Popular Science Summary

Weather forecasting has been of interest for millennia and will be of importance in the future with increasing extreme weather conditions. Modern forecasting techniques were developed in the 20th century and have been continuously improved since then. Weather forecast is one important example of an application in *computational fluid dynamics (CFD)*, where problems involving fluid flow are analyzed and solved. Other examples are the aerodynamic design of airplanes and cars, fire development in tunnels and blood flow simulations. CFD is based on *scientific computing*, where complex problems are studied and solved with the help of advanced computing techniques.

Even though mathematical models of fluid systems have been developed almost 200 years ago and CFD calculations have been steadily improved over the last 100 years, accurate fluid simulations are still a challenging research topic. Scientists are continuously increasing the accuracy of CFD simulations. The difference in the scale of magnitude in real world applications is a big challenge: A weather forecast for a region is done on the scale of kilometers, while windstorms can consist of wind shears and eddies with sizes ranging in diameter from centimeters to hundreds of kilometers. The goal of this thesis is to contribute to the improvement of CFD simulations by constructing efficient solvers for CDF problems.

To express CFD problems mathematically, *partial differential equations (PDEs)* are very often used. With the help of PDEs we can describe the change of a quantity in a domain over time, e.g. the temperature in a city over a day. These PDEs are often very complex and need to be solved using computers and *numerical methods*, which are approximation techniques for solving mathematical problems. It is an ongoing research topic in computational sciences to develop efficient numerical methods adapted to state of the art computing devices, which have increased computing power and require methods where many calculations are carried out in parallel. This is especially of interest when large problems can be divided into smaller ones which can be solved simultaneously.

The first step to solve PDEs with numerical methods is to choose a *discretization method*. These are based on dividing the problem domain into pieces which are small enough to accurately approximate important quantities of the problem as e.g. eddies. Different discretization methods exist, having specific advantages and varying accuracy. Methods with high accuracy are called *high order methods* and are used in this thesis. These are of advantage for problems with e.g. turbulence. It is an open problem to construct efficient solvers to be applicable to large real world problems.

Discretizing PDEs results in a system of linear equations which need to be solved with an efficient *iterative method*. In this context, we are interested in efficiency w.r.t. the number of iterations and the computing time while at the same time giving an accurate solution. To achieve a precise weather forecast, a lot of measurements need to be taken into account in the calculations. More measurements and in consequence more data increase the accuracy of the mathematical model, but also the number of unknowns in the equation system. It is common to have more than hundreds of million unknowns in typical CFD applications. Thus, very accurate calculations can take several weeks to run. This is of course not a reasonable option for weather forecasts and emphasizes the need for fast solvers for these equation systems.

The solution to this problem are *preconditioners*, which allow to transform the equation system to be solved into one more suitable for the iterative method used. The construction of good preconditioners is not easy since they depend both on the PDE to be solved and the discretization method applied. One main focus of this thesis is the construction of good preconditioners. The idea presented here is to use a combination of low order numerical methods which are simpler to construct, but work well as preconditioners given several other computational and numerical constraints we restrict ourselves to.

The other main topic of this thesis are so-called *space-time* problems. The traditional way to solve time dependent PDEs numerically is the following: for one specific time point, e.g. each second, the solution is calculated before everything is repeated for the next time point. The idea of space-time methods is to consider all time steps simultaneously in the calculations while still following the causality principle that a solution later in time is depending on a solution earlier in time. The main difference to the traditional methods is that the equation system becomes even larger. The interest for space-time methods has increased with the changing development in computer architecture. State of the art processors do not become faster any longer, instead the number of processors increases. This motivates to parallelize calculations even more. We compare different ways to implement space-time methods and present a method to analyze space-time solvers using established frameworks.