A Schuler-tuned Three-Gyro Platform System

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1960

Handlingen avhemligas den 10/4 1964
enligt beslut av C FOA 2.
Se dnr 2473-234

~~HEMLIG~~
~~12 APR 1960~~
~~Försvarets Forskningsanstalt Avdelning 2~~

FOA 2 RAPPORT
A 2267-234
~~AH 2051 - 2032~~
April 1964
~~April 1960~~
(TTN-rapport 591202)

Referensexemplar

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FÖRSVARETS FORSKNINGSANSTALT
AVDELNING 2
Stockholm

FÖRSVARETS FORSKNINGSANSTALT
Avdelning 2
Stockholm 80

FOA 2 rapport
4 2267 - 23 Y
~~AH 2051 - 2032~~
April ~~1960~~ 1961
(TTN-rapport 591202)

HEMLIG
12 APR 1960
Försvarets Forskningsanstalt

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A SCHULER-TUNED THREE-GYRO PLATFORM SYSTEM

In this report the possibilities of Schuler-tuning a three-gyro platform are discussed. The work is a direct continuation of the work reported in reference 1. It is assumed that the reader is familiar with the notations and the main scope of reference 1.

The platform is suspended in a vehicle which moves in a gravity field with spherical symmetry. In section 1 the equations of motion are derived, the conditions for ideal Schuler-tuning are also given. In section 2 the stability of the system is analysed. It is found that the gyros must be arranged in a special way if the system should be stable. The possibilities of obtaining position information from the system are discussed in section 3. It is shown that it is possible to obtain the position of the carrying vehicle without using any accelerometers. Some questions concerning the synthesis of a three-gyro Schuler-tuned platform system are briefly discussed in section 4. For an analysis of the practical problems, the requirements on the components etc. we refer to reference 3 where the instrumentational problems of a single-axis loop are discussed. Some experiments with a single axis system based on the german surplus gyro KZ-14 have been successfully performed.

Utsändes enligt särskild utsändningslista.

TTN-GRUPPEN
Rapport 591202

A SCHULER-TUNED THREE-GYRO PLATFORM SYSTEM

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1. A short description of the system. The equation of motion of the platform

It is a wellknown fact that an ordinary physical pendulum can be Schuler-tuned if the distance between the pivot point of the pendulum and its center of mass is sufficiently small, and the moment of inertia of the pendulum sufficiently great. It is also wellknown that it is impossible to mechanize such a pendulum with the technological means today available. The concept of platform introduced in section 7 of reference 1 has many properties similar to an ordinary physical pendulum, but advances the ordinary physical pendulum to a great extent. The moment of inertia of the platform, e.g., is not at all related to the geometrical structure of the platform. It therefore seems reasonable to assume that it is easier to mechanize a Schuler-tuned platform than to mechanize an ordinary physical pendulum with the Schuler period*.

The basic philosophy is thus to provide the platform with an unbalance of reasonable size and to obtain the high moment of inertia by the proper choice of the internal feedback. Compare reference 1, section 7.

Suppose the stable element to be suspended at a point P, fixed in a vehicle which moves in a gravity field with spherical symmetry. Let O be the center of the gravity field. The vector OP is denoted by $\bar{r}$, and the vector from the point of suspension to the center of mass of the stable element is denoted by $\bar{h}$. Further, let the vector from the center of mass to the center of gravity be $\bar{h}'$. See fig. 1.1.

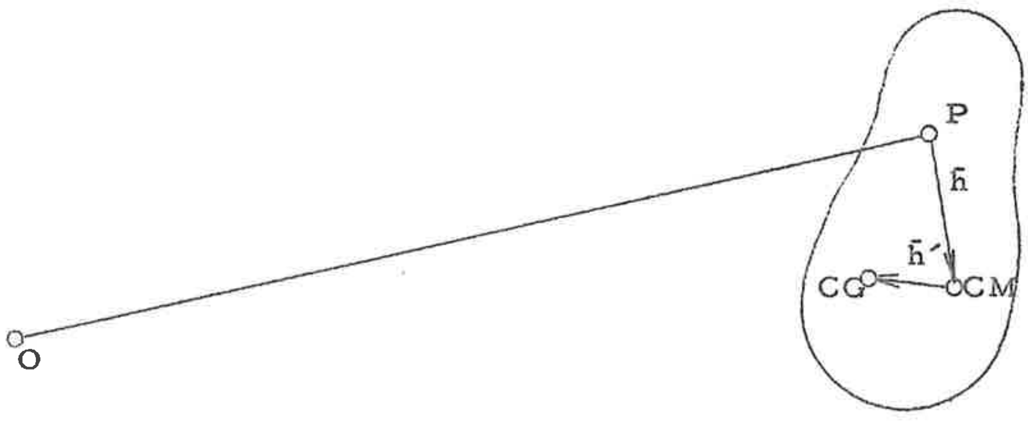

Figure 1.1

*This is also confirmed with experiments.

Newtons Laws of motion gives

$$\begin{cases} \dot{\bar{H}}_{CM} = -\bar{h} \times \bar{F} + \bar{h}' \times \bar{G} & 1.01 \\ m(\ddot{\bar{r}} + \ddot{\bar{h}}) = \bar{F} + \bar{G} & 1.02 \end{cases}$$

where

$\bar{H}_{CM}$ the angular momentum of the platform with respect to its center of mass

$\bar{H}_P$ the angular momentum of the platform with respect to the point of suspension

$\bar{F}$ the force acting on the platform at the point of suspension

$\bar{G}$ the gravity force acting on the stable element

m the mass of the stable element

Equations (1.01) and (1.02) gives

$$\dot{\bar{H}}_P = (\bar{h} + \bar{h}') \times \bar{G} - m\,\bar{h} \times \ddot{\bar{r}}$$

Neglecting the difference between the CM and CG and introducing

$$\bar{G} = m \cdot \bar{g}$$

we get

$$\dot{\bar{H}}_P = m\,\bar{h} \times (\bar{g} - \ddot{\bar{r}}) \qquad 1.03$$

Let ξ_1, ξ_2, ξ_3 be an orthogonal coordinate set fixed to inertial space. Introduce the η_i-set with the origin at the point P and the η_3-axis coincident with the vector PO. The orientation of the η-set around the η_3-axis is specified later.

The ζ-set is fixed to the stable element with the origin at the point P and the axes parallel to the input axes of the gyros. The ζ_3-axis coincides with the vector $\bar{h}$.

The transformations between the coordinate sets are

$$\bar{\eta} = A\,\bar{\xi}\,, \qquad A = \{a_{ik}\} \tag{1.04}$$

$$\bar{\zeta} = B\,\bar{\eta}\,, \qquad B = \{b_{ik}\} \tag{1.05}$$

Assuming the angular velocity of the stable element to be small, we obtain for the time derivative of the angular momentum of the platform.

$$\dot{\bar{H}}_P = J\,K'_{nm}\,\Omega_m\,\hat{\zeta}_n \tag{1.06}$$

where $\bar{\Omega}$ is the angular velocity of the stable element

$$\bar{\Omega} = \Omega_i\,\hat{\zeta}_i$$

and

$$\Omega_m = \frac{1}{2}\left[\dot{b}_{ij}\,b_{lj} + b_{ij}\,b_{ls}\,a_{sk}\,\dot{a}_{jk}\right]\epsilon_{mli} \tag{1.07}$$

Further is

$$\bar{g} = g\,\hat{\eta}_3$$

$$\bar{r} = -\,r\,\hat{\eta}_3$$

Evaluating the cross-product we get for the right membrum of equation (1.03)

$$m\,\bar{h}\,x(\bar{g} - \ddot{\bar{r}}) =$$

$$= mh\left[(g+\ddot{r})b_{13}+2\dot{r}\dot{a}_{3s}b_{lj}a_{js}+r\ddot{a}_{3s}b_{lj}a_{js}\right]\epsilon_{3ln}\,\hat{\zeta}_n \tag{1.08}$$

The equation of motion of the platform is then

$$K'_{nm}(D)\,\Omega_m =$$

$$= \frac{mh}{J}\left[(g+\ddot{r})b_{13}+2\dot{r}\dot{a}_{3s}b_{lj}a_{js}+r\ddot{a}_{3s}b_{lj}a_{js}\right]\epsilon_{3ln} \tag{1.09}$$

Assume that it is possible to choose the differential operators $K'_{mn}(D)$ in such a way that the η_i-set differs from the ζ_i-set only by a small rotation. Hence

$$b_{ij} = \delta_{ij} + \gamma_{ij}$$

where

$$|\gamma_{ij}| \ll 1$$

Neglecting the terms of the equation (1.09) which are of the second order in γ_{ij} and $\mathring{a}_{ij}$ we get

$$\frac{1}{2} K'_{nm}(D) \left\{ \dot{\gamma}_{il} \cdot \epsilon_{mli} \right\} - \frac{mh}{J}(g+\ddot{r}) \gamma_{13} \epsilon_{3nl} =$$

$$= \frac{mh}{J} \left[(g+\ddot{r}) \delta_{13} + 2\dot{r}\mathring{a}_{3s} a_{1s} + r\ddot{a}_{3s} a_{1s} \right] \epsilon_{3ln} - \qquad 1.10$$

$$- \frac{1}{2} K'_{nm}(D) \left\{ \mathring{a}_{ik} a_{lk} \right\} \epsilon_{mli}$$

The transformation (1.05)

$$\zeta_i = b_{ik} \eta_k$$

can be interpreted as the Euler rotations around three successive axes, see figure 1.2.

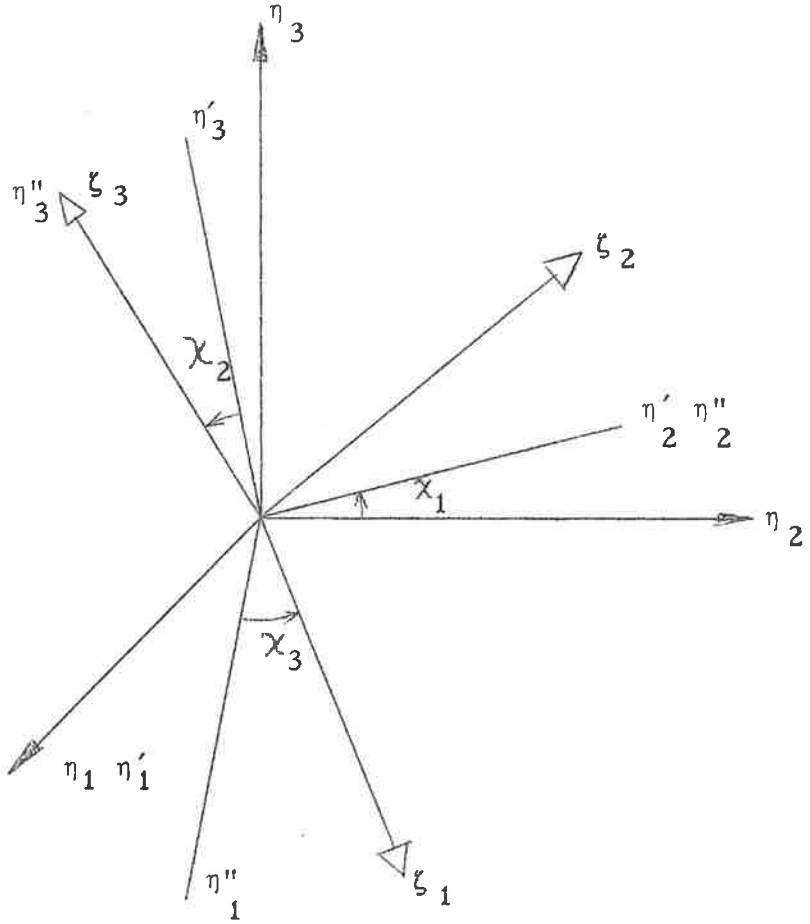

Figure 1.2

When the rotations are small they commute and we get

$$\begin{cases} \chi_1 = \gamma_{23} = -\gamma_{32} \\ \chi_2 = \gamma_{31} = -\gamma_{13} \\ \chi_3 = \gamma_{12} = -\gamma_{21} \end{cases} \tag{1.11}$$

Let ψ be the vertical indication error i.e the angle between the η_3-axis and the ζ_3-axis then

$$\cos\psi = \cos\chi_1 + \cos\chi_2 \tag{1.12}$$

For small angles this reduces to

$$\psi^2 = \chi_1^2 + \chi_2^2 \tag{1.13}$$

Equations (1.10) and (1.11) give

$$\begin{cases} \left[DK'_{11} + \frac{mh}{J}(\ddot{r}+g)\right]\chi_1 + DK'_{12}\chi_2 + DK'_{13}\chi_3 = -\frac{mh}{J}\left[2\mathring{r}\mathring{a}_{2s}a_{3s} + r\ddot{a}_{2s}a_{3s}\right] + K_{11}\mathring{a}_{2s}a_{3s} + K_{12}\dot{a}_{3s}a_{1s} + K_{13}\dot{a}_{1s}a_{2s} \\ DK'_{21}\chi_1 + \left[DK'_{22} + \frac{mh}{J}(\ddot{r}+g)\right]\chi_2 + DK'_{23}\ {}_3 = -\frac{mh}{J}\left[2\mathring{r}\mathring{a}_{3s}a_{1s} + r\ddot{a}_{3s}a_{1s}\right] + K_{21}\mathring{a}_{2s}a_{3s} + K_{22}\dot{a}_{3s}a_{1s} + K_{23}\dot{a}_{1s}a_{2s} \\ DK'_{31}\chi_1 + DK'_{32}\chi_3 + DK'_{33}\chi_3 = +K_{31}\dot{a}_{2s}a_{3s} + K_{32}\dot{a}_{3s}a_{1s} + K_{33}\dot{a}_{1s}a_{2s} \end{cases} \tag{1.14}$$

Introduce the notations

$$\bar{\chi} = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}$$

$$\mathbb{I}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbb{I}_4 = \mathbb{I} - \mathbb{I}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\bar{\omega}(t) = \begin{pmatrix} \dot{a}_{2s} & a_{3s} \\ \dot{a}_{3s} & a_{1s} \\ \dot{a}_{1s} & a_{2s} \end{pmatrix}$$

It is easily seen that ω_i is the η_i-component of the angular velocity of the η-set.

Equation (1. 14) becomes

$$\left[D\,\mathbb{K}'(D) + \frac{mh}{J}(r+g)\mathbb{I}_3\right]\bar{X}(t) = \left[\mathbb{K}'(D) - \frac{mh}{J}(2\dot{r}+rD)\mathbb{I}_3\right]\bar{\omega}(t) \qquad 1.\,14'$$

If it is required that the $\bar{\zeta}$-set and the $\bar{\eta}$-set shall coincide, at least when the vehicle moves at constant height, i.e. r=R=const. we get

$$K'_{ij} = 0 \qquad i \neq j \qquad 1.\,15$$

$$\left[K'_{11} - \frac{mRh}{J}D\right]\omega_1 = 0 \qquad 1.\,16$$

$$\left[K'_{22} - \frac{mRh}{J}D\right]\omega_2 = 0 \qquad 1.\,17$$

$$K'_{33} \cdot \omega_3 = 0 \qquad 1.\,18$$

Equation (1. 15) means that the platform system is diagonal. Equations (1. 16) and (1. 17) are satisfied if

$$K'_{11} = K'_{22} = \frac{mRh}{J}D = \lambda D \qquad 1.\,19$$

This means that the moments of inertia of the platform with respect to the axes ζ_1 and ζ_2 are mRh.

The platform is thus Schuler-tuned with respect to the ζ_1 and ζ_2 axes. Considering the order of magnitude of K'_{11} and K'_{22} we conclude that the high moments of inertia of the platform must be obtained by the proper choice of the internal feedback and not by making a heavy stable element. Compare section 4.

Equation (1. 18) means that the operator K'_{33} should give zero when acting on ω_3. This can be obtained in many ways. A few examples are given below.

(i) Choose the η-set in such a way that ω_3 is zero. This implies that the platform system should be inertial stabilized with respect to the ζ_3-axis, e.g. by choosing

$$K'_{33}(D) = a_1 D + a_2 + \frac{a_3}{D}$$

(ii) Choose the η-set in such a way that ω_3 is constant. Equation (1.18) is then satisfied if

$$K'_{33} = a_1 D$$

Considering disturbing torques this system is not very attractive as a constant disturbing torque will give an angular error increasing with t^2.

(iii) Choose the η-set in such a way that one axis is always pointing to the north. The angular velocity component ω_3 and K'_{33} will then depend on the position and the velocity of the vehicle. It will thus be necessary to feed the torquemotor on the ζ_3-axis from a computer.

How much deviation from the ideal conditions we can allow for is determined by the disturbances and the tolerable indication error. These questions are dealt with elsewhere. See references 2 and 3. Let it suffice by mentioning a few things.

Taking into account the disturbing torques on the stable element, and on the floats of the gyros, equation (1.14') becomes

$$\left[D\,\mathbb{K}'(D) + \frac{mh}{J}(\ddot{r}+g)\mathbb{I}_3\right]\bar{X}(t) = \left[\mathbb{K}'(D) - \frac{mh}{J}(2r+rD)\mathbb{I}_3\right]\bar{\omega}(t) \qquad 1.20$$

$$+ \bar{M}(t) + \frac{1}{A_{22}}\,\mathbb{G}(D)\,\mathbb{S}^{-1}(D)\,\bar{m}(t)$$

Compare equation (6.13), of reference 1.

Assume that the massdistributions of the gyrofloats are symmetric with respect to the output axes, i.e.

$$A_{21} = A_{23} = 0; \qquad A_{22} = a$$

Assume further that the vehicle moves at constant height, i.e. r=R=constant. Equation (1.20) then has constant coefficients. Laplace-transforming with respect to the timecoordinate and solving for $\bar{\chi}(p)$ we get

$$\bar{\chi}(p) = \mathbb{Y}_1(p)\,\bar{\omega}(p) + \mathbb{Y}_2(p)\,\bar{M}(p) + \mathbb{Y}_3(p)\,\bar{m}(p) \tag{1.21}$$

where

$$\mathbb{Y}_1(p) = \left[p\,\mathbb{K}'(p) + \lambda\omega_s^2\,\mathbb{I}_3\right]^{-1}\left[\mathbb{K}'(p) - \lambda p\,\mathbb{I}_3\right] \tag{1.22}$$

$$\mathbb{Y}_2(p) = \left[p\,\mathbb{K}'(p) + \lambda\omega_s^2\,\mathbb{I}_3\right]^{-1} \tag{1.23}$$

$$\mathbb{Y}_3(p) = \frac{1}{a}\left[p\,\mathbb{K}'(p) + \lambda\omega_s^2\,\mathbb{I}_3\right]^{-1}\mathbb{G}(p)\,\mathbb{S}^{-1}(p) \tag{1.24}$$

and

$$\lambda = \frac{mRh}{J}$$

$$\omega_s^2 = \frac{g}{R}$$

Compare reference 2, equations (6.207), (6.208), (6.209) and (6.210).

2. The stability of a Schuler-tuned three-gyro platform system

We will now give some conditions on the stability of the system. Analogous to section 8.1 of reference 1 we introduce.

Definition 2.1

A platform system is said to be stable if a proper torque-puls acting on the stable element or on the float of a gyro gives a finite angular displacement of the stable element.

By a proper torque-puls we mean a torque puls, of such a magnitude that the servos are not saturated, acting for a short time.

This definition and equations (1.21), (1.23) and (1.24) gives the following lemma

Lemma 2.1

A Schuler-tuned platform system is stable if the equations

$$\det \left\{ p\, \mathbb{K}'(p) + \lambda \omega_s^2\, \mathbb{I}_3 \right\} = 0 \qquad 2.01$$

$$\det \left\{ \mathbb{S}(p)\, \mathbb{G}^{-1}(p) \left[p\, \mathbb{K}'(p) + \lambda \omega_s^2\, \mathbb{I}_3 \right] \right\} = 0 \qquad 2.02$$

are stable.

The first equation is the characteristic equation of the system. The stability of this equation implies that the disturbing moments acting on the stable element do not give errors increasing with time. The stability of the second of the above equations means that disturbing moments acting on the gyrofloats do not give errors encreasing with time.

We further obtain the following sufficient condition on the stability of the system.

Corollarium 2.1

A Schuler-tuned platform system is stable if

(i) The arrangement of the gyros is choosen in such a way that $s \geq 3$ and $l = 0$.

(ii) The characteristic equation

$$\det \left\{ p\, \mathbb{K}'(p) + \lambda\, \omega_s^2\, \mathbb{I}_3 \right\} = 0$$

is stable.

(iii) The function $\det \left[\mathbb{K}'(p) - \mathbb{F}(p) \right]$ has no poles in the right half plane.

Proof

Equation (6.14) of reference 1 gives

$$\mathbb{S}(p)\, \mathbb{G}^{-1}(p) = \mathbb{W}(p) \left[\mathbb{K}(p) - \mathbb{F}(p) \right]$$

hence

$$\det \left\{ \mathbb{S}(p)\, \mathbb{G}^{-1}(p) \left[p\, \mathbb{K}'(p) + \lambda \omega_s^2\, \mathbb{I}_3 \right] \right\} =$$

$$= \det \mathbb{W}(p) \cdot \det \left[\mathbb{K}(p) - \mathbb{F}(p) \right]^{-1} \det \left\{ p\, \mathbb{K}'(p) + \lambda \omega_s^2\, \mathbb{I}_3 \right\}$$

The function det $\mathbb{W}(p)$ is stable according to (i) and det $\left[\mathbb{K}(p) - \mathbb{F}(p)\right]^{-1}$ according to (iii). Further is the characteristic equation stable according to (ii). Hence the system is stable according to the lemma 2.1.

If the condition (i) is not satisfied i.e., the function det $\mathbb{W}(p)$ has zeros in the right half plane, the system must be heavily restricted in order to be stable. This fact is illustrated by the following lemma.

<u>Lemma 2.2</u>

For a stable system the equations det $\mathbb{W}(p)$ and det $\left\{\mathbb{K}(p) - \mathbb{F}(p)\right\}$ has the same zeros in the right half plane.

The proof is left for the reader.

3. The position indication loop

An interesting feature of the Schuler-tuned platform system described, is that the output signals of the gyros are functionals of the angular velocity of the vehicle. This fact makes it possible to obtain position indication without using accelerometers. The output signal of the gyros is

$$\bar{\varphi}(t) = \mathbb{S}^{-1}(D)\,\mathbb{W}(D)\,\bar{\Omega}(t) - \frac{1}{A_{22}}\,\mathbb{S}^{-1}(D)\,\bar{m}(t) \qquad 3.01$$

The angular velocity of the stable element $\bar{\Omega}(t)$ is given by equation (1.07)

$$\Omega_m = \frac{1}{2}\left[\mathring{b}_{ij}b_{lj} + b_{ij}b_{ls}\;a_{sk}\mathring{a}_{jk}\right]\epsilon_{mli}$$

Introducing

$$b_{ij} = \delta_{ij} + \gamma_{ij}$$

and neglecting terms of the second order in γ_{ij} and $\mathring{a}_{ij}$ we get

$$\Omega_m = \frac{1}{2}\left[\mathring{\gamma}_{il} + a_{lk}\,\mathring{a}_{ik}\right]\epsilon_{mli}$$

hence

$$\Omega_m = \mathring{\chi}_m + \omega_m$$

Introducing this into equation (3.01) we get the output signal of the gyros

$$\bar{\varphi}(t) = \mathbb{S}^{-1}(D)\ \mathbb{W}(D) \left[\dot{\bar{\chi}}(t) + \bar{\omega}(t)\right] - \frac{1}{A_{22}}\ \mathbb{S}^{-1}(D)\ \bar{\mathbb{m}}(t) \qquad 3.02$$

By feeding this signal through a linear network with the transfer function

$$\mathbb{W}^{-1}(D)\ \mathbb{S}(D) \qquad 3.03$$

we get

$$\begin{aligned} \bar{\omega}^*(t) &= \mathbb{W}^{-1}(D)\ \mathbb{S}(D)\ \bar{\varphi}(t) = \\ &= \bar{\omega}(t) + \dot{\bar{\chi}}(t) - \frac{1}{A_{22}}\ \mathbb{W}^{-1}(D)\ \bar{m}(t) \end{aligned} \qquad 3.04$$

By this procedure we obtain $\bar{\omega}^*(t)$ which is an estimate of the angular velocity of the η-set and thus also an estimate of the velocity of the vehicle. The accuracy of the estimate is given by equations (1.20) and (3.04).

The orientation of the η-set is given by the transformation (1.04). Let $\mathbb{A}(t)$ denote the matrix formed by the a_{ij}:s of equation (1.04), hence

$$\bar{\eta} = \mathbb{A}(t)\ \xi$$

We obtain the following equation for the matrix $\mathbb{A}(t)$

$$\begin{cases} \dfrac{d\ \mathbb{A}(t)}{dt} = \Omega(t)\ \mathbb{A}(t) \\ \mathbb{A}(o) = \mathbb{I} \end{cases} \qquad 3.05$$

where

$$\Omega(t) = \left\{\omega_j(t)\ \epsilon_{ijk}\right\}$$

Let η^* denote the estimate of the η-set formed in the following way

$$\bar{\eta}^* = \mathbb{A}^*(t)\ \bar{\xi}$$

where the matrix $\mathbb{A}^*(t)$ is calculated from the estimated values of the angular velocity $\bar{\omega}^*(t)$ i.e.

$$\begin{cases} \dfrac{d\ \mathbb{A}^*(t)}{dt} = \Omega^*(t)\ \mathbb{A}^*(t) \\ \mathbb{A}^*(o) = \mathbb{I} \end{cases} \qquad 3.06$$

where

$$\Omega^*(t) = \left\{ \omega_j^*(t) \; \epsilon_{ijk} \right\} \qquad 3.07$$

The transformation matrix from the η-set to the η^*-set is denoted by $\mathbb{E}(t)$ i.e.

$$\bar{\eta}^*(t) = \mathbb{E}(t)\bar{\eta}(t)$$

Equations (3.05) and (3.06) gives

$$\mathbb{E}(t) = \mathbb{I} + \int_0^t \left[\Omega^*(t')\,\mathbb{E}(t') - \mathbb{E}(t')\,\Omega(t') \right] dt' \qquad 3.08$$

Introduce the matrix sequence

$$\mathbb{E}_o = \mathbb{I}$$

$$\mathbb{E}_n(t) = \mathbb{I} + \int_0^t \left[\Omega^*(t')\,\mathbb{E}_{n-1}(t') - \mathbb{E}_{n-1}(t')\,\Omega(t') \right] dt' \qquad 3.09$$

This sequence converges to a limit $\mathbb{E}(t)$, which is the solution of the equation (3.08), at least when all the elements of $\Omega^*(t)$ and $\Omega(t)$ are bounded in any compact set including $(0,t)$. Compare section (9.1) of reference 1.

According to Eulers theorem of a rigid body the transformation matrix $\mathbb{E}(t)$ can be interpreted as a rotation around the eigenvector of the matrix. Let the angle of rotation $\Psi(t)$.

The positional error of the system is then $r\Psi(t)$.

Definition 3.1

By the navigation error of the system we mean the angle $\Psi(t)$.

The navigation error $\Psi(t)$ is related to the matrix $\mathbb{E}(t)$ by the relation

$$\Psi(t) = \arccos \frac{1}{2}\left[\mathrm{Tr}\;\mathbb{E}(t) - 1 \right] \qquad 3.10$$

4. Some remarks concerning the synthesis of a Schuler-tuned three-gyro platform system

4.1 The design procedure can follow the scheme given in section 10 of reference 1. The first step is thus to choose a $\mathbb{K}'(p)$-matrix. In case of Schuler-tuned platform systems the $\mathbb{K}'(p)$-matrix should be choosen according to equations (1.15) and (1.19). The elements of the $\mathbb{K}'(p)$-matrix will thus depend on the magnitude of the unbalance mh. The unbalance is determined from considerations of navigation accuracy, bearing friction, torque-capacity of the gimbal torques, maximum acceleration of the vehicle etc. A detailed discussion of these questions is given in reference 3.

Because of the limited accuracy of the available components we have to allow for deviations from the conditions given by the equations (1.15) and (1.19). The $\mathbb{K}'(p)$-matrix may have small nondiagonal elements and the diagonal elements may deviate from the desired values. How much deviation we can allow for is given by the desired navigation accuracy and the disturbances. These questions are discussed in reference 3.

4.2 We will now discuss some methods of synthesizing a system with the desired $\mathbb{K}'(p)$-matrix. According to equations (1.15) and (1.19) the system should have

$$\mathbb{K}'(p) = \frac{mRh}{J}\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & k_3(p) \end{pmatrix} \qquad 4.21$$

This means that the platform with respect to the axes ζ_1 and ζ_2 should have the moment of inertia mRh. Considering actual magnitudes of mh, the number mRh will be so large that the inertia of the platform must be obtained by the proper choice of the internal feedback and not by using a stable element with extremely high moments of inertia.

This means that

$$\left\{\mathbb{K}'(p)\right\}_{ij} \gg \left\{\mathbb{F}\right\}_{ij}$$

According to Lemma 2.2 a stable system then must have det $\mathbb{W}(p)$ stable which means that the gyros must be arranged in such a way that $l = 0$ and $s = 3$.

We will now analyse some methods available for the synthesis of platforms with high moments of inertia.

Consider e.g. a system according to section 8.1 of reference 1. The diagonal elements of the $\mathbb{K}(p)$-matrix are

$$bp + \frac{\omega_o}{ab} \cdot \frac{\tau(p) + \omega_o p}{p^2 + \sigma(p)} \tag{4.22}$$

According to reference 1, section 7, moment of inertia of the platform equals the coefficient of p in the above expression. The moment of inertia of the platform can thus be obtained either by making b large or by choosing the second part of the above expression in such a way that it contains a term linear in p. One way to obtain this is by choosing

$$\begin{cases} \sigma(p) = \mathcal{H} + ap \\ \tau(p) = 0 \end{cases} \tag{4.23}$$

For small values of p the expression (4.22) then reduces to

$$(b + \frac{\omega_o^2}{ab\mathcal{H}})\, p$$

This means that the moment of inertia of the platform is

$$J(b + \frac{\omega_o^2}{ab\mathcal{H}})$$

which can be very large for sufficiently small values of $\mathcal{H}$.

This method of synthesizing a platform with a very high moment of inertia is e.g. used in the Anschütz Gyro Compass.

Another possibility of obtaining a platform with a high moment of inertia is by choosing

$$\tau(p) = A\frac{ab}{\omega_o}\left[p^2 + \sigma(p)\right] p - \omega_o p \tag{4.24}$$

The expression 4.22 then reduces to

$$(b + A)\,p$$

The moments of inertia of the platform is then

$$J(b + A)$$

This method of synthesizing a platform with high moments of inertia is extensively discussed in reference 3. Some experiments with the german surplus gyro KZ-14 showing the possibilities of this method have been successfully performed. The difference between the two schemes (4.23) and (4.24) of synthesizing a platform with high moments of inertia is that in the system according to equation (4.23) the gyros are used as actuating as well as sensing devices. In the last mentioned system the gyros are only used as sensing devices and the torque is supplied by the torque motors on the gimbals.

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