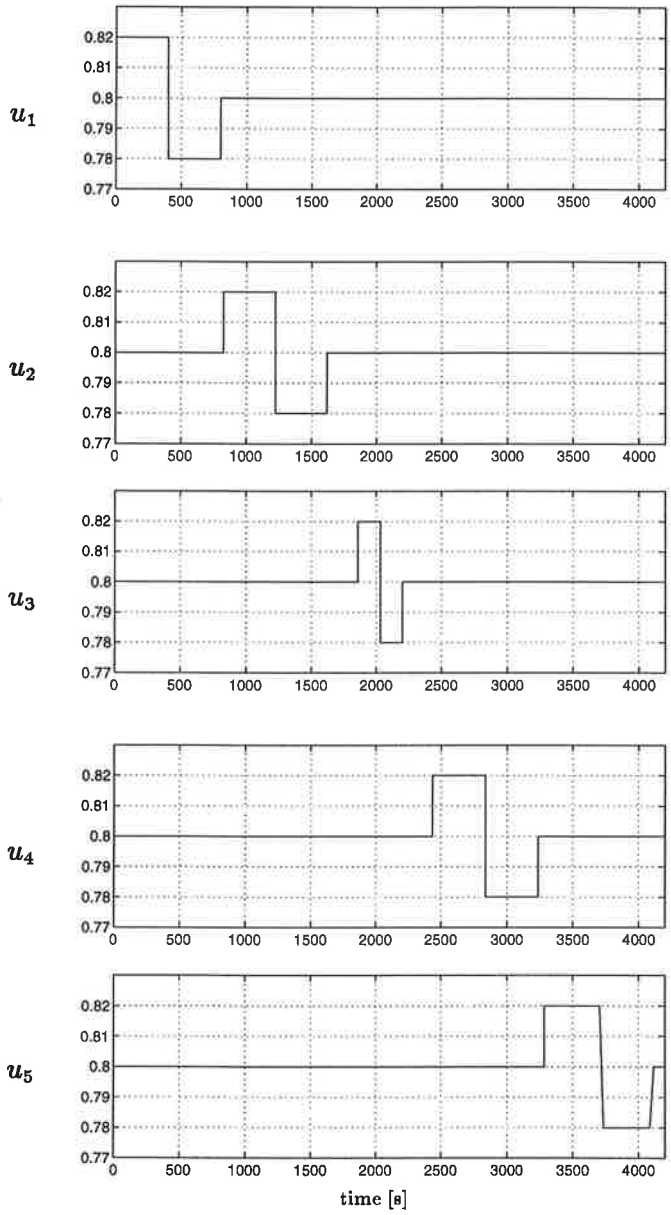


**Figure 7.** Input signals during the step-response experiments.

| | |
|---|---|
| Sample interval | 1 [s] |
| Step amplitude | 2% for all inputs |
| Step time | 400 [s] for all inputs |
| Maximum output deviation | 2% for $y_2$ and 20% for the others |
| Degree of criticality | Not specified |
| Known dynamics | None |

### Step-Response Experiments

In Figures 7 and 8, the results from the step-response experiments are shown. The whole experiment takes about an hour. We notice that the double step in $u_3$ is shorter than $2 \cdot 400 = 800$ seconds. This is due to an estimated upcrossing in $y_2$. We did not allow the fibre concentration in the wire pit to deviate more than 2%. The spikes in $y_1$, $y_2$, and $y_3$ are due to the direct terms in the process.

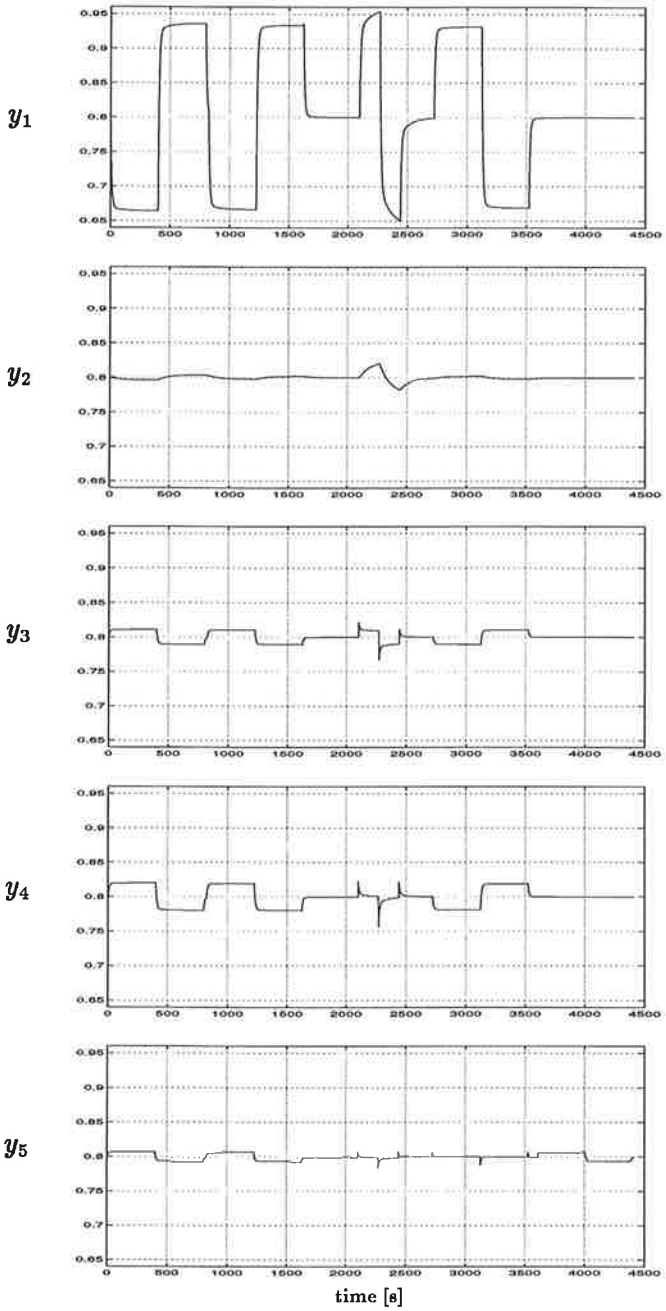


**Figure 8.** Measured output signals from the step-response experiment.

## Interaction Analysis

The method of weighted moments, run on the data shown above, gives $K$, $L$, and $T$. All elements of the delay matrix $L$ are zero except for one element, but that one relates to a very low steady-state gain. Hence, the matrix $L$ is unnecessary for the interaction analysis of this process (which is not surprising since the process does not contain any delays). The time constant matrix $T$ may include negative elements, since the basic version of the method of weighted moments does not check for stability. If these elements are simply

set to zero, the following process model will result

$$K = \begin{pmatrix} -6.8 & -6.6 & 7.3 & 6.6 & -0.0 \\ -0.2 & 0.0 & 1.2 & 0.1 & -0.0 \\ 0.5 & 0.5 & 0.6 & -0.5 & 0.0 \\ 1.0 & 1.0 & 0.2 & -1.0 & 0.0 \\ 0.3 & 0.3 & 0.1 & 0.0 & 0.3 \end{pmatrix} \qquad T = \begin{pmatrix} 9 & 10 & 12 & 9 & 0 \\ 92 & 0 & 89 & 0 & 0 \\ 9 & 10 & 0 & 9 & 0 \\ 9 & 10 & 0 & 9 & 0 \\ 9 & 10 & 0 & 0 & 0 \end{pmatrix}$$

A comparison of the step-responses between the process and the model, given by $K$ and $T$, shows a good accordance.

The RGA is

$$\Lambda(0) = \begin{pmatrix} 61.19 & 12.78 & 0.53 & -73.32 & -0.19 \\ 0.96 & 0.05 & -0.00 & -0.01 & -0.00 \\ 104.00 & 12.79 & 0.44 & -115.87 & -0.37 \\ -164.61 & -24.64 & 0.01 & 190.23 & 0.01 \\ -0.53 & 0.01 & -0.00 & -0.03 & 1.55 \end{pmatrix}$$

Let us assume that we are restricted to control the paper machine process with five SISO controllers. By permuting $\Lambda(0)$ we get the following suggested input–output pairing.

$$\begin{aligned} u_1 &\text{ — } y_2 \\ u_2 &\text{ — } y_3 \\ u_3 &\text{ — } y_1 \\ u_4 &\text{ — } y_4 \\ u_5 &\text{ — } y_5 \end{aligned}$$

This pairing seems intuitively reasonable, compare Figure 6.

## 6. Conclusions

We have discussed some topics that might be included in a start-up procedure for multivariable control systems. This procedure consists of three main parts: operator inputs, step-response experiments, and interaction analysis.

A start-up procedure to be used in industrial applications is inevitably very complex. But if we restrict a discussion about future developments to the three blocks discussed in this report, some specific remarks can be made.

The upcrossing estimation used during the step-response experiments can easily be improved. By modeling simple processes recursively while the step is applied, it is possible to get more reliable estimates if an upcrossing will occur or not.

The model estimation in the interaction analysis can be done in a large number of ways. If the model class is determined to be the one used in this report, we can compare the method of weighted moments with, for instance, a least-squares fit of the step-response to the given model.

RGA is a well-accepted tool in chemical process control. Its main drawback is that it does not contain any dynamic information about the process. It would be beneficial to have interaction measures built upon all three of the matrices $K$, $L$, and $T$. These should then give criteria as to when SISO controllers are sufficient or when a MIMO controller has to be used.

One start-up procedure for SISO processes is based on the autotuner. Relay feedback is used to tune the parameters of a PID controller. This start-up procedure has been successfully used in practice, see [Åström and Hägglund, 1984] and [Modén, 1994]. As mentioned in the introduction, a crude generalization of the relay method to the MIMO case is not possible, for example by simply connecting $m$ relays between the inputs and outputs for an $m \times m$ process. However, if the number of inputs and outputs is small (less than 5–15 as in our specifications), a number of SISO relay experiments can be performed on the MIMO process. This approach has been described in for instance [Friman and Waller, 1994].

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# Appendix A. A Bunch of Textbook Processes

In the following, eleven process examples are presented. All are linear and multi-input multi-output. Very brief comments are given together with reference notes.

EXAMPLE 1—Binary Distillation Column
This is a model of a binary distillation column where pressure variation is included. The process has 4 inputs (including one disturbance $u_4$) and 3 outputs. It is given in state-space form.

$$\dot{x} = \begin{pmatrix} A_1 & A_2 \end{pmatrix} x + Bu$$
$$y = Cx$$

where

$$A_1 = \begin{pmatrix} -0.0140 & 0.00430 & 0 & 0 & 0 \\ 0.00950 & -0.0138 & 0.00460 & 0 & 0 \\ 0 & 0.00950 & -0.0141 & 0.00630 & 0 \\ 0 & 0 & 0.00950 & -0.0158 & 0.0110 \\ 0 & 0 & 0 & 0.00950 & -0.0312 \\ 0 & 0 & 0 & 0 & 0.0202 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0.0255 & 0 & 0 & 0 & \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.00 \\ 0 & 0 & 0 & 0 & 0 & 2.00 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0150 & 0 & 0 & 0 & 0 & 0 \\ -0.0352 & -0.0220 & 0 & 0 & 0 & 0 \\ 0.0202 & -0.0422 & 0.0280 & 0 & 0 & 0 \\ 0 & 0.0202 & -0.0482 & 0.0370 & 0 & 2.00 \cdot 10^{-4} \\ 0 & 0 & 0.0202 & -0.0572 & 0.0420 & 5.00 \cdot 10^{-4} \\ 0 & 0 & 0 & 0.0202 & -0.0483 & 5.00 \cdot 10^{-4} \\ 0 & 0 & 0 & 0 & 0.0255 & -0.0185 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 5.00 \cdot 10^{-6} & -4.00 \cdot 10^{-5} & 0.00250 & 0 \\ 2.00 \cdot 10^{-6} & -2.00 \cdot 10^{-5} & 0.00500 & 0 \\ 1.00 \cdot 10^{-6} & -1.00 \cdot 10^{-5} & 0.00500 & 0 \\ 0 & 0 & 0.00500 & 0.0100 \\ 0 & 0 & 0.00500 & 0 \\ -5.00 \cdot 10^{-6} & 1.00 \cdot 10^{-5} & 0.00500 & 0 \\ -1.00 \cdot 10^{-5} & 3.00 \cdot 10^{-5} & 0.00500 & 0 \\ -4.00 \cdot 10^{-5} & 5.00 \cdot 10^{-6} & 0.00250 & 0 \\ -2.00 \cdot 10^{-5} & 2.00 \cdot 10^{-6} & 0.00250 & 0 \\ 4.60 \cdot 10^{-4} & 4.60 \cdot 10^{-4} & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Reference: "Benchmark problems for control system design," IFAC 1990.

EXAMPLE 2—Drum Boiler
This is a model of a drum boiler which is unstable and non-minimum phase. The process has 4 inputs (including one disturbance $u_4$) and 2 outputs. The state-space model is given by

$$A_1 = \begin{pmatrix} -3.93 & -0.00315 & 0 & 0 \\ 368 & -3.05 & 3.03 & 0 \\ 27.4 & 0.0787 & -0.0596 & 0 \\ -0.0647 & -5.2 \cdot 10^{-5} & 0 & -0.255 \\ 3850 & 17.3 & -12.8 & -12600 \\ 22400 & 18.0 & 0 & -35.6 \\ 0 & 0 & 0.00234 & 0 \\ 0 & 0 & 0 & -1.27 \\ -2.20 & -0.00177 & 0 & -8.44 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 4.03 \cdot 10^{-5} & 0 & 0 & 0 \\ 0 & -0.00377 & 0 & 0 & 0 \\ 0 & -2.81 \cdot 10^{-4} & 0 & 0 & 0 \\ 3.35 \cdot 10^{-6} & 3.6 \cdot 10^{-7} & 6.33 \cdot 10^{-5} & 1.94 \cdot 10^{-4} & 0 \\ -2.91 & -0.105 & 12.7 & 43.1 & 0 \\ -1.04 \cdot 10^{-4} & -0.414 & 90.0 & 56.9 & 0 \\ 0 & 2.22 \cdot 10^{-4} & -0.203 & 0 & 0 \\ 0.00100 & 7.86 \cdot 10^{-5} & 0 & -0.0717 & 0 \\ -1.11 \cdot 10^{-4} & 1.38 \cdot 10^{-5} & 0.00149 & 0.00602 & -1.00 \cdot 10^{-10} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & -0.0100 \\ 0 & 0 & 0 & 0 \\ 1.56 & 0 & 0 & 0 \\ 0 & -5.13 \cdot 10^{-6} & 0 & 0 \\ 8.28 & -1.50 & 0.0395 & 52.0 \\ 0 & 1.78 & 0 & 0 \\ 2.33 & 0 & 0 & 0 \\ 0 & -0.0245 & 0.00284 & 0 \\ 0 & 2.94 \cdot 10^{-5} & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Reference: "Benchmark problems for control system design," IFAC 1990.

EXAMPLE 3—The Shell Control Problem
This is a model of a heavy oil fractionator shown in Figure 9. The plant has three product draws and three side circulating loops. The process has 5 inputs (including two disturbances $u_4$ and $u_5$) and 7 outputs. The transfer function matrix $G$ of the model is given by the elements

$$G_{ji}(s) = \frac{K_{ji}}{1 + sT_{ji}} e^{-sL_{ji}}$$

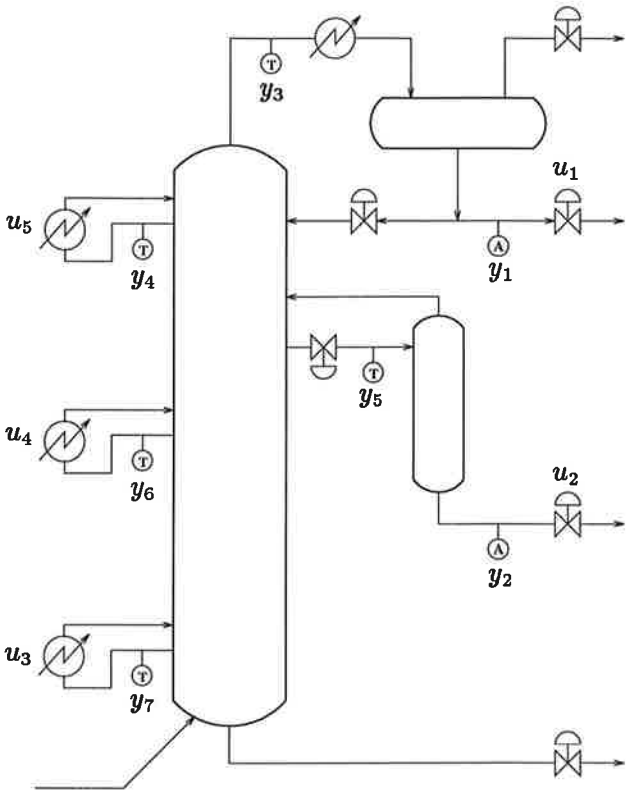


**Figure 9.** The Shell process.

where

$$K = \begin{pmatrix} 4.05 & 1.77 & 5.88 & 1.20 & 1.44 \\ 5.39 & 5.72 & 6.90 & 1.52 & 1.83 \\ 3.66 & 1.65 & 5.53 & 1.16 & 1.27 \\ 5.92 & 2.54 & 8.10 & 1.73 & 1.79 \\ 4.13 & 2.38 & 6.23 & 1.31 & 1.26 \\ 4.06 & 4.18 & 6.53 & 1.19 & 1.17 \\ 4.38 & 4.42 & 7.20 & 1.14 & 1.20 \end{pmatrix} \quad T = \begin{pmatrix} 50 & 60 & 50 & 45 & 40 \\ 50 & 60 & 40 & 25 & 20 \\ 9 & 30 & 40 & 11 & 6 \\ 12 & 27 & 20 & 5 & 19 \\ 8 & 19 & 10 & 2 & 22 \\ 13 & 33 & 9 & 19 & 24 \\ 33 & 44 & 19 & 27 & 32 \end{pmatrix}$$

$$L = \begin{pmatrix} 27 & 28 & 27 & 27 & 27 \\ 18 & 14 & 15 & 15 & 15 \\ 2 & 20 & 2 & 0 & 0 \\ 11 & 12 & 2 & 0 & 0 \\ 5 & 7 & 2 & 0 & 0 \\ 8 & 4 & 1 & 0 & 0 \\ 20 & 22 & 0 & 0 & 0 \end{pmatrix}$$

Reference: "Benchmark problems for control system design," IFAC 1990.

EXAMPLE 4—Gas-Fired Furnace
This is a model of a gas-fired furnace. The process has 4 inputs and 4 outputs. The transfer function matrix $G(s)$ of the model is given by the elements

$$G_{ji}(s) = \frac{K_{ji}}{1 + sT_{ji}}$$

where

$$K = \begin{pmatrix} 1.0 & 0.7 & 0.3 & 0.2 \\ 0.6 & 1.0 & 0.4 & 0.35 \\ 0.35 & 0.4 & 1.0 & 0.6 \\ 0.2 & 0.3 & 0.7 & 1.0 \end{pmatrix} \quad T = \begin{pmatrix} 4 & 5 & 5 & 5 \\ 5 & 4 & 5 & 5 \\ 5 & 5 & 4 & 5 \\ 5 & 5 & 5 & 4 \end{pmatrix}$$

Reference: Munro, E. N., "Modern approaches to control system design," Institution of Electrical Engineers (1979), pp 83–.

EXAMPLE 5—Air Compressor
This is a model of an air compressor. The process has 2 inputs and 2 outputs. The transfer function matrix is

$$G(s) = \begin{pmatrix} \frac{0.1133}{1.783s^2+4.48s+1.0}e^{-0.715s} & \frac{0.9222}{2.071s+1} \\ \frac{0.3378}{0.361s^2+1.09s+1.0}e^{-0.299s} & \frac{-0.321}{0.104s^2+2.463s+1.0}e^{-0.945s} \end{pmatrix}$$

Reference: Munro, E. N., "Modern approaches to control system design," Institution of Electrical Engineers (1979), pp 87–.

EXAMPLE 6—Distillation Column
This is a model of a pilot-scale distillation column. The process has 2 inputs and 2 outputs. The transfer function matrix is

$$G(s) = \begin{pmatrix} \frac{12.8}{16.7s+1}e^{-s} & \frac{-18.9}{21s+1}e^{-3s} \\ \frac{6.6}{10.9s+1}e^{-7s} & \frac{-19.4}{14.4s+1}e^{-3s} \end{pmatrix}$$

Reference: Wood, R. K., and M. W. Berry, "Terminal composition control of a binary distillation column," Chem. Eng. Sci. 28, 1707 (1973).

EXAMPLE 7—Distillation Column
This is a model of a distillation column. The process has 2 inputs and 2 outputs. The transfer function matrix is

$$G(s) = \begin{pmatrix} \frac{-2.16}{8.5s+1}e^{-s} & \frac{1.26}{7.05s+1}e^{-0.3s} \\ \frac{-2.75}{8.25s+1}e^{-1.8s} & \frac{4.28}{9.0s+1}e^{-0.35s} \end{pmatrix}$$

Reference: Luyben, W. L. and C. Vinante, "Experimental studies of distillation decoupling," Kem. Teollisuus 29, 499 (1972).

EXAMPLE 8—Distillation Column
This is a model of a distillation column. The process has 4 inputs and 4 outputs. The transfer function matrix is

$$G(s) = \begin{pmatrix} G_1(s) & G_2(s) \\ G_3(s) & G_4(s) \end{pmatrix}$$

$$G_1(s) = \begin{pmatrix} \frac{2.22}{(36s+1)(25s+1)}e^{-2.5s} & \frac{-2.94(7.9s+1)}{(23.7s+1)^2}e^{-0.05s} \\ \frac{-2.33}{(35s+1)^2}e^{-5s} & \frac{3.46}{32s+1}e^{-1.01s} \end{pmatrix}$$

$$G_2(s) = \begin{pmatrix} \frac{0.017}{(31.6s+1)(7s+1)}e^{-0.2s} & \frac{-0.64}{(29s+1)^2}e^{-20s} \\ \frac{-0.51}{(32s+1)^2}e^{-7.5s} & \frac{1.68}{(28s+1)^2}e^{-2s} \end{pmatrix}$$

$$G_3(s) = \begin{pmatrix} \frac{-1.06}{(17s+1)^2}e^{-22s} & \frac{3.511}{(12s+1)^2}e^{-13s} \\ \frac{-5.73}{(8s+1)(50s+1)}e^{-2.5s} & \frac{4.32(25s+1)}{(50s+1)(5s+1)}e^{-0.01s} \end{pmatrix}$$

$$G_4(s) = \begin{pmatrix} \frac{4.41}{16.2s+1}e^{-1.01s} & \frac{-5.38}{17s+1}e^{-0.5s} \\ \frac{-1.25}{(43.6s+1)(9s+1)}e^{-2.8s} & \frac{4.78}{(48s+1)(5s+1)}e^{-1.15s} \end{pmatrix}$$

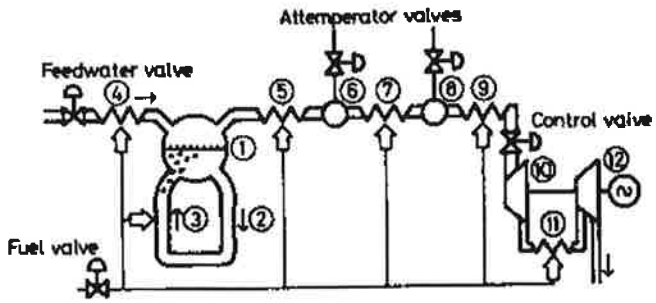


**Figure 10.** Drum boiler process.

Reference: Luyben, W. L., "Simple method for tuning SISO controllers in multivariable systems," Ind. Eng. Chem. Proc. Des. Dev., 1986, 25, pp. 654–660.

EXAMPLE 9—Packed Bed Chemical Reactor
This is a model of a packed bed chemical reactor. The process has 2 inputs and 2 outputs. The transfer function matrix is

$$G(s) = \begin{pmatrix} \frac{-2.265e^{-1.326s}}{0.786s+1} & \frac{0.746e^{-2.538s}}{0.092s+1} \\ \frac{1.841e^{-0.411s}}{0.917s+1} & \frac{-0.654e^{-0.768s}}{0.870s+1} \end{pmatrix}$$

Reference: Marino-Galarraga, McAvoy, Marlin, Ind. Eng. Chem, 26, pp 521–531 (1987).

EXAMPLE 10—Lime-Kiln
This is a model of a lime-kiln process. The process has 2 inputs and 2 outputs. The transfer function matrix is

$$G(s) = \begin{pmatrix} \frac{1.66}{39s+1} & \frac{-1.74}{4.4s+1}e^{-2s} \\ \frac{0.34}{8.9s+1}e^{-s} & \frac{1.4}{3.8s+1}e^{-s} \end{pmatrix}$$

Reference: Charos, G. N., Y. Arkun, and R. A. Taylor, "Model predictive constrained control of an industrial lime kiln," TAPPI J., 1991, pp. 203–211.

EXAMPLE 11—Drum Boiler Turbine Model
This is a 10th order state-space model of a drum boiler turbine, see Figure 10. The process has 5 inputs and 7 outputs. It is given by

$$\dot{x} = \begin{pmatrix} A_1 & A_2 \end{pmatrix} x + Bu$$
$$y = \begin{pmatrix} C_1 & C_2 \end{pmatrix} x + Du$$

where

$$A_1 = \begin{pmatrix} -0.044 & 0 & 0 & 0.019 & 0.050 \\ -1.0 \cdot 10^{-4} & 0 & 0 & 3.4 \cdot 10^{-4} & 2.3 \cdot 10^{-4} \\ -9.9 \cdot 10^{-3} & 0 & -0.2 & 0.015 & 0.017 \\ 0.011 & 0 & 0 & -0.023 & 2.5 \cdot 10^{-3} \\ 0.028 & 0 & 0 & 0 & -0.050 \\ -9.9 \cdot 10^{-5} & 0 & 0 & 1.5 \cdot 10^{-4} & 1.7 \cdot 10^{-4} \\ -0.013 & 0 & 0 & 0 & 0 \\ -0.010 & 0 & 0 & 0 & 0 \\ -9.5 \cdot 10^{-3} & 0 & 0 & 0 & 0 \\ -3.1 \cdot 10^{-3} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -0.51 & 7.3 \cdot 10^{-5} & 1.4 \cdot 10^{-4} & 5.1 \cdot 10^{-4} & 0 \\ -0.25 & -1.0 \cdot 10^{-6} & -1.9 \cdot 10^{-6} & -6.9 \cdot 10^{-6} & 0 \\ -8.6 & -3.4 \cdot 10^{-5} & -6.4 \cdot 10^{-5} & -2.4 \cdot 10^{-4} & 0 \\ -3.9 & -1.1 \cdot 10^{-5} & -2.1 \cdot 10^{-5} & -7.8 \cdot 10^{-5} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.086 & -3.4 \cdot 10^{-7} & -6.4 \cdot 10^{-7} & -2.4 \cdot 10^{-6} & 0 \\ 0 & -7.6 \cdot 10^{-3} & 1.4 \cdot 10^{-4} & 5.3 \cdot 10^{-4} & 0 \\ 0 & 0.016 & -0.017 & 3.8 \cdot 10^{-4} & 0 \\ 0 & 4.5 \cdot 10^{-3} & 8.4 \cdot 10^{-3} & -0.012 & 0 \\ 0 & 2.1 \cdot 10^{-4} & 4.0 \cdot 10^{-4} & 1.5 \cdot 10^{-3} & -2.3 \cdot 10^{-3} \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & -1.6 \cdot 10^{-3} & 0 & 0 & -0.17 \\ 0 & 3.4 \cdot 10^{-5} & 0 & 0 & 2.4 \cdot 10^{-5} \\ 0 & -9.1 \cdot 10^{-4} & 0 & 0 & 0.080 \\ 0 & -3.3 \cdot 10^{-3} & 0 & 0 & -0.026 \\ 0.12 & 0 & 0 & 0 & 0 \\ 0 & -9.1 \cdot 10^{-6} & 0 & 0 & 8.0 \cdot 10^{-4} \\ 0.21 & 0 & 0.013 & 0.012 & -0.18 \\ 0.23 & 0 & -0.095 & -8.5 \cdot 10^{-3} & -0.13 \\ 0.26 & 0 & -0.027 & -0.079 & -0.12 \\ 0.056 & 0 & -1.4 \cdot 10^{-3} & -4.0 \cdot 10^{-3} & -4.9 \cdot 10^{-4} \end{pmatrix}$$

$$C_1 = \begin{pmatrix} 1.1 & 0 & 0 & 0 & 0 \\ 0.11 & 0 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0.79 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1000 & 1000 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ -0.070 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_2 = \begin{pmatrix} 0 & 1.7 \cdot 10^{-3} & 3.2 \cdot 10^{-3} & 0.012 & 0.098 \\ 0 & 7.6 \cdot 10^{-3} & 0.014 & 0.053 & 0 \\ 0 & -5.8 \cdot 10^{-3} & -0.011 & -0.041 & 0.098 \\ -5.4 \cdot 10^{-3} & -0.010 & -0.038 & 0 & \\ 10000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.10 & 0.19 & 0.72 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0.046 & 0.073 & 19 \\ 0 & 0 & -0.040 & -0.12 & 1.9 \\ 0 & 0 & 0.086 & 0.20 & 17 \\ 0 & 0 & 0.072 & 0.17 & 13 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.63 & -1.8 & -0.28 \end{pmatrix}$$

Reference: Åström, K. J., R. D. Bell, "A 10th order linear drum boiler turbine model", 1979, Technical report, Dept. of Automatic Control, Lund Institute of Technology.