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Published in:
Journal of Astronomical History and Heritage

2015

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Gislén, L. (2015). Burmese eclipse calculations. *Journal of Astronomical History and Heritage*, *18*(1), 52-64.

Total number of authors:
1

BURMESE ECLIPSE CALCULATIONS

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Abstract: Two Burmese eclipse calculations, one lunar and one solar, are analysed using examples from a Burmese manuscript. The fundamental parameters are with some important exceptions the same as in the *Suryasiddhanta*, but the handling of, for instance, parallax in the solar eclipse is different and much simplified. Specific to Burma are also the shadow calculations.

Keywords: Burmese astronomy, eclipse calculations, eclipse parallax.

1 INTRODUCTION

The traditional astronomical calculation procedures in Southeast Asia are little known in the West. Although they clearly show a great influence from traditional methods in India, their actual implementation is very different. The procedures are laid out as a kind of computer program, a set of rules without explanation that once followed, generate a set of numbers that describe the astronomical phenomenon in question. Clearly, this set of rules has been established by some competent agent who knew very well their astronomical background but who intended the rules to be used by other agents whose only function was to compute by rote. This presents some difficulties for the analysis as many of the intermediate steps in the calculation are omitted and have to be reconstructed.

There are many clever calculation shortcuts, as will be seen below. The parallax calculation procedures, for instance, are extremely simplified but still quite adequate. In an earlier paper (Gislén and Eade, 2014) we explained the Burmese shadow calculations, and they will reappear in the present paper and will vindicate the conclusions we made there.

2 THE SOURCE

The main source for this investigation is a Burmese astrology dissertation (U Thar-Tha-Na, 1937).

3 PARAMETERS

Hindu and Southeast Asian astronomers use sidereal longitudes, i.e. longitudes measured eastward along the ecliptic from a fixed origin on the ecliptic. Modern astronomy uses tropical longitudes where this origin is slowly moving westward along the ecliptic due to precession and is determined as the point on the ecliptic where the Sun crosses the celestial equator on the vernal equinox. In Hindu astronomy the fixed origin for longitude is the point where the mean Sun, the mean Moon and the mean planets were assumed to gathered together at the start of the Kaliyuga epoch, which was midnight on 18 February 3102 BCE.

The [Old] *Suryasiddhanta* has 1577917800 days in 4320000 solar revolutions, the *Kaliyuga*. As is usual in Southeast Asian astronomy these numbers are divided by 5400 to have the more handy 292207 days in 800 years. In the Burmese *Thandeikhta* scheme (Irwin, 1909), used in the calculations, this is corrected by 28 extra days (4320000/193/800) in a *Kaliyuga*, making the mean solar theory precisely that of the modern Hindu *Suryasiddhanta*.

The longitude of the solar apogee, the point on the ecliptic where the solar longitude is such that the distance between the Sun and the Earth is largest, is assumed to be 4640′. The apogee value (*mandocca*) in modern *Suryasiddhanta* is 4636′.

The Moon is assumed to make 5775336 revolutions in a *Kaliyuga* or exactly 7219167′ in 25 years. The mean daily motion of the Moon is then 790.5811288′ ≈ 790′ 35″. These are *Suryasiddhanta* values.

The lunar apogee is as in *Suryasiddhanta* (without *bija*) and makes 488203 revolutions in 4230000 solar years or precisely 488203′ in 200 years. The daily motion is then 6.682974624′ ≈ 6′ 41″.

The lunar node is as in *Suryasiddhanta* (with *bija*) and makes 232242 retrograde revolutions in a *Kaliyuga* or precisely 116121′ in 100 years or 3.179143493′ ≈ 3′ 11″ per day.

The epoch of *Thandeikhta* is BE (Burmese Era) 1100.

The calculations are made for *Amarapura* (Mandalay) with a geographical latitude given as 21° 32′.

For the Moon, the lunar apogee, and the lunar node, the epoch is *Kaliyuga* 0 (i.e. midnight on 18 February 3101 BCE). The Moon has an extra epoch correction of –12′ as compared with *Suryasiddhanta*. This could be a way to account for the difference in geographical longitude between India and Burma. The lunar apogee is assumed to have a *Kaliyuga* epoch value of 5140′ and the lunar node has an epoch value of 10945′. The *Suryasiddhanta* values are 5400′ and 10800′ respectively.

4 UNITS AND NOTATION

The time unit used in Southeast Asia is the *nadi* (Burmese *nayi*) that is 1/60 of a day and night. The *nadi* is further divided into 60 *vinadis* (Burmese *bizana*). A hexadecimal notation is used: xx:yy, meaning xx + yy/60. For longitudes the same notation means arc minutes and arc seconds. We sometimes use the notation xx:yy:zz for signs, degrees, minutes. The Burmese calculations are presented in grids that we denote by A, B, C … Within each grid we give the column number and the row number. Thus [B 2,8] denotes grid B, column 2, and row 5. The transcribed grids are given in the Appendix. The Burmese grids have labels specifying the different numbers, some of these labels can be identified as somewhat distorted Sanskrit or Pali words but most labels are obscure Burmese technical terms.

5 LUNAR ECLIPSE CALCULATION

5.1 Preliminaries

One of the model calculations made in the source is the partial lunar eclipse of 26 July 1934. The input for the calculation is the BE year (1296) and the *sutin* (102), the elapsed days of the Burmese year. The Burmese calculations are typically arranged such that all operations involve entire numbers. The Burmese division algorithm is rather involved and for this we have used Western notation instead without affecting the final result. In two cases we have used decimal notation for simplicity.

First the *thawana*, the elapsed days since the *Thandeikhta* epoch BE 1100, is calculated (Irwin, 1909: 16) by:

{(y – 1100)·292207 + 17742 + int((y – 1100)/193)}/800 = a + b/800

292207 = 1577917800/5400 is the number of uncorrected days in 4320000/5400 = 800 solar years. It is a standard number in Southeast Asian calendar computations (Eade, 1995).

int(x) means the integer part of the number x. This term gives an extra correction of 28 days in a *Kaliyuga*.

y is the Burmese year, and *a* the integer part of the division and *b* the remainder. The New Year *thawana* then is *a* + 1.

The New Year weekday is given by the remainder of (*thawana* – 2)/7 with 1 = Sunday. 800 – *b* is the *kyammat*, the mean solar longitude at the New Year midnight (24 hours) expressed in 800 parts of a mean solar day. The *Kaliyuga* year is the Burmese year + 3739.

Thus we have

[A 1,1] 1296, the Burmese year.
[A 1,2] 1296 + 3739 = 5035, the *Kaliyuga* year.
(196·292207 + 17742 + 1)/800 = 71612 + 715/800
[A 1,3] 800 – 715 = 85, the New Year *kyammat.*
thawana = 71612 + 1 = 71613
[A 1,4] (71613 – 2)/7 = 10230 + 1/7, the New Year weekday = 1, Sunday.
[A 1,5] 102, the *sutin.*

5.2 Mean Solar Longitude

The total elapsed *kyammat* at midnight on the New Year is 85. To this we add the *kyammat* resulting from the elapsed days from the New Year to the eclipse day, 102, the *sutin*. Each day corresponds to a *kyammat* of 800. Thus we have in total 85 + 102·800 = 81685. Now the mean daily motion of the Sun in arc minutes is 21600·800/292207 = 59:8′. By multiplying the *kyammat* by 21600 and dividing by 292207 we arrive at the mean solar longitude in arc minutes, skipping whole rotations (21600) if possible: 6038:10 [A 1,6].

5.3 Mean Lunar Longitude

Multiply the *Kaliyuga* year by 7219167, the motion of the mean Moon in arc minutes in 25 years and divide by 25. Skip whole rotations. The result is 1033:48. Subtract the epoch correction of 12′ to get 1021:48. This is the New Year mean position.

For the mean daily motion we use 79058′ in 100 days. The correct value is 79058.11287. This introduces an error 79058.11287 – 79058 = 0.11287 ≈ 1/9 in 100 days. The *kyammat* gives a contribution 85·79058/800/100 = 84:0. The *sutin* gives a contribution 102·79058/100 = 80639:9.6. The extra correction gives 102·(1/9)/100 = 0:6.6. In total 80723:16, with rounding. Adding the New Year longitude and skipping whole rotations we finally get 16945:4 [A 1,7].

5.4 Lunar Apogee

Multiply the *Kaliyuga* year by 488203, the motion of the lunar apogee in 200 years and divide by 200. The result is 12290510:31. Add the epoch value 5140 and skip whole rotations to get 5250:31. For the mean daily motion during 101 days we use 675. The correct value is 6.682974624·101 = 674.980437. The error is 674.980437 – 675 = –0.019563 ≈ –1/51 in 101 days. The *kyammat* gives a contribution 85·675/800/101 = 0:43. The *sutin* gives a contribution 102·675/101 = 681:41. The extra correction gives –102·(1/51)/101 = –0:1. In total 682:23. Adding the New Year longitude and skipping whole rotations we finally get 5932:54 [A 1,8]. The source has 5932:53, either a typo or the result of a different rounding.

5.5 Lunar Node

Multiply the *Kaliyuga* year by 116121, the motion of the lunar node in 100 years and divide by

100. The result is 5846692:21. Add the epoch value 10945 and skip whole rotations to get 4037:21. For the mean daily motion during 100 days we use 318. The correct value is 317.9143493. The error is 317.9143493 – 318 = –0.0857 ≈ –1/12 in 100 days. The *kyammat* gives a contribution 85·318/800/100 = 0:20. The *sutin* gives a contribution 102·318/100 = 324:22. The extra correction gives –102·(1/12)/100 = –0:5. In total 324:37. Adding the New Year longitude and skipping whole rotations we finally get 4361:58 [A 1,9].

5.6 True Longitudes

The true longitudes are calculated from

$$\lambda = \lambda_M + e\cdot\sin(\omega - \lambda_M) \qquad (1)$$

where λ is the true longitude, λ_M the mean longitude, ω the apogee longitude and e the eccentricity.

This expression uses a constant eccentricity, a difference from *Suryasiddhanta* that uses an eccentricity dimension that varies with the anomaly. For the Sun the *Suryasiddhanta* dimensions of the epicycle vary between 14° and 13:40° (Burgess, 2000: 70). The Burmese calculation uses a fixed dimension that corresponds to 13:44°. For the Moon the *Suryasiddhanta* epicycle dimension varies between 32° and 31:40° (ibid.) while the Burmese dimension is fixed at 31:44°.

The second term of the formula, the equation, is taken from Table 1 (Solar *chaya)* with argument in steps of 10°. The anomaly $\alpha = \omega - \lambda_M$ is reduced to lie in the interval [0 , 90°] using the symmetry of the sine function, i.e.

If 0′< α ≤ 5400′ do nothing Quadrant 0
If 5400′ < α ≤ 10800′ replace α with 10800′ – α. Quadrant 1
If 10800′ < α ≤ 16200′ replace α with α – 10800′ Quadrant 2
If 16200′ < α ≤ 21600′ replace α with 21600′ – α Quadrant 3

The equation is positive for anomaly in quadrants 0 and 1, otherwise negative.

For the Sun we have the fixed value of the apogee ω = 4640′ and with λ_M = 6038′ [A 6,1] and get an anomaly of 4640 – 6038 = –1398. As this is negative we add a whole turn, 21600′, to get 21600 – 1398 = 20202.

This anomaly is in quadrant 3, [A, 1,10] and the reduced value is 21600 – 20202 = 1398′. Each 10° interval corresponds to 600′ so the entry point in the table is 1398/600 = 2 + 198/600. Interpolation in the table gives the equation

45 + (66 – 45)·198/600 = 51:56 [A 2,1].

Table 1: Solar *chaya*.

Arg/10	0	1	2	3	4	5	6	7	8	9
Equation	0	23	45	66	85	101	113	123	129	131

The table difference 66 – 45 = 21 [A 1,11] is saved for the computation of the true daily motion of the Sun below. As we are in quadrant 3, the equation is subtractive and we get the true solar longitude

6038:10 – 51:56 = 5986:14 [A 2,2].

For the Moon we have ω = 5932:53 [A 1,8] and λ_M 16945:4 [A 1,7]. The Moon's mean longitude is given an extra correction of the solar equation (including sign) divided by 27, in our case –51:56/27 = –1:55. We have no explanation for this correction.

The corrected mean longitude is then 16945 – 1:55 = 16943:9 [A 2,3]. Thus the corrected anomaly is 5932:53 – 16943:9 + 21600 = 10589:44. This is in quadrant 1 [A 2,4], and the reduced anomaly is 10800 – 10589:44 = 210:16 ≈ 210.

With the same procedure as for the Sun, but with a lunar *chaya* (Table 2), we get a difference of 53 [A 2, 5], and an equation of +18:34.

The true longitude is then 16943:9 + 18:34 = 16961:43 [A 2,6].

The motion of the node is retrograde and its longitude is 21600 – 4361:58 = 17238:2 [A 2,7].

5.7 True Daily Motions

The mean daily motion of the Sun is 59:8 [A 2,8].

Using the table difference in [A 1,11] we compute the difference correction to the mean motion: 59:8·21/600 = 2:4.

The correction is positive for the anomaly in [90°, 270°], quadrants 1 and 2, otherwise negative. The true daily motion of the Sun is thus 59:8 – 2:4 = 57:4 [A 2,10].

The correctness of this procedure can be seen by taking the time derivative of the true longitude equation: $d\lambda/dt = d\lambda_M/dt + d(e\cdot\sin\alpha)/d\alpha\cdot d\alpha/dt$. Here, $d\lambda/dt$ is the true daily motion in longitude, $d\lambda_M/dt$ the mean daily motion in longitude, $d(e\cdot\sin\alpha)/d\alpha$ is the *chaya* table difference, and $d\alpha/dt$ the mean daily change in anomaly.

As the apogee of the Sun is fixed, its motion in anomaly is the same as the mean motion.

The mean daily motion of the Moon is 790:35 [A 2,9]. For the Moon we use the table difference

Table 2: Lunar *chaya*.

Arg/10	0	1	2	3	4	5	6	7	8	9
Equation	0	53	104	152	195	232	262	285	298	303

Table 3: Excess day time.

Sign				Excess
0	6		12	0
1	5	7	11	48
2	4	8	10	86
3		9		102

ence [A 2,5] and a mean motion of the anomaly that is the difference between the mean lunar motion and the motion of the apogee 790:35 – 6:41 = 783:54. The correction to the mean motion is then 783:54·53/600 = 69:15. As we are in quadrant 1 the correction is positive. True daily lunar motion 790:35 + 69:15 = 859:50 [A 2,11].

The daily motion of the node is 3:11 (retrograde) [A 3,1].

The true daily motion in elongation is 859:50 – 57:4 = 802:46 = 48166 [A 3,2]. The last number comes from multiplying 802:46 by 60.

5.8 Precession

The precession is identical with the one used in Hindu astronomy. It is a zigzag function with a period of 7200 years and an amplitude of 27° = 1620′ = 1800·9/10. The precession is positive from AD 412 to AD 2212.

Add the epoch value 88 to the *Kaliyuga* year and divide by 1800: (5035 + 88)/1800 = 2 + 1523/1800. The number 2 [A 3, 3] tells us that the precession is positive and has the value 1523·9/10 = 1370:42 [A 3,4]. The tropical true solar longitude is thus 5986:14 + 1370:42 = 122° 37′ = 4:2:37 [A 3,5]. The tropical true lunar longitude is thus 16961:43 + 1370:42 = 305° 32′ = 10:5:32 [A 3,6].

5.9 Length of Day and Night

Again a table is used for this calculation. Table 3 gives the varying excess day times in *vinadis* over the mean half day of 15 *nadis*. For zodiacal signs above 6 the excess is negative. The solar tropical longitude is 4:2:37. Interpolating in the table gives 86 – (86 – 48)·2:37/30 ≈ 82 = 1:22.

A half day is then 15 + 1:22 = 16:22 [A 3,7].
A half night is 30 – 16:22 = 13:38 [A 3,8].
A quarter day is 8:11 [A 3,9].
A quarter night is 6:49 [A 3,10].

5.10 Noon Shadow

The equinoctial shadow for a location with geographical latitude φ is given by

Table 4: Shadow table.

Sign		Excess	Sign		Excess
0	6	0	6	12	0
1	5	92	7	11	110
2	4	159	8	10	214
3		183	9		262

$$S_{eq} = G \tan\varphi \quad (2)$$

where G is the height of the gnomon, in Burmese astronomy taken as 420 units or 7:0. The noon shadow at another date is

$$S_{noon} = G \tan(\varphi - \delta) \quad (3)$$

where δ is the declination of the luminary. The Burmese method is to state the equinoctial shadow for the particular location, for *Amarapura* 165 (S_{eq} = 420 tan (21:32) ≈ 165), and then a table of the quantity $|G \tan(\varphi - \delta) - G \tan\varphi|$ for different zodiacal signs. For *Amarapura* we have Table 4.

Using the table with the Sun we have the tropical true longitude 4:2:37. Interpolating in the table gives 159 – (159 – 92)·2:37/30 = 153. Subtracting this from the equinoctial shadow gives 165 – 153 = 12. Thus the solar noon shadow is 0:12 [A 3,11].

The Moon has a true tropical longitude of 10:5:32. Again interpolation gives 214 – (214 – 110)·5:32/30 = 194. In this case we add the equinoctial shadow and the table number to get the Moon 'noon' shadow: 165 + 194 = 359 = 5:59 [A 3,11].

The reason for calculating both the solar and lunar noon shadow is, as we will see, that this eclipse starts before sunset.

5.11 Conjunction Time

The longitude of the Earth's shadow will be exactly 180° away from the Sun. In our case we get 5986:14 + 10800 = 16786:14 [B 1,1].

The longitude of the Moon is 16961:43, thus it has moved 16961:43 – 16786:14 = 175:29 too far. This is converted into time using the daily motion in elongation, 175:29·3600/48166 = 13:7 *nadis before* midnight (24 hours) [B 1,2]. This is 30 – 13:7 = 16:53 *after* noon [B 1,3]. The Moon will move 859:50·13:7/60 = 187:58 backwards during this time and have an opposition longitude of 16961:43 – 187:58 = 16773:45 [B 1,4].

In the same way we can calculate the opposition position of the node, remembering its retrograde motion, and get 17238:44 [B 1,5]. The Moon has not yet reached the ascending node and its latitude is south. The number 0 [B 1,6] is the quadrant relative to the ascending node: (17238:44 – 16773:45)/5400 = 0: 464:59. The number 464:59 is the distance of the Moon from the ascending node.

5.12 Latitude of the Moon

The inclination of the lunar orbit in many Indian texts is taken as 4.5°. In the neighbourhood of the node we can approximate a spherical triangle by a planar one and get the lunar latitude by multiplying the distance from the node by tan(4.5°) = 0.0787 ≈ 1/12.7. Some Southeast Asian astronomical schemes use the small angle approximation of the tangent function tan(4.5) = tan(9°/2) ≈ 9/2·π/180 ≈ 9/2·3/180 = 9/2·1/60 ≈ 1/13.3 (Faraut, 1910; Gislén and Eade, 2001; Wisandarukorn, 1997). The Burmese scheme uses the factor 1/13 that may be a rounded value of either of these previous numbers. In the present case the distance from the node is 464:59, thus the latitude is 464:59/13 = 35:46 [B 1,7]. There can be an eclipse only if the distance of the Moon from the node is less than about 10°. The resulting difference in latitude between the above schemes is less than about 1 arc minute.

5.13 Diameters and Radii

As in Hindu astronomy, the diameter of the luminaries is assumed to be proportional to their true daily motions with a mean diameter of 31′. The diameter of the shadow is taken as 2.5 times the lunar diameter, a value also used in other Southeast Asian schemes (Gislen and Eade, 2001).

Diameter of the Moon 31·859:50/790:35 = 33:43 [B 1,8].
Diameter of the shadow 33:43·2:30 = 84:18 [B 2,1].
Sum of the radii (84:18 + 33:43)/2 = 59: 1 [B 2,2].
Difference of radii (84:18 – 33:43)/2 = 25:18 [B 2,3].
Eclipsed part 59:1 – 35:46 = 23:15 [B 2,4].
Crescent 33:43 [B 1,7] – 23:15 = 10:28 [B 2,5].

5.14 Eclipse Duration

$(59{:}\,1\cdot 60)^2$ = 12538681 [B 3,5].
$(35{:}46\cdot 60)^2$ = 4605316 [B 3,6].
$(25{:}18\cdot 60)^2$ = 2304324 [B 3,7].
This is smaller than the number on the row above, thus the eclipse is partial.
$\sqrt{\{(59.1\cdot 60)^2 - (35{:}46\cdot 60)^2\}}$ = 2816:37

See Figure 1 for the geometry.

We convert this value to time using the daily motion in elongation 2816·60/48166 = 3:30 [B 2,6], the preliminary half duration.

5.15 Correction for the Change in Latitude During the Eclipse

The relative daily motion of the Moon relative to the node is 859:50 + 3:11 = 863:1.
This motion during 3:30 *nadis* is 863:1·3:30/60 = 50:21.
The change in latitude is 50:21/13 = 3:52.
The Moon is approaching the ascending node. Thus its south latitude decreases during the eclipse.
The latitude at the start of the eclipse is 35:46 + 3:52 = 39:38 (south) [B 2,7].
The latitude at the end of the eclipse is 35:46 – 3:52 = 31:54 (south) [B 2,8].

The eclipsed part is 59:1 – 39:38 = 19:23 [B 3,1] and 59:1 – 31:54 = 27:7 [B 3,2] respectively.
$(39{:}38\cdot 60)^2$ = 5654884 [B 3,8].
$(31{:}54\cdot 69)^2$ = 3663396. This number is written below grid B.

The formula $R + r - \beta$, used for the 'eclipsed part' above, is only correct at the conjunction. The correct formula to use is $R + r - \sqrt{(\beta^2 + \Delta^2)}$, where Δ is the Moon's distance from the conjunction longitude. The numbers in [B 3,1] and [B 3,2] are purely nominal.

5.16 Corrected Duration Times

We repeat the duration calculations with the new set of latitudes. The duration from the start to the middle eclipse is then 3:16 [B 3,3] and from the middle eclipse to the end, 3:43 [B 3,4]. In the *Suryasiddhanta* scheme this iteration is repeated several times.

5.17 Eclipse Times

Start of the eclipse 16:53 – 3:16 = 13:37 after noon [C 1,1]. This is before sunset as the half day length is 16:22.

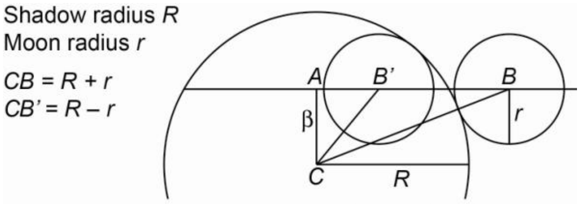


Figure 1: Eclipse geometry.

Middle eclipse 16:53 after noon [C 1,2], shortly after sunset.
End of the eclipse 16:53 + 3:43 = 20:36 after noon [C 1,3].
13:37 – 8:11 (quarter day) [A 3,9] = 5:26 [C 2,1].
The middle eclipse is 16:53 – 16:22 = 0:31 after sunset [C 2,2].
The end of the eclipse is 20:36 – 16:22 = 4:14 after sunset [C 2,3].

Column [C3] contains the times converted to Western hour:minute:second. This is done by multiplying the times in *nadis* by 2/5.

13:37·2/5 = 05:26:48 (p.m.) [C 3,1].
16:53·2/5 = 06:45:20 [C 3,2].
20:36·2/5 = 08:14:24 [C 3,3].

Modern calculations give 05:24 p.m., 06:51 p.m., and 08:6 p.m. respectively, local true solar time.

5.18 Shadow Calculations

The Burmese method of calculating shadows for times other than noon/midnight is based on the following model. At noon the deviation from the

Table 5: Multipliers.

Nadis		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Capricorn		61	124	168	216	251	280	307	325	338	345	349	350	325			
Aquarius	Sagittarius	63	133	176	220	250	280	305	321	331	335	337	337	310			
Pisces	Scorpio	79	134	187	232	267	291	309	320	324	325	320	312	301	288		
Aries	Libra	89	162	236	283	312	330	339	340	335	328	317	303	286	267	268	
Taurus	Virgo	178	304	363	395	408	415	402	391	375	313	338	318	296	309	251	
Gemini	Leo	571	599	582	563	590	509	484	456	430	409	375	349	322	301	270	229
Cancer		455	522	539	530	515	494	471	450	427	402	372	351	326	302	267	241

noon shadow has to be zero. At set/rise the deviation is infinite. A mathematical function with the required behaviour is

$(H \cdot M)/(D - H)$

or the equivalent expression

$(D \cdot M)/(D - H) - M$

where H is the time from noon/midnight, D is the length of a half day/night, and M is a suitably chosen multiplier. Actually this multiplier is a rather complicated function of H, the geographical latitude of the location and the tropical longitude of the luminary. Ignoring the rather weak geographical latitude dependence, we arrive at a multiplier, that depends on two arguments: H and the tropical longitude of the luminary. A table with such multipliers is given in the present and several other Burmese sources (Gislén and Eade, 2014); see Table 5 above.

When using the table, only the zodiacal sign is used and H is rounded to the nearest *nadi*; there is no interpolation involved. The total shadow, S, will then be

$$S = S_{noon} + (D \cdot M)/(D - H) - M \qquad (4)$$

Grid D treats the shadow calculations. Column [D 1] shows a solar shadow calculation as the Sun has not yet set at the start of the eclipse and the Moon is below the horizon. [D 2] and [D 3] are lunar shadow calculations. We have

Start eclipse:

The solar tropical longitude is 4:2:37 [A 3,5].
D = 16:22 = 982.
H =13:37 from solar noon = 817 [D 1,1], [D 1,2].
Zodiacal sign = 4 (Sun) H = 14 gives M = 301 [D 1,3].
$D \cdot M$ = 982 × 301 = 295582 [D 1,4].
$D - H$ = 982 – 817 = 165 [D 1,5].
S_{noon} = 0;12 = 12 from [A 3,11].

Shadow:

$S = S_{noon} + (D \cdot M)/(D - H) - M$ = 12 + 1791 – 301 = 1502 = 25:2 [D 1,6].

Middle eclipse:

The lunar tropical longitude is 10:5:32 [A 3,6].
The time is 0:31 from sunset [D 2,1].
The time is 16:53 from solar noon, H = 30 – 16:53 = 13:7 = 787 from midnight [D 2,2].
D = 13:38 = 818 (half night).
Zodiacal sign = 10, H = 13 gives M = 310 [D 2,3].
$D \cdot M$ = 818 × 310 = 253580 [D 2,4].
$D - H$ = 818 – 787 = 31 [D 2,5].
S_{noon} = 5:59 = 359 from [A 3,11].
$S = S_{noon} + (D \cdot M)/(D - H) - M$ = 359 + 8180 – 310 = 8229 = 137:9 [D 2,6].

End eclipse:

The time is 4:14 from sunset [D 3,1].
The time is 20:36 from solar noon, H = 30 – 20:36 = 9:24 = 564 from midnight [D 3,2].
Zodiacal sign = 10, H = 9 gives M = 331 [D 3,3].
$D \cdot M$ = 818 × 331 = 270758 [D 3,4].
$D - H$ = 818 – 564 = 254 [D 3,5].
S_{noon} = 5:59 = 359 from [A 3,11].
$S = S_{noon} + (D \cdot M)/(D - H) - M$ = 359 + 1066 – 331 = 1094 = 18:14 [D 3,6].

6 SOLAR ECLIPSE CALCULATION

The solar eclipse calculation example is for the annular eclipse on BE 1295, *sutin* 128, 21 August 1933. By coincidence this eclipse is also calculated for Calcutta (Burgess, 2000: 392g). The procedures for calculating the longitudes are the same as for a lunar eclipse but the *kyammat* is reduced by 400 in the calculations in order to get the longitudes at noon. In Burma the eclipse was partial.

For grid A, we only give the numbers without a detailed derivation.

6.1 Fundamental Data

[A 1,1] The Burmese year, 1295.
[A 1,2] The *Kaliyuga* year, 5034.
[A 1,3] The New Year *kyammat*, 292.
[A 1,4] The New Year weekday, 0, Saturday.
[A 1,5] The *sutin*, 128.

6.2 Mean Longitudes

[A 1,6] Mean solar longitude, 7561:27.
[A 1,7] Mean lunar longitude, 7742:47.
[A 1,8] Lunar apogee, 3664:2.
[A 1,9] Lunar node, 3242:38.

6.3 True Longitudes

[A 1,10] Quadrant 3 for the solar equation.
[A 1,11] Difference in solar equation table, 16.
[A 2,1] Solar equation, 98:54, negative.
[A 2,2] Solar true longitude, 7462:33.
[A 2,3] Lunar mean longitude corrected by –98:54 / 27, gives 7739:7.

[A 2,4] Quadrant 3 for the lunar equation.
[A 2,5] Difference in lunar equation table, 24.
[A 2,6] Lunar equation 280:13, negative.
[A 2,7] True lunar longitude, 7458:54.
[A 2,8] Lunar node, 21600 – 3242:38 = 18317:22.

6.4 True Daily Motions

[A 2,9] Mean solar daily motion, 59:8.
[A 2,10] Mean lunar daily motion 790:35.
[A 2,11] True solar daily motion, 57:33.
[A 3,1] True lunar daily motion, 760:32.
[A 3,2] Daily motion of the node, retrograde, 3:11.
[A 3,3] True daily motion in elongation, 42179.

6.5 Precession, Day Length and Noon Shadow

[A 3,4] Quadrant for precession, 2.
[A 3,5] Precession, positive, 1369:48.
[A 3,6] True solar tropical longitude, 147° 12′ = 4:27:12.
[A 3,7] Half day, 15:52.
[A 3,8] Half night 14:8.
[A 3,9] Quarter day, 7:56.
[A 3,10] Quarter night, 7:4.
[A 3,11] Solar noon shadow, 1:7.

6.6 Parallax of the Middle Eclipse

The difference in longitude between the Sun and the Moon at noon is 7462:33 – 7458:54 = 3:39. This is the distance the Moon still has to travel. We convert this into time using the daily motion in elongation [A 3,3]: 3:39·3600/42179 = 0:19 *nadis* [B 1,1]. This is the time of the true conjunction after noon. The time after midnight is 30 + 0:19 = 30:19 [B 1,2].

The procedure used to calculate the parallax in longitude (time) is to take the true conjunction time after noon and express it in *vinadis*, *v*. The parallax in *nadis*, *n*, is then computed from the formula

$$n = (7 \cdot v)/(600 + |v|), \qquad (5)$$

$|v|$ being the positive value of v.

The correct parallax is a complicated function of the latitude of the luminary, the time of day, and geographical latitude. However, the formula above gives a surprisingly good approximation to the true parallax at least for locations not too far from the equator (see below). For times before noon the parallax correction in longitude is negative. Applying the formula with 19 *vinadis* above, we arrive at the longitudinal parallax (in time measure) of 0:13 [B 1,3]. Thus the true apparent conjunction time, corrected for parallax is at 0:19 + 0:13 = 0:32 [B 1,4] after noon.

During the time from noon the Moon moves 0:32·760:32/60 = 6:46 [B 1,5]. The movement of the node is in the same way 0:2, backwards [B 1,6]. The longitude of the Moon is then

Table 6: Declination table.

λ_N	0	10	20	30	40	50	60	70	80	90
δ_N	0	4	8	12	15	18	20	22	23	24

7465:40 [B 1,7], and of the node 17317:20 [B 1,8].

The descending node is at 17317:20 – 10800 = 6517:20. The Moon has not reached the descending node; its longitude is still north. Relative to the ascending node it is in quadrant 1 [B 1,9]. The distance from the descending node is 51:40, giving a latitude of 51:40/13 = 3:58 (north) [B 1,10].

The procedure for calculating the parallax in latitude is as follows. The time in *nadis* of the apparent conjunction after noon is multiplied by 6 in order to convert it to degrees (360° = 60 *nadis*). The result is then added to the true tropical longitude of the Sun. The result is an approximation to the longitude of the *nonagesimal*, λ_N, the highest point of the ecliptic. From Table 6 we then calculate the declination of the *nonagesimal*, δ_N. The table uses the standard Southeast Asian value of 24° for the obliquity of the ecliptic. The table is based on the formula $\sin \delta_N = \sin 24° \cdot \sin \lambda_N$.

In our case we have λ_N = 147° 12′ + 6·0:32 = 150° 24′. Reducing to the first quadrant we get by interpolation δ_N = 11° 50′ [B 1,11], positive.

The parallax in latitude is then

$$\pi_\beta = 49' \sin(\delta_N - \varphi) \qquad (6)$$

where φ = 21° 32′ is the actual geographical latitude. The value, 49′, is the effective horizontal parallax of the Moon and the Sun used in Southeast Asian astronomy. This is also the mean value used in *Suryasiddhanta* (Burgess, 2000(V): 11).

We have $\delta_N - \varphi$ = 11° 50′ – 21° 32′ = –9° 42′ [B 1,12]. The parallax is taken from a new table (Table 7); we get by interpolation 7:45, negative [B 1,13].

The latitude corrected for parallax is then 3:58 – 7:45 = –3:47 (south) [B 1,14].

6.7 Diameters and Radii

[B 2,1] True diameter of the Sun, 30:10.
[B 2,2] True diameter of the shadow (Moon), 29:49 .
[B 2,3] Sum of radii, 29:59.
[B 2,4] Difference of radii, 0:10.
[B 2,5] Eclipsed part, 26:12.

Table 7: Parallax table.

$\delta_N - \varphi$	0	10	20	30	40	50	60	70	80	90
π_β	0	8	17	24	31	37	42	46	48	49

6.8 Preliminary Data for the Beginning and End of the Eclipse

[E 1,1] Square of sum of radii, $(29{:}59\cdot 60)^2$ = 3236401.
[E 1,2] Square of middle apparent latitude $(3{:}47\cdot 60)^2$ = 51529.
[E 2,1] Square root of the difference: 1785.
[B 2,6] Convert to time 1785·60/42179 = 2:32, preliminary half duration.

During this time the Moon moves relative to the node 2:32·(760:32 + 3:11)/60 = 32:15.
This corresponds to a change in latitude of 32:15/13 = 2:28. During the eclipse the lunar latitude will decrease as the Moon is approaching the descending node.
The true latitude at the beginning is 3:58 + 2:28 = 6:26 (north) [B 2,7].
The true latitude at the end is 3:58 – 2:28 = 1:30 (north) [B 2,8].
Preliminary time of the beginning 0:32 – 2:32 = –2:0, before noon [B 2,9].
Preliminary time of the end 0:32 + 2:32 = 3:4, after noon [B 2,10].

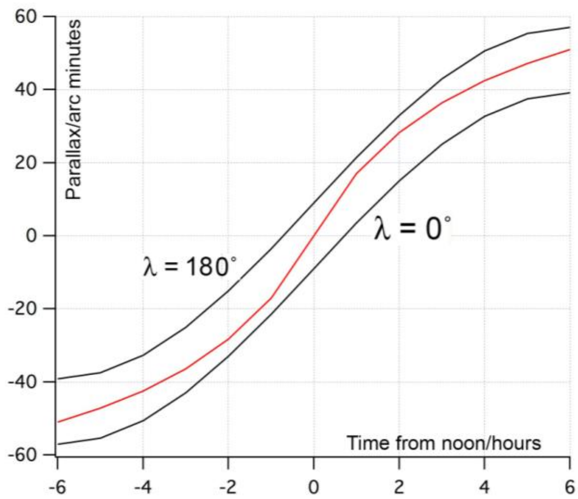


Figure 2: Parallax in longitude.

6.9 Latitude Parallax Corrections

Repeating the procedures for parallax in latitude we have:
Nonagesimal longitude at the beginning 147° 12′ – 6·2:0 = 135° 12′.
Nonagesimal declination 16° 26′ [B 2, 11].
Nonagesimal longitude at the end 147° 12′ + 6·3:4 = 165° 36′.
Nonagesimal declination 5° 45′ [B 2, 12].
Argument for beginning parallax 16° 26′ – 21° 32′ = –5° 6′ (south) [B 2,13].
Argument for end parallax 5° 45′ – 21° 32′ = –15° 47′ (south) [B 2,14].
Parallax correction at the beginning 4:4 (south) [B 3,1].
Parallax correction at the end 13:12 (south) [B 3,2].
Apparent latitude at the beginning 6:26 – 4:4 = 2:22 (north) [B 3,3].
Apparent latitude at the end 1:30 – 13:12 = –11:42 (south) [B 3,4].

6.10 Time Corrections

[E 1,3] Square of latitude at the beginning $(2{:}22\cdot 60)^2$ = 20164.
[E 1,4] Square of latitude at the end $(11{:}42\cdot 60)^2$ = 492804.
[E 2,3] √{3236401 – 20164} = 1793. Beginning of the eclipse.
[E 2,4] √{3236401 – 492804} = 1656. End of the eclipse.
[B 3,5] Eclipsed part at the beginning 29:59 – 2:22 = 27:37.
[B 3,6] Eclipsed part at the end 29:59 – 11:42 = 18:17.
[B 3,7] Conversion to time from the true conjunction 1793·60/42179 = 2:33.
[B 3,8] Conversion to time from the true conjunction 1656·60/42179 = 2:21.
[B 3,9] 30:19 – 2:33 = 27:46, time from midnight
[B 3,10] 30:19 + 2:21 = 32:40, time from midnight

As counted from noon the times in [B 3,7] and [B 3,8] become
[B 3,11] 2:33 – 0:19 = 2:14 = 134
[B 3,12] 2:21 + 0:19 = 2:40 = 160

Using the formula (1) to compute the longitudinal parallax correction we get
[B 3,13] 1:17
[B 3,14] 1:28
The total duration of the beginning phase is then 2:14 + 1:17 = 3:31 and of the end phase 2:40 + 1:28 = 4:8.

[C 1,1] Beginning of the eclipse after midnight 30:0 – 3:31 = 26:29.
[C 1,2] Middle eclipse 30:32.
[C 1,3] End of the eclipse 30:0 + 4:8 = 34:8.

[C 2,1] Quarter day – beginning phase = 7:56 – 3:31 = 4:25.
[C 2,2] Middle time from noon 0:32.
[C 2,3] End time from noon 4:8.

[C 3,1] Western time of the beginning 10:35:36
[C 3,2] Western time of the middle 12:12:48
[C 3,1] Western time of the end 1:39:12

Modern calculations give 10:23 a.m., 12:7 a.m., and 1:43 p.m. respectively, local true solar time.

6.11 Grid D, Shadow Calculations

The tropical longitude of the Sun is 147° 12′, zodiacal sign 4.
Half day D = 15:52 = 952.
[D 1,1] The time of the beginning from sunrise 15:52 – 3:31 = 12:21
[D 1,2] The time of the beginning before noon 3:31.
[D 1,3] H = 3:31 = 211, sign 4, 4 *nadis* gives

M = 563.
[D 1,4] $D\cdot M$ = 952·563 = 535976.
[D 1,5] $D - H$ = 952 – 211 = 741.
[D 1,6] Shadow 3:47.

[D 2,1] The time of the middle eclipse from noon 0:32
[D 2,2] The time of the middle eclipse after noon 0:32.
[D 2,3] H = 0:32 = 32, sign 4, 1 *nadi* gives M = 571.
[D 2,4] $D\cdot M$ = 952·571 = 543292.
[D 2,5] $D - H$ = 952 – 32 = 920.
[D 2,6] Shadow 1:27.

[D 3,1] The time of the end from noon 4:8
[D 3,2] The time of the beginning after noon 4:8.
[D 3,3] H = 4:8 = 248, sign 4 , 4 *nadi* 2 gives M = 563.
[D 3,4] $D\cdot M$ = 952·563 = 535976.
[D 3,5] $D - H$ = 952 – 248 = 704.
[D 3,6] Shadow 4:25.

7 THE BURMESE SOLAR ECLIPSE PARALLAX CORRECTIONS

The Burmese parallax corrections represent a large simplification as compared with the analogous *Suryasiddhanta* corrections and it is therefore interesting to compare them with parallaxes computed using modern astronomy. How accurate are they?

The parallax in latitude and longitude of the Moon in general depends on the observer's latitude, φ the longitude of the Sun, λ, that in a solar eclipse is equal to that of the Moon, the time from noon h, and the obliquity ε of the ecliptic. We here neglect the dependence on the distance of the Moon from the Earth.

7.1 The Parallax in Longitude

As shown above, for the parallax in longitude the Burmese use a very simple function:

$$n = 7\cdot v / (600 + |v|) \quad (7)$$

where v is the time in *vinadis* after noon, and n the parallax in *nadis*. Since 1 *vinadi* = 1/150 hour and the mean elongation motion is 730′ per day we can rewrite this formula as

$$\delta\lambda \approx 85.2\cdot h/(4 + |h|) \quad (8)$$

where now h is the time after noon in hours and $\delta\lambda$ the longitudinal parallax in arc minutes. The Burmese formula is explicitly independent of the longitude of the Sun.

We compare the value of this function with the corresponding modern function for parallax in longitude (Meeus, 2000: 98–100) being a function of the geographical latitude φ, the solar longitude λ, and the time from noon h.

We use φ = 23.2°, the geographical latitude of Ujjain, but our result is quite insensitive of this choice. All latitudes of Burmese sites are quite close to 20°, and the manuscript has 21° 32′. The graph of the parallax function versus the time in hours from noon (Figure 2) for a fixed solar longitude is a sigmoid curve. Different solar longitudes give similar parallel curves but displaced slightly upwards or downwards. For simplicity we only show the extreme cases, the bottom black curve being the parallax for solar longitude 0° and the upper black curve the parallax for solar longitude 180°. The red curve is the Burmese longitudinal parallax, which is a kind of average of the modern values and quite closely approximates the modern parallax for solar longitudes 90° and 270°. The horizontal axis in this diagram is the time in hours from noon, and the vertical axis is the parallax in arc minutes. Negative hours are times before noon.

The error of the Burmese longitudinal parallax is at most about 10′, which corresponds to about 20 minutes in time.

7.2 The Parallax in Latitude

The Burmese parallax in latitude depends, as does the modern parallax, on the solar longitude λ, the observer's geographical latitude φ and the time h from noon. We have again taken the observer's latitude as 23.2° and compare parallaxes for λ = 0°, 30°, 60°, 90°, 180°, 210°, 240°, 270° (see Figures 3a–h). Graphs for other longitudes that are multiples of 30° are identical to the others but with the sign of the time axis reversed. The black curves show the modern parallax and the red curves the Burmese one.

The Burmese algorithm has a maximum discrepancy of about 5′. The main part of this discrepancy is due to the Burmese use of a horizontal parallax value of 49′ instead of the modern value of 57′.

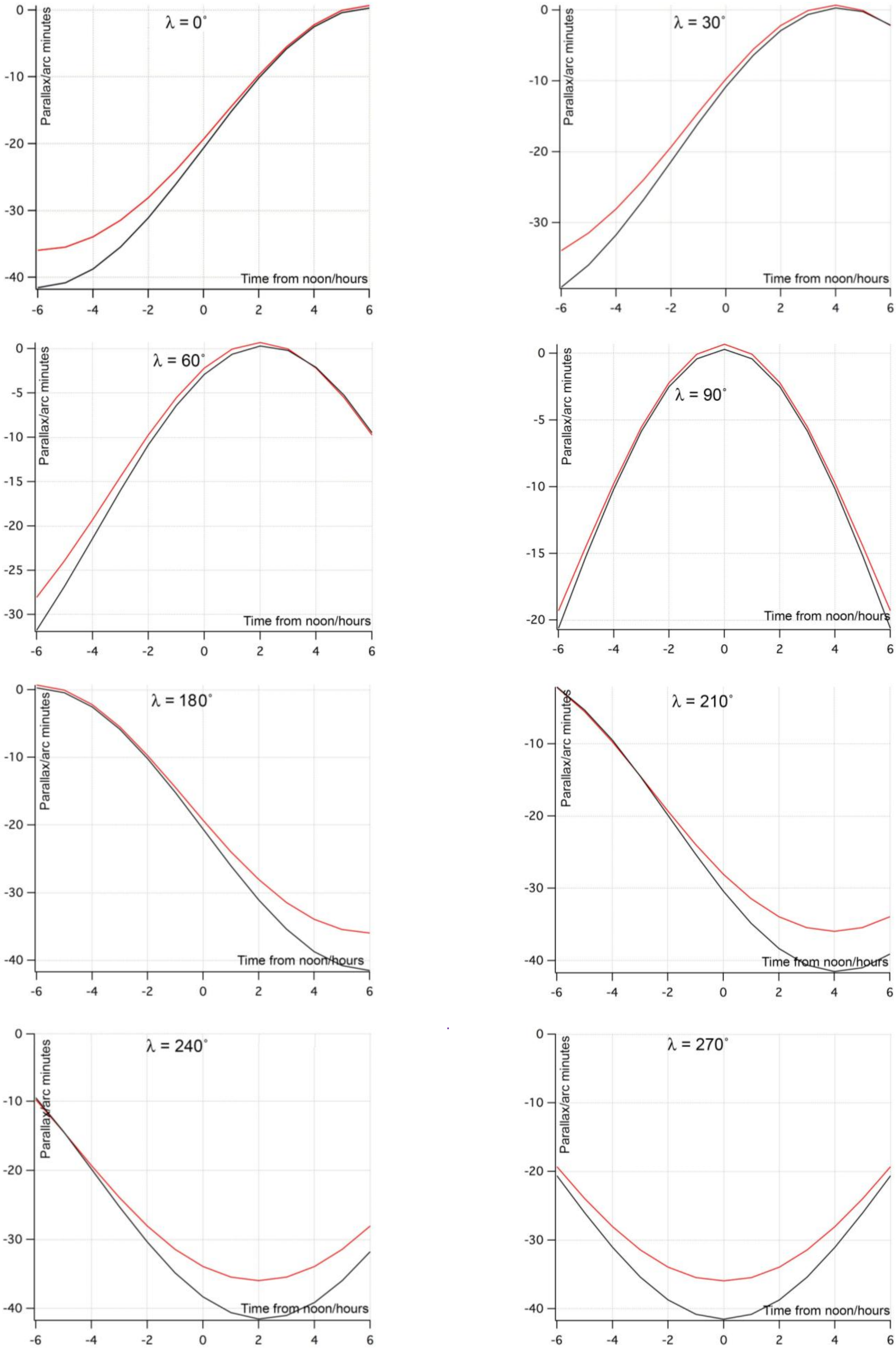
8 CONCLUSIONS

A general impression one gets from the calculations above is that they are of surprisingly good quality. There are very few computational or typographical errors. The computational methods show a delicate balance between the need for accuracy and clever approximation shortcuts for simplifying the calculations. This is especially evident in the solar eclipse calculation and in the corrections for parallax.

The shadow calculations seem to be a characteristic of Burma and give a very simple and interesting solution to a complicated mathematical problem.

9 ACKNOWLEDGEMENTS

I am grateful to two anonymous referees for their helpful comments on the original version of this paper.



Figures 3a–3h: Parallax in latitude.

10 REFERENCES

Burgess, E., 1860, *The Sûrya Siddhânta*. Delhi, Motilal Banarsiddass (reprinted 2000).

Eade, J.C., 1995. *The Calendrical Systems of Mainland South-East Asia*. Leiden, Brill.

Faraut, F.G., 1910. *Astronomie Cambodgienne*. Pnom Penh, F.-H. Schneider.

Gislén, L., and Eade, J.C., 2001. South East Asian eclipse calculations. *Centaurus*, 43, 278–307.

Gislén, L., and Eade, C.J., 2014. Burmese shadow calculations. *Journal of Astronomical History and Heritage*, 17: 258–266.

Irwin, A.M.B., 1909. The *Burmese and Arakanese Calendars*. Rangoon, Hanthawaddy Printing Works. Also possible to download a reprint at https://archive.org/details/burmesearakanese00irwiiala

Meeus, J., 2000. *Astronomical Algorithms*. Richmond, Willman-Bell.

U Thar-Tha-Na, 1937. *Hsin-temu thara-kjan-thi*. Mandalay. A pdf version of the original can be found on: http://home.thep.lu.se/~larsg/Site/UTharThaNa.pdf

Wisandarunkorn, L., 1997. *Khampi Horasat*. Bangkok (in Thai).

11 APPENDICES

11.1 Appendix 1

Partial lunar eclipse grids Burmese Era 1296 *sutin* 102, 26 July 1934.

See Tables 8A–D.

Table 8 A–D: Trancription of lunar eclipse panels A – D.

A	1	2	3
1	1296	51:56	3:11
2	5035	5986:14	48166
3	85	16943:9	2
4	1	1	1370:42
5	102	53	122:37
6	6038:10	16961:43	305:32
7	16945:4	17238:2	16:22
8	5932:53	59:08	13:38
9	4361:58	790:35	8:11
10	3	57:04	6:49
11	21	859:50	0:12 5:59

B	1	2	3
1	16786:14	84:18	19:23
2	13:07	59:01	27:07
3	16:53	25:18	3:16
4	16773:45	23:15	3:43
5	17238:44	10:28	12538681
6	0	3:30	4605316
7	35:46	39:38	2304324
8	33:43	31:54	5654884

C	1	2	3
1	13:37	5:26	5:26:48
2	16:53	0:31	6:45:20
3	20:36	4:14	8:14:24

D	1	2	3
1	13:37	0:31	4:14
2	13:37	13:07	9:24
3	301	310	331
4	295582	253580	270758
5	165	31	254
6	25:02:00	137:09	18:14

11. 2 Appendix 2

Annular solar eclipse grids Burmese Era 1295, *sutin* 128, 21 August 1933.

See Tables 9 A–E.

Table 9 A – E: Transcription of solar eclipse panels A – E.

A	1	2	3
1	1295	98:54	760:32
2	5034	7462:33	3:11
3	292	7739:07	42179
4	0	3	2
5	128	23	1369:48
6	7561:27	280:13	147:12
7	7742:47	7458:54	15:52
8	3664:02	18317:22	14:08
9	3242:38	59:08	7:56
10	3	790:35	7:04
11	16	57:33	1:07

B	1	2	3
1	0:19	30:10	4:04
2	30:19	29:49	13:12
3	0:13	29:59	2:22
4	0:32	0:10	11:42
5	6:46	26:12	27:37
6	0:2	2:32	18:17
7	7465:40	6:26	2:33
8	18317:20	1:30	2:21
9	1	2:00	27:46
10	3:58	3:04	32:40
11	11:50	16:26	2:14
12	9:42	5:45	2:40
13	7:45	5:06	1:17
14	3:47	15:47	1:28

C	1	2	3
1	26:29	4:25	10:35:36
2	30:32	0:32	12:12:48
3	34:08	4:08	1:39:12

D	1	2	3
1	12:21	0:32	4:08
2	3:31	0:32	4:08
3	563	571	563
4	535976	543592	535976
5	741	920	704
6	3:47	1:27	4:25

E	1	2
1	3236401	1785
2	51529	
3	20164	1793
4	492804	1656

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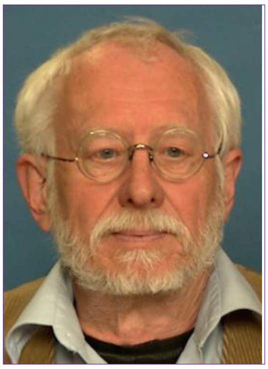

Theoretical Physics at the University of Lund, Sweden, and retired in 2003. In 1970 and 1971 he studied for a Ph.D. in the Faculté des Sciences, at Orsay, France. He has been doing research in elementary particle physics, complex systems with applications of physics in biology and with atmospheric optics. During the last fifteen years he has developed several computer programs and Excel sheets implementing calendars and medieval astronomical models from Europe, India and Southeast Asia (see http://home. thep.lu.se/~larsg/).