An Improved Stochastic Send-on-Delta Scheme for Event-Based State Estimation

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Introduction

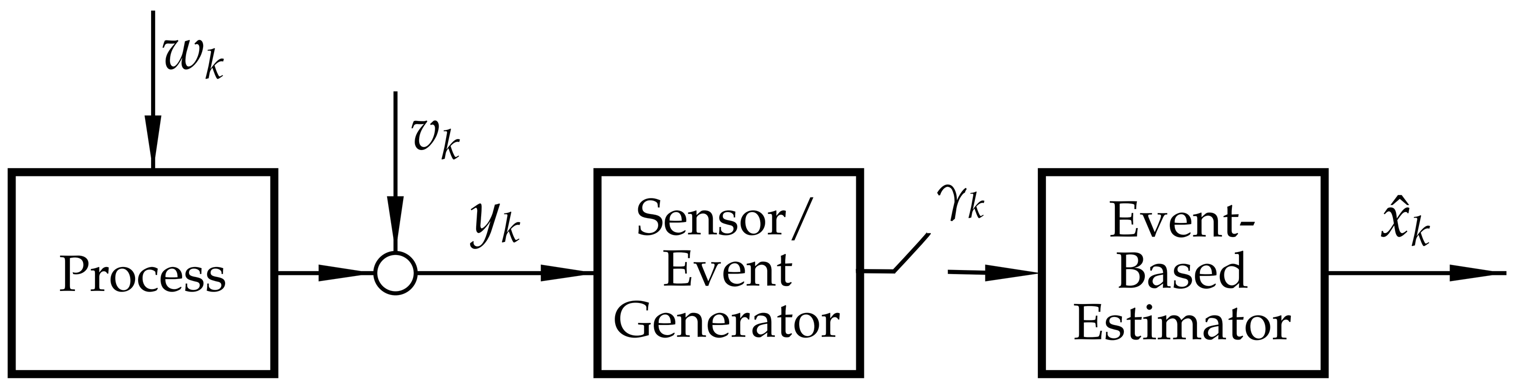
Event-based sensing and communication holds the promise of lower resource utilization and/or better performance for remote state estimation applications in e.g networked control systems (NCS).

However, the problem of designing an optimal event-based state estimator often becomes untractable due to nonlinear measurements. This complexity is avoided with stochastic event-triggering.

In this work [1], we extend the work on stochastic triggering in [2] by proposing a simple predictor in the sensor to further improve the estimation performance.

The Remote Estimation Problem

Compute optimal estimates both with and without transmission:



Process:

$$x_{k+1} = Ax_k + w_k$$
$$y_k = Cx_k + v_k$$
$$w_k \sim \mathcal{N}(0, Q)$$
$$v_k \sim \mathcal{N}(0, R)$$

Two Cases:

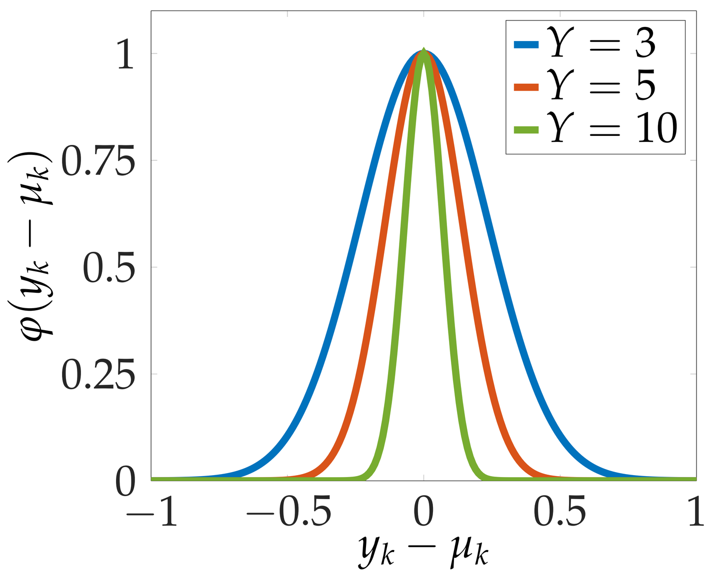
$$\gamma_k = \begin{cases} 1 \implies \text{Transmission} \\ 0 \implies \text{No transmission} \end{cases}$$

Stochastic Event-Triggering

Trigger transmission with certain probability:

Decision Function:

$$\varphi(y_k - \mu_k) = e^{-\frac{1}{2}(y_k-\mu_k)^T Y (y_k - \mu_k)}$$



Event-Generator:

$$\zeta_k \sim \mathcal{U}(0, 1)$$

$$\gamma_k = \begin{cases} 1, \text{ if } \zeta_k > \varphi(y_k - \mu_k) \\ 0, \text{ else} \end{cases}$$

Property of Scheme:

$$\Pr(\gamma_k = 0) = \varphi(y_k - \mu_k)$$

A Simple Prediction

Proposed μ_k in [2] with no estimator-to-sensor feedback are:

Open-Loop (OL): $\mu_k = 0$

Stoch. Send-on-Delta (SSOD): $\mu_k = y_{k-l}$ (Transmit l steps ago)

Based on stationarity, we instead propose:

Stoch. Send-on-Delta with Simple Prediction (SSOD-P):

$$\mu_k = \mathbb{E}\{y_k | y_{k-l}\} = S_l y_{k-l}$$

$$S_l = CA^l \Sigma C^T [C \Sigma C^T + R]^{-1}, \quad \Sigma = \mathrm{Cov}(x_k) \text{ in stationarity}$$

The MMSE Estimator

Bayes' theorem gives case dependent Kalman filter:

Time Update:

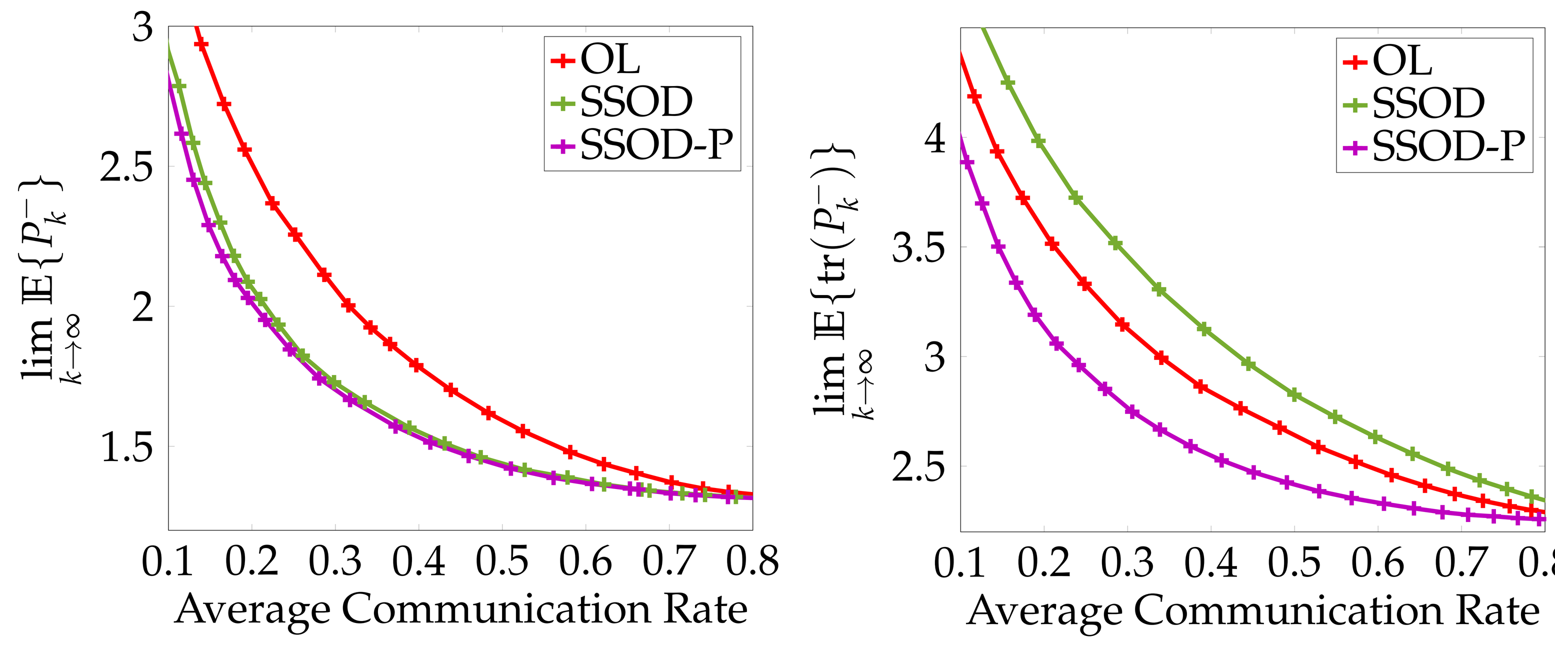
$$\hat{x}_k^- = A\hat{x}_{k-1}$$
$$\hat{y}_k^- = C\hat{x}_k^-$$
$$P_k^- = AP_{k-1}A^T + Q$$

Measurement Update:

$$\hat{x}_k = \hat{x}_k^- + K_k[\gamma_k y_k + (1-\gamma_k)S_l y_{k-l} - \hat{y}_k^-]$$
$$P_k = (I - K_k C)P_k^-$$
$$K_k = P_k^- C^T [CP_k^- C^T + R + (1-\gamma_k)Y^{-1}]^{-1}$$

Numerical Performance Comparison

Performance of SSOD and OL depends on process, while SSOD-P takes the process configuration into account:



Slow 1st-order process:

$$A = 0.95,\ C = 1$$
$$Q = 0.8, \quad R = 1$$

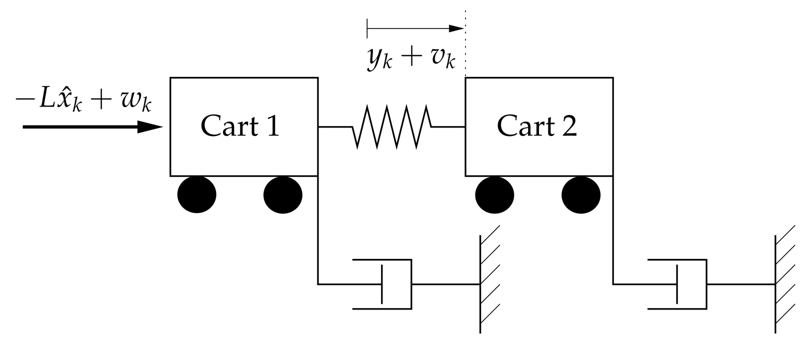
Highly oscillatory 2nd-order process

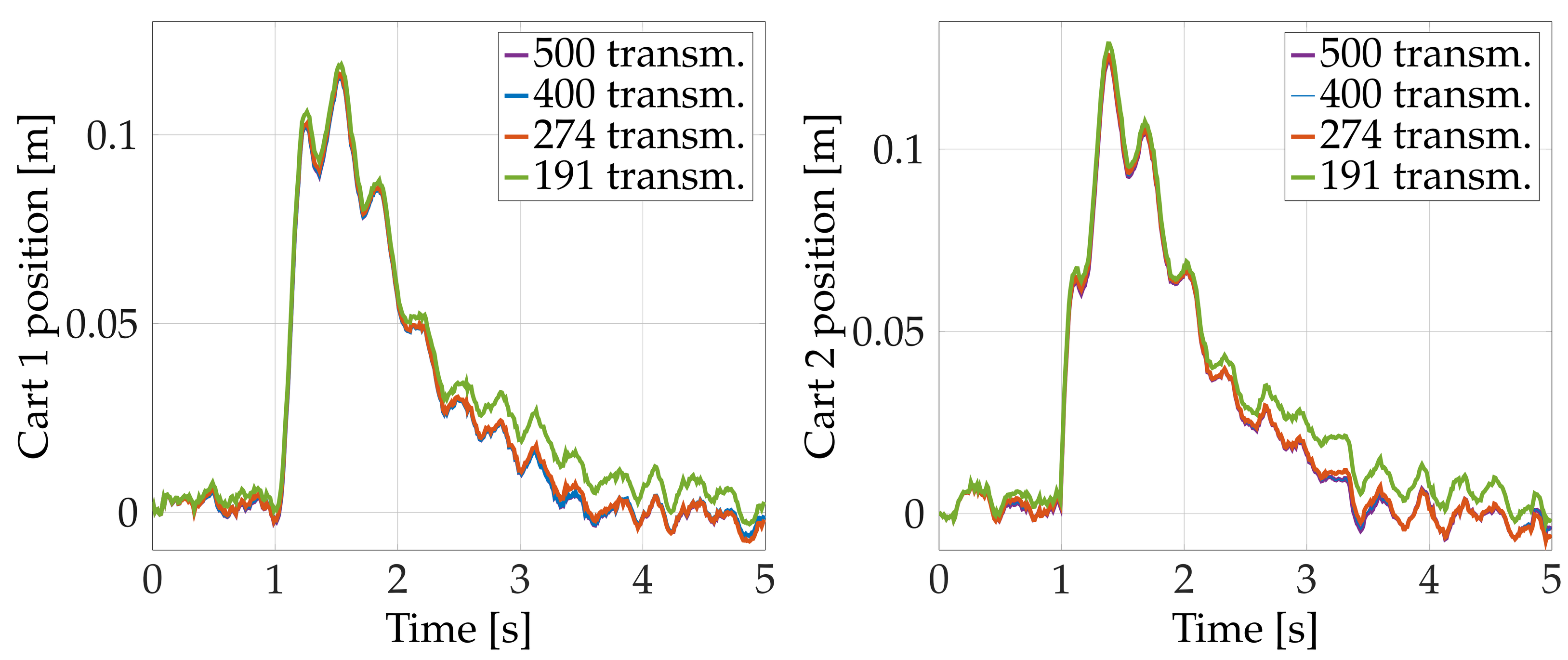
$$A = \begin{bmatrix} -0.85 & -0.35 \\ 0.35 & -0.85 \end{bmatrix}, C = [1\ 0]$$
$$Q = \mathrm{diag}(10^{-3}, 1), \quad R = 0.1$$

Simulation Study

Position control of two carts with state-feedback and SSOD-P:

- System discretized with time step 0.01 s
- Impulse in cart 1 velocity at time 1 s
- Degradation in control performance small even at low communication rates





Conclusions

- **Stochastic Triggering** enables simple remote estimator design
- We propose **a simple sensor prediction** for improved performance
- Prediction implies a **scaling of last transmitted value**
- Scaling factors can be **pre-computed offline**
- Proposed scheme **compares well** in numerical examples

Acknowledgments

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References

[1] M. Thelander Andrén and A. Cervin *Event-Based State Estimation Using an Improved Stochastic Send-on-Delta Sampling Scheme* In *2nd Int. Conf. on Event-Based Control, Communication and Signal Processing (EBCCSP)* (Accepted), Krakow, Poland, June, 2016.

[2] Shi, D., Shi, L. and Tongwen, C. *Event-Based State Estimation – A Stochastic Perspective* Springer, 2016.