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# An Improved Stochastic Send-on-Delta Scheme for Event-Based State Estimation

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## Introduction

Event-based sensing and communication holds the promise of lower resource utilization and/or better performance for remote state estimation applications in e.g networked control systems (NCS).

However, the problem of designing an optimal event-based state estimator often becomes untractable due to nonlinear measurements. This complexity is avoided with stochastic event-triggering.

In this work [1], we extend the work on stochastic triggering in [2] by proposing a simple predictor in the sensor to further improve the estimation performance.

## The MMSE Estimator

Bayes' theorem gives case dependent Kalman filter:

### Time Update:

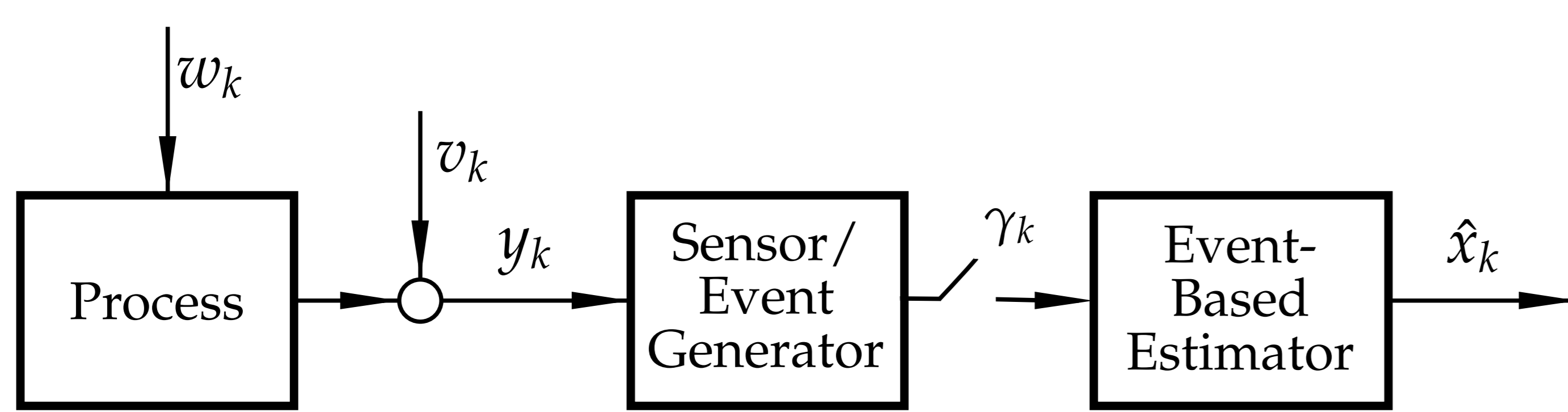
$$\begin{aligned}\hat{x}_k^- &= A\hat{x}_{k-1} \\ \hat{y}_k^- &= C\hat{x}_k^- \\ P_k^- &= AP_{k-1}A^T + Q\end{aligned}$$

### Measurement Update:

$$\begin{aligned}\hat{x}_k &= \hat{x}_k^- + K_k[\gamma_k y_k + (1 - \gamma_k)S_l y_{k-l} - \hat{y}_k^-] \\ P_k &= (I - K_k C)P_k^- \\ K_k &= P_k^- C^T [C P_k^- C^T + R + (1 - \gamma_k)Y^{-1}]^{-1}\end{aligned}$$

## The Remote Estimation Problem

Compute optimal estimates both with and without transmission:



### Process:

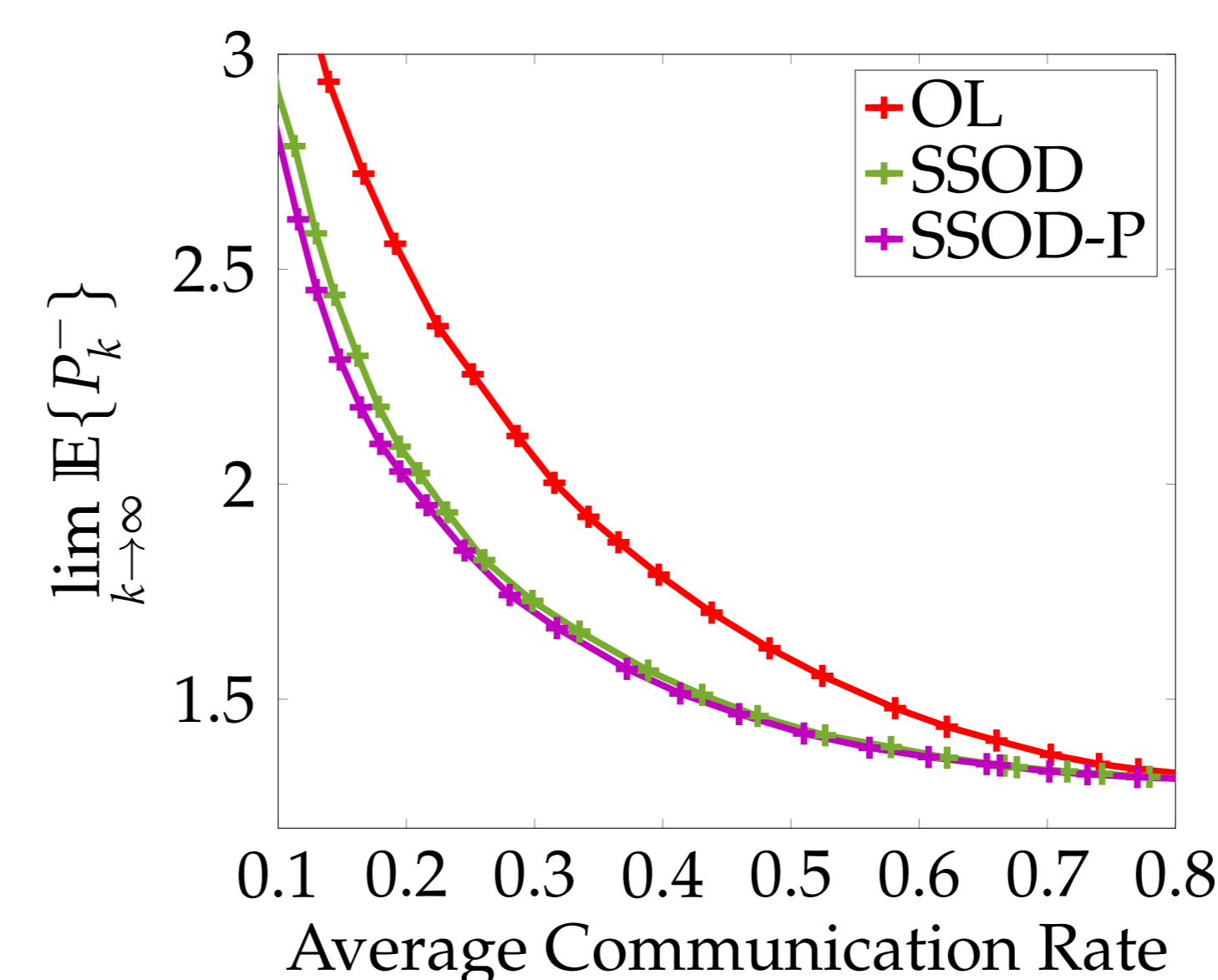
$$\begin{aligned}x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \\ w_k &\sim \mathcal{N}(0, Q) \\ v_k &\sim \mathcal{N}(0, R)\end{aligned}$$

### Two Cases:

$$\gamma_k = \begin{cases} 1 & \Rightarrow \text{Transmission} \\ 0 & \Rightarrow \text{No transmission} \end{cases}$$

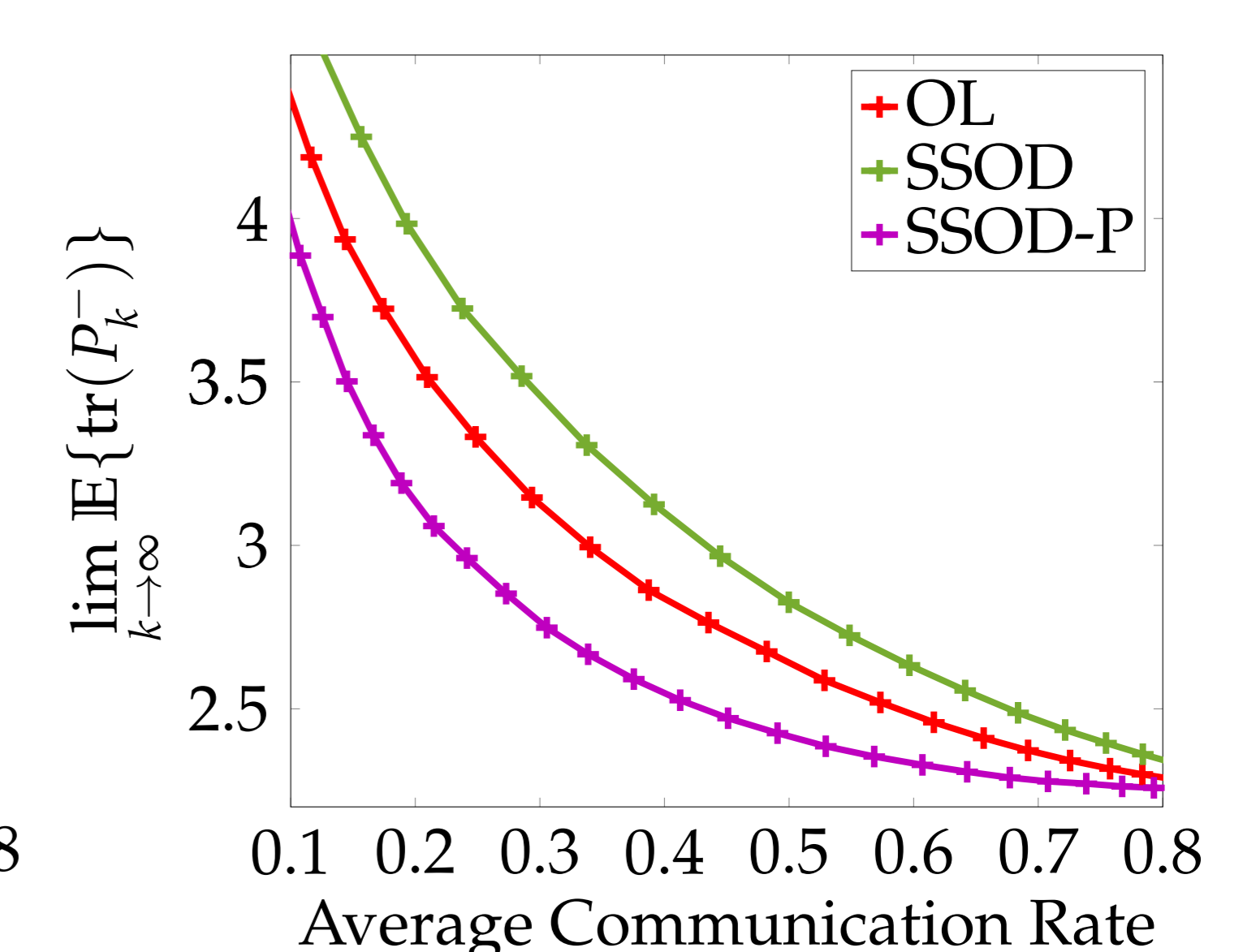
## Numerical Performance Comparison

Performance of SSOD and OL depends on process, while SSOD-P takes the process configuration into account:



### Slow 1<sup>st</sup>-order process:

$$\begin{aligned}A &= 0.95, C = 1 \\ Q &= 0.8, R = 1\end{aligned}$$



### Highly oscillatory 2<sup>nd</sup>-order process

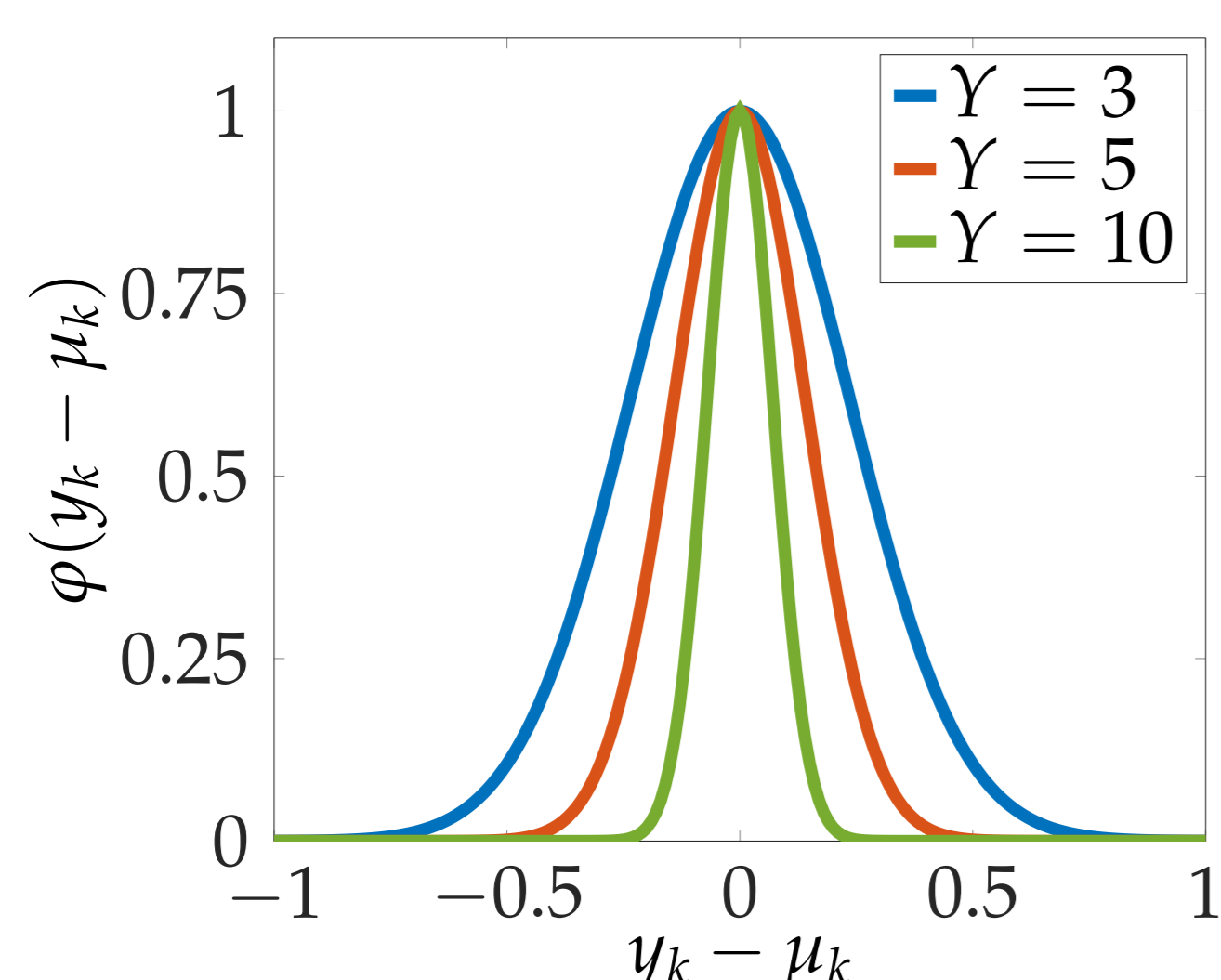
$$\begin{aligned}A &= \begin{bmatrix} -0.85 & -0.35 \\ 0.35 & -0.85 \end{bmatrix}, C = [1 \ 0] \\ Q &= \text{diag}(10^{-3}, 1), R = 0.1\end{aligned}$$

## Stochastic Event-Triggering

Trigger transmission with certain probability:

### Decision Function:

$$\varphi(y_k - \mu_k) = e^{-\frac{1}{2}(y_k - \mu_k)^T Y (y_k - \mu_k)}$$



### Event-Generator:

$$\zeta_k \sim \mathcal{U}(0, 1)$$

$$\gamma_k = \begin{cases} 1, & \text{if } \zeta_k > \varphi(y_k - \mu_k) \\ 0, & \text{else} \end{cases}$$

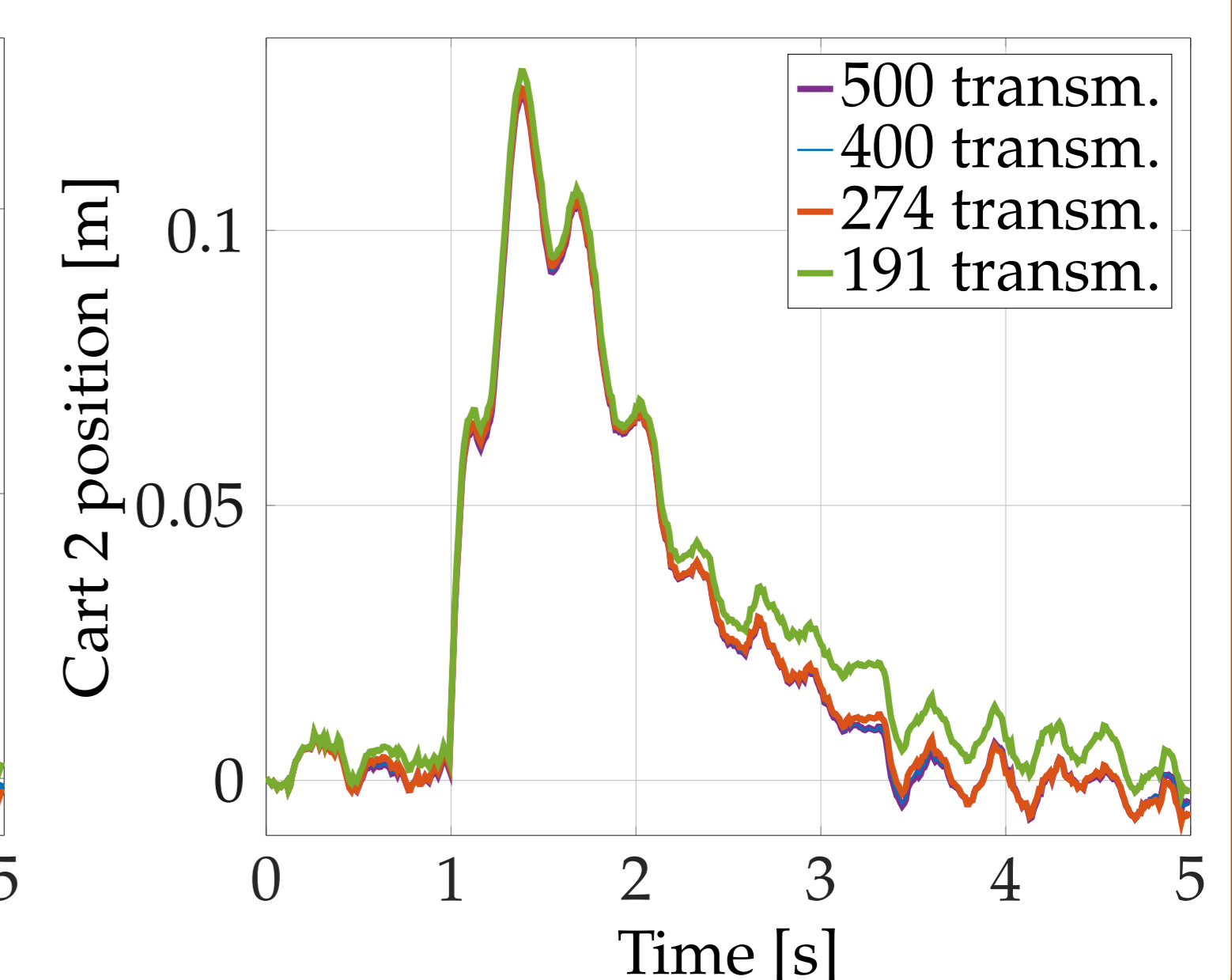
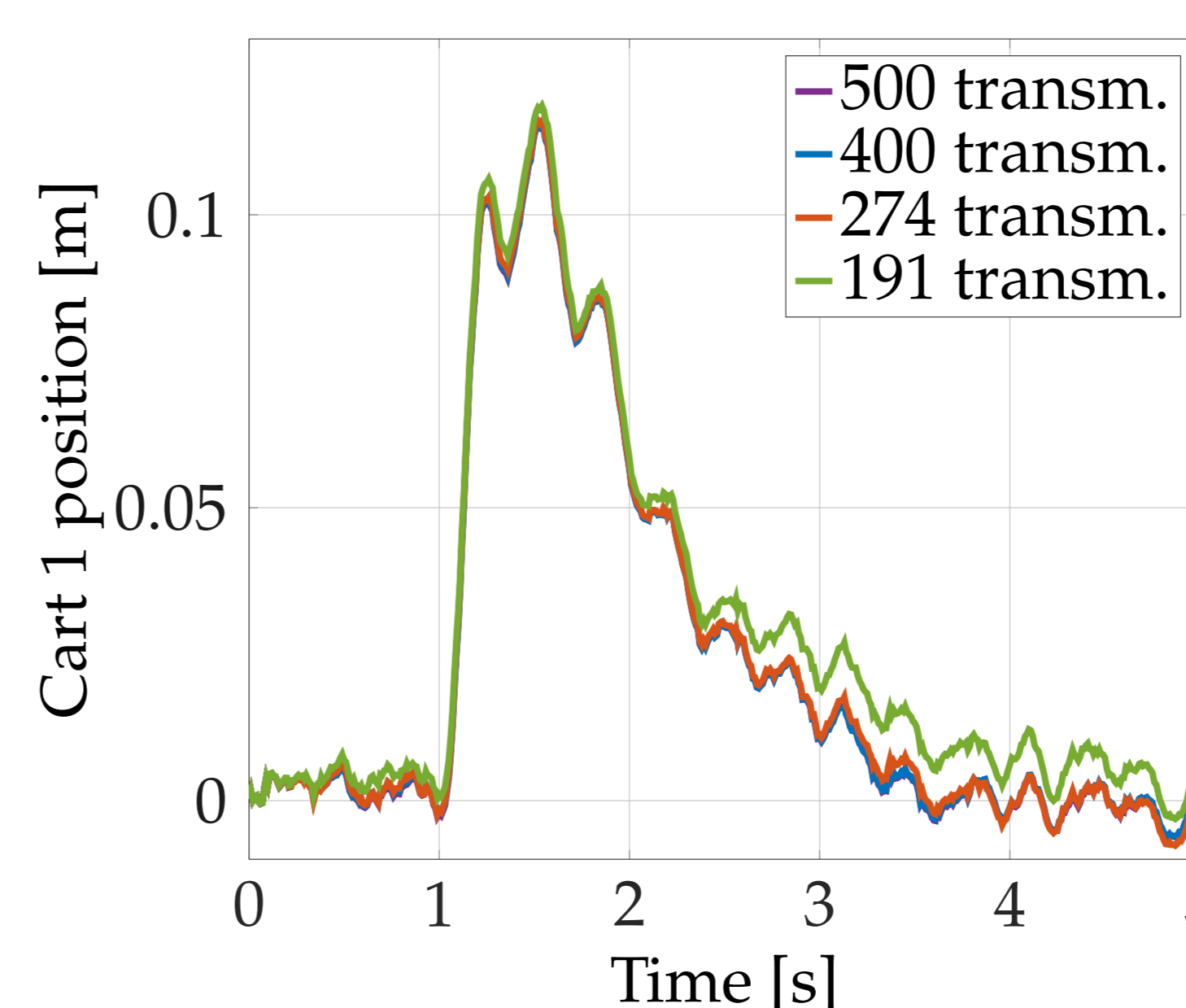
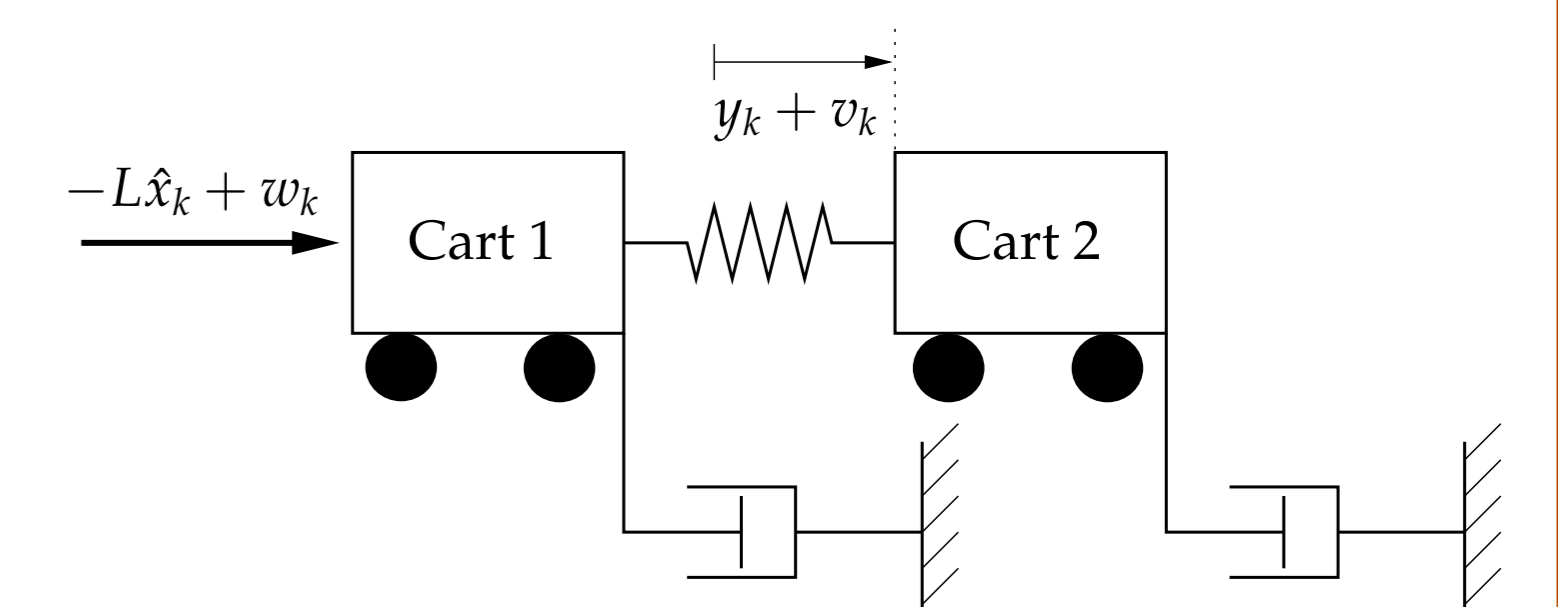
### Property of Scheme:

$$\Pr(\gamma_k = 0) = \varphi(y_k - \mu_k)$$

## Simulation Study

Position control of two carts with state-feedback and SSOD-P:

- System discretized with time step 0.01 s
- Impulse in cart 1 velocity at time 1 s
- Degradation in control performance small even at low communication rates



## A Simple Prediction

Proposed  $\mu_k$  in [2] with no estimator-to-sensor feedback are:

### Open-Loop (OL):

$$\mu_k = 0$$

### Stoch. Send-on-Delta (SSOD):

$$\mu_k = y_{k-l} \text{ (Transmit } l \text{ steps ago)}$$

Based on stationarity, we instead propose:

### Stoch. Send-on-Delta with Simple Prediction (SSOD-P):

$$\mu_k = \mathbb{E}\{y_k | y_{k-l}\} = S_l y_{k-l}$$

$$S_l = CA^l \Sigma C^T [C \Sigma C^T + R]^{-1}, \quad \Sigma = \text{Cov}(x_k) \text{ in stationarity}$$

## Conclusions

- **Stochastic Triggering** enables simple remote estimator design
- We propose a **simple sensor prediction** for improved performance
- Prediction implies a **scaling of last transmitted value**
- Scaling factors can be **pre-computed offline**
- Proposed scheme **compares well** in numerical examples

## Acknowledgments

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## References

- [1] M. Thelander Andrén and A. Cervin *Event-Based State Estimation Using an Improved Stochastic Send-on-Delta Sampling Scheme* In 2nd Int. Conf. on Event-Based Control, Communication and Signal Processing (EBCCSP) (Accepted), Krakow, Poland, June, 2016.
- [2] Shi, D., Shi, L. and Tongwen, C. *Event-Based State Estimation – A Stochastic Perspective* Springer, 2016.