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SIMULATION OF SHIP STEERING

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SIMULATION OF SHIP STEERING

Claes Källström

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SIMULATION OF SHIP STEERING

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Abstract

Computer simulations of ship steering are presented. The ship model describes a tanker. An adaptive autopilot, consisting of a Kalman filter, a self-tuning regulator for steady state course keeping, and a turning regulator is tested in different load and speed conditions. Straight course keeping as well as turning is simulated. Comparisons to a conventional autopilot based on a PID-regulator is also performed.

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APPENDIX - PROGRAM LISTINGS

1. INTRODUCTION

Simulations of straight course keeping and yawing in different load and speed conditions with an autopilot consisting of a stationary Kalman filter, a self-tuning regulator and a yaw regulator are presented in this report. The simulations are performed on the computer UNIVAC 1108 by use of the interactive program SIMNON (see Elmqvist (1975)). The ship model used describes a 350 000 tdw tanker of Kockums' design.

The self-tuning regulator for straight course keeping is based on least squares identification and minimum variance control. A descrete, fixed gain PID-regulator is also used for comparison. The yaw regulator consists of different discrete fixed gain PID-regulators. The reference values used by the yaw regulator are the yaw rate and the heading angle. Either the non-filtered measurements or the Kalman filter estimates are used by the different regulators.

Simulation of ship steering is also discussed in Aspernäs and Foisack (1975), Aspernäs and Källström (1975), Källström (1976a) and (1976b). Full-scale experiments on 255 000 tdw tankers are described in Källström (1974) and (1975).

Listings of the programs used are given in the Appendix.

2. SHIP STEERING DYNAMICS

The following model, which describes a 350 000 tdw tanker of Kockums' design, is used in the simulations (cf. Norrbin (1970)):

$$\dot{\delta} = -\frac{1}{T_r} \delta + \frac{1}{T_r \cdot CRG} \delta_C$$

$$|\dot{\delta}| \leq \frac{1}{CRG} \delta_{lim}$$

$$\left(1 - Y_{v}^{"}\right) \dot{v} = \left(Y_{ru}^{"} - 1\right) ru + \frac{1}{\sqrt{gL}} Y_{ru|u|}^{"} ru|u| + \frac{1}{L} Y_{|u|v}^{"} |u|v + \frac{1}{\sqrt{gL}^{3}} Y_{u|u|v}^{"} u|u|v + \frac{1}{L} Y_{v|v|}^{"} v|v| + Y_{r|v|}^{"} r|v| + \frac{1}{\sqrt{gL}^{3}} Y_{u|u|v}^{"} u|u|v + \frac{1}{L} Y_{v|v|}^{"} v|v| + Y_{r|v|}^{"} r|v| + \frac{1}{\sqrt{gL}^{3}} Y_{u|u|v}^{"} u|v|v + \frac{1}{L} Y_{uu\delta}^{"} u^{2}\delta + \frac{1}{L} Y_{u|u|\delta}^{"} u|u|\delta + \frac{1}{\sqrt{gL}^{3}} (T/m)\delta + k_{TY}(T/m) - F_{w} \sin\left(\frac{\alpha}{CRG} - \psi\right) + w_{1}$$

$$\dot{\psi} = r \tag{2.1}$$

It is assumed that the number of propeller revolutions n is kept constant to a specified value n_{O} by a regulator during all the simulations. The propeller thrust per mass unit (T/m) is computed as:

$$J = \frac{u (1-w) \cdot 60}{nD}$$

$$J' = \frac{J}{\sqrt{1+J^{2'}}}$$

$$K_{T}' = -0.33 \cdot J'^{2} - 0.38 \cdot J' + 0.35$$

$$T = K_{T}' \left(\frac{J}{J'}\right)^{2} \rho_{S} n^{2} D^{4}/3600$$

$$(T/m) = \frac{T}{\rho_{S}} \nabla$$
(2.2)

Notice that the terms $(T/m)\delta$ in (2.1) always are limited by the value $(T/m)_{O}\delta$, where $(T/m)_{O}$ is computed from (2.2) with the stationary forward speed corresponding to $n = n_{O}$ rpm.

Input signals:

| rudder | command | (or | rudder | servo | position) | δ _C | [deg] |
|--------|----------|-------|--------|--------|-----------|----------------|-------|
| number | of prope | ellei | revolu | utions | | n | [rpm] |

States:

| rudder angle | δ | [rad] |
|------------------|-----|---------|
| forward velocity | u | [m/s] |
| sway velocity | v | [m/s] |
| yaw rate | r | [rad/s] |
| heading angle | 1/2 | [rad] |

Disturbances:

| sway acceleration disturbance | w_1 | $[m/s^2]$ |
|---------------------------------------|-------|-----------------------|
| disturbance of yaw angle acceleration | v_2 | [rad/s ²] |

Other notations:

| time constant of rudder servo | $^{\mathtt{T}}\mathtt{r}$ | [s] |
|--------------------------------|---------------------------|-----------|
| limit of rudder turning rate | $\delta_{\texttt{lim}}$ | [deg/s] |
| length of ship | L | [m] |
| acceleration of gravity | g | $[m/s^2]$ |
| propeller thrust per mass unit | T/m | $[m/s^2]$ |
| wind force per mass unit | $F_{\mathbf{w}}$ | $[m/s^2]$ |
| lever arm of wind force | e w | [m] |
| angle of wind direction | α | [deg] |
| conversion factor rad - deg | CRG | [deg] |

The following parameter values are used:

$$T_r = 5 \text{ s}$$
 $\delta_{lim} = 2.32 \text{ deg/s}$
 $L = 350 \text{ m}$
 $g = 9.80665 \text{ m/s}^2$
 $F_w = 0.002 \text{ m/s}^2$
 $\ell_w = 25 \text{ m}$
 $\alpha = 90 \text{ deg}$
 $CRG = 57.2958 \text{ deg}$

The values of the other parameters are given in Dyne and Trägårdh (1975). Two different load conditions are considered corresponding to the mean draught $T=22.3~\mathrm{m}$ (full load, forward and aft draught equal to 22.3 m) and $T=10.5~\mathrm{m}$ (ballast, forward and aft draught equal to

9.0 m and 12.0 m, resp.). The forward speed u which corresponds to n = 87.6 rpm is equal to 15.8 knots when T = 22.3 m and equal to 17.25 knots when T = 10.5 m. If the model (2.1) and (2.2) is linearized, the following transfer function relating the yaw rate r to the rudder angle δ is obtained:

$$G(s) = \frac{K(1 + sT_3)}{(1 + sT_1)(1 + sT_2)}$$
 (2.3)

If the forward speed u is assumed to be constant and equal to 15.8 knots, then the following parameter values of (2.3) are obtained when T = 22.3 m:

$$K = -0.0161 \text{ l/s}$$
 $T_1 = -110.1 \text{ s}$
 $T_2 = 18.3 \text{ s}$
 $T_3 = 54.3 \text{ s}$

(2.4)

The corresponding values when u = 17.25 knots and T = 10.5 m are:

$$K = -0.0707 \text{ 1/s}$$
 $T_1 = -337.1 \text{ s}$
 $T_2 = 19.9 \text{ s}$
 $T_3 = 69.5 \text{ s}$

(2.5)

Notice that the sign of the rudder angle in the model is chosen in such a way that a positive rudder angle (starboard rudder) gives a positive yaw rate (starboard yaw). From (2.4) and (2.5) it can be concluded that the tanker is unstable in full load condition as well as in ballast condition.

The disturbance signals w_1 and w_2 are obtained as white, gaussian noise. The covariance matrix of the white noise vector, which generates w_1 and w_2 , is

$$R_{W} = \begin{pmatrix} 0.8 \cdot 10^{-4} & 0 \\ 0 & 0.53 \cdot 10^{-10} \end{pmatrix}$$
 (2.6)

The measured outputs from the model (2.1) and (2.2) are

$$\delta_{m} = CRG \cdot \delta + d_{\delta} + e_{1}$$

$$v_{1} = \overline{v}_{1} + d_{v} + e_{2}, \qquad \overline{v}_{1} = CMK \cdot (v + \ell_{1}r)$$

$$r_{m} = CRG \cdot r + d_{r} + e_{3}$$

$$\psi_{m} = CRG \cdot \psi + e_{4}$$

$$u_{m} = CMK \cdot u$$

$$n_{m} = n$$

where e_1 , e_2 , e_3 and e_4 are white, gaussian measurement noise with covariance matrix

$$R_{e} = \begin{pmatrix} 0.04 & 0 & 0 & 0 \\ 0 & 0.0025 & 0 & 0 \\ 0 & 0 & 0.0004 & 0 \\ 0 & 0 & 0 & 0.0025 \end{pmatrix}$$
 (2.7)

The measured rudder angle $\delta_{\rm m}$ [deg], sway velocity of bow v_1 [knots], yaw rate $r_{\rm m}$ [deg/s], heading angle $\psi_{\rm m}$ [deg], forward speed $u_{\rm m}$ [knots] and number of propeller revolutions $n_{\rm m}$ [rpm] are used by the autopilot. The conversion factor from m/s to knots CMK is equal to 1.943844 and the lever arm ℓ_1 is equal to 164.35 m. The measurement biases are assigned the following values:

$$\begin{aligned} &d_{\delta} = 2 \text{ deg} \\ &d_{V} = 0.5 \text{ knots} \\ &d_{r} = 0.05 \text{ deg/s} \end{aligned} \tag{2.8}$$

It should be pointed out that the model of the disturbances is extremely simplified. A more realistic approach is given in Berlekom, Trägårdh, and Dellhag (1975). Notice, however,

that the disturbances applied in the simulations describe a rather rough weather condition.

The program of the ship model, TANK3, is given in the Appendix. Almost the same model was used in the simulations of Källström (1976b).

3. AUTOPILOT

The structure of the autopilot is shown in Fig. 3.1. To obtain a good performance in all speeds, the autopilot performs a speed scaling using the speed $V_{\rm S}$ [m/s]. $V_{\rm S}$ is computed every second according to

$$V_{s} = 0.18 \cdot n_{m} / CMK$$
 (3.1)

The signals δ_m , v_1 , r_m , ψ_m , u_m and n_m are measured every second. The Kalman filtering as well as the computation of v_m [knots] according to

$$v_{m} = v_{1} - CMK \cdot \ell_{1} \cdot r_{m} / CRG$$
 (3.2)

are performed every second. Either the estimates from the Kalman filter \hat{v} , \hat{r} and $\hat{\psi}$ or the measurements v_m , r_m and ψ_m are used by the self-tuning regulator, the PID-regulator, and the yaw regulator. The rudder command δ_c is computed with sampling interval 10 s or 15 s. The PID-regulator for straight course keeping is only used for comparison. The reference course ψ_{ref} and the reference yaw rate r_{ref} for yawing as well as the rudder limit δ_{ℓ} are also used by the autopilot. The complete autopilot is implemented by the Fortran subroutines AUTP3 and STUR given in the Appendix.

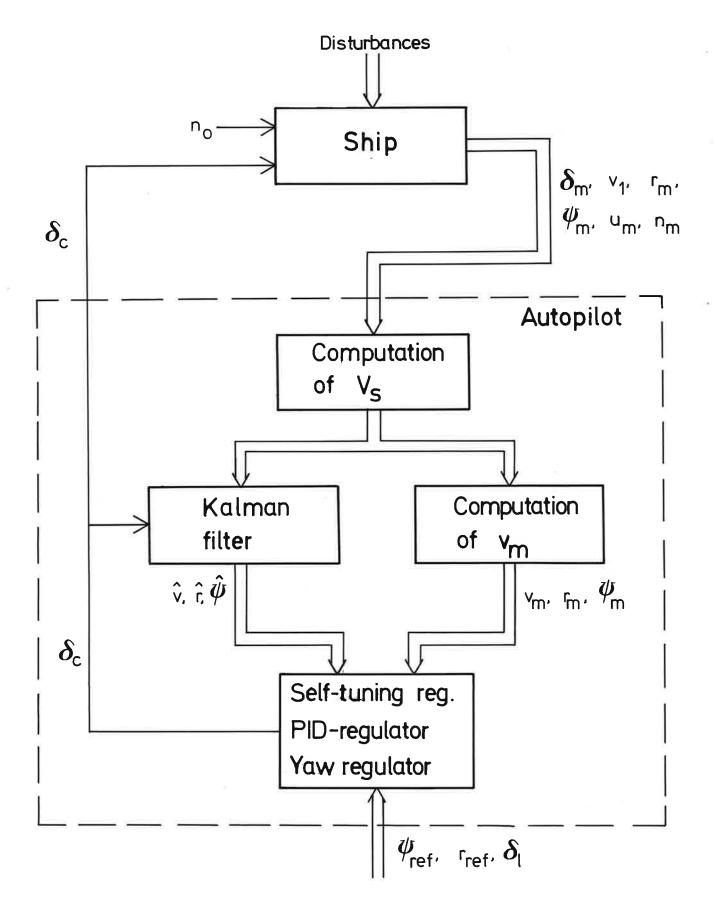


Fig. 3.1 - Structure of the autopilot.

3.1 Kalman Filter

The following linear model is used when designing the Kalman filter:

$$\begin{cases} dx = Ax \ dt + Bu \ dt + dw \\ y(t_k) = \theta x(t_k) + \tilde{e}(t_k), \quad k = 0, 1, 2, ... \end{cases}$$
 (3.3)

where { w(t), $t_0 \le t \le \infty$ } is assumed to be a wiener process with incremental covariance R_1 dt and the measurement errors { $\widetilde{e}(t_k)$ } are assumed to be independent and gaussian with zero mean and covariance \widetilde{R}_2 . It is furthermore assumed that the measurement errors are independent of { w(t), $t_0 \le t \le \infty$ }. The vectors and matrices of (3.3) are explained by:

$$\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{T_{\mathbf{r}}} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{e} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & \ell_{1} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{diag} \ (\mathbf{R}_{1}) = \begin{bmatrix} r_{1} & r_{2} & r_{3} & r_{4} & r_{5} & r_{6} & r_{7} & r_{8} \end{bmatrix}$$

$$\mathbf{diag} \ (\widetilde{\mathbf{R}}_{2}) = \begin{bmatrix} (\sigma_{\delta}/\mathbf{CRG})^{2} & (\sigma_{\mathbf{v}}/\mathbf{CMK})^{2} & (\sigma_{\mathbf{r}}/\mathbf{CRG})^{2} & (\sigma_{\psi}/\mathbf{CRG})^{2} \end{bmatrix}$$

where δ_0 [rad] is the rudder bias due to disturbances, δ_d [rad] is equal to $\delta-\delta_0$ and d_v [knots], d_r [deg/s] and d_δ [deg] are measurement biases (cf (2.8)). The total speed is denoted V [m/s]. The parameters are assigned the following values (cf Chapter 2):

CRG = 57.2958 deg

CMK = 1.943844 knots * s/m

L = 350 m

$$\ell_1$$
 = 164.35 m

 T_r = 5 s

 $a_{11} = Y''_{|u|v} / (1 - Y_v'') = -0.385$
 $a_{12} = (Y''_{ru} - 1) / (1 - Y_v'') = -0.451$
 $a_{21} = N''_{|u|} / (k''_{zz} - N_r'') = -3.398$
 $a_{22} = (N''_{r|u|} - x''_{0}) / (k''_{zz} - N_r'') = -1.583$
 $b_{11} = (Y''_{uu\delta} + Y''_{u|u|\delta}) / (1 - Y_v'') = -0.0967$
 $b_{21} = (N''_{uu\delta} + N''_{u|u|\delta}) / (k''_{zz} - N_r'') = 0.806$
 $r_1 = 2.5 \cdot 10^{-5} \text{ m}^2/\text{s}^3$
 $r_2 = 5 \cdot 10^{-9} \text{ 1/s}^3$
 $r_3 = 8 \cdot 10^{-8} \text{ 1/s}$

$$r_4 = 8 \cdot 10^{-8} \text{ 1/s}$$
 $r_5 = 3 \cdot 10^{-9} \text{ 1/s}$
 $r_6 = 9 \cdot 10^{-8} \text{ m}^2/\text{s}^3$
 $r_7 = 3 \cdot 10^{-12} \text{ 1/s}^3$
 $r_8 = 8 \cdot 10^{-11} \text{ 1/s}$
 $\sigma_{\delta} = 0.2 \text{ deg}$
 $\sigma_{v} = 0.05 \text{ knots}$
 $\sigma_{r} = 0.02 \text{ deg/s}$
 $\sigma_{v} = 0.05 \text{ deg}$

The dynamics of (3.3) is equivalent to the linearization of the model (2.1) when parameter values for full load condition are used. By performing some calculations it can be concluded that the model (3.3) is observable, if the heading measurement ψ_m is one of the outputs. This means that one or more of the measurement signals δ_m , v_1 and r_m can be rejected and still it is possible to obtain good state estimates, at least if the dynamics and the disturbances are modelled reasonably well.

The model (3.3) is now normalized using the length of the ship L as unit of length and the time to cover the length L, i.e. L/V, as unit of time:

$$\begin{cases} dx' = A'x'dt' + B'u'dt' + dw' \\ y'(t'_k) = \theta'x'(t'_k) + \tilde{e}'(t'_k), & k = 0, 1, 2, ... \end{cases}$$
(3.4)

where

$$(x')^T = \begin{bmatrix} v' & r' & \psi' & \delta_d' & \delta_0' & d_v' & d_r' & d_\delta' \end{bmatrix}$$

$$u' = \delta_C'$$

$$\begin{aligned} \text{diag } (\mathbf{R_1'}) &= \left[\begin{array}{ccc} \frac{\mathbf{L}}{\mathbf{V}^3} \mathbf{r}_1 & \frac{\mathbf{L}^3}{\mathbf{V}^3} \mathbf{r}_2 & \frac{\mathbf{L}}{\mathbf{V}} \mathbf{r}_3 & \frac{\mathbf{L}}{\mathbf{V}} \mathbf{r}_4 & \frac{\mathbf{L}}{\mathbf{V}} \mathbf{r}_5 & \frac{\mathbf{L}}{\mathbf{V}^3} \mathbf{r}_6 & \frac{\mathbf{L}^3}{\mathbf{V}^3} \mathbf{r}_7 & \frac{\mathbf{L}}{\mathbf{V}} \mathbf{r}_8 \end{array} \right] \\ \text{diag } (\widetilde{\mathbf{R}_2'}) &= \left[(\sigma_{\delta}/\mathsf{CRG})^2 & (\sigma_{\mathbf{V}}/\mathsf{(CMK^*V)})^2 & (\sigma_{\mathbf{r}^*}\cdot\mathbf{L}/\mathsf{(CRG^*V)})^2 & (\sigma_{\psi}/\mathsf{CRG})^2 \end{array} \right] \end{aligned}$$

The normalized variables are obtained as

$$t' = \frac{V}{L} \cdot t$$

$$t'_{k} = \frac{V}{L} \cdot t_{k}$$

$$v' = \frac{1}{V} v$$

$$r' = \frac{L}{V} r$$

$$\psi' = \psi$$

$$\delta'_{0} = \delta_{0}$$

$$\delta'_{0} = \delta_{0}$$

$$d'_{v} = \frac{L}{CMK \cdot V} d_{v}$$

$$d'_{r} = \frac{L}{CRG \cdot V} d_{r}$$

$$d'_{\delta} = \delta_{\delta}/CRG$$

$$\delta'_{c} = \delta_{c}/CRG$$

$$\delta'_{m} = \delta_{m}/CRG$$

$$v'_{1} = \frac{L}{CMK \cdot V} v_{1}$$

$$r'_{m} = \frac{L}{CRG \cdot V} r_{m}$$

$$\psi'_{m} = \psi_{m}/CRG$$
(3.5)

The normalized model (3.4) is now transformed to a discrete model with sampling interval $h^{\, \prime} = \frac{V}{L} \, h$, where h = 1 s:

$$\begin{cases} x'(t'+h') = \Phi'x'(t') + F'u'(t') + \widetilde{w}'(t') \\ y'(t') = \theta'x'(t') + \widetilde{e}'(t') \end{cases}$$
(3.6)

Since the value of h' is rather small (e.g. h' = 0.023 if V = 8 m/s), the following approximations are not too bad:

$$\Phi' \approx I + A'h'$$

$$\Gamma' \approx B'h'$$

$$\widetilde{R}_{1} \approx R_{1}'h'$$
(3.7)

It can be concluded using (3.7) that the speed dependence of Φ' , Γ' and θ' in (3.6) is rather insignificant. Some

elements of the covariance matrices $\widetilde{R}_1^{\, \prime}$ and $\widetilde{R}_2^{\, \prime}$, however, are dependent on the speed V.

If it is assumed that V = 8 m/s, the following matrices are obtained (no approximations):

$$\Phi' = \begin{bmatrix} 0.99163 & -1.00810 \cdot 10^{-2} & 0 & -2.08207 \cdot 10^{-3} & 2.12358 \cdot 10^{-4} & 0 & 0 & 0 \\ -7.59537 \cdot 10^{-2} & 0.96485 & 0 & 1.64706 \cdot 10^{-2} & -1.70991 \cdot 10^{-3} & 0 & 0 & 0 \\ -8.74514 \cdot 10^{-4} & 2.24515 \cdot 10^{-2} & 1 & 1.95418 \cdot 10^{-4} & -1.32723 \cdot 10^{-5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.81873 & -0.18127 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(\Gamma')^{\mathrm{T}} = \begin{bmatrix} -2.12358 \cdot 10^{-4} & 1.70991 \cdot 10^{-3} & 1.32723 \cdot 10^{-5} & 0.18127 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta^{\bullet} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0.46957 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

 $\widetilde{R}_1' = \text{(see page 16)}$

diag
$$(\widetilde{R}_{2}') = \begin{bmatrix} 1.21847 \cdot 10^{-5} & 1.03380 \cdot 10^{-5} & 2.33223 \cdot 10^{-4} & 7.61544 \cdot 10^{-7} \end{bmatrix}$$

| 0 | 0 | 0 | 0 | 0 | 0 | 8.00000.10-11 |
|---------------------------|--|---|---|--|--|--|
| 0 | 0 | 0 | 0 | 0 6- | 5.74218.10 ⁻⁹ | 0 |
| 0 | 0 | 0 | 0 | 0625.10 | 0 | 0 |
| .10-12 | .10-14 | .10-10 | -108 | 1.4 | | |
| -1.74199 | -1.00657 | -2.80961 | 3.00000 | 0 | 0 | 0 |
| 6.00259.10 ⁻¹⁰ | 4.57698.10 ⁻¹² | 6.59705.10 ⁻⁸ | -2.80961.10-10 | 0 | 0 | 0 |
| 1.05534.10 ⁻⁷ | 8.16225.10 ⁻⁸ | 4.57698.10 ⁻¹² | -1.00657.10-14 | 0 | 0 | 0 |
| 9.23544.10-6 | 1.05534.10 ⁻⁷ | 6.00259.10-10 | -1.74199.10-12 | 0 | 0 | 0 |
| -6.23157.10 ⁻⁸ | -8.42885.10-10 | -7.45031.10-11 | 2.14470.10 ⁻¹³ | 0 | 0 | 0 |
| | 9.23544°10 ⁻⁶ 1.05534°10 ⁻⁷ 6.00259°10 ⁻¹⁰ -1.74199°10 ⁻¹² 0 | $9.23544 \cdot 10^{-6}$ $1.05534 \cdot 10^{-7}$ $6.00259 \cdot 10^{-10}$ $-1.74199 \cdot 10^{-12}$ 0 $1.05534 \cdot 10^{-7}$ $8.16225 \cdot 10^{-8}$ $4.57698 \cdot 10^{-12}$ $-1.00657 \cdot 10^{-14}$ 0 | 9.23544·10 ⁻⁶ 1.05534·10 ⁻⁷ 6.00259·10 ⁻¹⁰ -1.74199·10 ⁻¹² 0 0 0 1.05534·10 ⁻⁷ 8.16225·10 ⁻⁸ $4.57698\cdot10^{-12}$ -1.00657·10 ⁻¹⁴ 0 0 0 1 6.00259·10 ⁻¹⁰ $4.57698\cdot10^{-12}$ 6.59705·10 ⁻⁸ -2.80961·10 ⁻¹⁰ 0 0 | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | $\begin{array}{cccccccccccccccccccccccccccccccccccc$ |

Using (3.6) the Kalman filter equations are obtained as (cf. Aström (1970)):

where

$$(\epsilon')^{\mathrm{T}} = \begin{bmatrix} \epsilon_{\delta}' & \epsilon_{\mathbf{v}}' & \epsilon_{\mathbf{r}}' & \epsilon_{\psi}' \end{bmatrix}$$
 (3.9)

and

$$K = \begin{bmatrix} -1.29 \cdot 10^{-4} & 9.73 \cdot 10^{-2} & -2.40 \cdot 10^{-2} & -0.369 \\ 5.31 \cdot 10^{-4} & 0.481 & 9.61 \cdot 10^{-2} & 1.01 \\ 1.22 \cdot 10^{-5} & 7.90 \cdot 10^{-3} & 3.12 \cdot 10^{-3} & 0.326 \\ 1.60 \cdot 10^{-2} & -2.13 \cdot 10^{-4} & 1.65 \cdot 10^{-3} & 1.89 \cdot 10^{-2} \\ -5.82 \cdot 10^{-12} & 3.08 \cdot 10^{-4} & -1.63 \cdot 10^{-3} & -1.88 \cdot 10^{-2} \\ -3.69 \cdot 10^{-5} & 9.39 \cdot 10^{-3} & 1.61 \cdot 10^{-4} & 8.17 \cdot 10^{-5} \\ -1.40 \cdot 10^{-6} & -1.56 \cdot 10^{-3} & 3.01 \cdot 10^{-3} & -5.78 \cdot 10^{-2} \\ 2.54 \cdot 10^{-3} & 2.57 \cdot 10^{-6} & 3.31 \cdot 10^{-6} & 3.66 \cdot 10^{-5} \end{bmatrix}$$

The speed $V_{\rm S}$ (cf. (3.1)) is used to normalize the measurement vector, i.e.

$$(y')^{T} = \begin{bmatrix} \delta_{m}/CRG & \frac{1}{CMK \cdot V_{S}} v_{1} & \frac{L}{CRG \cdot V_{S}} r_{m} & \psi_{m}/CRG \end{bmatrix}$$
 (3.11)

The state estimate vector

$$(\overset{\wedge}{\mathbf{x}})^{\mathrm{T}} = \begin{bmatrix} \overset{\wedge}{\mathbf{v}} & \overset{\wedge}{\mathbf{r}} & \overset{\wedge}{\mathbf{v}} & \overset{\wedge}{\mathbf{d}}_{\mathbf{d}} & \overset{\wedge}{\mathbf{d}}_{\mathbf{d}} & \overset{\wedge}{\mathbf{d}}_{\mathbf{d}} & \overset{\wedge}{\mathbf{d}}_{\mathbf{d}} \\ & & & & & & & & \end{bmatrix}$$
 (3.12)

is updated every second. The initial state estimate vector is equal to

$$\left(\stackrel{\wedge}{\mathbf{x}} \cdot (\mathbf{t}_0) \right)^{\mathrm{T}} = \left[\begin{array}{cccc} \frac{1}{\text{CMK} \cdot \mathbf{V}_{\mathbf{S}}} \, \mathbf{v}_{\mathbf{m}} & \frac{\mathbf{L}}{\text{CRG} \cdot \mathbf{V}_{\mathbf{S}}} \, \mathbf{r}_{\mathbf{m}} & \psi_{\mathbf{m}}/\text{CRG} & \delta_{\mathbf{m}}/\text{CRG} & 0 & 0 & 0 \end{array} \right]$$

where \boldsymbol{v}_{m} is obtained from (3.2) and all the measurements are from the time $\boldsymbol{t}_{0}.$

Ιf

$$|\epsilon_{\delta}^{i}| > t_{\delta}^{i}$$

$$|\varepsilon_{V}^{1}| > t_{V}^{1}$$

$$|\epsilon_{r}^{\dagger}| > t_{r}^{\dagger}$$

or

$$|\epsilon_{\psi}^{\dagger}| > t_{\psi}^{\dagger}$$
,

where

$$t_{\delta}^{I} = 0.75$$
 $t_{V}^{I} = 0.06$
 $t_{r}^{I} = 0.25$
 $t_{\psi}^{I} = 0.015$, (3.13)

then the corresponding measurement or measurements are rejected, when the state estimate vector is updated. A measurement signal is definitively rejected when 10 consecutive measurements are rejected. However, during the first 900 s after the Kalman filter is initialized, no measurements are rejected because the bias states of the Kalman filter must be fairly estimated to avoid incorrect rejectings. The rejecting of a measurement is performed by putting the corresponding column of K (see (3.10)) equal to zero. Notice that the values of (3.13) are chosen very large to avoid rejectings when the Kalman filter is tested.

The non-normalized state estimate vector is obtained as (cf. (3.12)):

$$(\hat{x})^{T} = \begin{bmatrix} \hat{v} & \hat{r} & \hat{\psi} & \hat{\delta}_{d} & \hat{\delta}_{0} & \hat{d}_{v} & \hat{d}_{r} & \hat{d}_{\delta} \end{bmatrix}$$
(3.14)

where

$$\dot{V} = CMK \cdot V_{S} \cdot \dot{V}' \quad [knots]$$

$$\dot{\hat{\Gamma}} = \frac{CRG \cdot V_{S}}{L} \dot{\hat{\Gamma}}' \quad [deg/s]$$

$$\dot{\hat{V}} = CRG \cdot \dot{\hat{V}}' \quad [deg]$$

$$\dot{\hat{\delta}}_{d} = CRG \cdot \dot{\hat{\delta}}_{d}' \quad [deg]$$

$$\dot{\hat{\delta}}_{0} = CRG \cdot \dot{\hat{\delta}}_{0}' \quad [deg]$$

$$\dot{\hat{C}}_{V} = CMK \cdot V_{S} \cdot \dot{\hat{C}}_{V}' \quad [knots]$$

$$\dot{\hat{C}}_{r} = \frac{CRG \cdot V_{S}}{L} \dot{\hat{C}}_{r}' \quad [deg/s]$$

$$\dot{\hat{C}}_{\delta} = CRG \cdot \dot{\hat{C}}_{\delta}' \quad [deg]$$

Notice the following expressions, too:

$$\hat{\mathbf{v}}_{1} = \hat{\mathbf{v}} + CMK \cdot \ell_{1} \cdot \hat{\mathbf{r}} / CRG \quad [knots]$$

$$\hat{\delta} = \hat{\delta}_{d} + \hat{\delta}_{0} \quad [deg]$$

3.2 Self-tuning Regulator

A simple self-tuning regulator based on least squares identification and minimum variance control is used for straight course keeping. The basic self-tuning regulator is described in Wittenmark (1973).

The following model of the ship is used by the self-tuning regulator:

where the design speed is denoted V_0 [m/s] and the speed V_s [m/s] is defined by (3.1). The minimum variance control is given by

$$\nabla \delta_{c}(t) = \left(\frac{V_{o}}{V_{s}(t)}\right)^{2} \frac{1}{b_{0}} \left[a_{1}(\mathring{\psi}(t) - \psi_{ref}) + \dots + a_{NA}(\mathring{\psi}(t - NA + 1) - \psi_{ref}) - \right.$$

$$- \left(V_{s}(t-1)/V_{0}\right)^{2} b_{0} b_{1} \nabla \delta_{c}(t-1) - \dots - \left. \left(V_{s}(t-NB)/V_{0}\right)^{2} b_{0} b_{NB} \nabla \delta_{c}(t-NB) - \left. \left(V_{s}(t-NB)/V_{0}\right)^{2} \right] b_{0} b_{NB} \nabla \delta_{c}(t-NB) - \left. \left(V_{s}(t-NB)/V_{0}\right)^{2} \right] b_{0} b_{NB} \nabla \delta_{c}(t-NB) - \left. \left(V_{s}(t-NB)/V_{0}\right)^{2} \right] c_{0} c_{$$

where

$$\nabla \delta_{c}(t) = \delta_{c}(t) - \delta_{c}(t-1)$$

$$\nabla \nabla (t) = \nabla (t) - \nabla (t-1)$$

$$\nabla r(t) = r(t) - r(t-1)$$

Notice that the speed scaling of (3.15) and (3.16) is introduced in such a way that the parameters a_1, \dots, a_{NA} , b_1, \dots, b_{NB} , c_1 and c_2 are approximately independent of the forward speed. This fact will, of course, simplify the mission of the self-tuning regulator.

The following standard values are used:

$$NA = 4$$
 $NB = 2$
 $k = 7$
 $T_s = 10 \text{ s}$
 $\lambda_f = 0.99$
 $b_0 = 1$
 $v_0 = 8 \text{ m/s}$

where T_S is the sampling interval and λ_f the exponential forgetting factor. Other values of k and T_S are sometimes used. The following initial values of the parameters and of the covariance matrix P are used when the Kalman filter estimates are fed into the self-tuning regulator:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -6.91 \\ 5.95 \\ 3.88 \\ -3.57 \\ 0.48 \\ 0.11 \\ -2.10 \\ 34.73 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3.17) \\ 0.01 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3.17) \\ 0.01 \\ 0 \\ 0 \end{bmatrix}$$

When non-filtered measurements are used instead of Kalman filter estimates, then $c_1 = c_2 = 0$, i.e. no feedforward signals. The following initial values are then used:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -7.64 \\ 8.44 \\ 1.74 \\ -3.15 \\ 0.00834 \\ 0.195 \end{bmatrix} P = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0.01 \end{bmatrix}$$

$$(3.18)$$

By use of the minimum variance control (3.16) the following criterion is minimized:

$$J_{1} = \sum_{n=k+1}^{N} \left(\sqrt[h]{(nT_{s})} - \psi_{ref} \right)^{2}$$
 (3.19)

If the criterion

$$J_{2} = \sum_{n=k+1}^{N} \left[\left(\hat{\psi}(nT_{S}) - \psi_{ref} \right)^{2} + q_{2} \left(\nabla \delta_{c} \left((n-k-1)T_{S} \right) \right)^{2} \right]$$
 (3.20)

is minimized instead, a penalty on the rudder motions is introduced by the parameter \mathbf{q}_2 . However, a proper solution

of this problem requires the solving of a Riccati equation. A self-tuning regulator, which performs this, is used in Källström (1976a).

If the criterion (3.20) is modified to read

$$J_{3}(n) = \left(\psi((n+k+1)T_{s}) - \psi_{ref} \right)^{2} + q_{2} \left(\nabla \delta_{c}(nT_{s}) \right)^{2}$$

$$n = 0, 1, ..., N-k-1$$
(3.21)

and if (3.21) is minimized at every sample event, then a simpler regulator is obtained. By inserting (3.15) into (3.21) and then performing the minimization, the following control is obtained:

$$\overline{\nabla \delta_{c}(t)} = \frac{(V_{s}(t)/V_{0})^{4} b_{0}^{2}}{(V_{s}(t)/V_{0})^{4} b_{0}^{2} + q_{2}} \nabla \delta_{c}(t)$$
(3.22)

where $\nabla \delta_{_{\bf C}}(t)$ is the minimum variance control given by (3.16). If ${\bf q}_2=0$, then minimization of (3.21) gives the same result as minimization of (3.19) and consequently the controls (3.22) and (3.16) are equivalent. Notice that (3.22) only is a very small modification of (3.16) and that the identification part of the self-tuning regulator is unchanged. However, the control (3.22) has the serious disadvantage that no guarantee of closed loop stability is obtained in the general case.

The minimum variance control (3.16) is approximately scaled by $\left(V_0/V_{\rm S}\left(t\right)\right)^2$ when the speed changes, i.e. (3.22) may be re-written

$$\overline{\nabla \delta_{c}(t)} = \frac{b_0^2}{b_0^2 + \left(\frac{V_0}{V_s(t)}\right)^4 q_2} \left(\frac{V_0}{V_s(t)}\right)^2 \left[\nabla \delta_{c}(t)\right]_{V_0} \tag{3.23}$$

where $\left[\forall \delta_{_{\mathbf{C}}}(\mathsf{t}) \right]_{V_{0}}$ denotes the minimum variance control when $v_{_{\mathbf{S}}} = v_{_{\mathbf{0}}}$. By introducing $q_{_{\mathbf{2}}}^* = \left(v_{_{\mathbf{0}}} / v_{_{\mathbf{S}}}(\mathsf{t}) \right)^4 q_{_{\mathbf{2}}}$ we obtain

$$\overline{\nabla \delta_{c}(t)} = \frac{b_{0}^{2}}{b_{0}^{2} + q_{2}^{*}} \left(\frac{V_{0}}{V_{s}(t)}\right)^{2} \left[\nabla \delta_{c}(t)\right]_{V_{0}} = \frac{b_{0}^{2}}{b_{0}^{2} + q_{2}^{*}} \nabla \delta_{c}(t) \qquad (3.24)$$

which is the actual control used in the autopilot. The standard value of ${\bf q_2}^{*}$ is equal to zero.

The estimates from the Kalman filter are used in all formulas of this section. Notice, however, that it is possible to use the non-filtered measurements instead.

3.3 PID-regulator

The following discrete PID-regulator for straight course keeping is also implemented for comparison:

$$\delta_{\mathbf{C}}(\mathbf{n}\mathbf{T}_{\mathbf{S}}) = -\left(\frac{\mathbf{V}_{0}}{\mathbf{V}_{\mathbf{S}}(\mathbf{n}\mathbf{T}_{\mathbf{S}})}\right)^{2} \left[k_{\mathbf{P}}(\mathring{\psi}(\mathbf{n}\mathbf{T}_{\mathbf{S}}) - \psi_{\mathbf{ref}}) + k_{\mathbf{D}} \mathring{\mathbf{r}}(\mathbf{n}\mathbf{T}_{\mathbf{S}}) + k_{\mathbf{D}}$$

n = 0, 1, 2, ...

The following standard values are used:

$$k_{p} = 3$$
 $k_{D} = 75 \text{ s}$
 $k_{I} = 0.02 \text{ l/s}$
 $T_{s} = 10 \text{ s}$
 $V_{0} = 8 \text{ m/s}$

(3.26)

The scaling speed $V_{\rm S}$ is obtained from (3.1). Notice that it is possible to use the non-filtered measurements instead of the Kalman filter estimates in (3.25). The special speed scaling used in (3.25) will approximately give the same course keeping performance independent of the forward speed. The rudder deviations, however, are increased proportional to $(V_{\rm O}/V_{\rm S})^2$ when the speed is decreased.

3.4 Yaw Regulator

A yaw performed by the yaw regulator consists of four different phases, viz. the initial phase (phase 1), the phase of constant yaw rate (phase 2), the checking rudder phase (phase 3) and the terminating phase (phase 4). However, if the requested heading change $\Delta \psi_{\rm ref}$ is small, one or more of the phases may be skipped. The Kalman filter estimates used by the yaw regulator are the yaw rate \hat{r} and the heading $\hat{\psi}$, and the reference values used are the requested yaw rate $r_{\rm ref}$ and the new requested heading $\psi_{\rm ref}$. The phase of straight course keeping is denoted phase 0.

Modified discrete, fixed gain PID-regulators are used in the different phases (note that n = 0, 1, 2, ...):

Phase 1:

$$\delta_{\mathbf{C}}(\mathbf{n}\mathbf{T}_{\mathbf{S}}) = -\left(\frac{\mathbf{V}_{0}}{\mathbf{V}_{\mathbf{S}}(\mathbf{n}\mathbf{T}_{\mathbf{S}})}\right)^{2} \mathbf{k}_{4} \left[\hat{\mathbf{r}}(\mathbf{n}\mathbf{T}_{\mathbf{S}}) - \mathbf{r}_{0}\right] + \overline{\delta}_{\mathbf{C}}$$

$$\left|\mathbf{k}_{4} \left[\hat{\mathbf{r}}(\mathbf{n}\mathbf{T}_{\mathbf{S}}) - \mathbf{r}_{0}\right]\right| \leq \left|\overline{\mathbf{c}}_{1} \mathbf{r}_{0}\right|$$

Phase 2:

$$\delta_{c}(nT_{s}) = -\left(\frac{V_{0}}{V_{s}(nT_{s})}\right)^{2} k_{5} \left[\hat{r}(nT_{s}) - r_{0}\right] - \left(\frac{V_{0}}{V_{s}(nT_{s})}\right)^{2} k_{6} T_{s} \sum_{i=0}^{n-1} \left[\hat{r}(iT_{s}) - r_{0}\right] + \overline{\delta}_{c}$$

Phase 3:

$$\delta_{c}(nT_{s}) = -\left(\frac{V_{0}}{V_{s}(nT_{s})}\right)^{2} k_{7} \left[\mathring{\psi}(nT_{s}) - \psi_{ref}\right] - \left(\frac{V_{0}}{V_{s}(nT_{s})}\right)^{2} k_{8} \mathring{r} (nT_{s})$$

$$\left| k_{7} \left[\mathring{\psi}(nT_{s}) - \psi_{ref}\right] + k_{8} \mathring{r} (nT_{s}) \right| \leq \left| \overline{c}_{3} r_{0} \right|$$

Phase 4:

$$\begin{split} \delta_{_{\mathbf{C}}}(\mathbf{n}\mathbf{T}_{_{\mathbf{S}}}) &= -\left(\frac{\mathbf{V}_{0}}{\mathbf{V}_{_{\mathbf{S}}}(\mathbf{n}\mathbf{T}_{_{\mathbf{S}}})}\right)^{2} \ \mathbf{k}_{1} \ \left[\mathbf{\hat{\psi}}(\mathbf{n}\mathbf{T}_{_{\mathbf{S}}}) - \mathbf{\psi}_{\mathbf{ref}} \right] - \\ &- \left(\frac{\mathbf{V}_{0}}{\mathbf{V}_{_{\mathbf{S}}}(\mathbf{n}\mathbf{T}_{_{\mathbf{S}}})}\right)^{2} \ \mathbf{k}_{2} \ \mathbf{\hat{r}} \ (\mathbf{n}\mathbf{T}_{_{\mathbf{S}}}) - \\ &- \left(\frac{\mathbf{V}_{0}}{\mathbf{V}_{_{\mathbf{S}}}(\mathbf{n}\mathbf{T}_{_{\mathbf{S}}})}\right)^{2} \ \mathbf{k}_{3} \ \mathbf{T}_{_{\mathbf{S}}} \ \mathbf{\hat{\Sigma}} \ \mathbf{\hat{\Sigma}} \ \left[\mathbf{\hat{\psi}}(\mathbf{i}\mathbf{T}_{_{\mathbf{S}}}) - \mathbf{\hat{\psi}}_{\mathbf{ref}} \right] \end{split}$$

The scaling speed V_S is obtained from (3.1). The moving average $\overline{\delta}_C$ of the rudder commands δ_C is only updated during phase 0 and phase 4:

$$\begin{split} \overline{\delta}_{\mathbf{C}}\big((\mathbf{k}+1)\mathbf{T}_{\mathbf{S}}\big) &= \overline{\delta}_{\mathbf{C}}(\mathbf{k}\mathbf{T}_{\mathbf{S}}) + \bigg(\frac{1-\gamma}{\mathbf{k}+1} + \gamma\bigg)\bigg(\delta_{\mathbf{C}}(\mathbf{k}\mathbf{T}_{\mathbf{S}}) - \overline{\delta}_{\mathbf{C}}(\mathbf{k}\mathbf{T}_{\mathbf{S}})\bigg), \\ \mathbf{k} &= 0, 1, 2, \dots \\ \overline{\delta}_{\mathbf{C}}(0) &= 0 \end{split}$$

The reference yaw rate \mathbf{r}_0 including sign is computed once, when the yaw is initiated, as

$$r_0 = r_{ref}$$
 if $\psi - \psi_{ref} \leq 0$

or as

$$r_0 = -r_{ref}$$
 if $\psi - \psi_{ref} > 0$

Notice that the value of r_{ref} always is positive.

The conditions to jump from one phase to another read:

Phase $0 \rightarrow \text{phase } 4$:

$$\psi_1 < \Delta \psi_{ref} \leq \psi_2$$

Phase $0 \rightarrow \text{phase } 1$:

$$\Delta \psi_{ref} > \psi_2$$

Phase 1 \rightarrow phase 2:

$$r_0 \ge 0$$
 and $\hat{r} - r_0 > -\epsilon_1$

or

$$r_0 < 0$$
 and $\hat{r} - r_0 < \epsilon_1$

or

(time in phase 1) > T_1

Phase 1 or 2 \rightarrow phase 3:

$$r_0 \ge 0$$
 and $-\overline{c}_2 \stackrel{\wedge}{r} < \stackrel{\wedge}{\psi} - \psi_{ref}$
or $r_0 < 0$ and $-\overline{c}_2 \stackrel{\wedge}{r} > \stackrel{\wedge}{\psi} - \psi_{ref}$

Phase $3 \rightarrow \text{phase } 4$:

or
$$r_0 \geqslant 0 \quad \text{and} \quad \mathring{\psi} - \psi_{\text{ref}} > -\varepsilon_3$$
 or
$$r_0 < 0 \quad \text{and} \quad \mathring{\psi} - \psi_{\text{ref}} < \varepsilon_3$$
 or
$$(\text{time in phase 3}) > T_3$$

Phase $4 \rightarrow \text{phase } 0$:

(time in phase 4) > T_4

The condition to remain in phase 0 is:

$$\Delta \psi_{\text{ref}} \leq \psi_1$$

If the reference yaw rate r_{ref} is changed during a yaw, the new value is immediately used, but no other changes. It is also possible to change the reference course ψ_{ref} during a yaw, and then a new yaw is initiated by entering phase 1, if

$$|\Delta\psi_{\text{ref}}| > \psi_3$$

and (3.27)

the actual phase is 3 or 4

or if

$$|\Delta\psi_{\text{ref}}| > \psi_3$$
,

the actual phase is 1 or 2,

and one of the two conditions

$$r_0 > 0$$
, $\hat{\psi} - \psi_{ref} > 0$ and

$$r_0$$
 < 0, $\dot{\psi}$ - ψ_{ref} < 0

is satisfied.

If neither condition (3.27) nor condition (3.28) is fulfilled, the new value of $\psi_{\mbox{ref}}$ is used, but no other changes.

The following parameter values of the yaw regulator are used:

| k ₁ = 5 | $\bar{c}_1 = 60 \text{ s}$ |
|-----------------------------------|---------------------------------|
| $k_2 = 200 \text{ s}$ | $\overline{c}_2 = 50 \text{ s}$ |
| $k_3 = 0.005 1/s$ | $\bar{c}_3 = 60 \text{ s}$ |
| k ₄ = 200 s | $T_1 = 30 s$ |
| $k_5 = 200 \text{ s}$ | $T_3 = 100 \text{ s}$ |
| $k_6 = 8$ | $T_4 = 300 s$ |
| $k_7 = 2$ | $\psi_1 = 0.35 \text{ deg}$ |
| k ₈ = 200 s | ψ_2 = 2.5 deg |
| $\epsilon_1 = 0$ deg/s | $\psi_3 = 2.5 \text{ deg}$ |
| $\epsilon_2 = 0.02 \text{ deg/s}$ | $v_0 = 8 \text{ m/s}$ |
| $\epsilon_3 = 1$ deg | $T_s = 10 s$ |
| | $\gamma = 0.05$ |

A special indicator M_y is used to describe the actual yaw phase, i.e. M_y = 0, 1, 2, 3, 4 corresponds to phase 0, 1, 2, 3, 4 respectively. Notice that it is possible to use the non-filtered measurements instead of the Kalman filter estimates in the yaw regulator. The special speed scaling used in the yaw regulator will approximately give the same performance of the yaw rate and the heading independent of the forward speed. The rudder deviations, however, are increased proportional to $(V_0/V_s)^2$ when the speed is decreased.

4. SIMULATIONS

To make it possible to compare the steering quality of different autopilot structures, a loss function is now introduced:

$$V_{\ell} = \frac{1}{\tau} \int_{0}^{\tau} \left[\left(\psi(t) - \psi_{\text{ref}} \right)^{2} + \lambda \left(\delta(t) - m_{\delta} \right)^{2} \right] dt$$
 (4.1)

where m_{δ} is the mean value of the rudder angle δ and the weighting factor λ is equal to 1/12 or 0. The duration of a simulation is denoted τ . The loss function is approximated by:

$$V_{\ell} = \frac{1}{N} \sum_{n=0}^{N-1} \left[\left(\psi(nh) - \psi_{ref} \right)^{2} + \lambda \left(\delta(nh) - m_{\delta} \right)^{2} \right]$$
 (4.2)

where Nh = τ and the sampling interval h always is equal to 1 s.

In the sequel the mean values m_{δ} and m_{ψ} , and the standard deviations σ_{δ} and σ_{ψ} , of the rudder angle δ and the course error $\psi^-\psi_{\rm ref}$, resp, will be presented as well as the loss function V_{ℓ} . Notice that the rudder angle δ and the heading angel ψ without measurement noise and without measurement bias are used. If nothing else is remarked, the standard values given in Chapters 2 and 3 are used in the simulations. Several plots are often shown in the same figure, and the plots are then slided in relation to each other. The corresponding straight line is the level zero. Notice that the initial forward speed is denoted u_{0} .

4.1 Kalman Filter Testing

Simulations of straight course keeping ($\psi_{\rm ref}$ = 0 deg) with the self-tuning regulator using estimates from the Kalman filter are presented in this section. The performance of the Kalman filter and the self-tuning regulator for different load conditions and different speeds are shown in Figs 4.1 - 4.6.

In Fig. 4.7 the initial covariance matrix of the self-tuning regulator is given as (cf.(3.17)):

diag (P) =
$$[10 \ 10 \ 10 \ 10 \ 0.1 \ 0.1 \ 10 \ 500]$$
 (4.3)

The only measurement signal used by the Kalman filter in Figs. 4.8 and 4.9 is the heading angle. The first three columns of the filter gain K (cf.(3.10)) are cancelled in Fig. 4.8, while the following filter gain, designed correctly for the case of heading measurements only, is used in Fig. 4.9:

$$K^{T} = [-0.481 \quad 2.4 \quad 0.39 \quad 0.072 \quad -0.0716 \quad 0 \quad 0 \quad 0] \quad (4.4)$$

The process noise is increased in Fig. 4.10 by use of the following covariance matrix (cf.(2.6)):

$$R_{W} = \begin{bmatrix} 3.2 \cdot 10^{-4} & 0 \\ 0 & 2.12 \cdot 10^{-10} \end{bmatrix}$$
 (4.5)

A modified filter gain is used in Fig. 4.11. The tuning rate of the state estimates δ_0 and d_v is increased by changing two elements of R₁:

$$r_5 = 1.2 \cdot 10^{-8} \ 1/s$$

$$r_6 = 4 \cdot 10^{-7} \text{ m}^2/\text{s}^3$$

The corresponding filter gain is (cf.(3.10)):

$$K = \begin{bmatrix} -9.27 \cdot 10^{-5} & 8.82 \cdot 10^{-2} & -2.40 \cdot 10^{-2} & -0.367 \\ 6.30 \cdot 10^{-4} & 0.481 & 9.65 \cdot 10^{-2} & 1.02 \\ 1.69 \cdot 10^{-5} & 7.86 \cdot 10^{-3} & 3.14 \cdot 10^{-3} & 0.326 \\ 1.66 \cdot 10^{-2} & -2.90 \cdot 10^{-3} & 5.65 \cdot 10^{-3} & 6.51 \cdot 10^{-2} \\ 1.03 \cdot 10^{-3} & 2.98 \cdot 10^{-3} & -5.63 \cdot 10^{-3} & -6.49 \cdot 10^{-2} \\ -1.26 \cdot 10^{-4} & 2.00 \cdot 10^{-2} & -6.69 \cdot 10^{-5} & -5.06 \cdot 10^{-3} \\ 1.63 \cdot 10^{-6} & -1.68 \cdot 10^{-3} & 3.02 \cdot 10^{-3} & -5.77 \cdot 10^{-2} \\ 2.54 \cdot 10^{-3} & 7.61 \cdot 10^{-6} & 1.24 \cdot 10^{-5} & 1.41 \cdot 10^{-4} \end{bmatrix}$$

All the simulations are summarized in Table 4.1. The reason why $\lambda=0$ (cf.(4.1)) when the initial speed u_0 is equal to 4 knots is that the course keeping is much more important than the rudder deviations at such a low speed.

From the simulations it can be concluded that the performance of the Kalman filter is very good for the initial speeds 15.8 knots and 10 knots in full load condition as well as in ballast condition (cf. Figs 4.1 - 4.4). Notice, however, that the tuning rate of the state estimates δ_0 and d_v is rather small. The performance of the Kalman filter for the initial speed $u_0 = 4$ knots (Figs 4.5 - 4.7) is not as good as for larger speeds, but quite acceptable. The small tuning rate of $\hat{d}_{_{\mathbf{V}}}$, however, is emphasized. Another difficulty is that the limited rudder turning rate has not been considered in the Kalman filter. This means large values of the residuals $\epsilon_{\delta}^{\prime}$ (see Figs 4.6j and 4.7j) when large rudder changes are requested, which is the case quite often when the speed is low. To avoid incorrect rejectings of the rudder angle measurements, the test value t_{δ}' (cf.(3.13)) must be chosen that large, that the checking is quite meaningless.

| Remarks | | | | | | | Initial P given by (4.3). | Only ∜ _m used by Kalman filter. | Only ψm used by Kalman filter. Correct K (4.4). | R_{W} given by (4.5). | K given by (4.6). |
|---------------------------------------|--------|--------|--------|--------|---------|---------|---------------------------|--|--|-------------------------|-------------------|
| Fig. | 4.1 | 4.2 | 4.3 | 4.4 | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 | 4.10 | 4.11 |
| γ | 1/12 | 1/12 | 1/12 | 1/12 | 0 | 0 | 0 | 1/12 | 1/12 | 1/12 | 1/12 |
| $V_{\mathcal{L}}$ [deg ²] | 0.55 | 0.29 | 2.05 | 1.06 | 1.46 | 0.46 | 2.11 | 0.79 | 0.74 | 1.91 | 0.54 |
| σ _ψ [deg] | 0.47 | 0.29 | 0.51 | 0.35 | 1.01 | 0.61 | 1.14 | 0.48 | 0.46 | 0.82 | 0.44 |
| m ψ [deg] | -0.09 | -0.01 | -0.16 | -0.01 | 0.66 | 0.30 | 0.90 | 0.03 | 0.03 | -0.14 | -0.06 |
| σ _δ [deg] | 2.04 | 1.55 | 4.61 | 3.36 | 19.96 | 19.24 | 22.53 | 2.58 | 2.52 | 3.83 | 2.04 |
| m [deg] | - 1.28 | - 0.15 | - 3.71 | - 1.18 | -21.62 | - 8.93 | -22.29 | - 1.30 | - 1.30 | - 1.38 | - 1.28 |
| δ_{ℓ} [deg] | 10 | 10 | 35 | 35 | 45 | 45 | 45 | 10 | 10 | 10 | 10 |
| u ₀ [knots] | 15.8 | 15.8 | 10 | 10 | 4 | 4 | 4 | 15.8 | 15.8 | 15.8 | 15.8 |
| n ₀ [rpm] | 87.6 | 87.6 | 55.443 | 55.443 | 22.1772 | 22.1772 | 22.1772 | 9.78 | 87.6 | 87.6 | 9.78 |
| _ [m] | 22.3 | 10.5 | 22.3 | 10.5 | 22.3 | 10.5 | 22.3 | 22.3 | 22.3 | 22.3 | 22.3 |

Table 4.1 - Summary of simulations for Kalman filter testing. The duration of each simulation is 2400 s, but the values of ${\rm m_S},~\sigma_{\rm S},~{\rm m_\psi},~\sigma_{\psi},$ and ${\rm V_L}$ are computed for the part 1200 - 2400 s. The straight course keeping ($\psi_{\rm ref}$ = 0 deg) is performed by the self-tuning regulator using estimates from the Kalman

If the only measurement signal used by the Kalman filter is the heading angle, the state estimates \hat{v} , \hat{r} , $\hat{\psi}$, $\hat{\delta}$ and $\hat{\delta}_0$ are nevertheless good, which can be concluded from Figs 4.8 and 4.9. The difference between cancelling the corresponding columns of K (Fig. 4.8) and designing a correct K (Fig. 4.9) is very small.

The quality of the Kalman filter estimates is only decreased very little, when the standard deviations of the process disturbances are doubled (see Fig. 4.10).

Finally a filter gain designed for increased tuning rate of $\hat{\delta}_0$ and \hat{d}_v is tested in the simulation of Fig. 4.11. It can be concluded that the tuning rate of \hat{d}_v still is small, although an improvement is obtained compared to Fig. 4.1.

The final parameter values of the self-tuning regulator are summarized in Table 4.7 of the next section. Some examples of the performance of the Kalman filter during yaws are shown in Section 4.3.

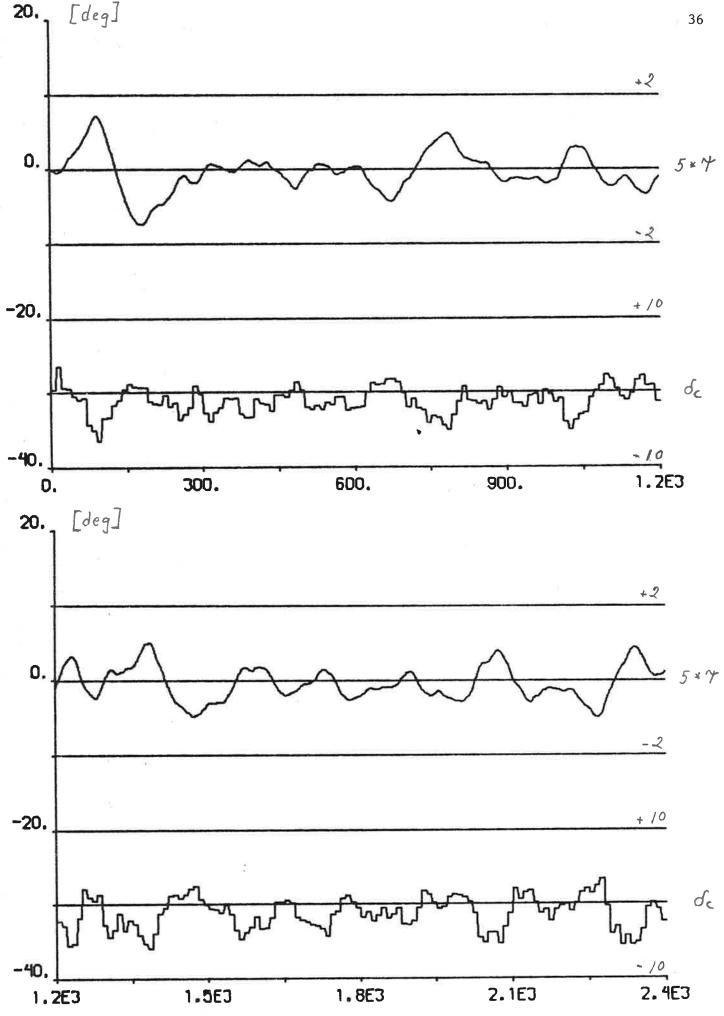
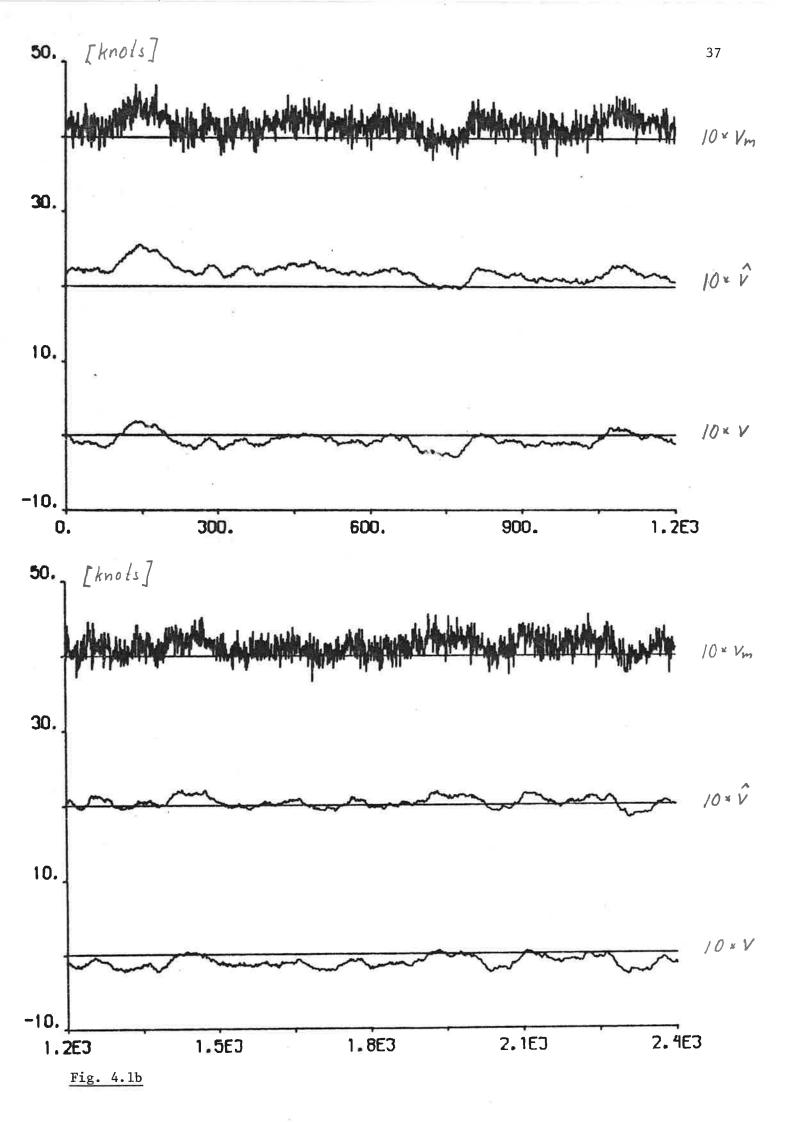
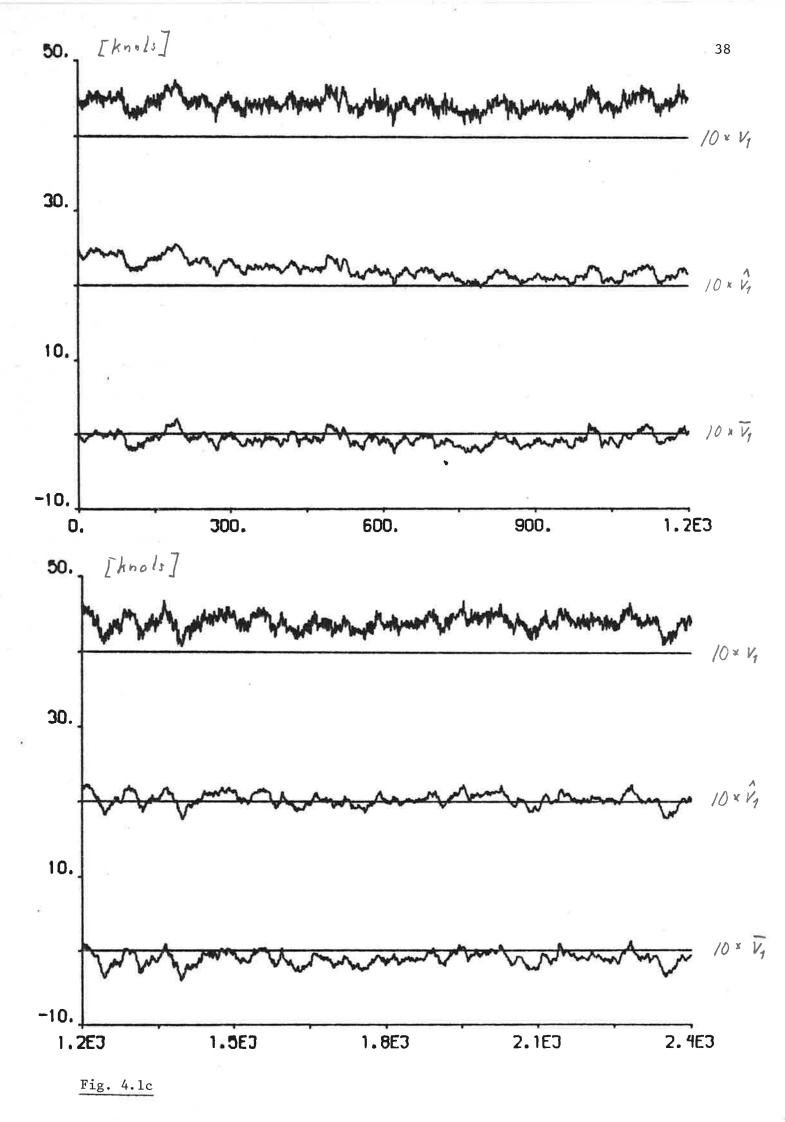


Fig. 4.1a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using estimates from the Kalman filter.





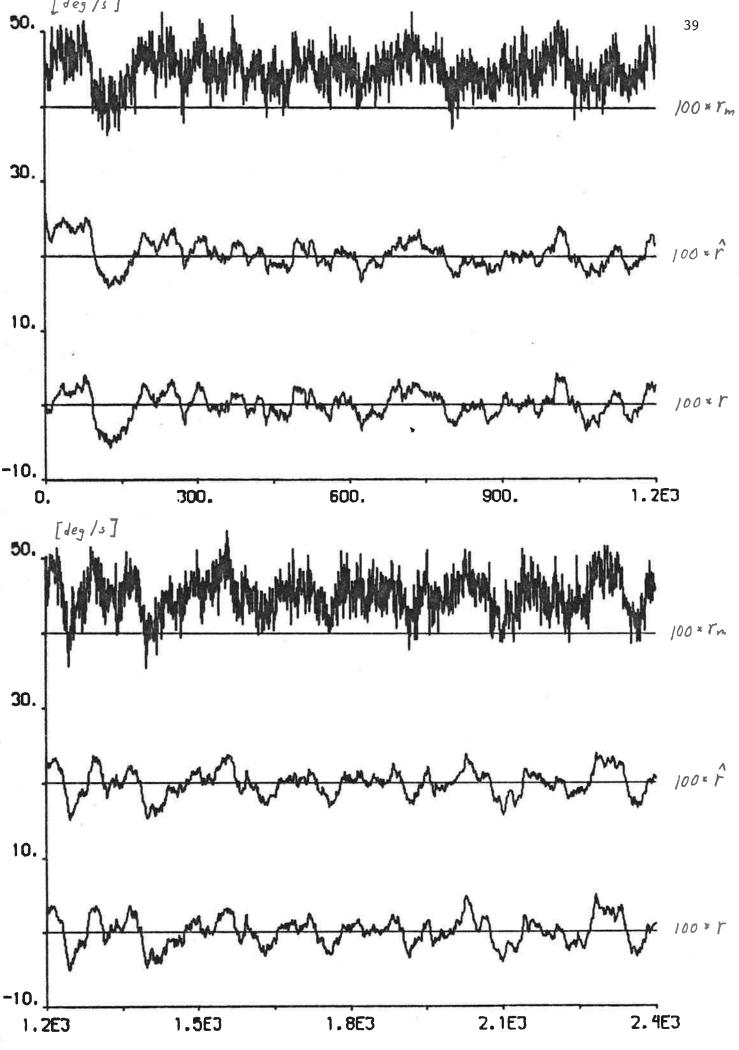


Fig. 4.1d

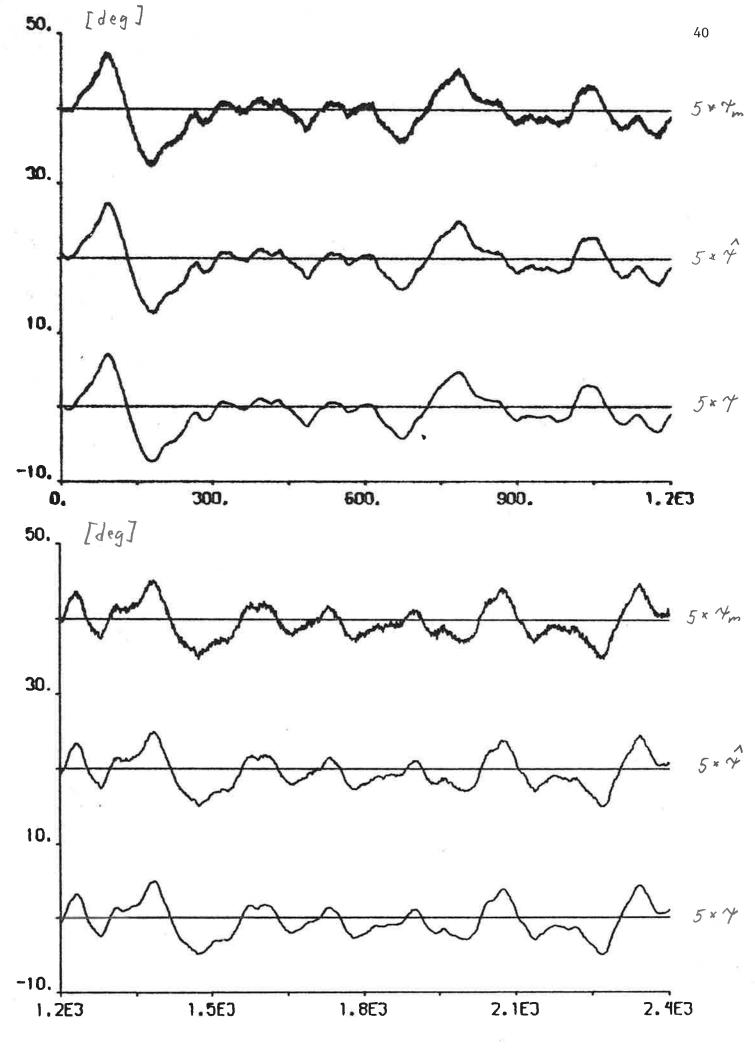
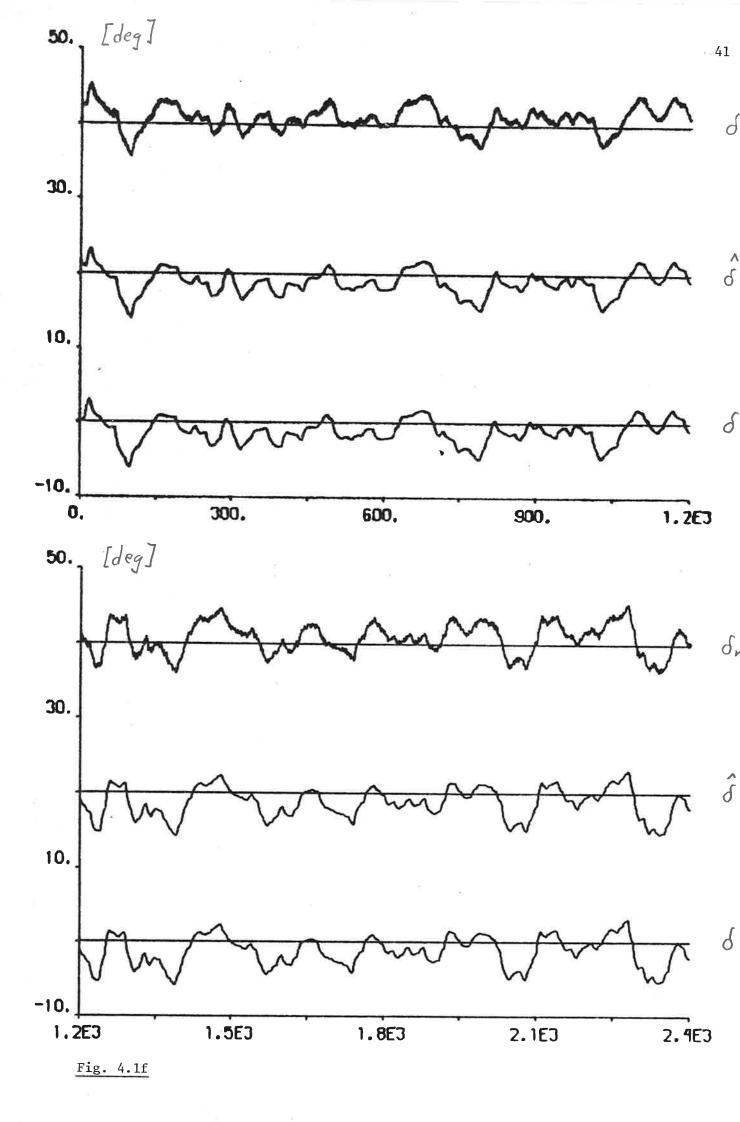
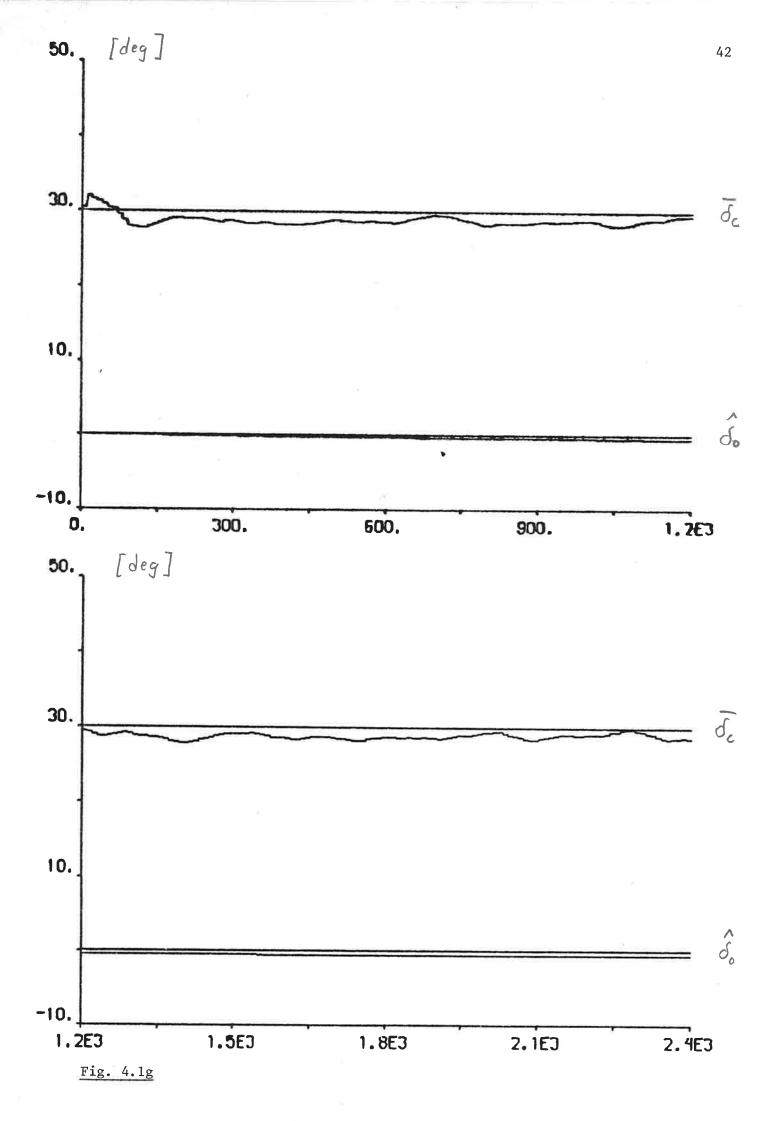
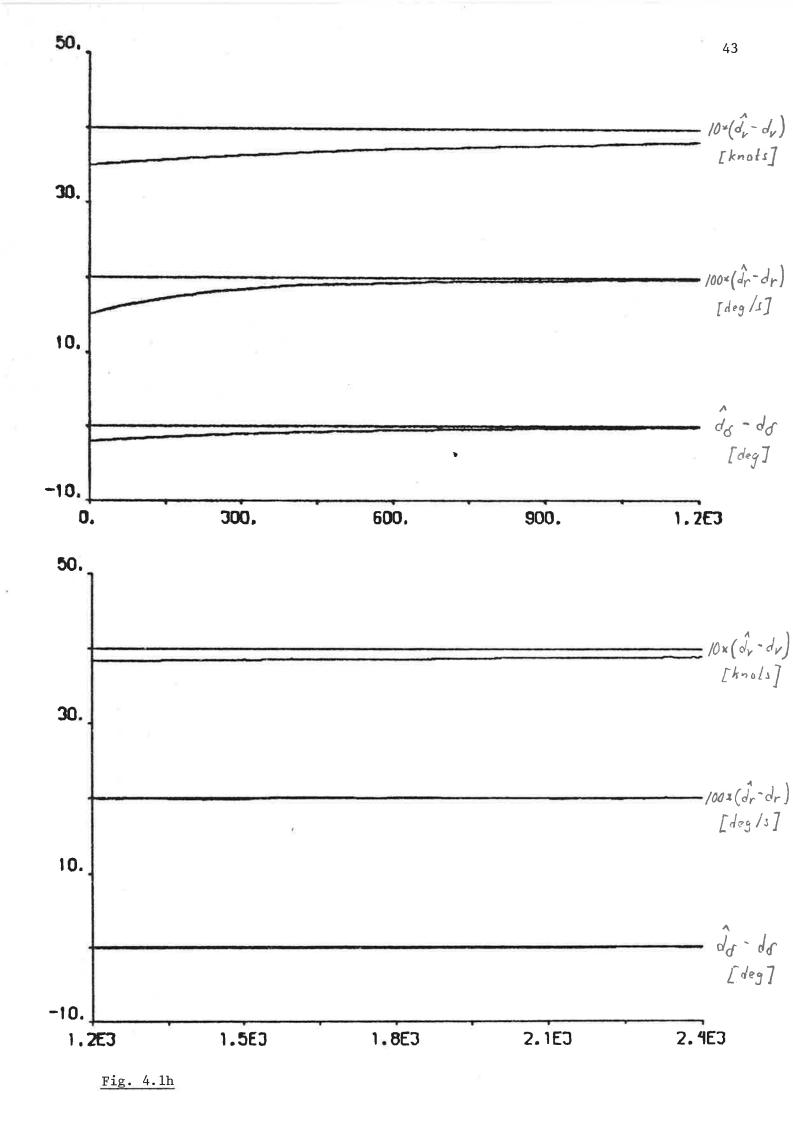
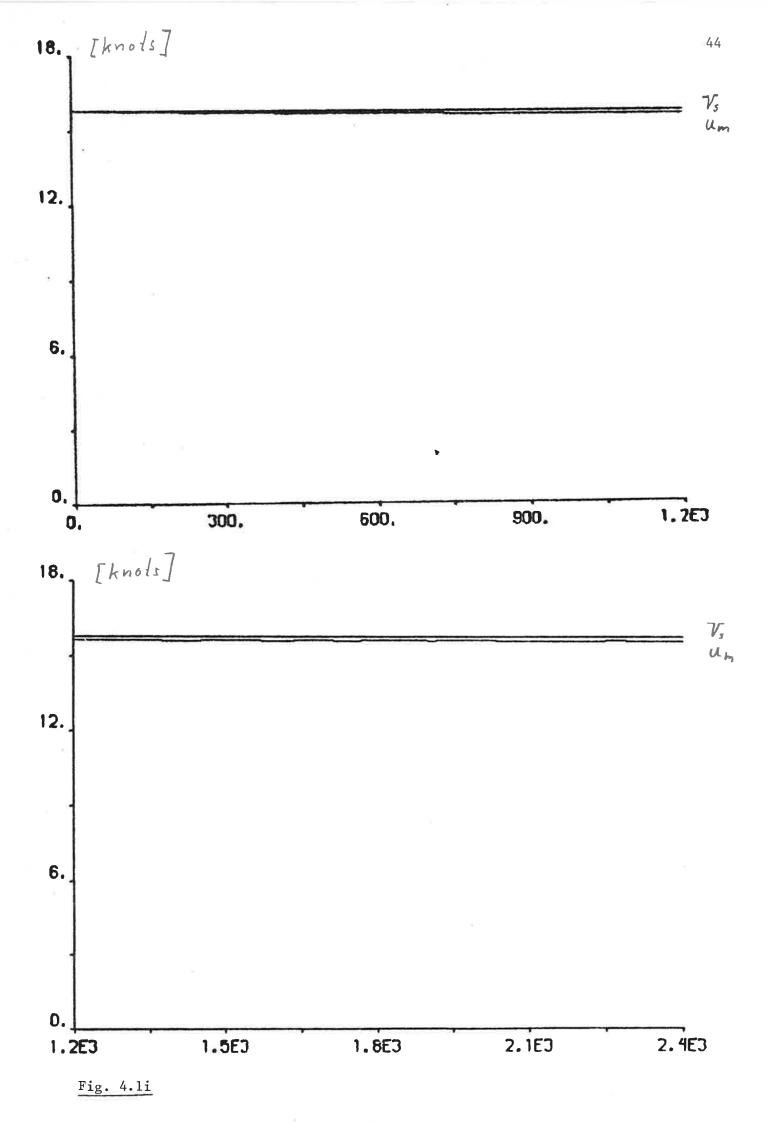


Fig. 4.1e











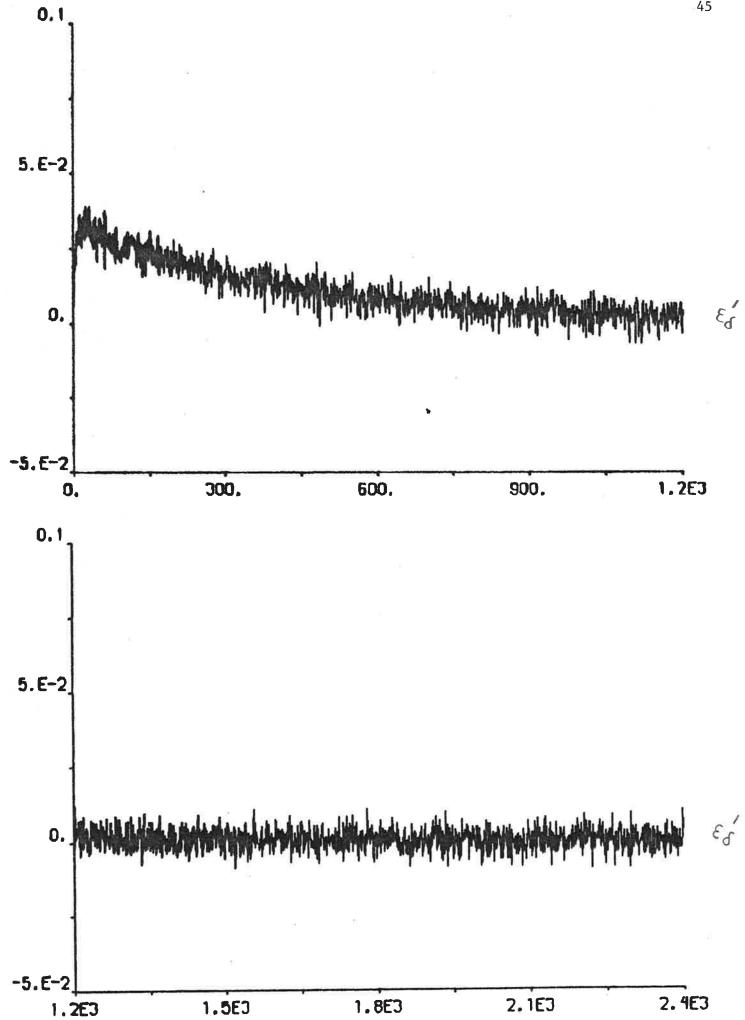


Fig. 4.1j



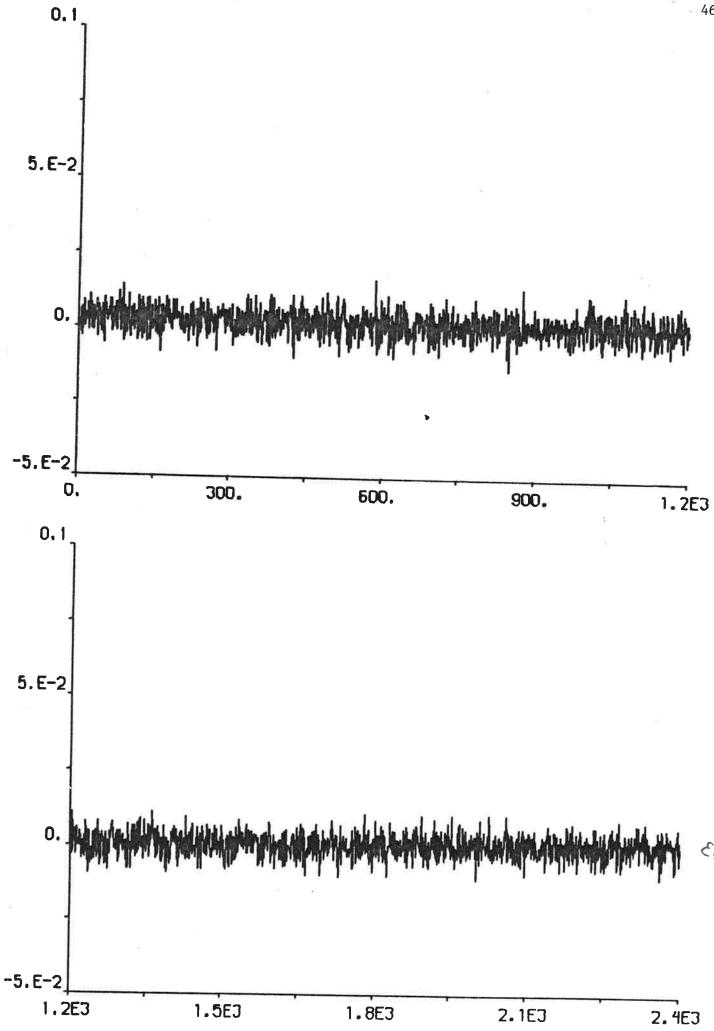


Fig. 4.1k



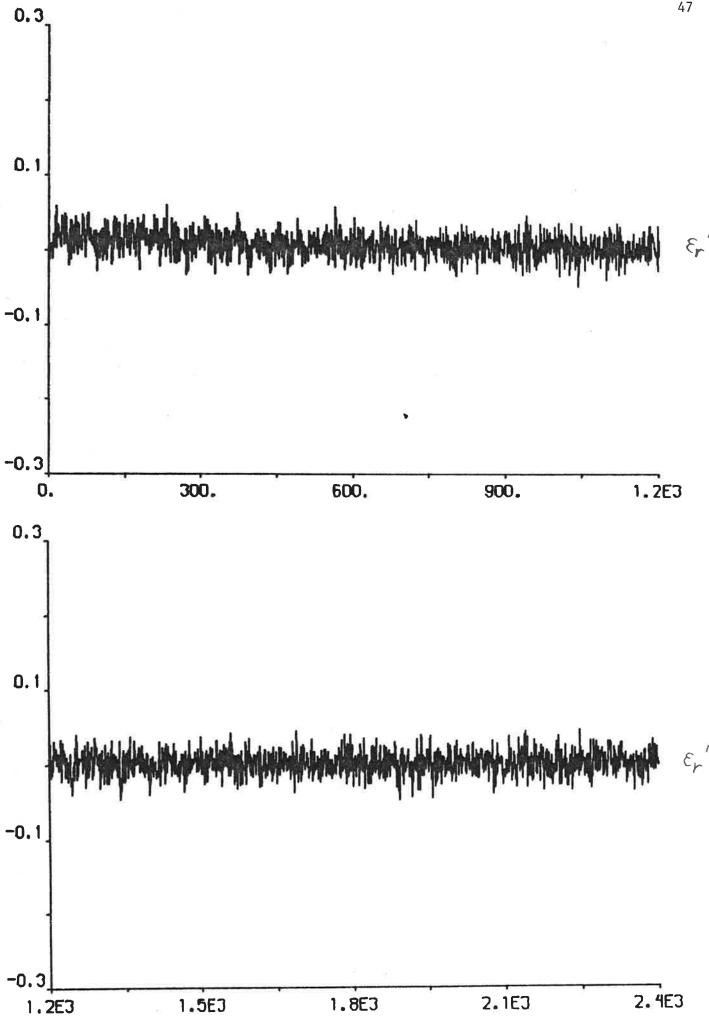
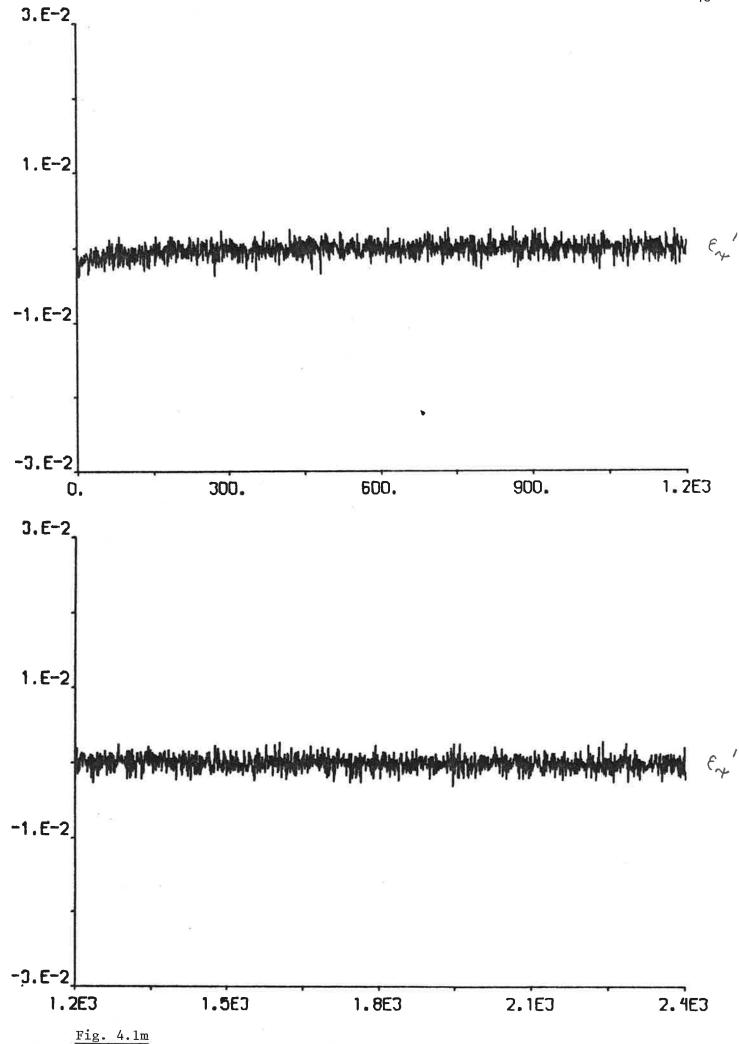
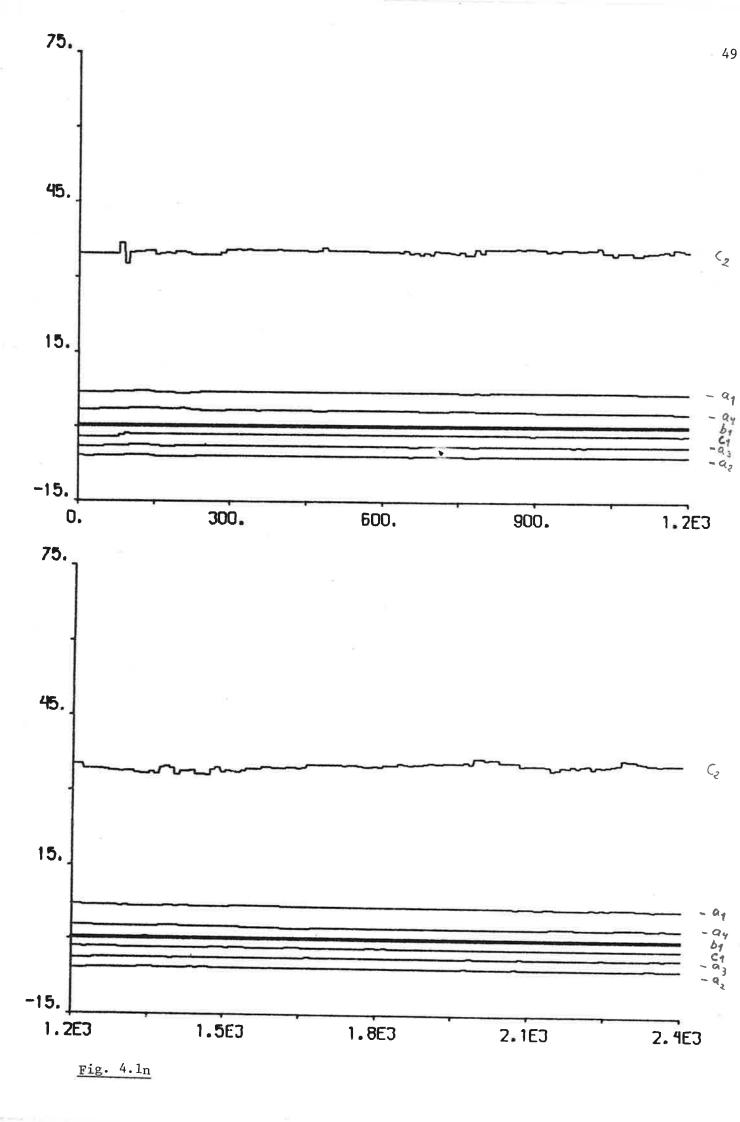
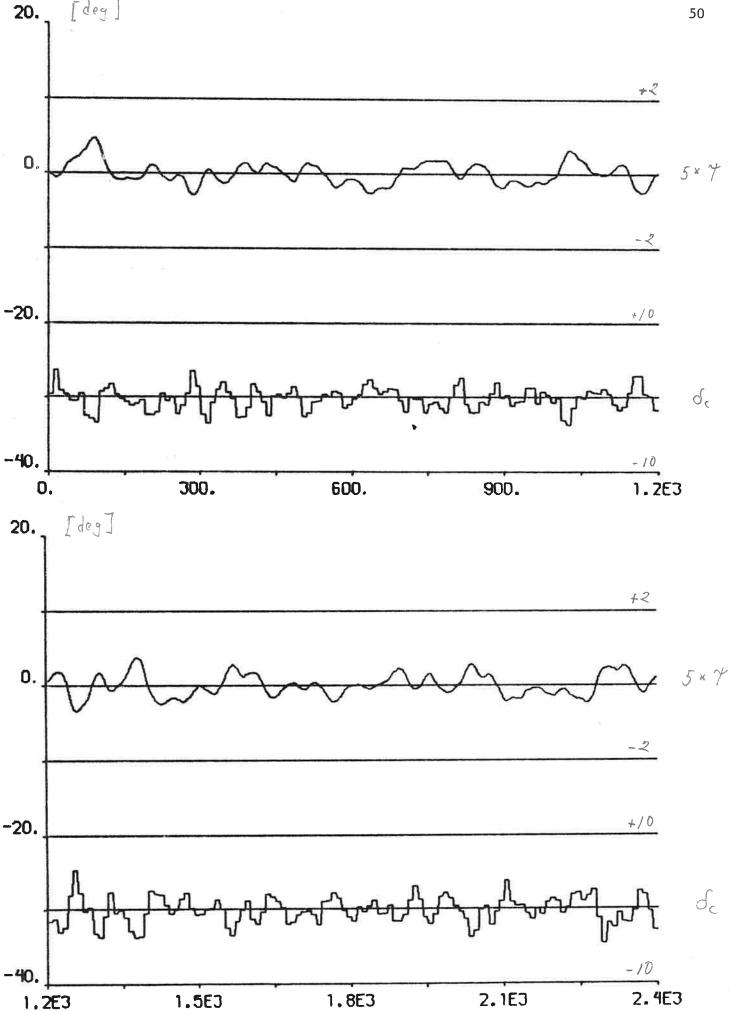


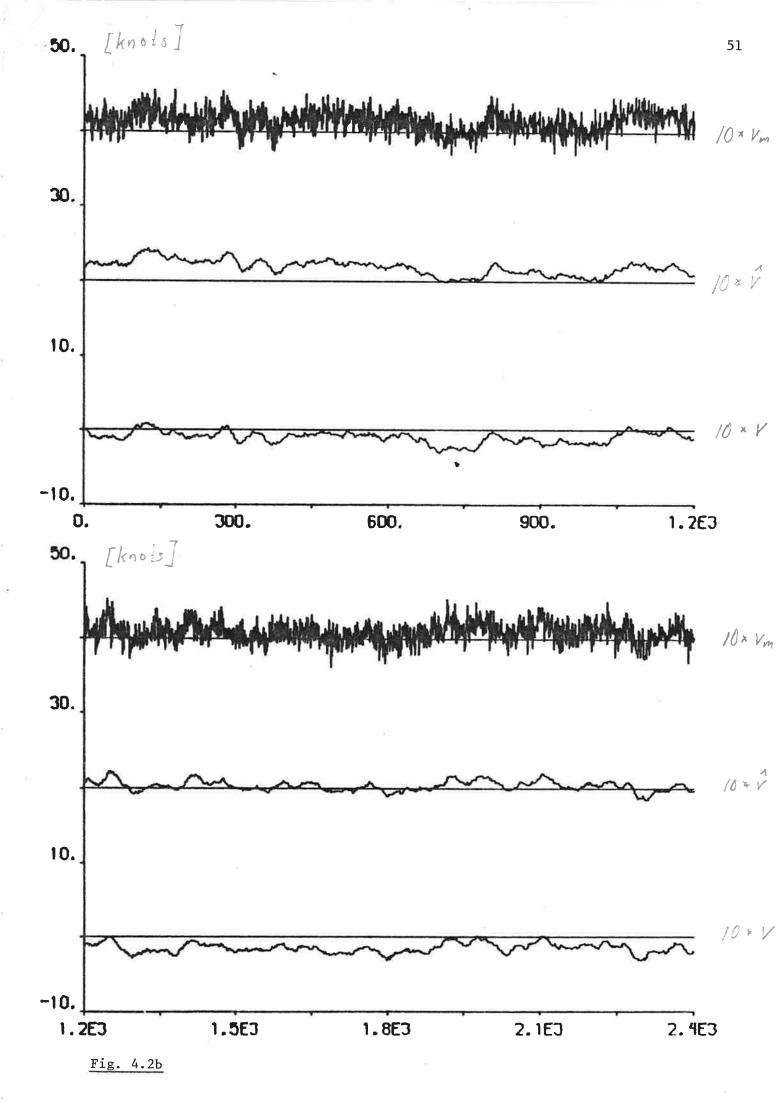
Fig. 4.1ℓ







<u>Fig. 4.2a</u> - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using estimates from the Kalman filter.



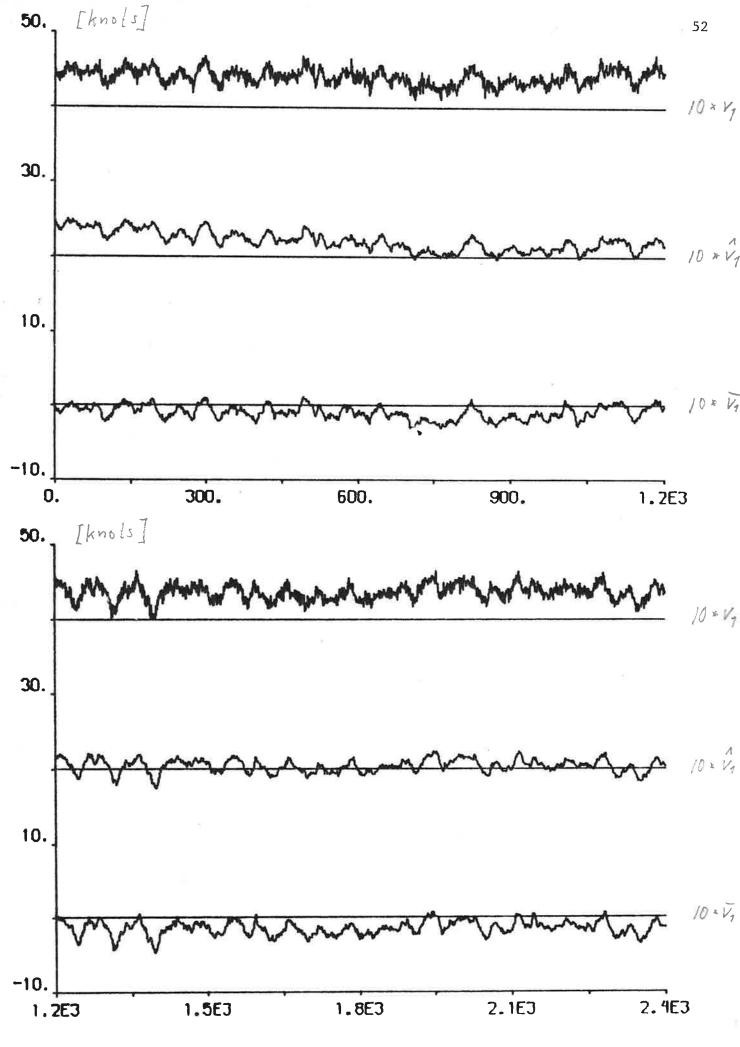


Fig. 4.2c

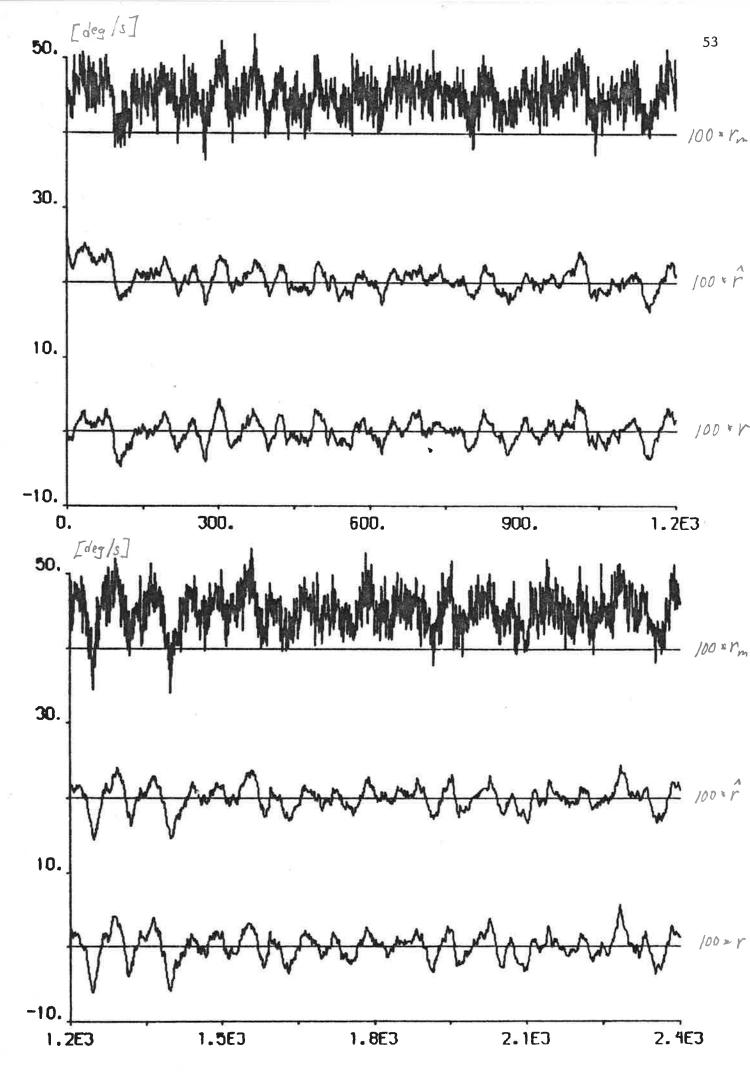


Fig. 4.2d

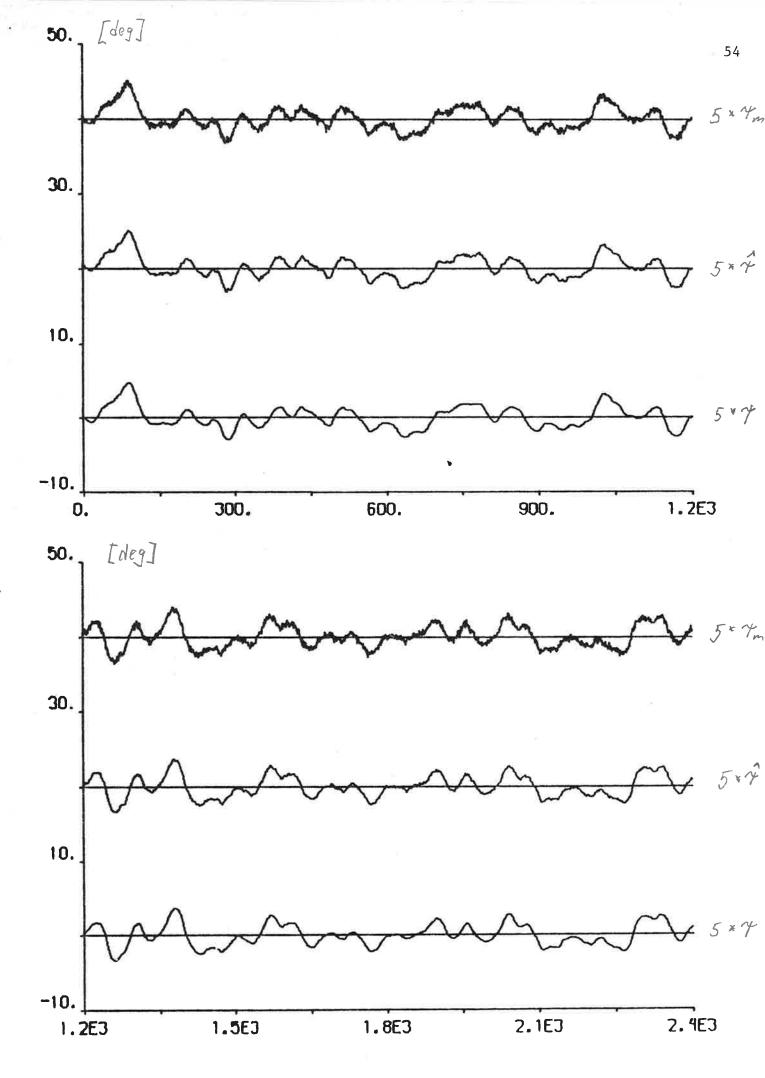
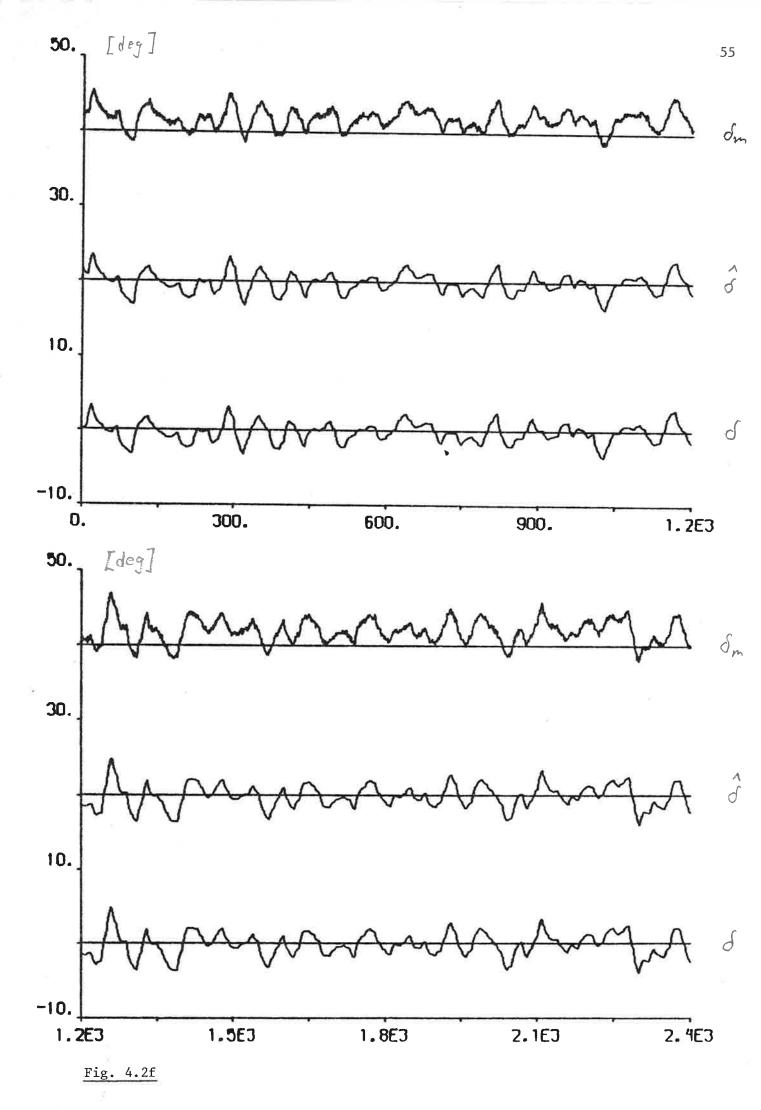
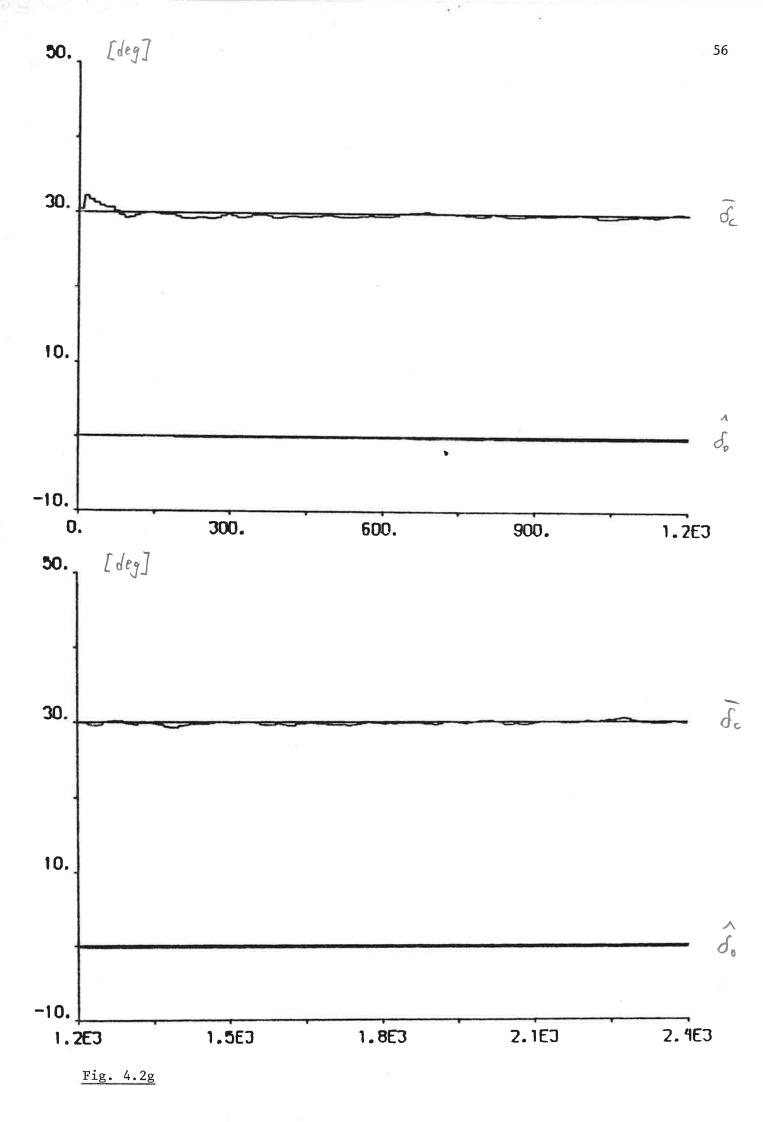
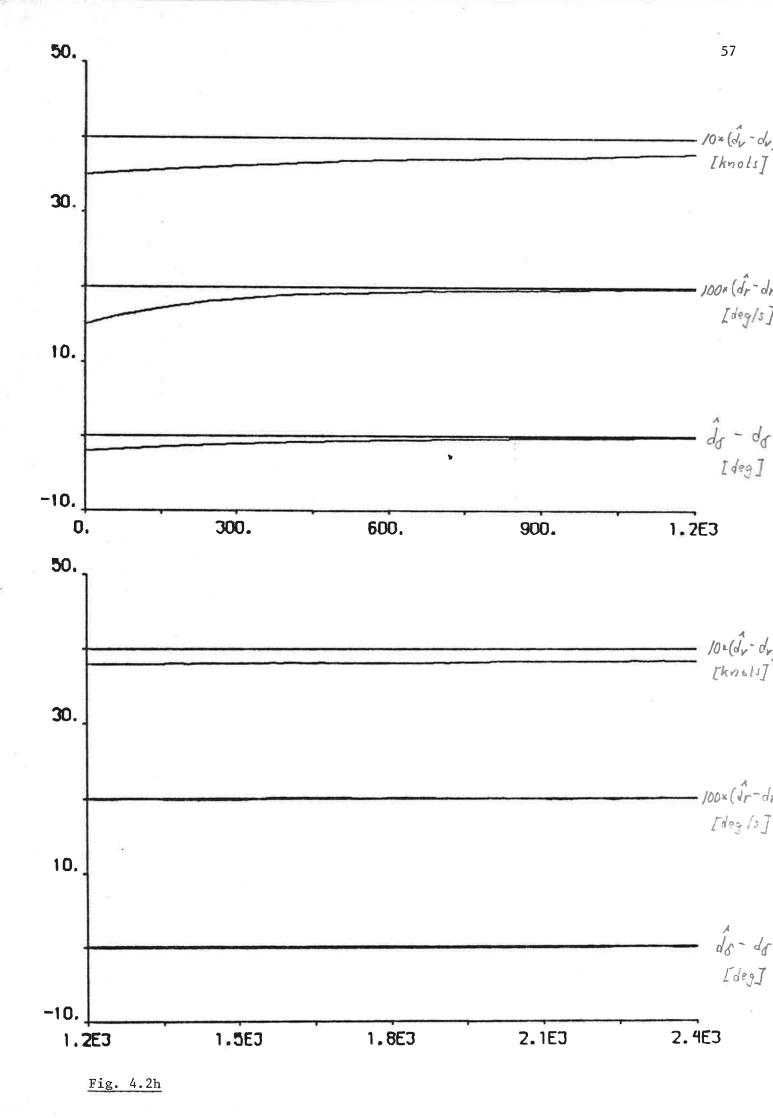


Fig. 4.2e







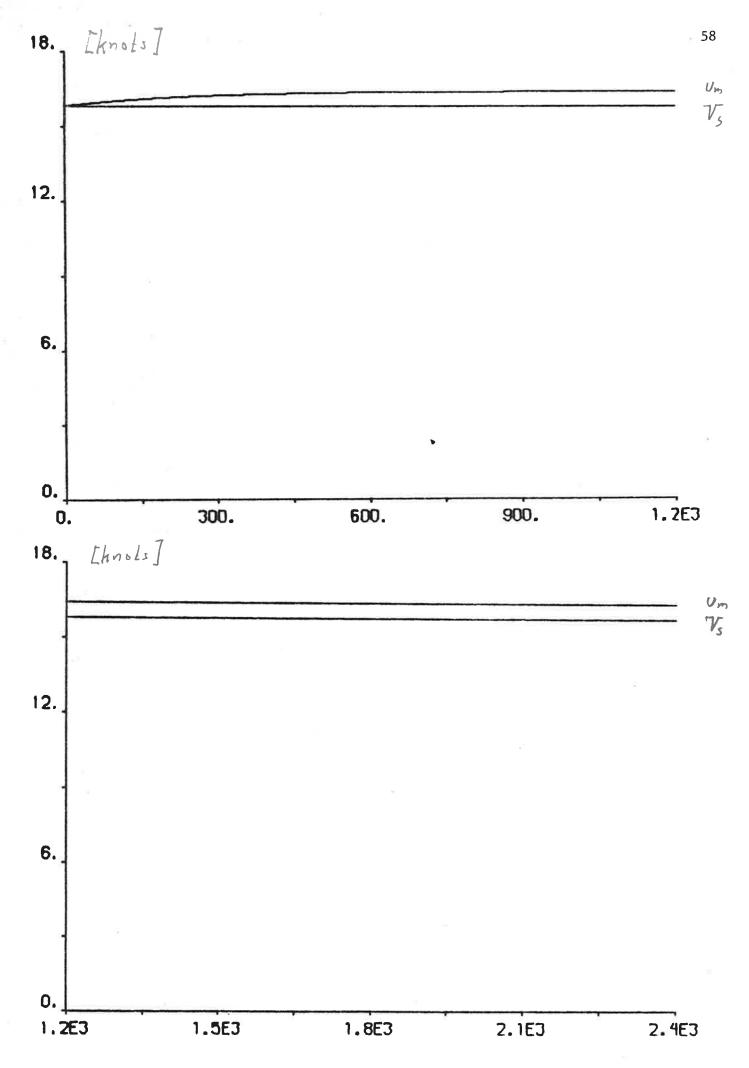


Fig. 4.2i



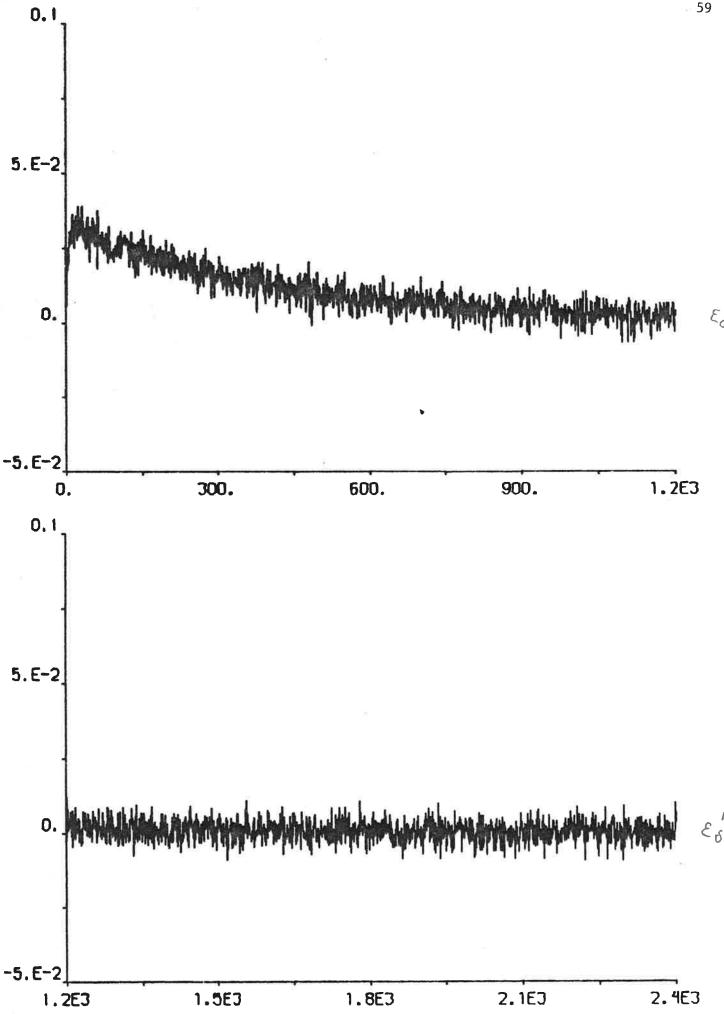


Fig. 4.2j

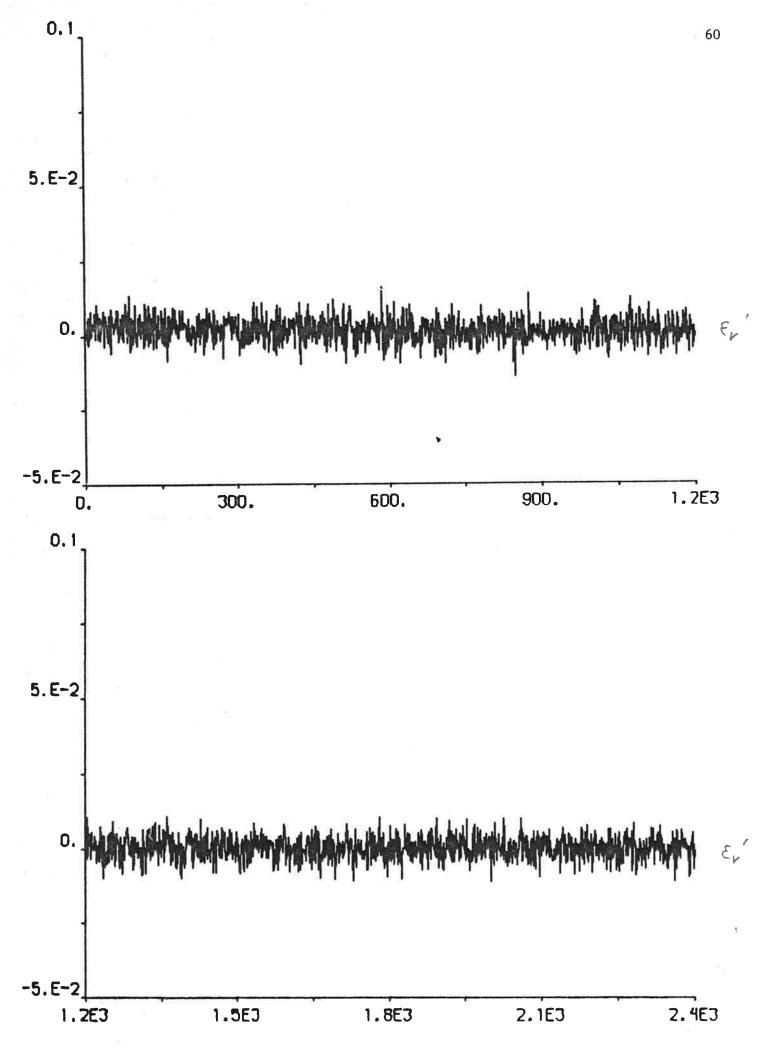


Fig. 4.2k



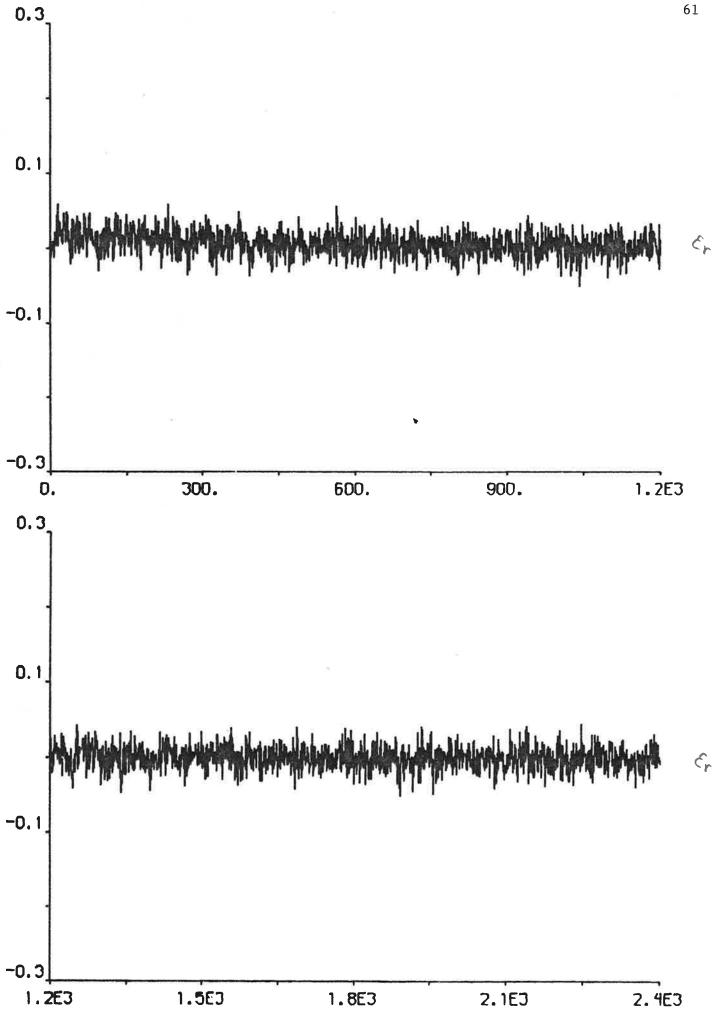
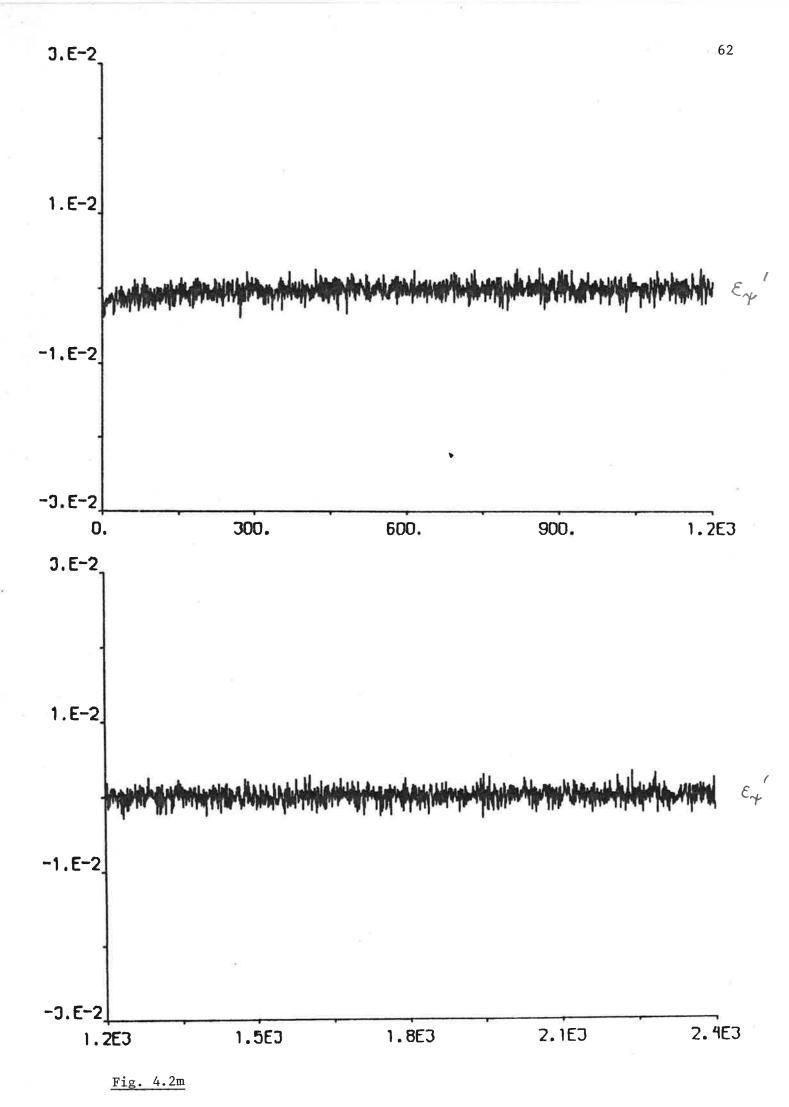


Fig. 4.2l



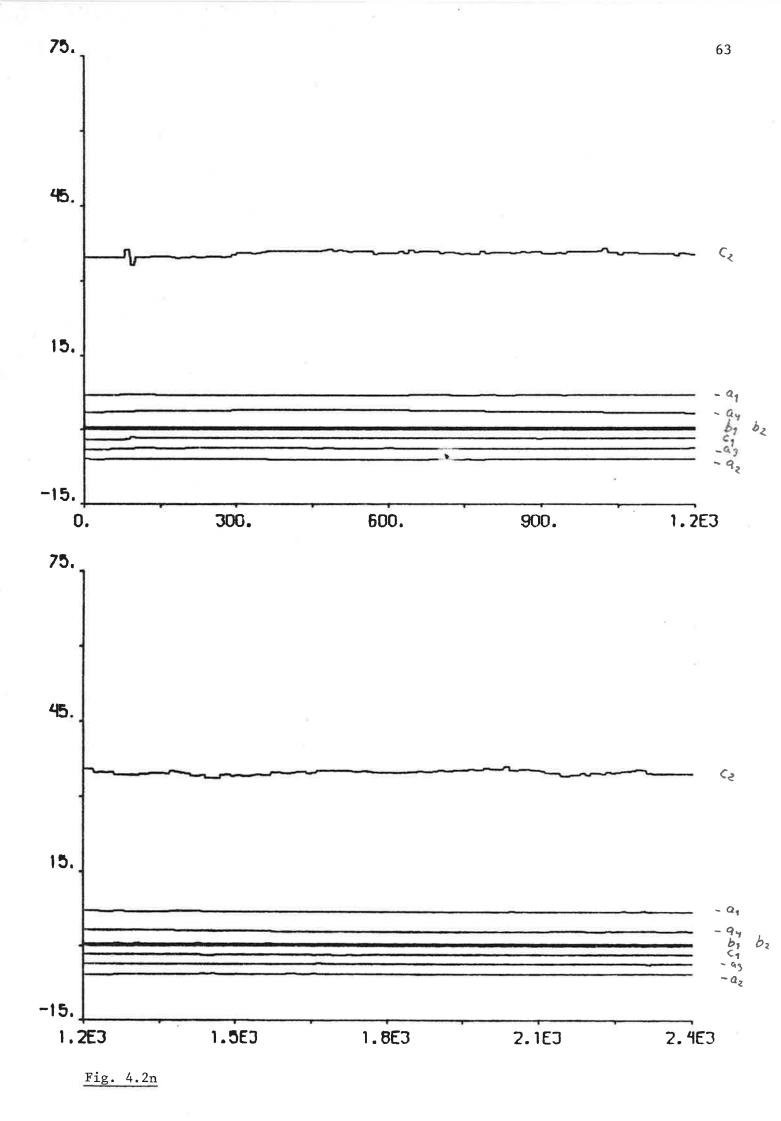
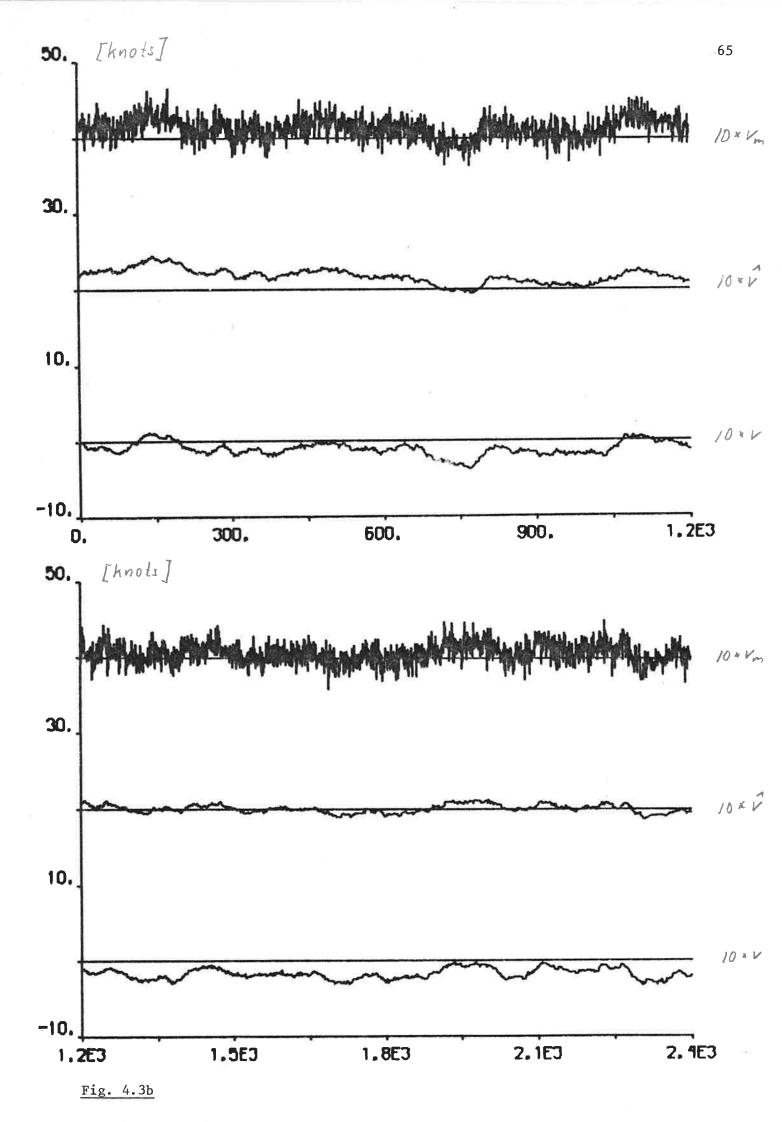
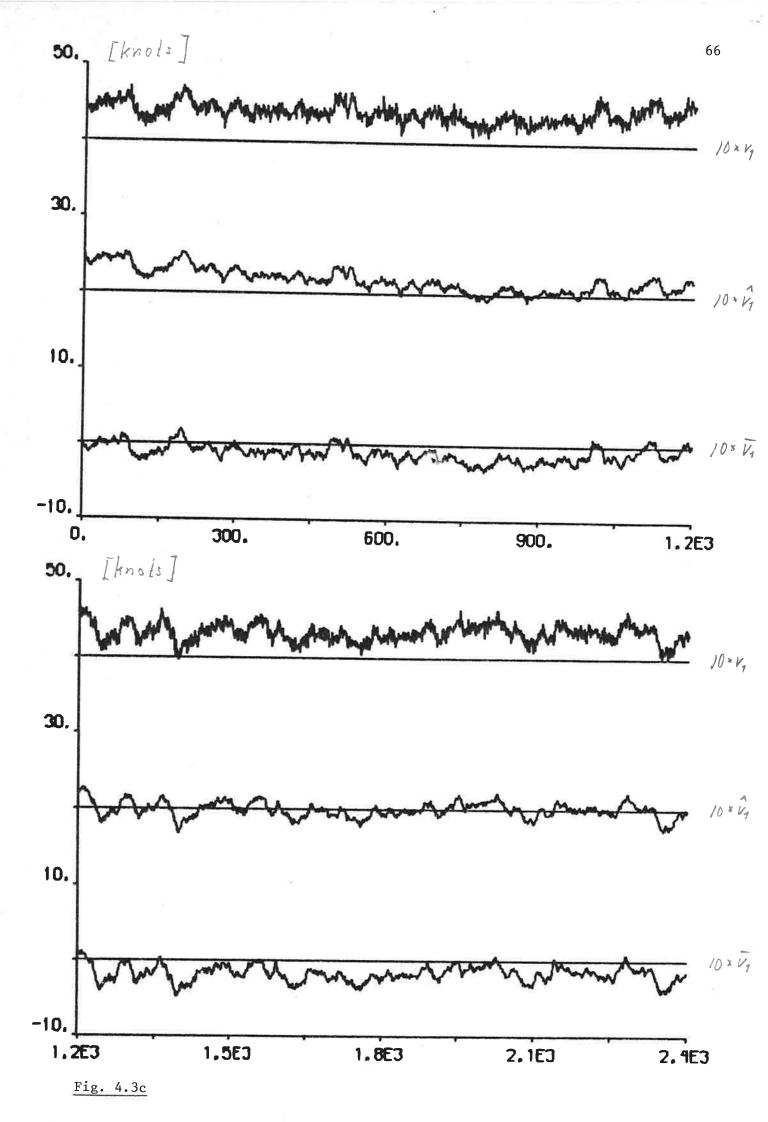
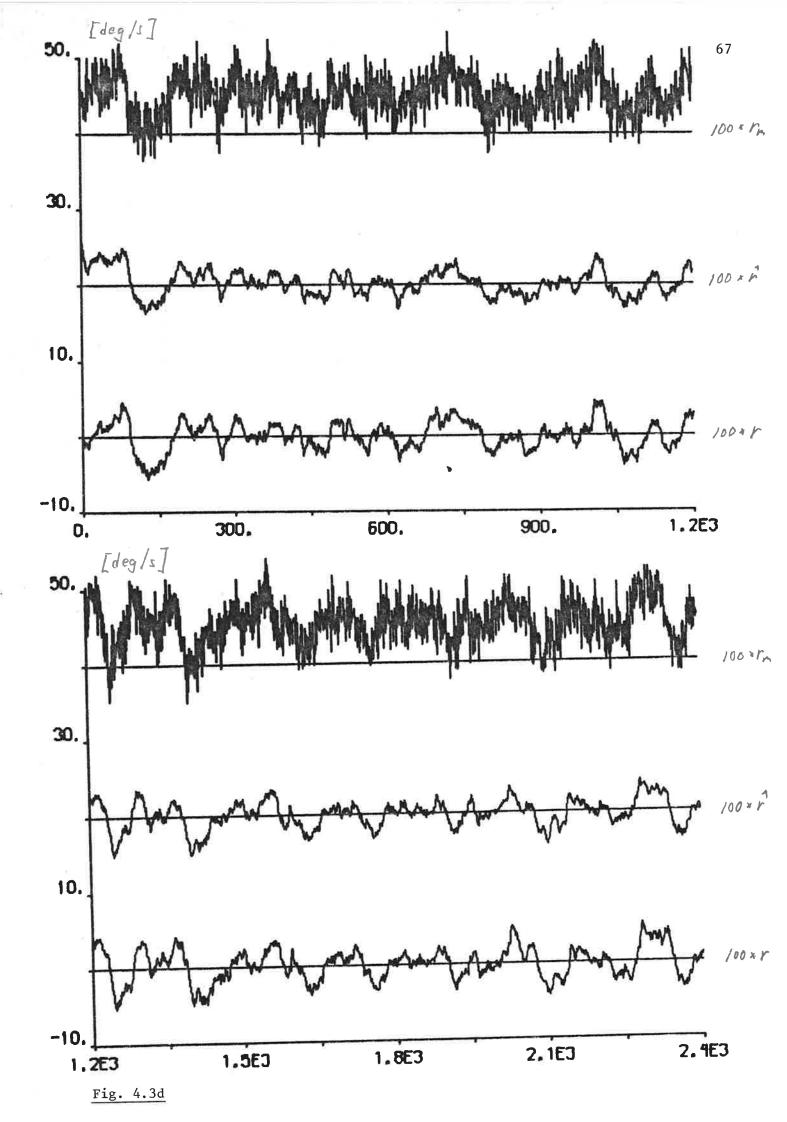


Fig. 4.3a - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, self-tuning regulator using estimates from the Kalman filter.







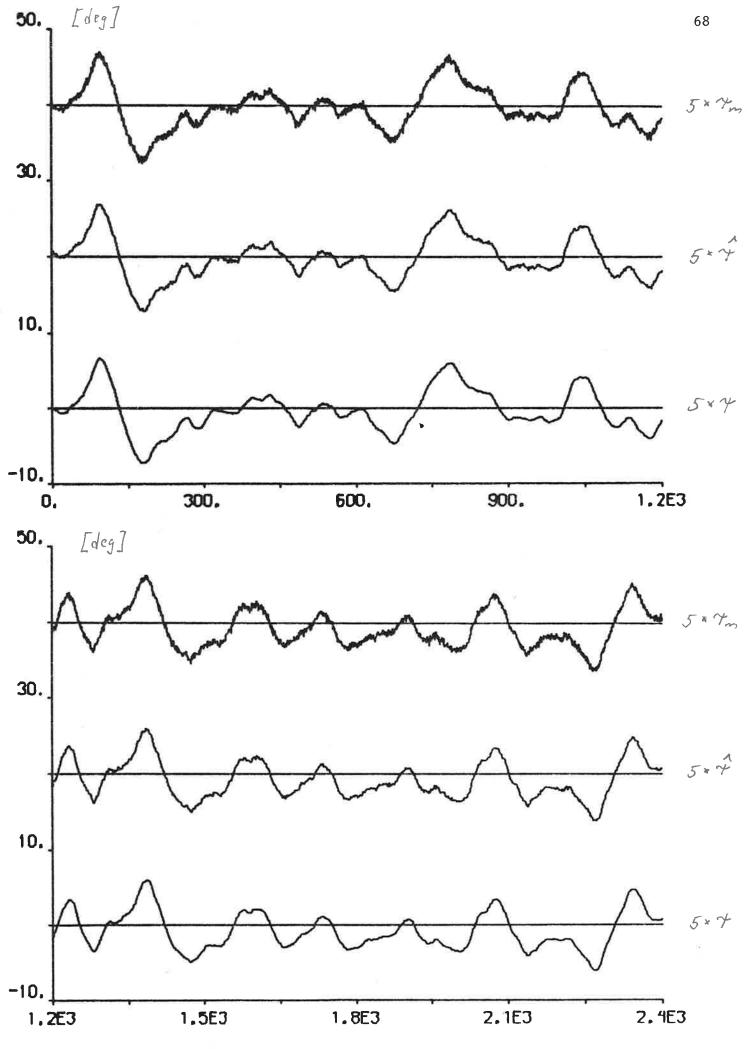
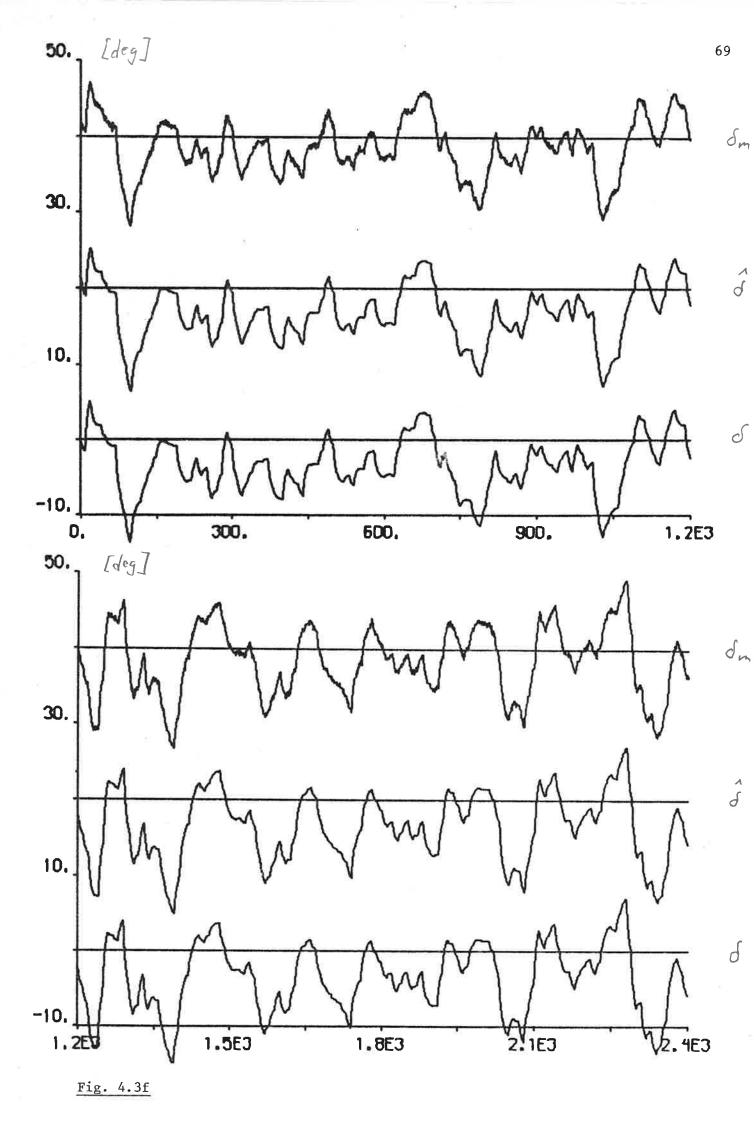
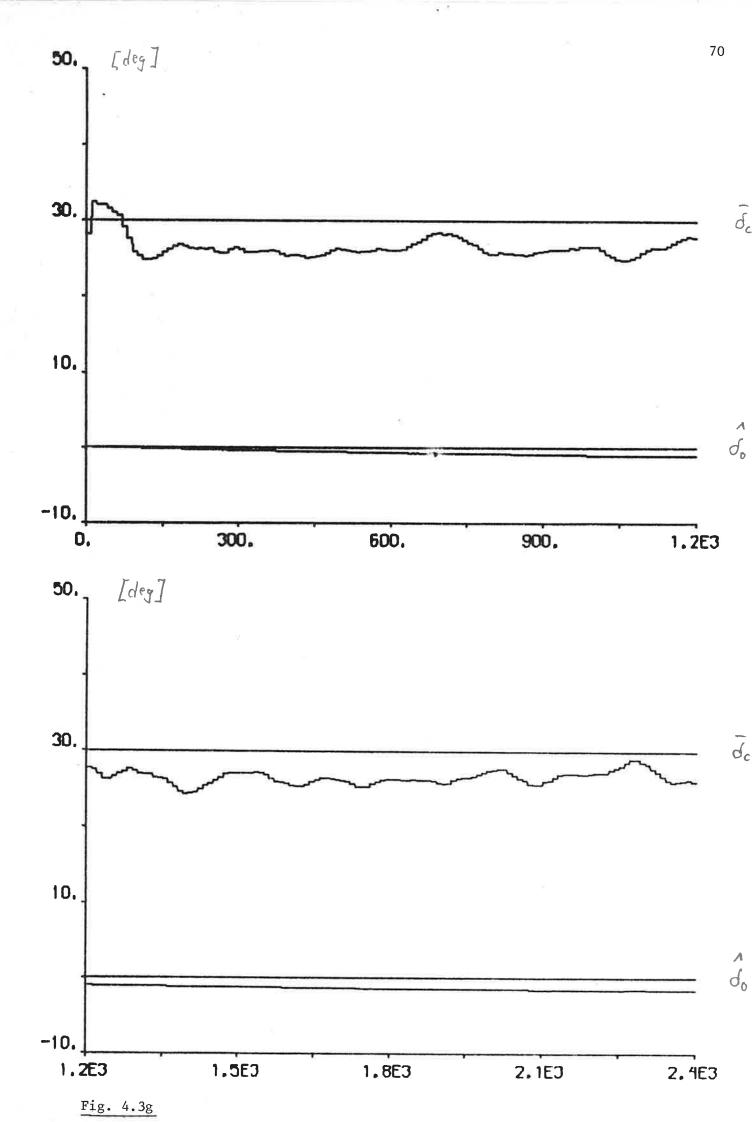
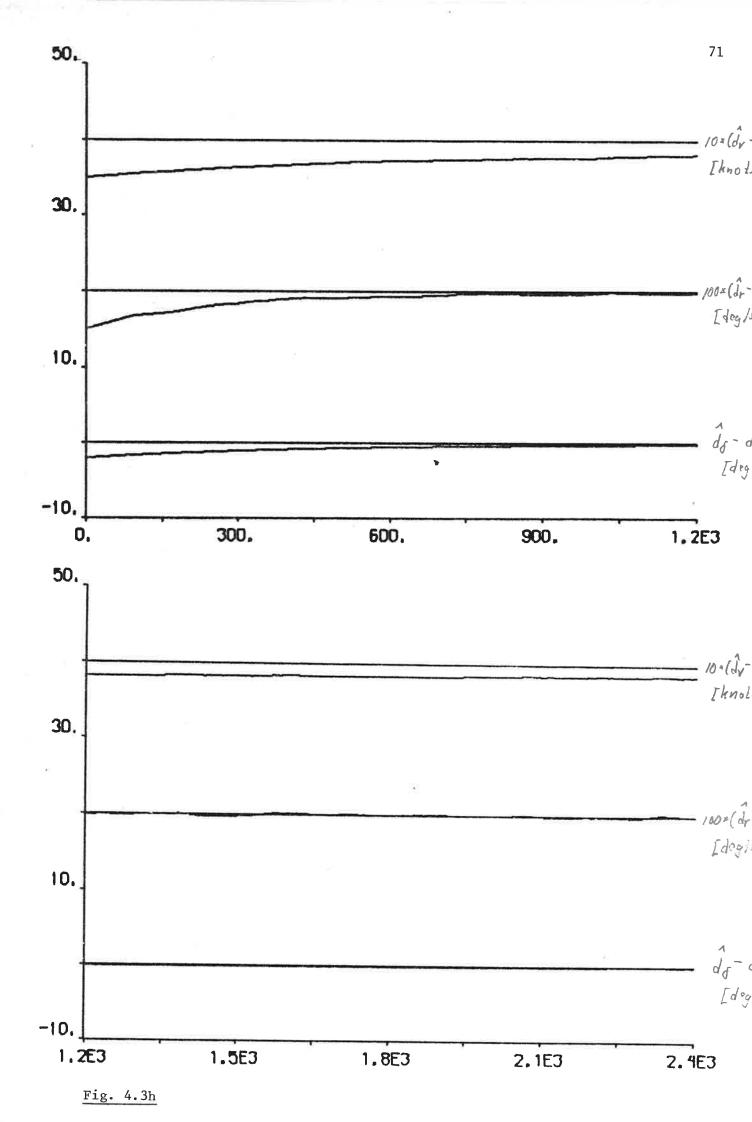
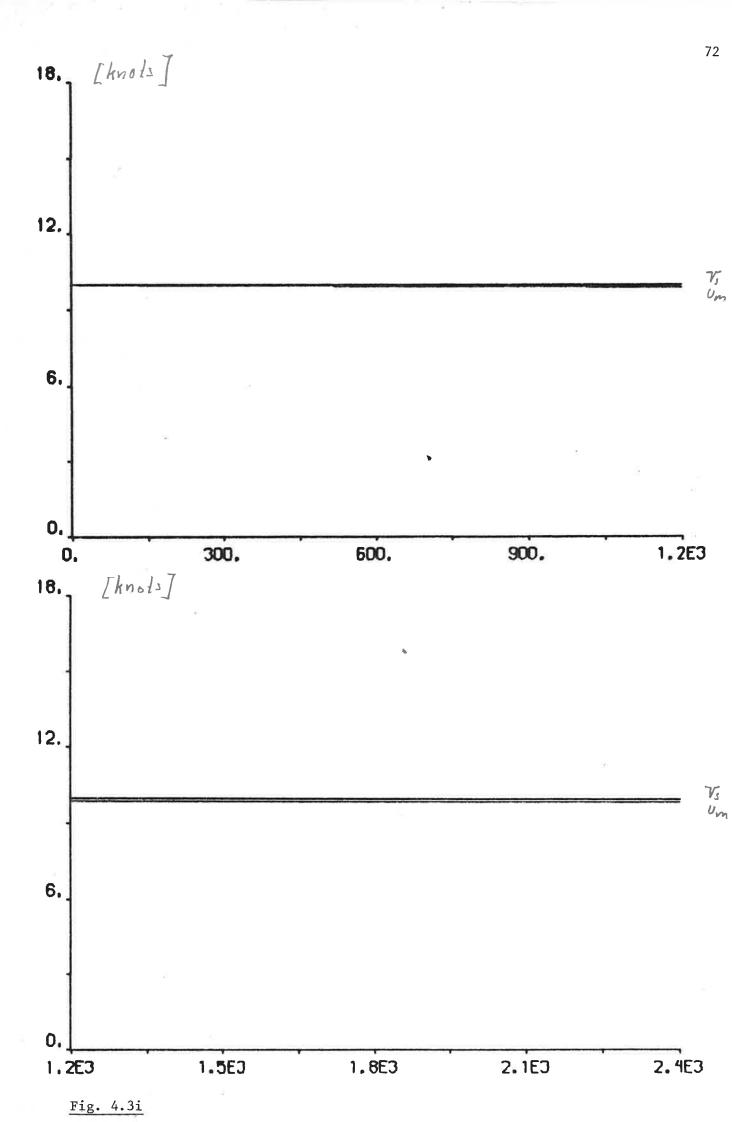


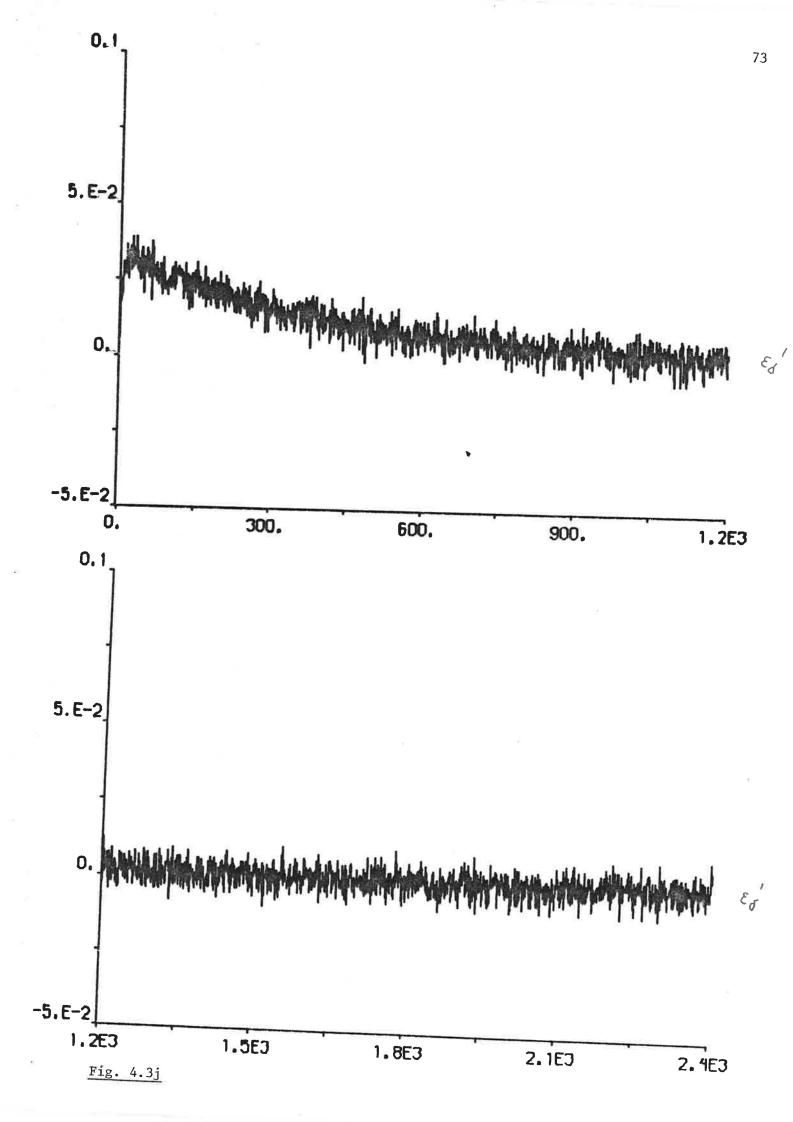
Fig. 4.3e



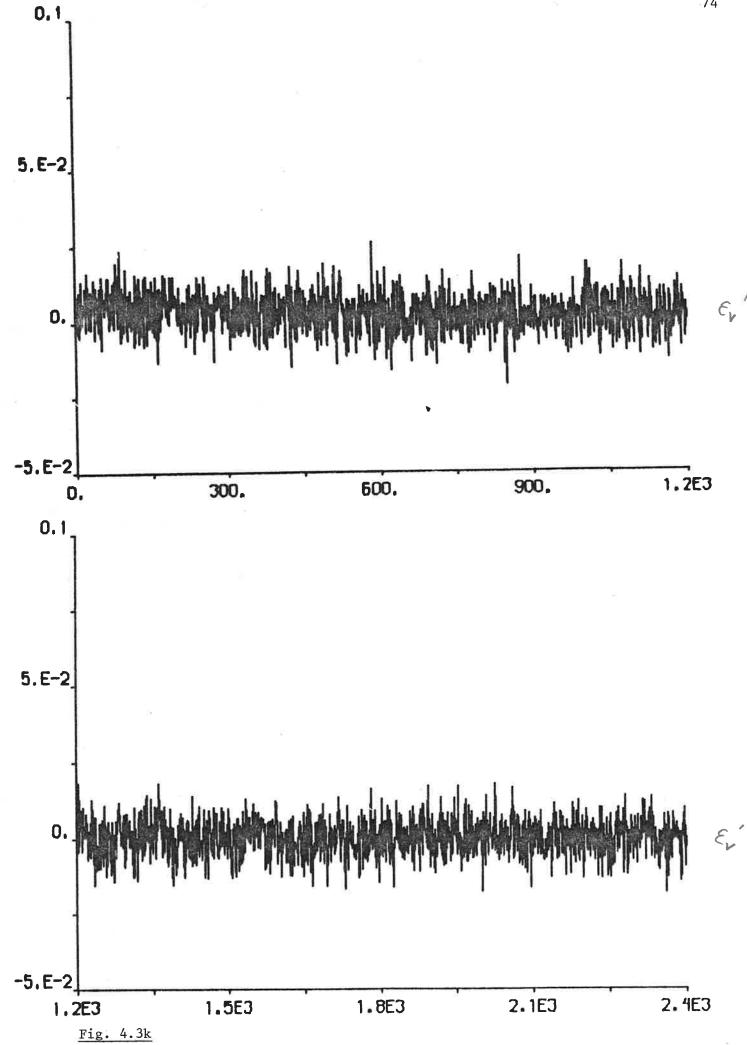












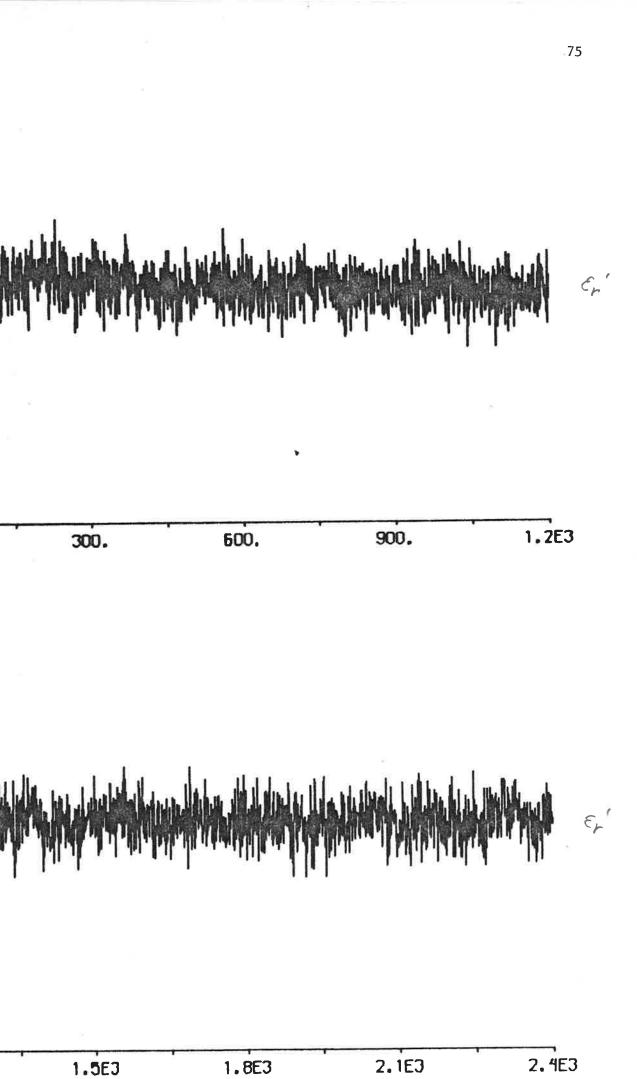


Fig. 4.3l

1.2E3

0.3

0.1

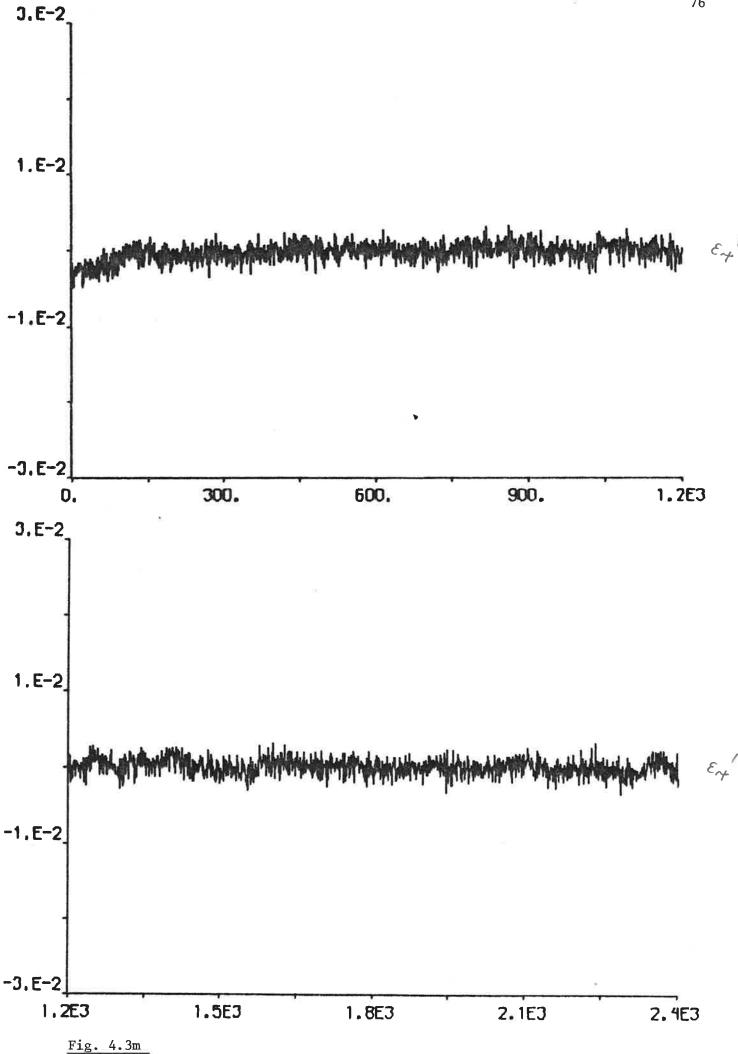
-0.1

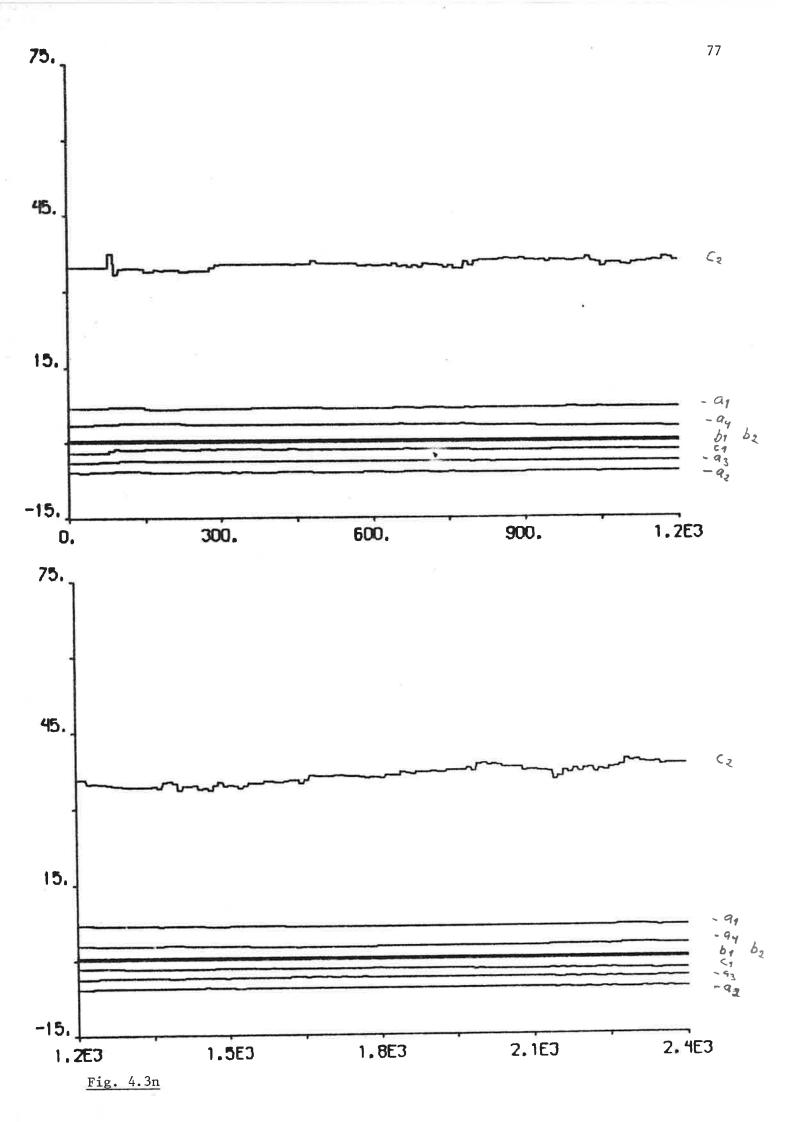
0.

0.3

0.1







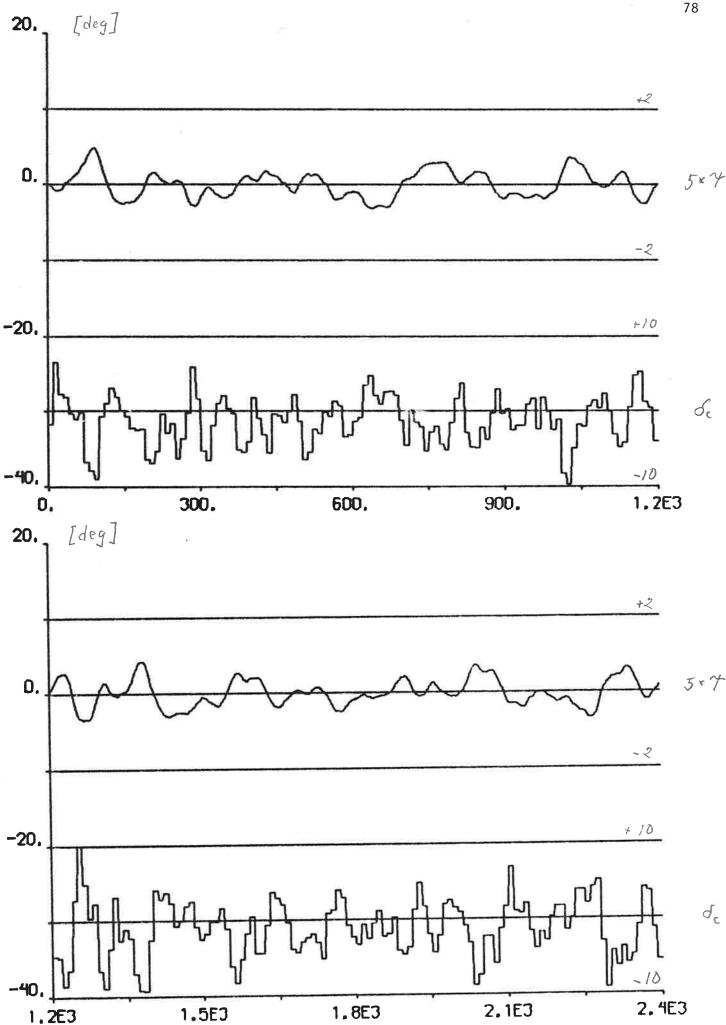
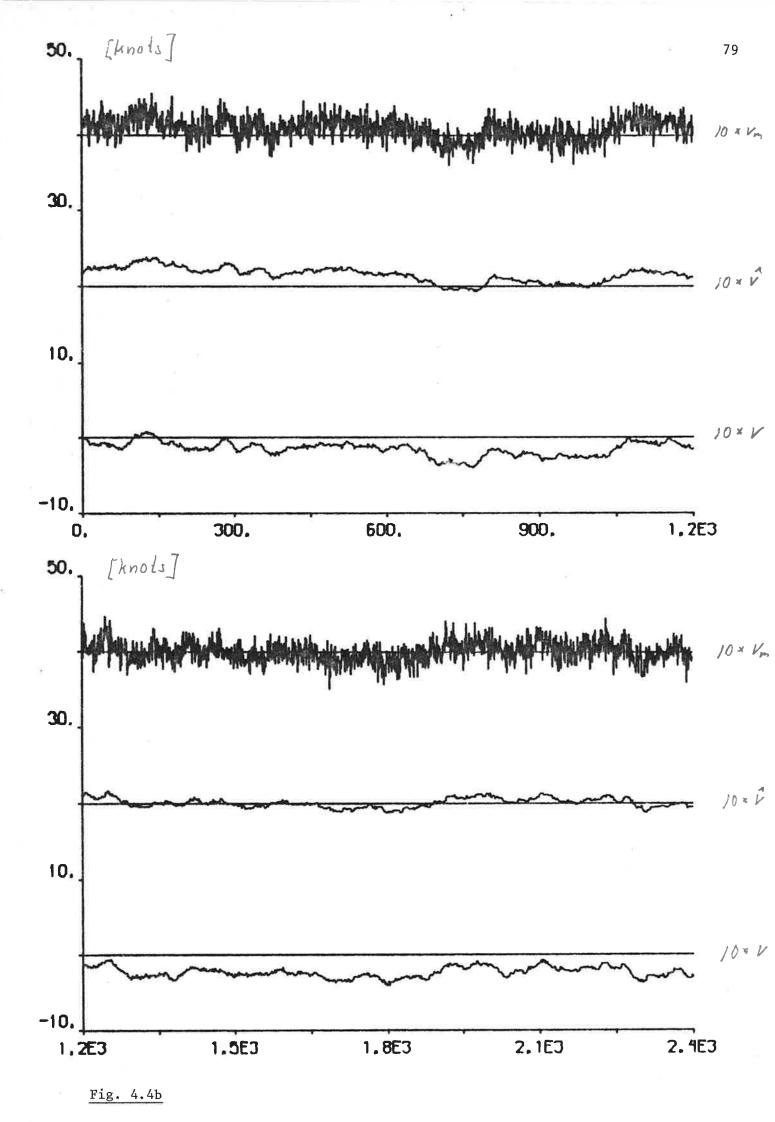
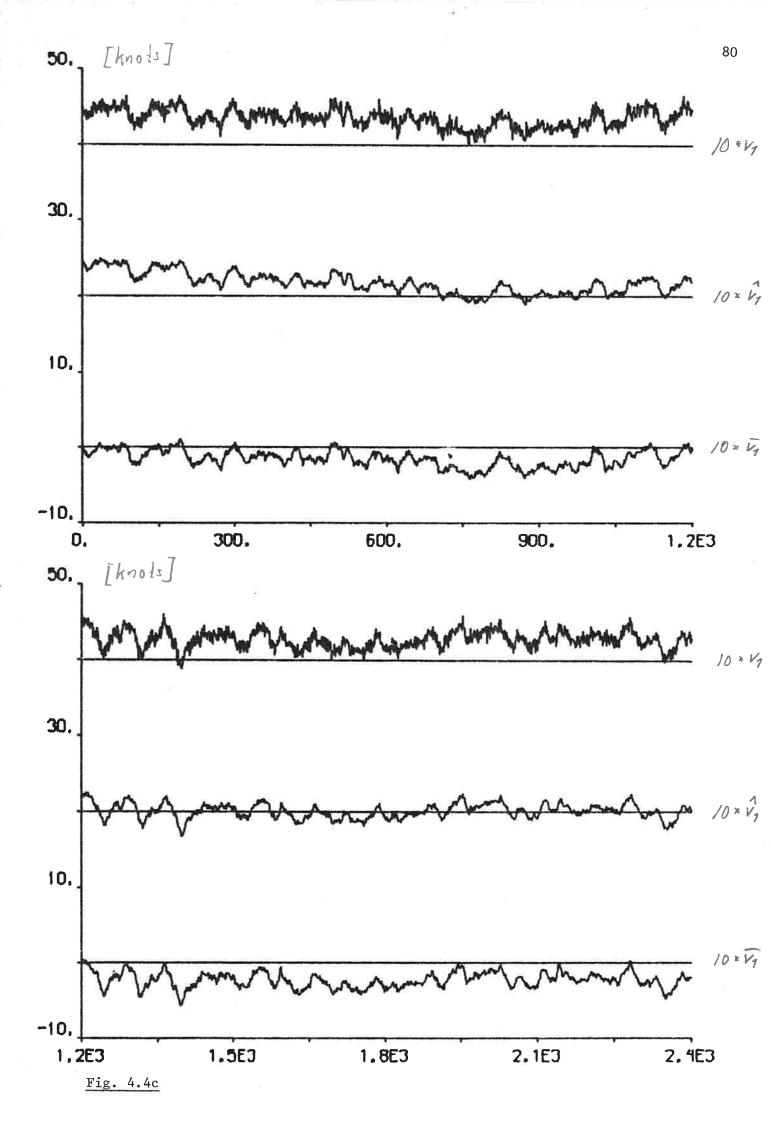


Fig. 4.4a - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, self-tuning regulator using estimates from the Kalman filter.





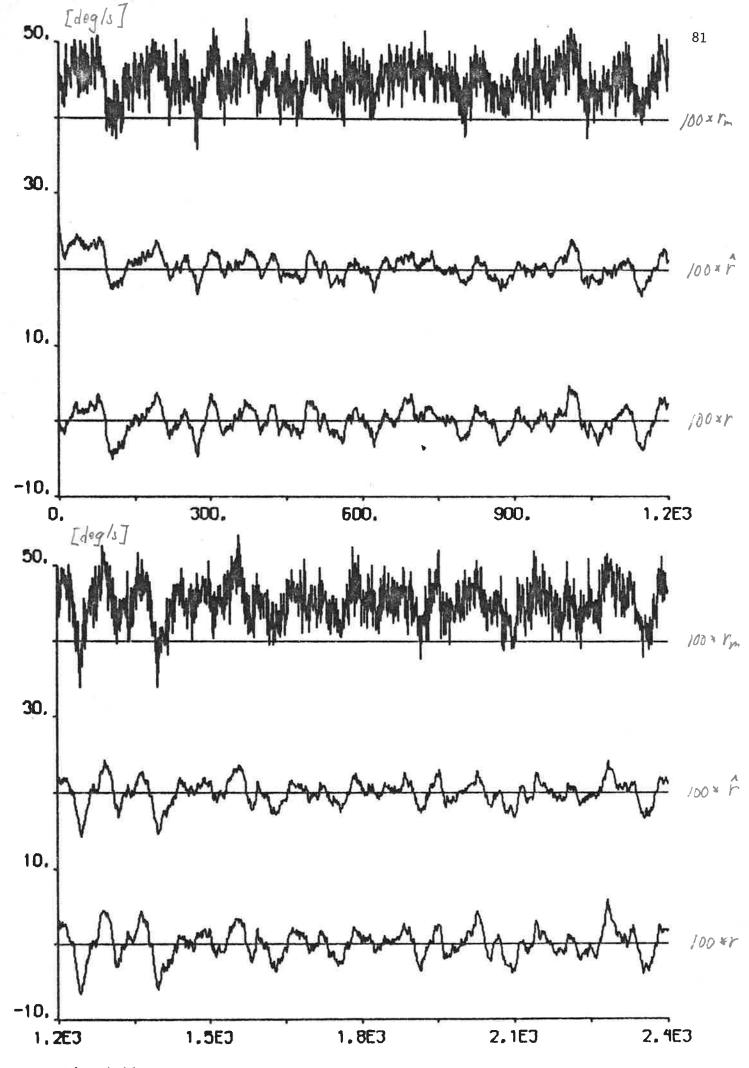
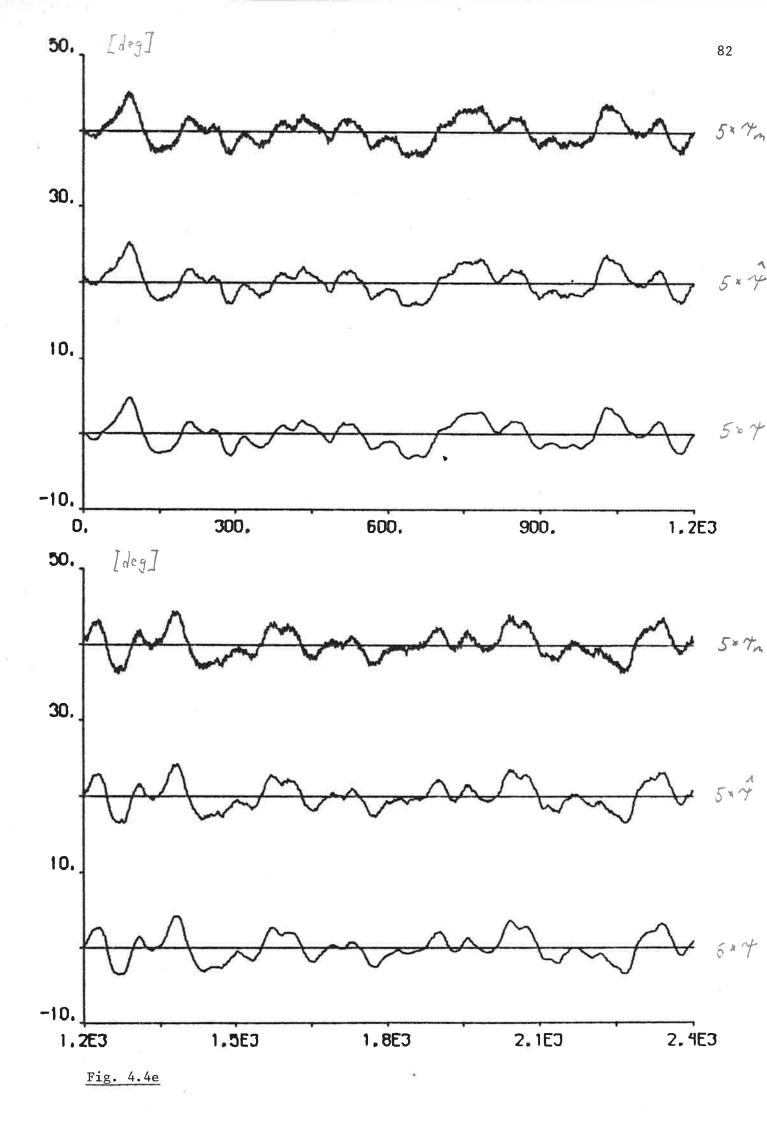


Fig. 4.4d



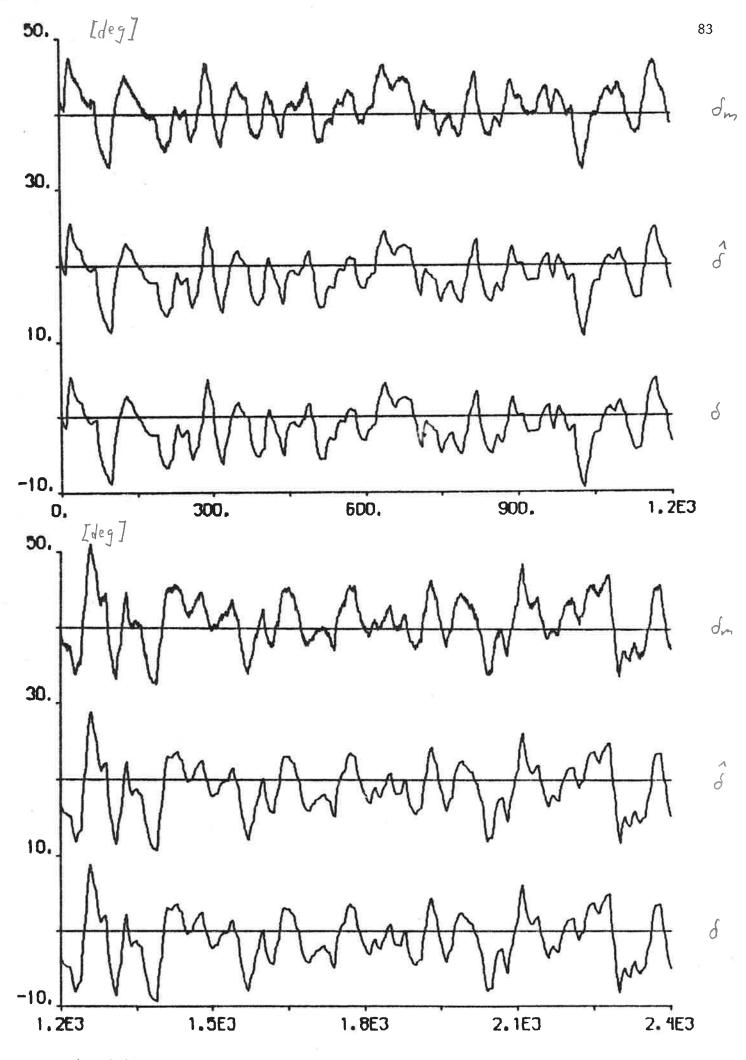
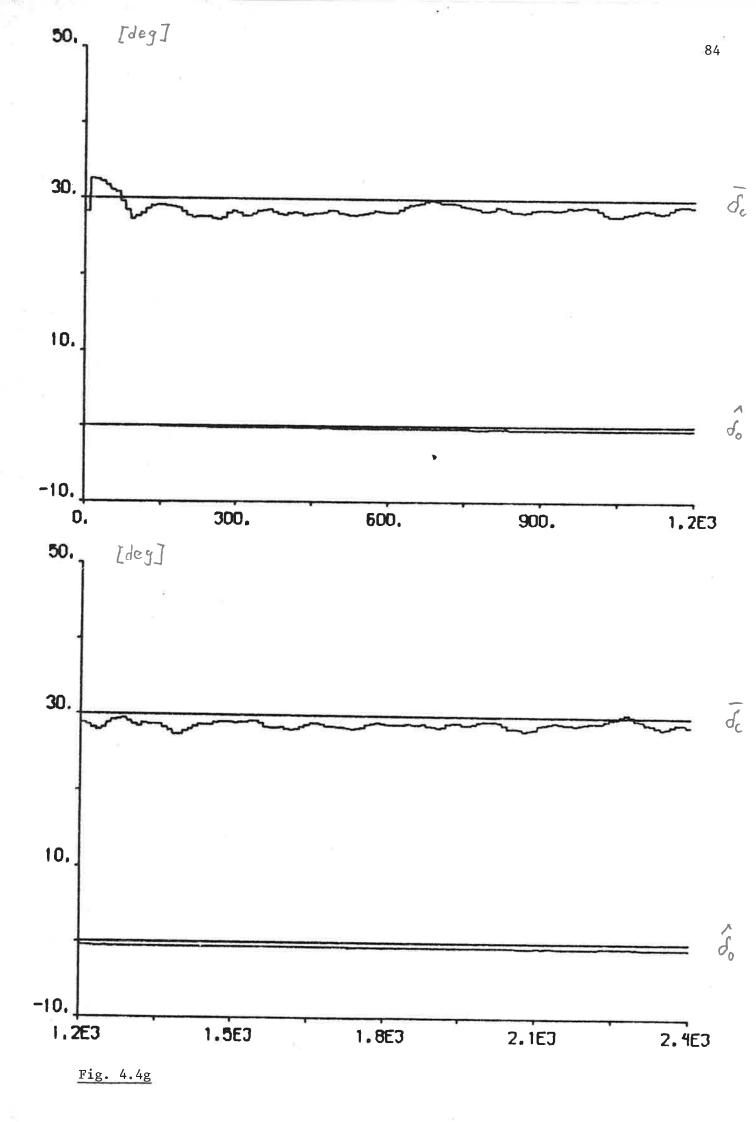
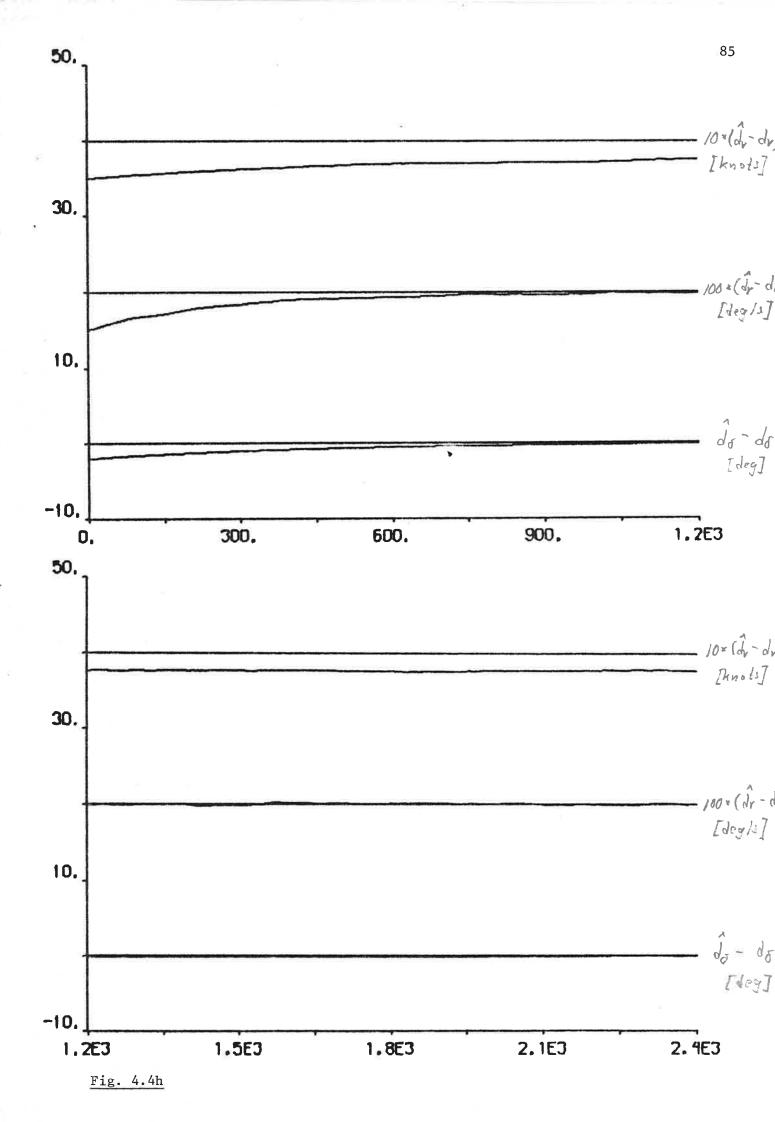
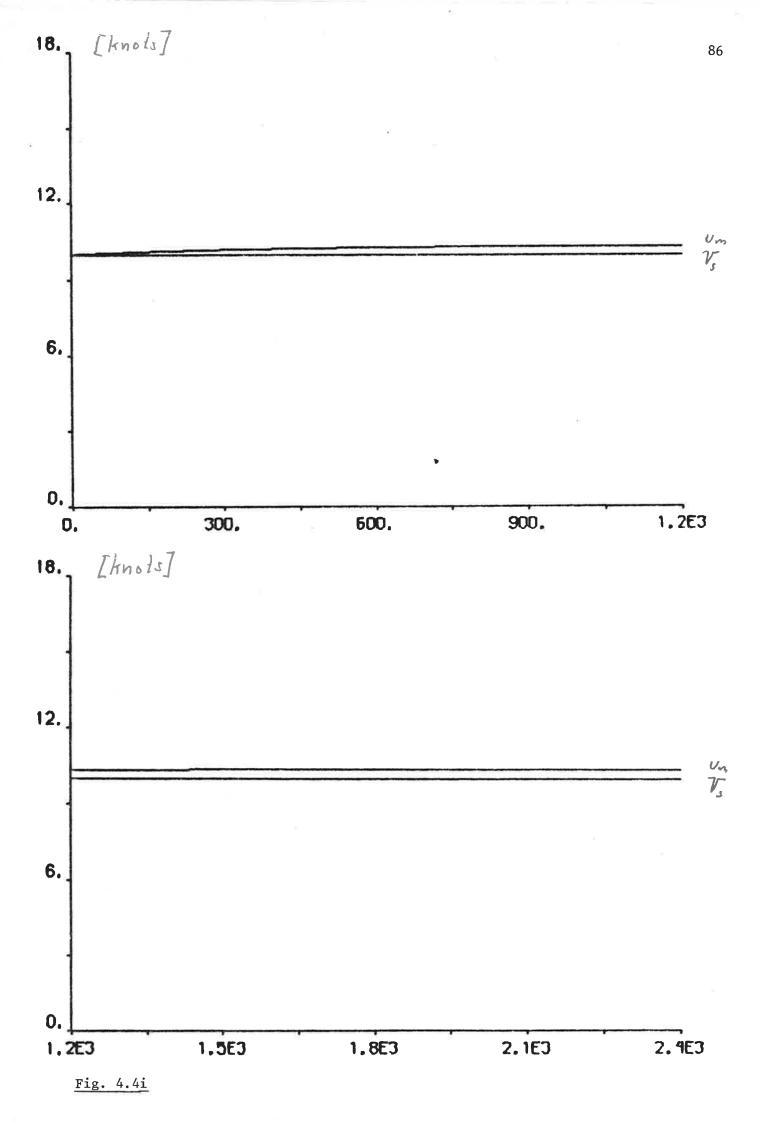


Fig. 4.4f









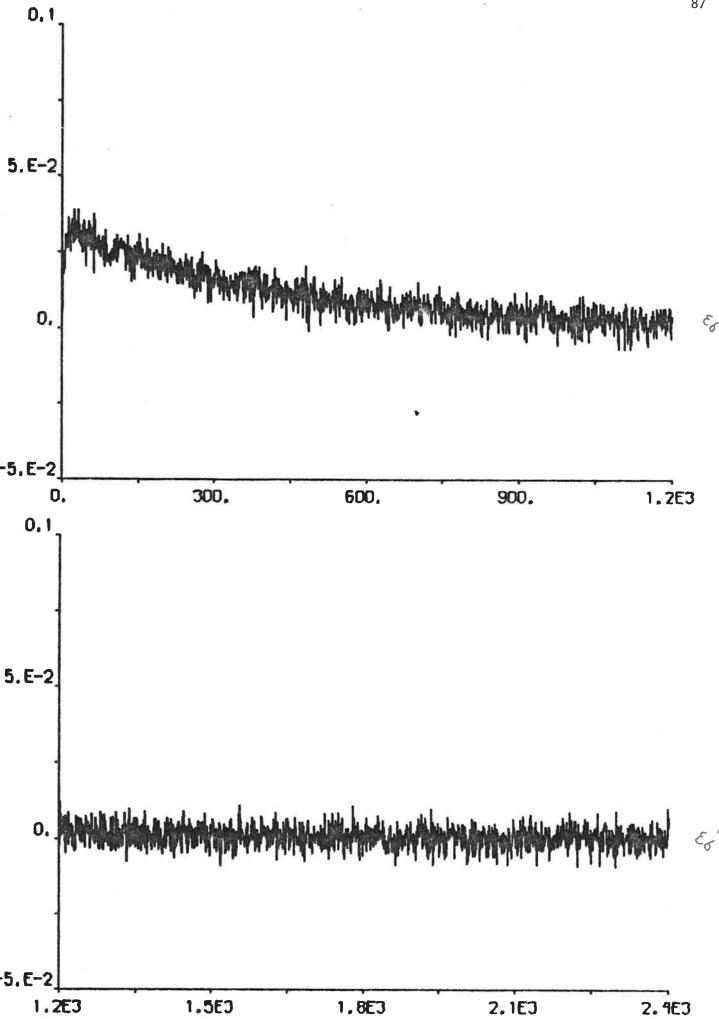
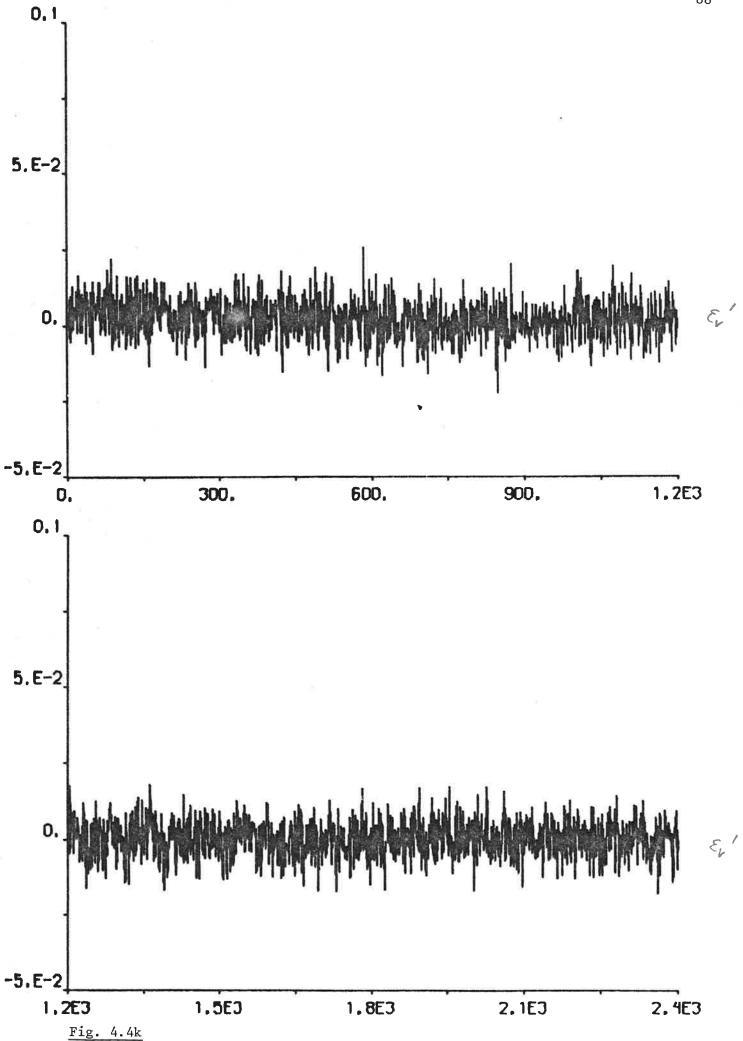


Fig. 4.4j





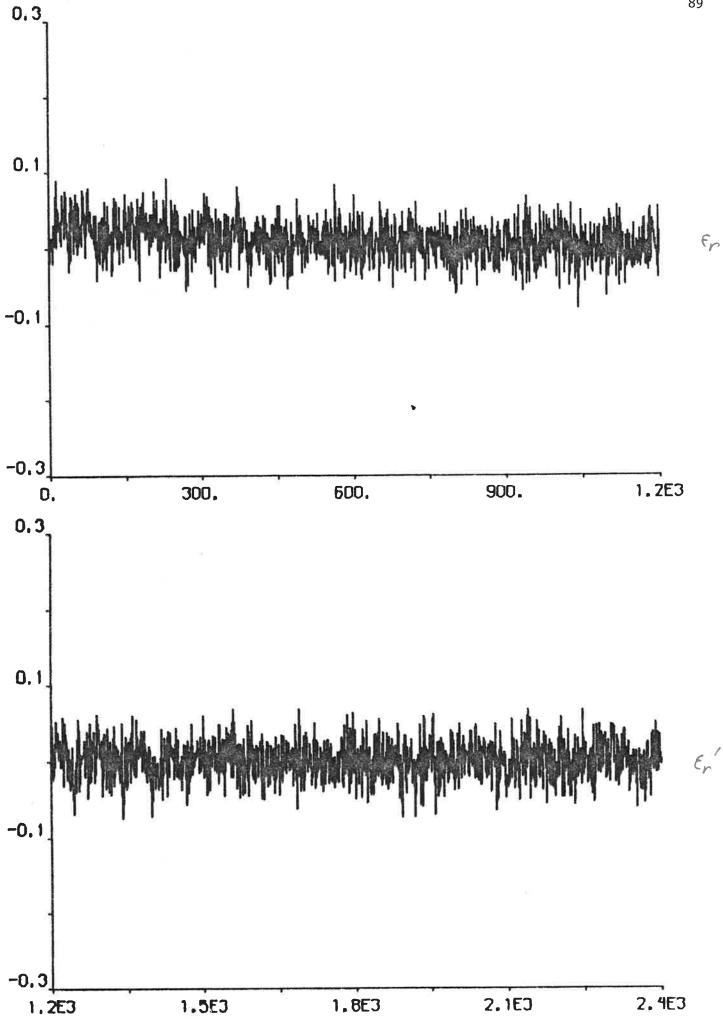
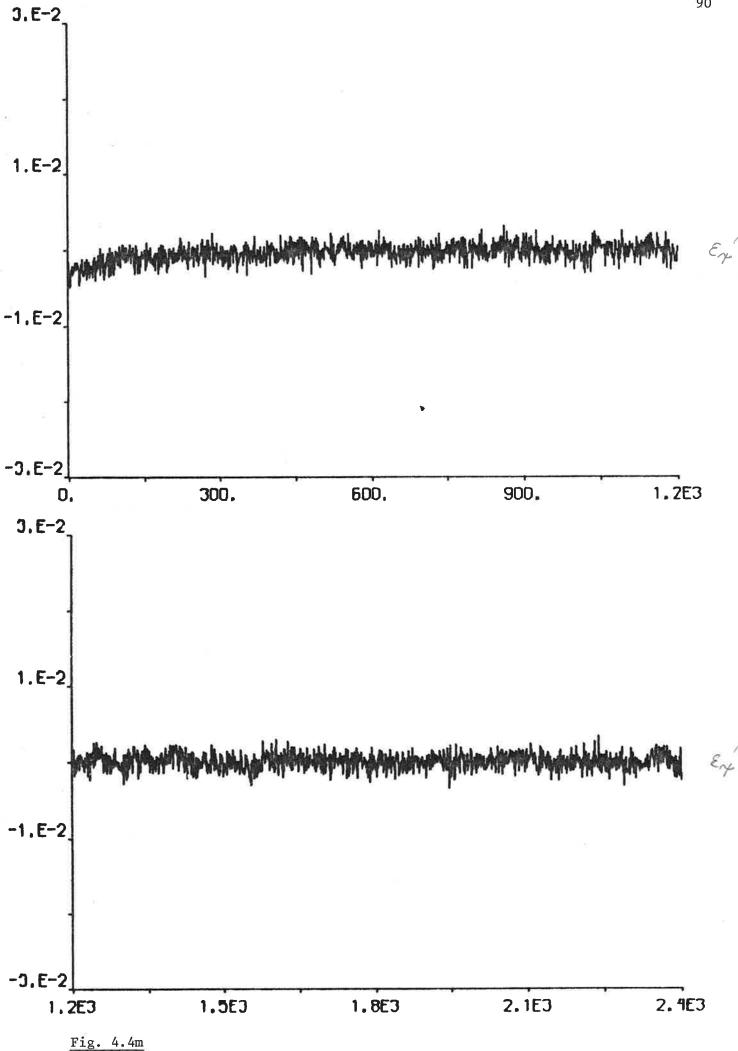
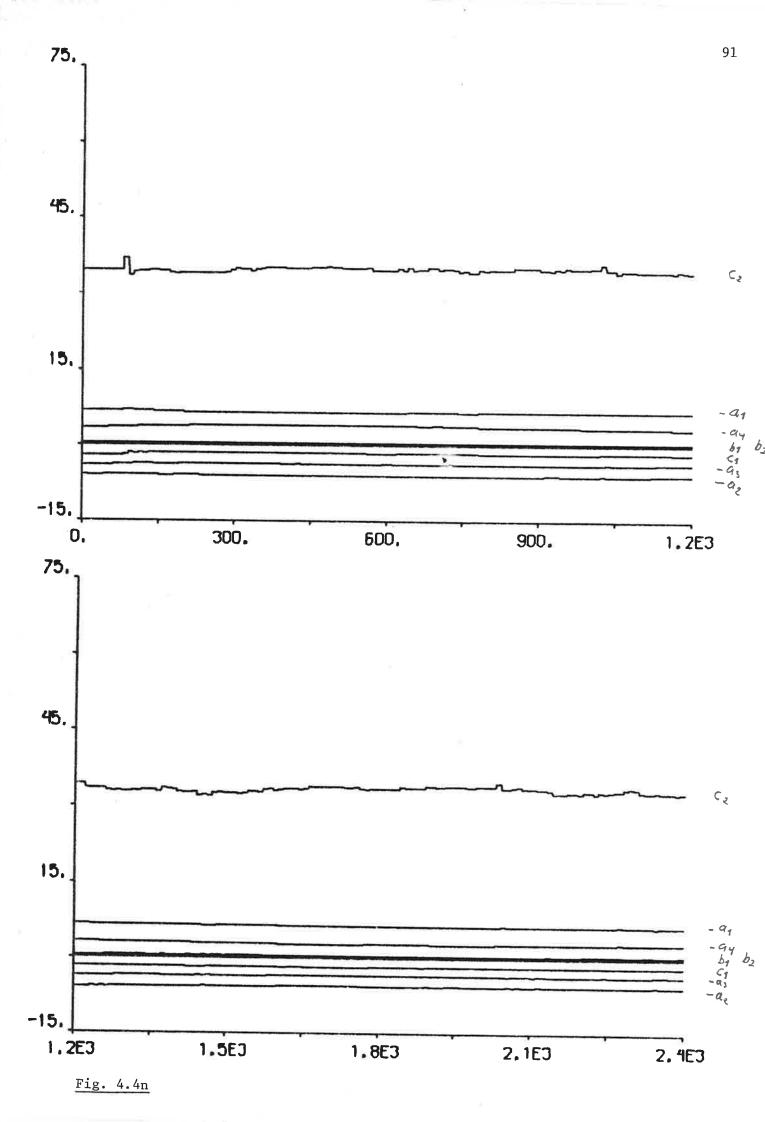


Fig. 4.4l







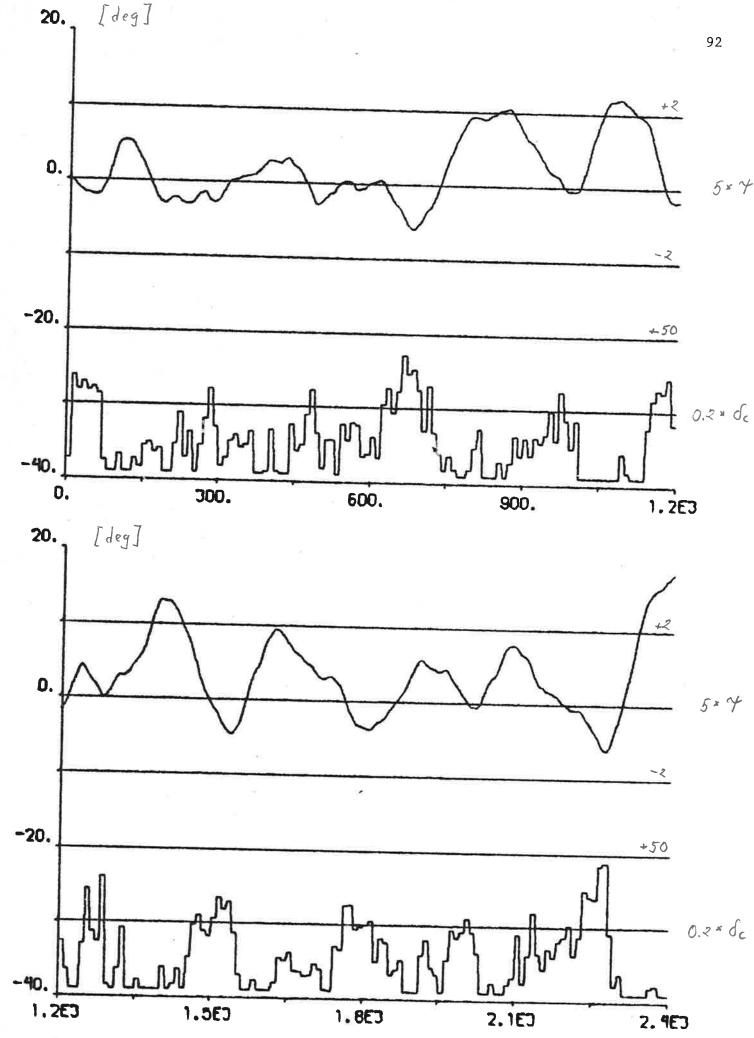


Fig. 4.5a - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, self-tuning regulator using estimates from the Kalman filter.

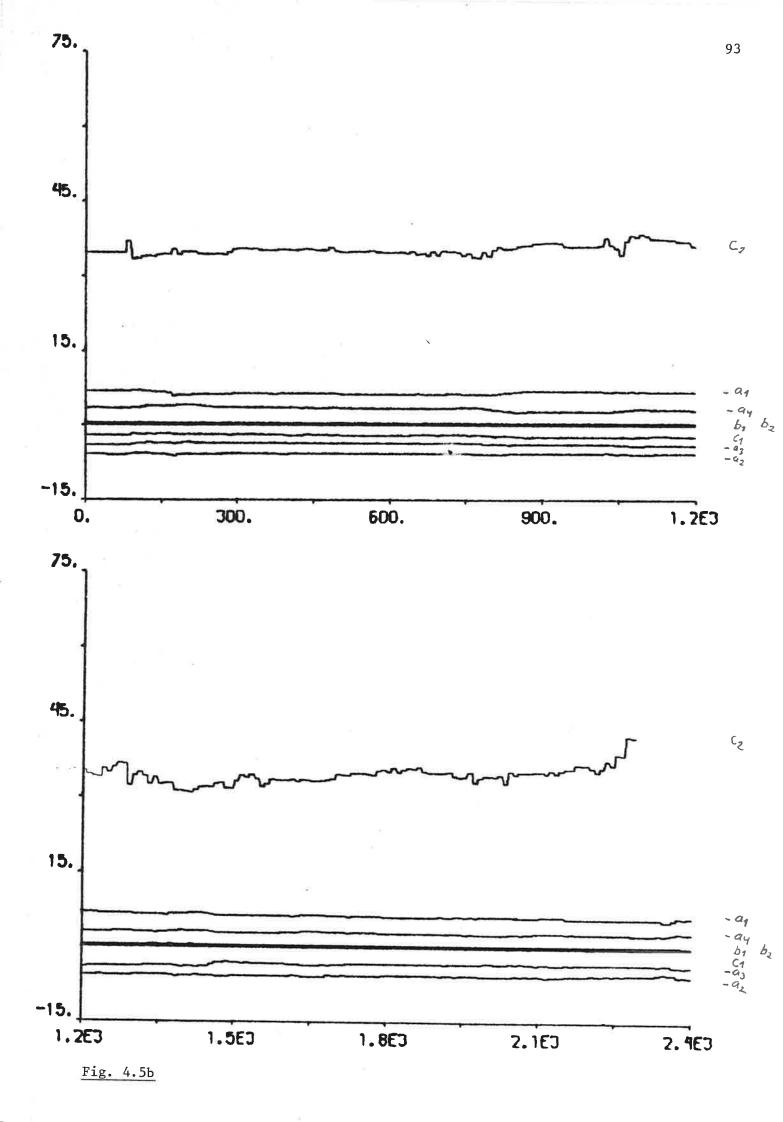


Fig. 4.6a - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, self-tuning regulator using estimates from the Kalman filter.

1.8E3

2.1EJ

2.4E3

1.5E3

1.2E3

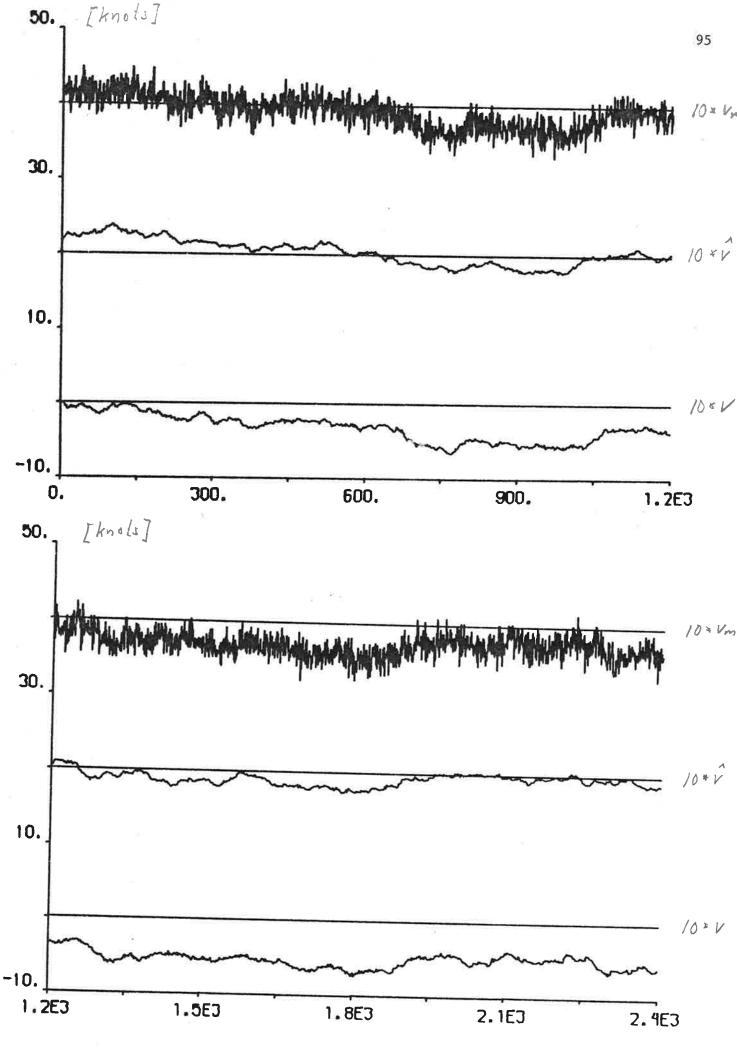
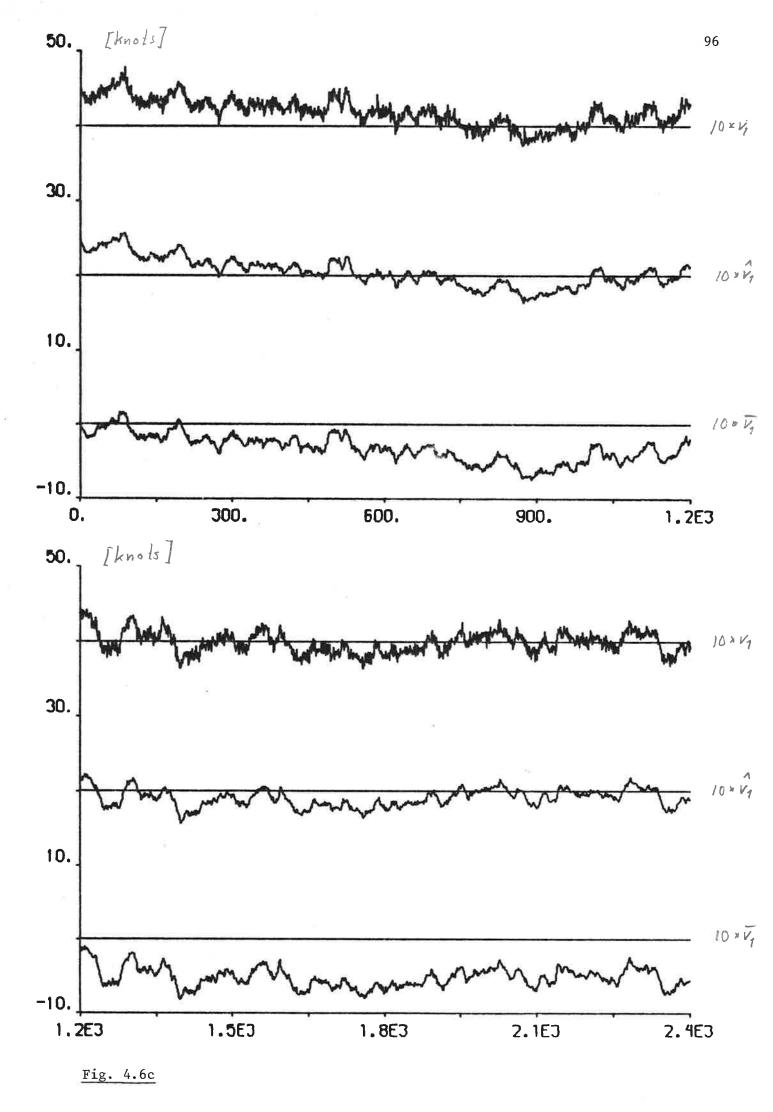
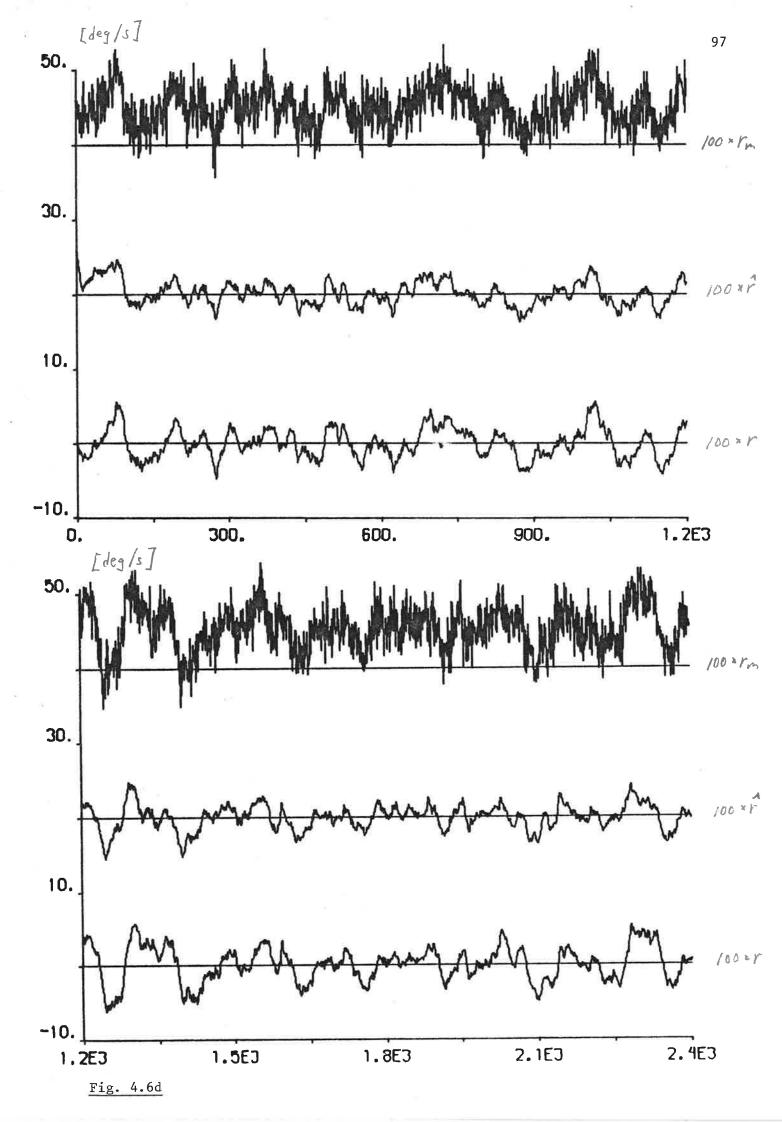
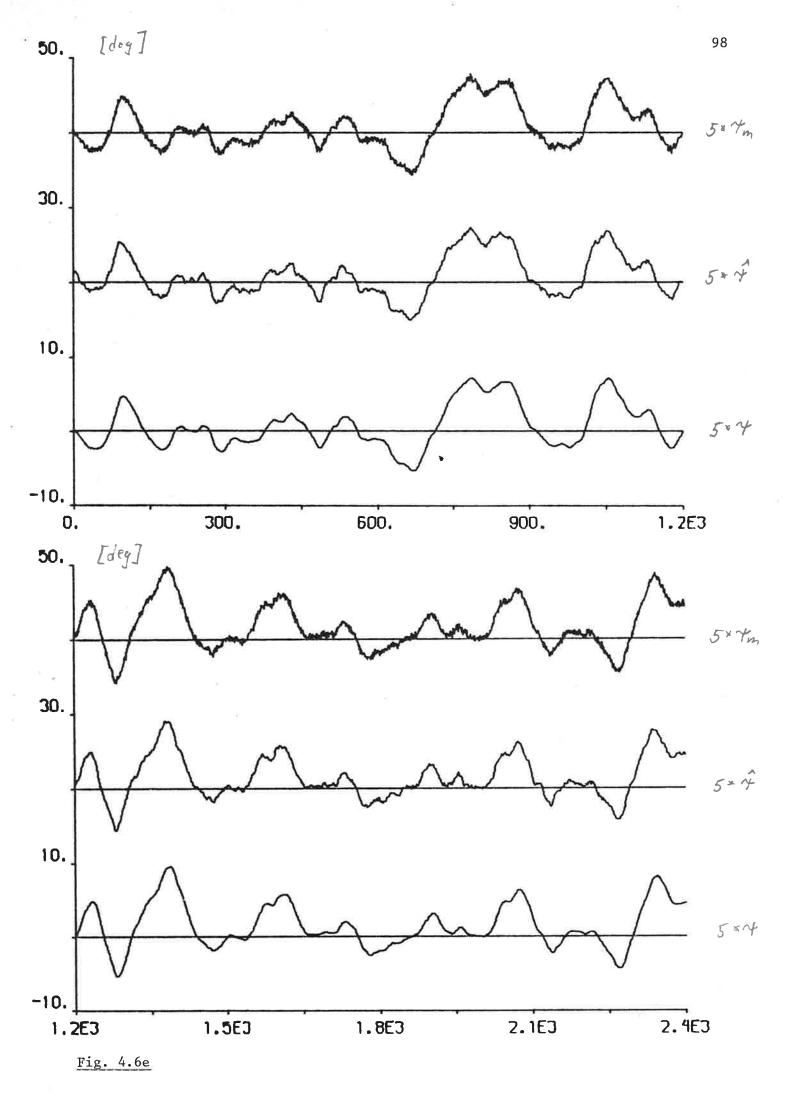
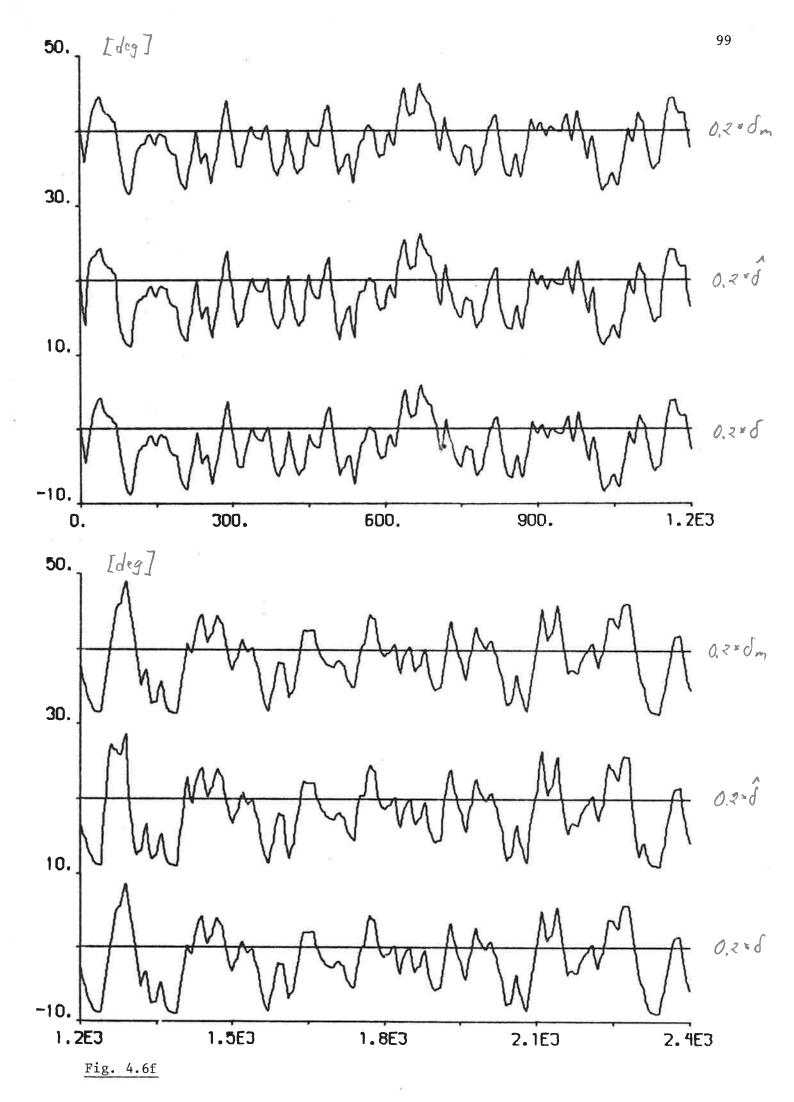


Fig. 4.6b

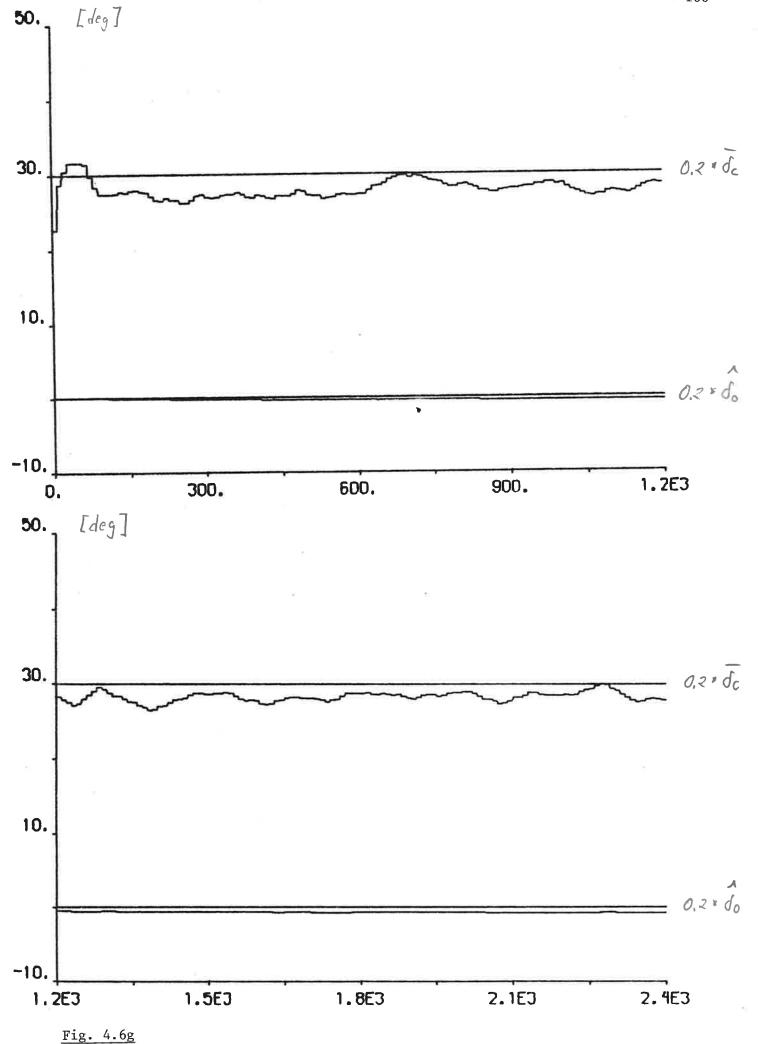


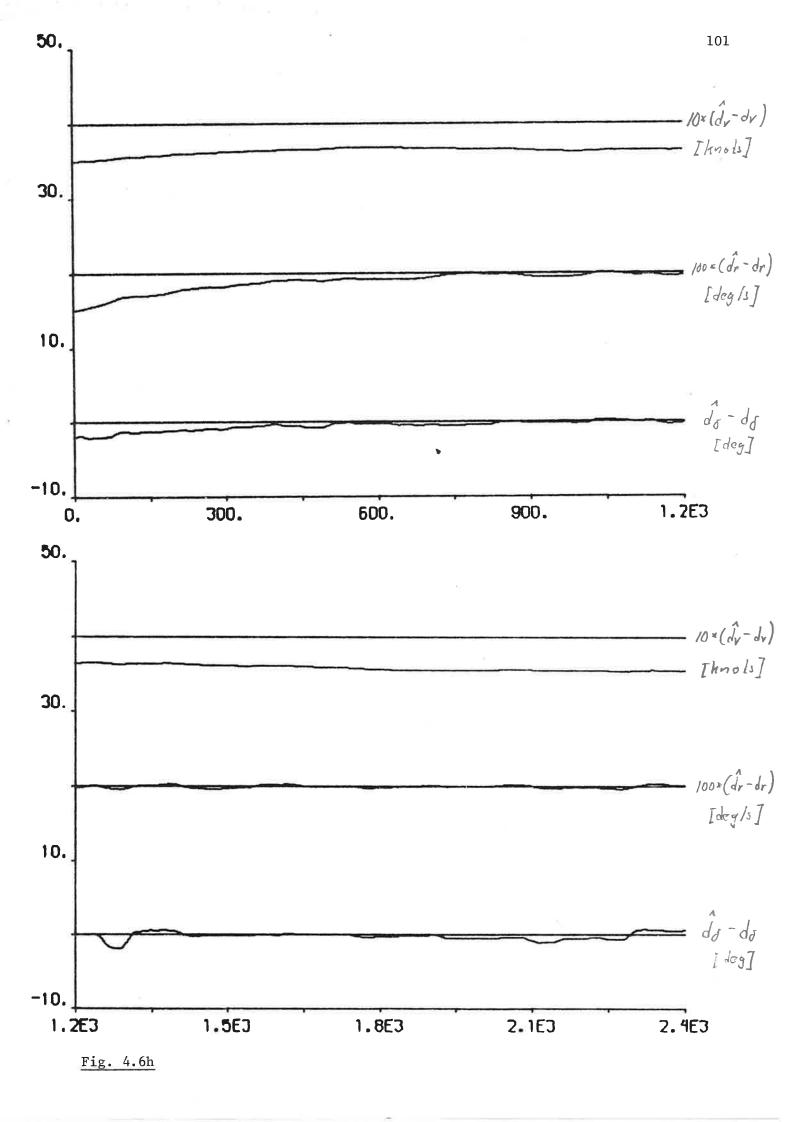


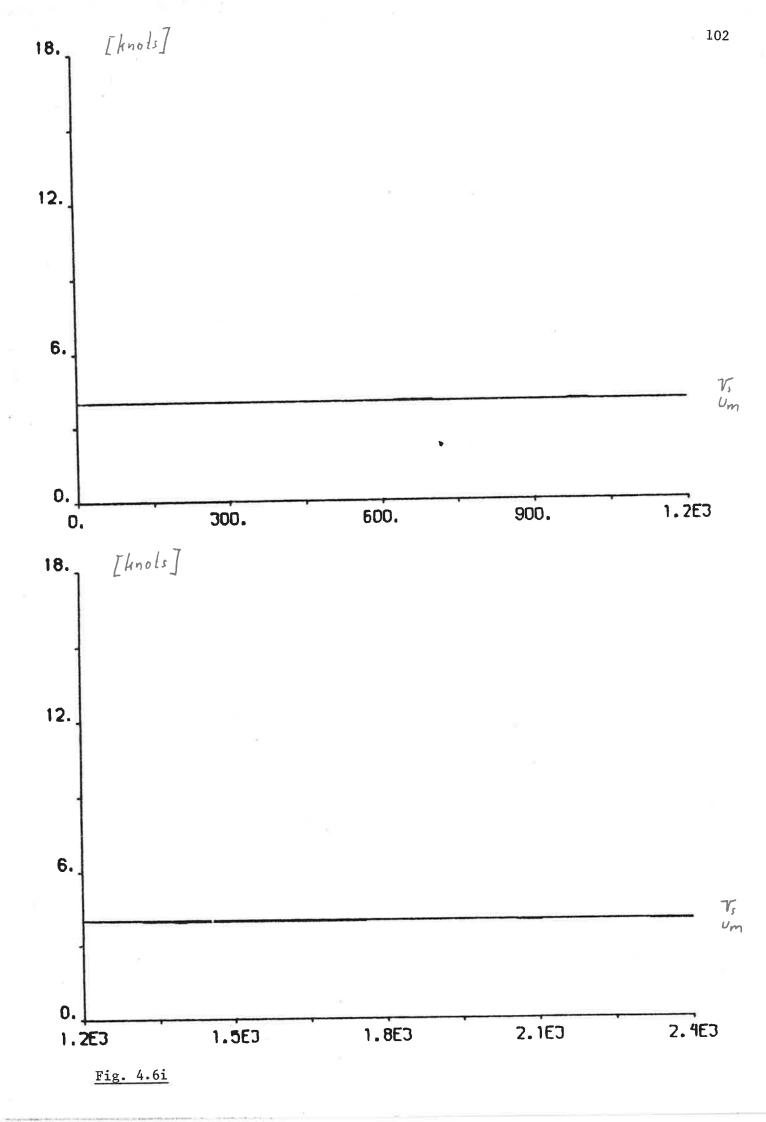














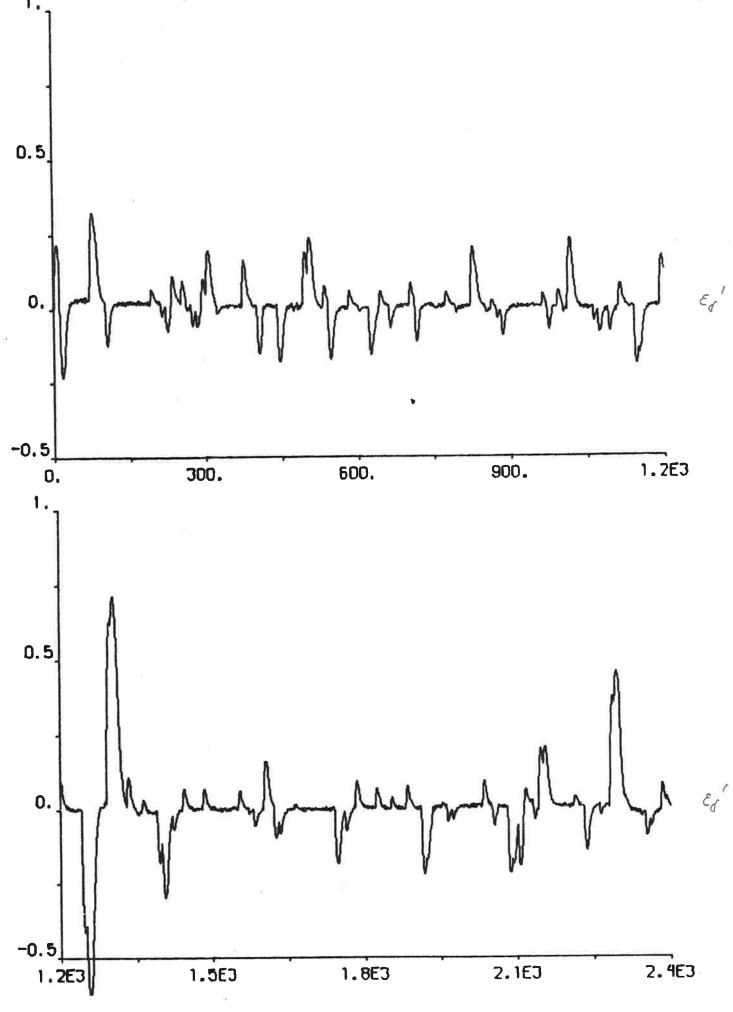


Fig. 4.6j



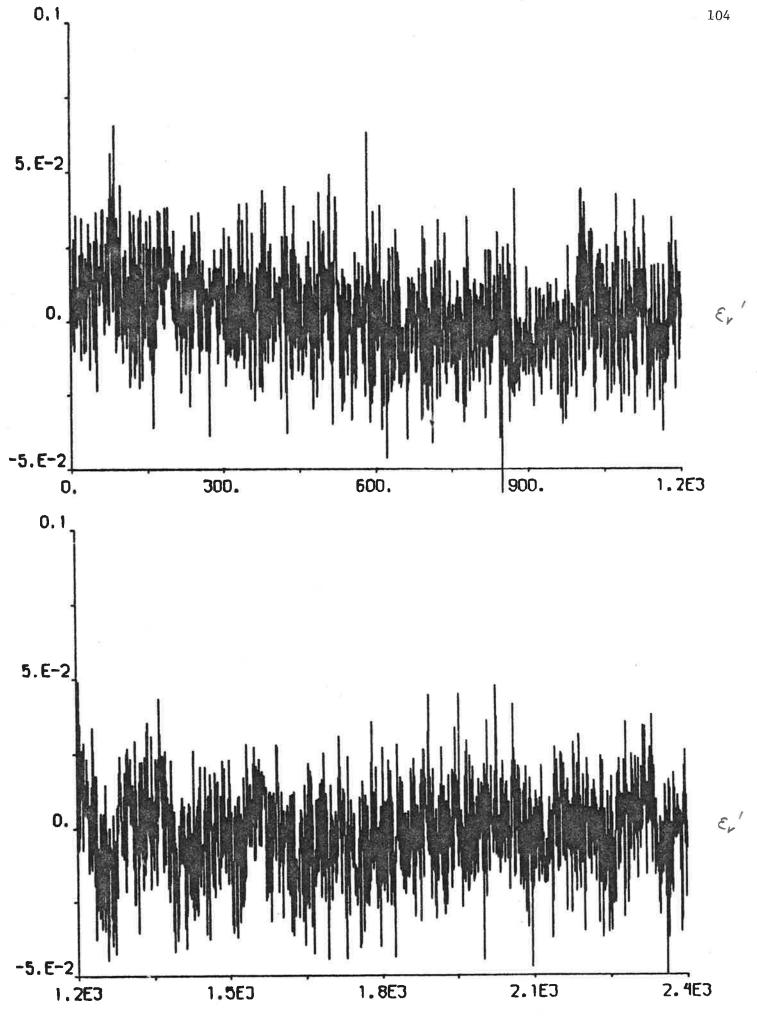
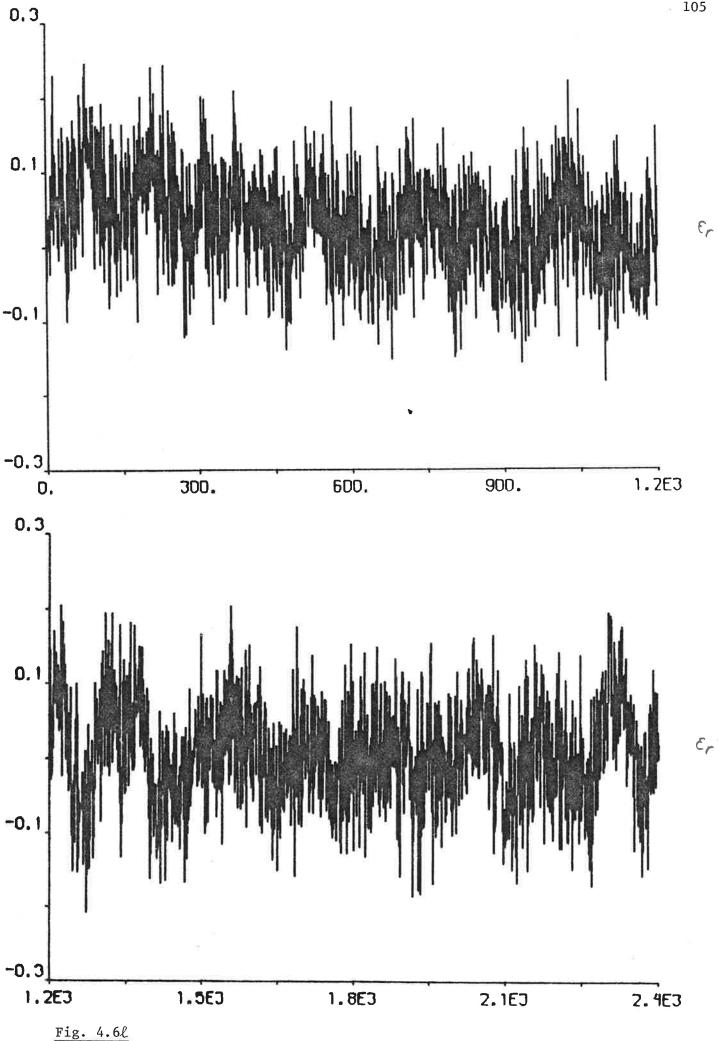


Fig. 4.6k





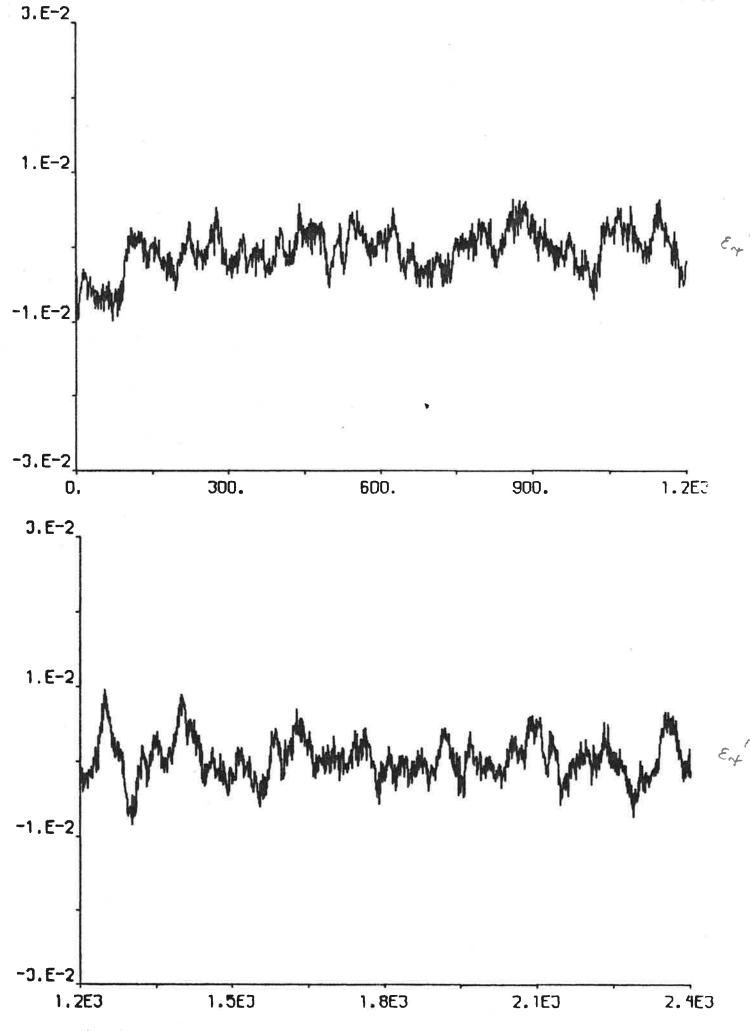
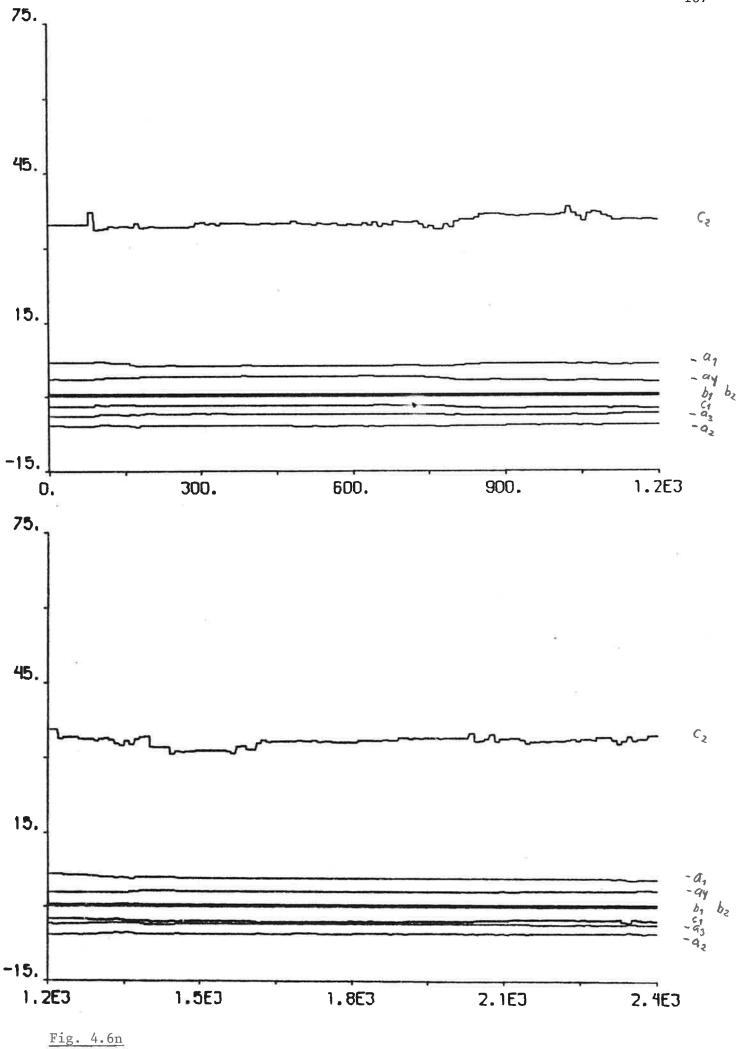


Fig. 4.6m



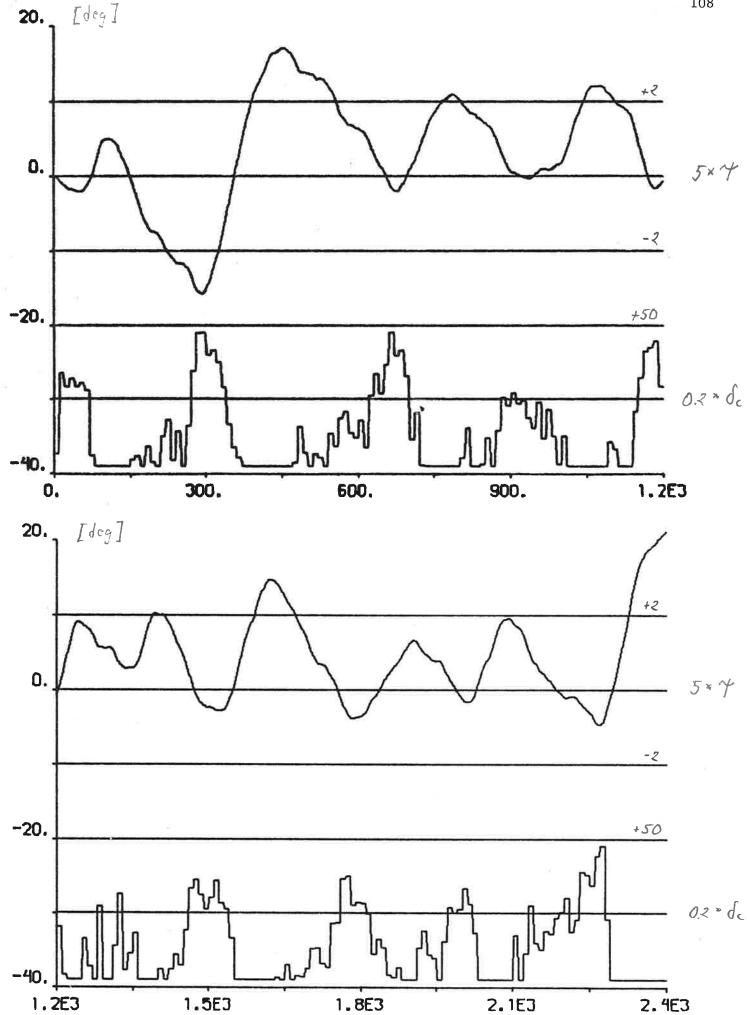
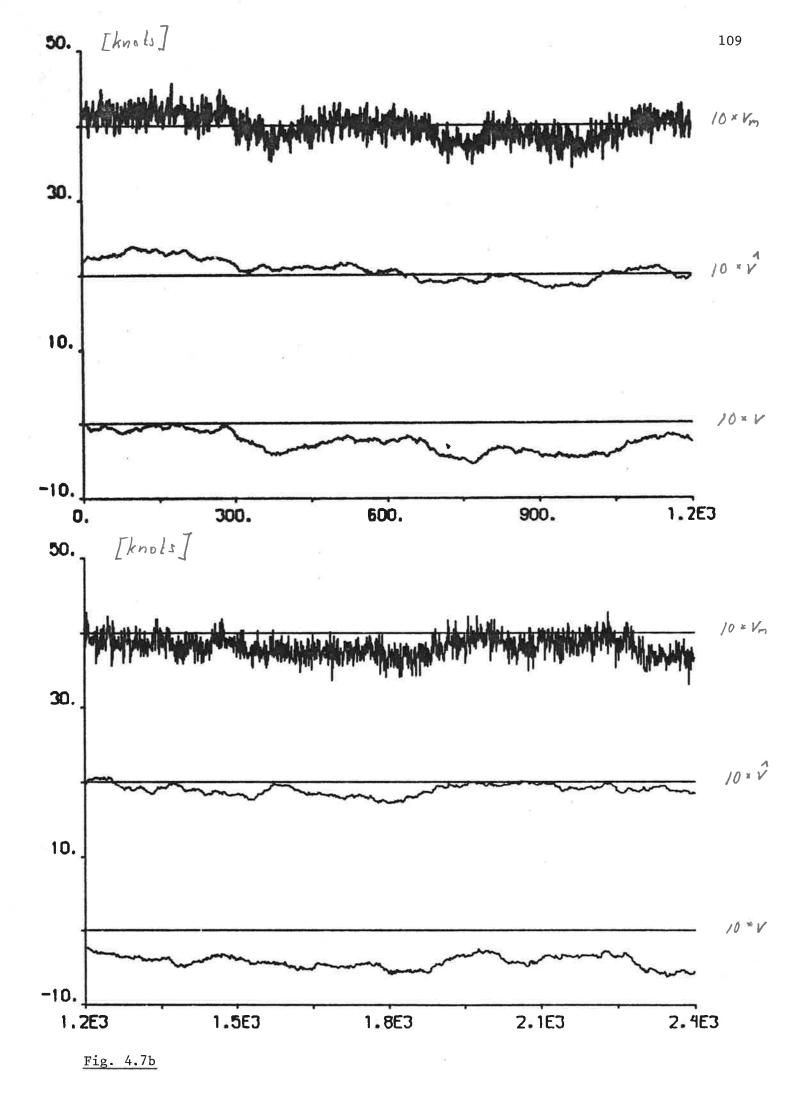


Fig. 4.7a - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, self-tuning regulator using estimates from the Kalman filter. The initial covariance matrix P of the self-tuning regulator is given by (4.3).



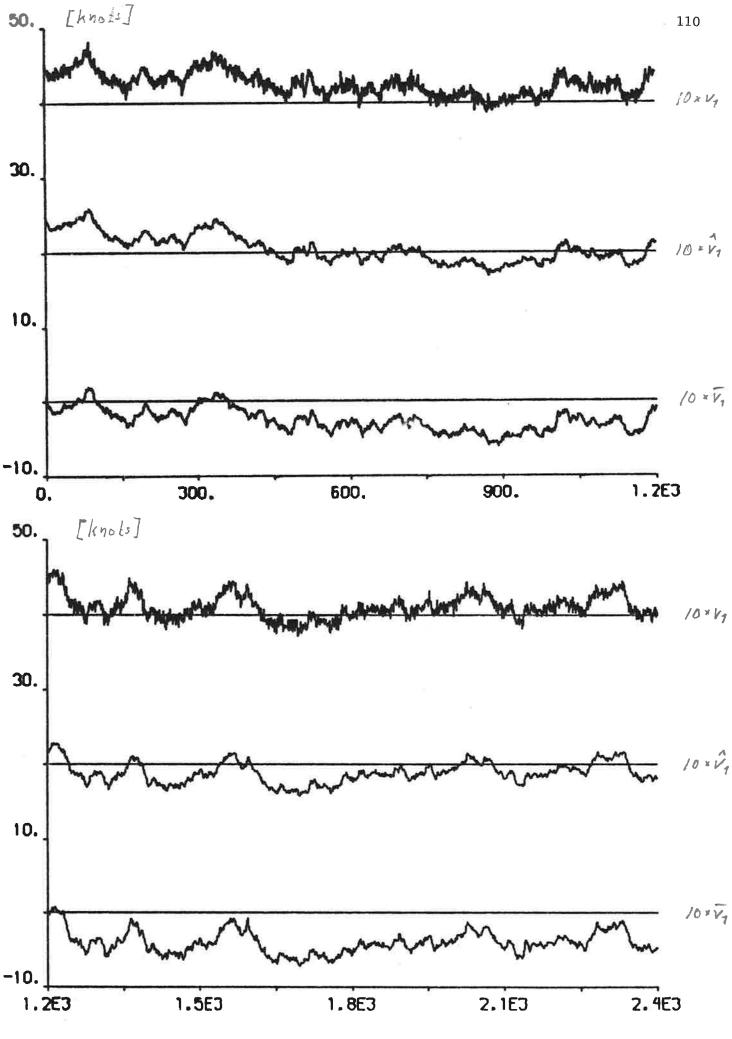
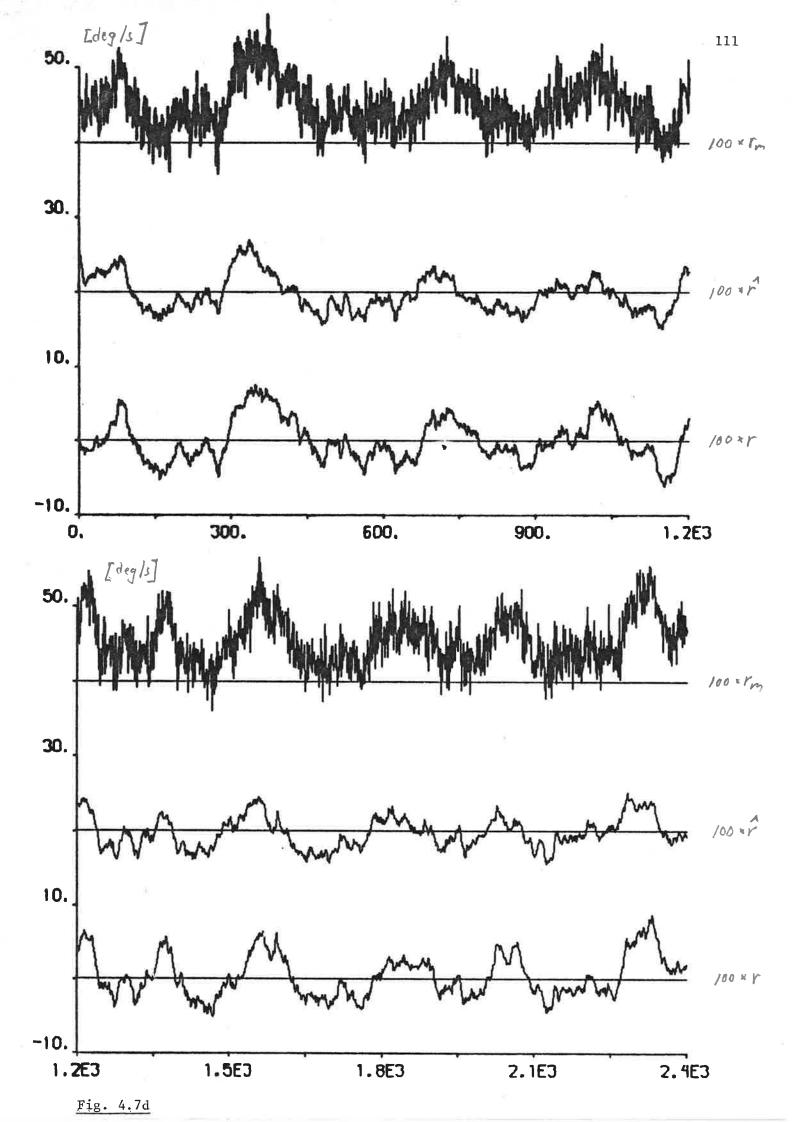
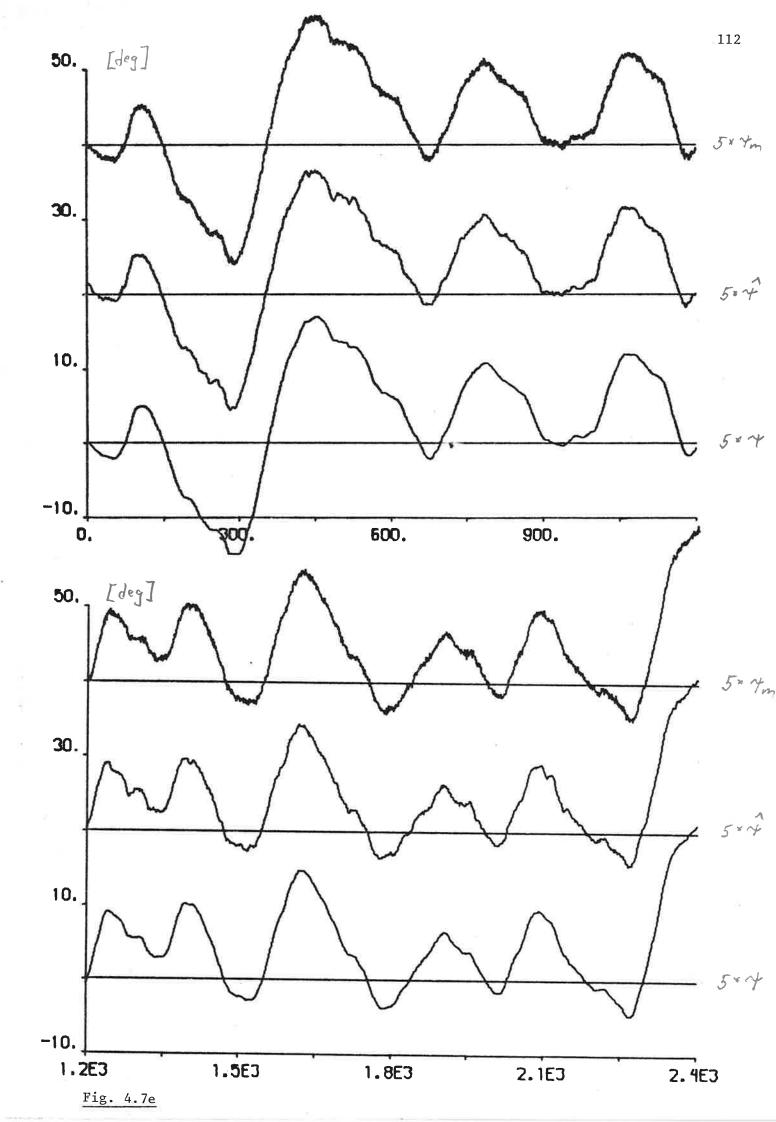


Fig. 4.7c





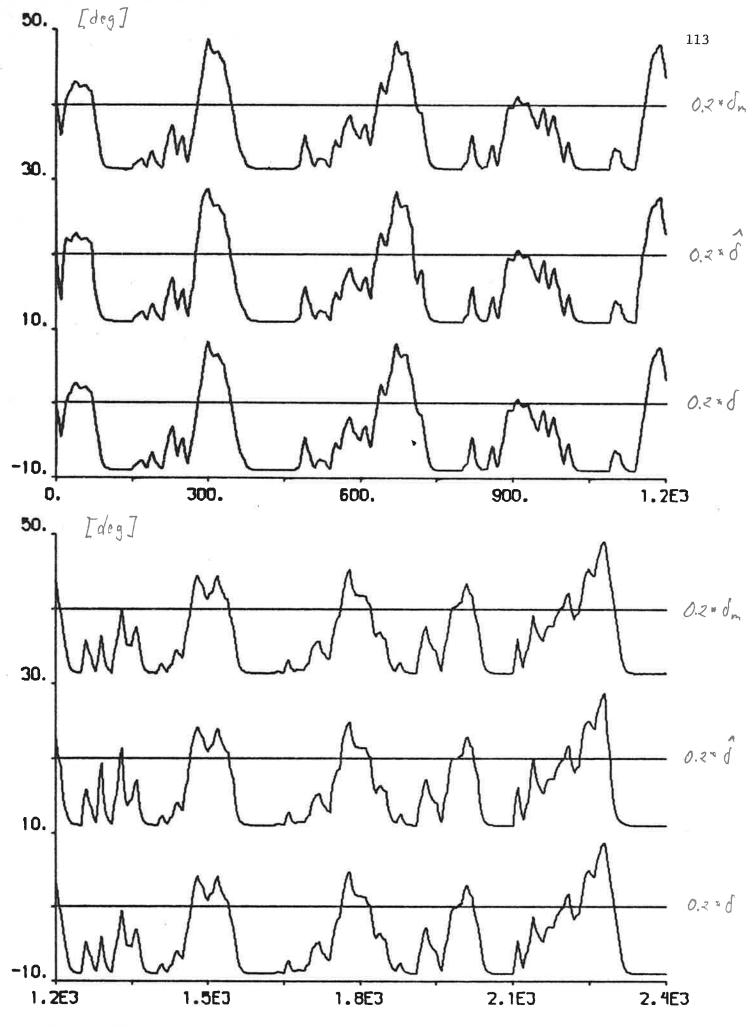
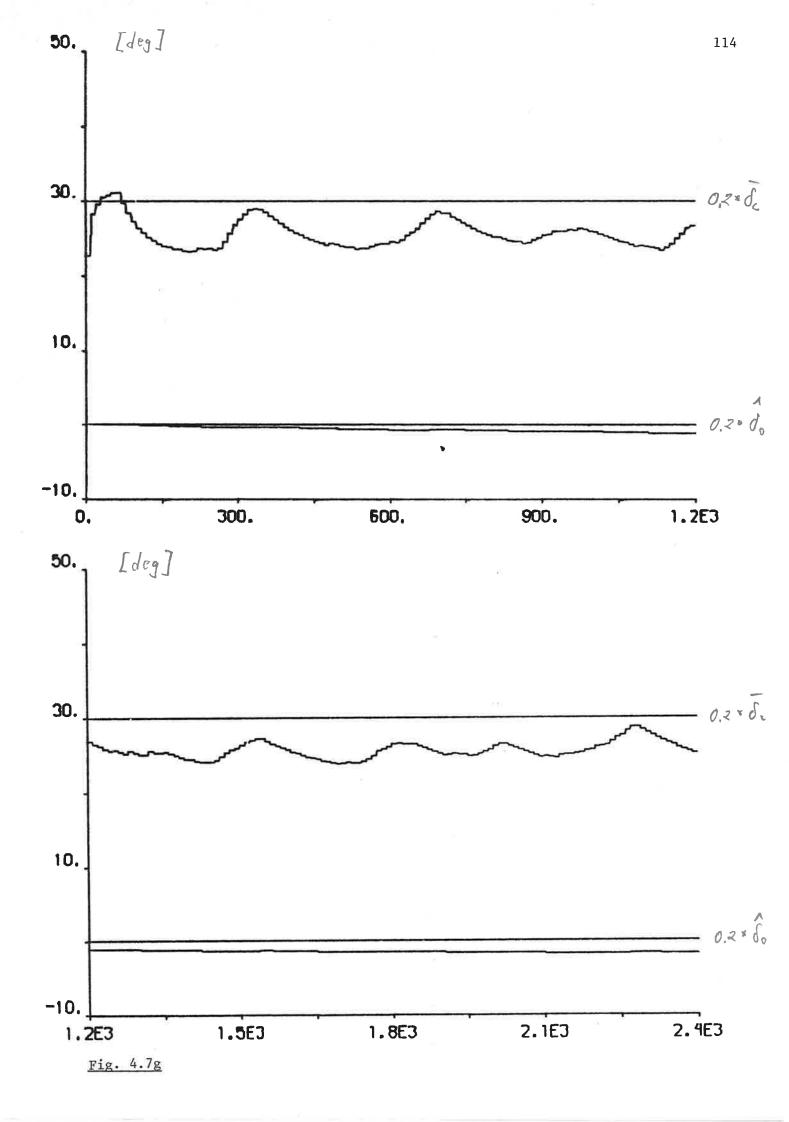
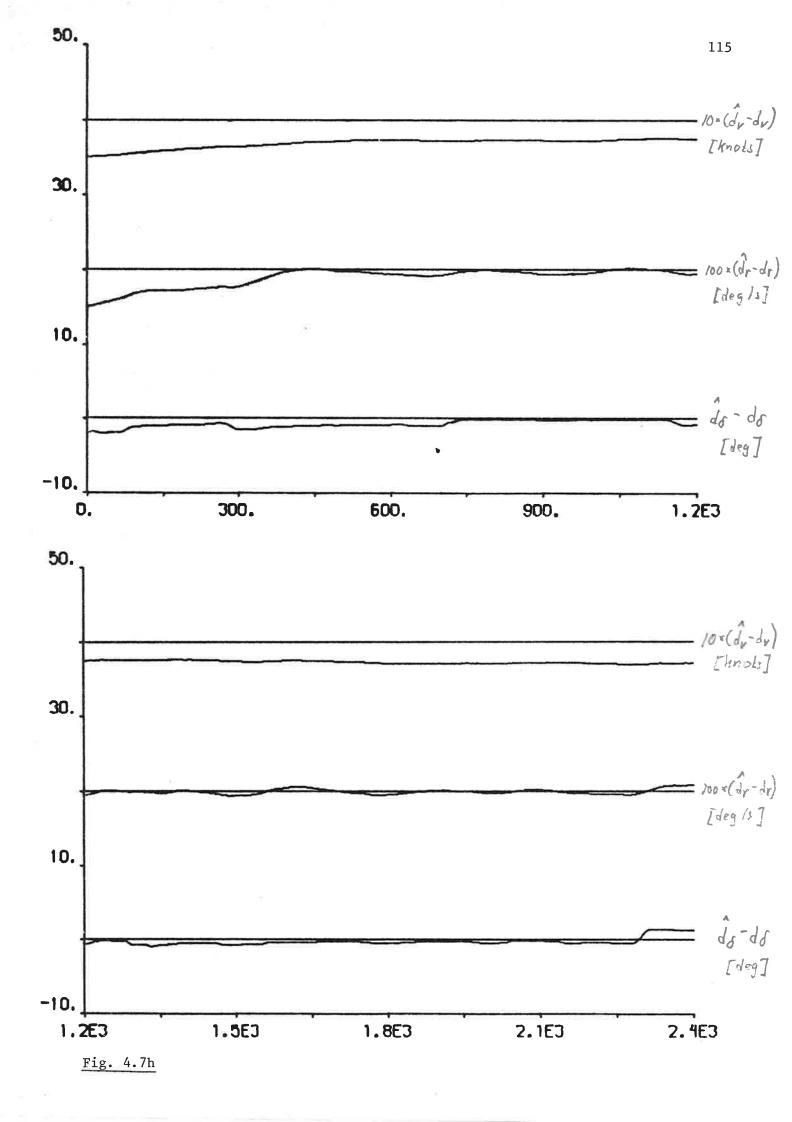
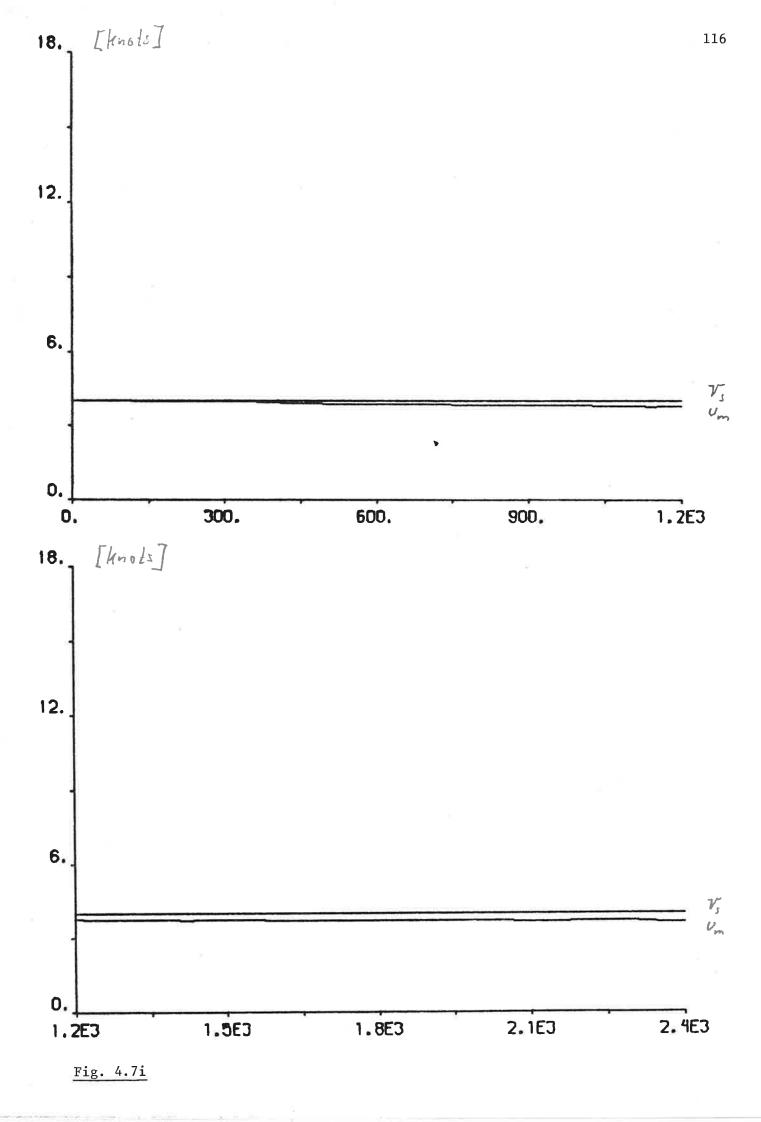


Fig. 4.7f









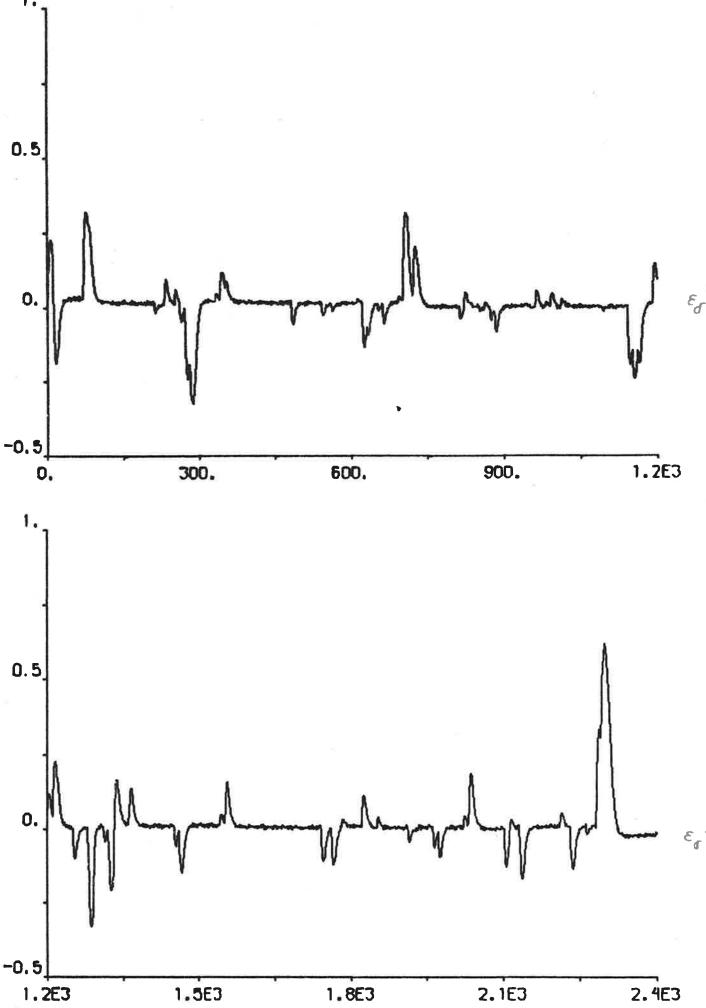


Fig. 4.7j

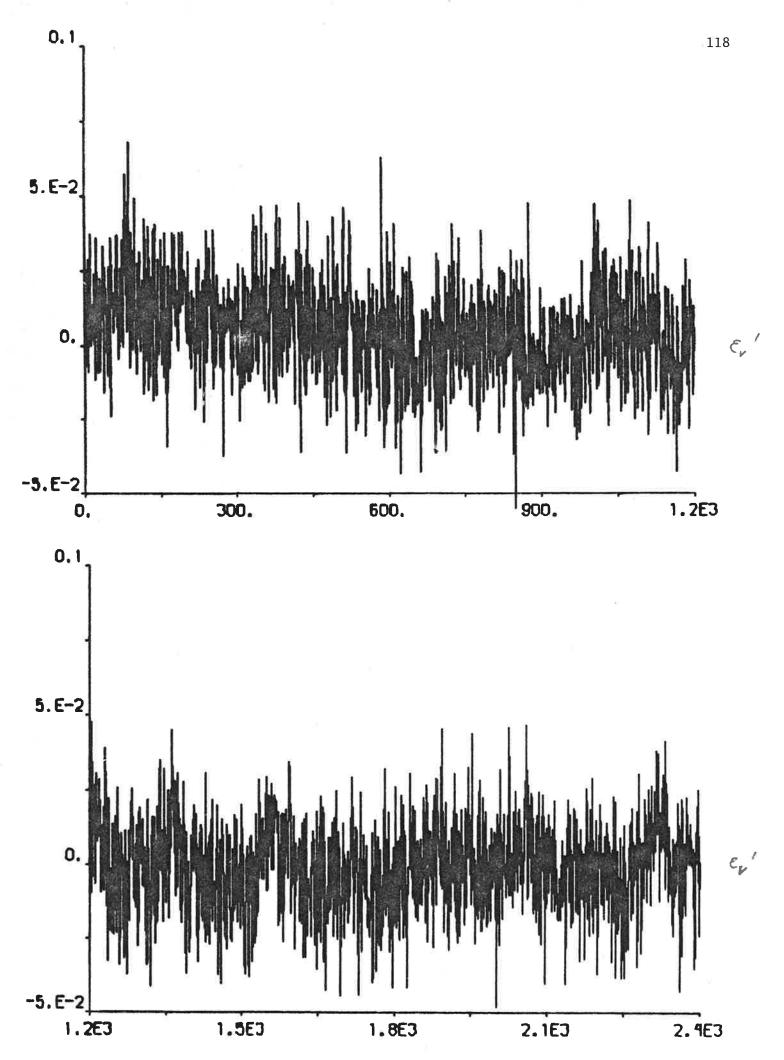
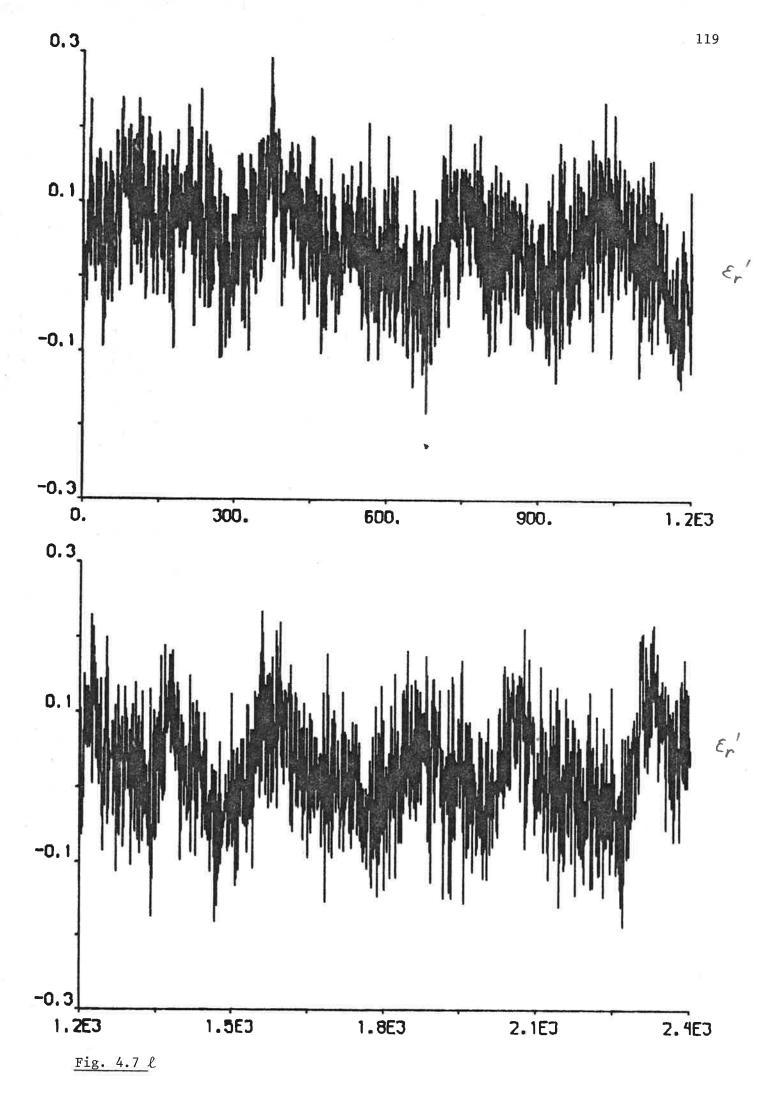
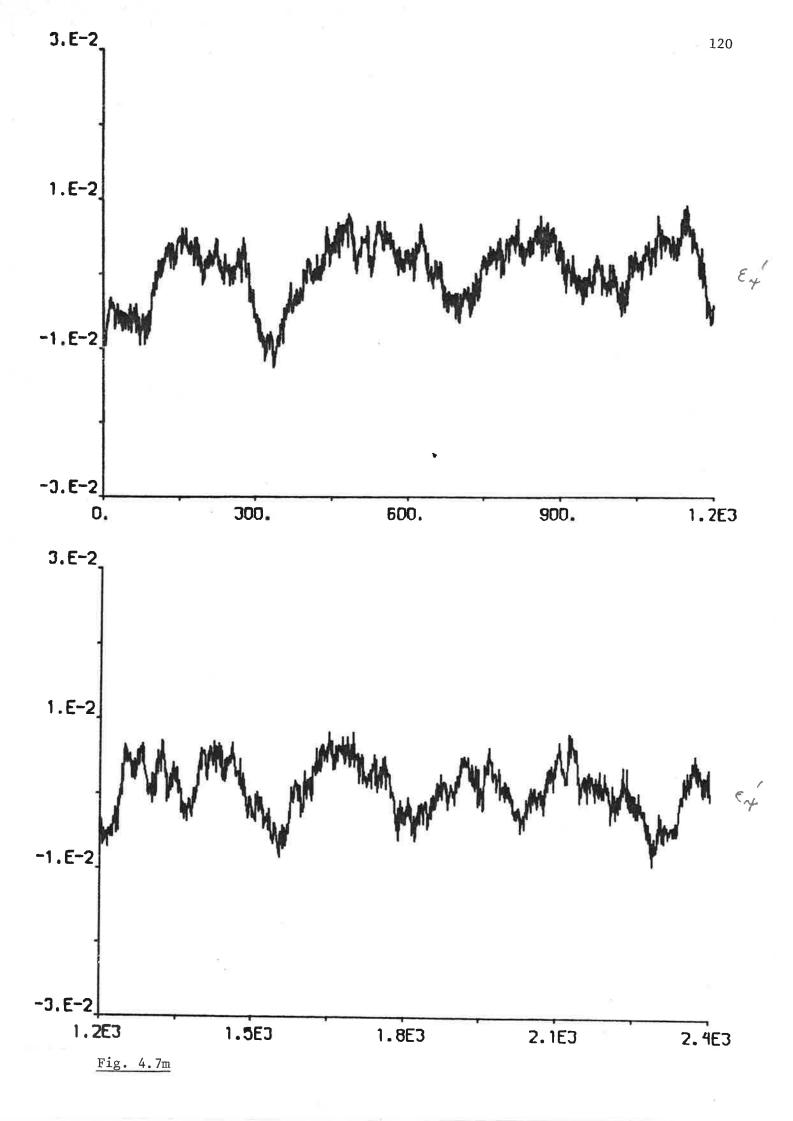
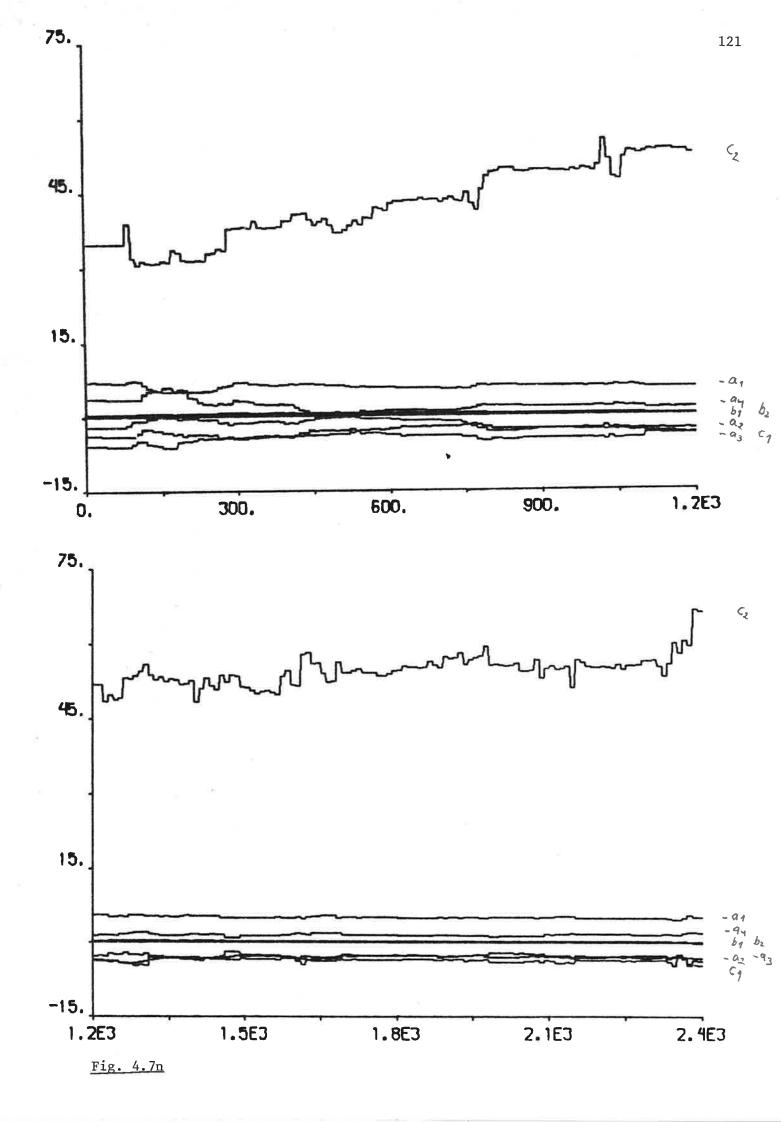


Fig. 4.7k







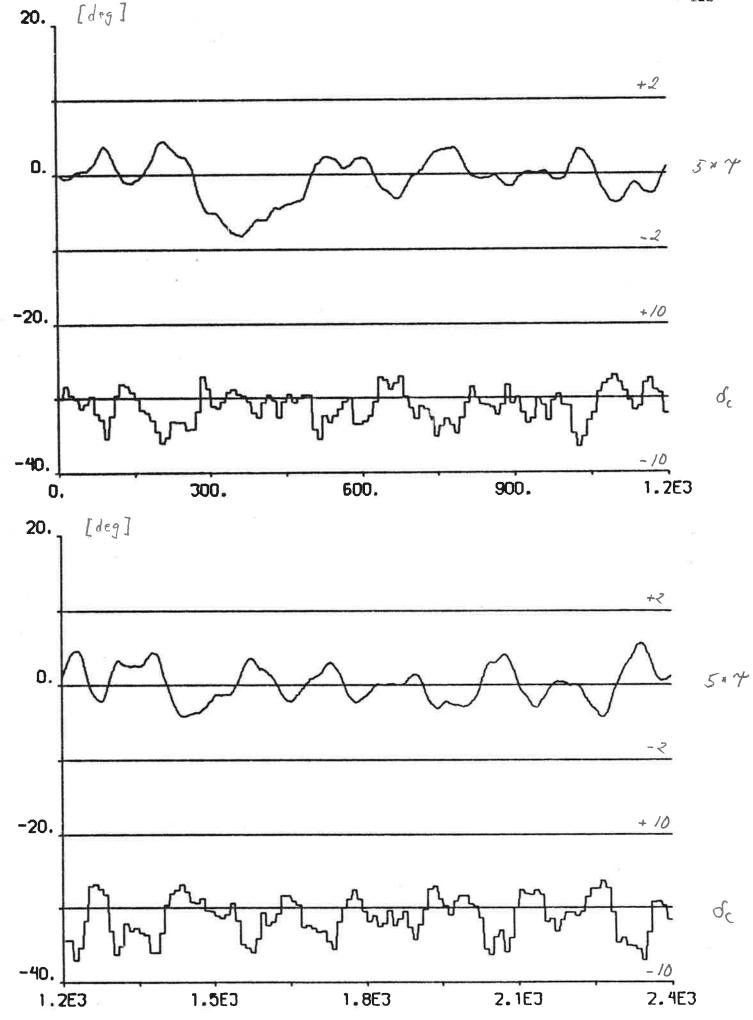
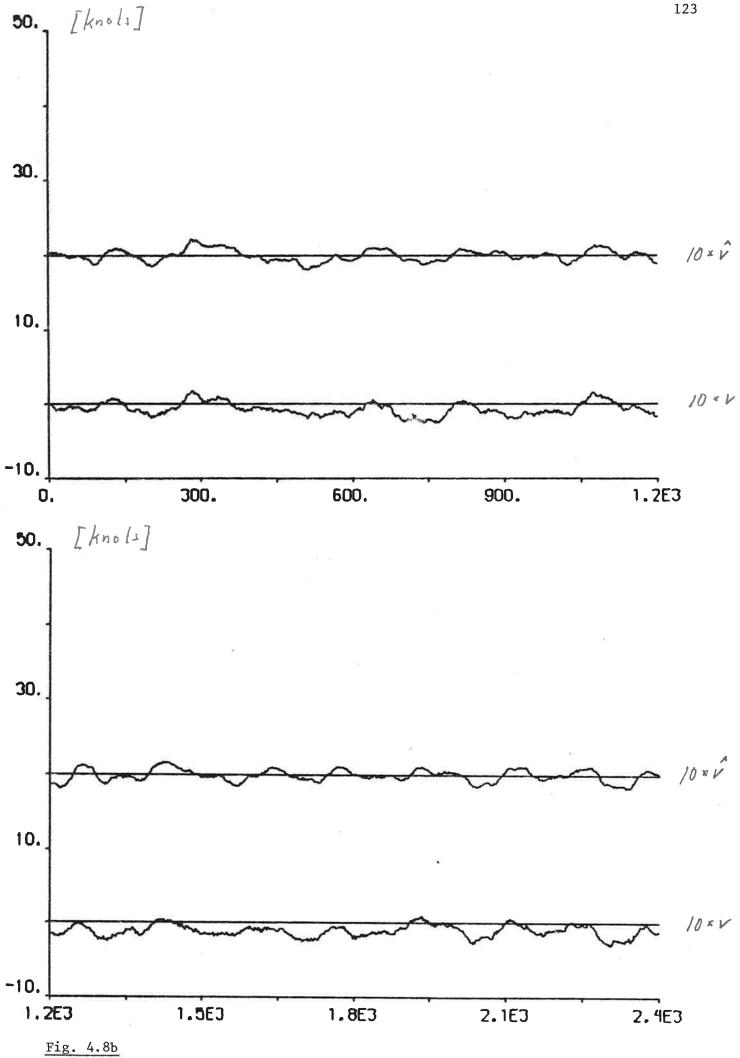


Fig. 4.8a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using estimates from the Kalman filter. The only measurement signal used by the filter is the heading angle.





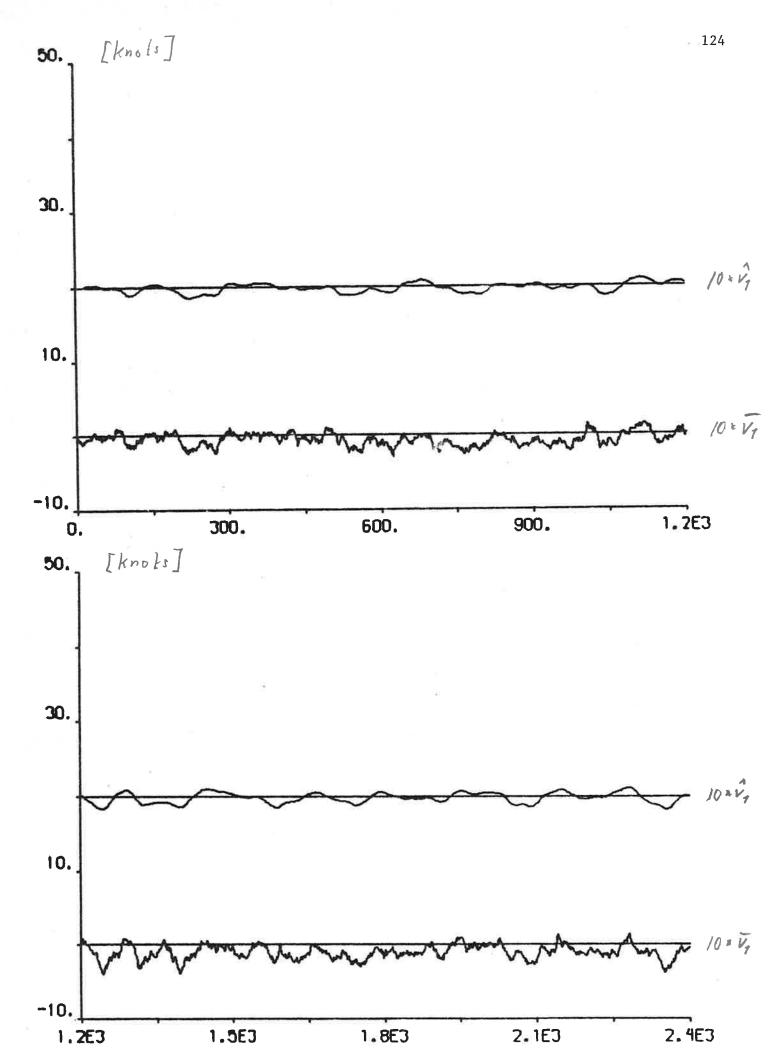


Fig. 4.8c

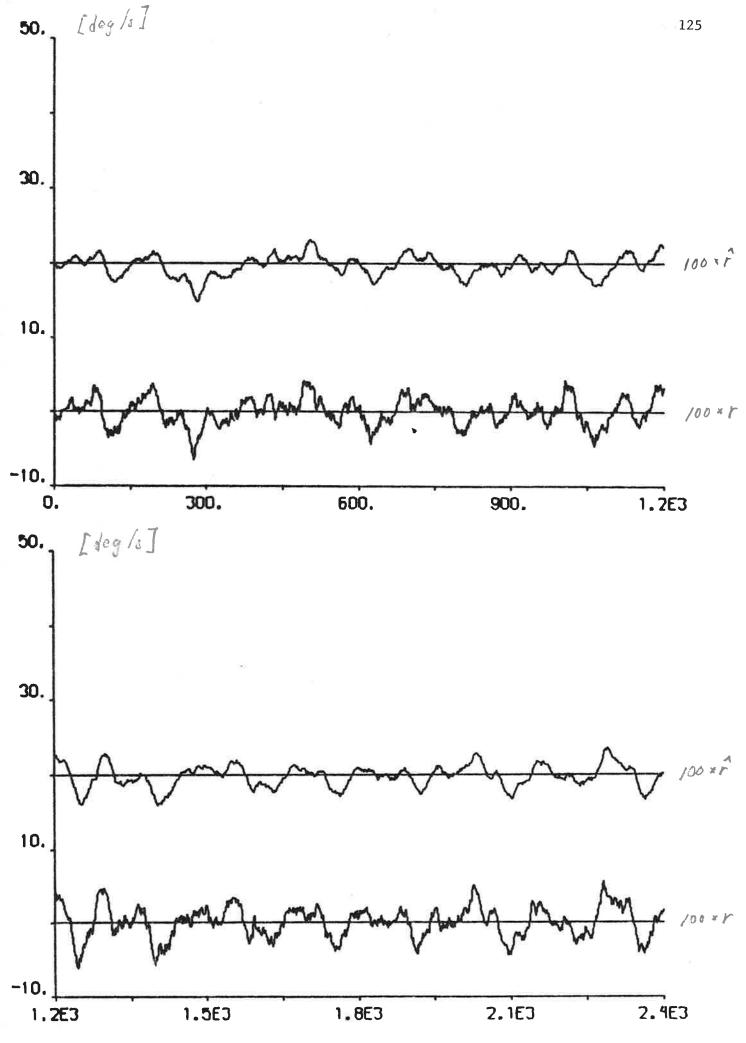
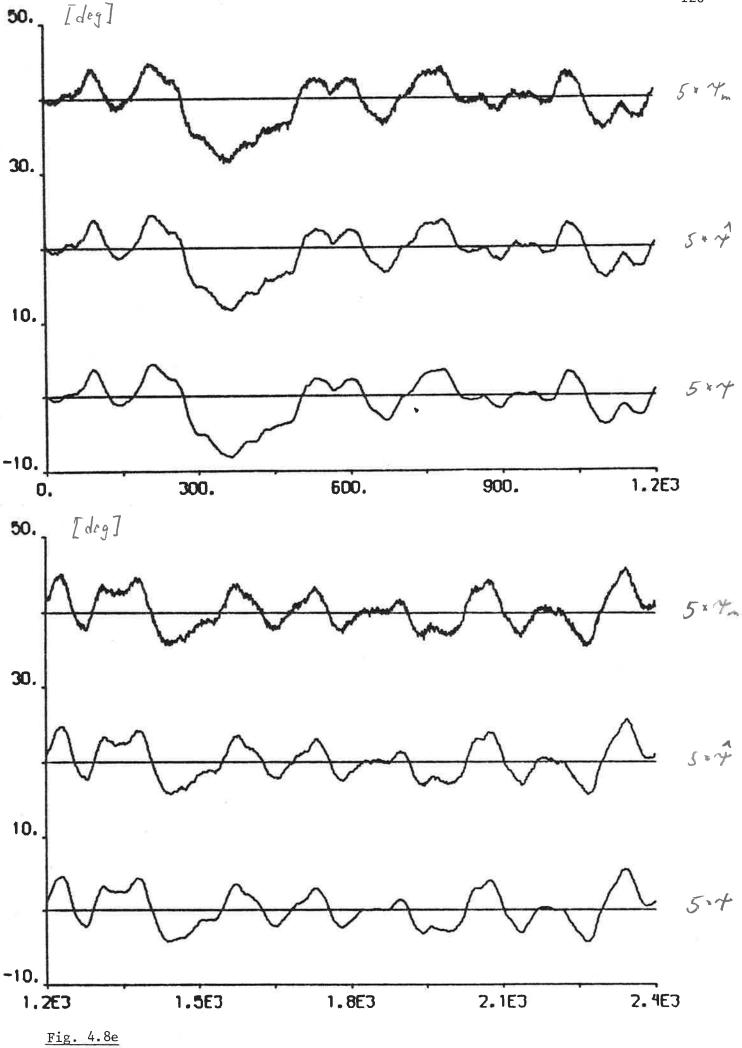
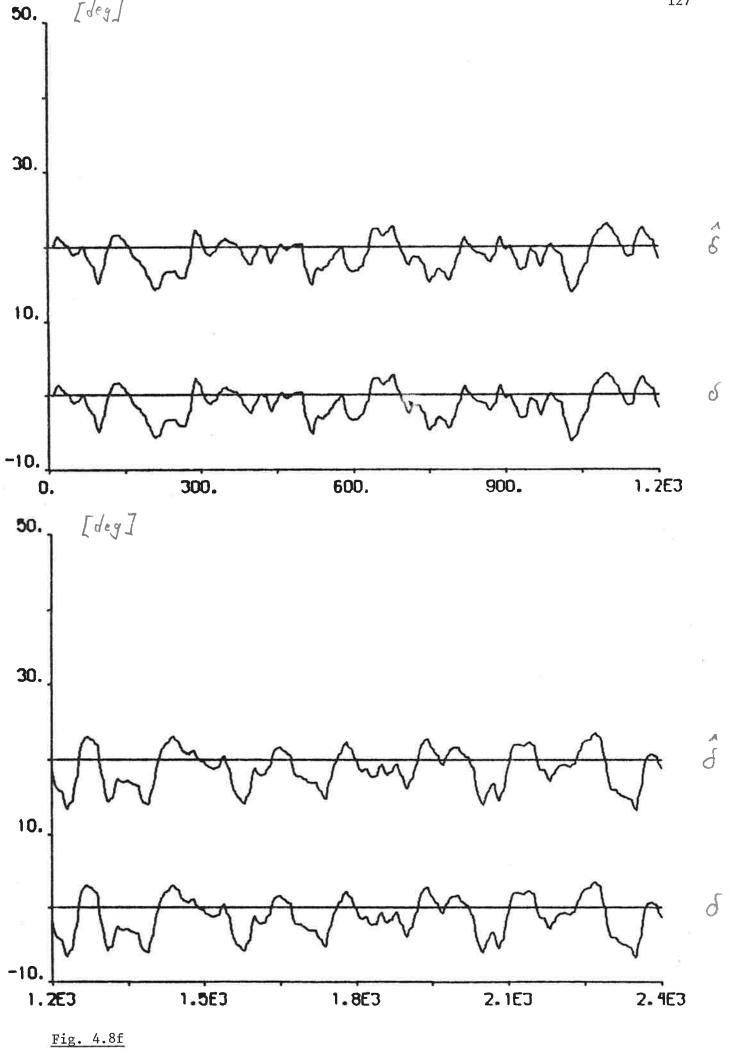
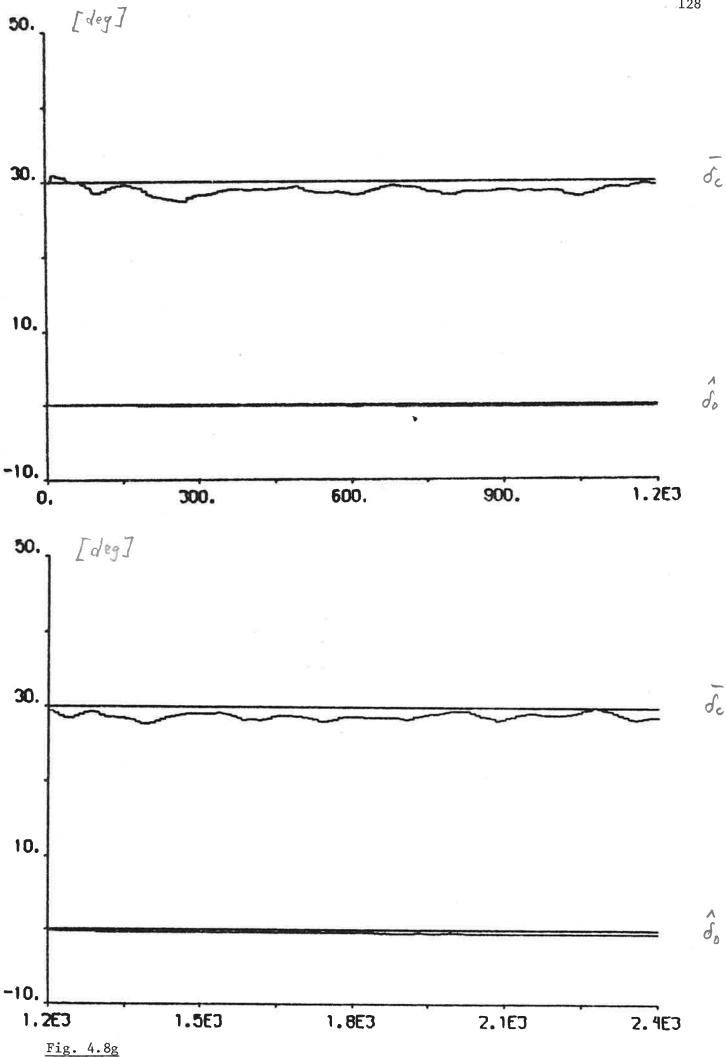


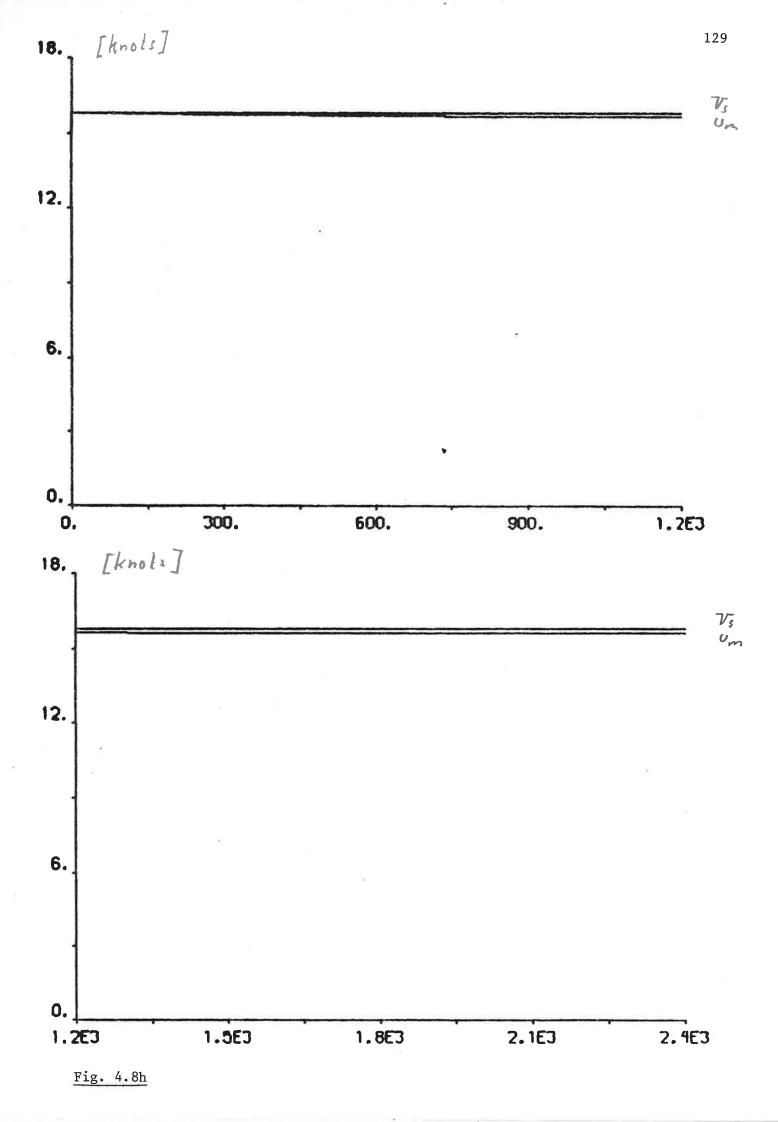
Fig. 4.8d



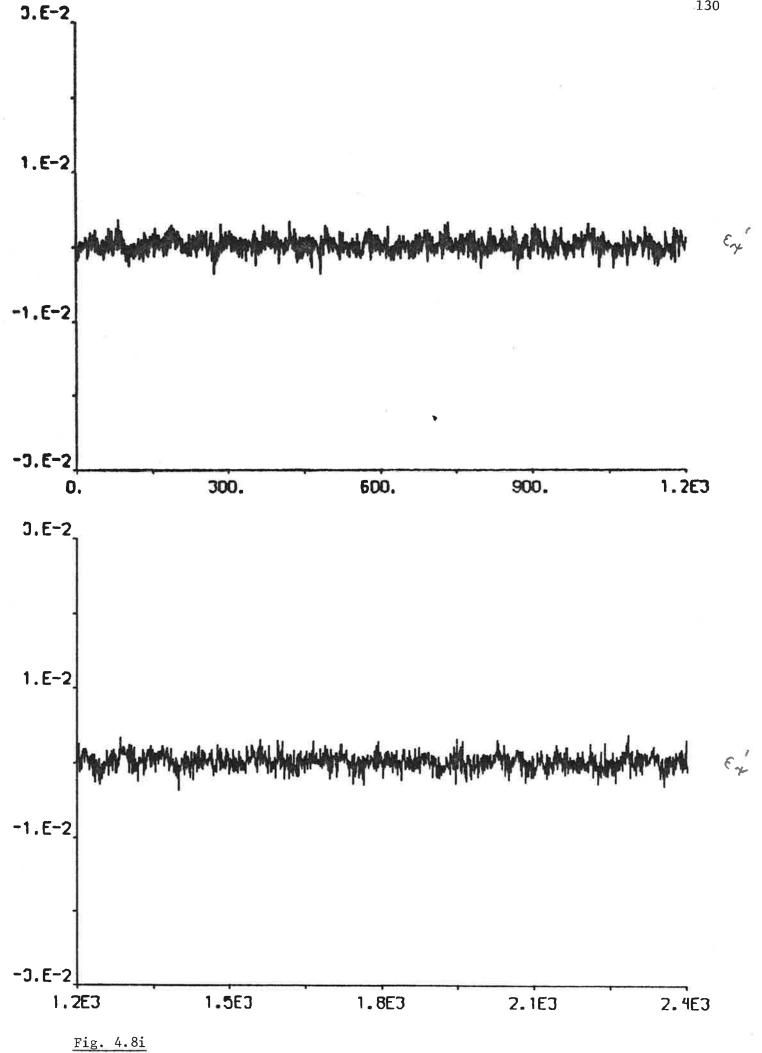


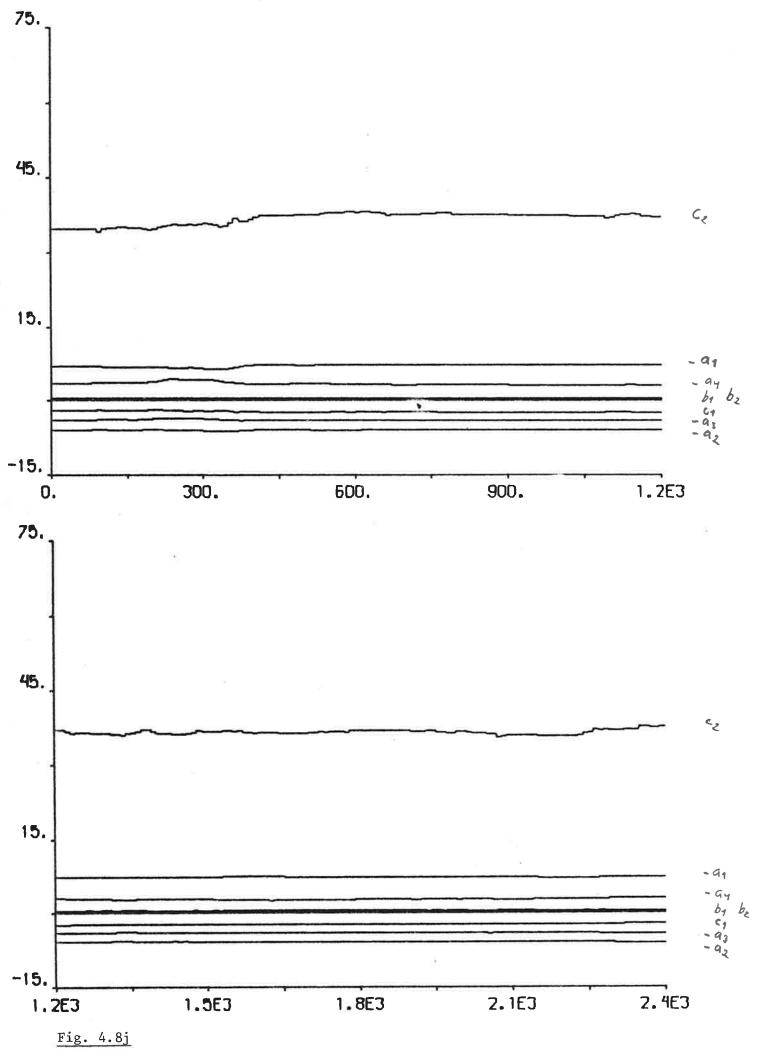














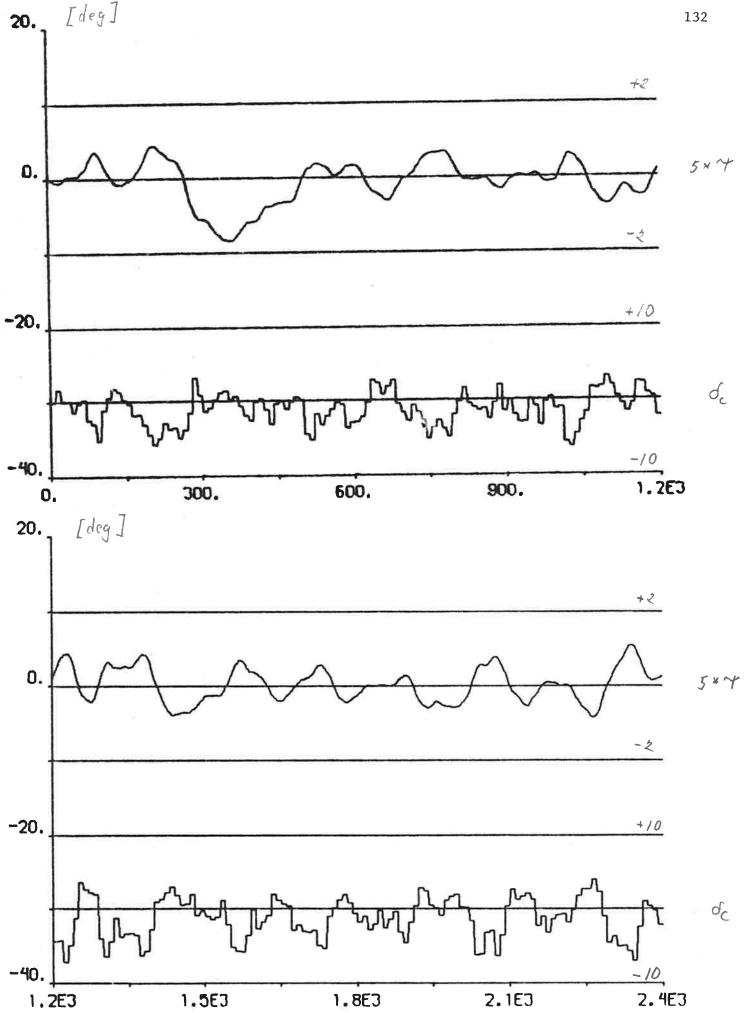
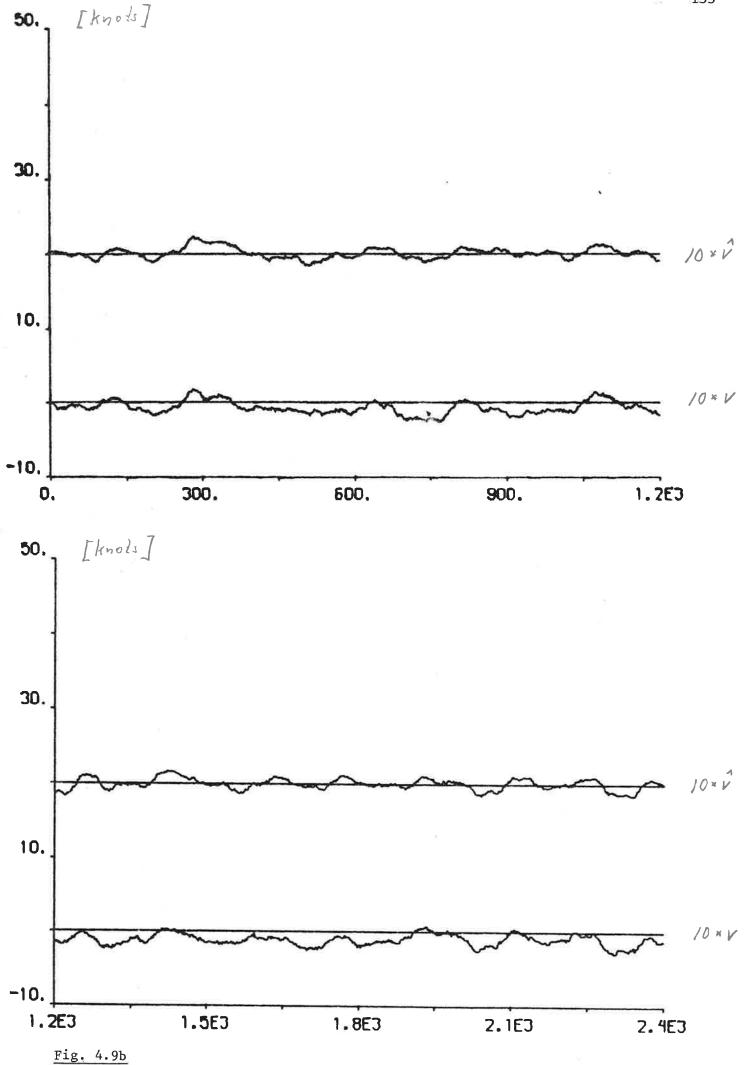
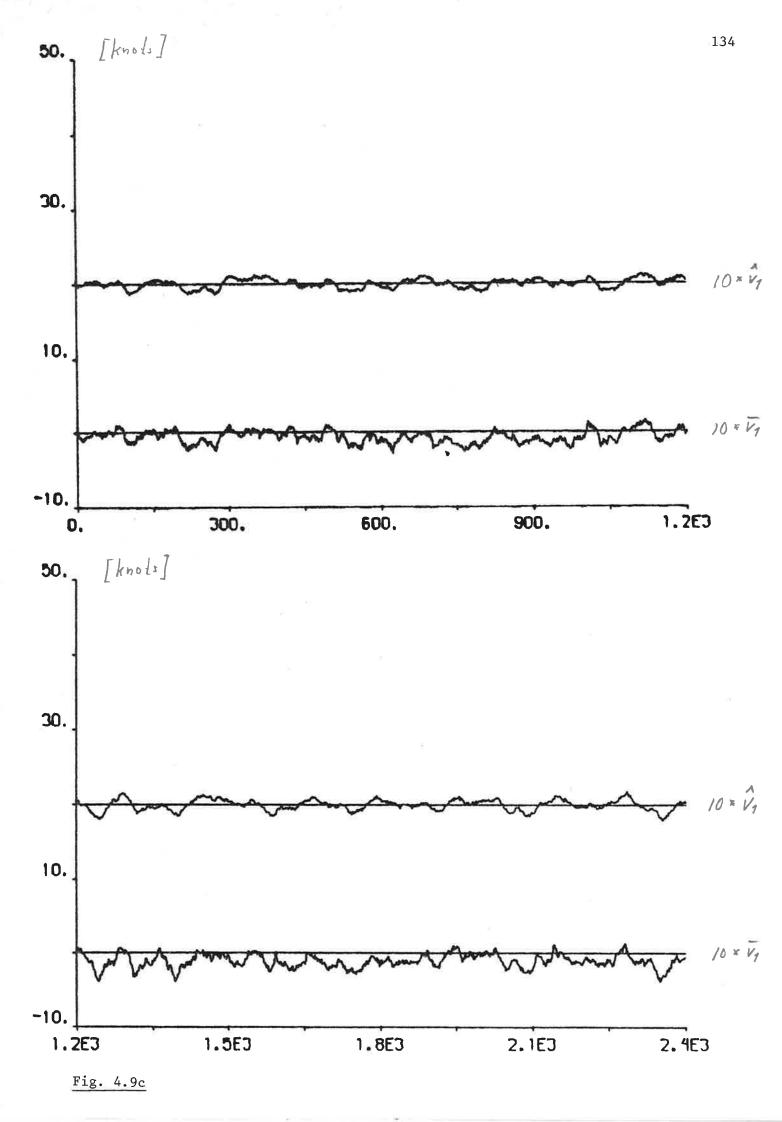


Fig. 4.9a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using estimates from the Kalman filter. The only measurement signal used by the filter is the heading angle. The correct filter gain K given by (4.4) is used.





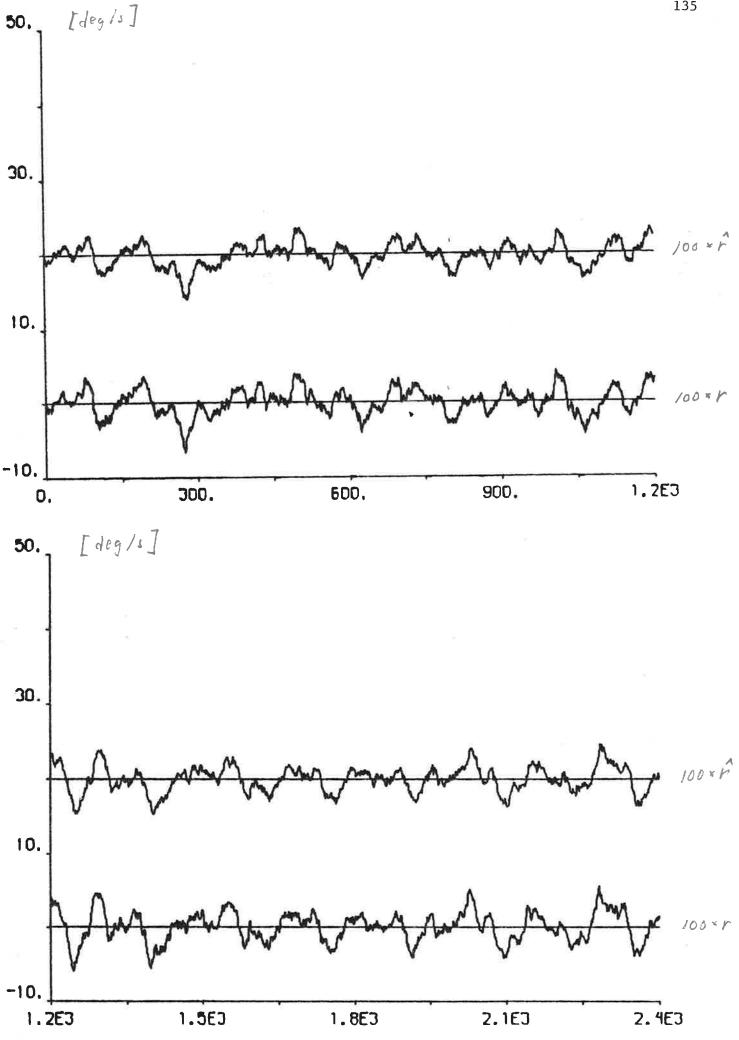


Fig. 4.9d

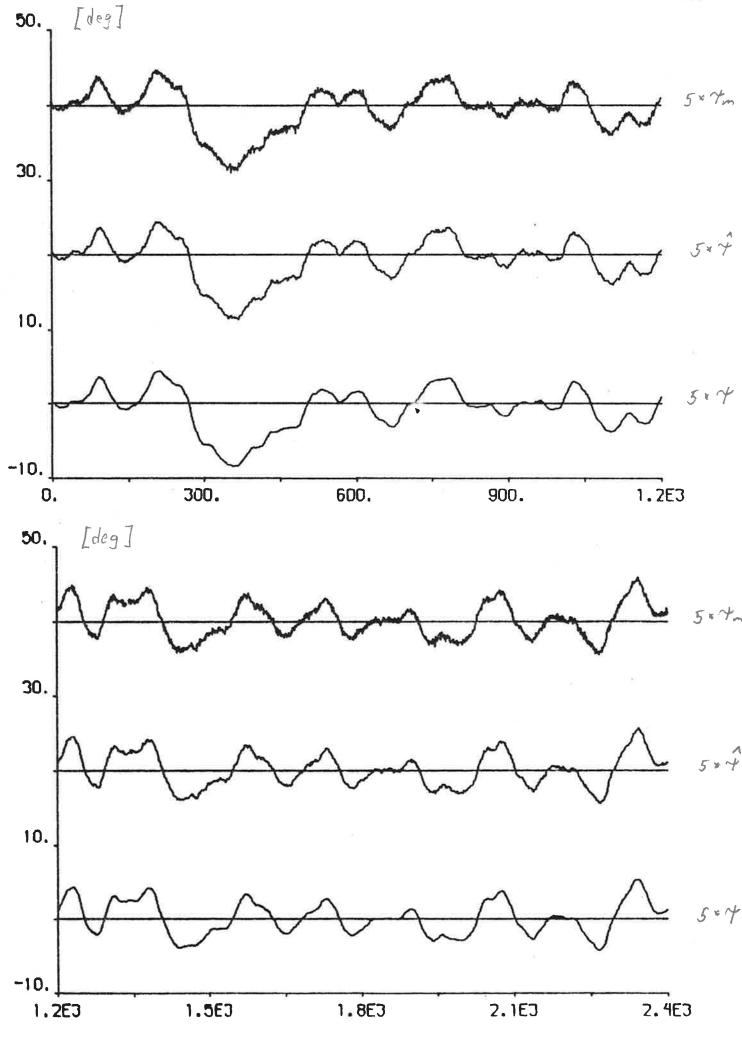
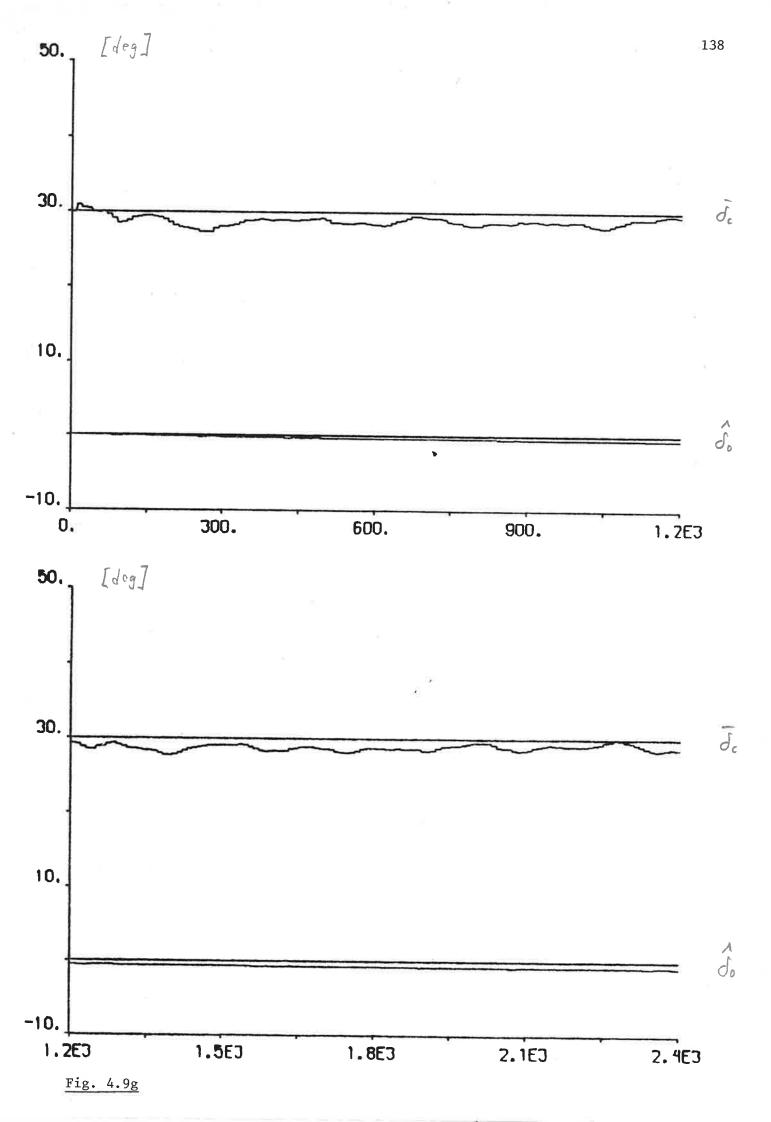
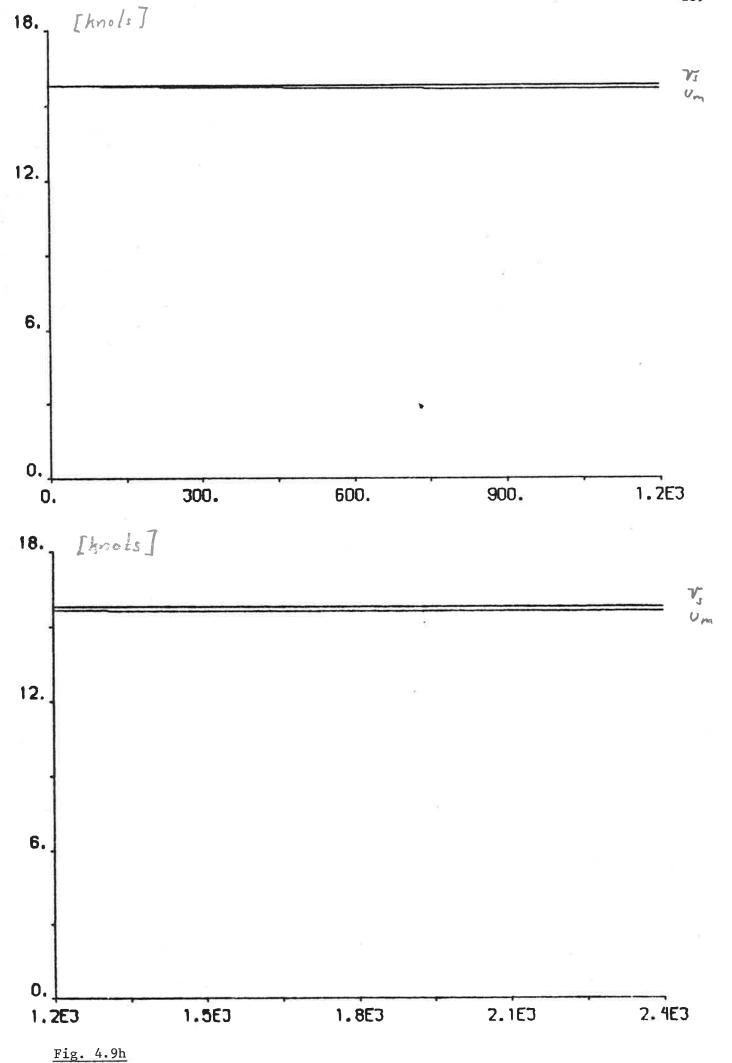
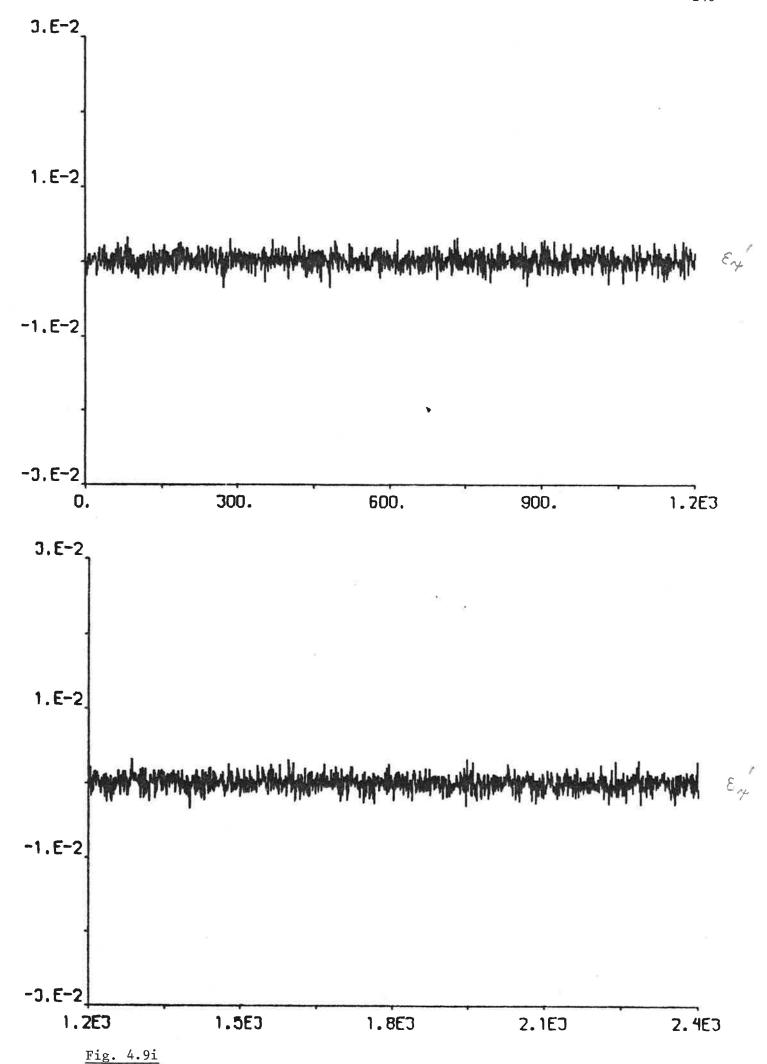


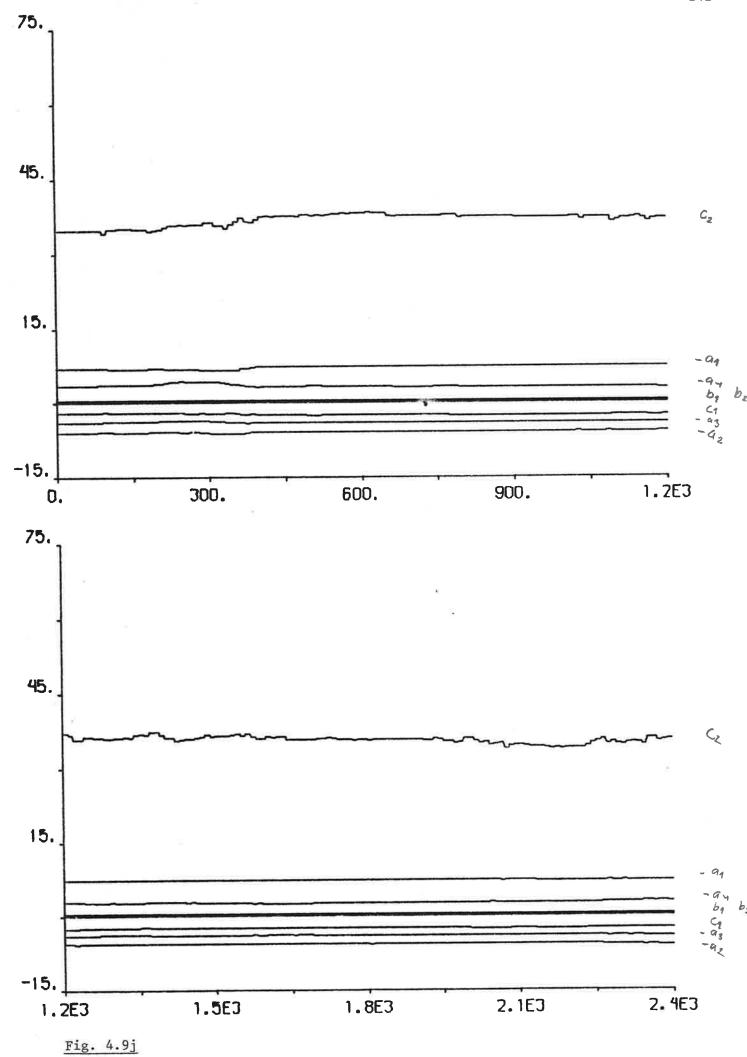
Fig. 4.9e











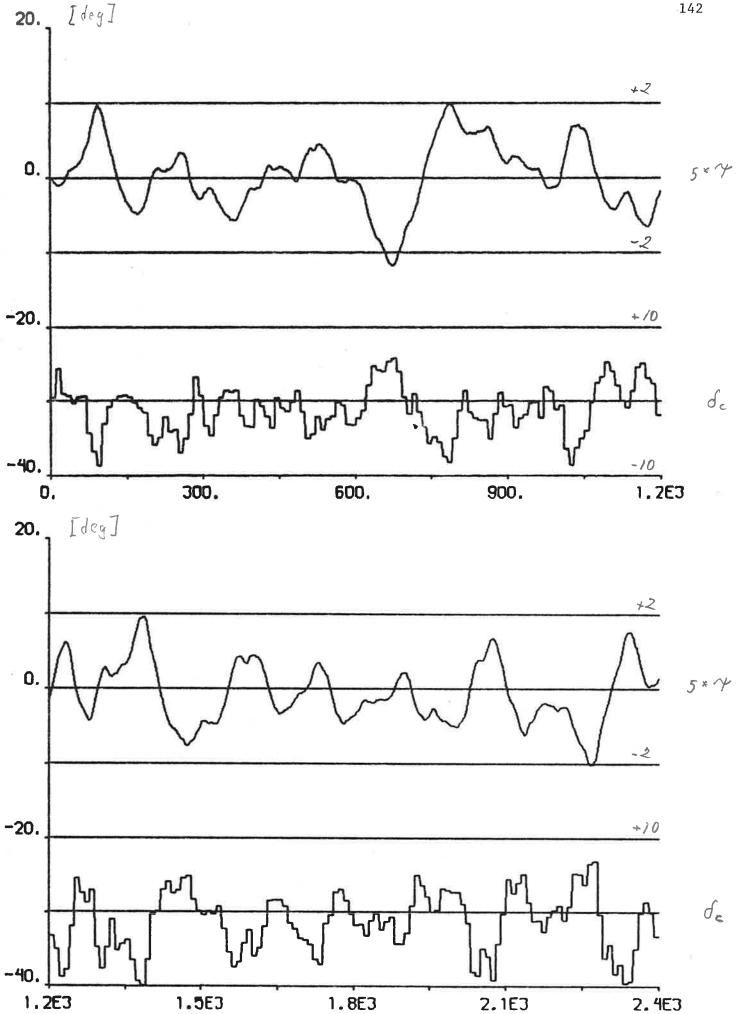
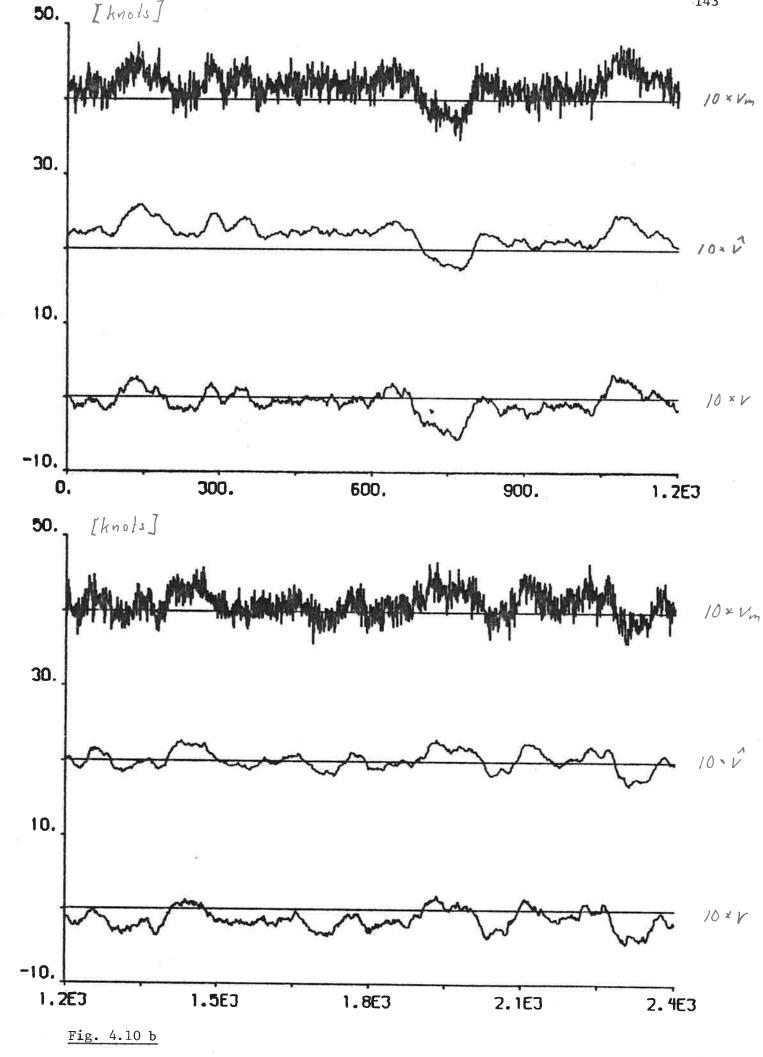


Fig. 4.10a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_{ℓ} = 10 deg, self-tuning regulator using estimates from the Kalman filter. The covariance matrix $R_{\rm w}$ is given by (4.5).





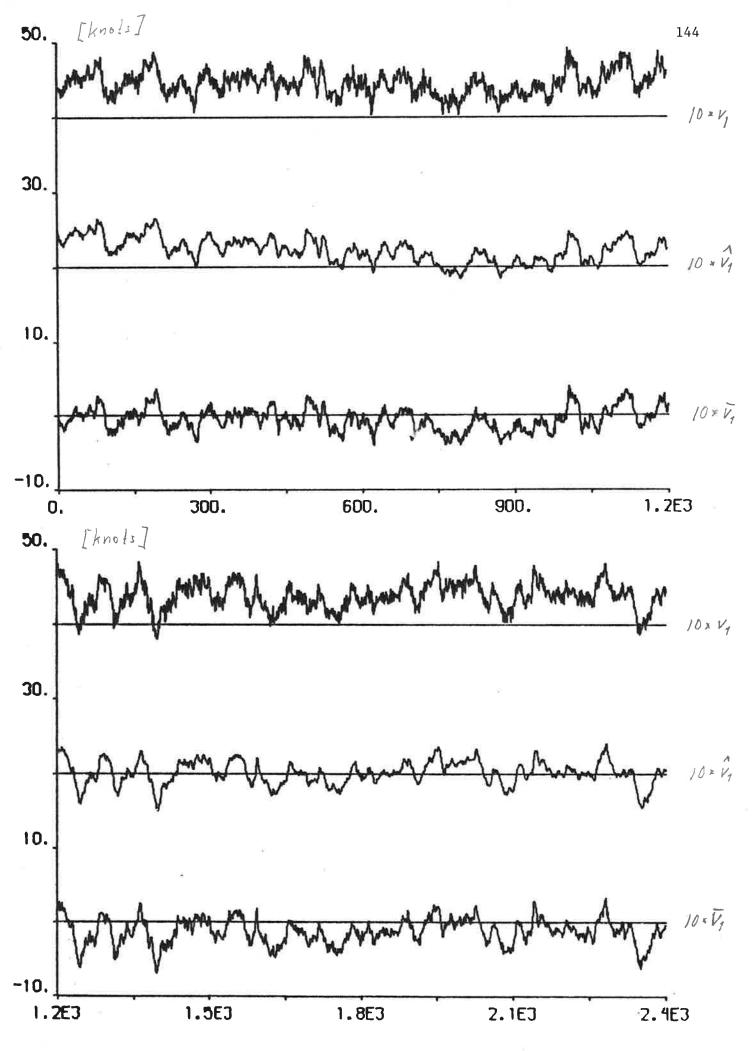
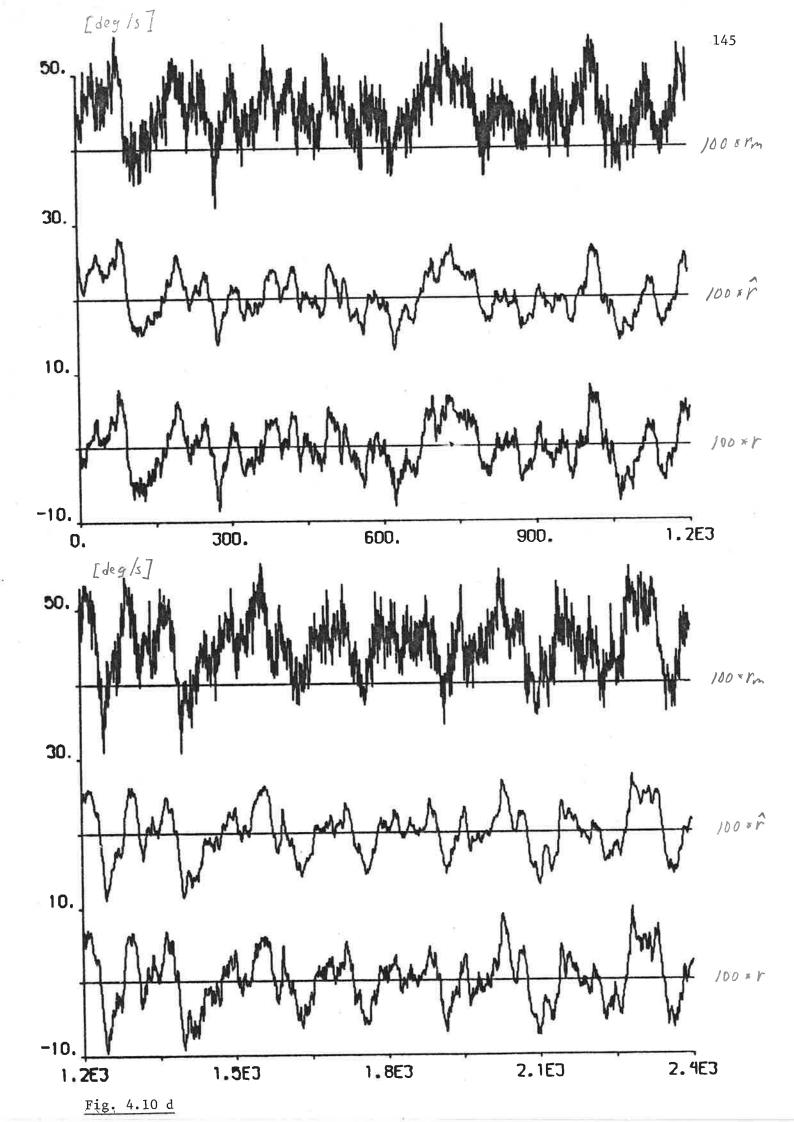


Fig. 4.10 c



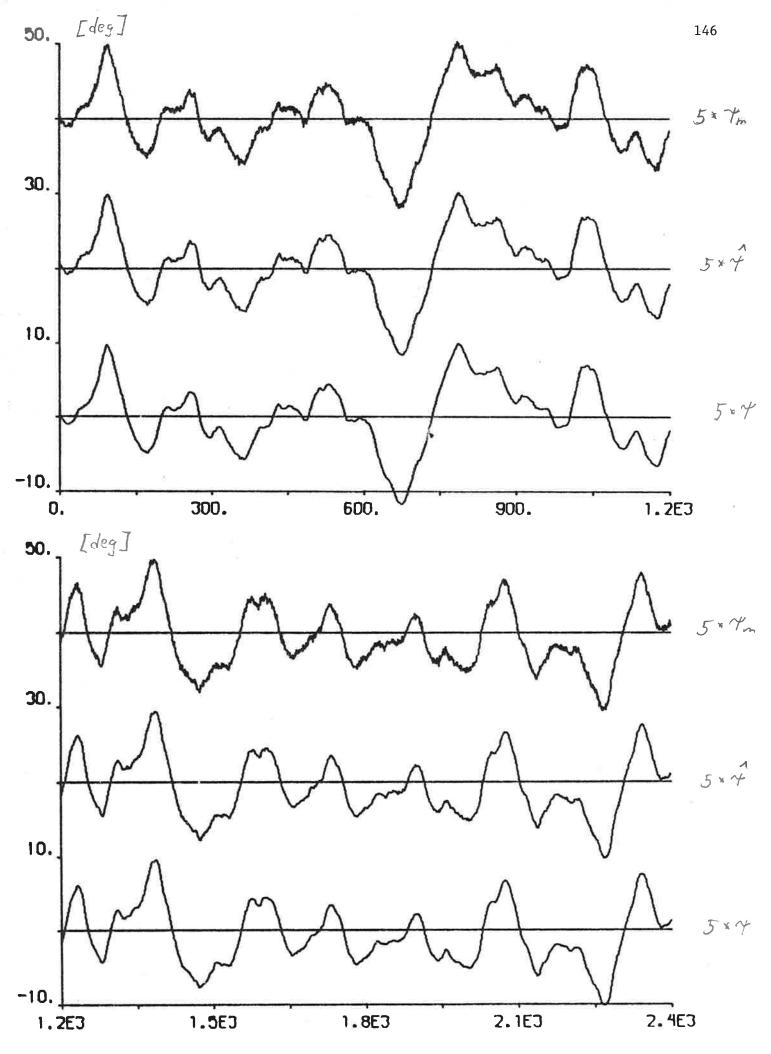
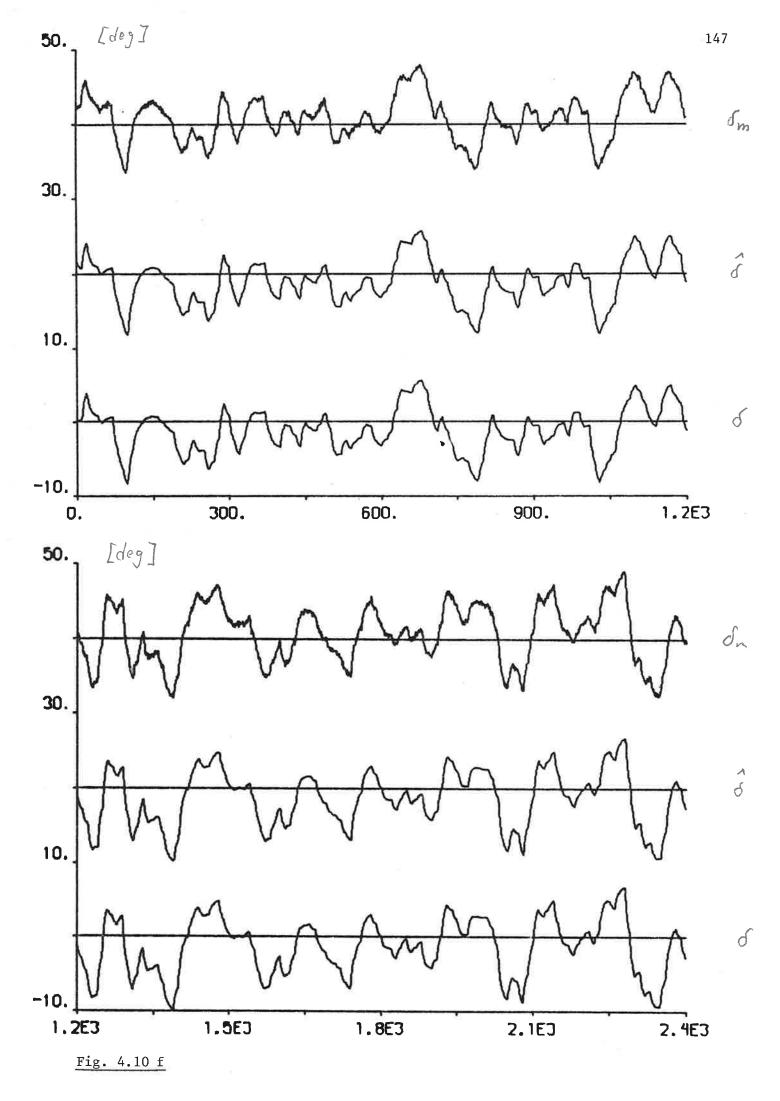
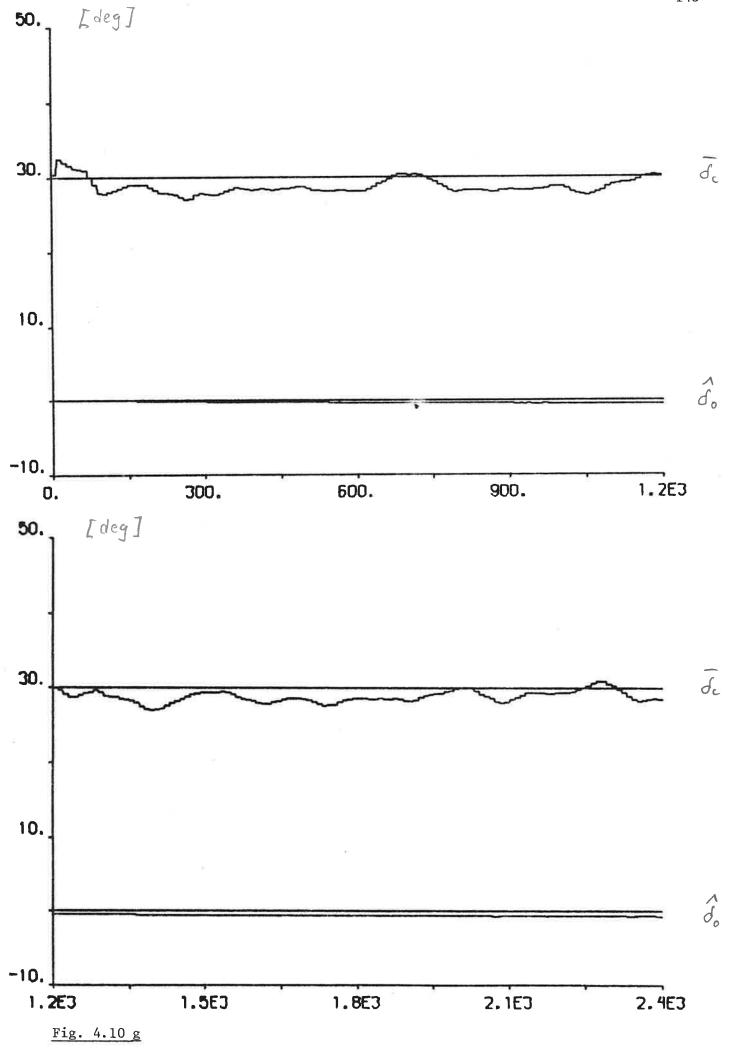
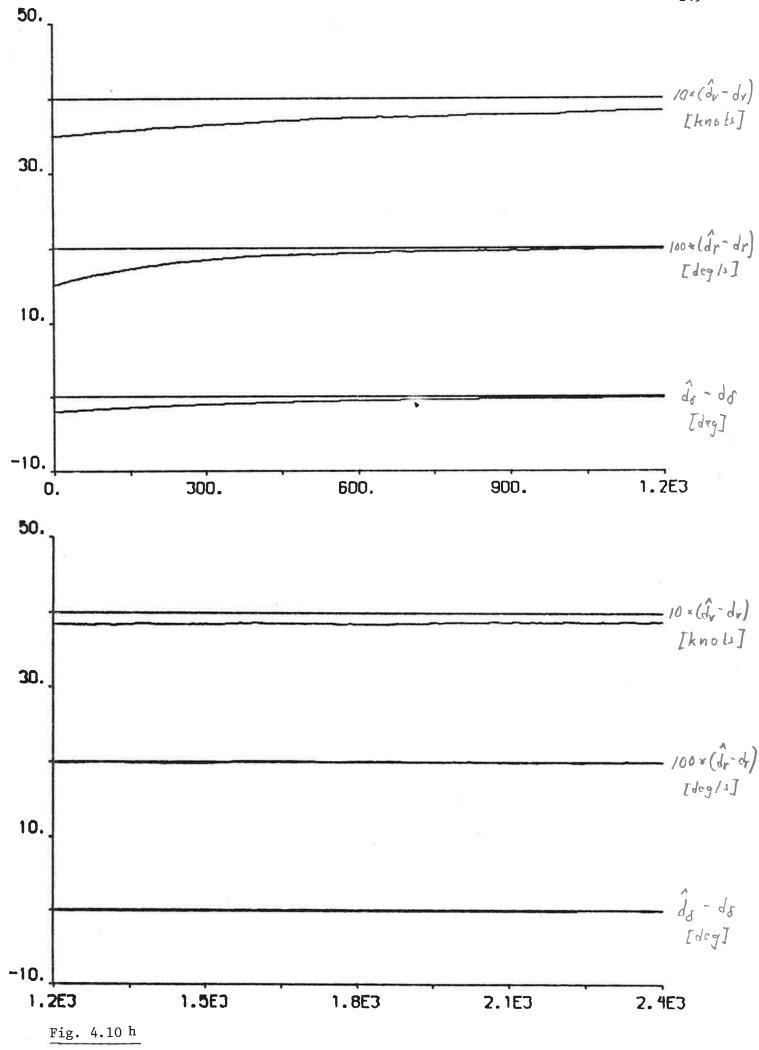


Fig. 4.10 e

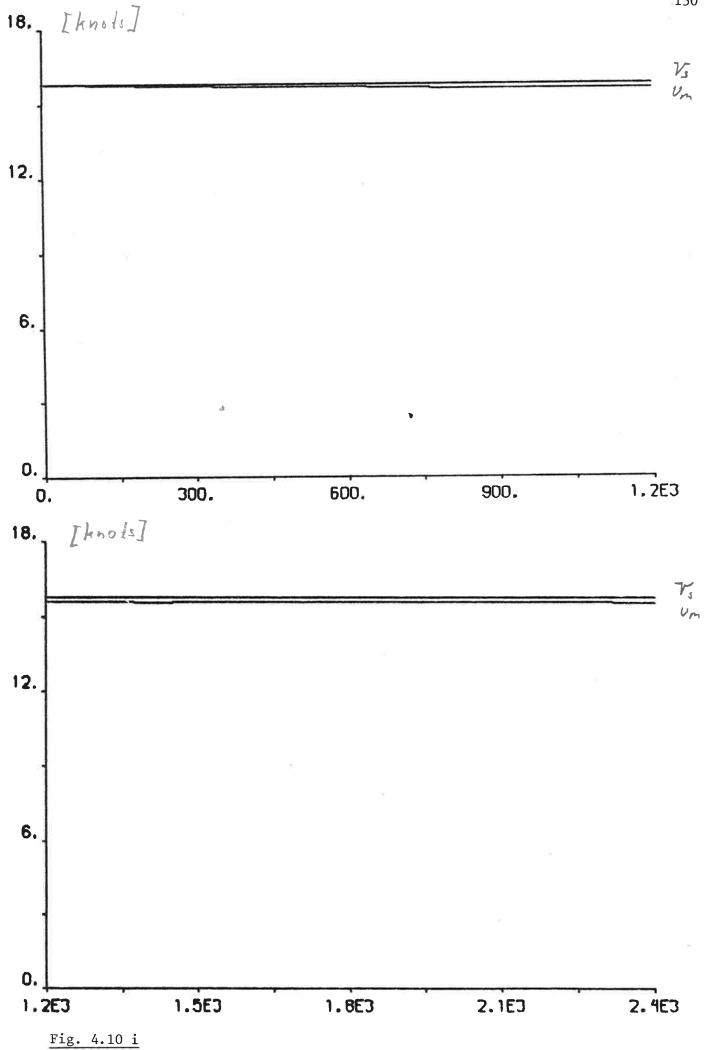












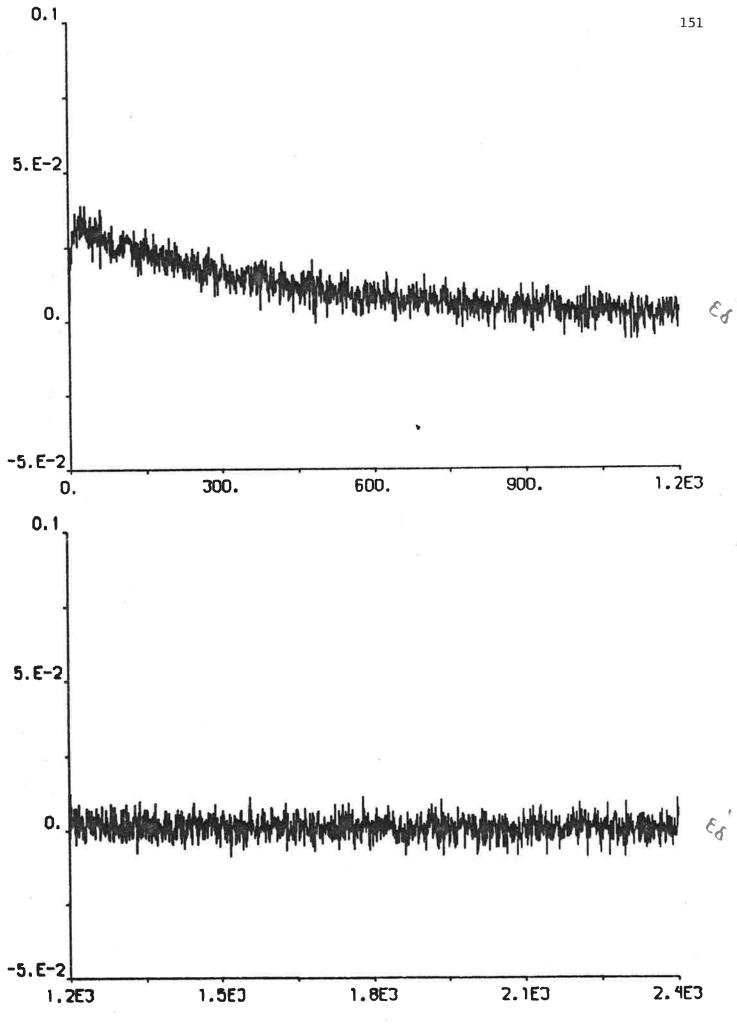


Fig. 4.10 j

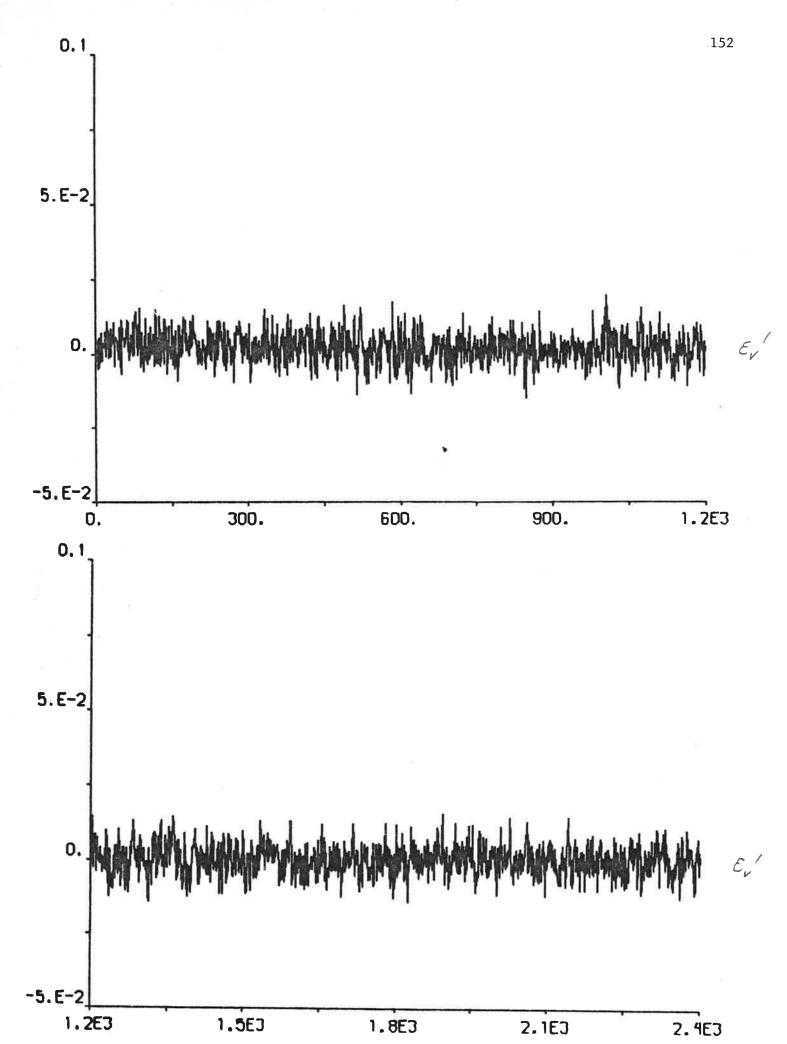
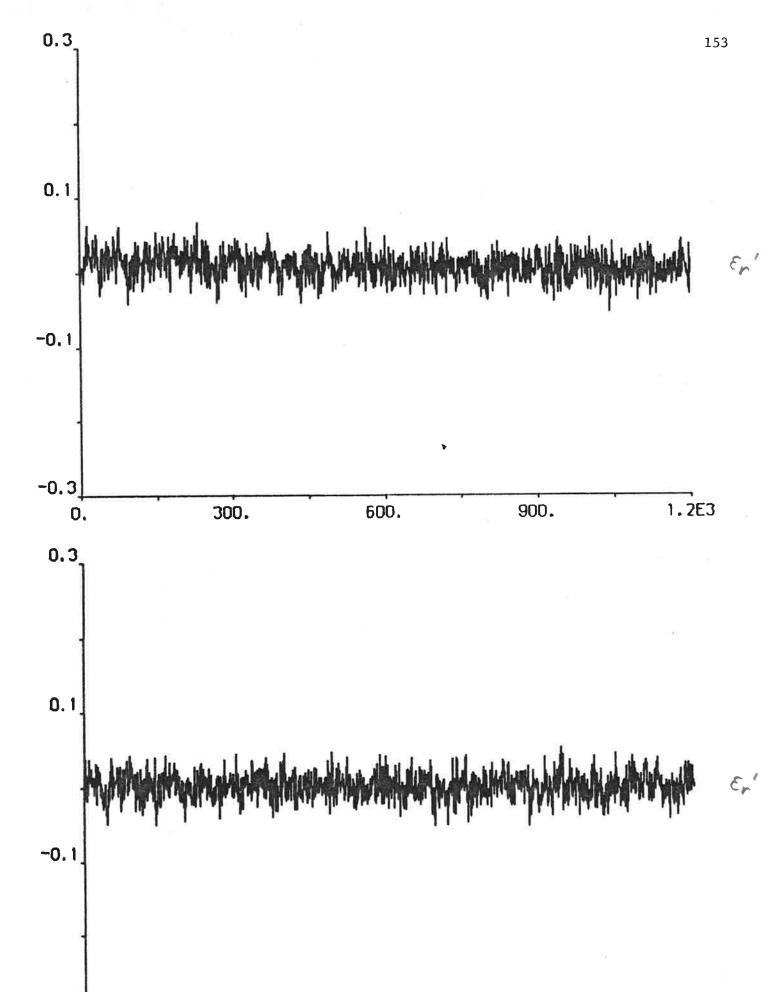


Fig. 4.10 k



1.8E3

2.1E3

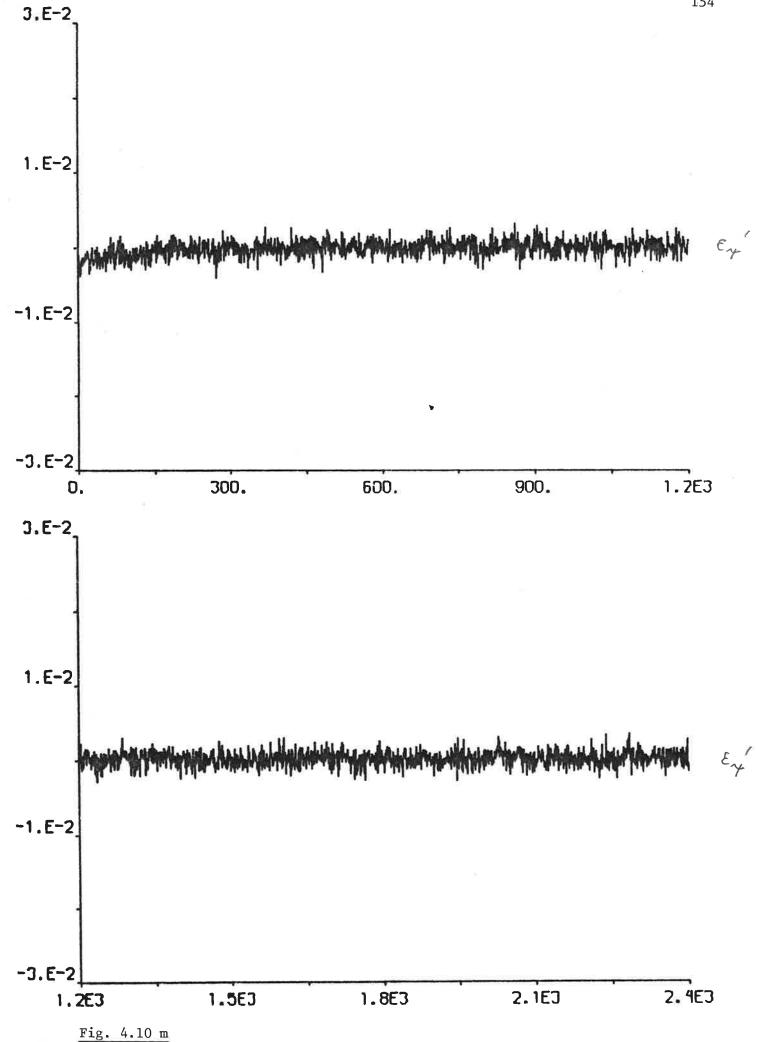
2.4E3

Fig. 4.10 ℓ

1.2E3

1.5E3





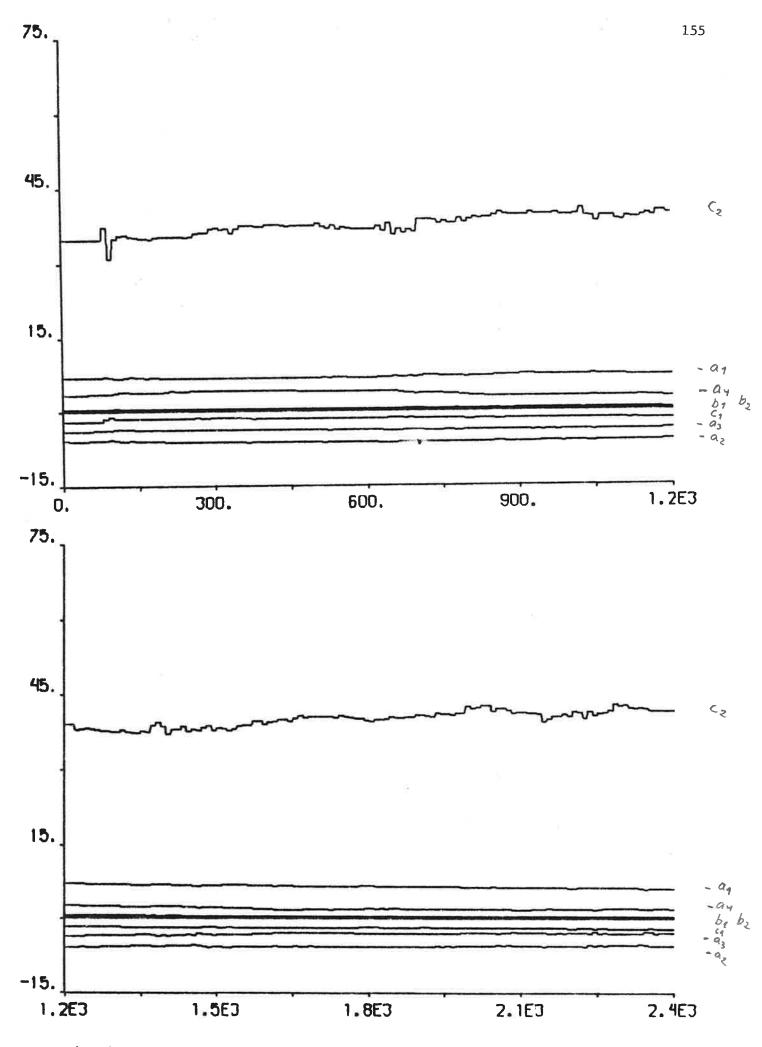


Fig. 4.10 n



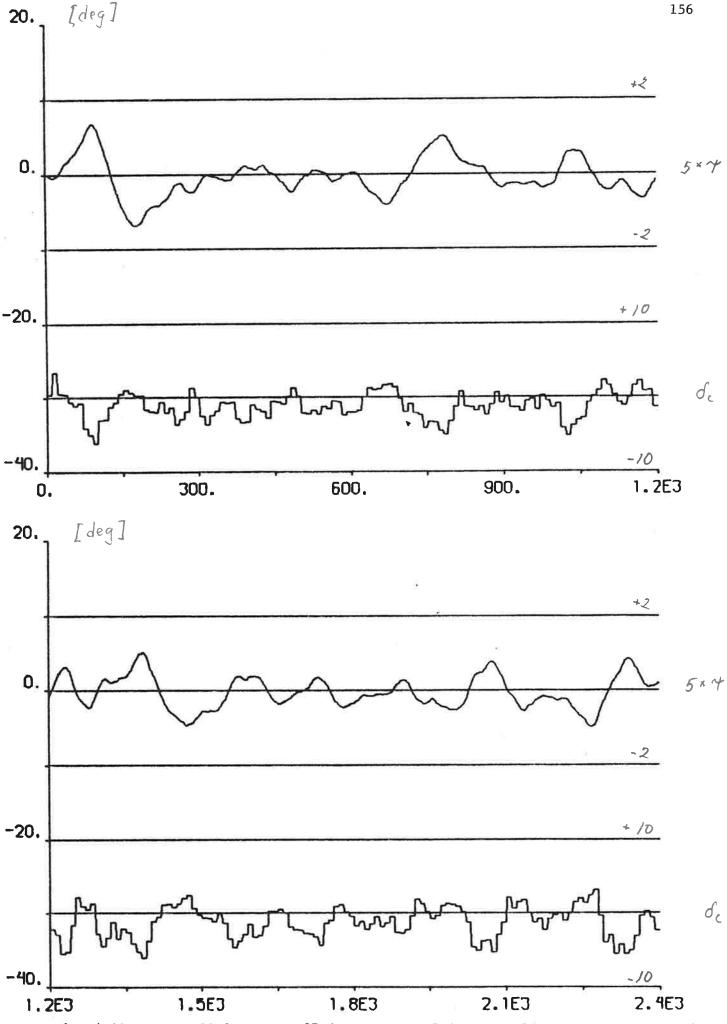
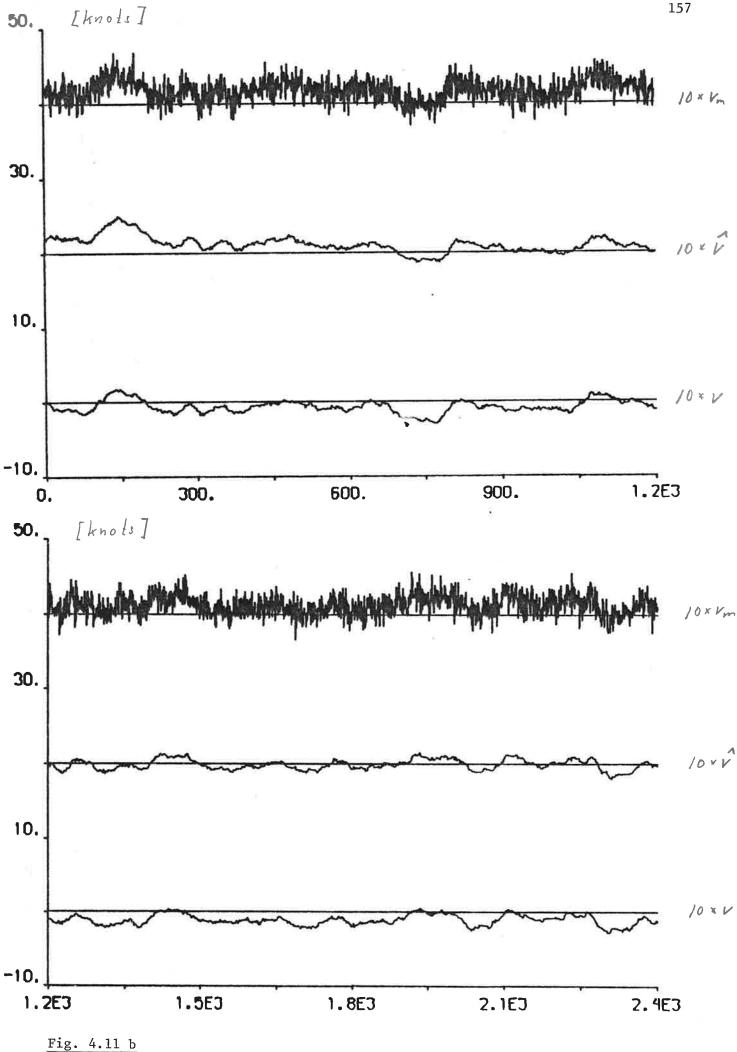
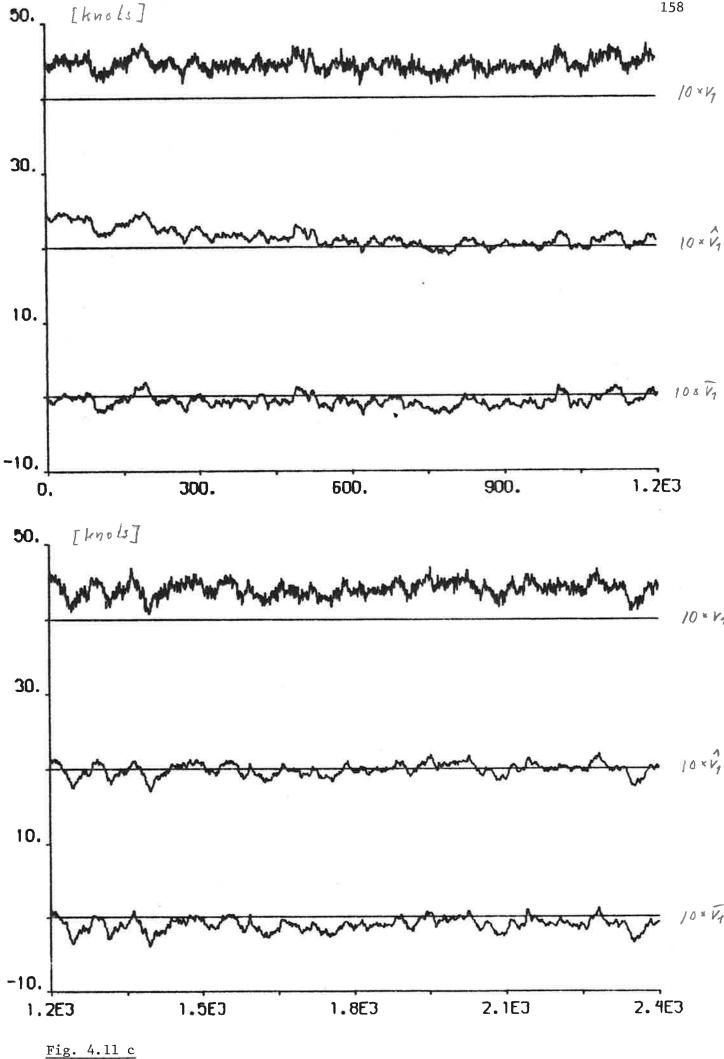


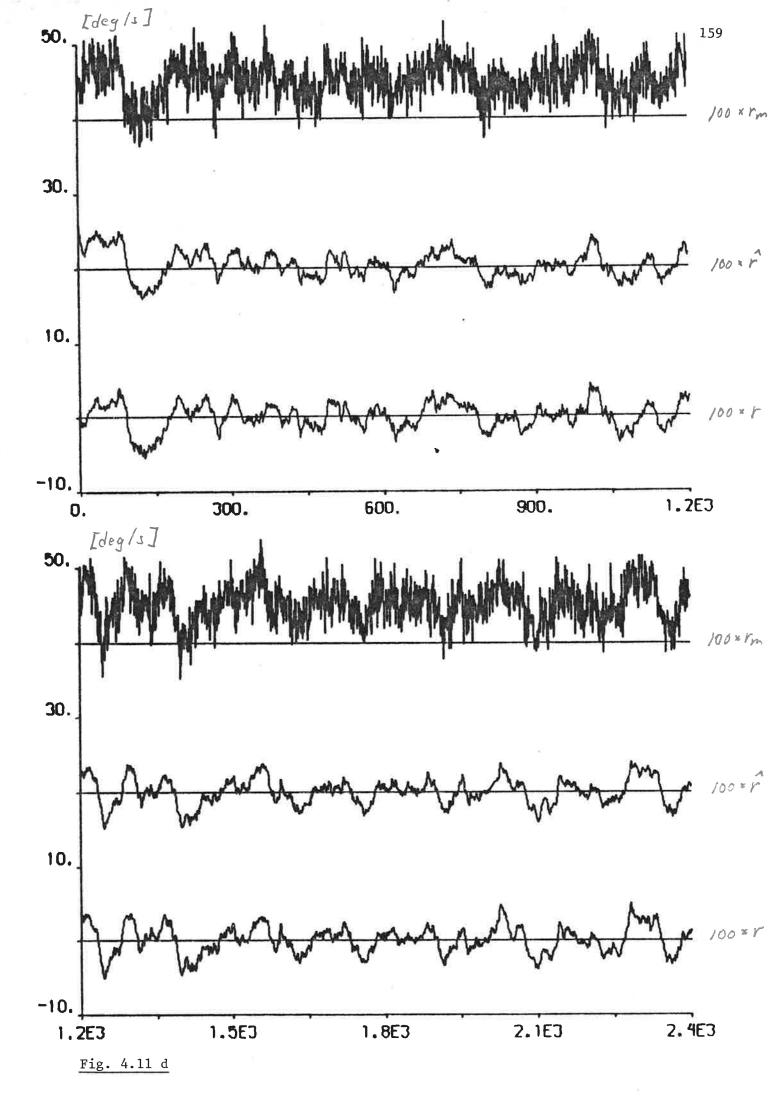
Fig. 4.11 a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using estimates from the Kalman filter. The filter gain K is given by (4.6).

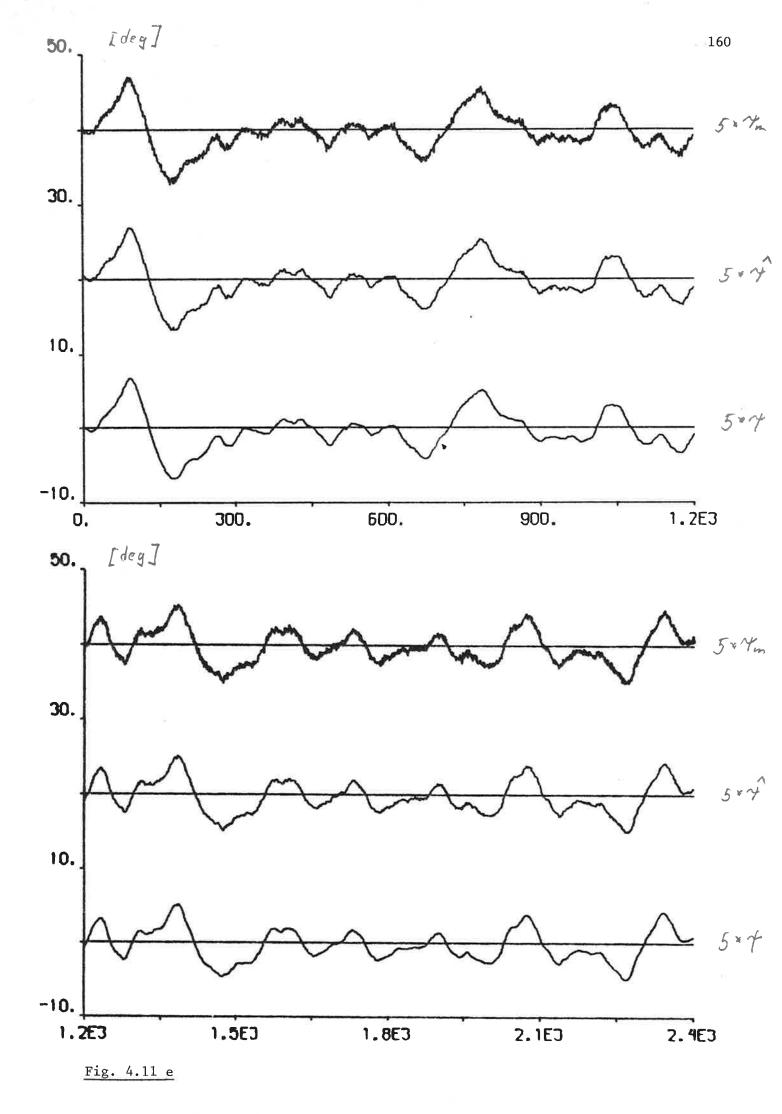


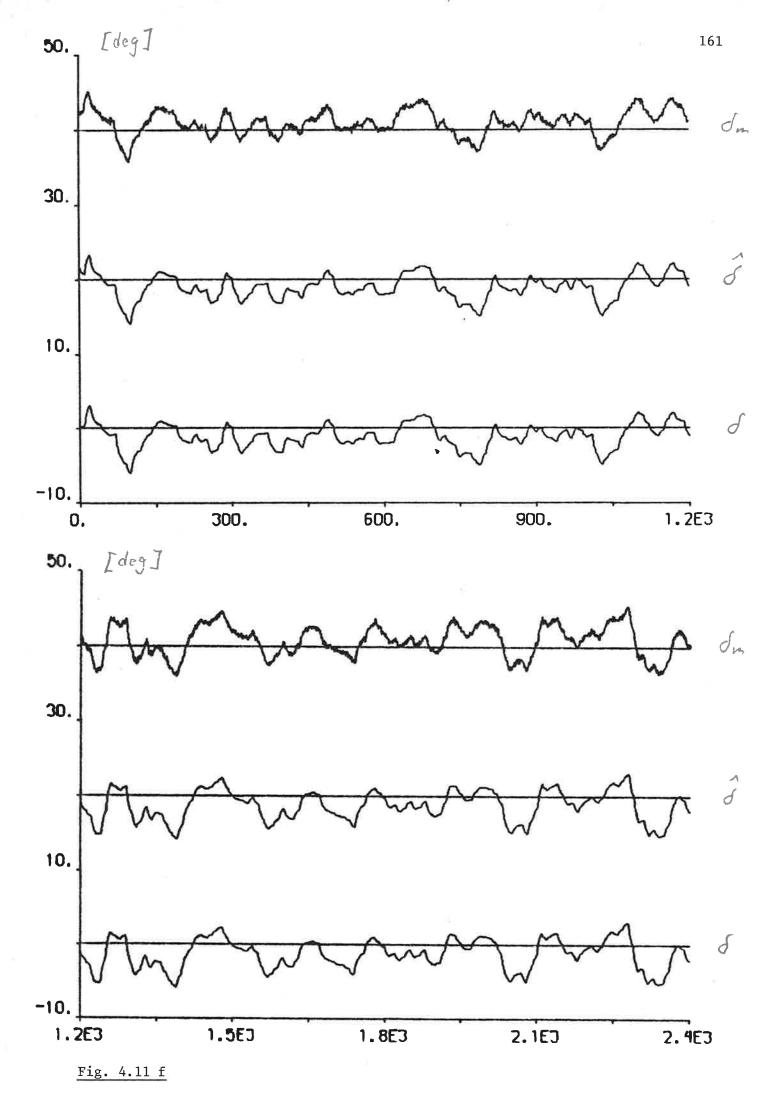




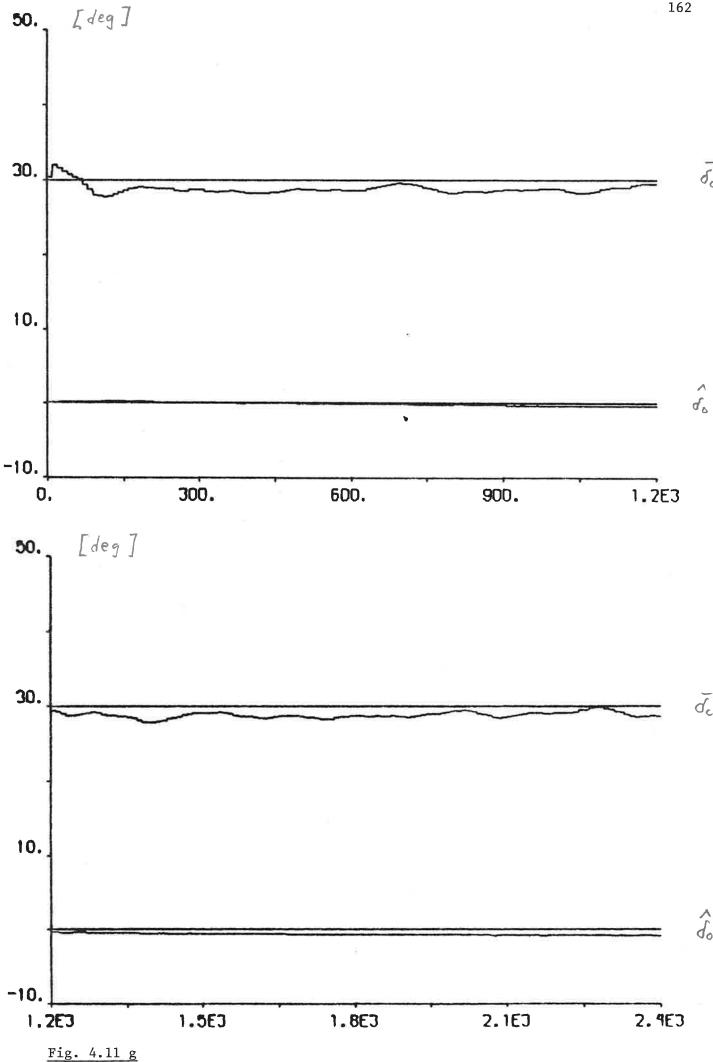


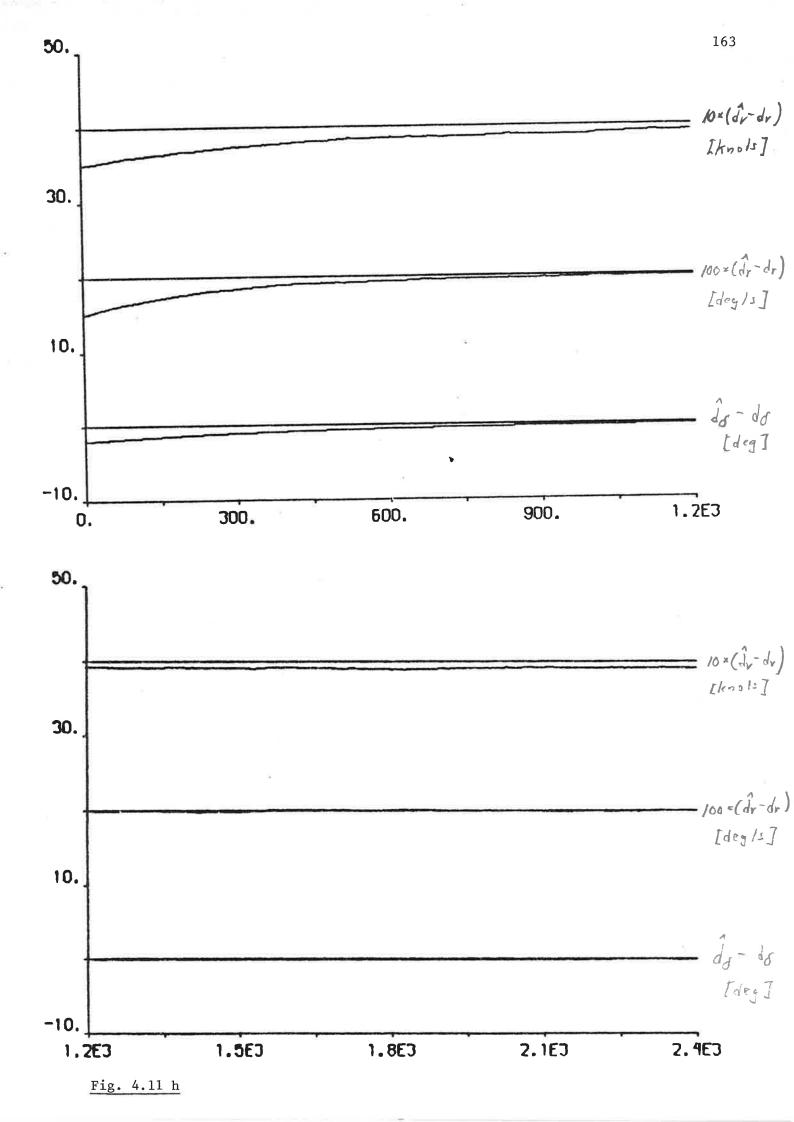


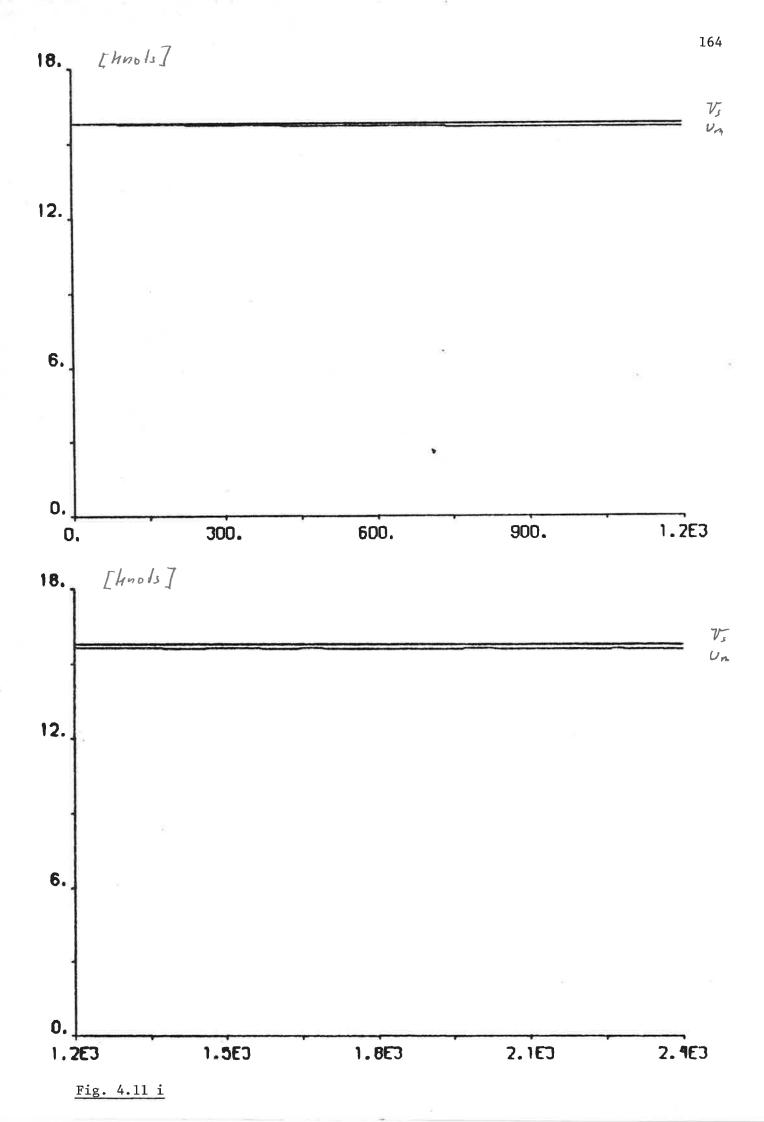


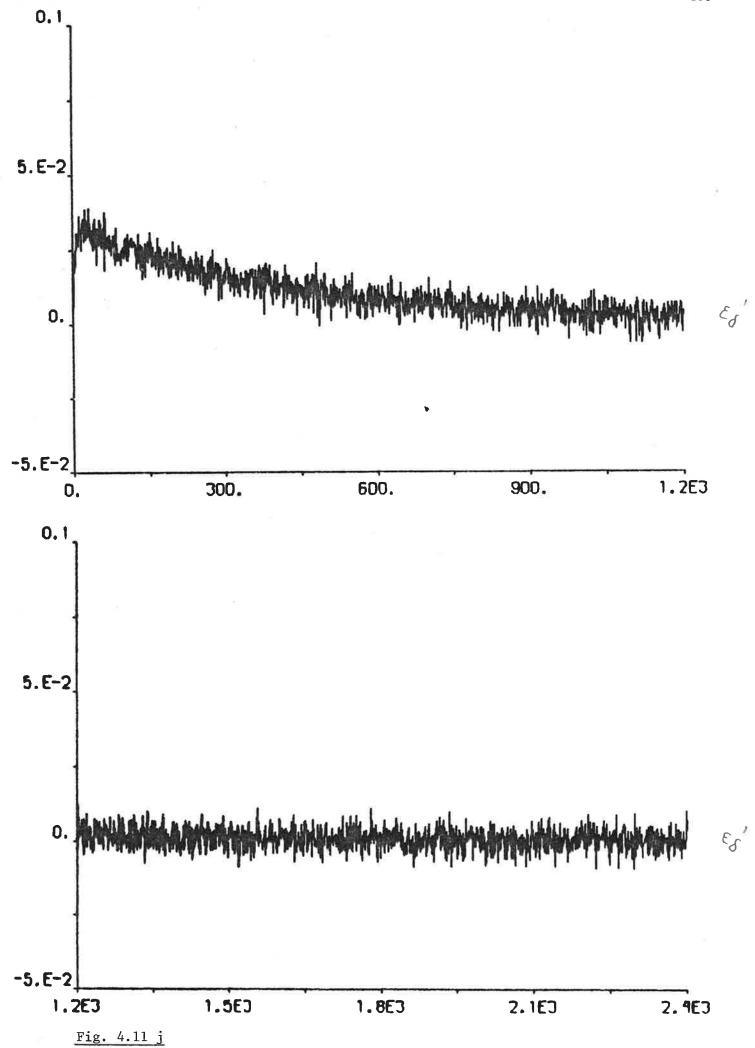


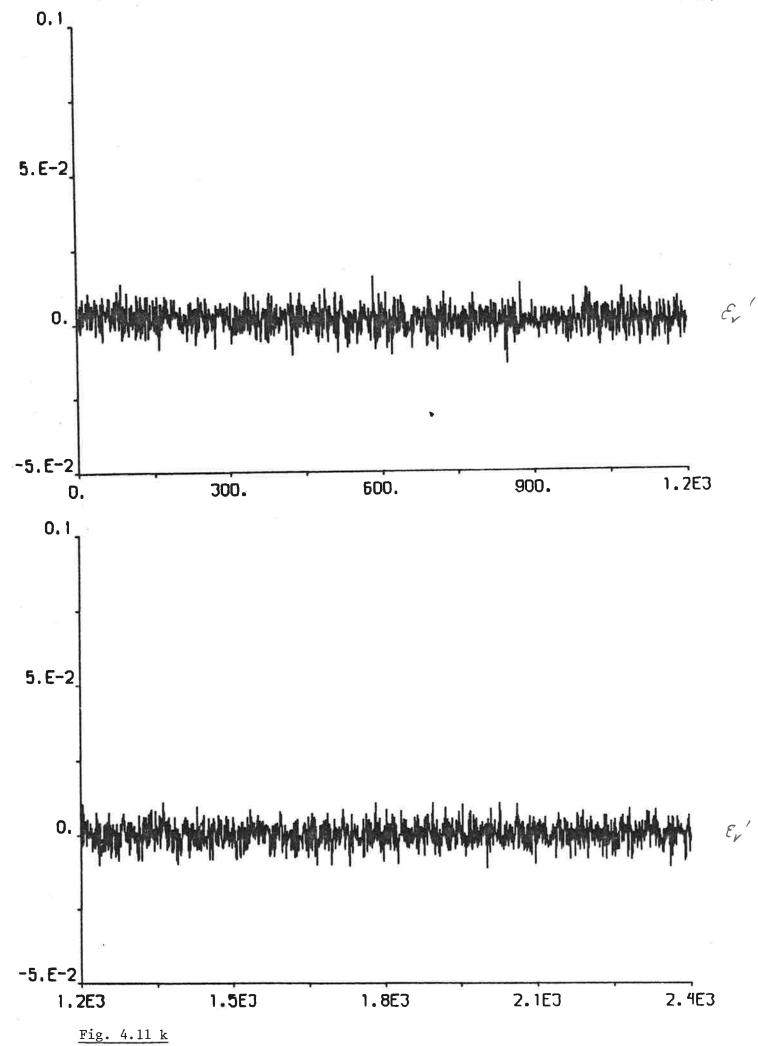




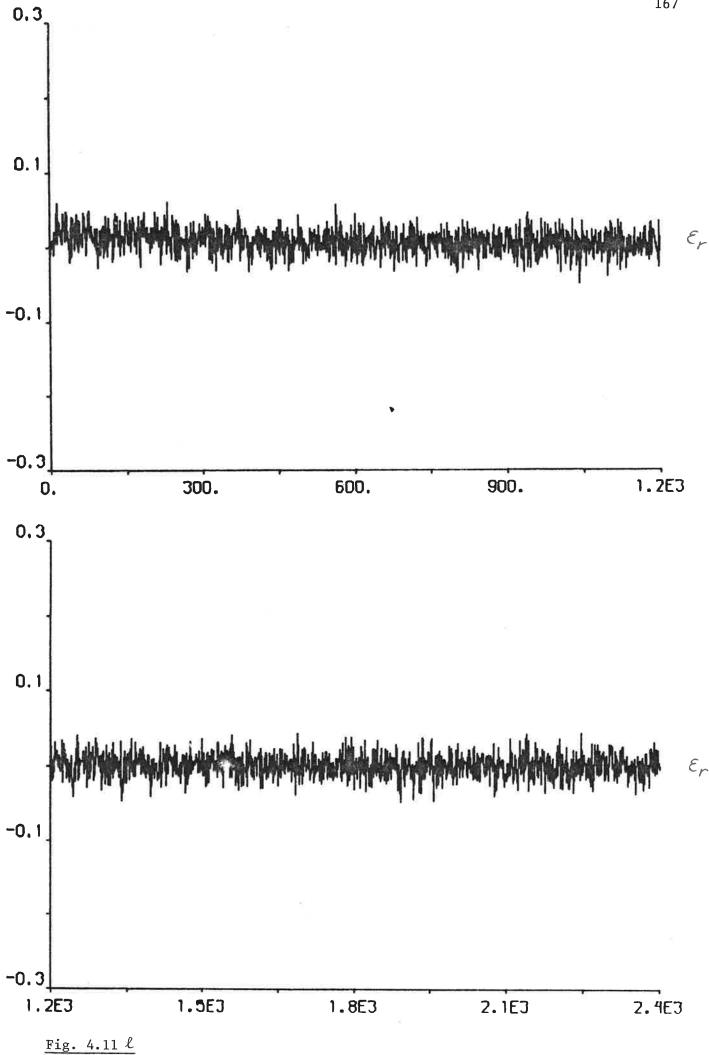


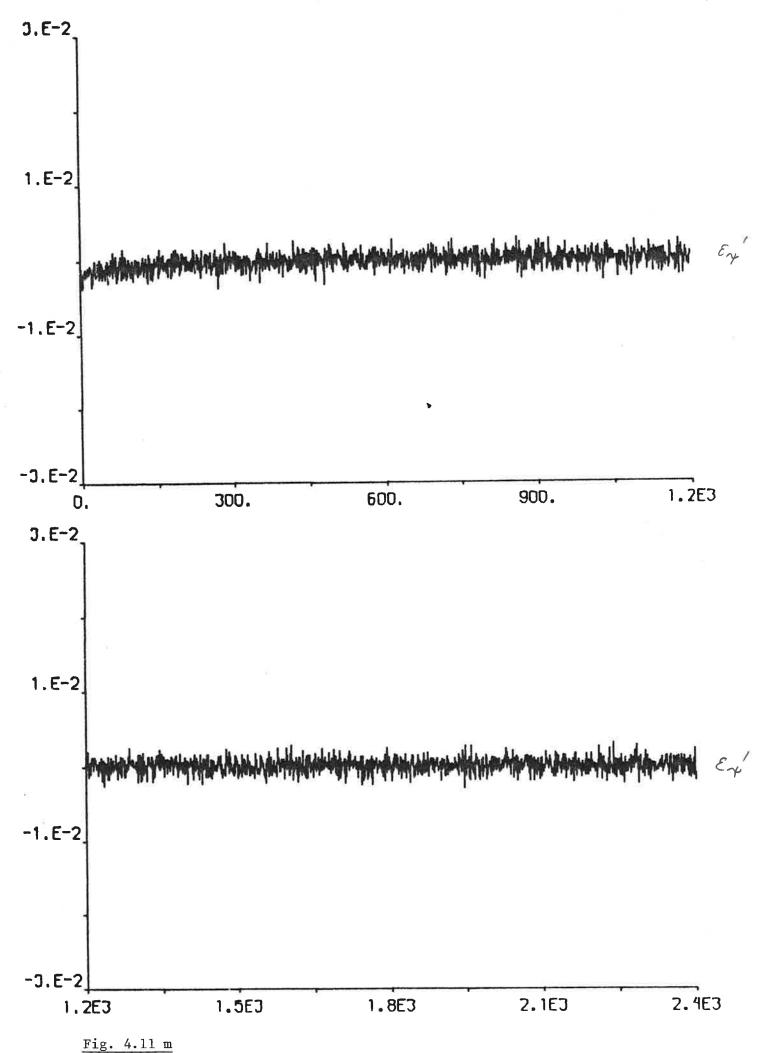


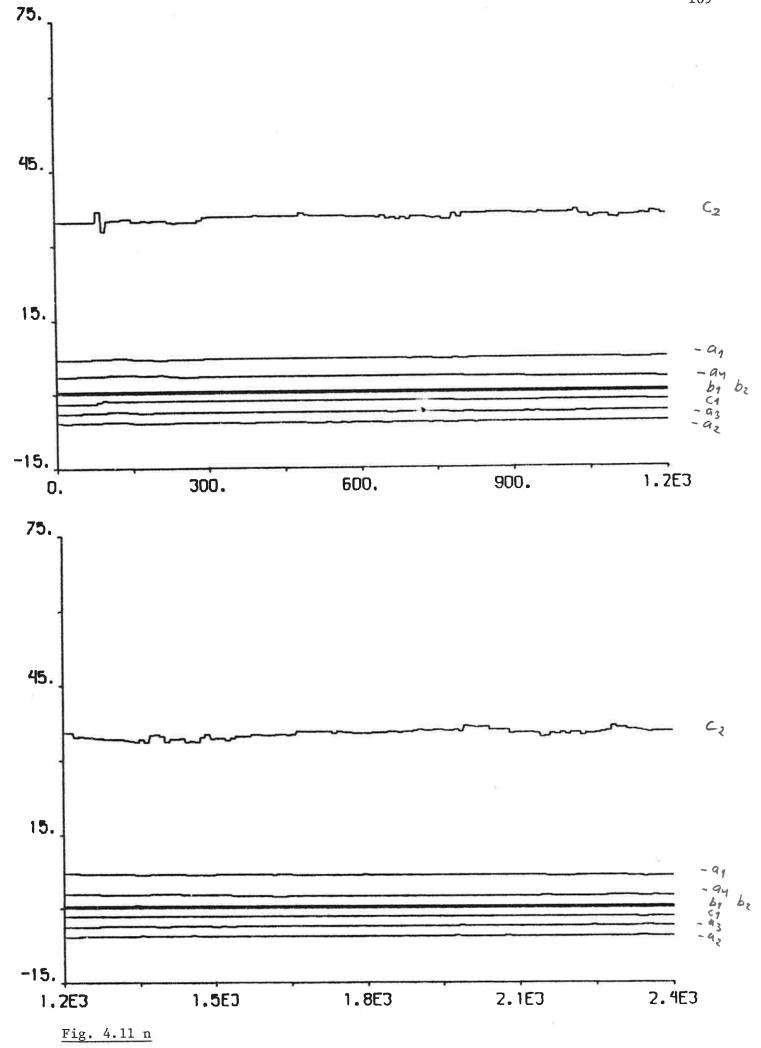












4.2 Straight Course Keeping

Simulations of straight course keeping with the self-tuning regulator using non-filtered measurements are shown in Figs 4.12 - 4.25, where the mean draught T is equal to 22.3 m and k, T_s and q_2^* are varied. Two different initial speeds, u_0 = 15.8 knots and u_0 = 4 knots, are used. The initial parameter values and the initial covariance matrix are

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -10.92 \\ 10.38 \\ 4.717 \\ -4.871 \\ 0.2482 \end{bmatrix} \qquad P = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 1 \end{bmatrix}$$

$$(4.7)$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -13.88 \\ 12.85 \\ 7.866 \\ -7.302 \\ 0.6101 \\ -0.01241 \end{bmatrix} \qquad P = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 1 \\ 1 \end{bmatrix}$$

$$(4.8)$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -8.105 \\ 8.189 \\ 2.845 \\ -3.605 \\ 0.4698 \\ 0.2522 \end{bmatrix} \qquad P = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

$$0.1$$

$$0.1$$

or

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -9.47 \\ 9.67 \\ 3.55 \\ -4.42 \\ 0.150 \\ 0.266 \end{bmatrix} \qquad P = \begin{bmatrix} 1 \\ 10 \\ 10 \\ 10 \\ 0.1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 \\ 10 \\ 10 \\ 0.1 \end{bmatrix}$$

$$0.1$$

$$0.1$$

Notice that $c_1 = c_2 = 0$. The speed V_s (cf (3.1)) is computed according to

$$V_{S} = V_{C} \tag{4.11}$$

The simulations are summarized in Table 4.2. When $u_0=15.8$ knots, it can be concluded that the lowest value of V_ℓ (0.75) is obtained for k=7, $T_s=10$ s and $q_2^*=0$. These values are also chosen as standard values. Notice, however, that k=6, $T_s=10$ s, $q_2^*=0$ ($V_\ell=0.76$) and k=5, $T_s=15$ s, $q_2^*=0$ ($V_\ell=0.79$) are good alternatives. It can also be concluded that the performance of the self-tuning regulator is not improved when $q_2^*\neq 0$. When $u_0=4$ knots, the best choice is k=8, $T_s=10$ s, $q_2^*=0$. The only difference compared to the standard values is that k=8 instead of k=7.

The performance of the self-tuning regulator using estimates from the Kalman filter, when the speed is decreased and increased, is shown in Figs 4.26 and 4.27, resp. A summary is given in Table 4.3. The standard parameter values are used, but the speed $\rm V_S$ (cf (3.1)) is computed according to

$$V_{s} = u_{m} / CMK$$
 (4.12)

Notice that the parameters of the self-tuning regulator are changed very little and that the performance of the regulator is very good in both cases. This may be an indication that the speed scaling introduced in the self-tuning regulator is acceptable.

| | | | - | | | | | | | | | | | |
|-------------------------------|----------------------|---------|--------|----------------------|--------|--------|--------|--------|--------|--------|----------------------|---------|---------|-----------------------|
| Remarks | Initial values (4.7) | | | Initial values (4.8) | | | | | | | Initial values (4.9) | | | Initial values (4.10) |
| Rema | Ini | .! = | = | Ini | = | = | = | =: | = | = | Ini | = | = | Ini |
| Fig. | 4.12 | 4.13 | 4.14 | 4.15 | 4.16 | 4.17 | 4.18 | 4.19 | 4.20 | 4.21 | 4.22 | 4.23 | 4.24 | 4.25 |
| γ | 1/12 | 1/12 | 1/12 | 1/12 | 1/12 | 1/12 | 1/12 | 1/12 | 1/12 | 1/12 | 0 | 0 | 0 | 0 |
| $V_{\mathcal{k}}$ [deg 2] | . 0.87 | 0.79 | 1.57 | 1.01 | 0.76 | 0.75 | 1.50 | 1.04 | 0.78 | 0.84 | 3.65 | 3.71 | 3.08 | 3.47 |
| σ _ψ [deg] | 0.51 | 0.57 | 0.64 | 0.44 | 0.45 | 0.45 | 0.64 | 0.52 | 0.53 | 1.61 | 1.48 | 1.49 | 1.30 | 1.39 |
| m ∯ [deg] | 0.05 | 0.05 | 0.06 | 0.01 | -0.05 | 00.00 | 0.02 | 0.01 | -0.10 | -0.16 | 1.21 | 1.22 | 1.18 | 1.24 |
| ^م ح [deb] | 2.71 | 2.37 | 3.73 | 3.12 | 2.58 | 2.58 | 3.62 | 3.04 | 2.43 | 2.33 | 28.57 | 26.28 | 25.86 | 29.46 |
| m _S [deg] | - 1.31 | - 1.32 | - 1.27 | - 1.28 | - 1.29 | - 1.30 | - 1.29 | - 1.29 | - 1.30 | - 1.31 | -21.34 | -22.04 | -22.56 | -22.28 |
| V _C [m/s] | 8 | 8 | 8 | 8 | 8 | 8 | ∞ | 8 | 8 | 8 | 2 | 2 | 2 | 2 |
| d* | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.05 | 0.05 | 0.1 | 0 | 0 | 0 | 0.05 |
| T_S_[S] | 15 | 15 | 15 | 19 | 19 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| ~ | 4 | വ | 9 | വ | 9 | 7 | 8 | 2 | 9 | 9 | 9 | 7 | ∞ | 9 |
| δ_{ℓ} [deg] | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 45 | 45 | 45 | 45 |
| u ₀ [knots] | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 15.8 | 4 | 4 | 4 | 4 |
| n ₀ [rpm] | 87.6 | 87.6 | 9.78 | 9.78 | 9.78 | 87.6 | 97.6 | 9.78 | 9.78 | 87.6 | 22.1772 | 22.1772 | 22.1772 | 22.1772 |
| | | | | | | | | | | | | | | |

Table 4.2 - Summary of straight course keeping simulations ($\psi_{
m ref}$ = 0 deg) with the self-tuning regulator using non-filtered measurements, when k, $T_{\rm s}$, and $q_{\rm s}^{*}$ are varied. The mean draught T is equal to 22.3 m. The speed $V_{\rm s}$ is computed according to (4.11). The duration of each simulation is 2400 s, but the values of m $_{\mathcal{S}}$, $\sigma_{\mathcal{S}}$, m_{ψ} , σ_{ψ} , and $V_{\mathcal{L}}$ are computed for the part 1200 - 2400 s.

| T [m] | n ₀ [rpm] | u ₀ [knots] | δ _ℓ [deg] | ^m δ [deg] | ^σ δ [deg] | m _ψ [deg] | σ _ψ [deg] | V _ℓ [deg²] | λ | Fig. |
|----------|-------------------------|---------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|--------------------------|------|------|
| 22.3 | 22.1772 | 15.8 | 45 | -4.47 | 5.60 | -0.11 | 0.85 | 3.34 | 1/12 | 4.26 |
| 22.3 | 87.6 | 4 | 45 | -1.85 | 9.36 | -0.07 | 0.71 | 7.81 | 1/12 | 4.27 |

Table 4.3 - Summary of straight course keeping simulations (ψ_{ref} = 0 deg) with the self-tuning regulator using estimates from the Kalman filter, when the speed is changed significantly. The speed V_S is computed according to (4.12).

Simulations of straight course keeping with the self-tuning regulator and the PID-regulator, when the Kalman filter estimates as well as the non-filtered measurements are used, are shown in Figs 4.28-4.53. The initial speed \mathbf{u}_0 is equal to 15.8, 10 or 4 knots and the mean draught T is equal to 22.3 or 10.5 m. A summary is given in Tables 4.4, 4.5 and 4.6, where also simulations from Section 4.1 are included.

The speed scaling used in the PID-regulator is $(V_0/V_s)^2$ (cf (3.25)). By putting $V_0=6.3$ m/s when $V_s=5$ m/s, and $V_0=4$ m/s when $V_s=2$ m/s, the speed scaling V_0/V_s may be simulated (cf Figs 4.38, 4.43, 4.48 and 4.53). The speed scaling used in the self-tuning regulator is also $(V_0/V_s)^2$ (cf (3.24)), since $q_2^*=0$. It is, however, possible to obtain the speed scaling V_0/V_s instead, if $q_2^*=0.6$ when $V_s=5$ m/s and if $q_2^*=3$ when $V_s=2$ m/s (cf Figs 4.37, 4.42, 4.47, and 4.52).

| - | Kalman filter estimates used | ilter ; used | Non-filtered measurements | -filtered surements used | s _m | م | æ E | a The | V | ٧ | Fig. |
|------|---------------------------------|-------------------|------------------------------|-----------------------------|----------------|-------|--------|----------|---------------------|------|------|
| [m] | Self-tuning regulator | PID- regulator | Self-tuning regulator | PID- regulator | [ded] | [deg] | [ded] | [deg] | [deg ²] | | |
| 22.3 | × | | | | -1.28 | 2.04 | -0.09 | 0.47 | 0.55 | 1/12 | 4.1 |
| 22.3 | | × | | | -1.36 | 2.08 | 0.07 | 0.66 | 0.80 | 1/12 | 4.28 |
| 22.3 | | | × | | -1.32 | 2.34 | -0.22 | 0.64 | 0.91 | 1/12 | 4.29 |
| 22.3 | | | | × | -1.28 | 2.40 | 0.03 | 08.0 | 1.12 | 1/12 | 4.30 |
| 10.5 | × | | | | -0.15 | 1.55 | -0.01 | 0.29 | 0.29 | 1/12 | 4.2 |
| 10.5 | | × | | | -0.16 | 1.29 | 0.01 | 0.32 | 0.24 | 1/12 | 4.31 |
| 10.5 | | | × | | -0.17 | 1.94 | 0.01 | 0.43 | 0.50 | 1/12 | 4.32 |
| 10.5 | | 22 | | × | -0.13 | 1.63 | -0.01 | 0.42 | 0.39 | 1/12 | 4.33 |

Table 4.4 - Summary of straight course keeping simulations ($\psi_{\rm ref}$ = 0 deg), when η_0 = 87.6 rpm, u_0 = 15.8 knots, and δ_ℓ = 10 deg. The duration of each simulation is 2400 s, but the values of m_{δ}, σ_δ , m $_\psi$, σ_ψ , and V_ℓ are computed for the part 1200-2400 s.

| F | Kalman filter estimates used | ilter s used | Non-filtered measurements used | ed its used | ¶ 9 | g | E P | d d | Ve | ~ | Fig. | Remarks |
|------|---------------------------------|-------------------|-----------------------------------|-------------------|--------|-------|-------|--------|---------------------|------|------|-------------------------|
| [m] | Self-tuning regulator | PID- regulator | Self-tuning regulator | PID- regulator | [ded] | [deg] | [ded] | [ded] | [deg ²] | | | |
| 22.3 | × | | | | -3.71 | 4.61 | -0.16 | 0.51 | 2.05 | 1/12 | 4.3 | |
| 22.3 | | × | | | -3.82 | 4.62 | 0.07 | 0.65 | 2.21 | 1/12 | 4.34 | |
| 22.3 | | | × | | -3.76 | 5.64 | -0.19 | 0.66 | 3.12 | 1/12 | 4.35 | |
| 22.3 | | | | × | -3.65 | 5.33 | 0.01 | 0.65 | 2.79 | 1/12 | 4.36 | |
| 22.3 | × | | | | -3.77 | 4.43 | -0.26 | 0.83 | 2.38 | 1/12 | 4.37 | q2 = 0.6 |
| 22.3 | | × | | | -3.64 | 4.46 | 90.0 | 1.08 | 2.83 | 1/12 | 4.38 | $V_0 = 6.3 \text{ m/s}$ |
| 10.5 | × | | | | -1.18 | 3.36 | -0.01 | 0.35 | 1.06 | 1/12 | 4.4 | |
| 10.5 | | × | | | -1.19 | 3.32 | 0.01 | 0.37 | 1.06 | 1/12 | 4.39 | |
| 10.5 | | | × | | -1.19 | 5.54 | 00.00 | 0.53 | 2.83 | 1/12 | 4.40 | |
| 10.5 | | | | × | -1.12 | 4.47 | -0.01 | 0.42 | 1.84 | 1/12 | 4.41 | |
| 10.5 | × | | | | -1.19 | 2.31 | -0.18 | 0.70 | 0.97 | 1/12 | 4.45 | q* = 0.6 |
| 10.5 | | × | | | -1.22 | 2.56 | 0.03 | 0.52 | 0.81 | 1/12 | 4.43 | $V_0 = 6.3 \text{ m/s}$ |

Table 4.5 - Summary of straight course keeping simulations (ψ_{ref} = 0 deg), when η_0 = 55.443 rpm, u_0 = 10 knots, and δ_ℓ = 35 deg. The duration of each simulation is 2400 s, but the values of m₆, σ_6 , m $_\psi$, σ_ψ , and V_ℓ are computed for the part 1200 - 2400 s.

| Remarks | | | | | E) | q* = 3 | $V_0 = 4 \text{ m/s}$ | | | | | q* = 3 | $V_0 = 4 \text{ m/s}$ |
|-----------------------------------|--------------------------|--------|--------|--------|--------|--------|-----------------------|-------|-------|-------|-------|--------|-----------------------|
| Fig. | | 4.5 | 4.44 | 4.45 | 4.46 | 4.47 | 4.48 | 4.6 | 4.49 | 4.50 | 4.51 | 4.52 | 4.53 |
| χ | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ٧ ه | [deg ²] | 1.46 | 2.37 | 3.75 | 4.23 | 4.19 | 1.75 | 0.46 | 0.46 | 0.82 | 1.47 | 2.22 | 1.54 |
| ď | [ded] | 1.01 | 1.54 | 1.48 | 2.05 | 1.87 | 1.31 | 0.61 | 0.68 | 0.82 | 1.21 | 1.41 | 1.24 |
| E B | [deg] | 0.66 | -0.03 | 1.25 | -0.17 | 0.83 | -0.19 | 0.30 | 0.05 | 0.38 | 0.03 | -0.48 | -0.04 |
| d S | [ded] | 19.96 | 22.44 | 27.55 | 31.25 | 24.21 | 13.95 | 19.24 | 22.26 | 23.44 | 31.12 | 13.73 | 12.05 |
| E S | [deg] | -21.62 | -20.44 | -21.89 | -18.00 | -20.02 | -19.55 | -8.93 | -9.23 | -9.08 | -9.31 | -8.47 | -7.83 |
| ed ts used | PID- regulator | | | | × | | | | | | × | | |
| Non-filtered measurements used | Self-tuning regulator | | | × | | | | | | × | | | |
| Kalman filter estimates used | PID- regulator | | × | | | | × | | × | | | | × |
| | Self-tuning regulator | × | | | | × | | × | | | | × | |
| - | | 22.3 | 22.3 | 22.3 | 22.3 | 22.3 | 22.3 | 10.5 | 10.5 | 10.5 | 10.5 | 10.5 | 10.5 |

Table 4.6 - Summary of straight course keeping simulations ($\psi_{\rm ref}$ = 0 deg), when n_0 = 22.1772 rpm, u_0 = 4 knots, and δ_ℓ = 45 deg. The duration of each simulation is 2400 s, but the values of m_δ , σ_δ , m_ψ , σ_ψ , and V_ℓ are computed for the part 1200 - 2400 s.

The simulations show that the performance of both the self-tuning regulator and the PID-regulator is improved when Kalman filter estimates are used instead of non--filtered measurements. This is true for each of the three different initial speeds. When Kalman filter estimates are used, the performance of the self-tuning regulator is significantly better than the performance of the PID-regulator in full load condition (T = 22.3 m). However, when T = 10.5 m, the difference between the two regulators is hardly noticable. When non-filtered measurements are used, a comparison between the self-tuning regulator and the PID-regulator is rather confusing; sometimes the self-tuning regulator is to prefer in front of the PID--regulator and sometimes vice versa. However, the performance of the self-tuning regulator seems to be somewhat better compared to the PID-regulator when T = 22.3 m. It can also be concluded that the self-tuning regulator using Kalman filter estimates always is significantly better than the PID-regulator using non-filtered measurements, and that the PID-regulator using Kalman filter estimates always is significantly better than the self-tuning regulator using non-filtered measurements.

In general, the rudder deviations are decreased when the speed scaling V_0/V_s is used instead of $(V_0/V_s)^2$. When $u_0=10$ knots, the speed scaling V_0/V_s is to prefer if T=10.5 m, but the speed scaling $(V_0/V_s)^2$ is to prefer if T=22.3 m. When $u_0=4$ knots, the speed scaling $(V_0/V_s)^2$, with one exception, is preferable.

The final parameter values of the self-tuning regulator for all the simulations presented in Sections 4.1 and 4.2 are shown in Table 4.7. It can be concluded that the values don't change very much when the speed and the draught are changed.

| Fig. | ^a 1 | а ₂ | ^a 3 | a ₄ | ^b 1 | ь2 | c ₁ | ^c 2 | Σa _i |
|------|----------------|----------------|----------------|----------------|----------------|-------|----------------|------------------|-----------------|
| 4.1 | -6.53 | 5.67 | 3.43 | -2.56 | 0.50 | 0.15 | -1.65 | 36.28 | 0.01 |
| 4.2 | -6.93 | 5.70 | 3.58 | -2.97 | 0.53 | 0.20 | -1.64 | 34.93 | -0.62 |
| 4.3 | -6.49 | 5.76 | 3.61 | -2.94 | 0.47 | 0.12 | -2.08 | 38.72 | -0.06 |
| 4.4 | -6.28 | 5.61 | 3.45 | -2.90 | 0.53 | 0.20 | -1.60 | 33.07 | -0.12 |
| 4.5 | -6.34 | 5.31 | 4.01 | -3.24 | 0.36 | -0.01 | -3.42 | 43.48 | -0.26 |
| 4.6 | -5.60 | 5.11 | 3.79 | -3.37 | 0.51 | 0.11 | -2.56 | 35.57 | -0.07 |
| 4.7 | -5.32 | 3.33 | 3.83 | -2.29 | 0.08 | -0.04 | -4.32 | 67.13 | -0.45 |
| 4.8 | -7.26 | 6.10 | 4.11 | -3.01 | 0.53 | 0.18 | -2.07 | 37.39 | -0.06 |
| 4.9 | -7.20 | 5.95 | 4.06 | -2.96 | 0.52 | 0.18 | -2.36 | 35.55 | -0.15 |
| 4.10 | -6.20 | 5.46 | 2.80 | -2.02 | 0.48 | 0.23 | -1.97 | 42.28 | 0.04 |
| 4.11 | -6.53 | 5.69 | 3.46 | -2.55 | 0.50 | 0.16 | -1.69 | 35.92 | 0.07 |
| 4.12 | -12.15 | 13.10 | 2.62 | -4.14 | 0.83 | 0.22 | - | - | -0.57 |
| 4.13 | -10.68 | 11.86 | 2.18 | -3.63 | 0.86 | 0.28 | - | - / | -0.27 |
| 4.14 | -11.18 | 12.08 | 1.01 | -2.77 | 0.62 | -0.02 | - | - | -0.86 |
| 4.15 | -14.45 | 15.38 | 4.81 | -5.94 | 0.47 | 0.10 | - | - | -0.20 |
| 4.16 | -13.80 | 15.07 | 5.55 | -6.78 | 0.54 | 0.06 | - | - 1 | 0.04 |
| 4.17 | -12.66 | 12.82 | 5.84 | -5.88 | 0.61 | 0.15 | - | - | 0.12 |
| 4.18 | -12.76 | 17.64 | -3.86 | -1.75 | 0.09 | 0.23 | - | - | -0.73 |
| 4.19 | -12.31 | 11.34 | 6.77 | -5.95 | 0.52 | 0.09 | _ | _ | -0.15 |
| 4.20 | -11.43 | 11.22 | 6.52 | -6.29 | 0.55 | 0.05 | _ | - | 0.02 |
| 4.21 | -9.91 | 9.11 | 6.43 | -5.67 | 0.53 | 0.03 | - | - 1 | -0.04 |
| 4.22 | -9.47 | 9.67 | 3.55 | -4.42 | 0.15 | 0.27 | _ | - | -0.67 |
| 4.23 | -9.24 | 10.28 | 2.30 | -3.86 | 0.26 | 0.20 | - | - | -0.52 |
| 4.24 | -9.62 | 10.96 | 2.09 | -3.97 | 0.38 | 0.19 | - | - | -0.54 |
| 4.25 | -8.08 | 8.55 | 2.24 | -3.24 | -0.14 | 0.33 | - | - | -0.53 |
| 4.26 | -7.57 | 5.76 | 3.81 | -2.39 | 0.47 | 0.15 | -1.32 | 36.42 | -0.39 |
| 4.27 | -6.19 | 5.42 | 3.26 | -2.81 | 0.52 | 0.19 | -2.09 | 35.16 | -0.32 |
| 4.29 | -8.21 | 9.19 | 2.44 | -3.46 | 0.04 | 0.17 | - | - | -0.04 |
| 4.32 | -6.75 | 7.14 | 1.31 | -1.84 | 0.09 | 0.27 | - | _ | -0.14 |
| 4.35 | -7.66 | 8.77 | 2.25 | -3.46 | 0.07 | 0.18 | _ | = | -0.10 |
| 4.37 | -4.63 | 3.26 | 3.31 | -2.00 | 0.45 | 0.07 | -1.25 | 22.26 | -0.06 |
| 4.40 | -6.16 | 6.76 | 1.02 | -1.71 | 0.08 | 0.29 | \$ = \$ | 55 90 | -0.09 |
| 4.42 | -3.63 | 2.94 | 2.47 | -1.84 | 0.52 | 0.09 | -0.64 | 12.48 | -0.06 |
| 4.45 | -8.35 | 9.41 | 1.83 | -3.31 | 0.04 | 0.19 | - | - | -0.42 |
| 4.47 | -4.61 | 1.90 | 3.01 | -0.66 | 0.53 | 0.20 | -2.54 | 28.74 | -0.36 |
| 4.50 | -6.38 | 8.24 | 0.59 | -2.56 | 0.22 | 0.31 | - | - | -0.11 |
| 4.52 | -3.46 | 1.72 | 2.61 | -1.01 | 0.64 | 0.25 | -2.87 | 21.31 | -0.14 |

 $\underline{\text{Table 4.7}}$ - Final parameter values of the self-tuning regulator.

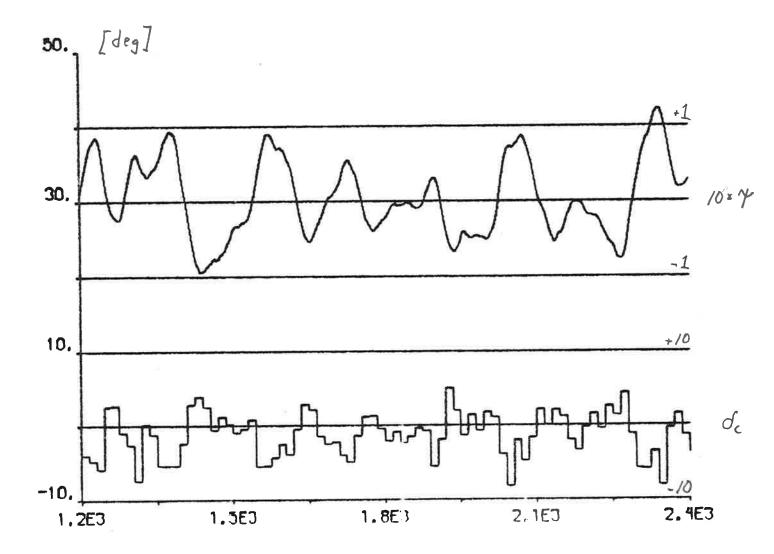


Fig. 4.12 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = \text{10 deg, self-tuning regulator using non-filtered}$ measurements (k = 4, T_s = 15 s, q_2^* = 0, V_c = 8 m/s).

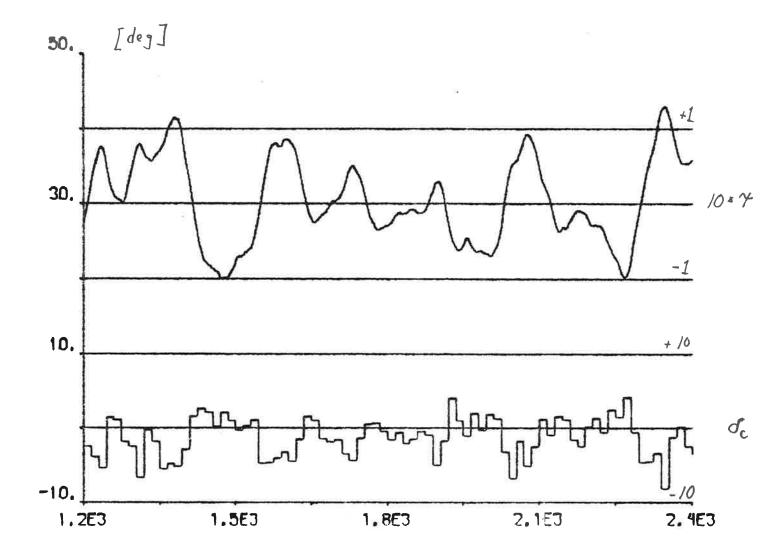


Fig. 4.13 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using non-filtered measurements (k = 5, T_s = 15 s, q_2^* = 0, V_c = 8 m/s).

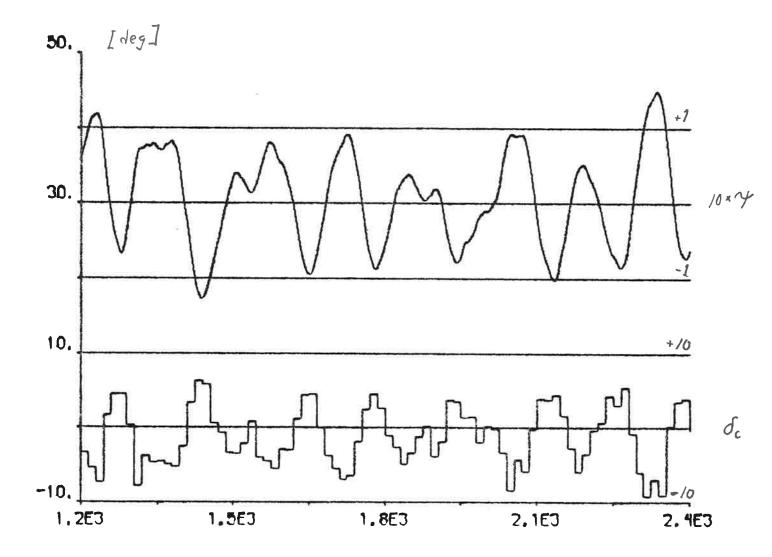


Fig. 4.14 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 10 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 6, T_S = 15 s, q_2^* = 0, V_c = 8 m/s).

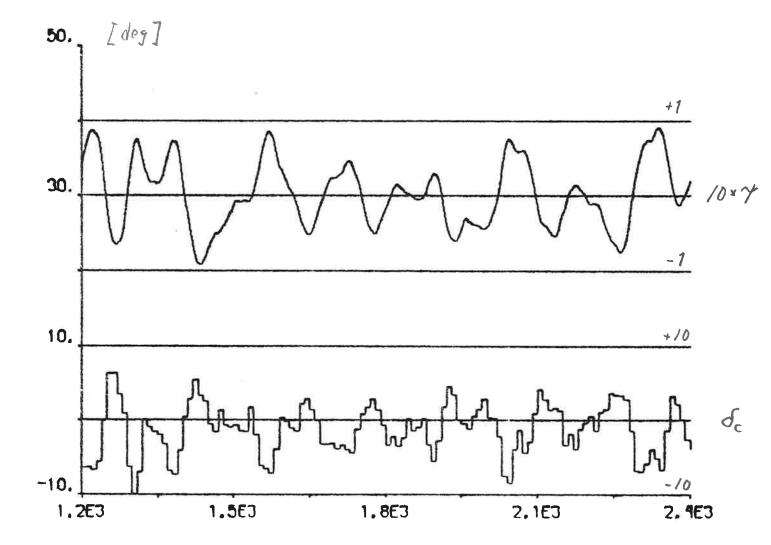


Fig. 4.15 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 10 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 5, T_s = 10 s, q_2^* = 0, V_c = 8 m/s).

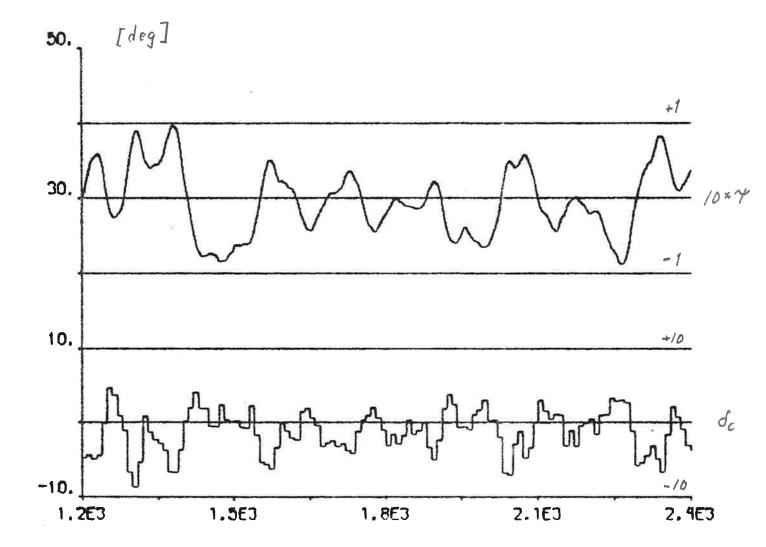


Fig. 4.16 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 10 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 6, T_S = 10 s, q_2^* = 0, V_c = 8 m/s).

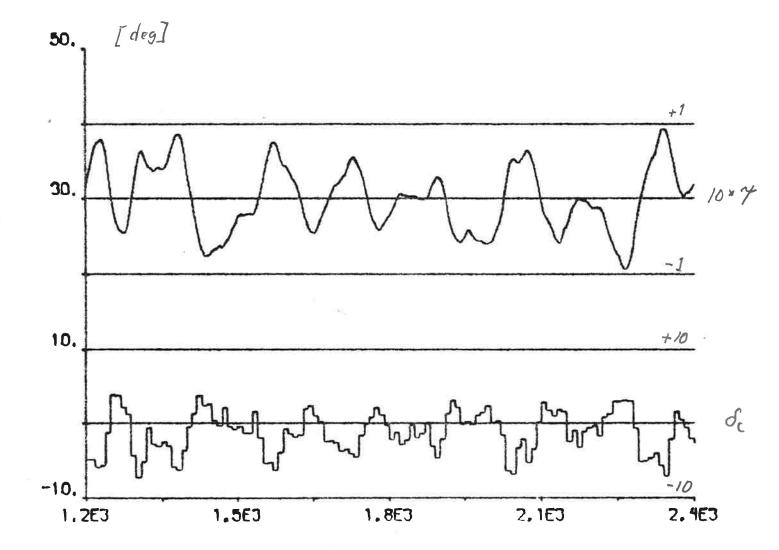


Fig. 4.17 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 10 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 7, T_s = 10 s, q_2^* = 0, V_c = 8 m/s).

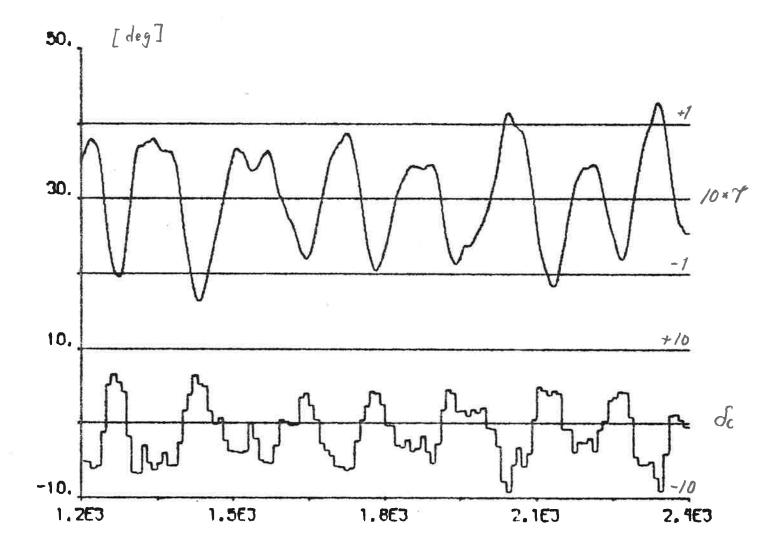


Fig. 4.18 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 10 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 8, T_s = 10 s, q_2^* = 0, V_c = 8 m/s).

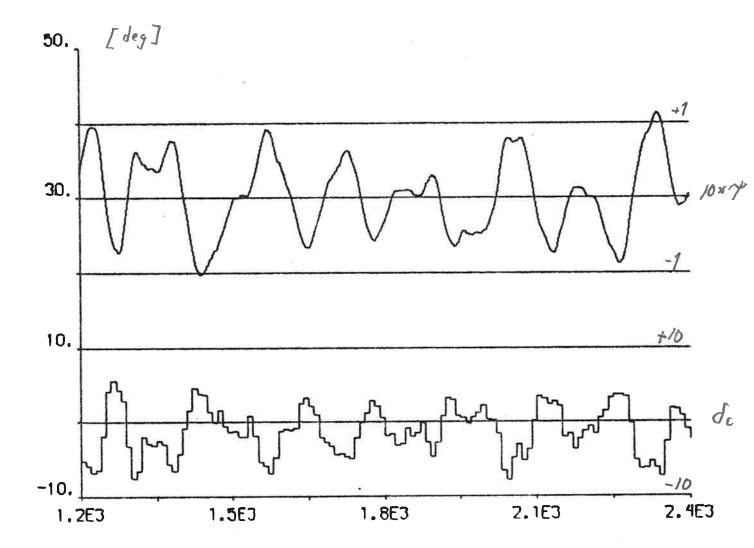


Fig. 4.19 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_{ℓ} = 10 deg, self-tuning regulator using non-filtered measurements (k = 5, T_s = 10 s, q_2^* = 0.05, V_c = 8 m/s).

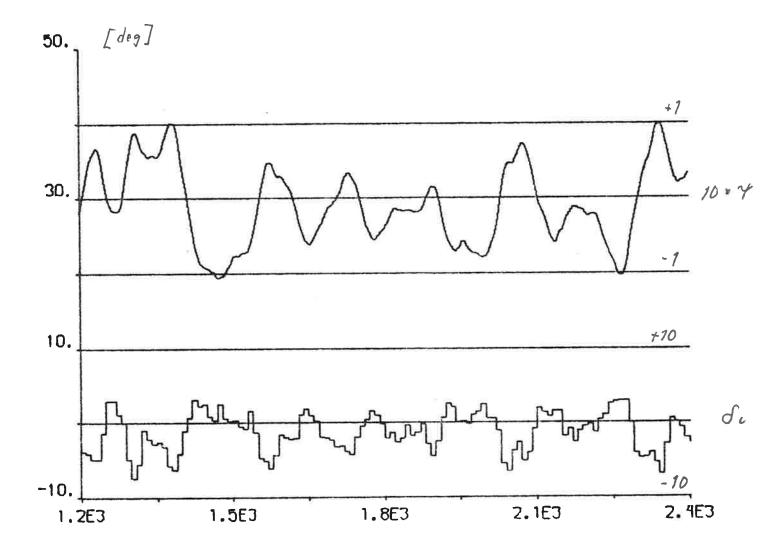


Fig. 4.20 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 10 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 6, T_S = 10 s, q_2^* = 0.05, V_c = 8 m/s).

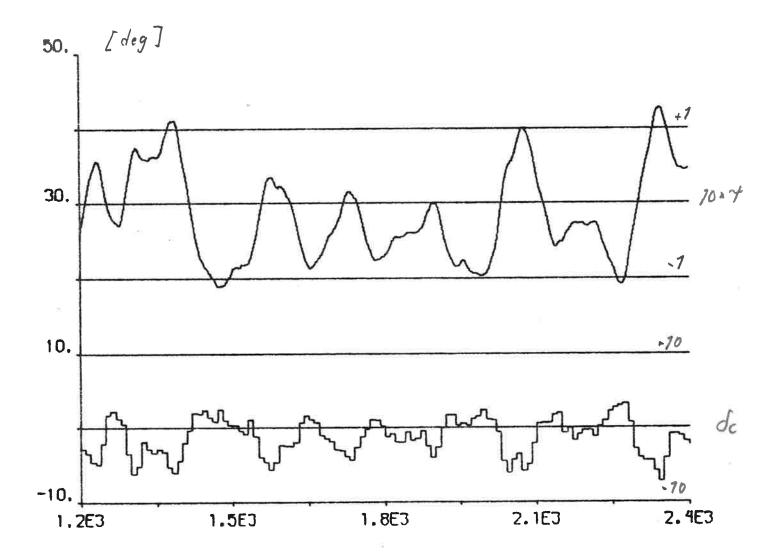


Fig. 4.21 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 10 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 6, T_S = 10 s, q_2^* = 0.1, V_c = 8 m/s).

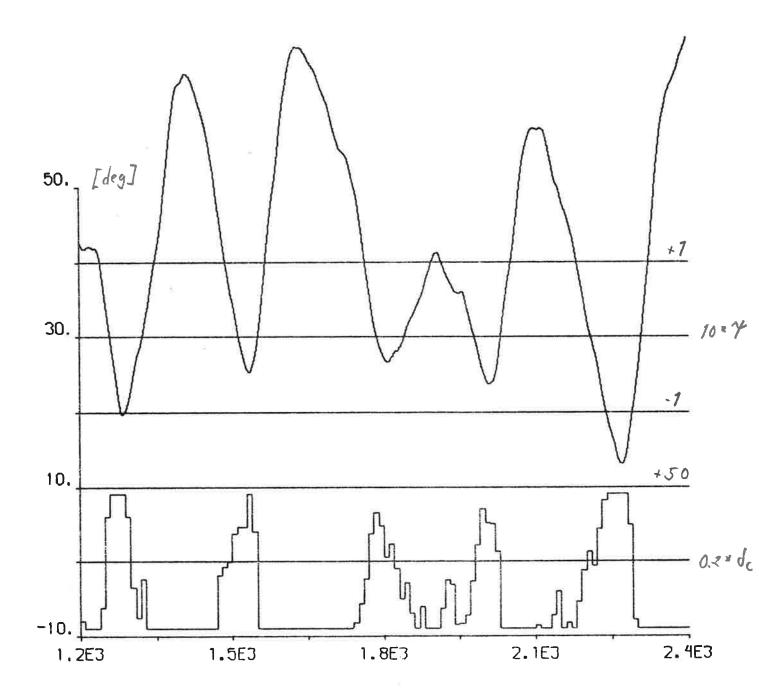


Fig. 4.22 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\delta_{\ell} = 45 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 6, T_S = 10 s, q_2^* = 0, V_c = 2 m/s).

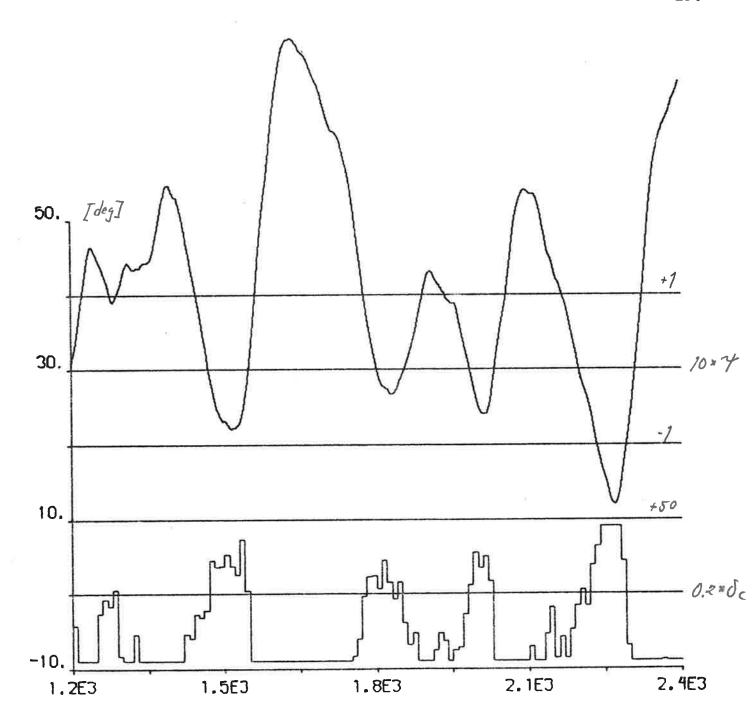


Fig. 4.23 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, self-tuning regulator using non-filtered measurements (k = 7, T_s = 10 s, q_2^* = 0, V_c = 2 m/s).

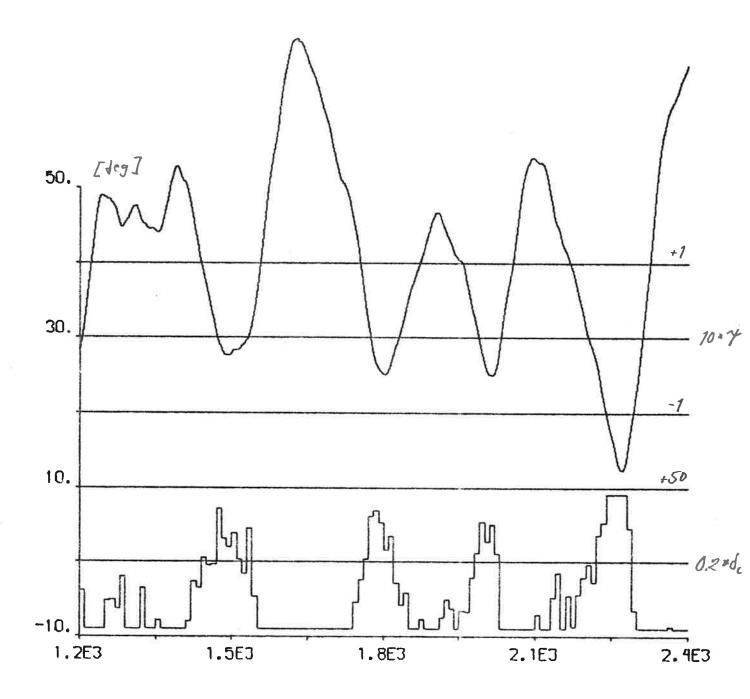


Fig. 4.24 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\delta_{\ell} = 45 \text{ deg, self-tuning regulator using}$ non-filtered measurements (k = 8, T_S = 10 s, q_2^* = 0, V_c = 2 m/s).

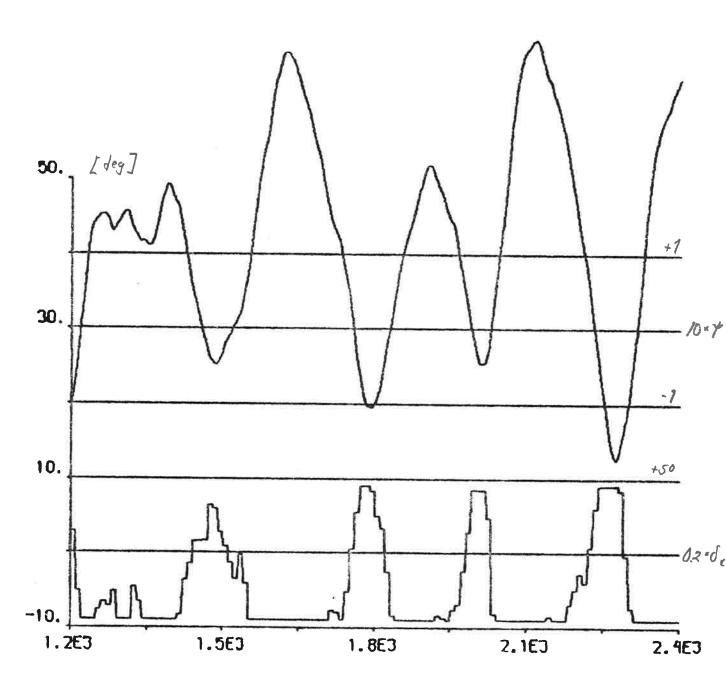


Fig. 4.25 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, self-tuning regulator using non-filtered measurements (k = 6, T_s = 10 s, q_2^* = 0.05, V_c = 2 m/s).

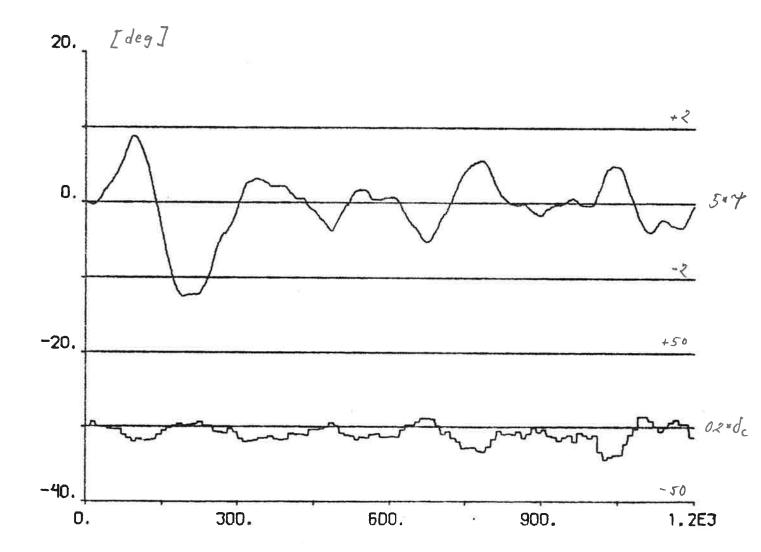
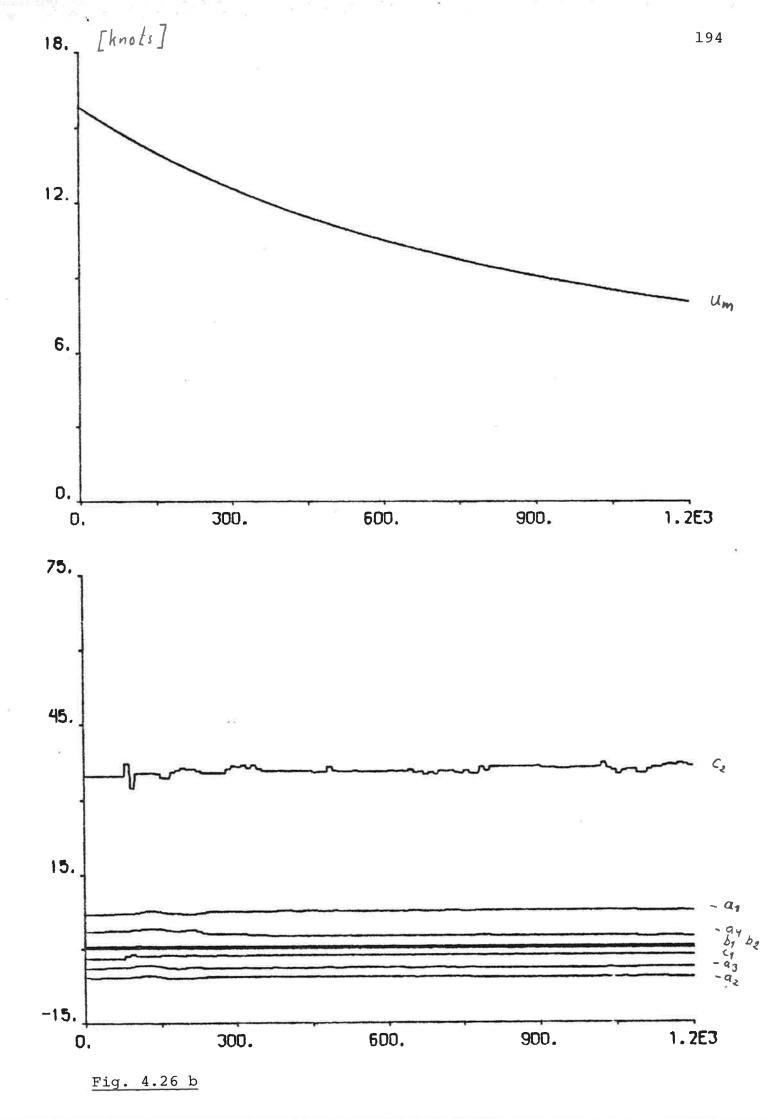


Fig. 4.26 a - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 15.8 knots, $\delta_{\ell} = 45 \text{ deg, self-tuning regulator using}$ estimates from the Kalman filter.



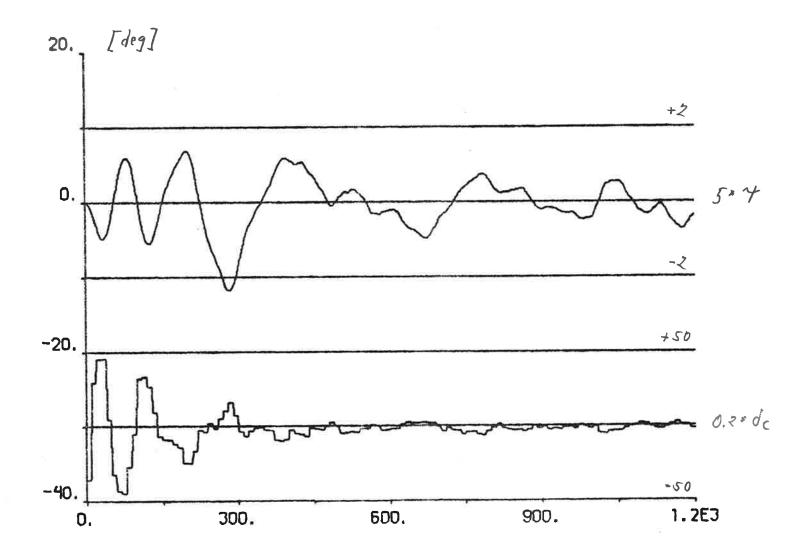
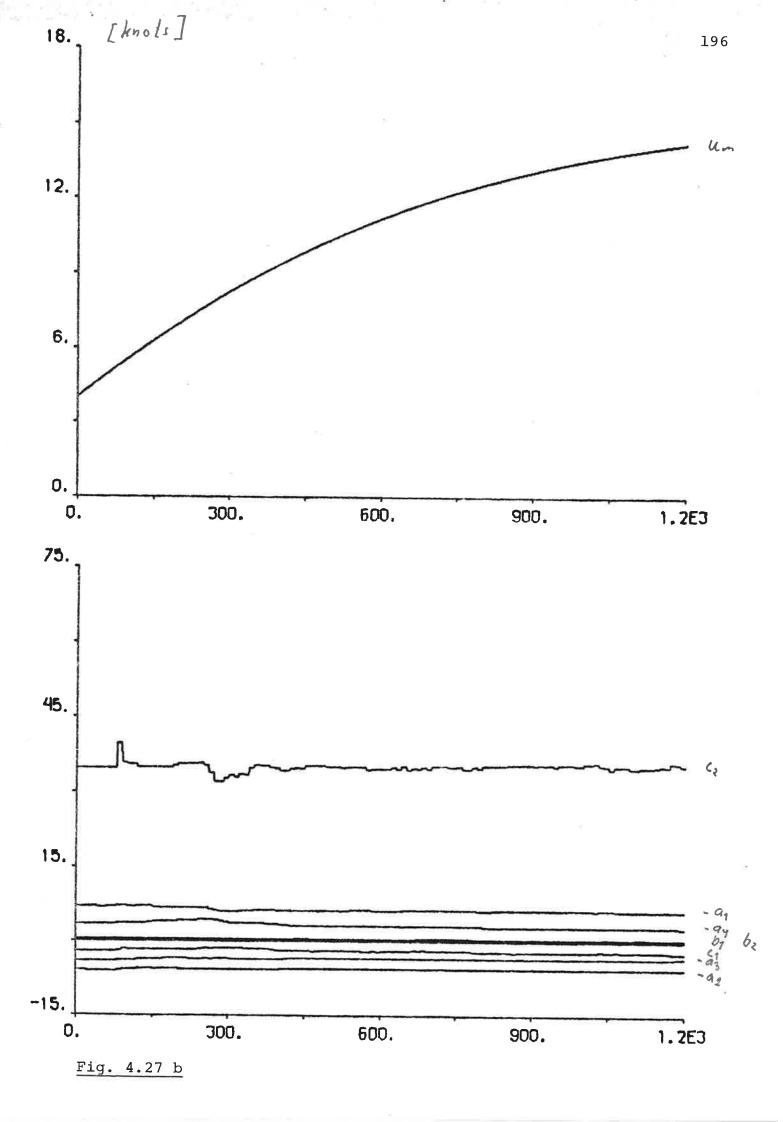
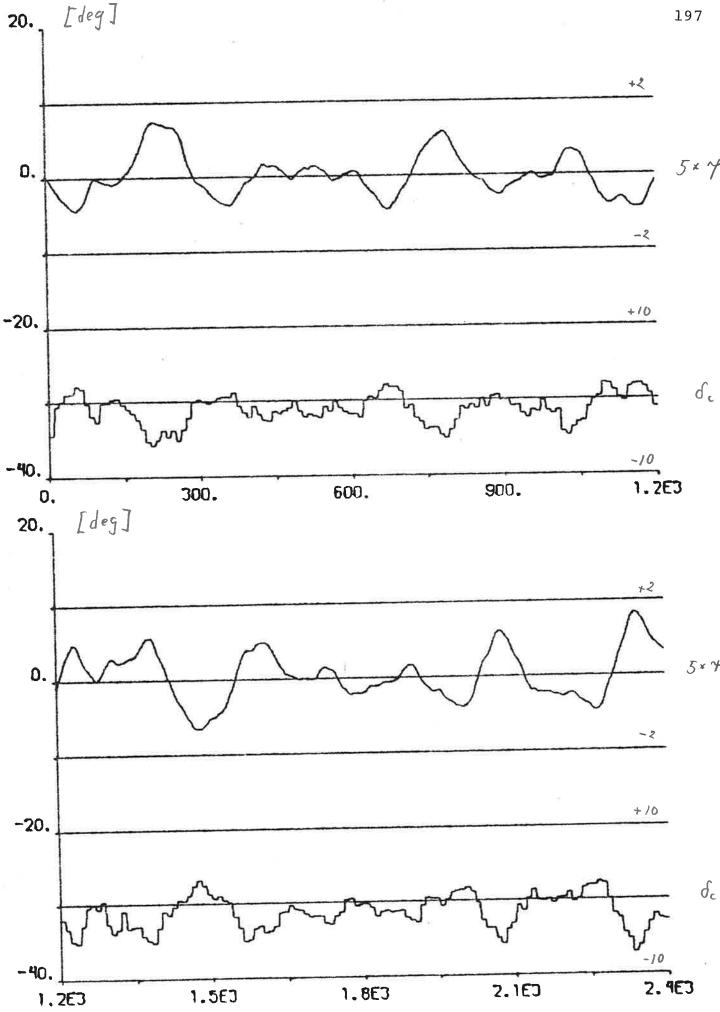


Fig. 4.27 a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, self-tuning regulator using estimates from the Kalman filter.





<u>Fig. 4.28</u> - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, PID-regulator using estimates from the Kalman filter.



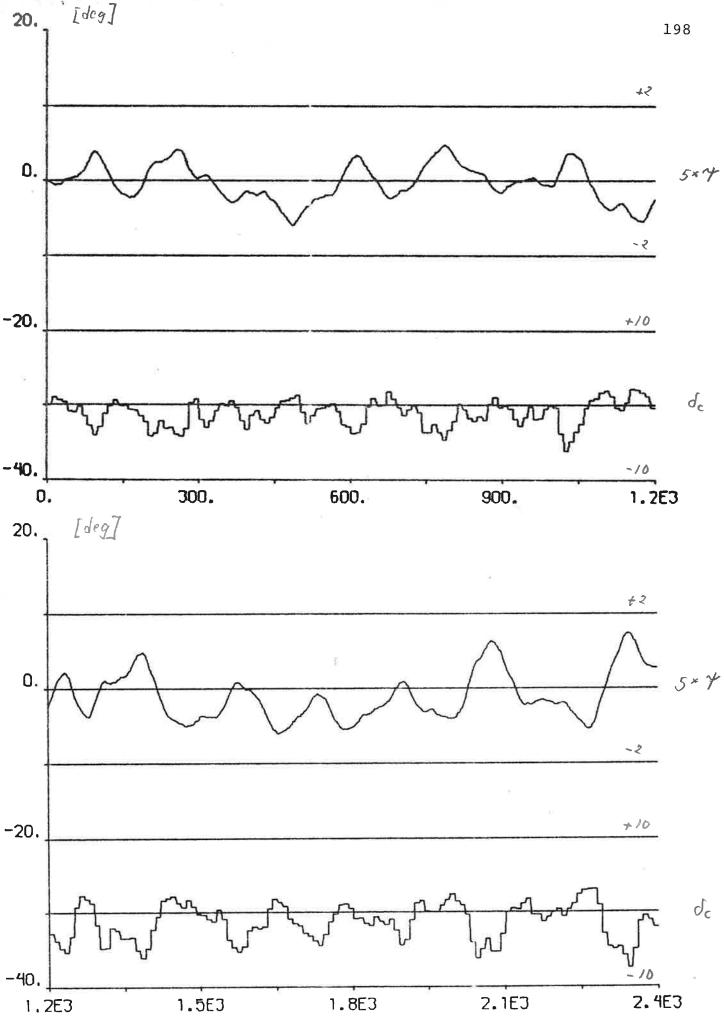
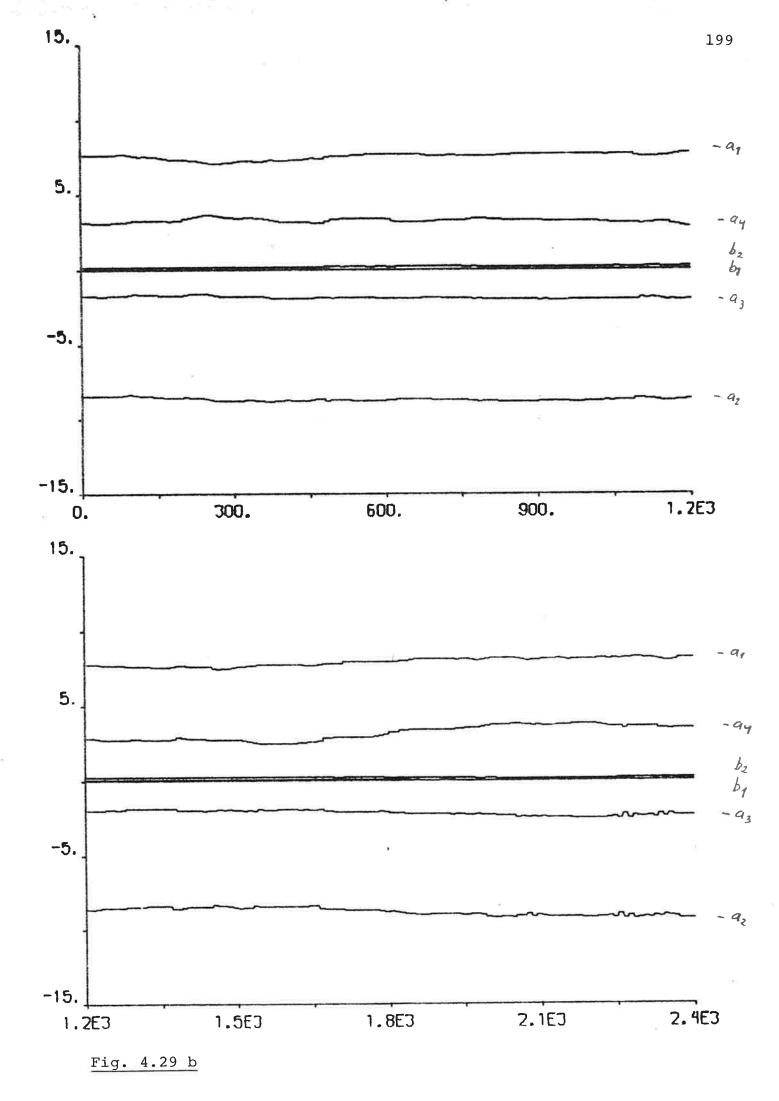


Fig. 4.29 a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using non-filtered measurements.



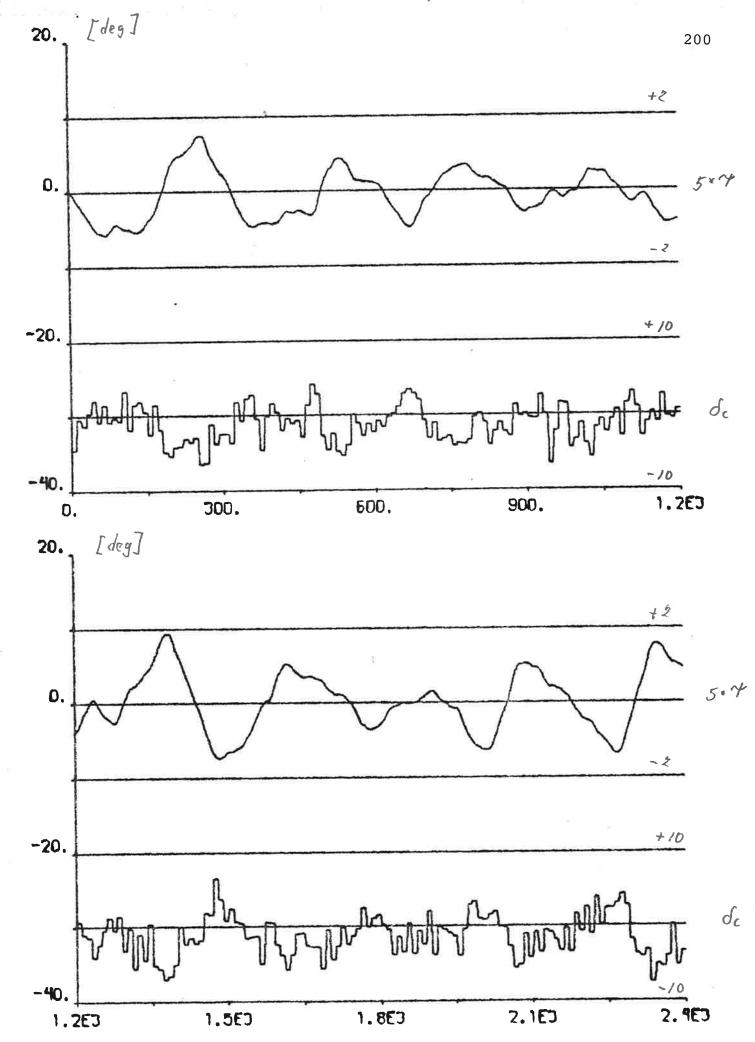
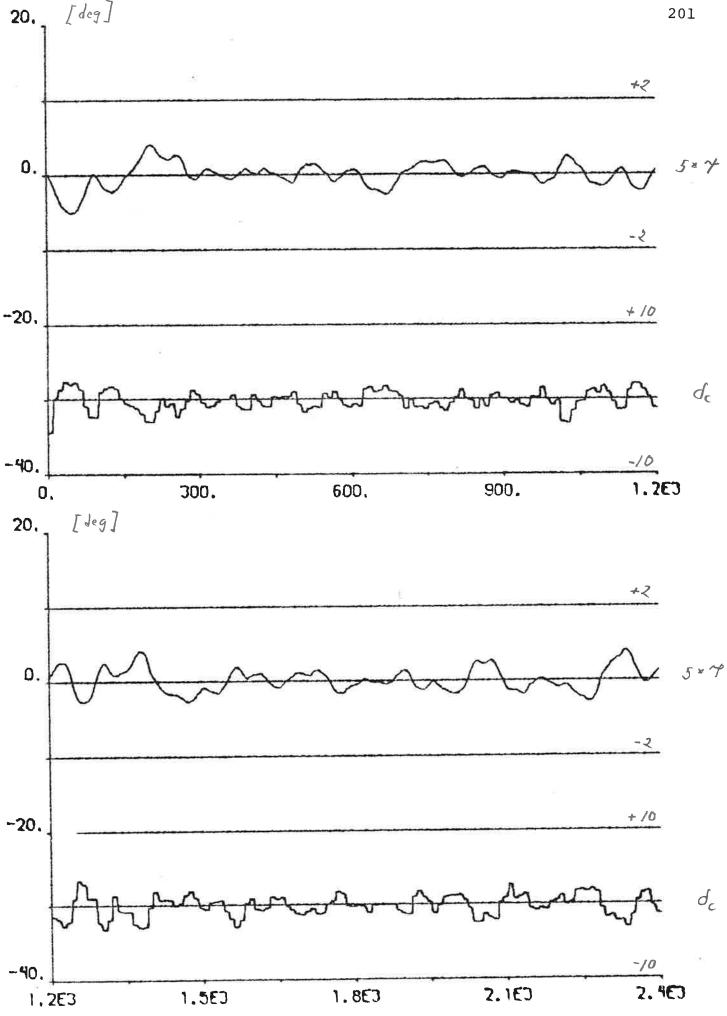


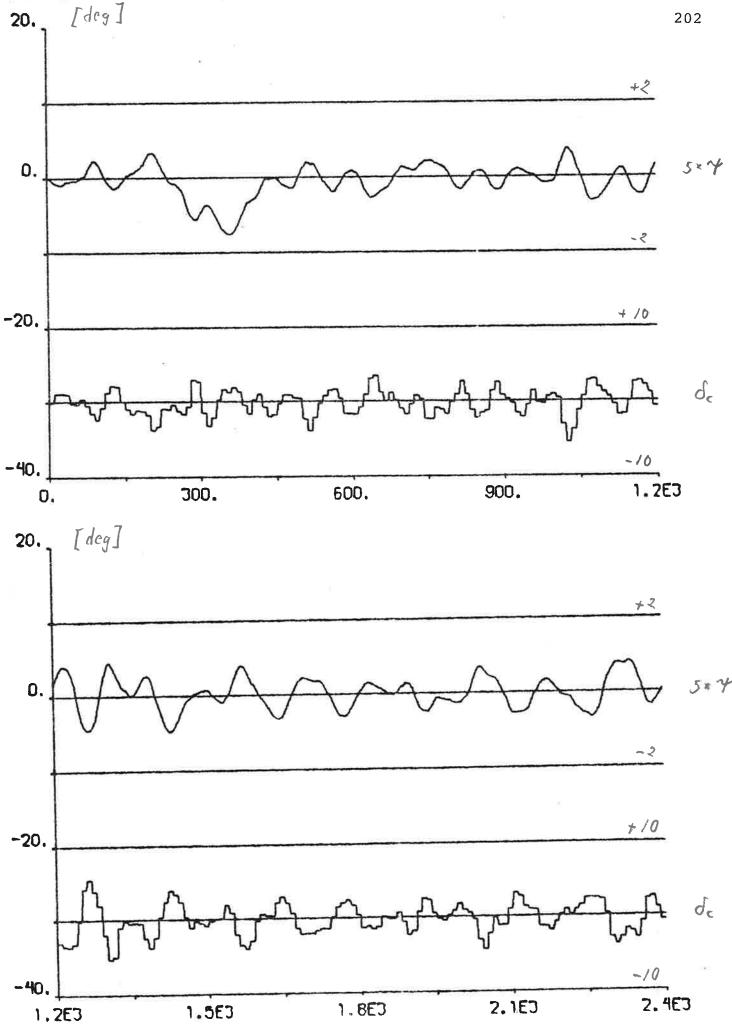
Fig. 4.30 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, PID-regulator using non-filtered measurements.





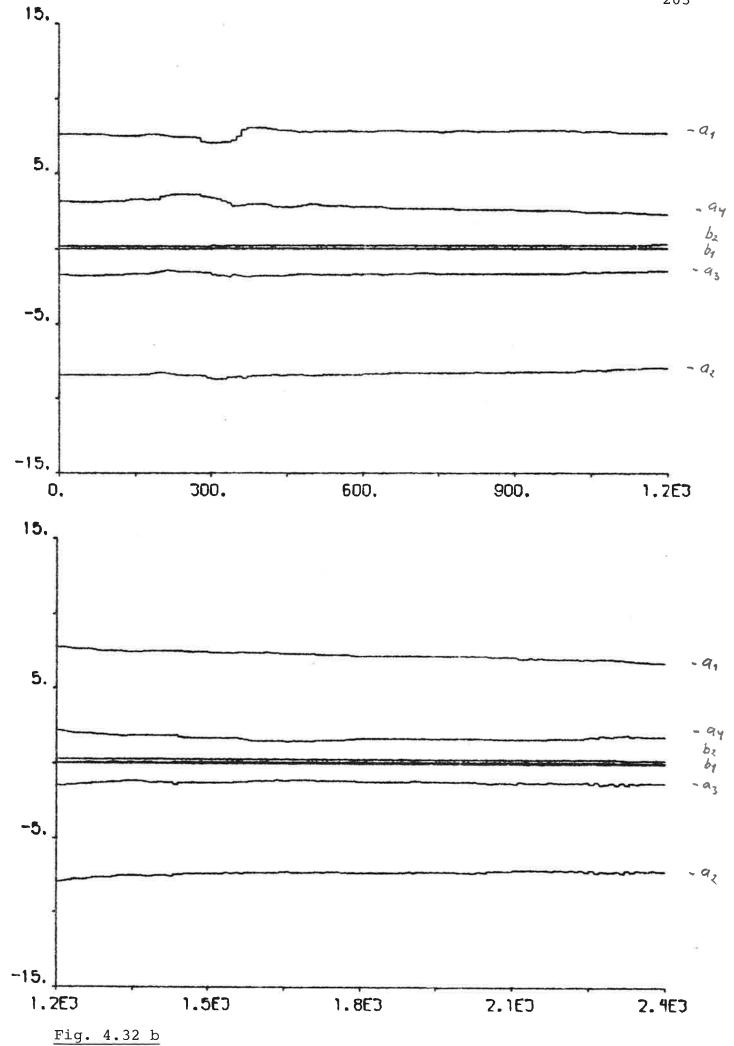
<u>Fig. 4.31</u> - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, PID-regulator using estimates from the Kalman filter.





<u>Fig. 4.32 a</u> - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, self-tuning regulator using non-filtered measurements.





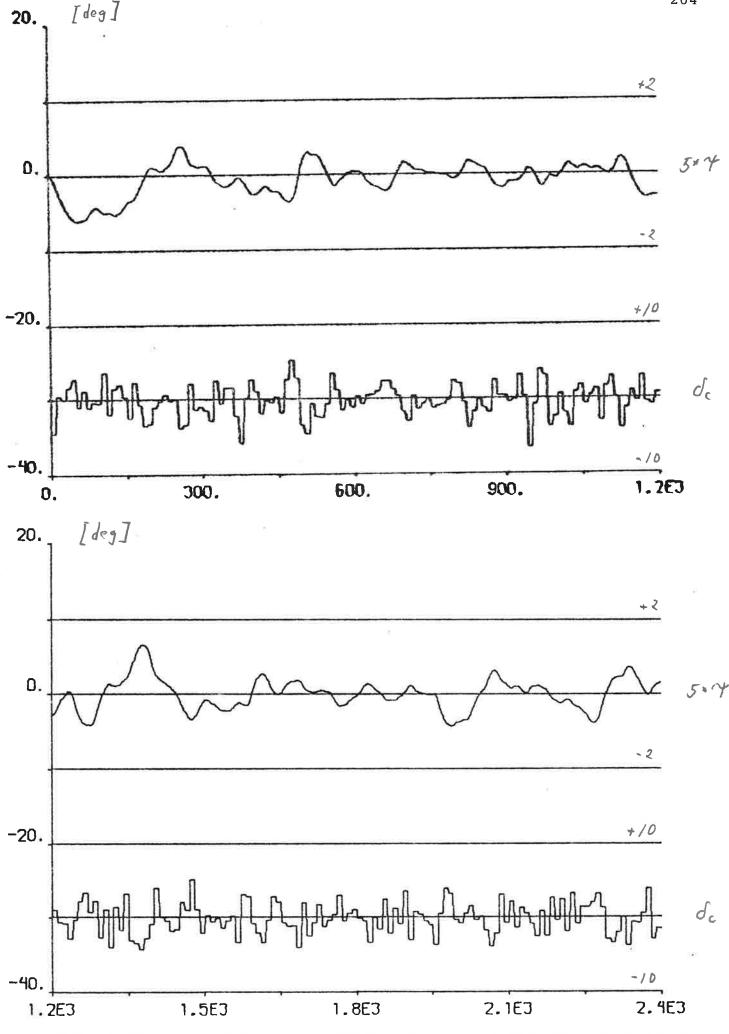


Fig. 4.33 - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, δ_ℓ = 10 deg, PID-regulator using non-filtered measurements.

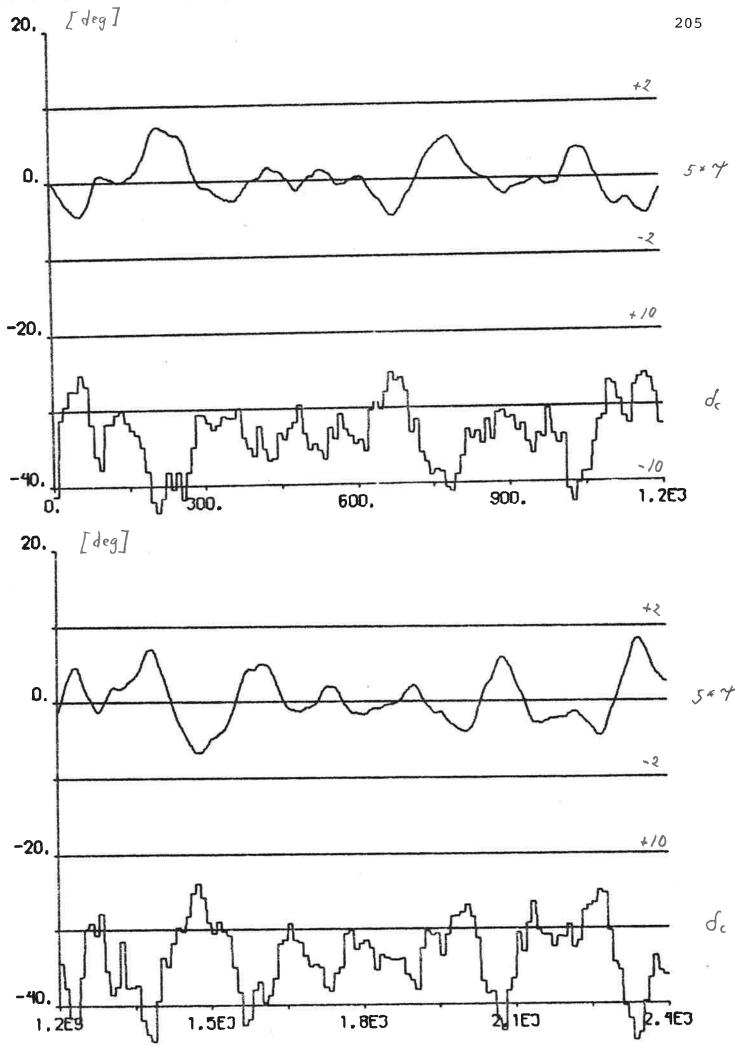
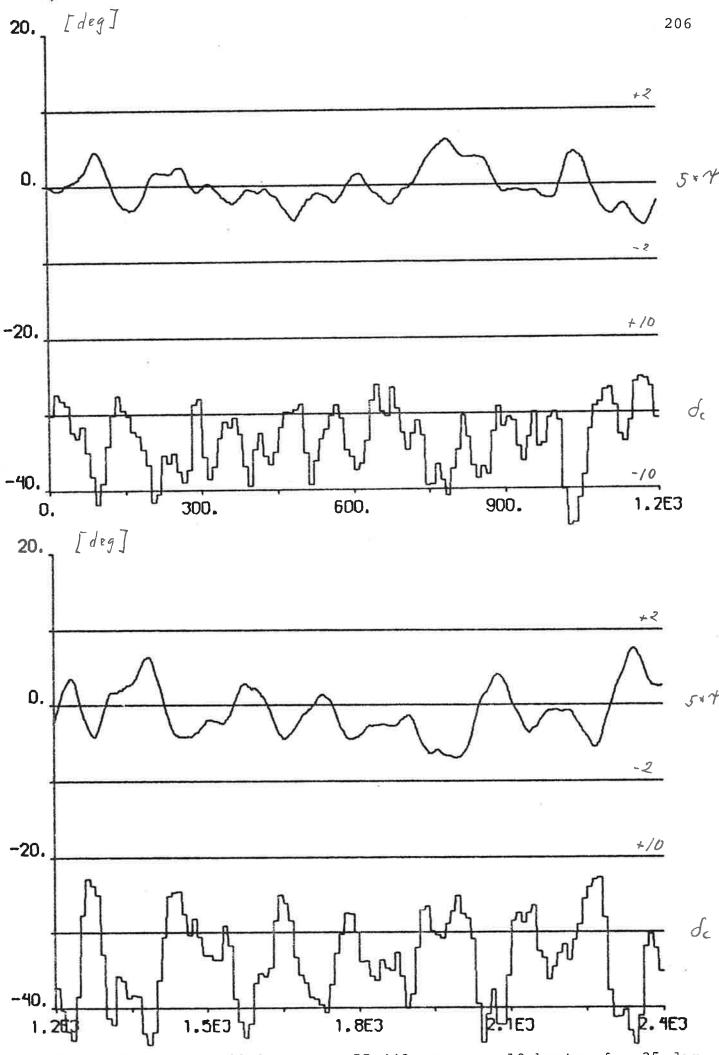
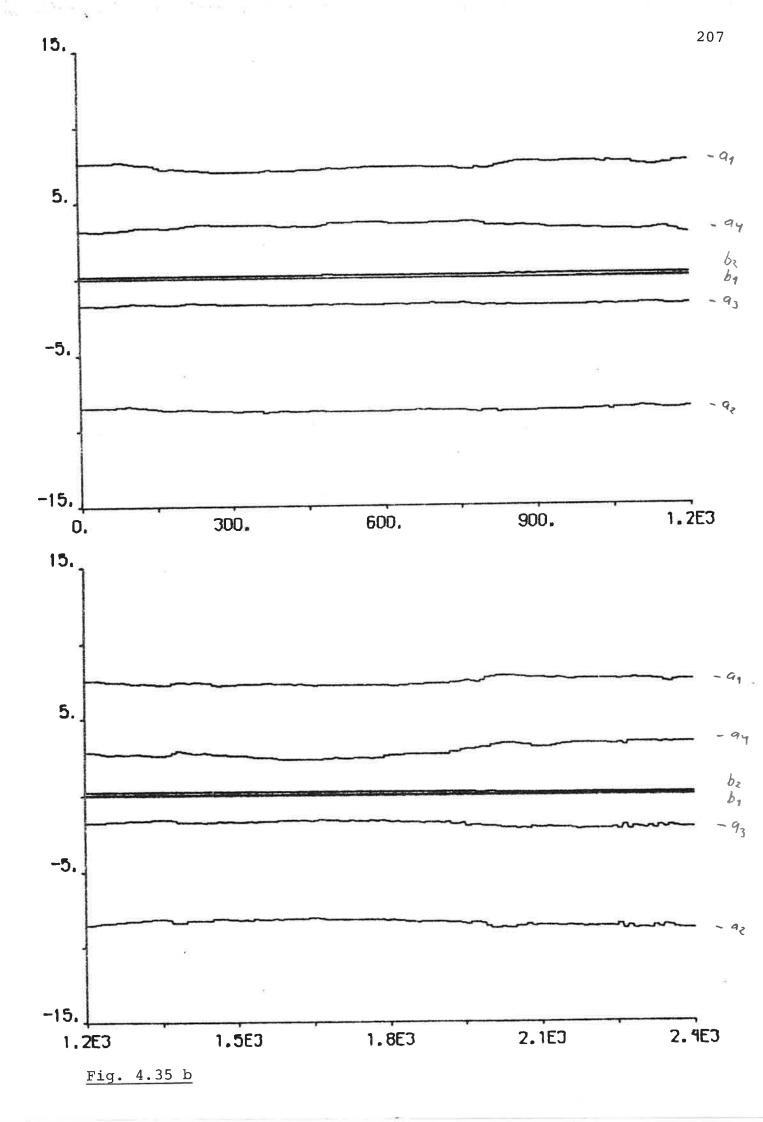


Fig. 4.34 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, PID-regulator using estimates from the Kalman filter.





T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, self-tuning regulator using non-filtered measurements.



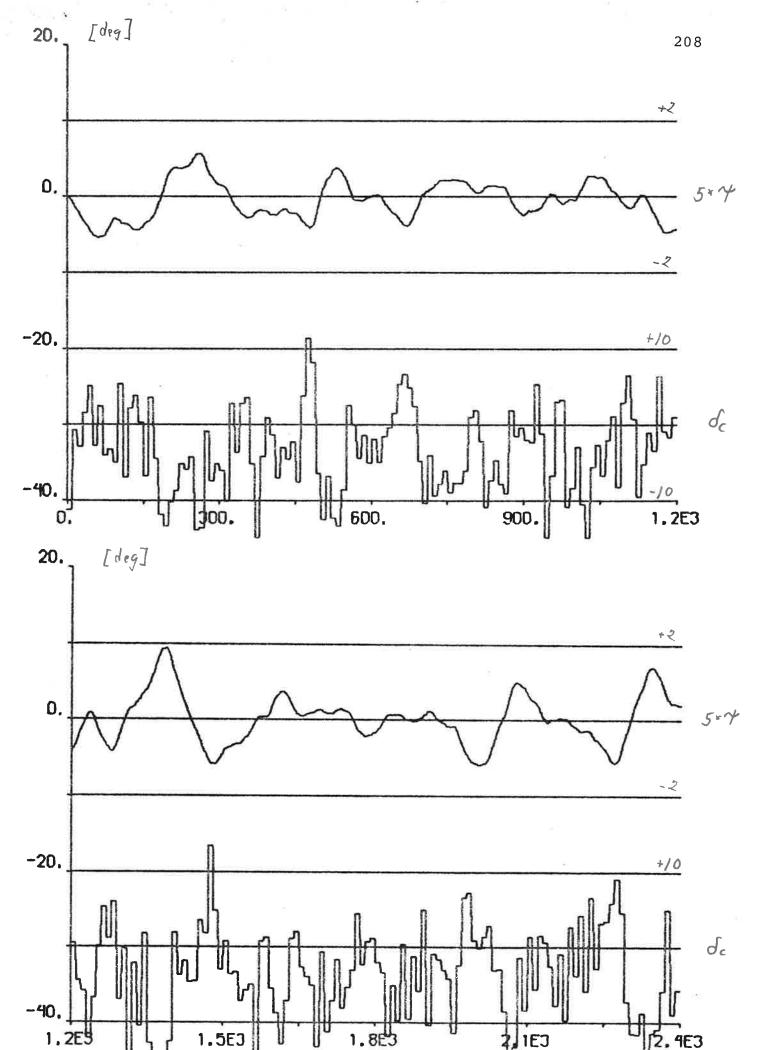


Fig. 4.36 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, PID-regulator using non-filtered measurements.

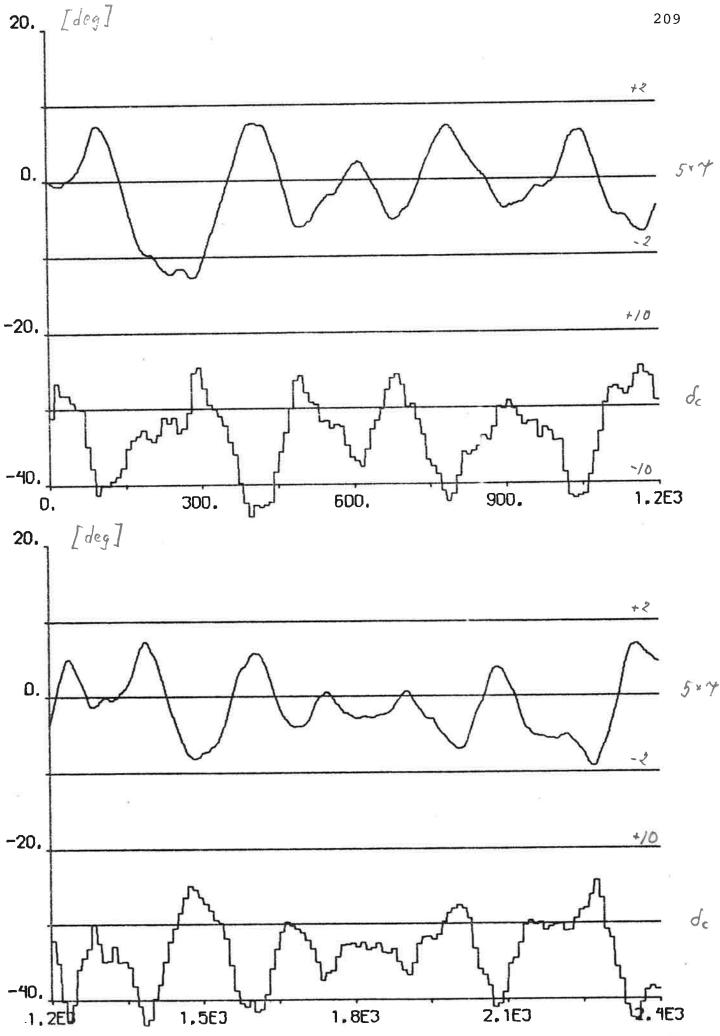
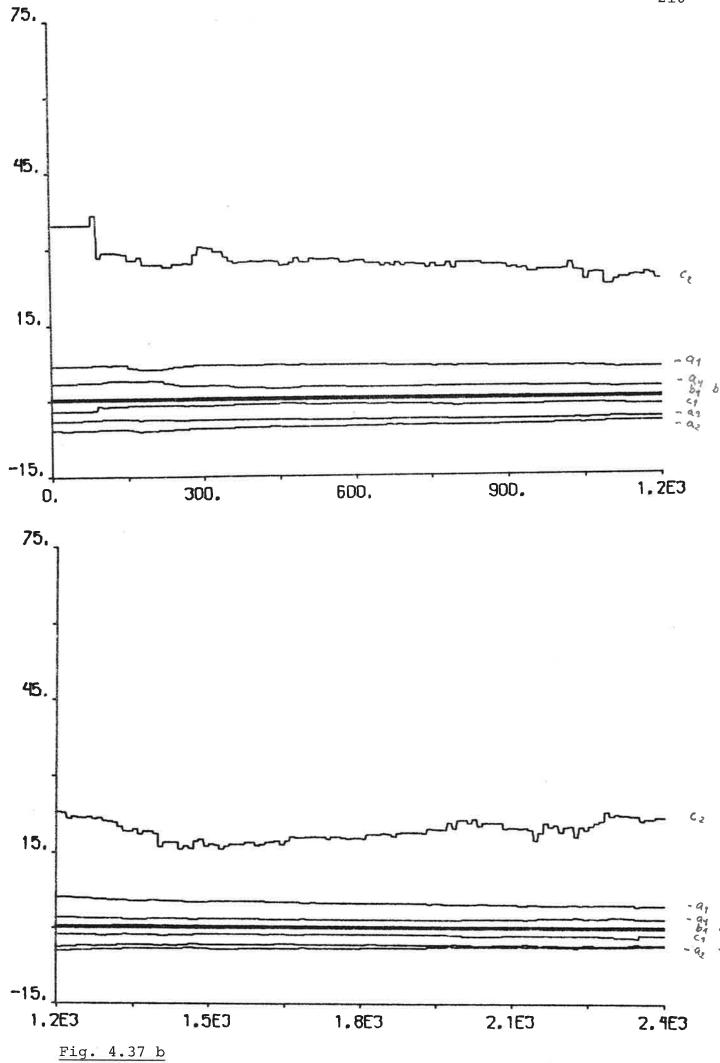


Fig. 4.37 a - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg self-tuning regulator using estimates from the Kalman filter (q_2^* = 0.6).



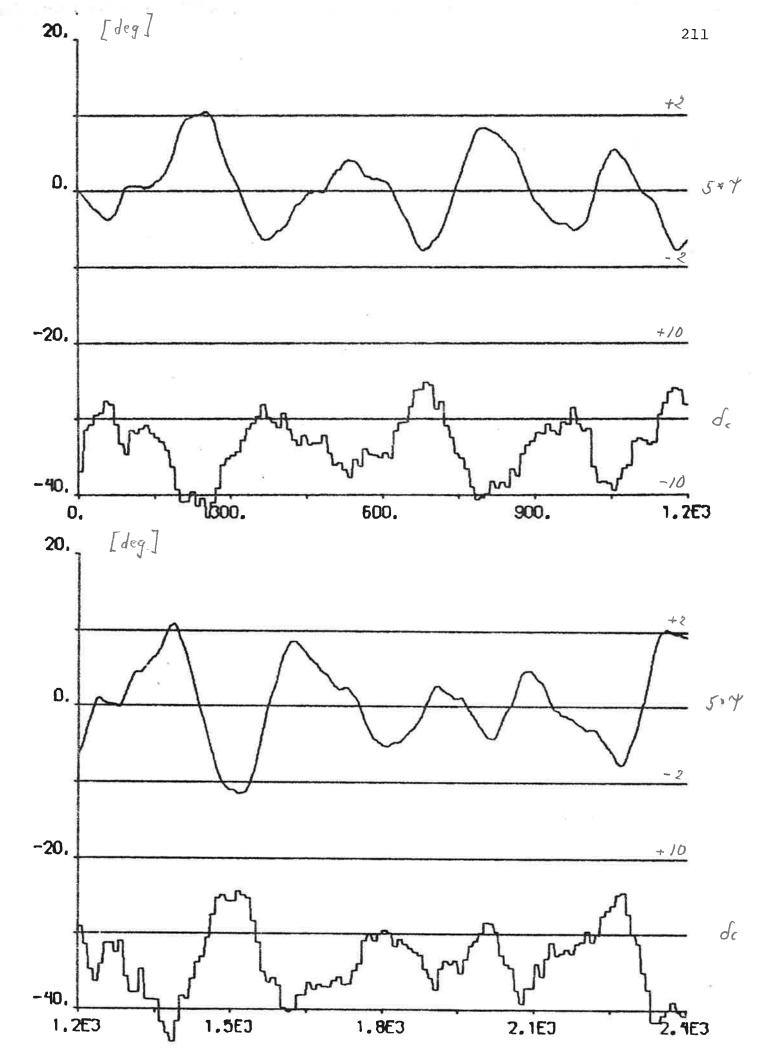
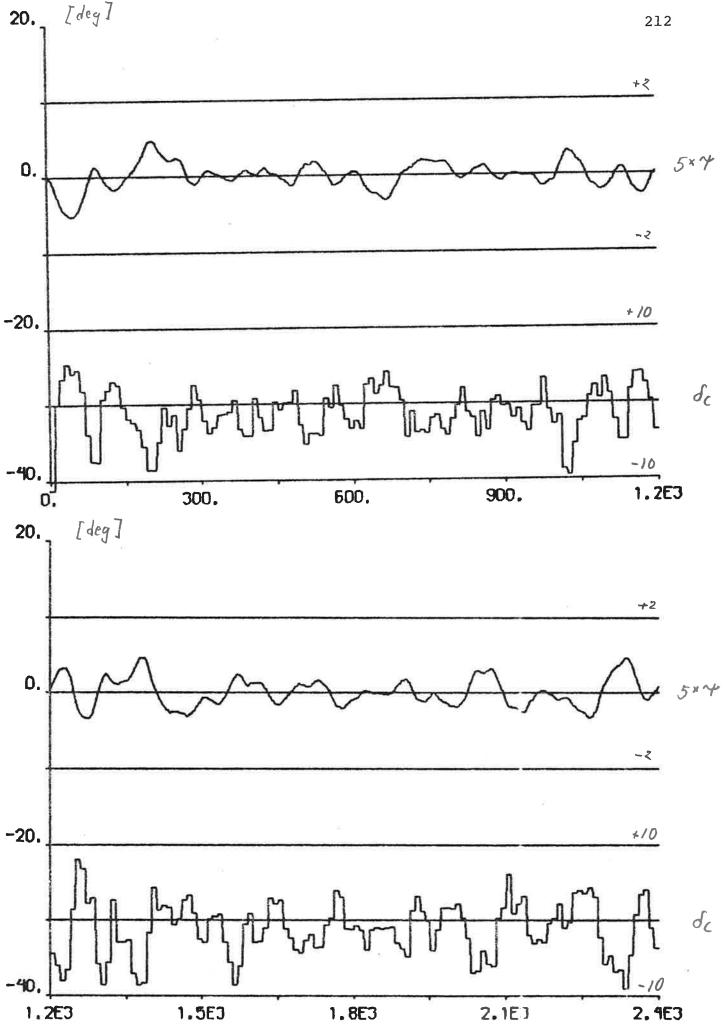


Fig. 4.38 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, PID-regulator using estimates from the Kalman filter (V_0 = 6.3 m/s).





<u>Fig. 4.39</u> - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, PID-regulator using estimates from the Kalman filter.



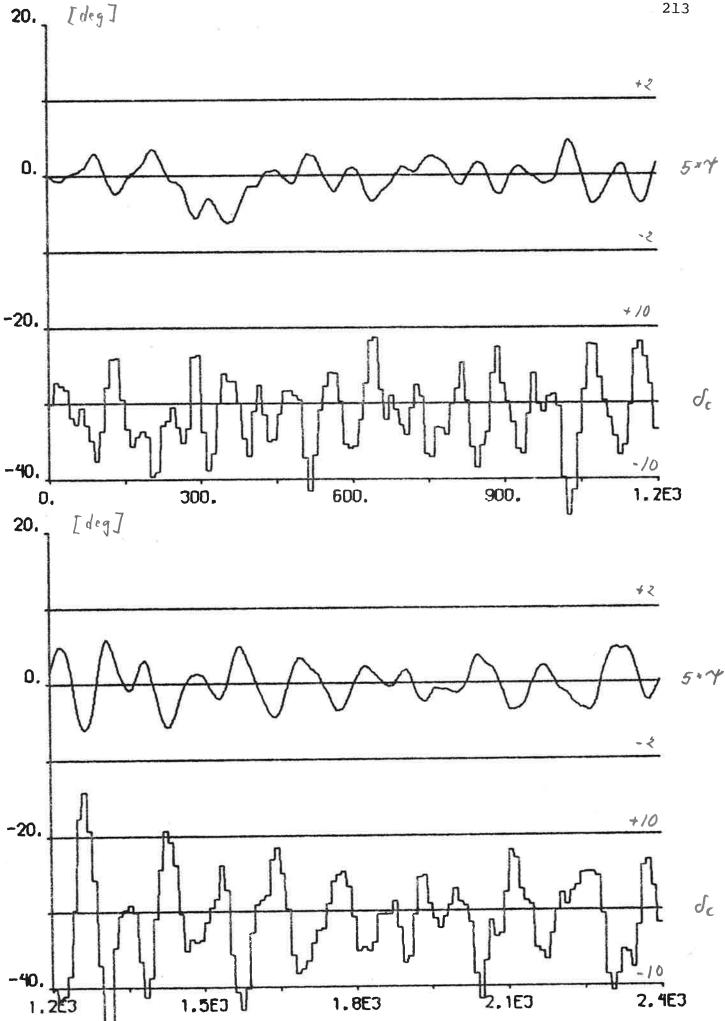
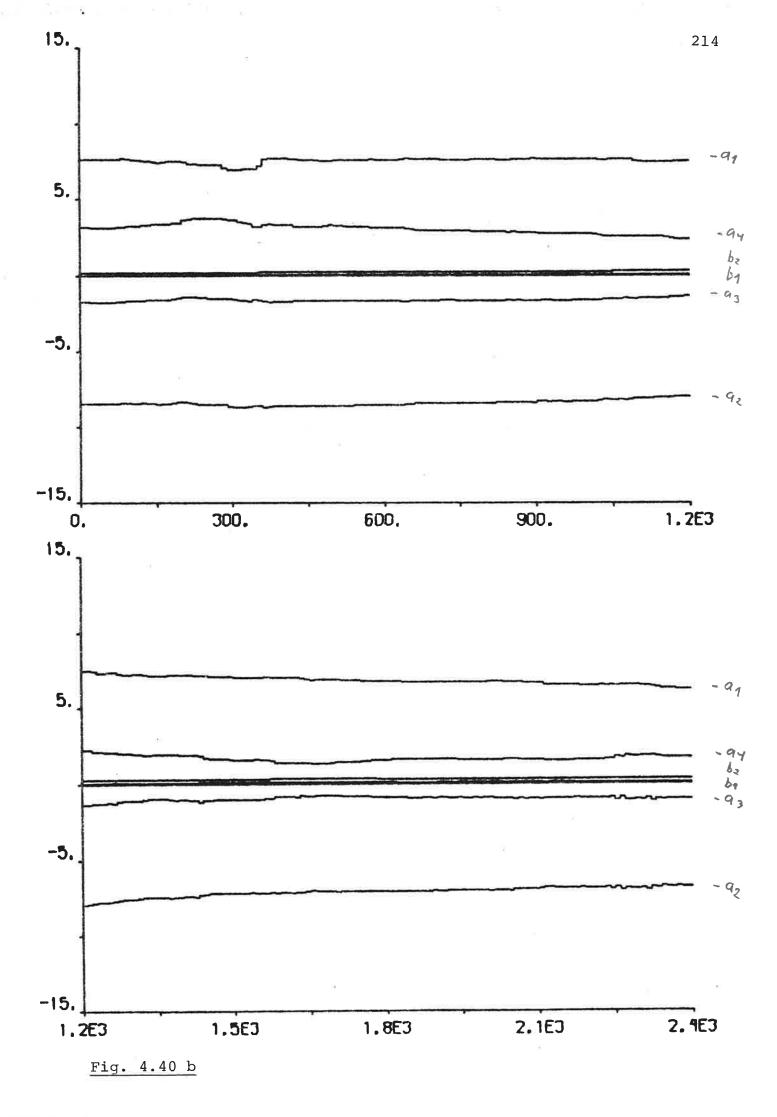
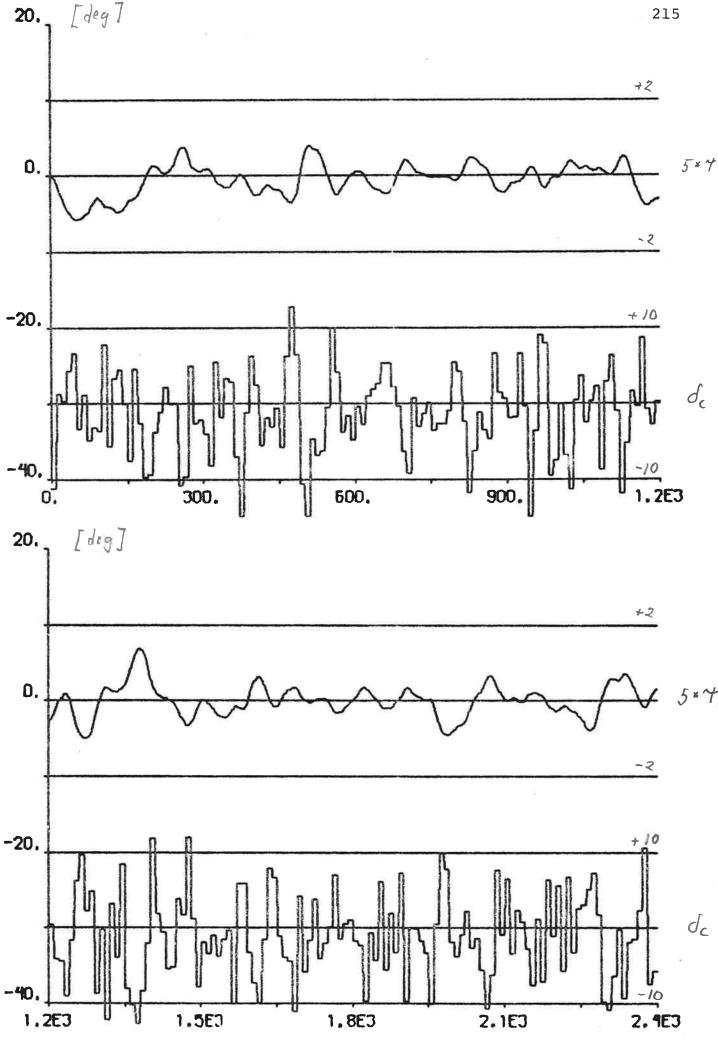


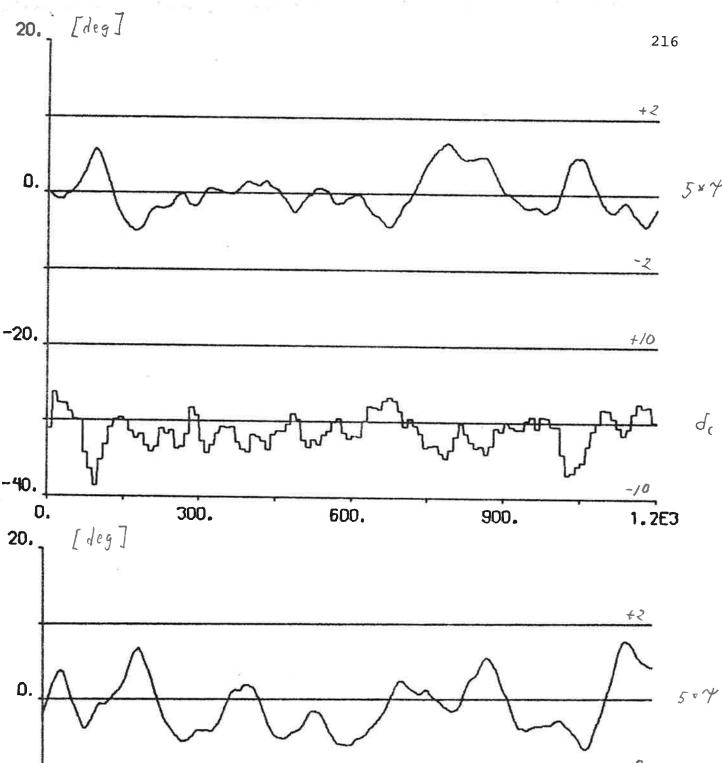
Fig. 4.40 a - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, self-tuning regulator using non-filtered measurements.







<u>Fig. 4.41</u> - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, PID-regulator using non-filtered measurements.



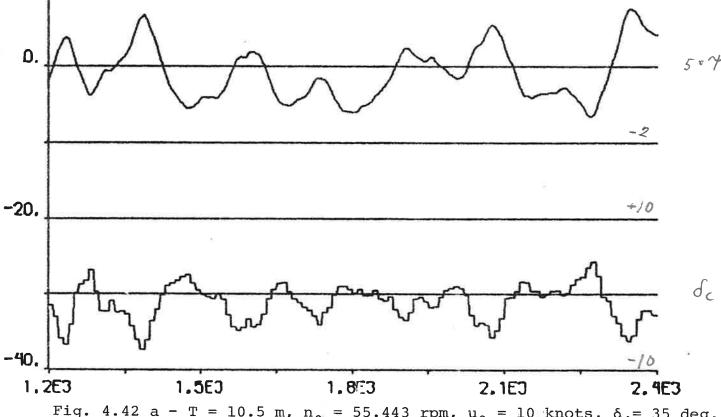


Fig. 4.42 a - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, self-tuning regulator using estimates from the Kalman filter (q_2^* = 0.6).



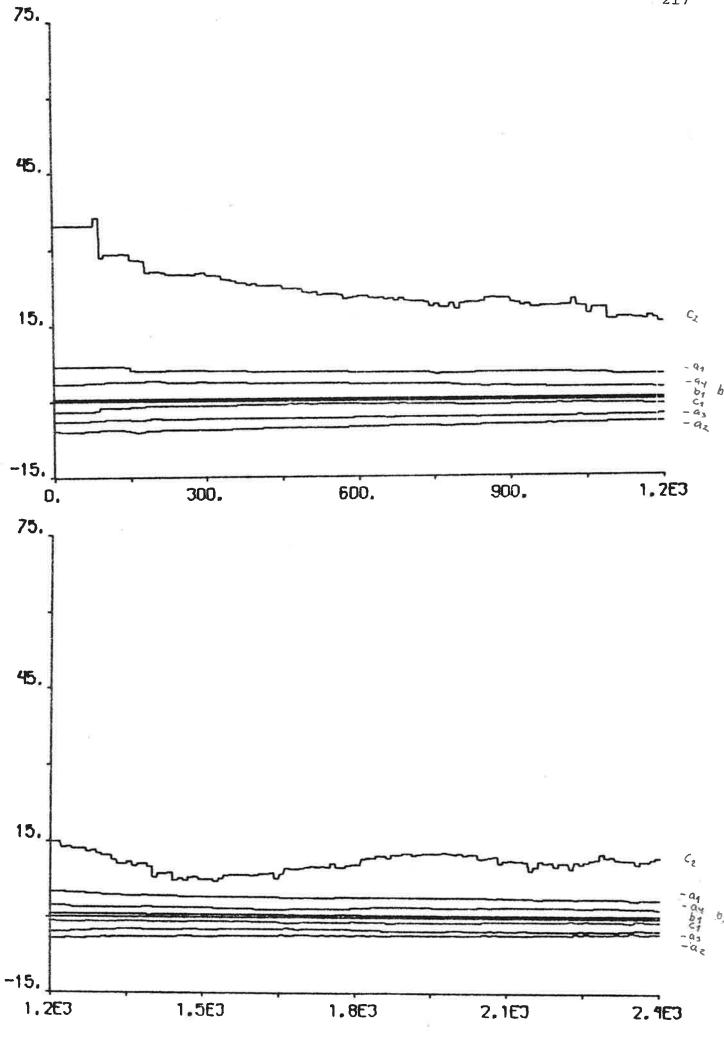


Fig. 4.42 b

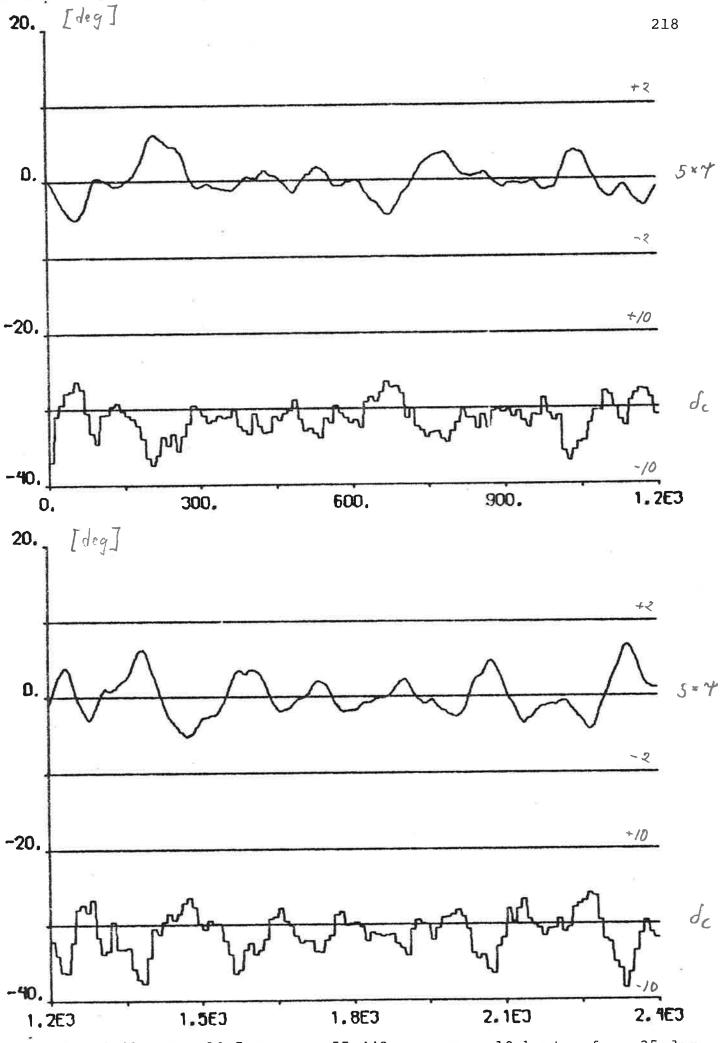


Fig. 4.43 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, δ_ℓ = 35 deg, PID-regulator using estimates from the Kalman filter (v_0 = 6.3 m/s).

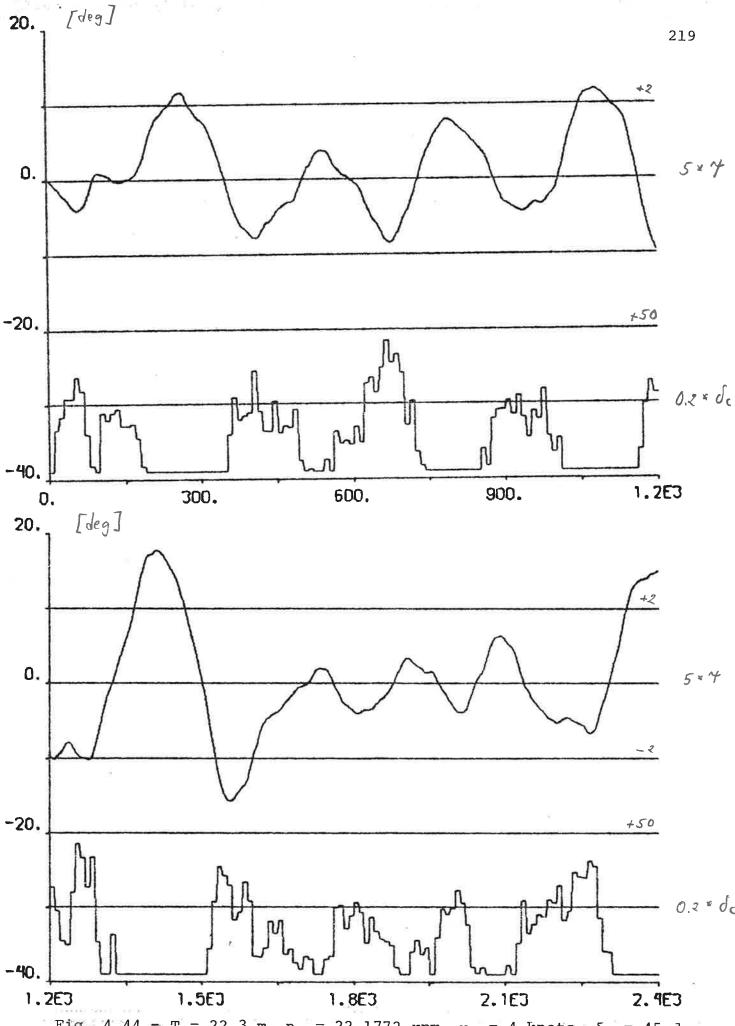


Fig. 4.44 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, PID-regulator using estimates from the Kalman filter.

Fig. 4.45 a - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, self-tuning regulator using non-filtered measurements.

2.1E3

2.4E3

1.8E3

-40.

1.2E3

1.5E3

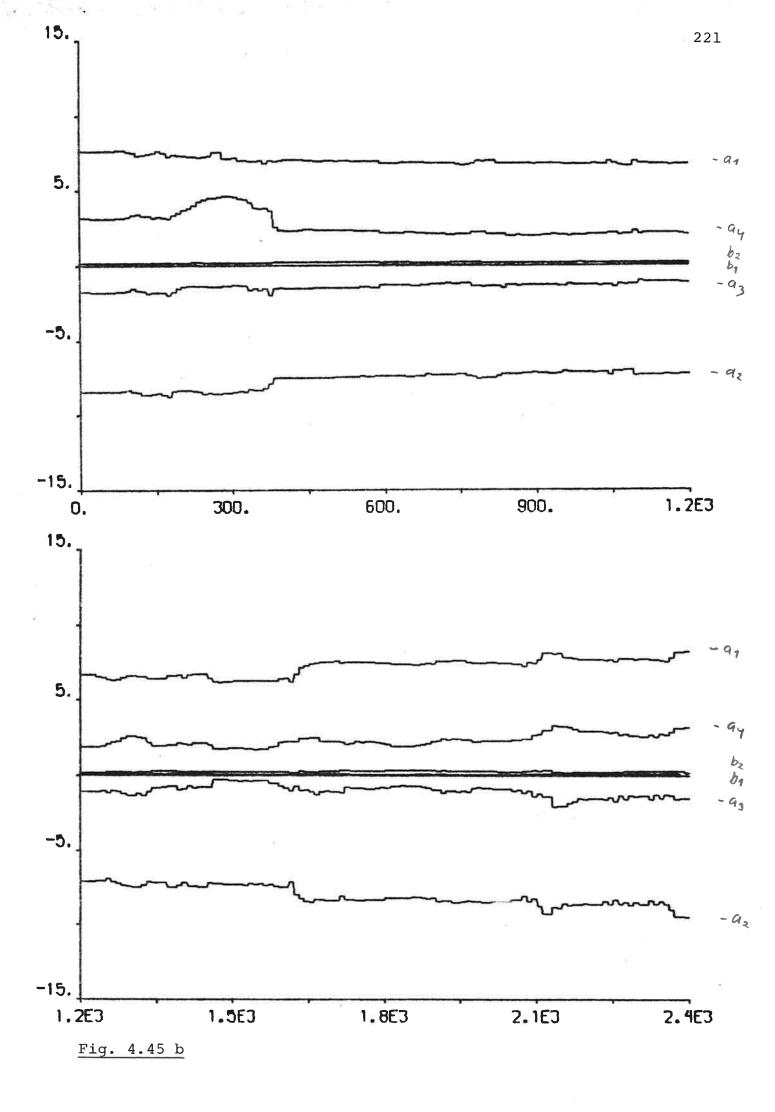
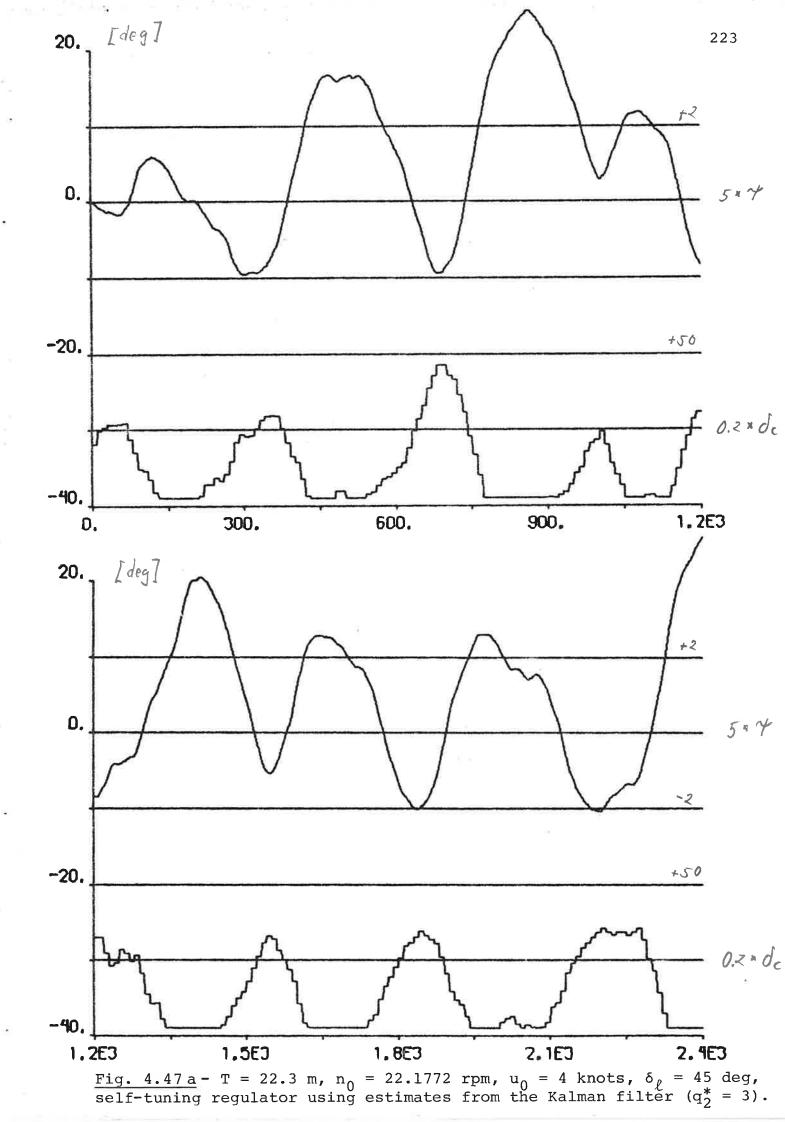
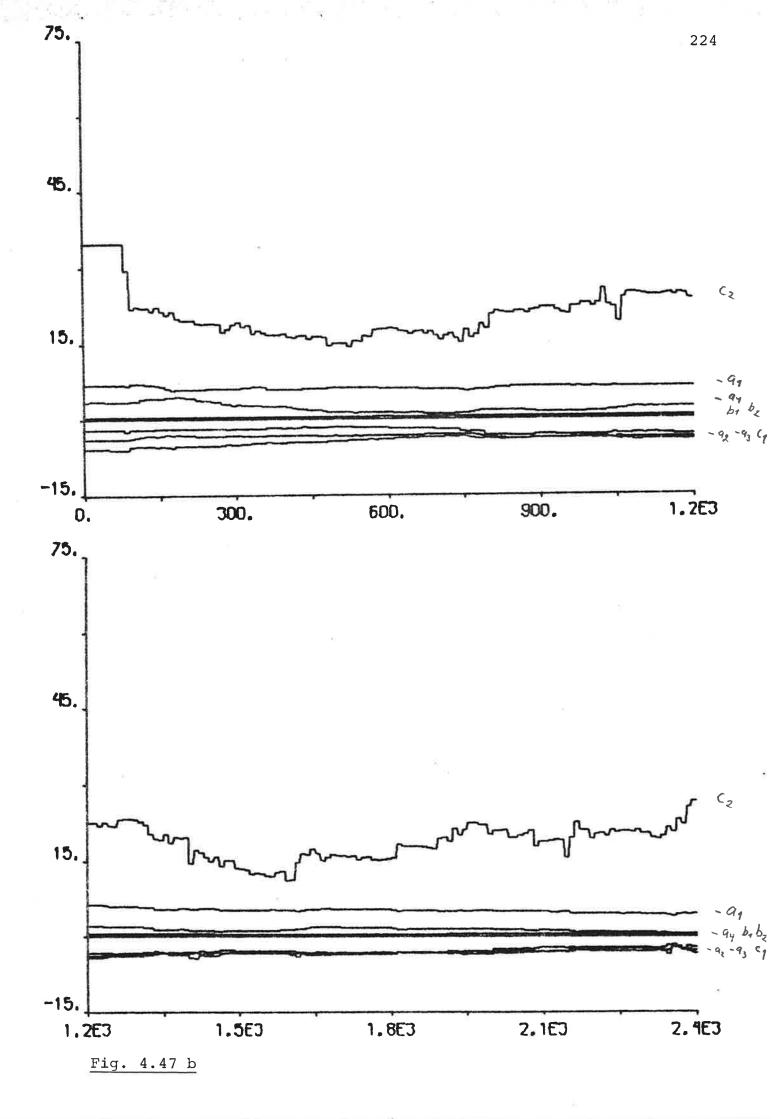


Fig. 4.46 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, PID-regulator using non-filtered measurements.





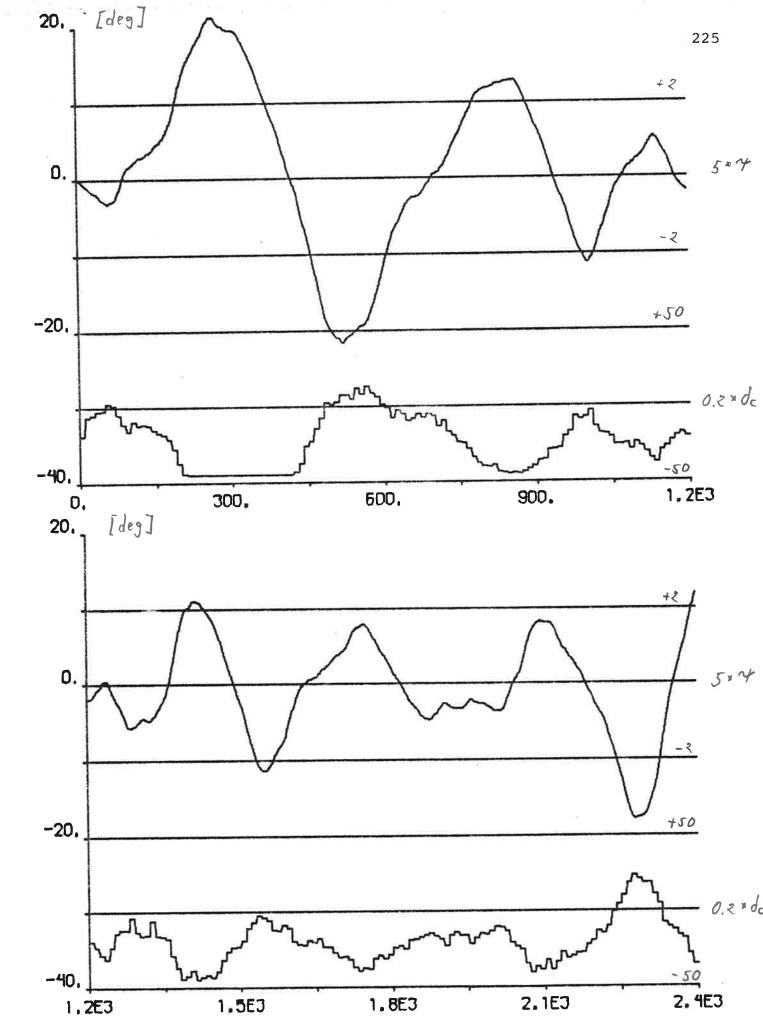
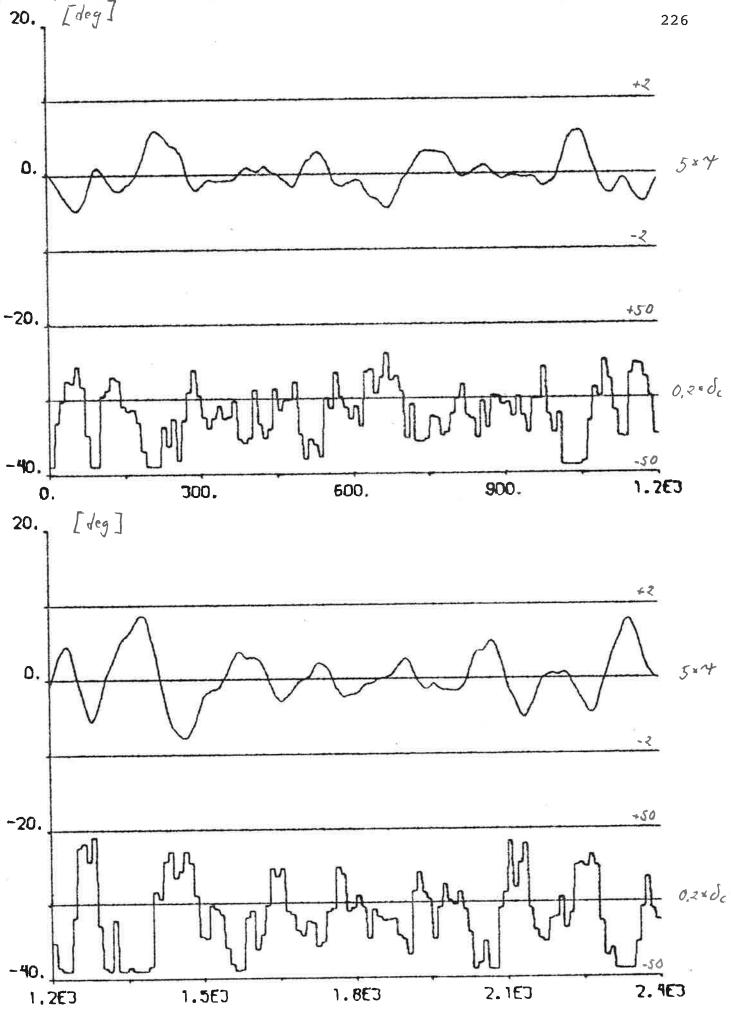


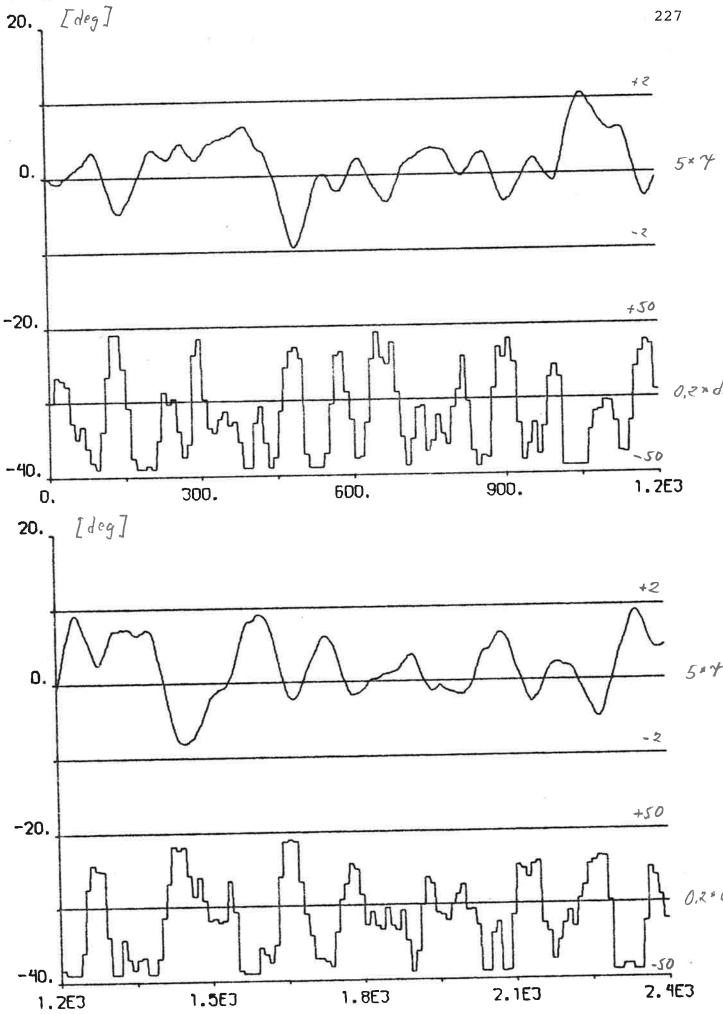
Fig. 4.48 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, PID-regulator using estimates from the Kalman filter (V_0 = 4 m/s).



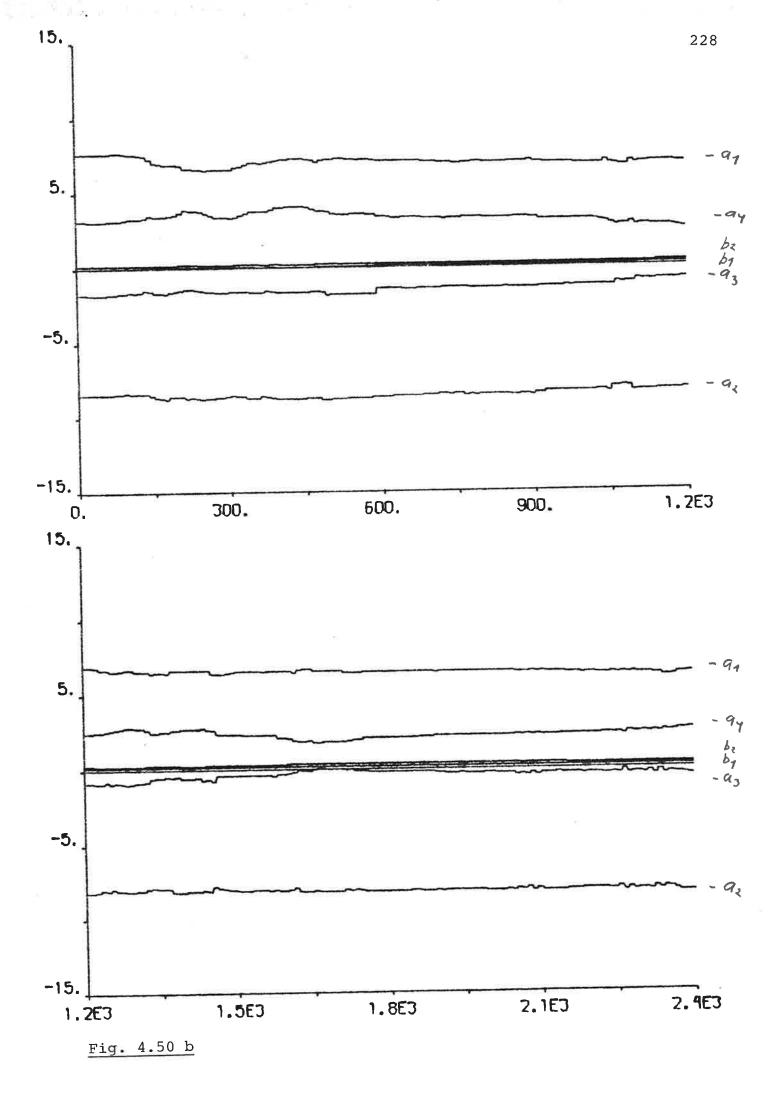


<u>Fig. 4.49</u> - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, PID-regulator using estimates from the Kalman filter.





<u>Fig. 4.50 a</u> - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg self-tuning regulator using non-filtered measurements.



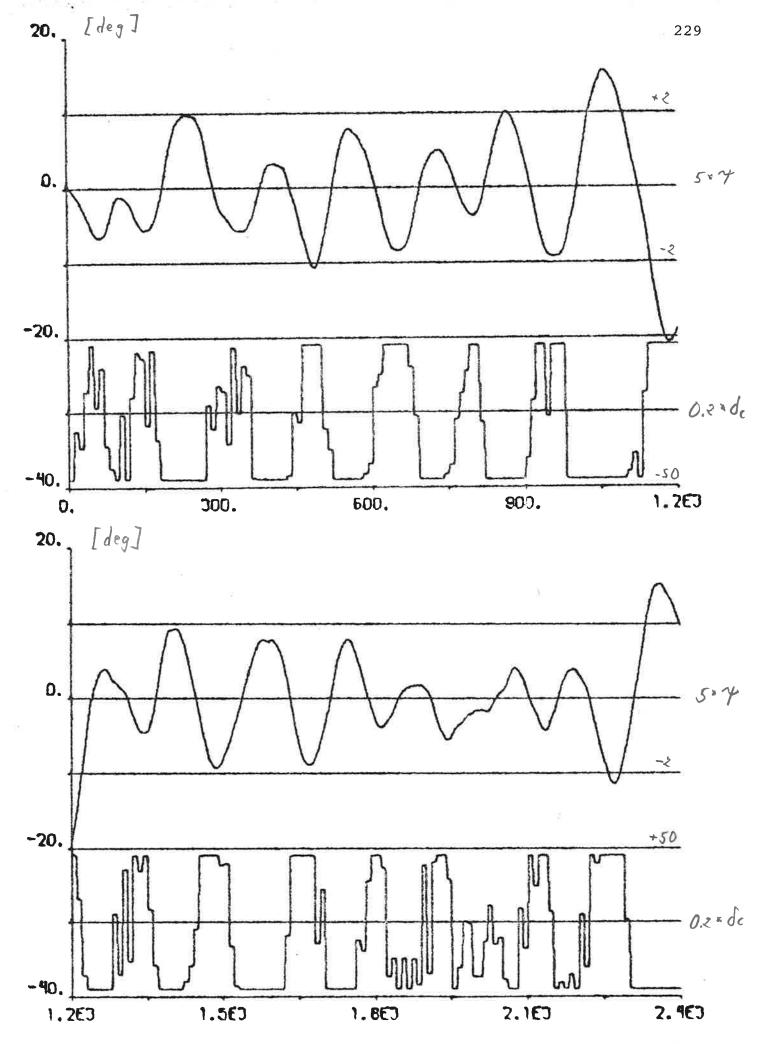


Fig. 4.51 - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, PID-regulator using non-filtered measurements.

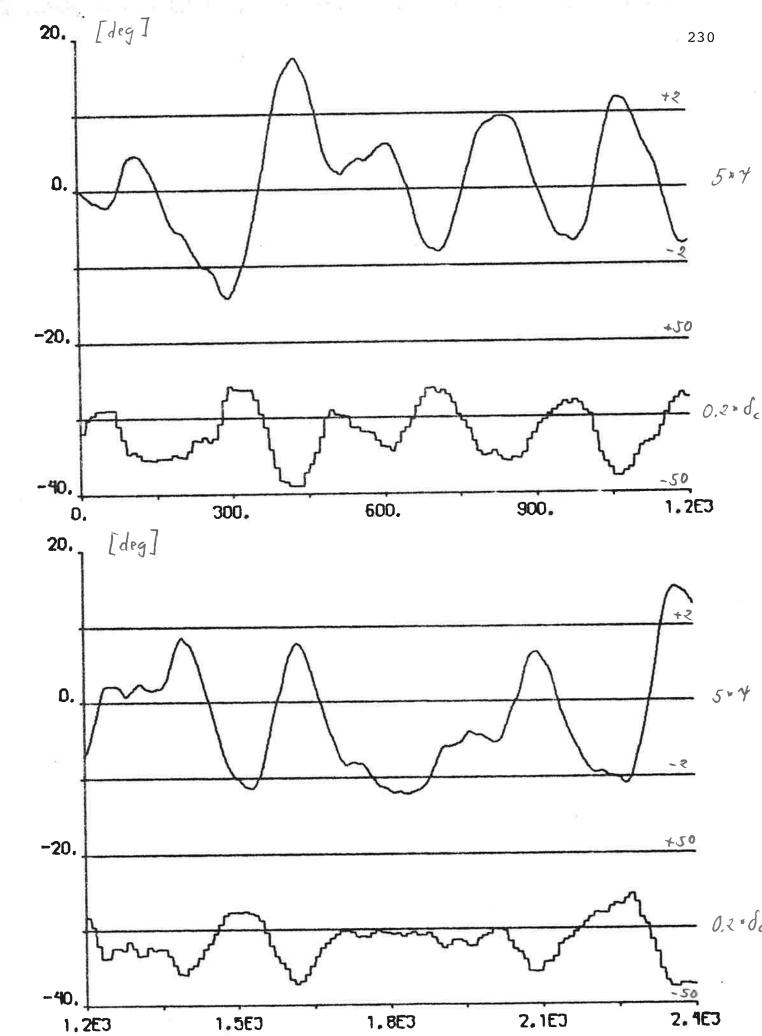
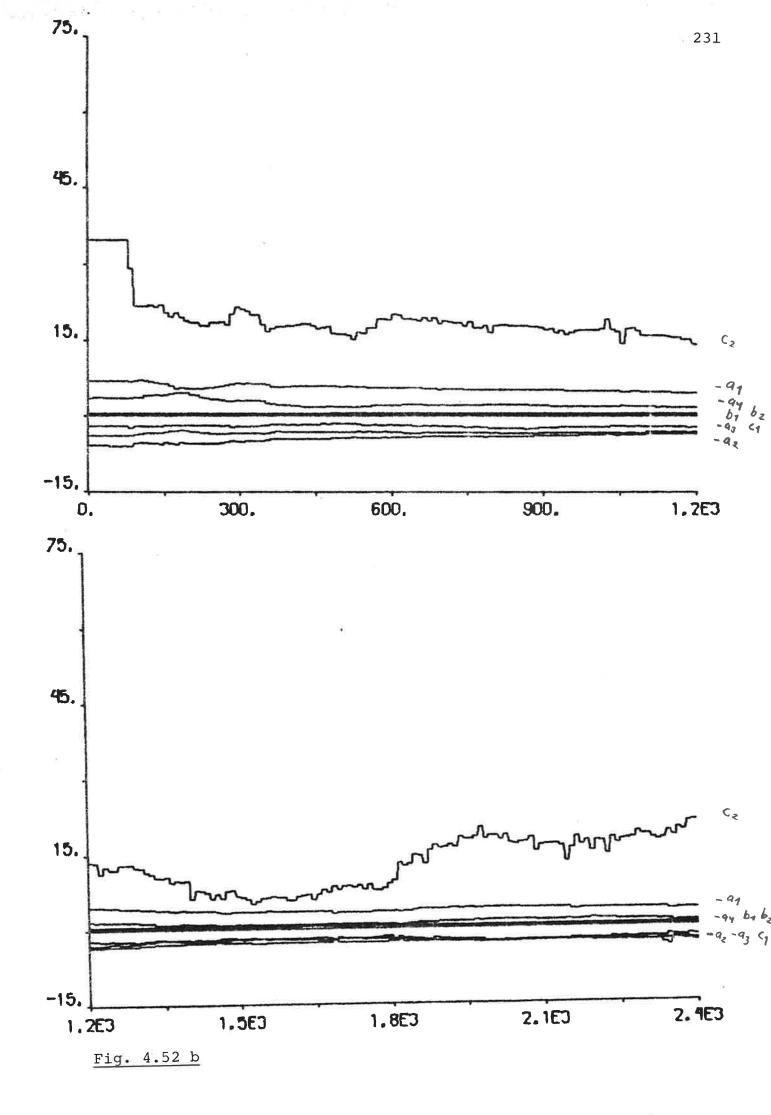


Fig. 4.52 a - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg self-tuning regulator using estimates from the Kalman filter (q_2^* = 3).



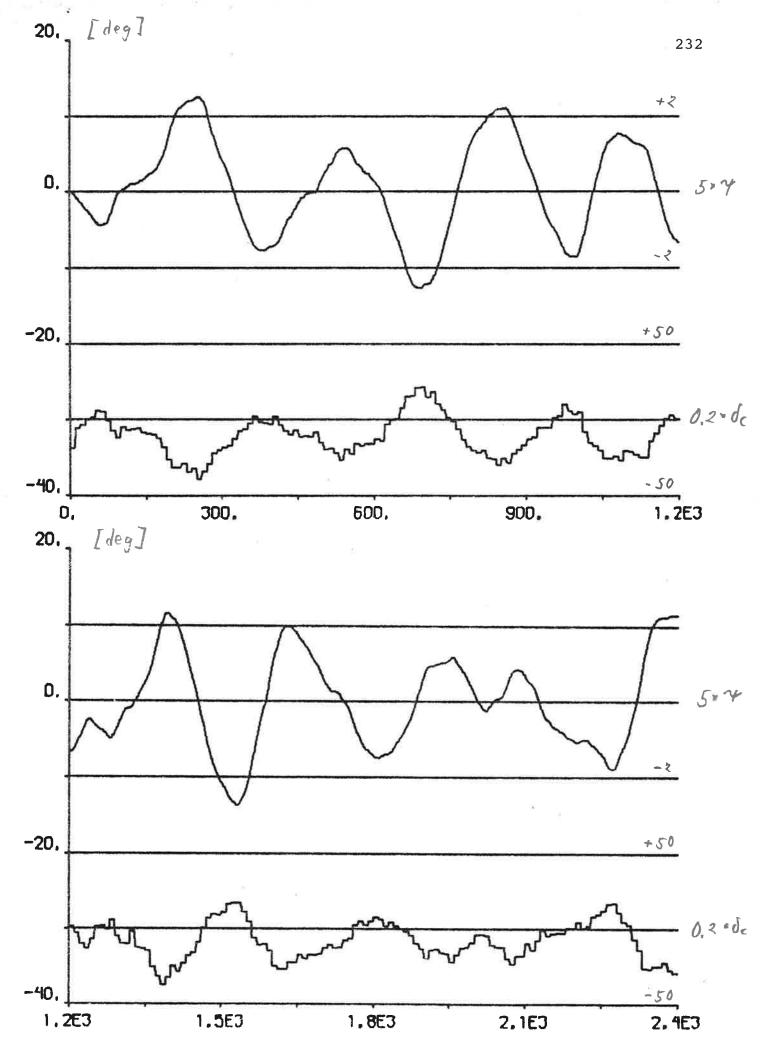


Fig. 4.53 - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, δ_ℓ = 45 deg, PID-regulator using estimates from the Kalman filter (V_0 = 4 m/s).

4.3 Yawing

Yawing simulations with the yaw regulator, using either the Kalman filter estimates or the non-filtered measurements, are shown in Figs 4.54 - 4.95. The initial speed \mathbf{u}_0 is equal to 15.8, 10 or 4 knots and the mean draught T is equal to 22.3 or 10.5 m. Course changes $\Delta\psi_{\text{ref}}$ of 2, 4, 45 and 120 deg are requested and the reference yaw rate \mathbf{r}_{ref} is equal to 0, 0.1 or 0.3 deg/s. The rudder limit δ_ℓ is always equal to 45 deg. Examples of Kalman filter estimates during yaws are shown in Figs 4.57, 4.58, 4.65, 4.66, 4.88 and 4.93. A summary of the simulations is given in Table 4.8.

It can be concluded that the performance of the Kalman filter, when the draught and the initial speed are varied, is approximately of the same quality during yawing as during straight course keeping (cf Section 4.1). Notice, however, that the estimates δ_0 , \hat{d}_v , \hat{d}_r and \hat{d}_δ are disturbed rather much during yaws. This implies that the correct yaw rate is not always kept when the yaw regulator uses the Kalman filter estimates. Of course, an incorrect yaw rate is always kept when non-filtered measurements are used, since the yaw rate measurement is biased in the simulations. A straightforward way to improve the Kalman filter is to skip the updating of $\hat{\delta}_0$, \hat{d}_v , \hat{d}_r and \hat{d}_δ during yaws.

The only parameter of the yaw regulator, that is changed when the speed is changed, is \overline{c}_2 . The following relation is found empirically:

$$\overline{c}_2 = \sqrt{\frac{V_0}{V_s}} c_2^*$$

where $V_0 = 8$ m/s and $c_2^* = 50$ s. This relation is used in the simulations of this section.

| T [m] | ⁿ 0 [rpm] | u ₀ [knots] | Self-tuning reg. and yaw reg. using Kalman filter estimates | Self-tuning reg. and yaw reg. using non-filtered measurements | -c ₂ [s] | Δψ _{ref} | r _{ref} [deg/s] | Fig. | Remarks |
|--|--|--|---|---|--|--|--|--|----------|
| 22.3 22.3 22.3 22.3 22.3 22.3 22.3 22.3 | 87.6 87.6 87.6 87.6 87.6 87.6 87.6 | 15.8 15.8 15.8 15.8 15.8 15.8 15.8 | x x x x | x x x | 50 50 50 50 50 50 50 50 | 2 4 45 120 4 45 120 | 0 0.1 0.3 0.1 0.3 0.1 0.1 0.3 | 4 . 54 4 . 55 4 . 56 4 . 57 4 . 58 4 . 59 4 . 60 4 . 61 | x) x) |
| 10.5 10.5 10.5 10.5 10.5 10.5 10.5 | 87.6 87.6 87.6 87.6 87.6 87.6 87.6 | 15.8 15.8 15.8 15.8 15.8 15.8 15.8 | x x x x | x x x | 50 50 50 50 50 50 50 50 | 2 4 4 45 120 4 45 120 | 0 0.1 0.3 0.1 0.3 0.1 0.1 0.3 | 4.62 4.63 4.64 4.65 4.66 4.67 4.68 4.69 | x) x) |
| 22.3 22.3 22.3 22.3 22.3 22.3 22.3 22.3 | 55.443 55.443 55.443 55.443 55.443 55.443 55.443 | 10 10 10 10 10 10 10 | x x x x | x x x | 63.25 63.25 63.25 63.25 63.25 63.25 63.25 63.25 | 2 4 4 45 120 4 45 120 | 0 0.1 0.3 0.1 0.3 0.1 0.1 0.3 | 4.70 4.71 4.72 4.73 4.74 4.75 4.76 4.77 | |
| 10.5 10.5 10.5 10.5 10.5 10.5 10.5 | 55.443 55.443 55.443 55.443 55.443 55.443 55.443 | 10 10 10 10 10 10 10 | x x x x | x x x | 63.25 63.25 63.25 63.25 63.25 63.25 63.25 63.25 | 2 4 4 45 120 4 45 120 | 0 0.1 0.3 0.1 0.3 0.1 0.1 0.3 | 4.78 4.79 4.80 4.81 4.82 4.83 4.84 4.85 | |
| 22.3 22.3 22.3 22.3 22.3 | 22.1772 22.1772 22.1772 22.1772 22.1772 | 4 4 4 4 4 | x x x | x x | 100 100 100 100 100 | 2 4 45 4 45 | 0 0.1 0.1 0.1 0.1 | 4.86 4.87 4.88 4.89 4.90 | х) |
| 10.5 10.5 10.5 10.5 10.5 | 22.1772 22.1772 22.1772 22.1772 22.1772 | 4 4 4 4 4 | x x x | x x | 100 100 100 100 100 | 2 4 45 4 45 | 0 0.1 0.1 0.1 0.1 | 4.91 4.92 4.93 4.94 4.95 | х) |

 $[\]mathbf{x}$) The Kalman filter estimates are shown.

Table 4.8 - Summary of yawing simulations. The initial reference course ψ_{ref} is equal to 0 deg and the course change $\Delta\psi_{\text{ref}}$ is requested after 100 s. The rudder limit δ_{ℓ} is equal to 45 deg.

It can be concluded from the simulations that the performance of the yaw regulator, when Kalman filter estimates are used, is very good for different load conditions and speeds, with one exception: the performance when T = 22.3 m and $u_0 = 4$ knots is rather bad (cf Figs 4.86 - 4.88). Maybe it is possible to increase the yawing quality in this case by introducing more speed dependent parameters of the yaw regulator. Notice, however, that the forward speed $\boldsymbol{u}_{\mathrm{m}}$ is decreased to the approximate value 1 knot, when T = 22.3 m, $u_0 = 4 \text{ knots}$, $\Delta \psi_{\text{ref}} = 45 \text{ deg and } r_{\text{ref}} = 0.1 \text{ deg/s}$ (cf Fig. 4.88 e). It is, of course, rather difficult to obtain a good performance in such an extremely small speed. When the initial speed un is equal to 10 knots, rather large rudder deviations are required compared to the case when $u_0 = 15.8$ knots. This is necessary, however, to obtain a good performance.

The performance of the yaw regulator, when non-filtered measurements are used, is not at all as good as the case when Kalman filter estimates are used. The rudder deviations are in general very large due to the noisy yaw rate measurements. It is probably possible to improve the performance by decreasing the values of the gain factors of the yaw regulator. See Källström (1976b). It can also be concluded that it is highly desired to filter the yaw rate in some way, if the measurements are very noisy. If the resolution of the heading measurements is good, it is also possible to perform a difference approximation to obtain a yaw rate estimate.

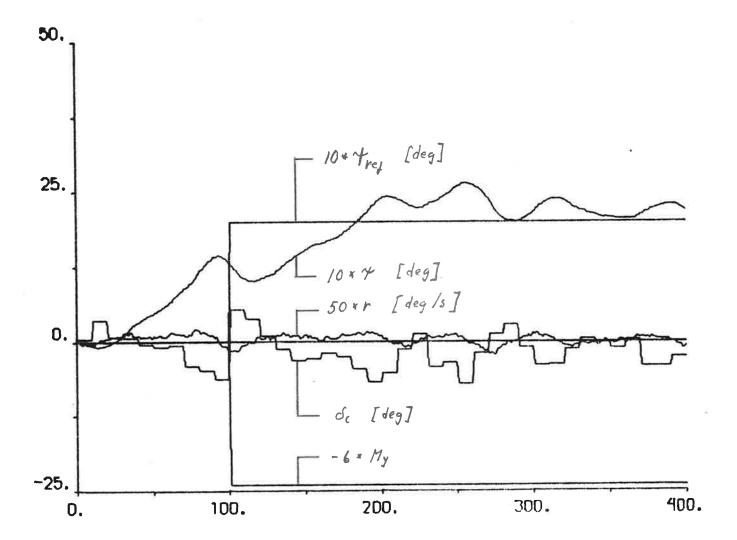


Fig. 4.54 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\text{ref}} = 2 \text{ deg, } r_{\text{ref}} = 0 \text{ deg/s, self-tuning regulator}$ and yaw regulator using estimates from the Kalman filter $(\overline{c}_2 = 50 \text{ s})$.

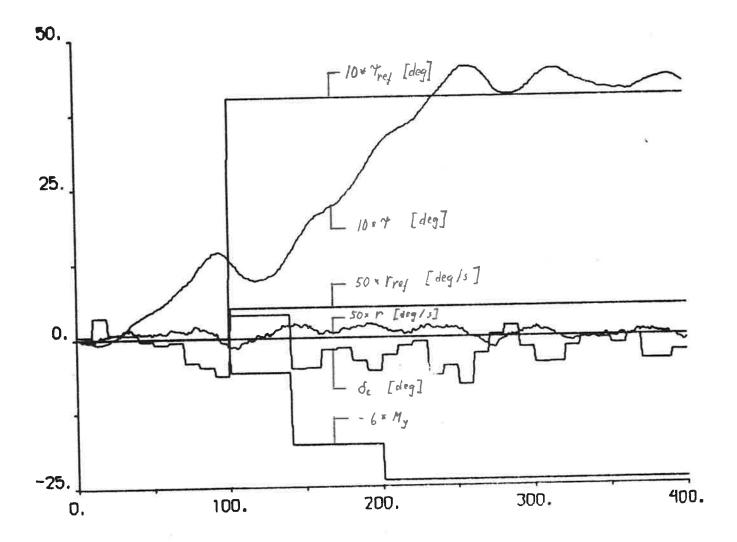


Fig. 4.55 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).

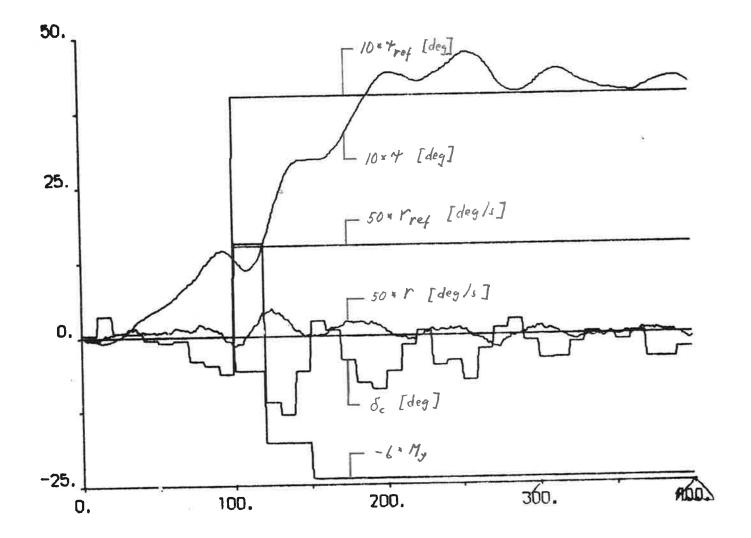
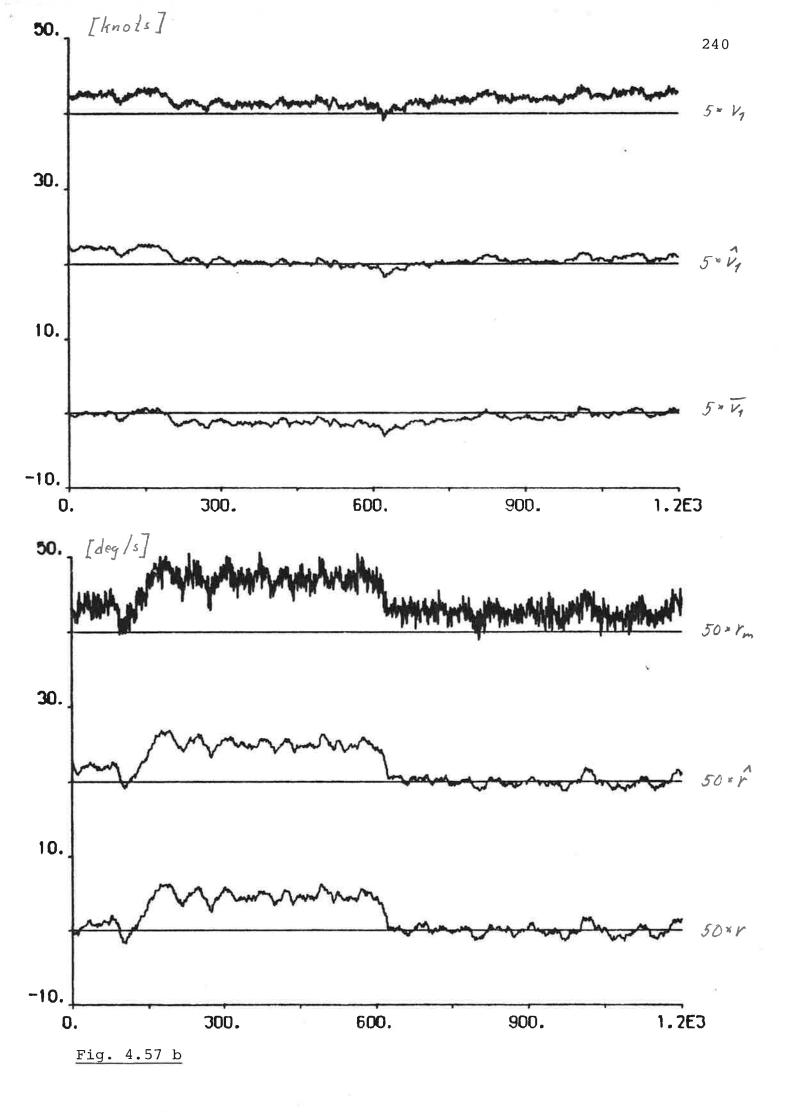
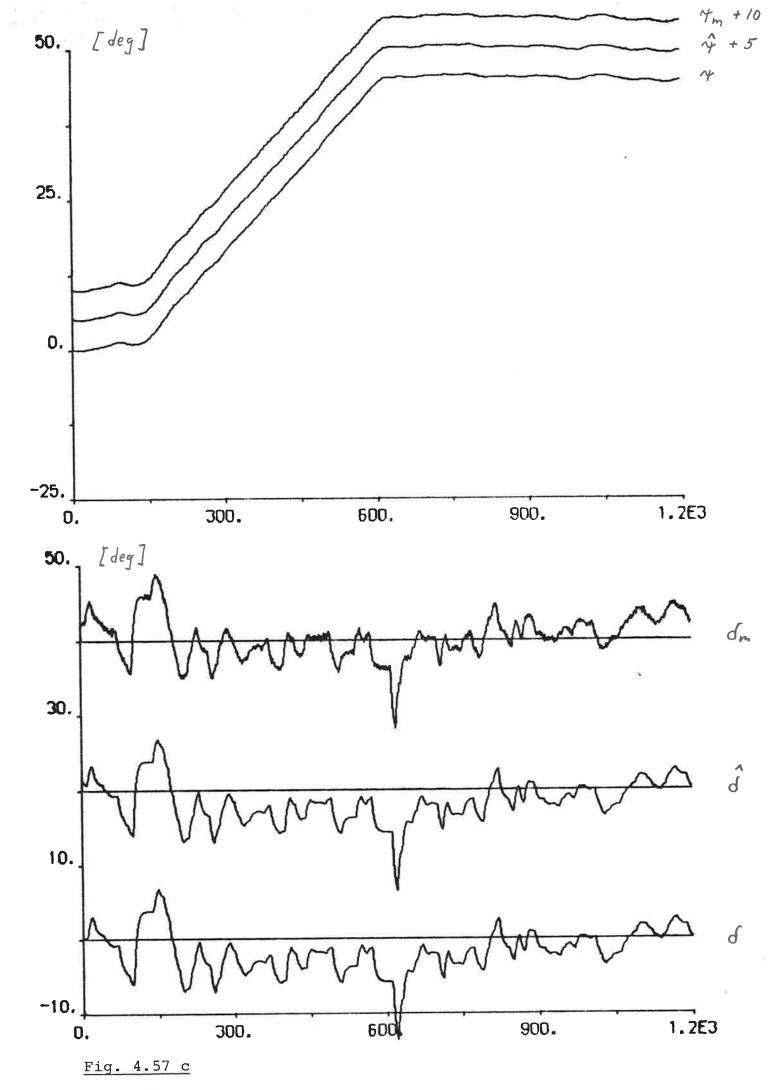
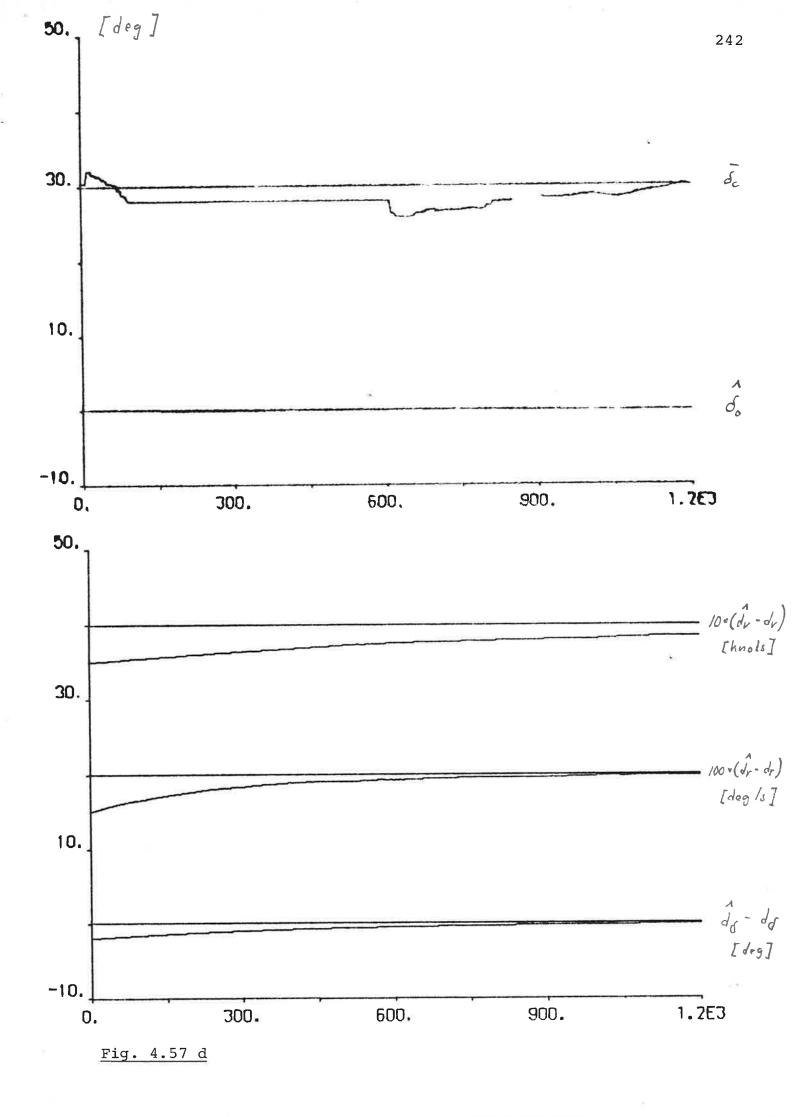


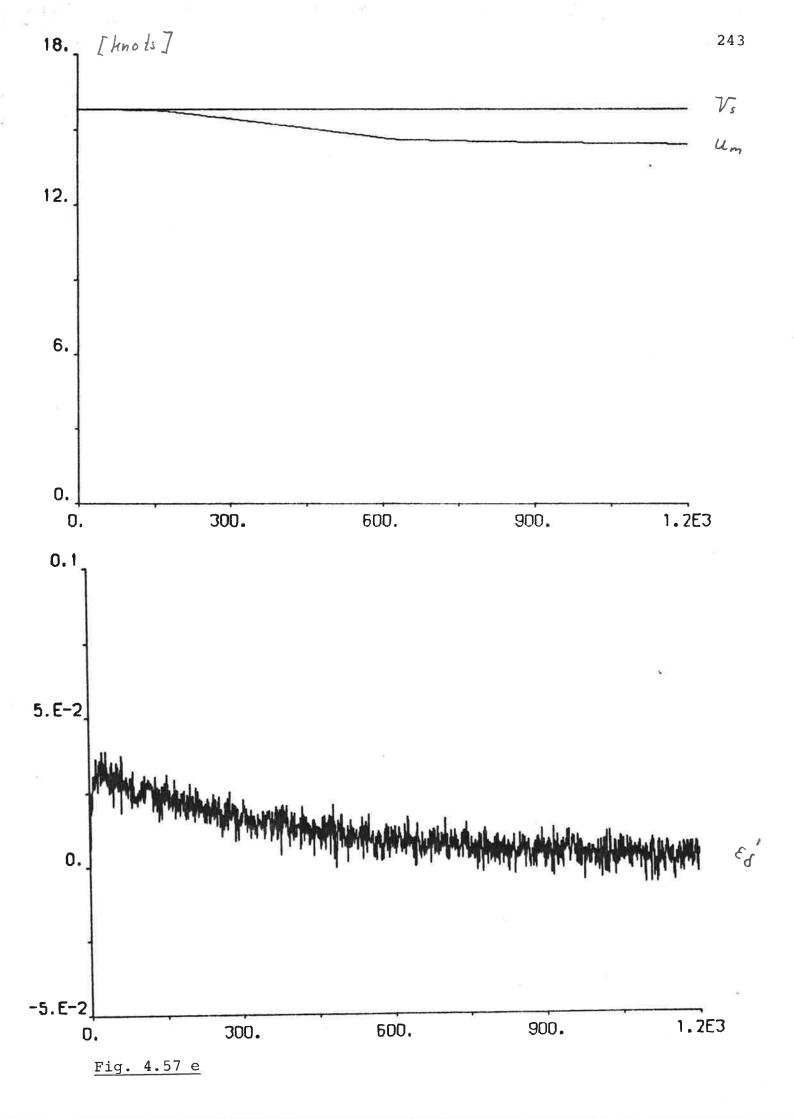
Fig. 4.56 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.3 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).

Fig. 4.57 a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (c_2 = 50 s).









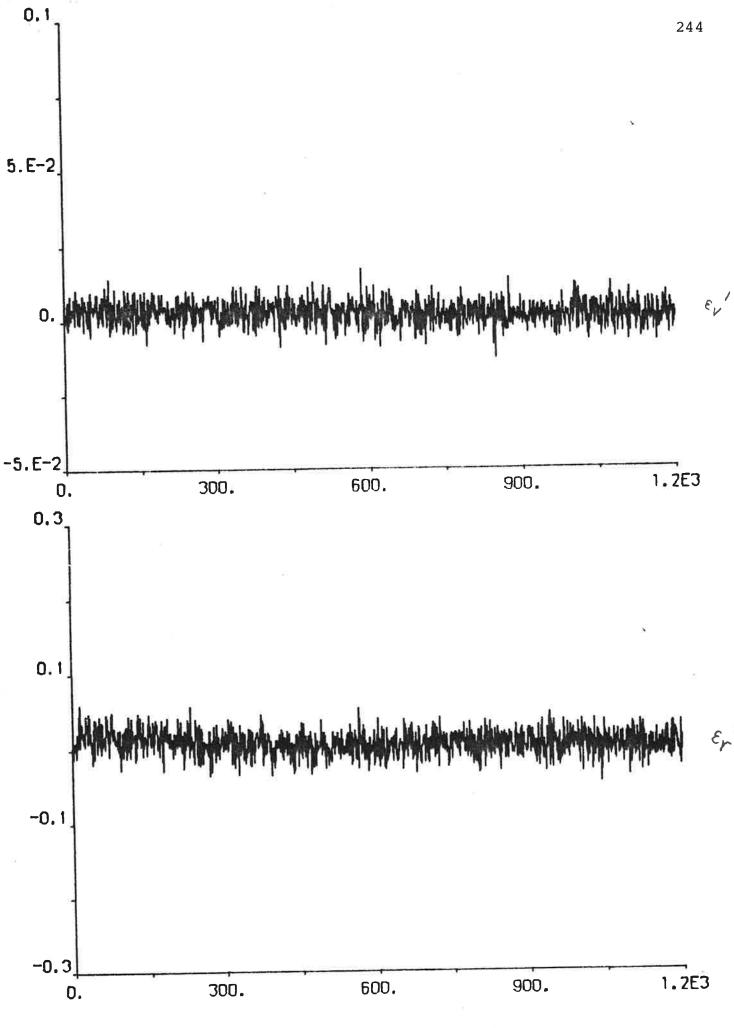


Fig. 4.57 f



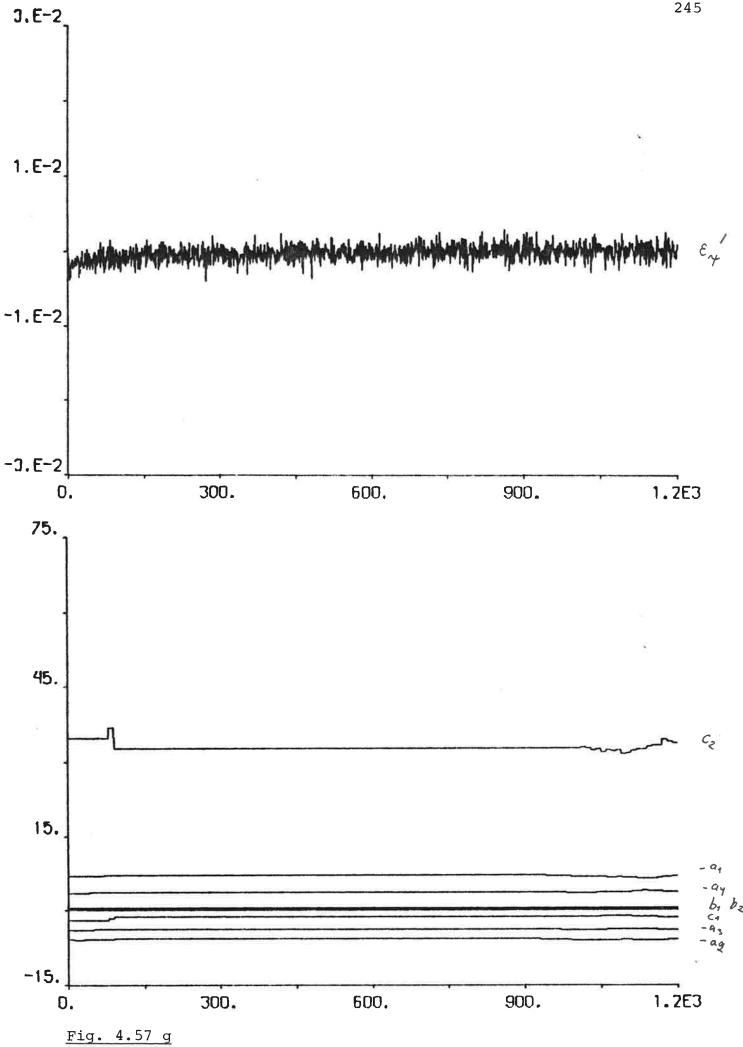
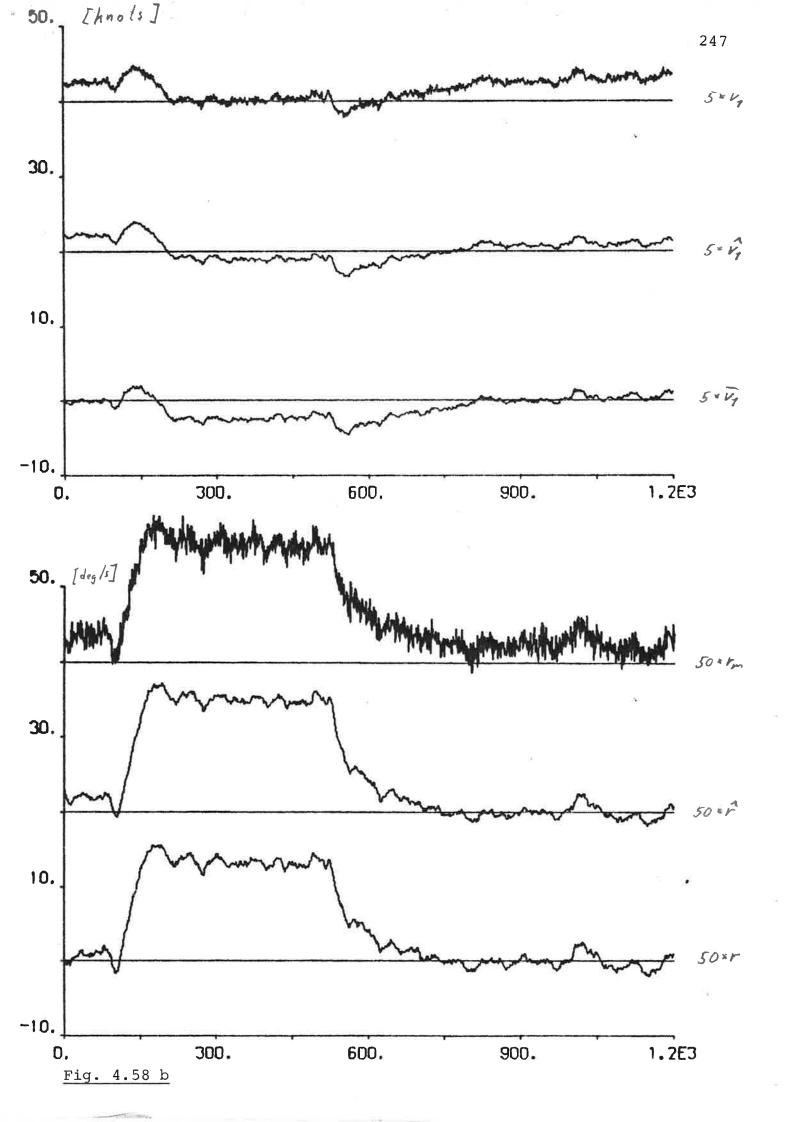
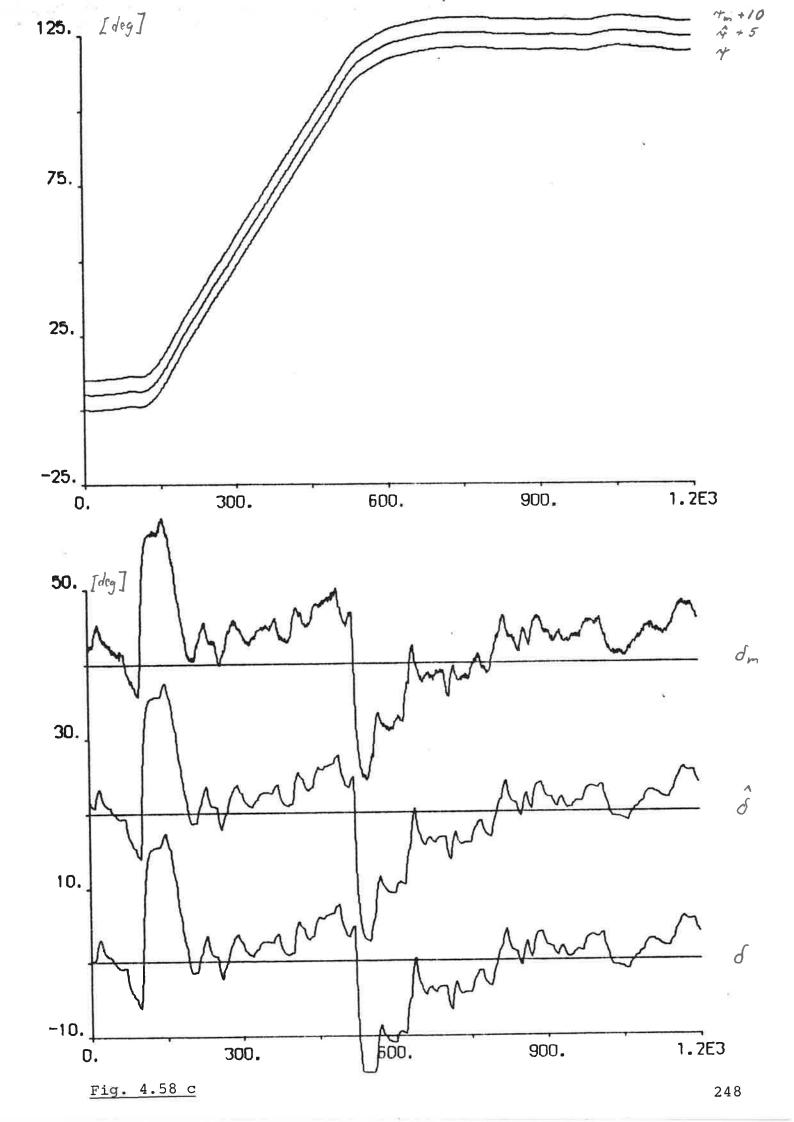


Fig. 4.58 a - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{ref}$ = 120 deg, r_{ref} = 0.3 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).





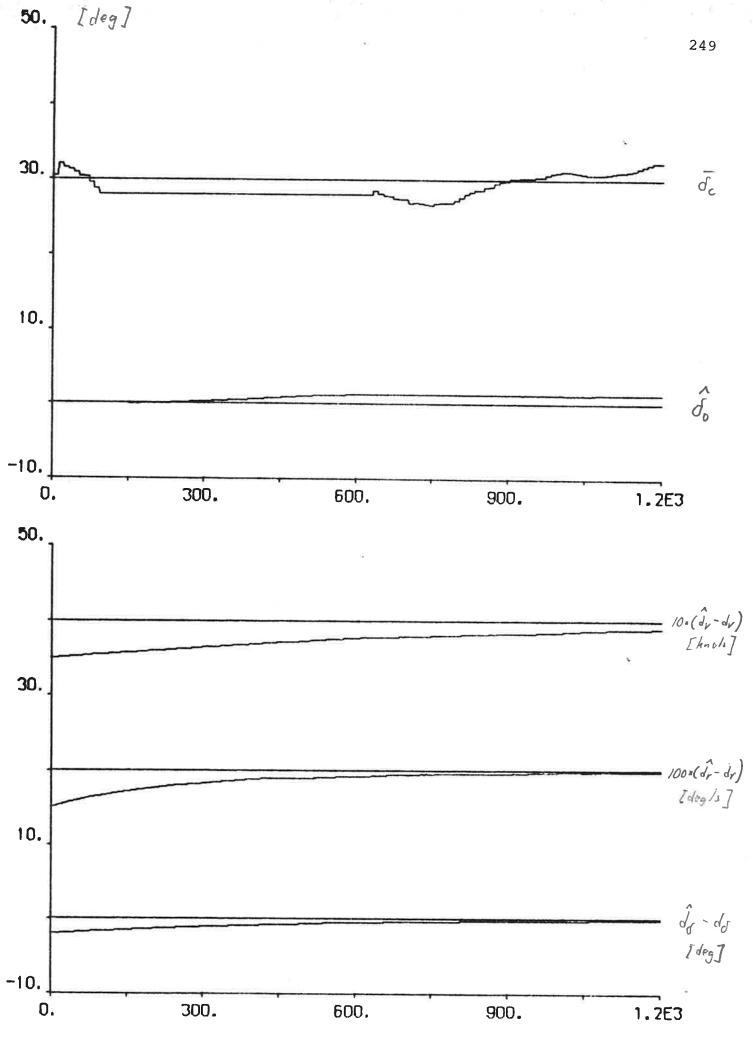
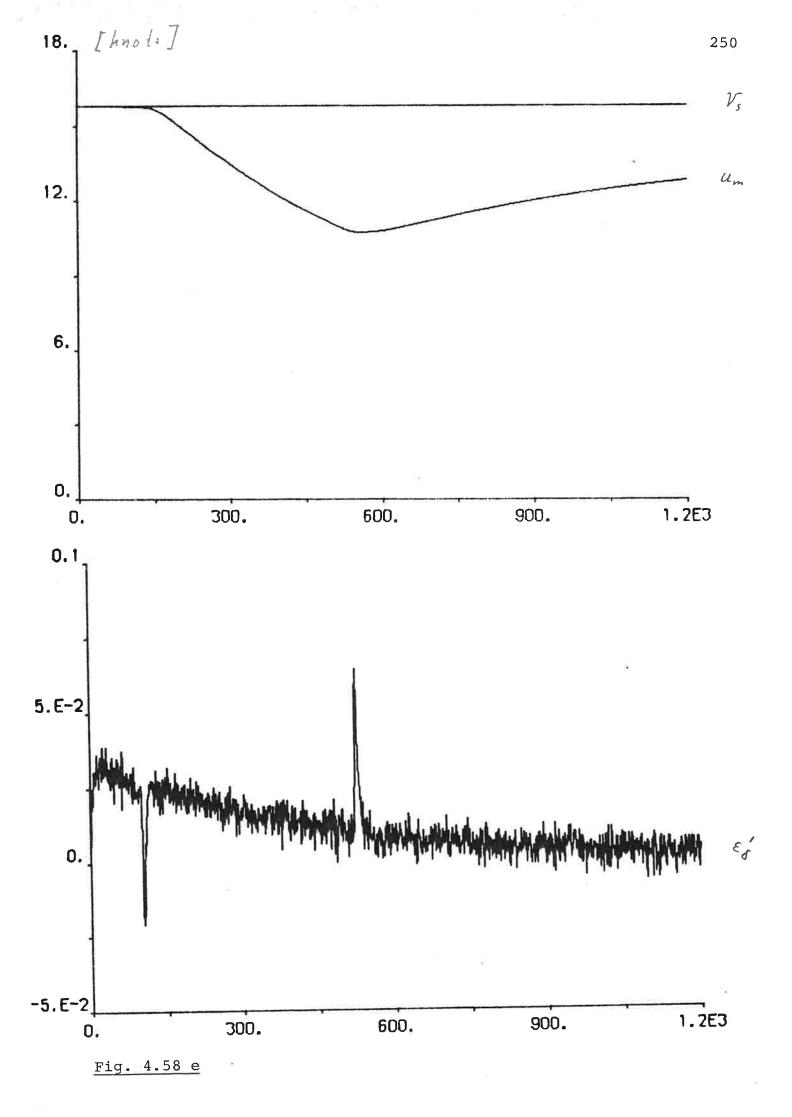
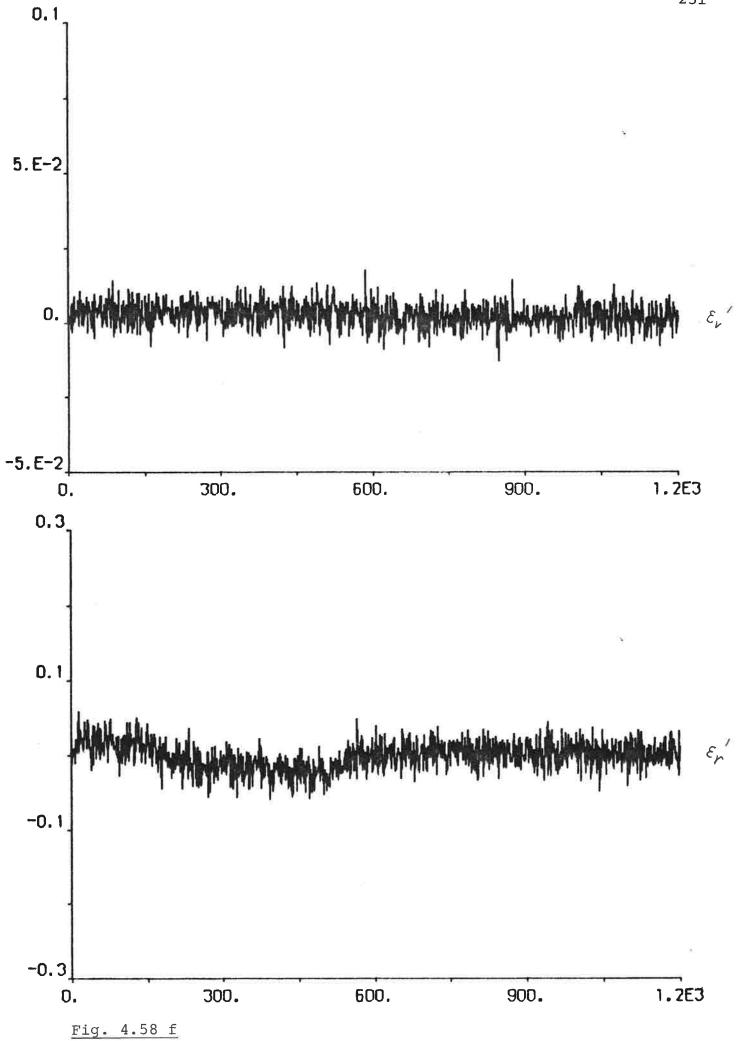
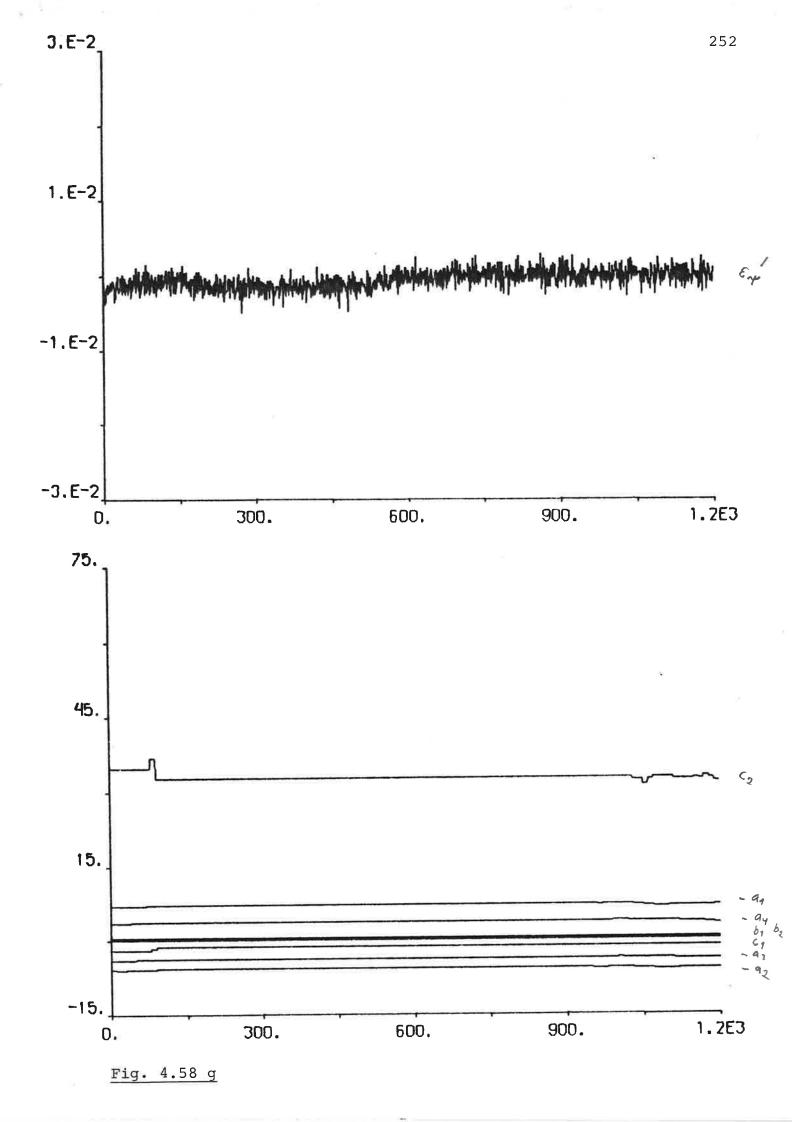


Fig. 4.58 d









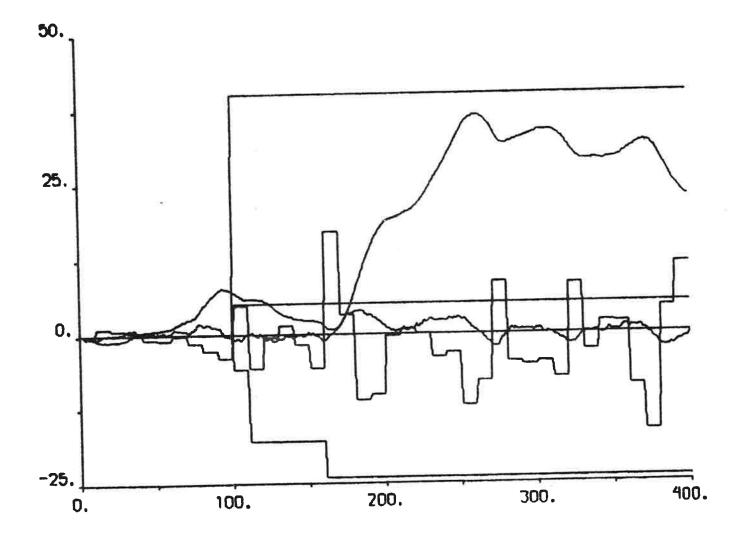


Fig. 4.59 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\rm ref} = 4 \ \rm deg, \ r_{\rm ref} = 0.1 \ \rm deg/s, \ self-tuning \ regulator \ and \ yaw \ regulator \ using \ non-filtered \ measurements (<math>\overline{c}_2$ = 50 s).

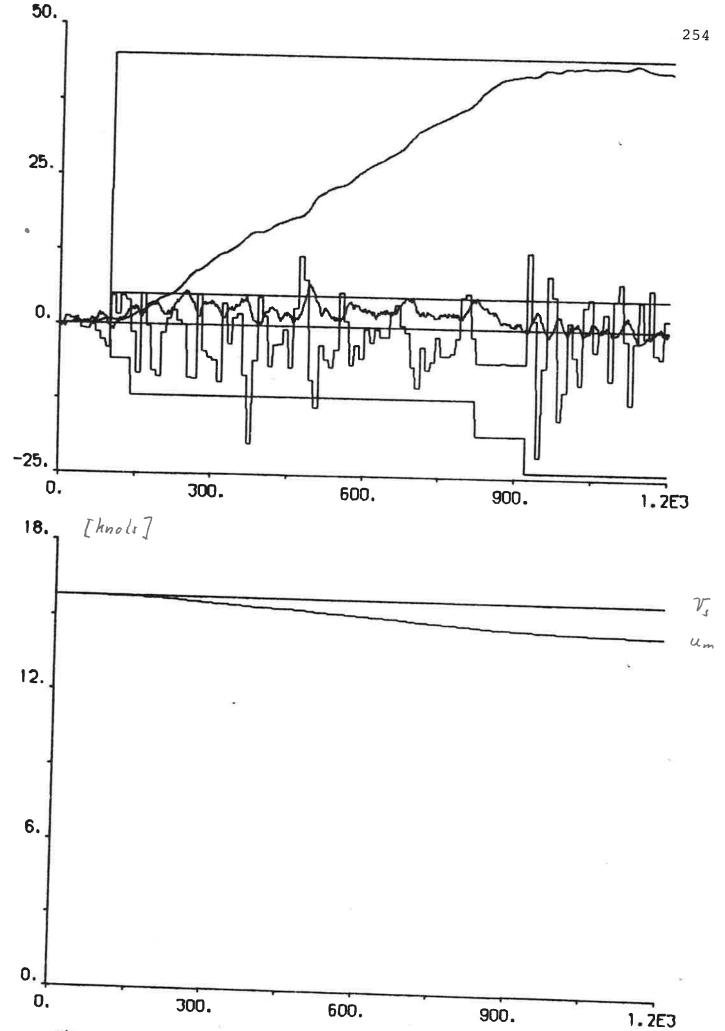


Fig. 4.60 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\text{ref}}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 50 s).

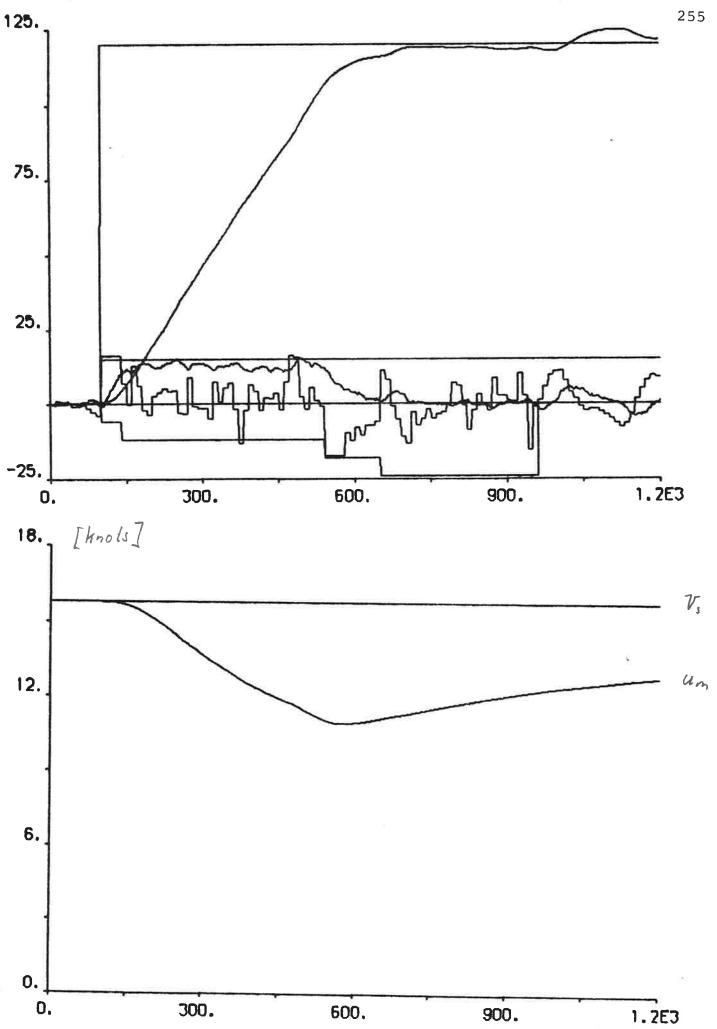


Fig. 4.61 - T = 22.3 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{ref}$ = 120 deg, r_{ref} = 0.3 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 50 s).

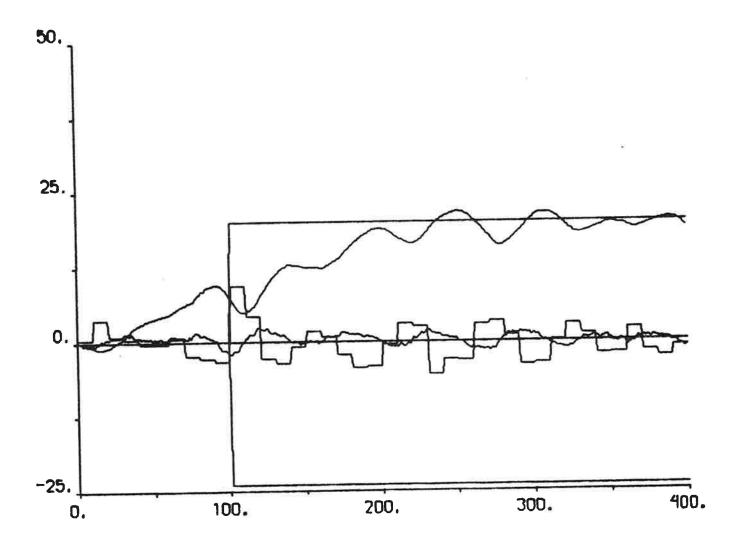


Fig. 4.62 - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\rm ref}$ = 2 deg, $r_{\rm ref}$ = 0 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).

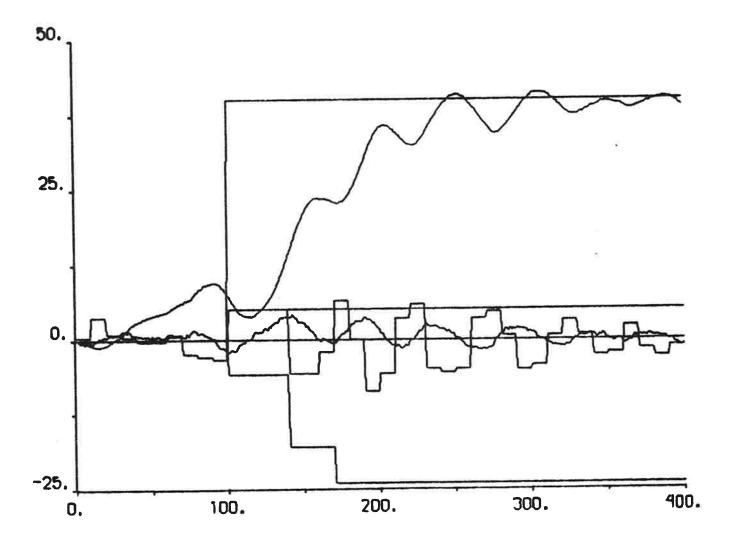


Fig. 4.63 - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).

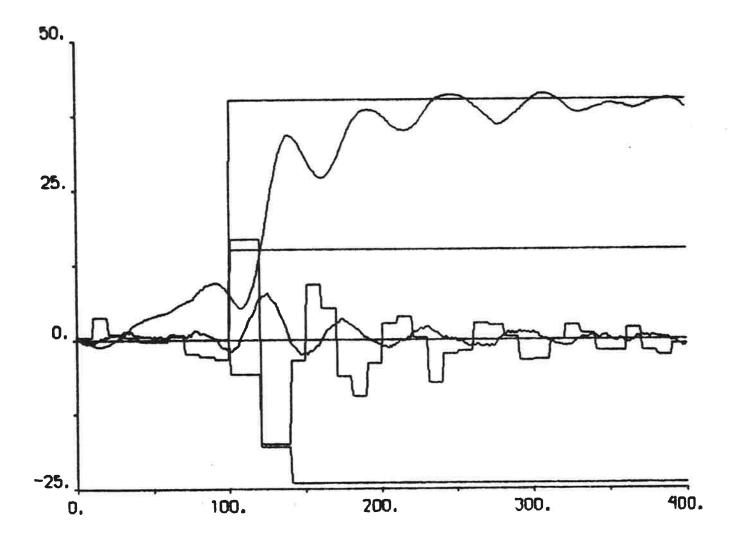


Fig. 4.64 - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.3 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).



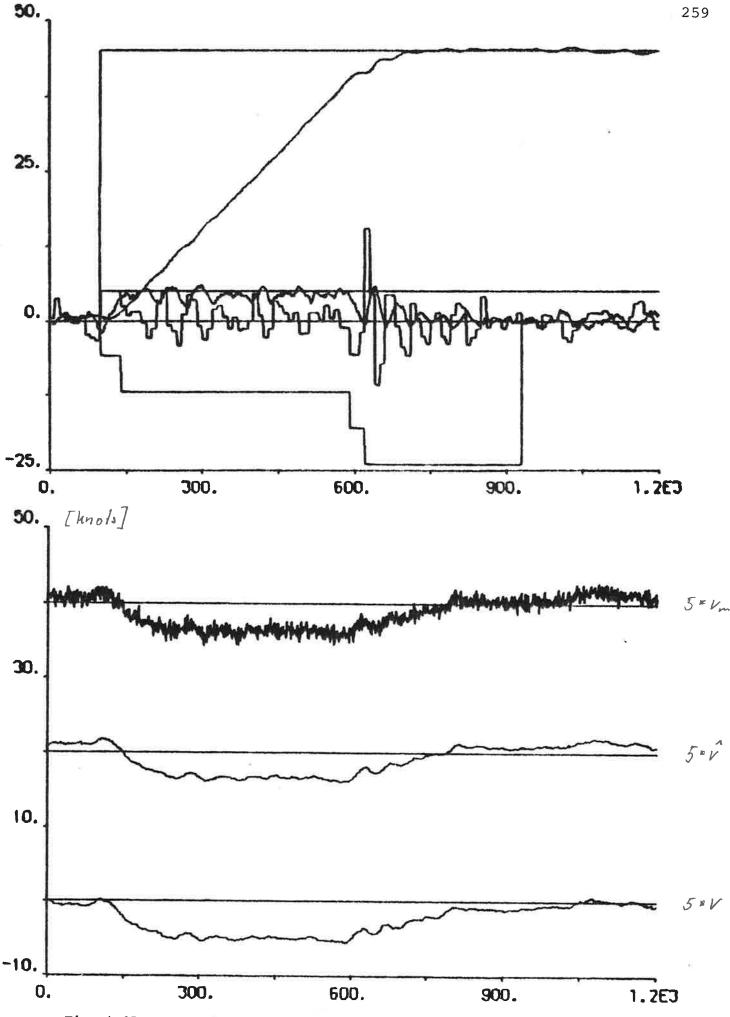
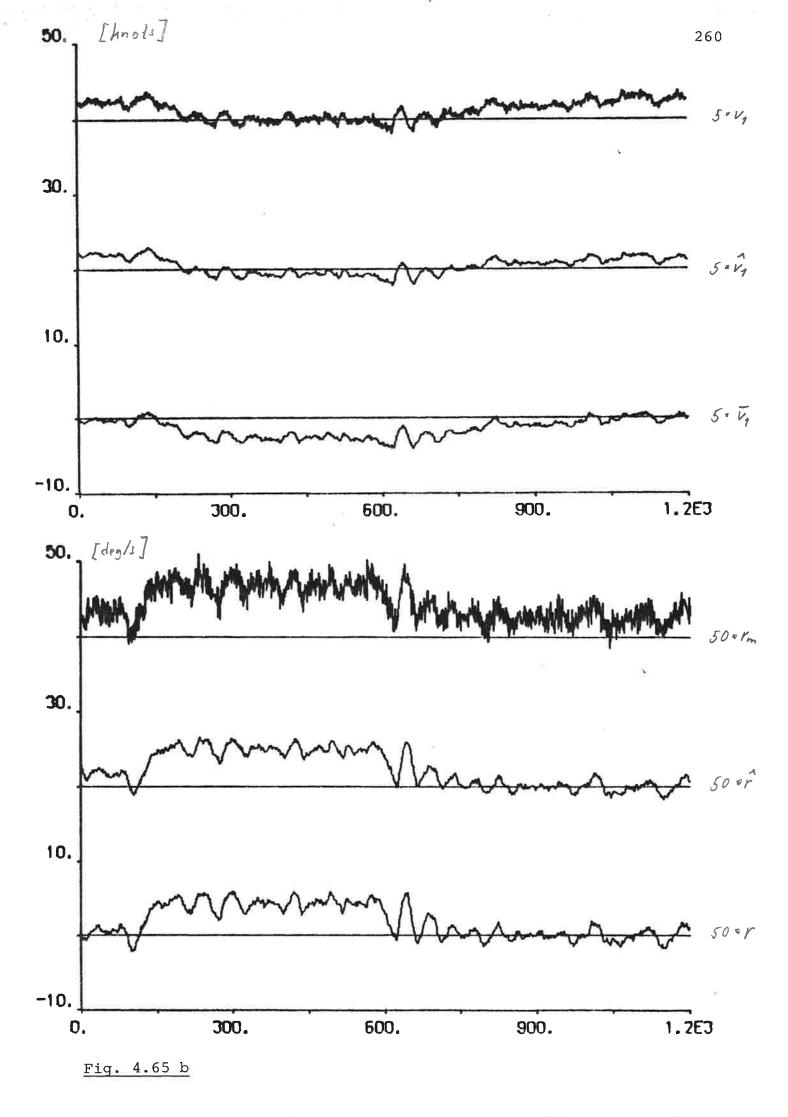
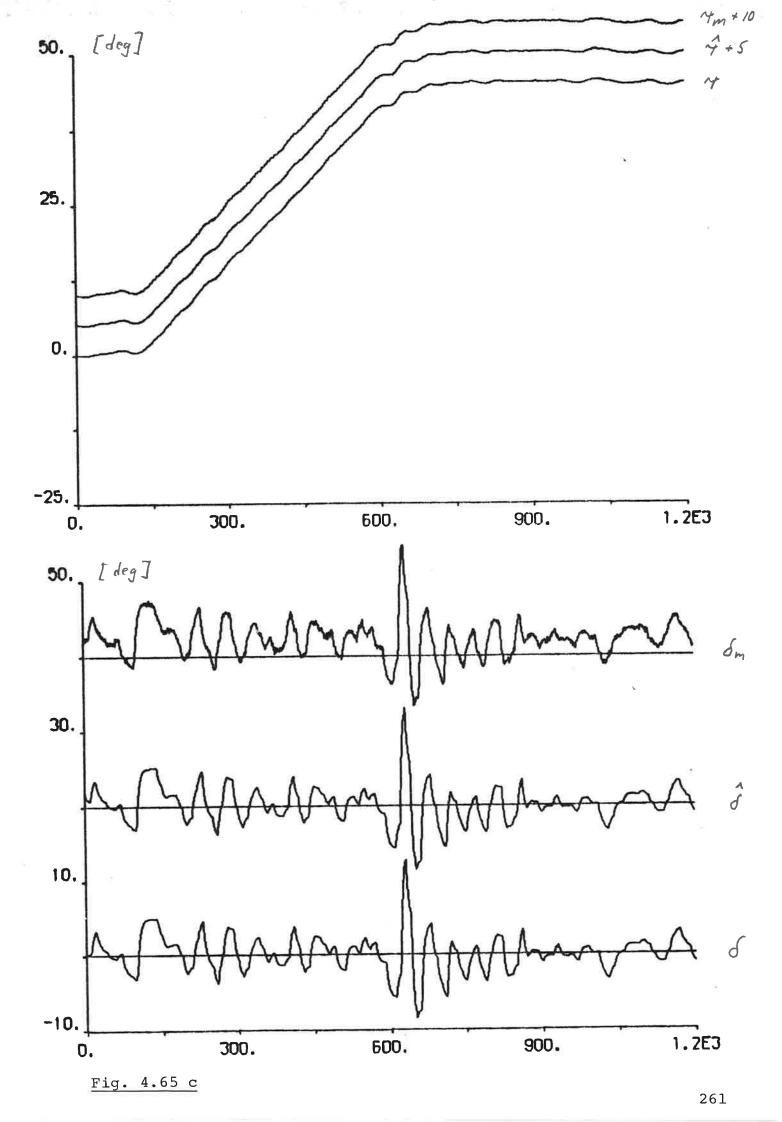
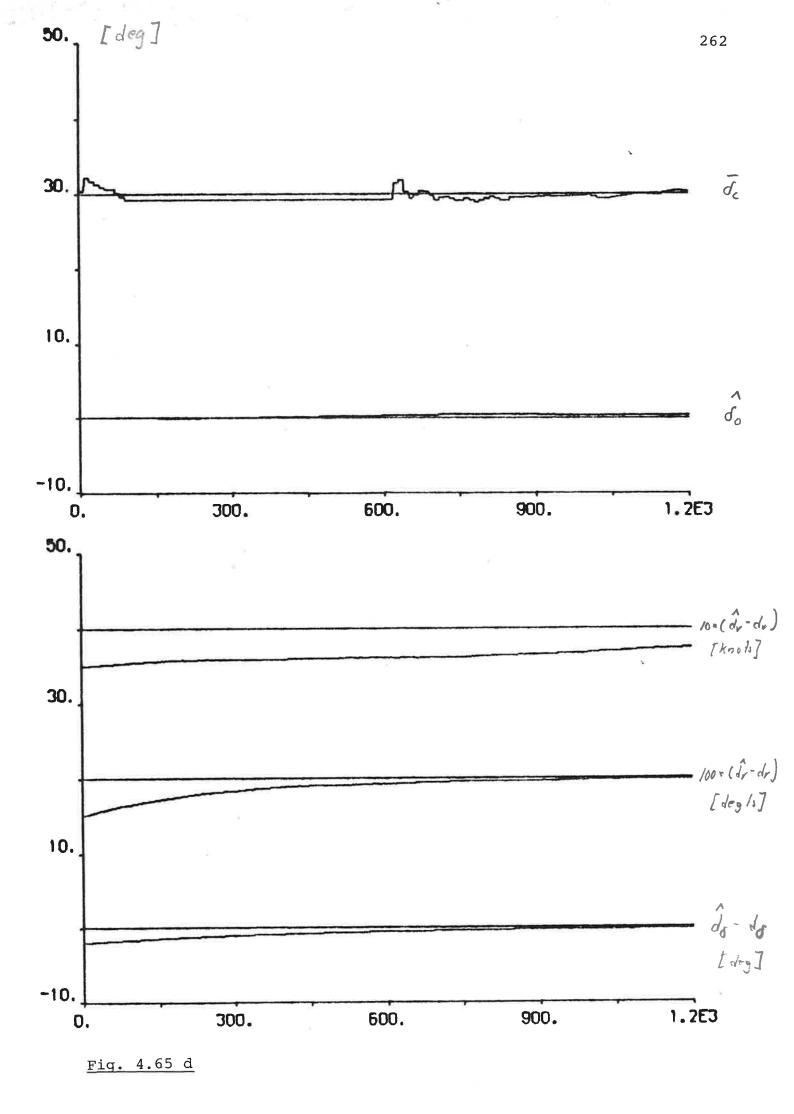
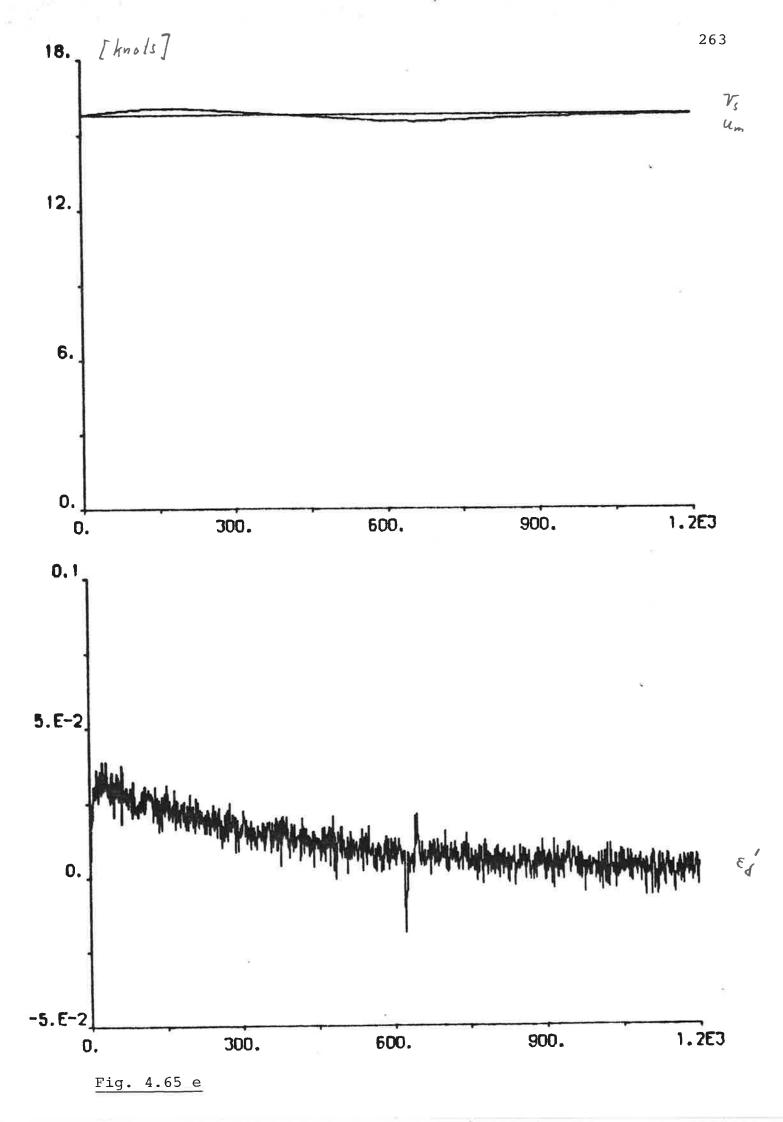


Fig. 4.65 a - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).











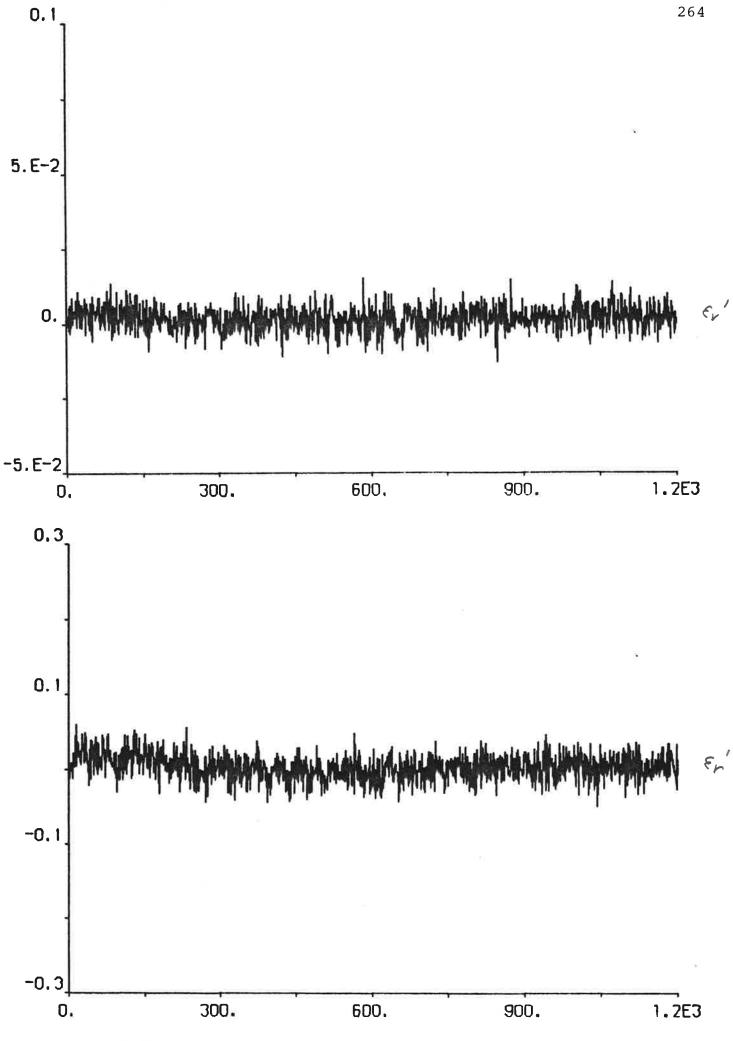
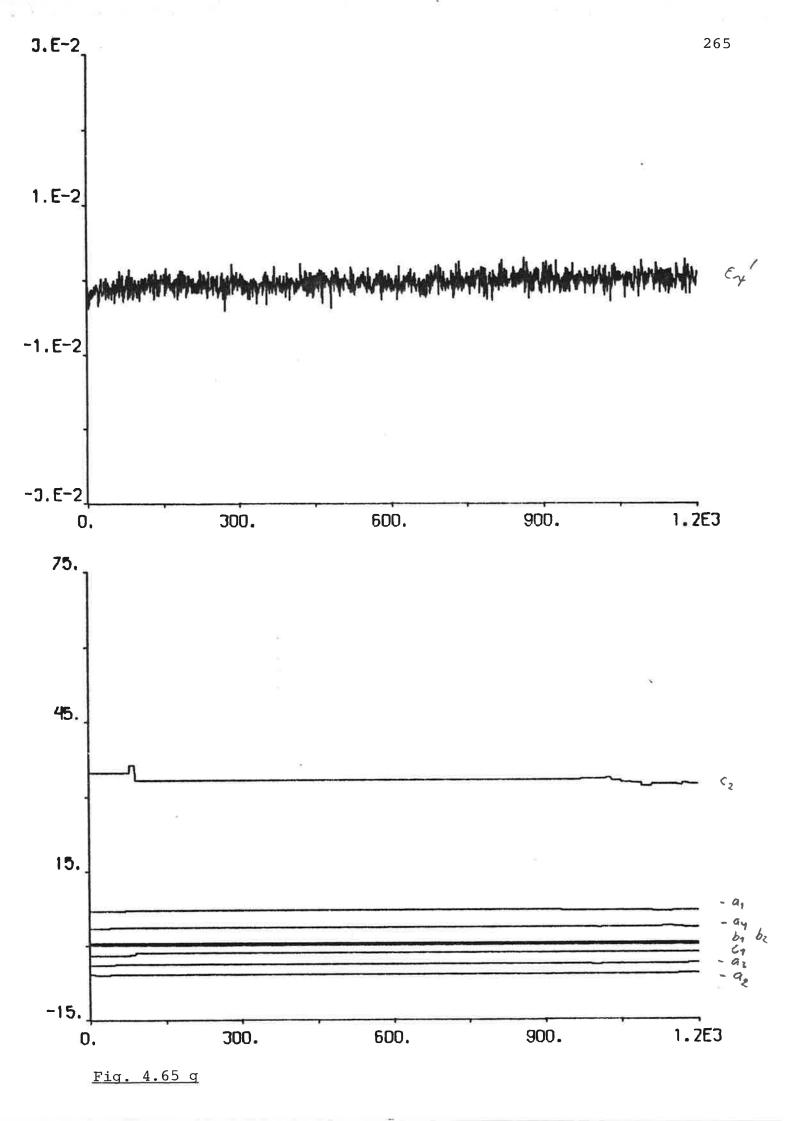


Fig. 4.65 f



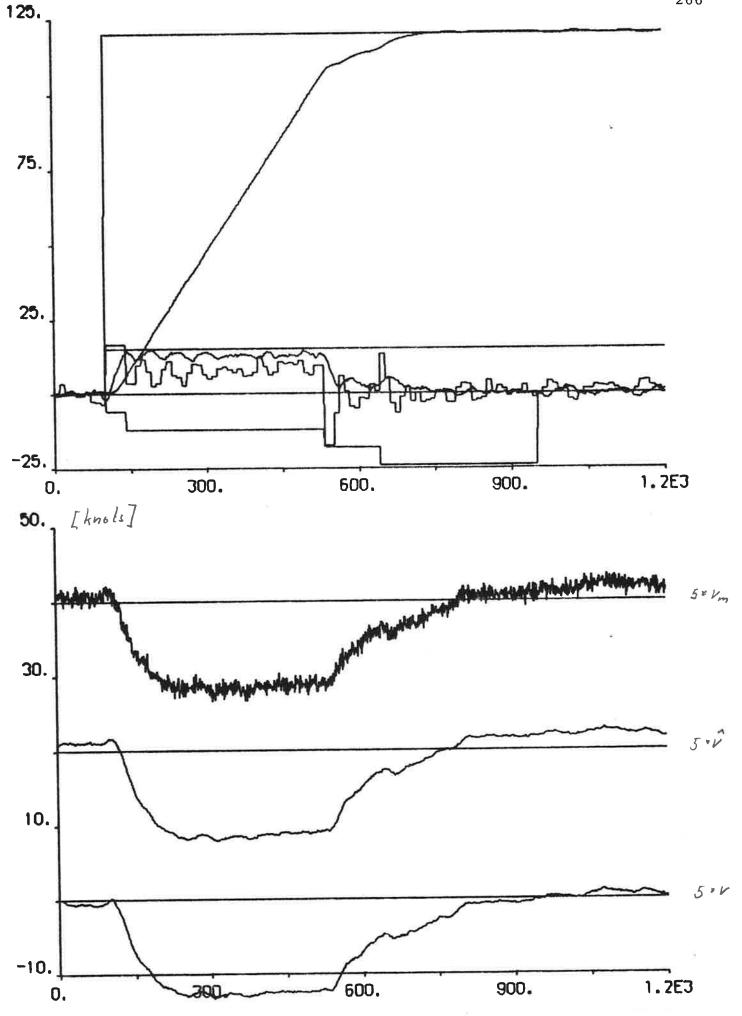
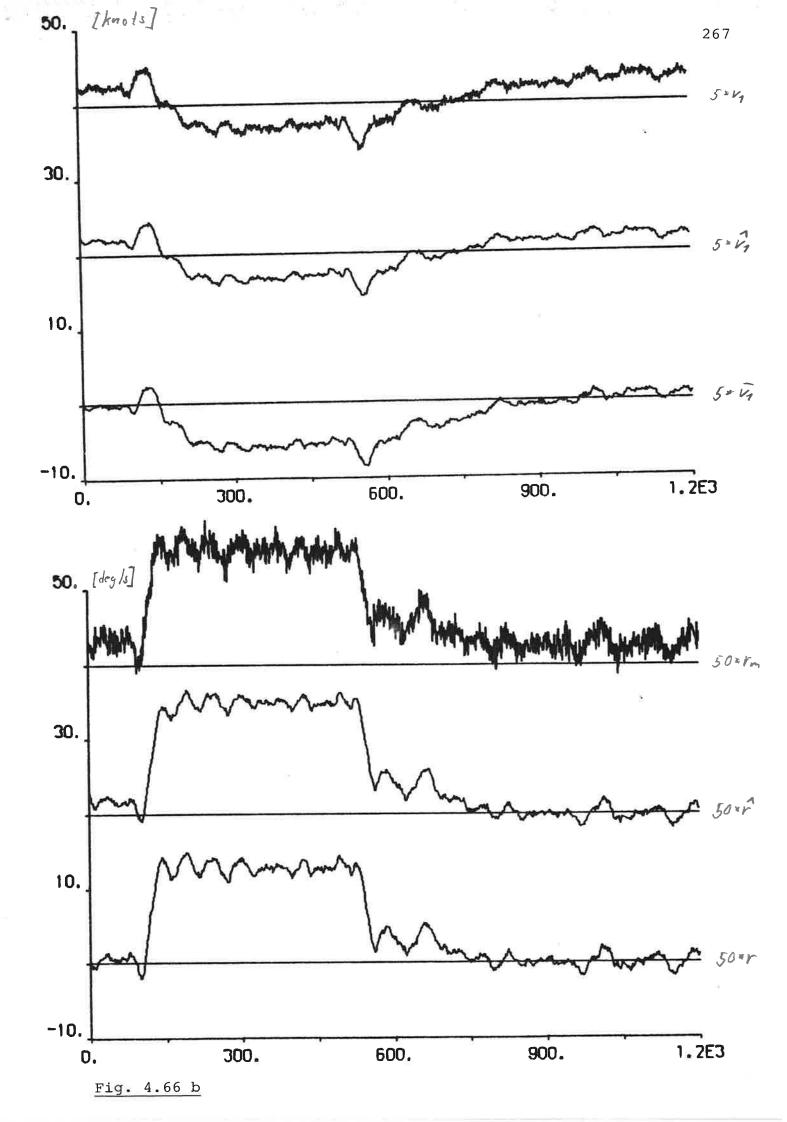
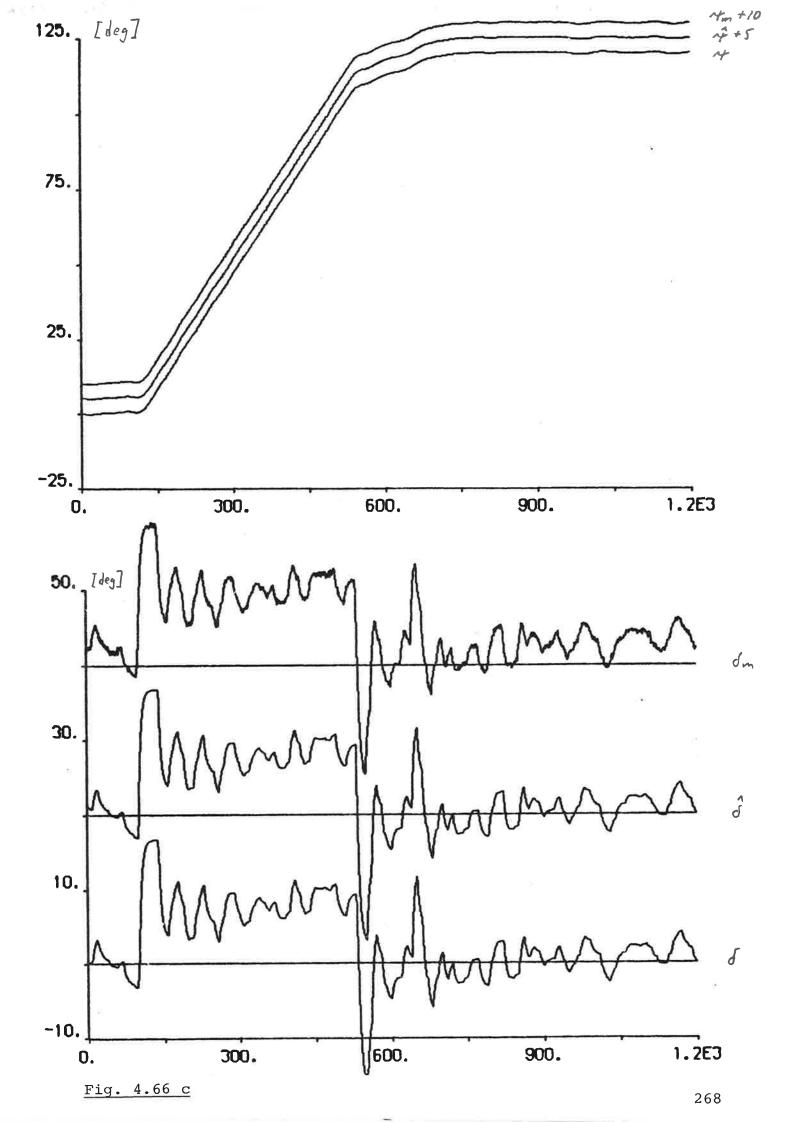


Fig. 4.66 a - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{ref}$ = 120 deg, r_{ref} = 0.3 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 50 s).





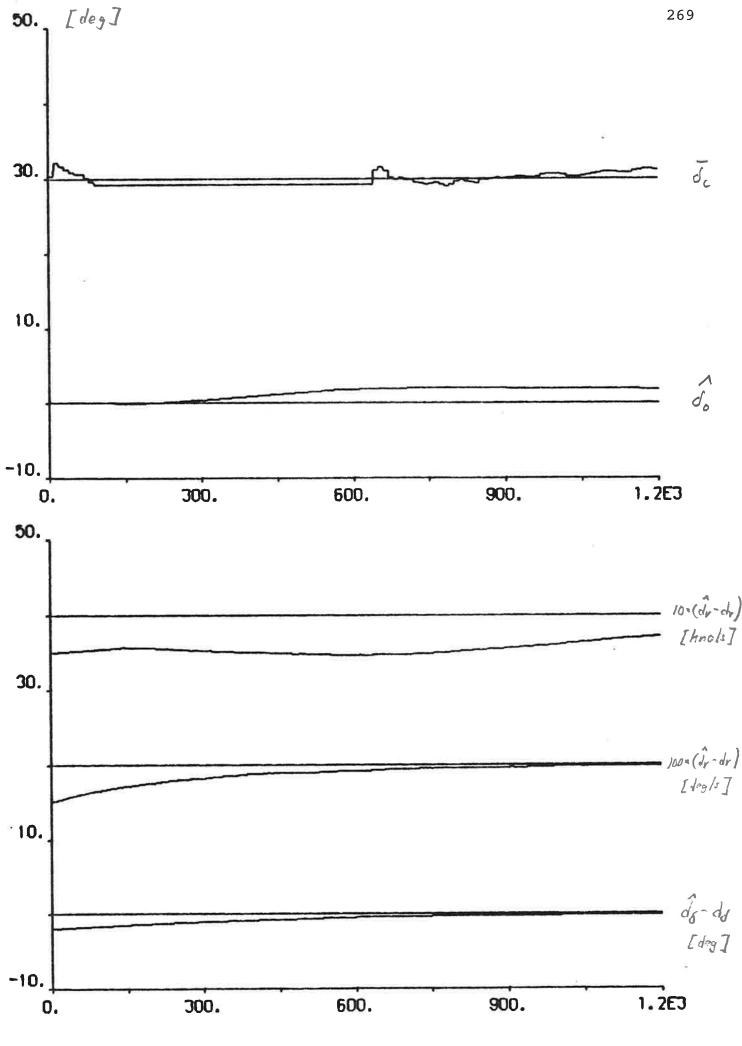
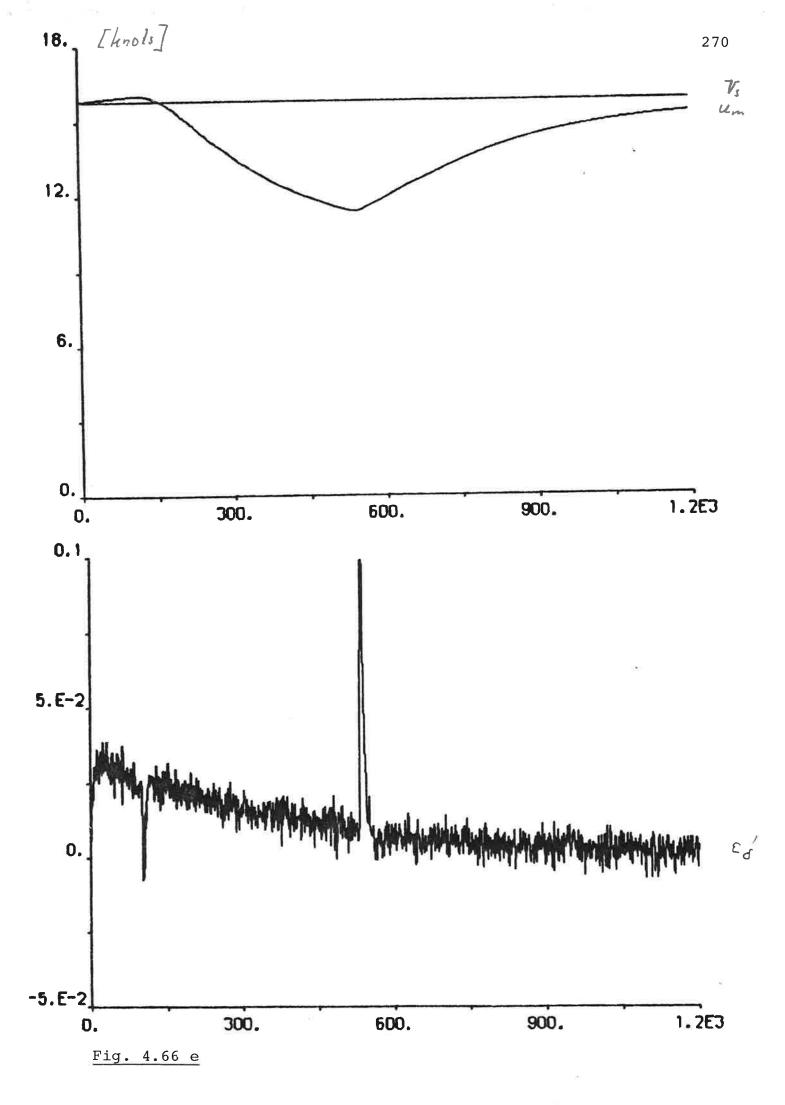


Fig. 4.66 d



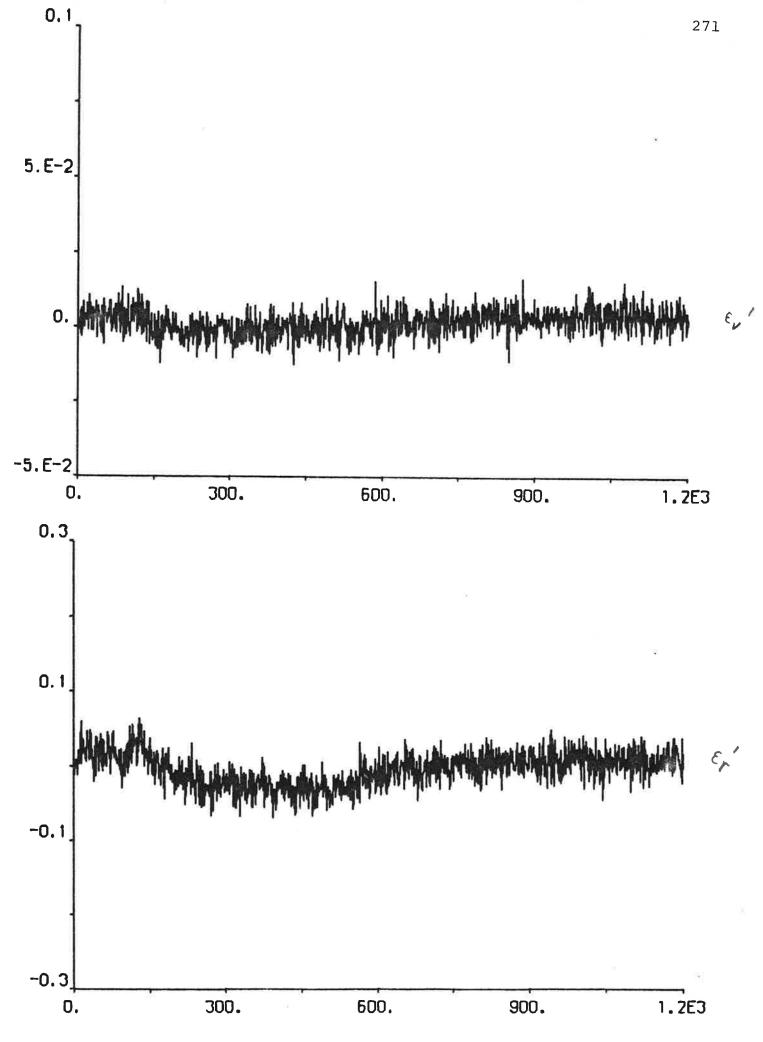
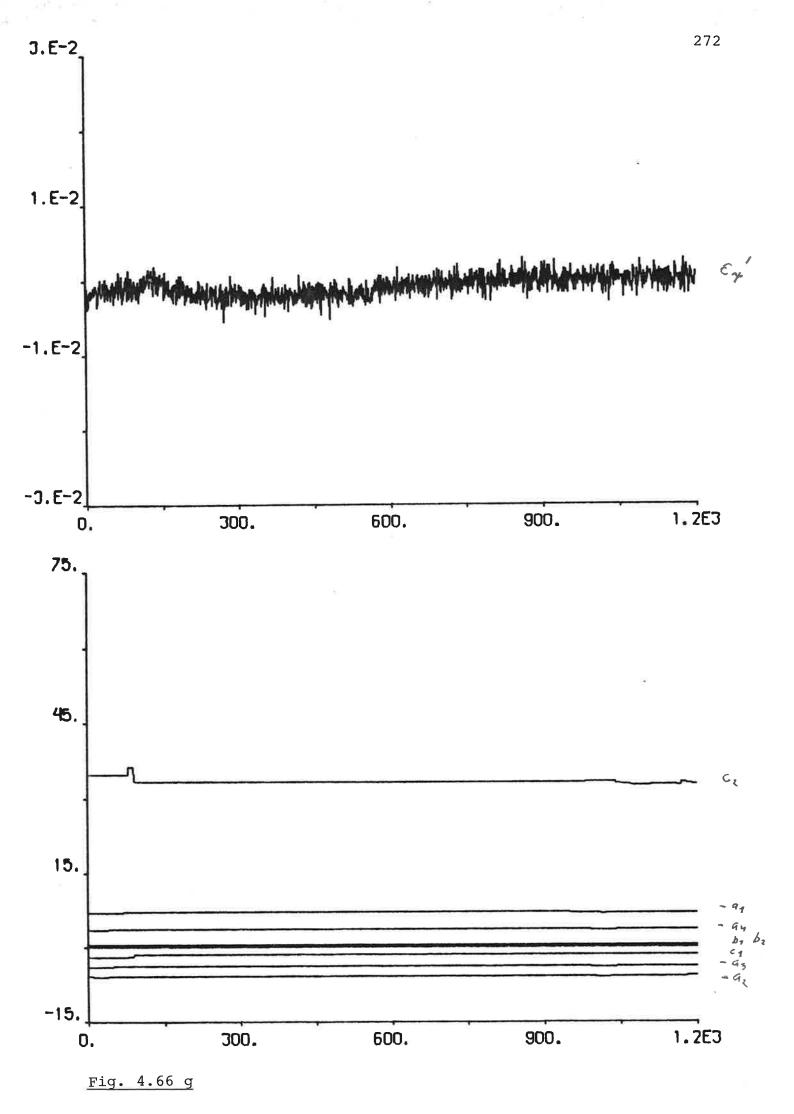


Fig. 4.66 f



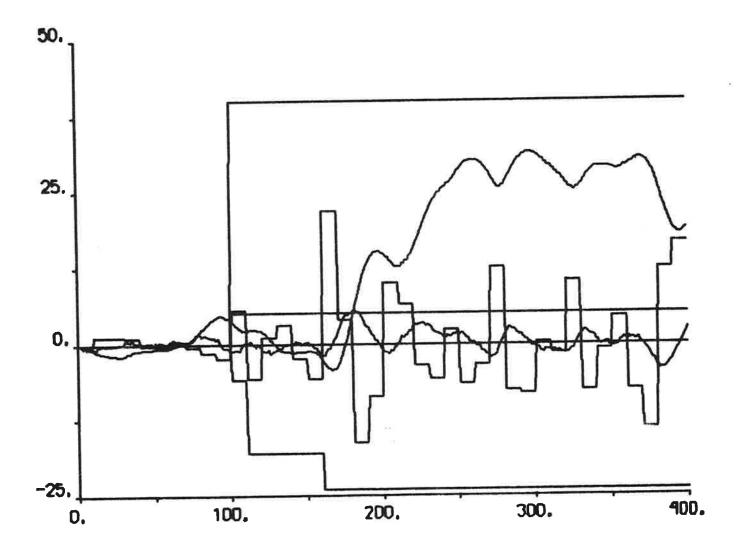


Fig. 4.67 - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{\rm ref} = 4 \ \rm deg, \ r_{\rm ref} = 0.1 \ \rm deg/s, \ self-tuning \ regulator \ and \ yaw \ regulator \ using \ non-filtered \ measurements (<math>\overline{c}_2$ = 50 s).



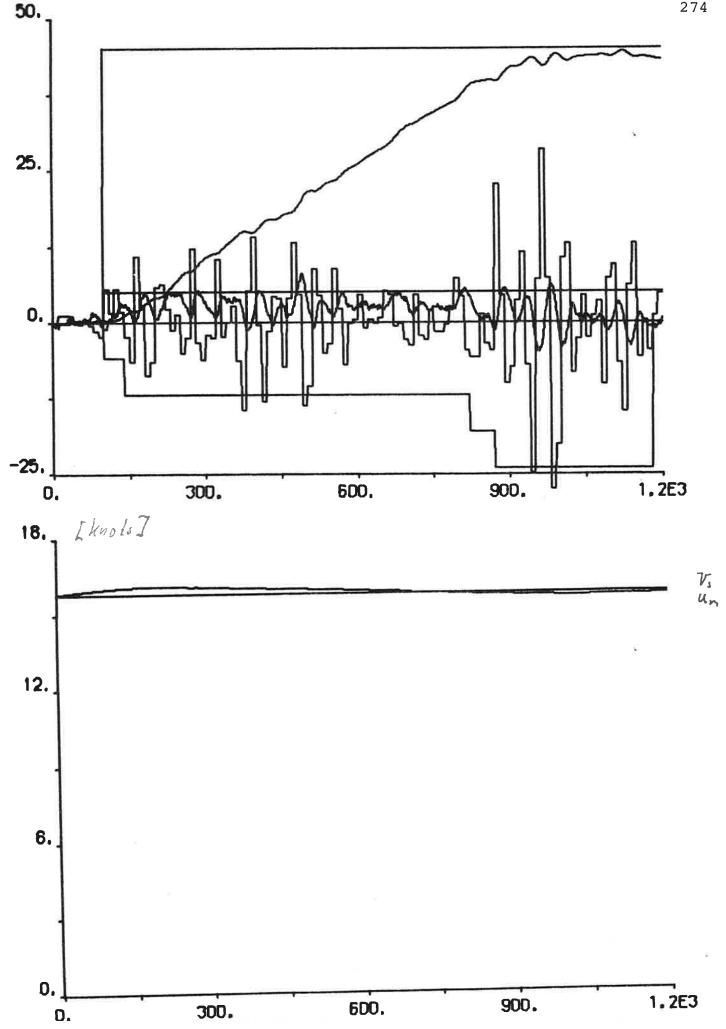
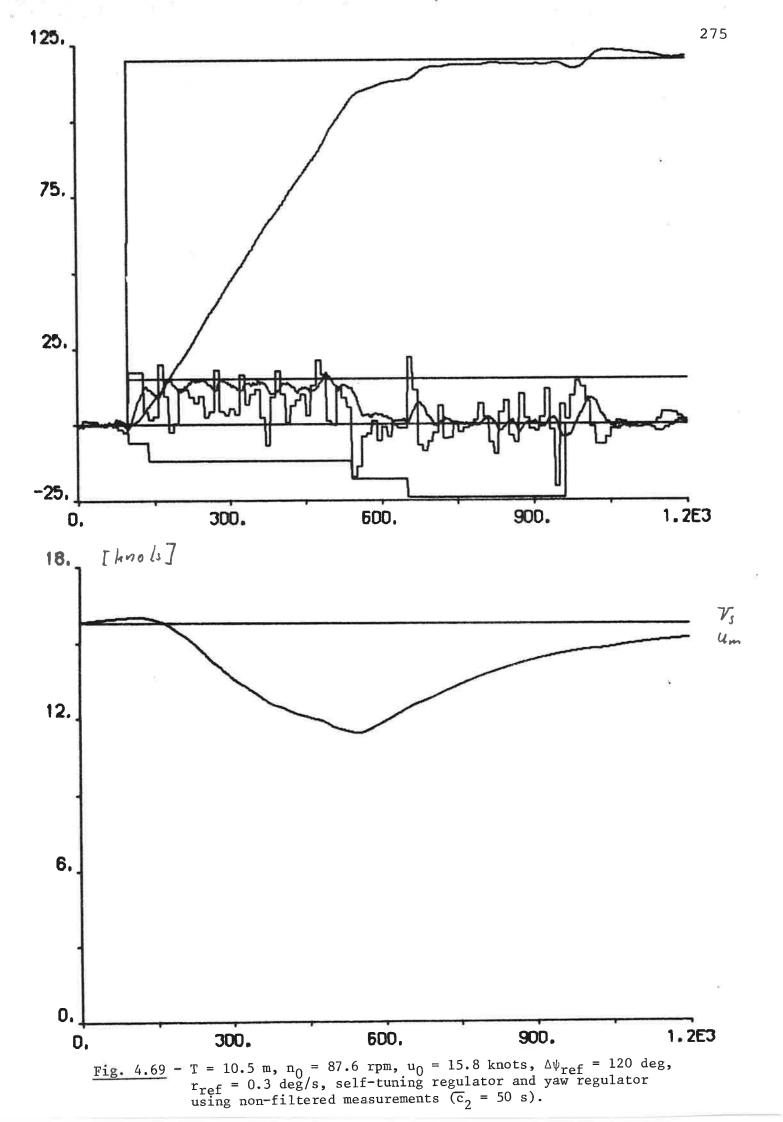


Fig. 4.68 - T = 10.5 m, n_0 = 87.6 rpm, u_0 = 15.8 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 50 s).



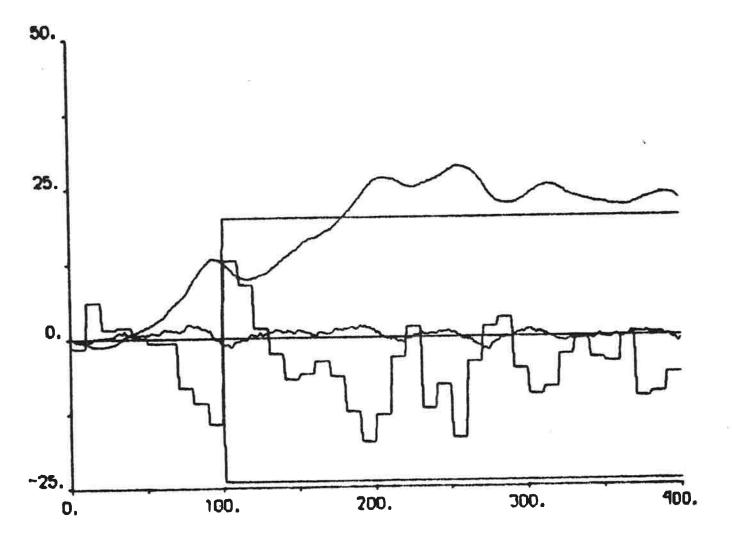


Fig. 4.70 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 2 \text{ deg, r}_{\text{ref}} = 0 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).

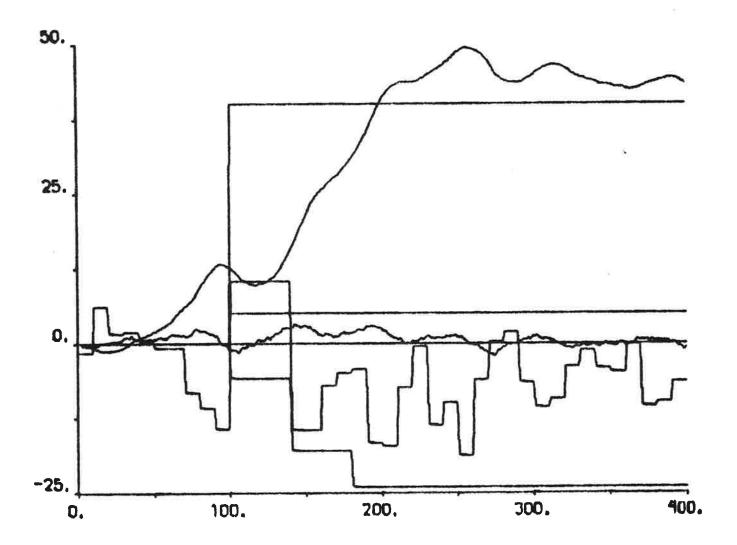


Fig. 4.71 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, r}_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).

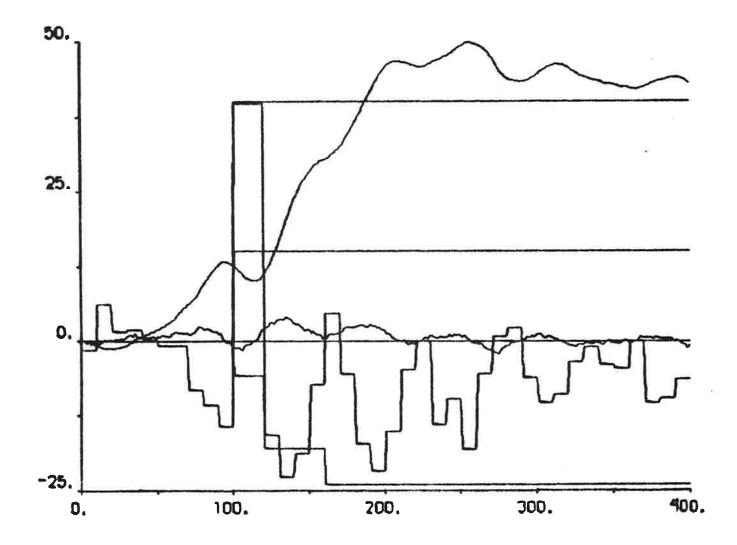


Fig. 4.72 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.3 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).



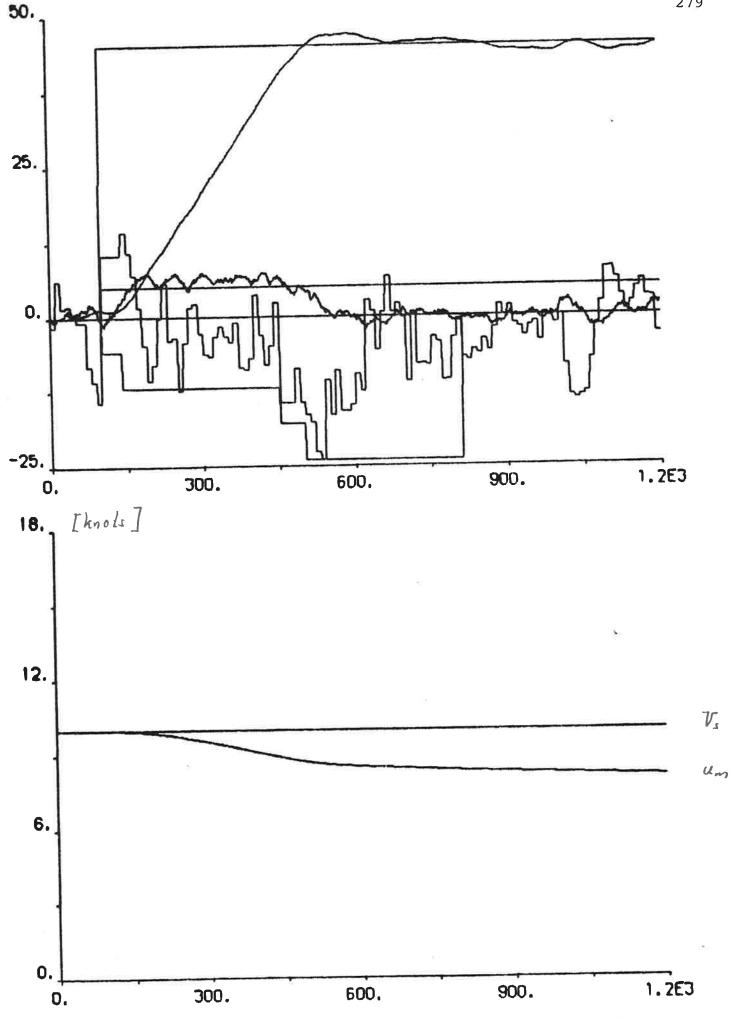
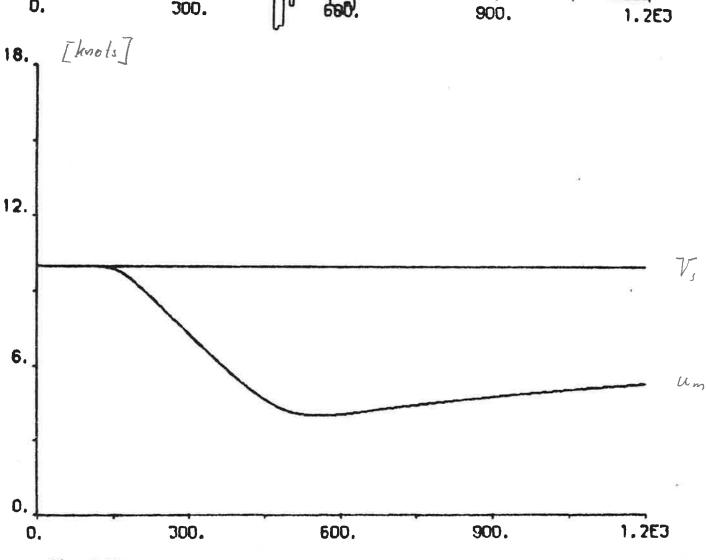


Fig. 4.73 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).



125.

*7*5.

25.

-25.

0.

300.

Fig. 4.74 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 120 deg, r_{ref} = 0.3 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).

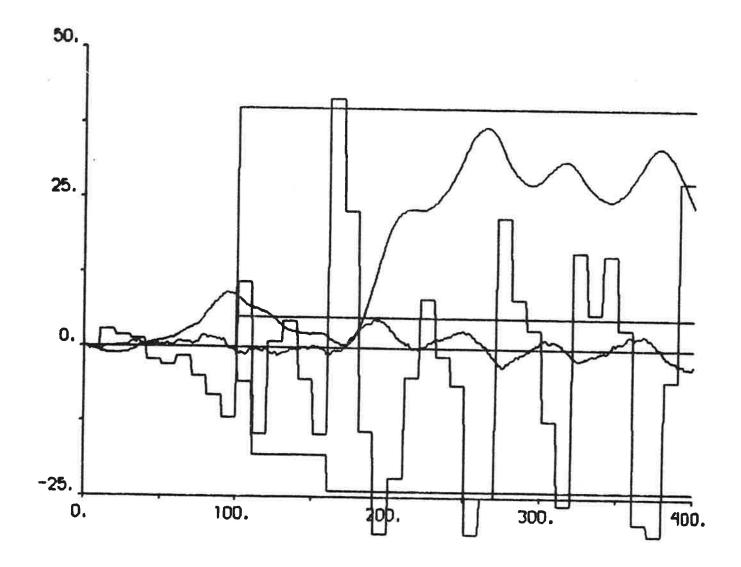


Fig. 4.75 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 63.25 s).

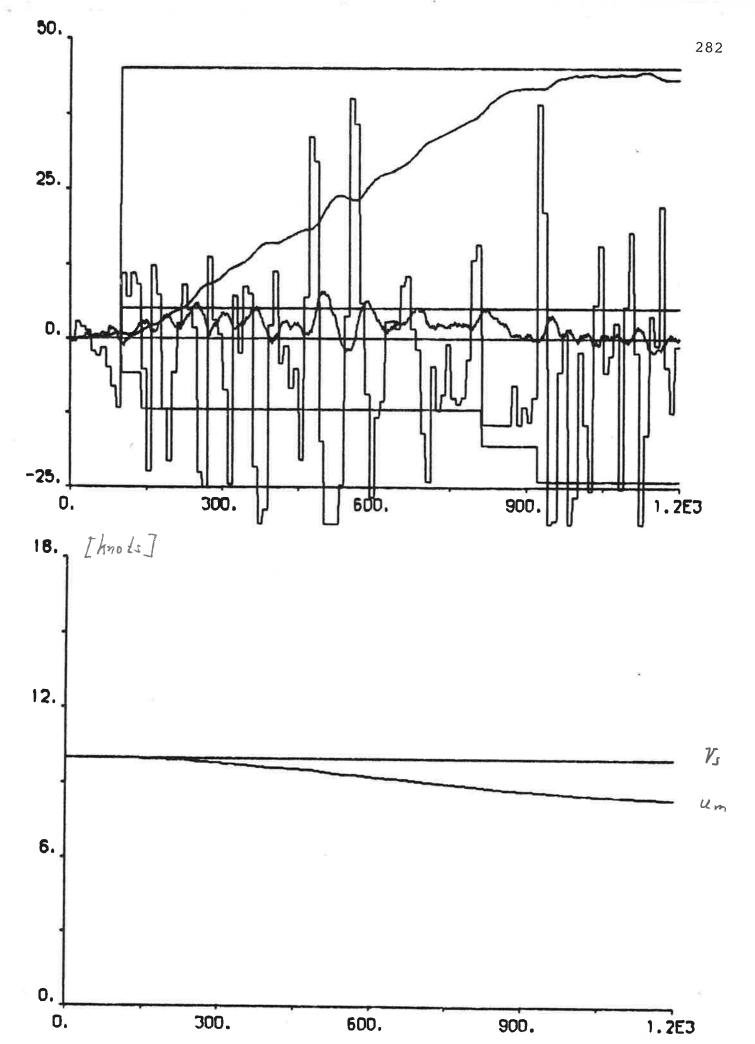


Fig. 4.76 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 63.25 s).

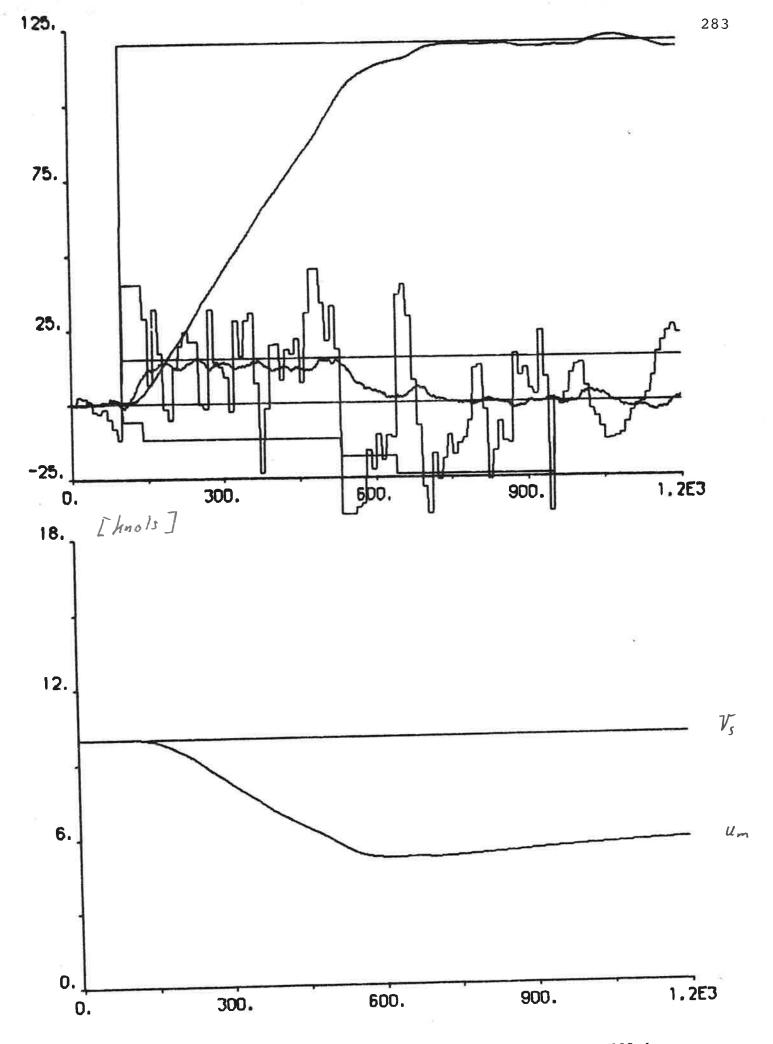


Fig. 4.77 - T = 22.3 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 120 deg, r_{ref} = 0.3 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 63.25 s).

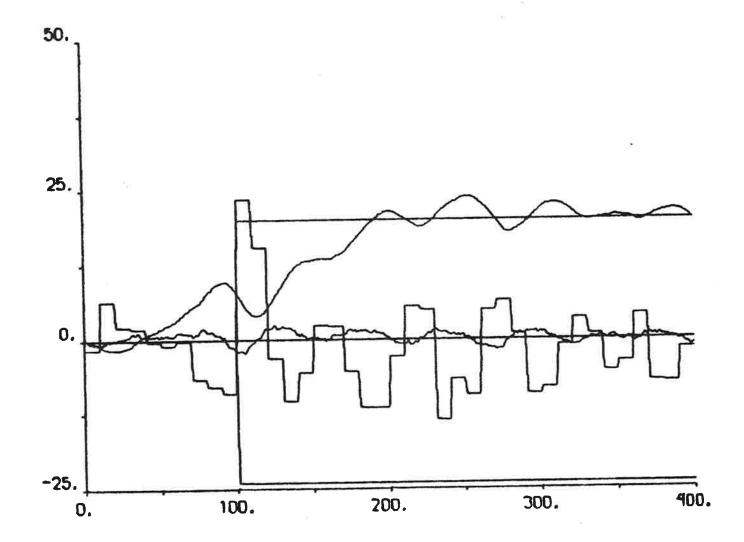


Fig. 4.78 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 2 \text{ deg, } r_{\text{ref}} = 0 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).

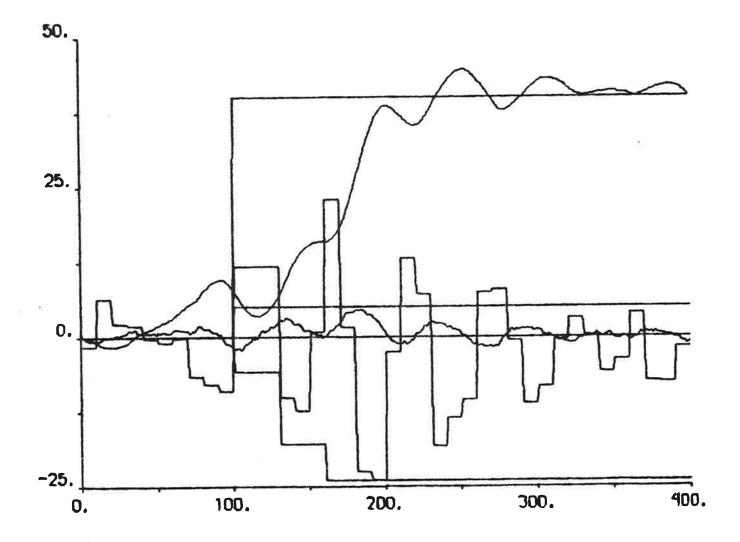


Fig. 4.79 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).

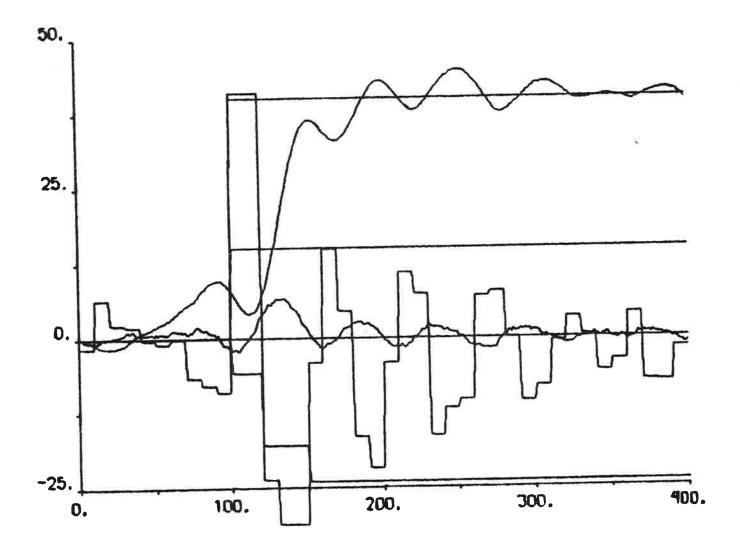


Fig. 4.80 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.3 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 63.25 s).

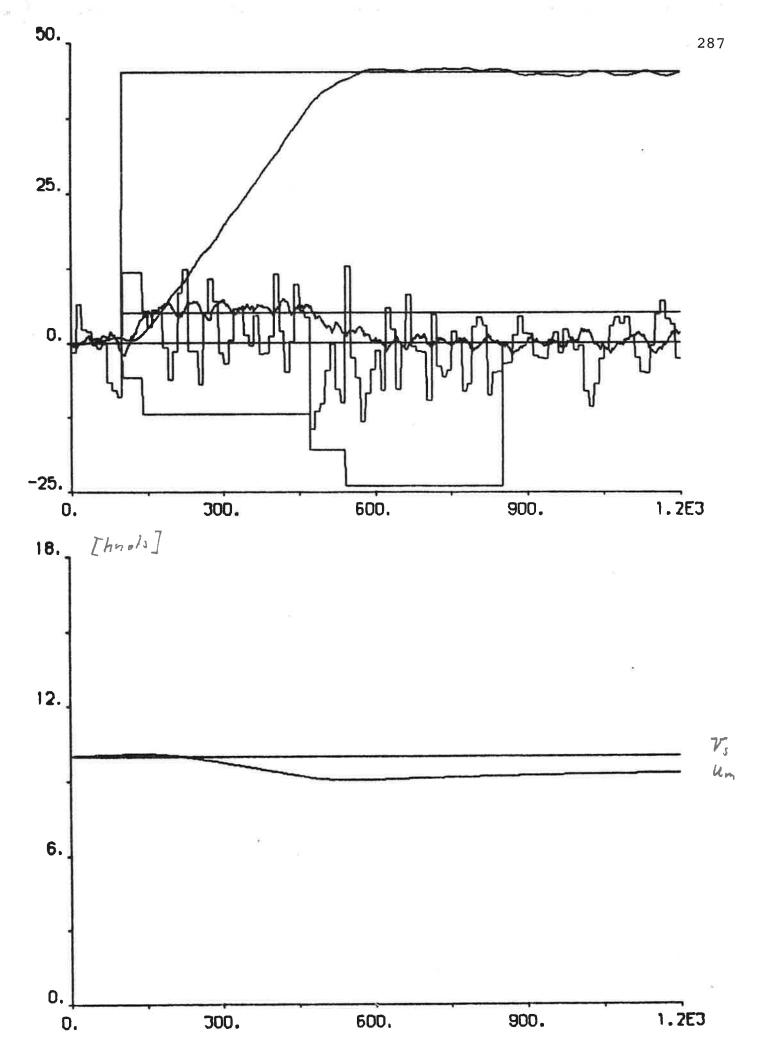


Fig. 4.81 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (c_2 = 63.25 s).

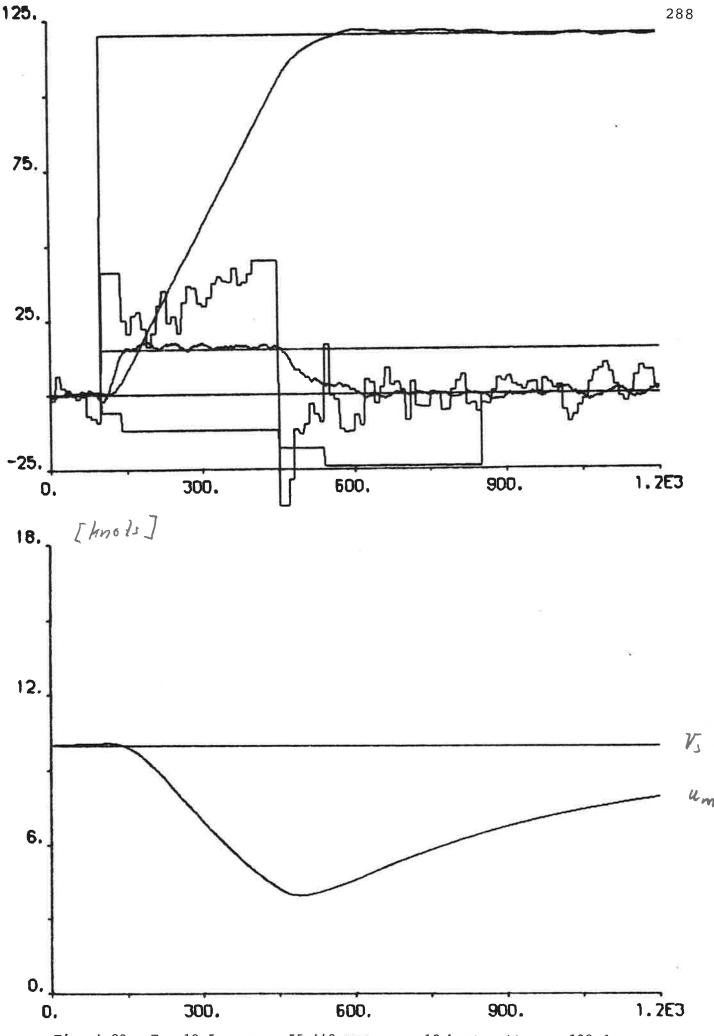


Fig. 4.82 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 120 deg, r_{ref} = 0.3 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (c_2 = 63.25 s).

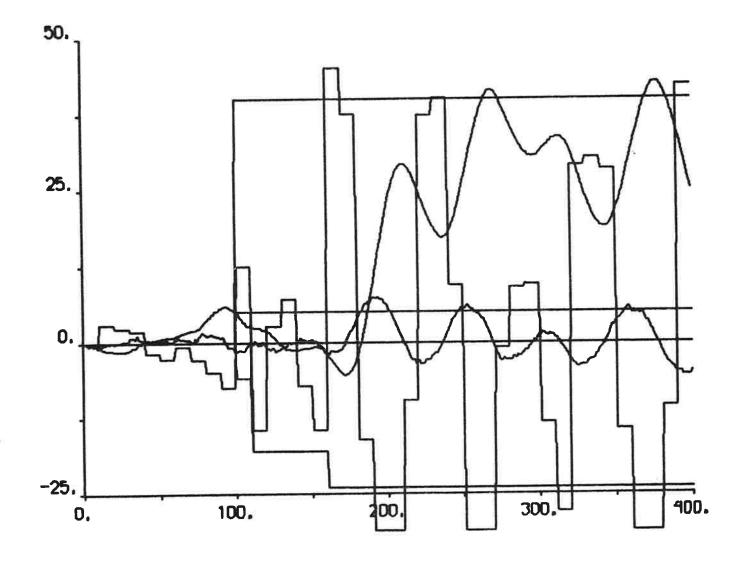


Fig. 4.83 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, r}_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 63.25 s).



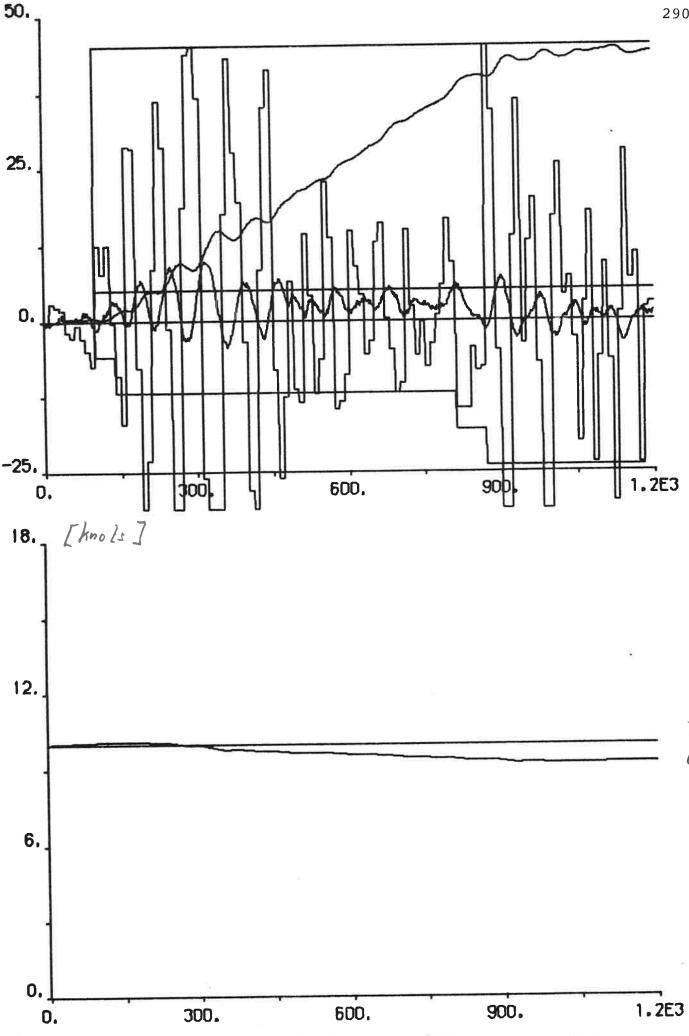
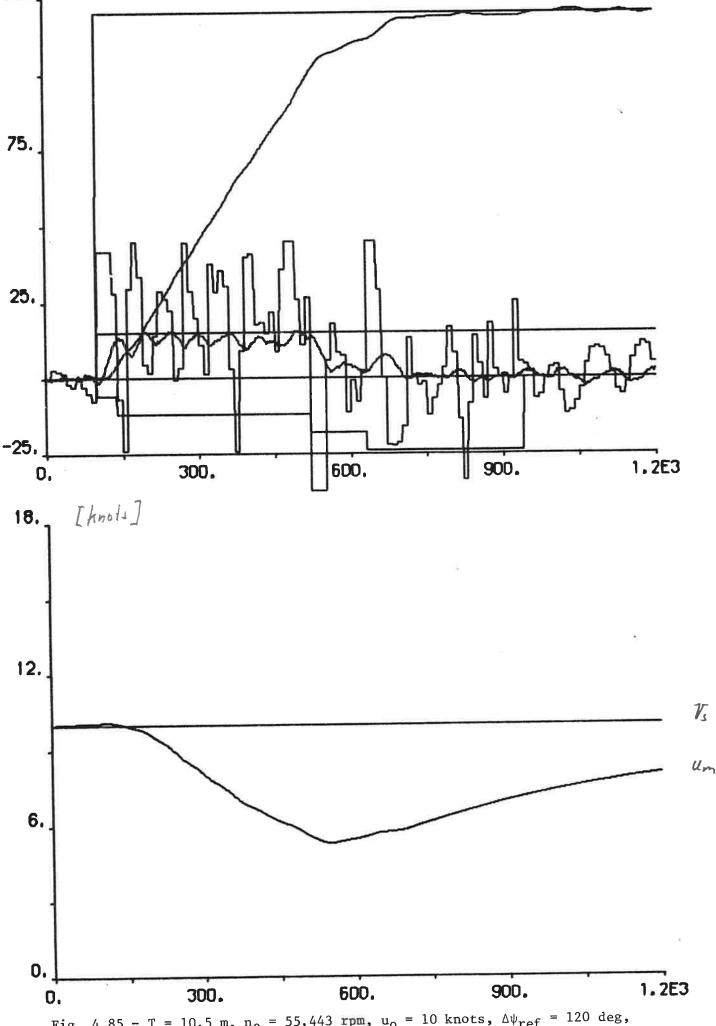


Fig. 4.84 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (c_2 = 63.25 s).





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Fig. 4.85 - T = 10.5 m, n_0 = 55.443 rpm, u_0 = 10 knots, $\Delta \psi_{ref}$ = 120 deg, r_{ref} = 0.3 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (c_2 = 63.25 s).

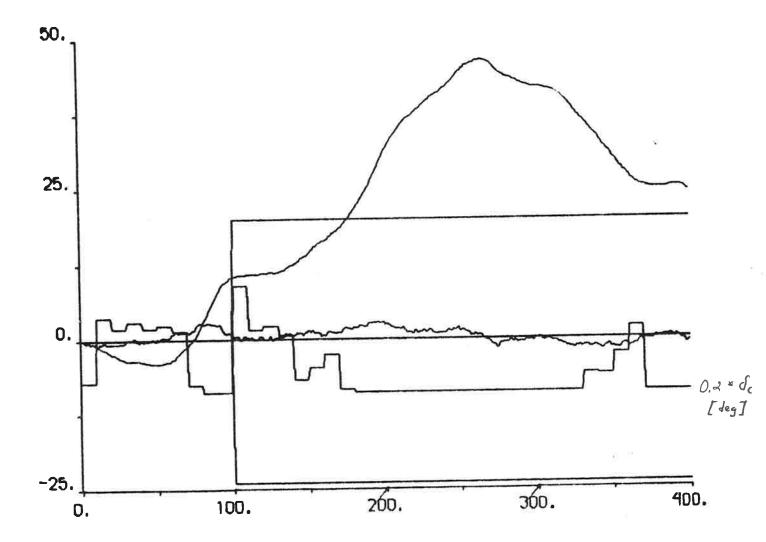


Fig. 4.86 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{\text{ref}} = 2 \text{ deg, r}_{\text{ref}} = 0 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 100 s).

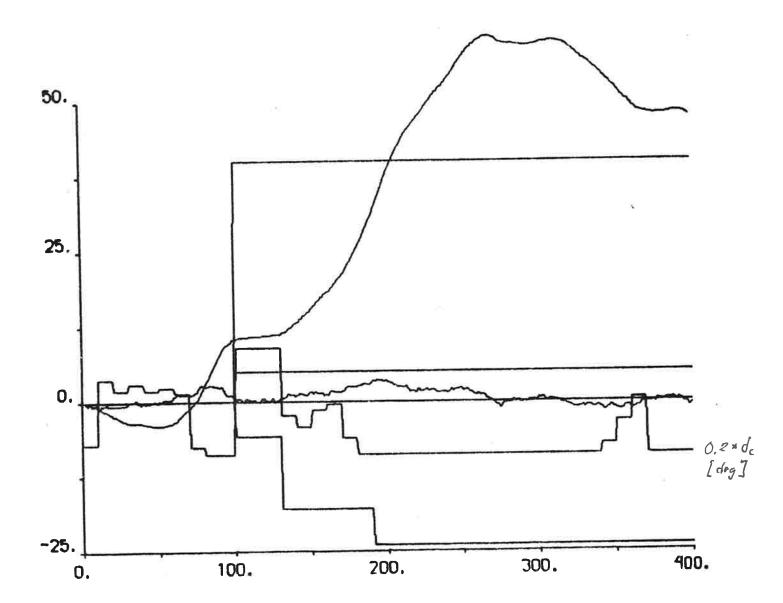
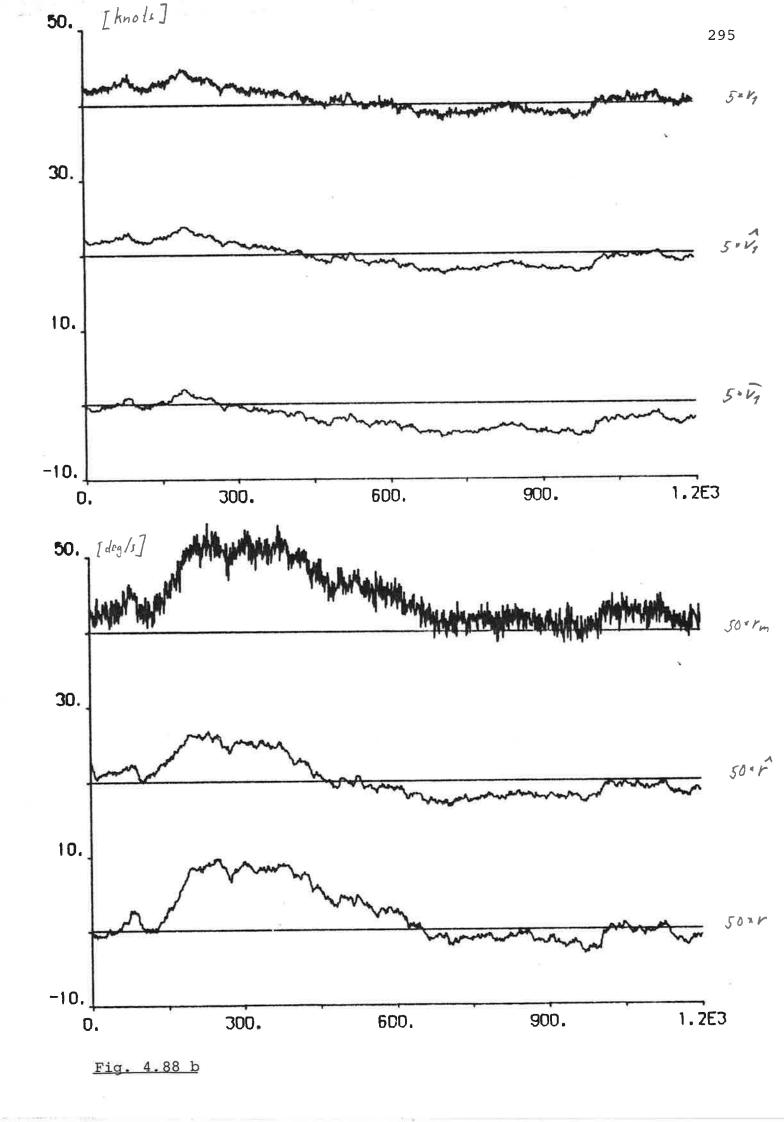
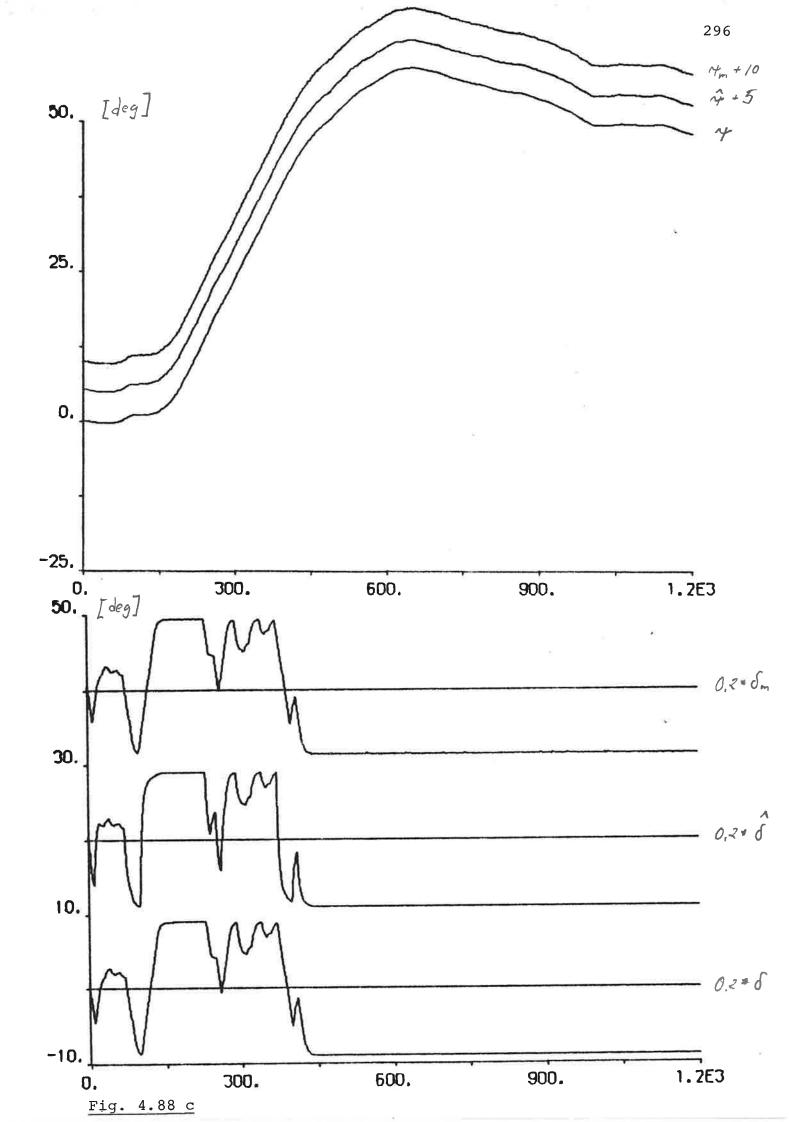


Fig. 4.87 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, r}_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 100 s).

Fig. 4.88 a - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 100 s).





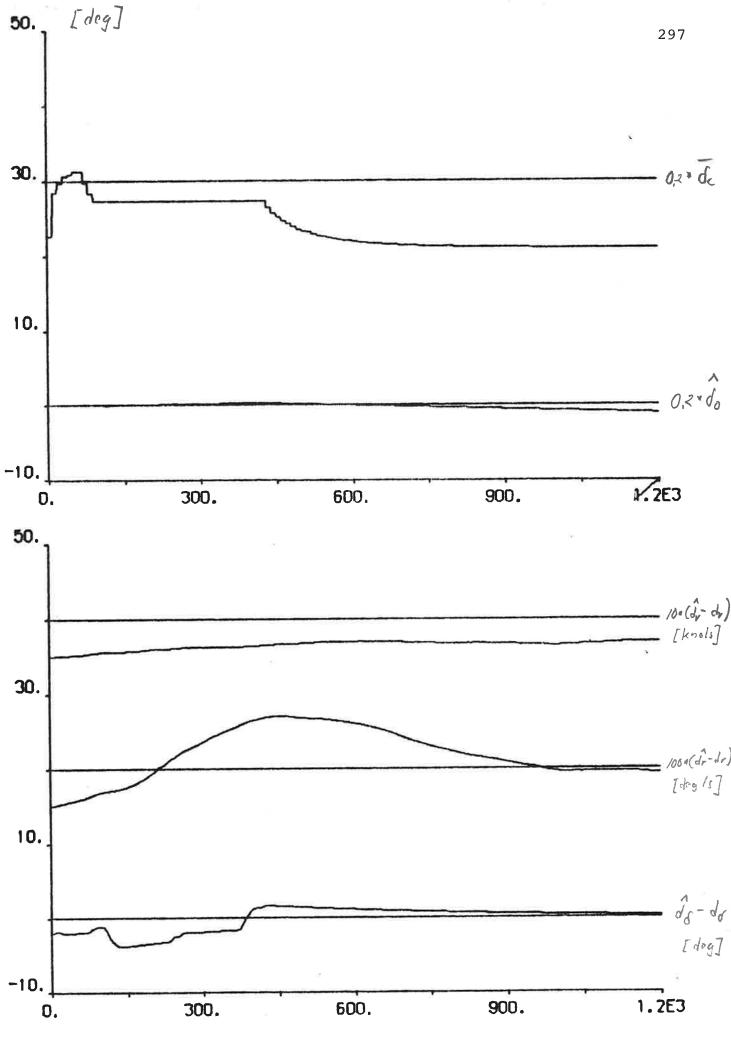
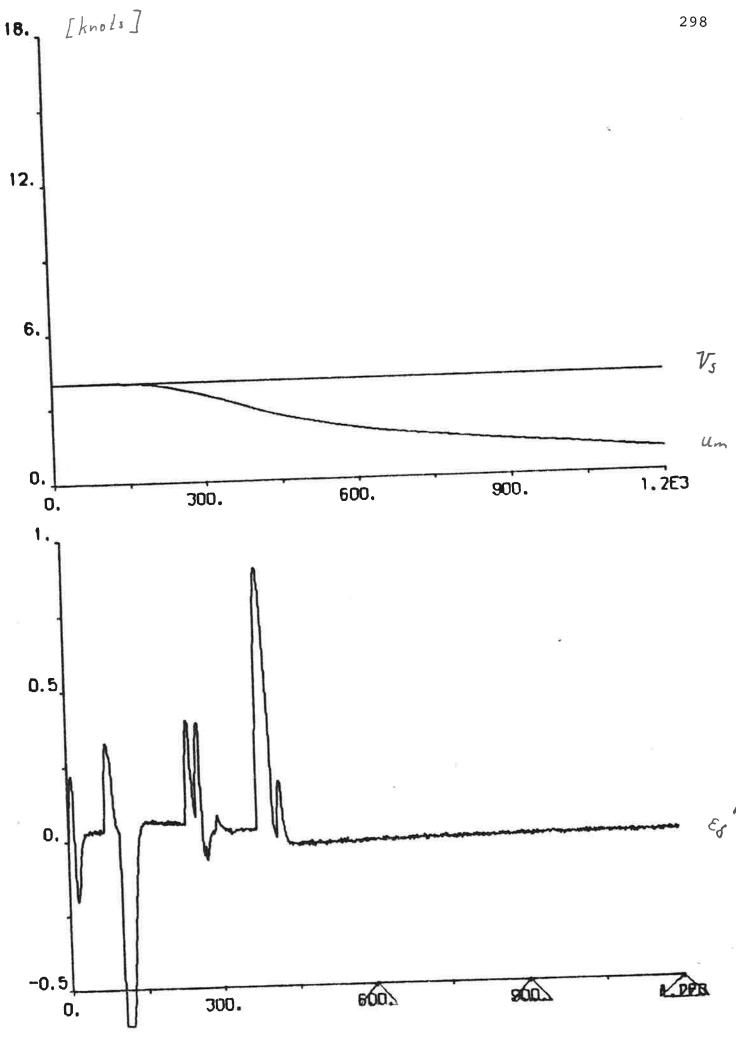


Fig. 4.88 d



<u>Fig. 4.88 e</u>

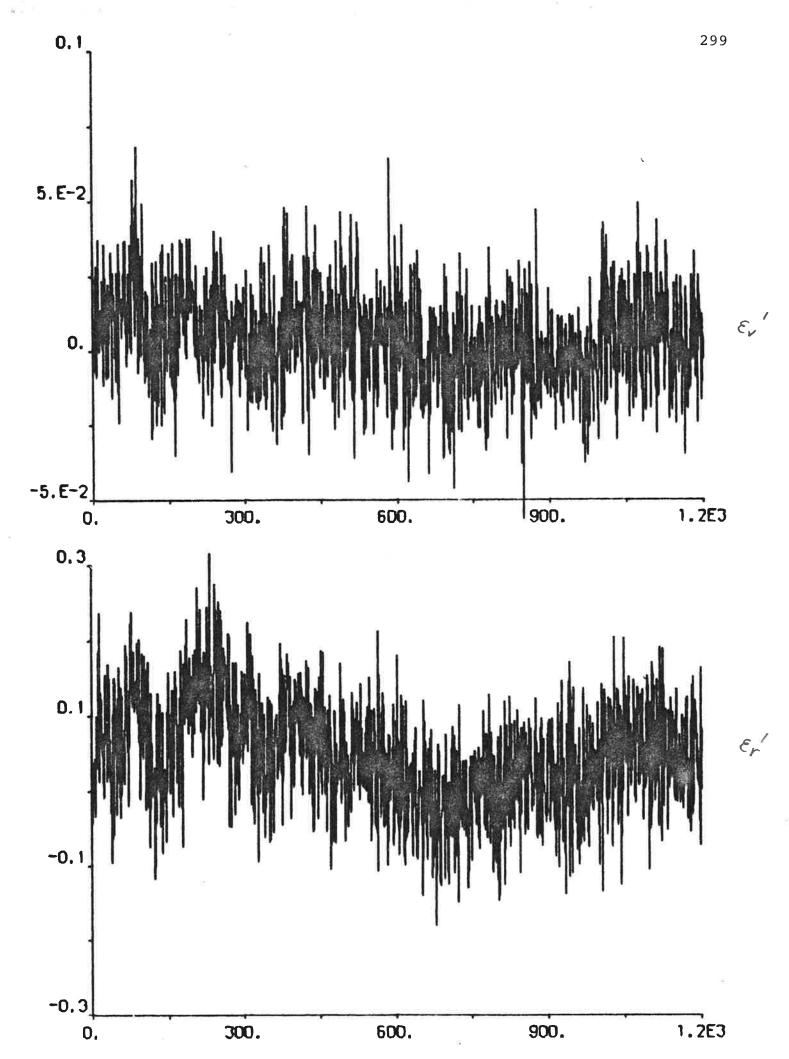
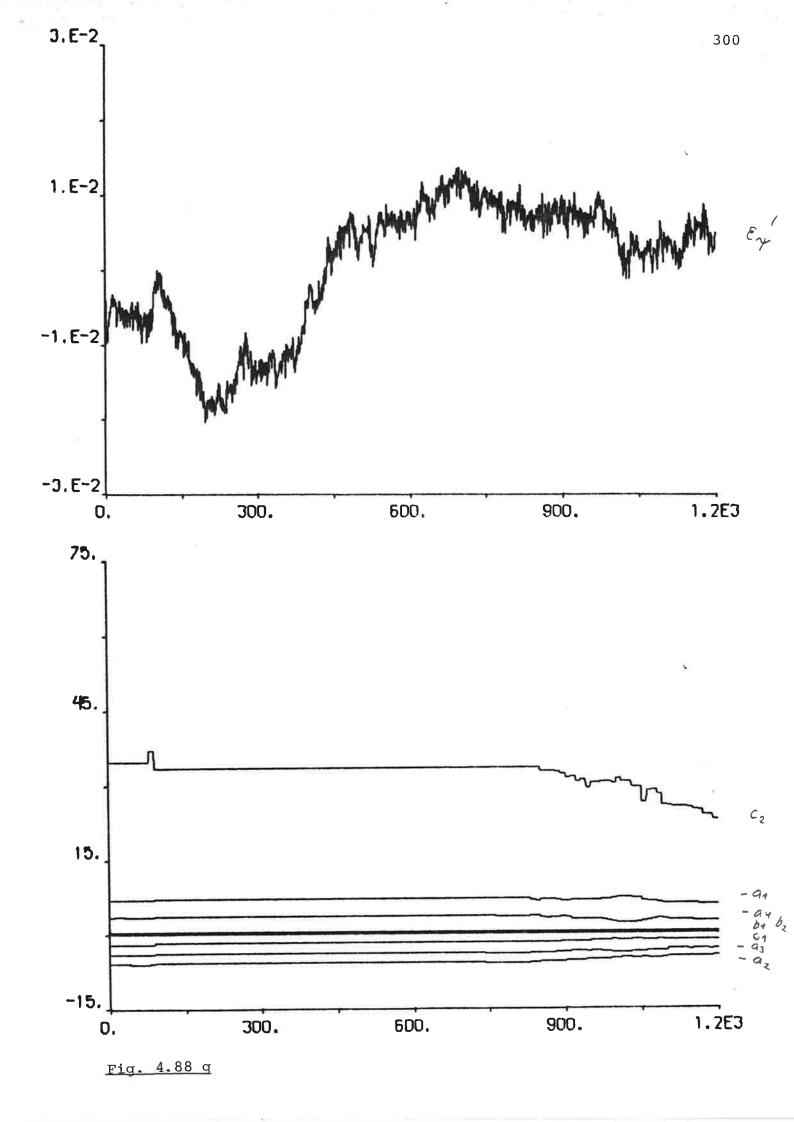


Fig. 4.88 f



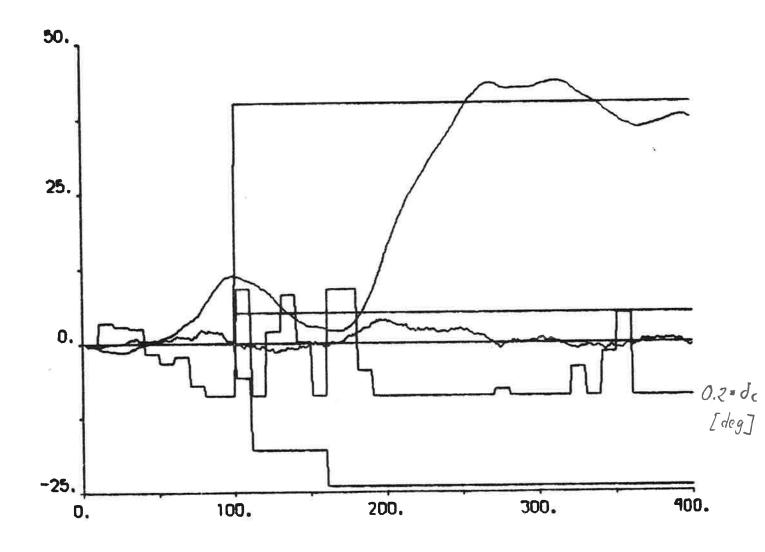


Fig. 4.89 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 100 s).

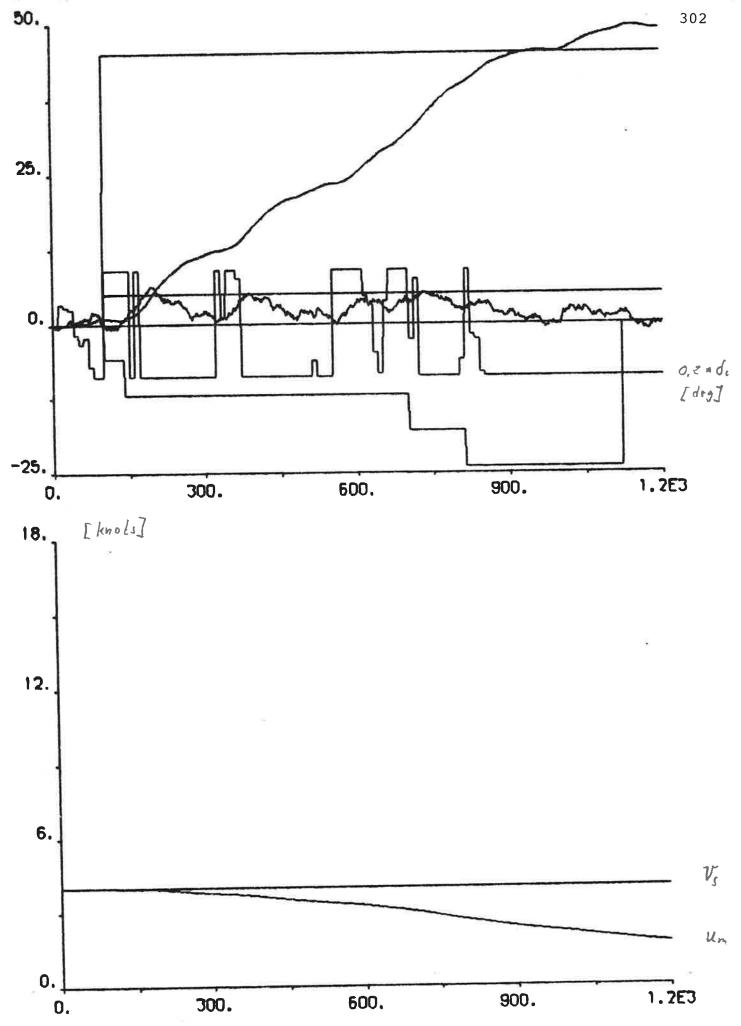


Fig. 4.90 - T = 22.3 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 100 s).

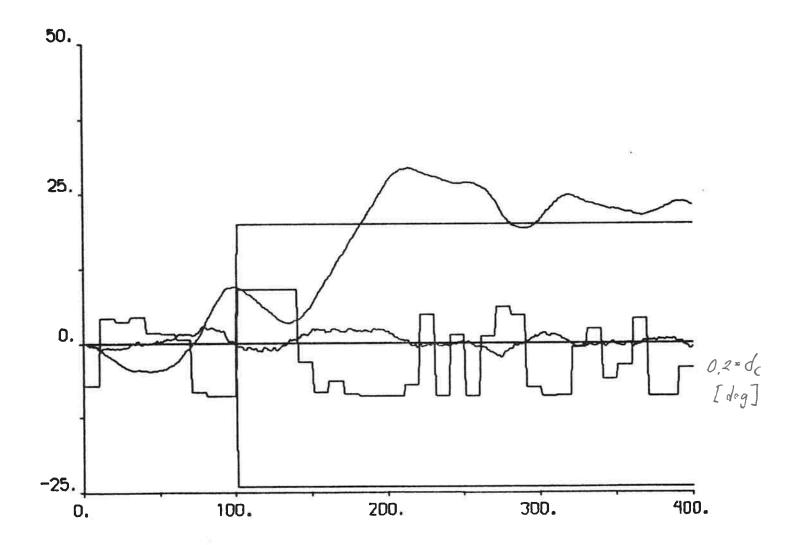


Fig. 4.91 - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{\text{ref}} = 2 \text{ deg, r}_{\text{ref}} = 0 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 100 s).

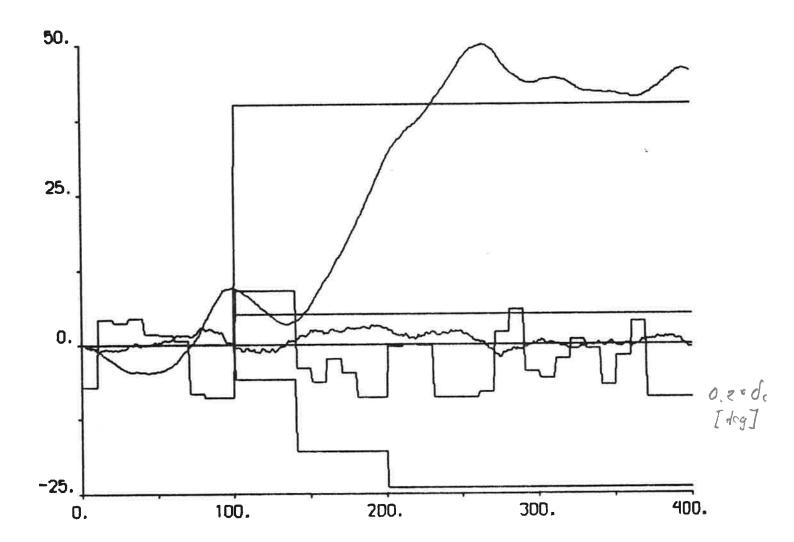
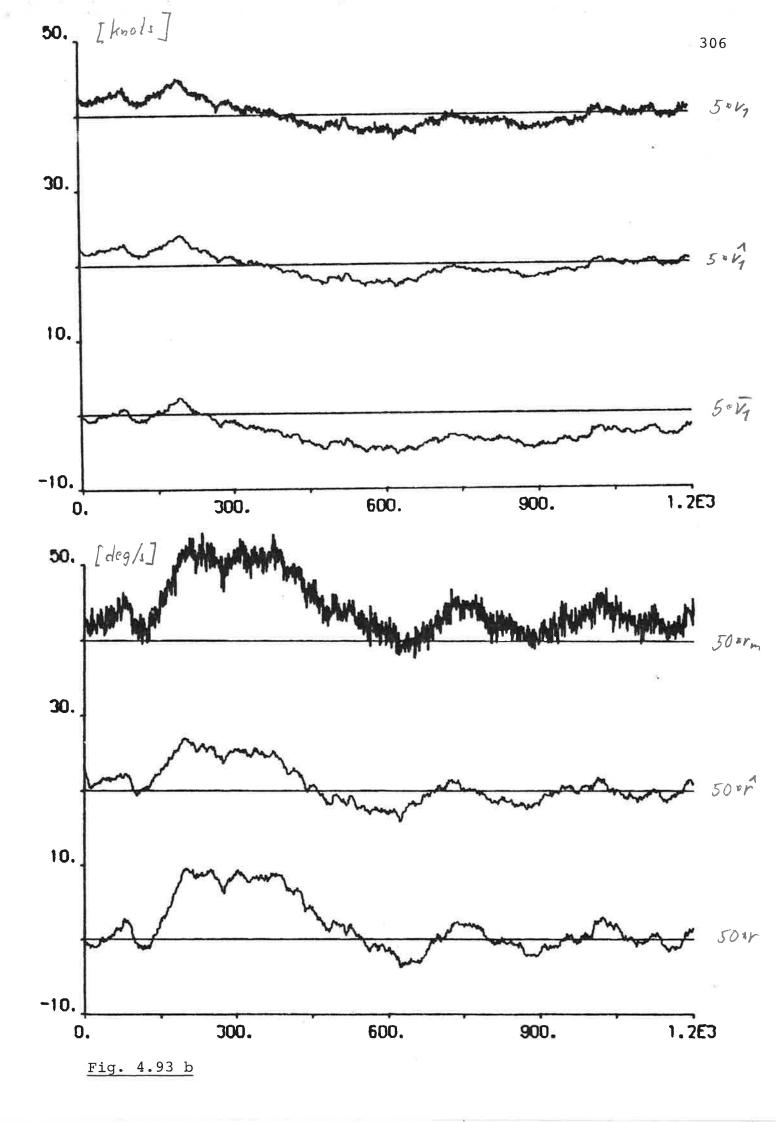


Fig. 4.92 - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, } r_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 100 s).

Fig. 4.93 a - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using estimates from the Kalman filter (\overline{c}_2 = 100 s).



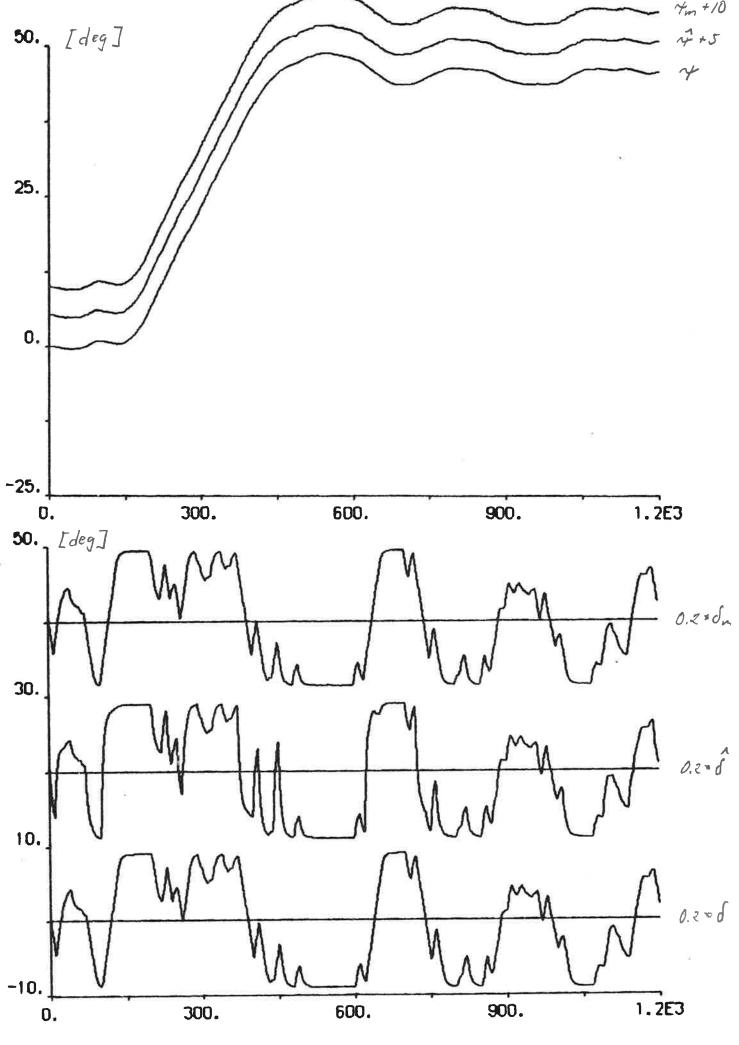
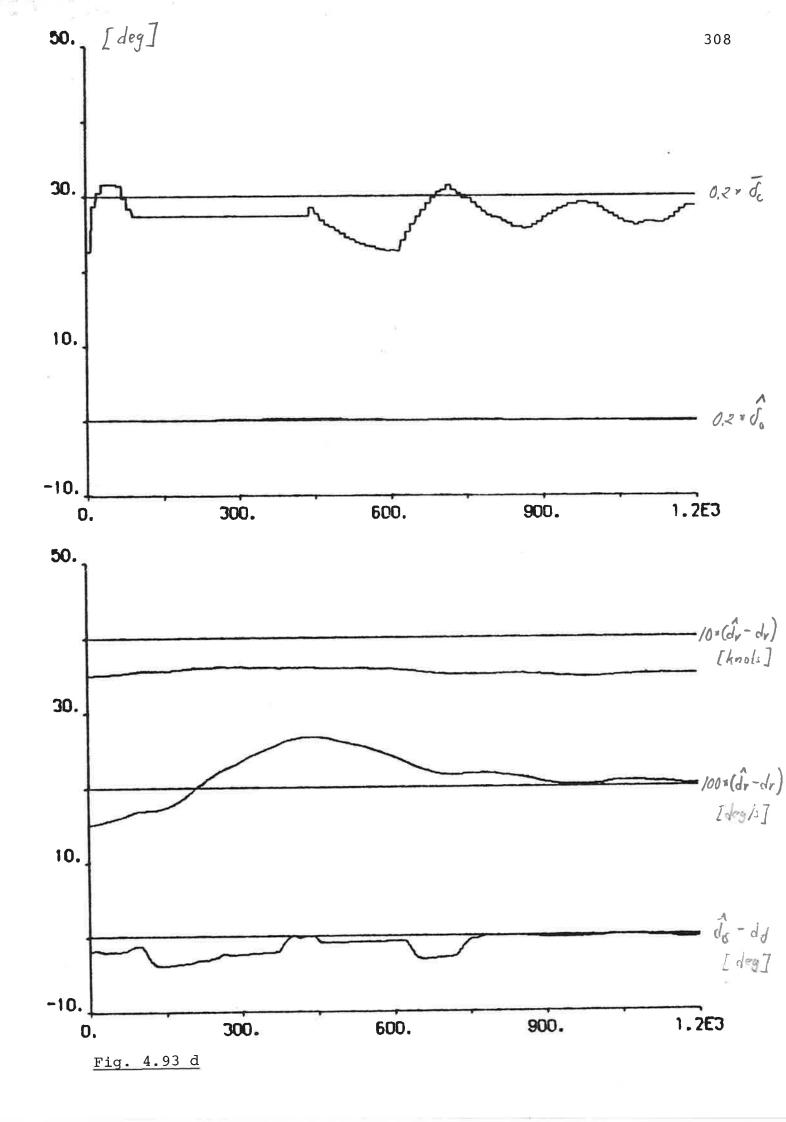


Fig. 4.93 c



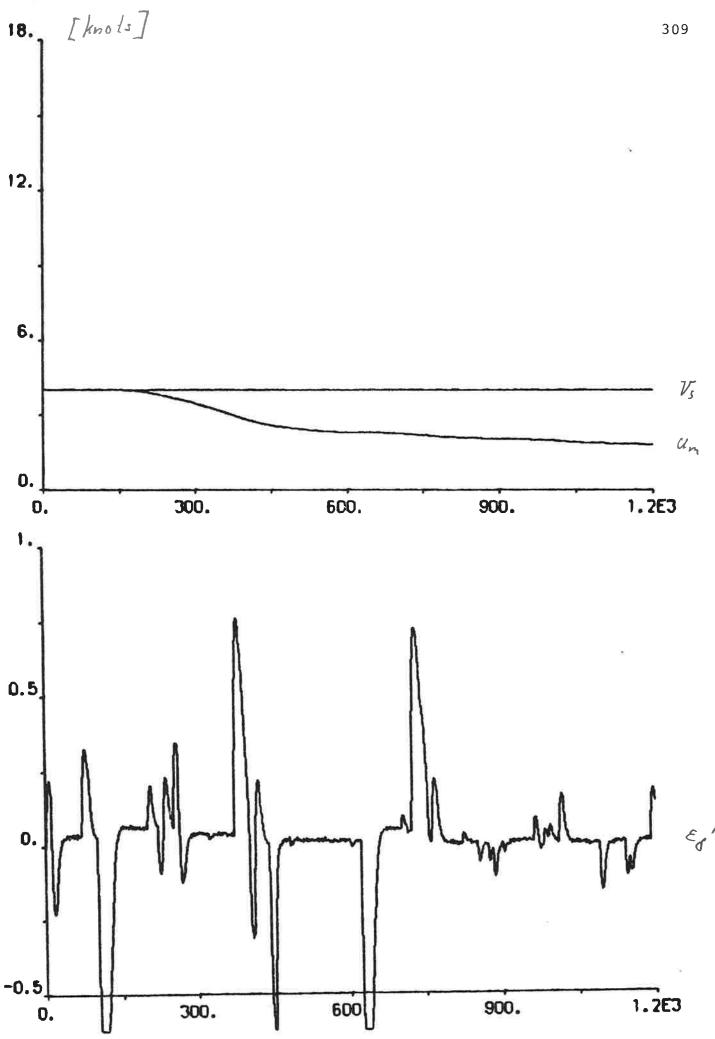
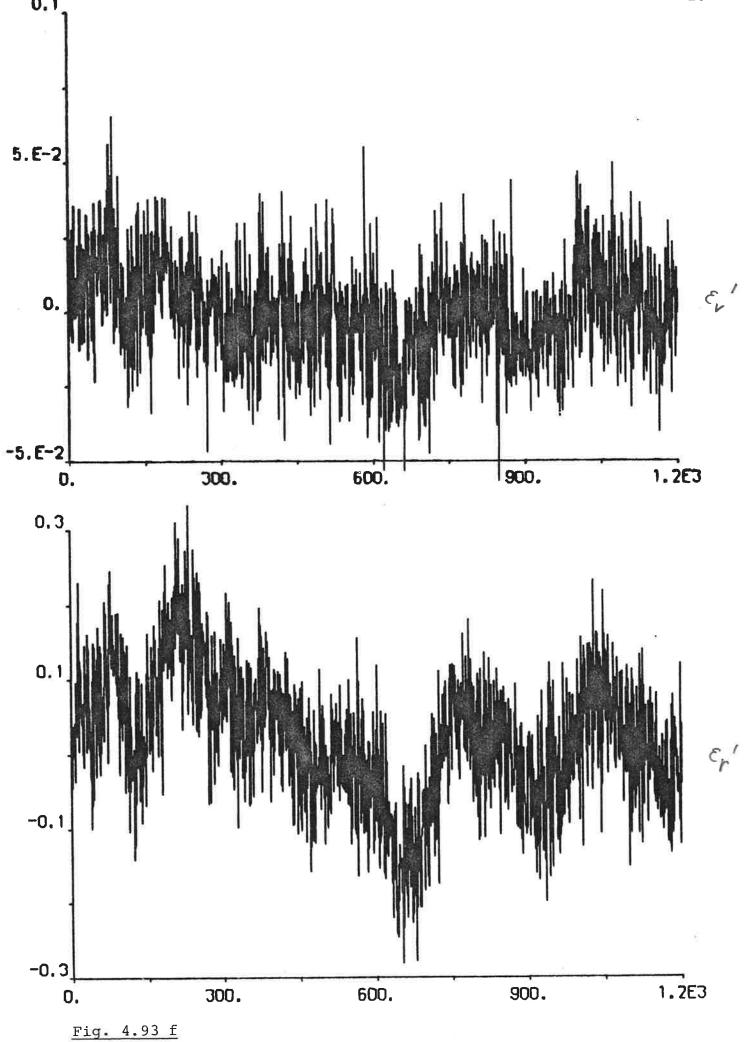


Fig. 4.93 e





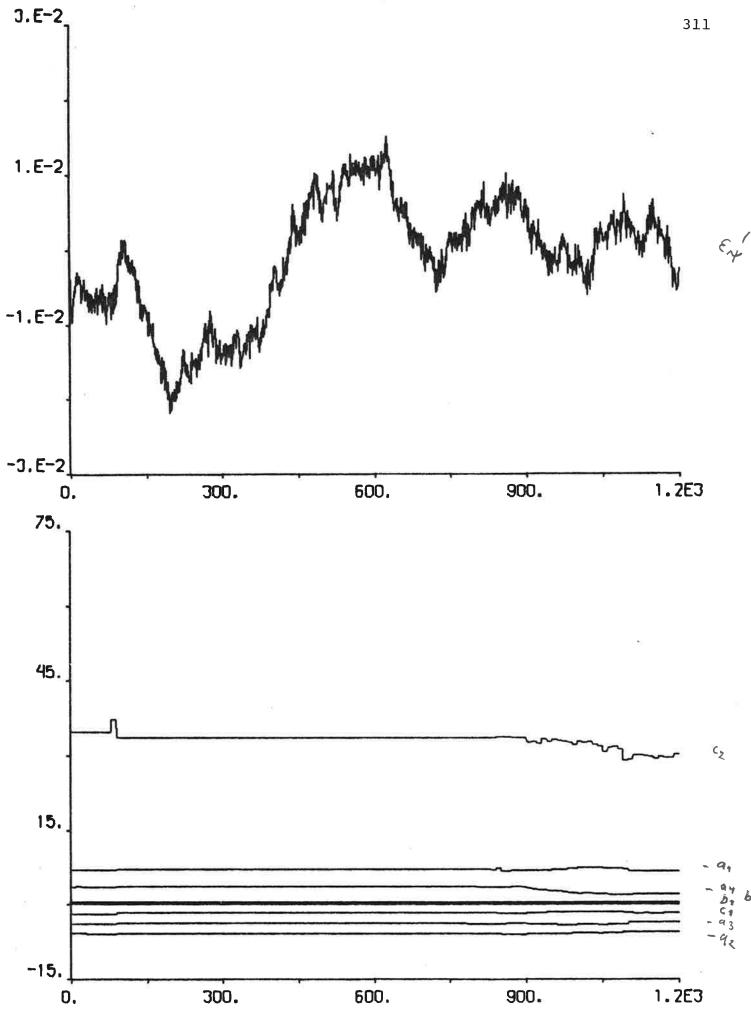


Fig. 4.93 g

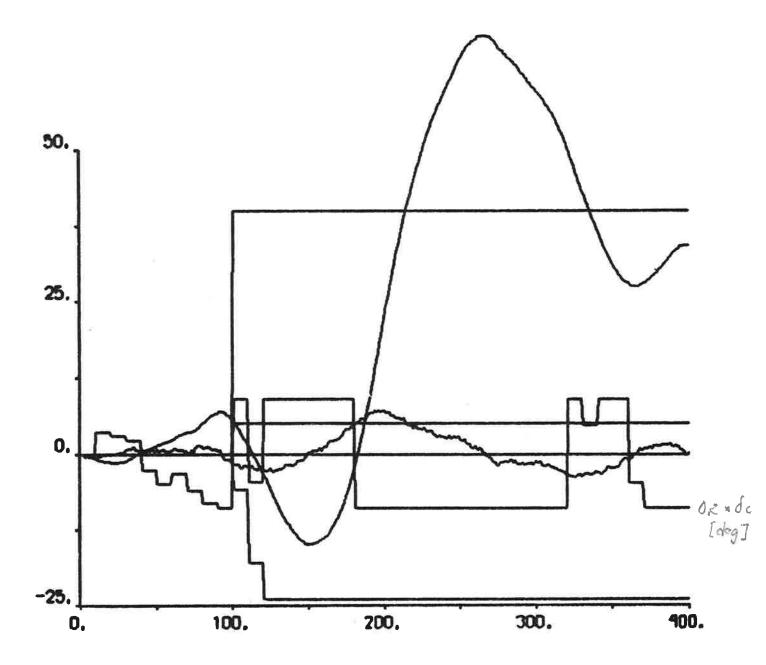


Fig. 4.94 - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{\text{ref}} = 4 \text{ deg, r}_{\text{ref}} = 0.1 \text{ deg/s, self-tuning}$ regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 100 s).

Fig. 4.95 - T = 10.5 m, n_0 = 22.1772 rpm, u_0 = 4 knots, $\Delta \psi_{ref}$ = 45 deg, r_{ref} = 0.1 deg/s, self-tuning regulator and yaw regulator using non-filtered measurements (\overline{c}_2 = 100 s).

5. CONCLUSIONS

Simulations of straight course keeping and yawing of a 350 000 tdw tanker in full load condition as well as in ballast condition and with three different initial speeds (15.8, 10 and 4 knots) are presented in this report. The disturbances applied are equivalent to a rather rough weather condition to obtain decisive comparisons. The steering is performed by an autopilot, which consists of a Kalman filter, a self-tuning regulator and a yaw regulator. A PID-regulator is also implemented for comparison. It is possible to use either the Kalman filter estimates or the non-filtered measurements in the different regulators.

The performance of the Kalman filter is very good in full load condition as well as in ballast condition, when the initial speed is 15.8 or 10 knots. The quality of the filter estimates is decreased, although quite acceptable, when the initial speed is 4 knots. A difficulty is that the limited rudder turning rate has not been considered in the Kalman filter. This means rather poor estimates of the rudder angle when large rudder deviations are requested, which is the case quite often when the speed is low. The performance of the Kalman filter is approximately of the same quality during yawing as during straight course keeping. Notice, however, that it is suitable to skip the updating of the bias estimates of the filter during yaws.

The simulations show that the performance of both the self-tuning regulator and the PID-regulator is improved when Kalman filter estimates are used instead of non-filtered measurements. When Kalman filter estimates are used, the performance of the self-tuning regulator is significantly better than the performance of the PID-regulator, if the ship is full-loaded. However, the difference between the two regulators is hardly noticable in ballast condition. It

is not possible to make any conclusions when non-filtered measurements are used by the self-tuning regulator and the PID-regulator, since the simulation results are rather confusing. However, the self-tuning regulator using Kalman filter estimates is always significantly better than the PID-regulator using non-filtered measurements, and the PID-regulator using Kalman filter estimates is always significantly better than the self-tuning regulator using non-filtered measurements.

The performance of the yaw regulator, when Kalman filter estimates are used, is very good for different load conditions and speeds, with one exception: the performance is rather bad when the ship is full-loaded and the initial speed is 4 knots. One explanation is that the actual forward speed is increased to the extreme value 1 knot during one of the simulations. When non-filtered measurements are used instead of Kalman filter estimates, the performance quality of the yaw regulator is significantly decreased. The reason is, of course, that the yaw rate measurements are very noisy. In such a case, it is highly desired to filter the yaw rate signal in some way, to obtain a better performance. Another possibility is to perform a difference approximation of the heading angle measurement to obtain a yaw rate estimate. This requires, however, a rather good resolution of the heading measurement.

6. REFERENCES

- Aspernäs, B, and Foisack, P (1975): "Simularing av styrsystem för tankfartyg", TFRT 5154, Dept of Automatic Control, Lund Institute of Technology.
- Aspernäs, B, and Källström, C (1975): "Simulering av adaptiv fartygsstyrning med Kalmanfilter", TFRT 3123, Dept of Automatic Control, Lund Institute of Technology.
- Aström, K J (1970): "Introduction to Stochastic Control Theory", Academic Press, New York.
- van Berlekom, W B, Trägårdh, P, and Dellhag, A (1975):

 "Large Tankers Wind Coefficients and Speed Loss Due
 to Wind and Sea", Trans RINA 117.
- Dyne, G and Trägårdh, P (1975): "Simuleringsmodell för 350 000 tdw tanker i fullast- och ballastkonditioner på djupt vatten", Report 2075-1, The Swedish State Shipbuilding Experimental Tank, Gothenburg, Sweden.
- Elmqvist, H (1975): "SIMNON An Interactive Simulation
 Program for Nonlinear Systems, User's Manual",
 TFRT 3091, Dept of Automatic Control, Lund Institute
 of Technology.
- Källström, C (1974): "The Sea Scout Experiments, October 1973", TFRT 7063, Dept of Automatic Control, Lund Institute of Technology.
- Källström, C (1975): "The Sea Swift Experiments, October 1974", TFRT 7078, Dept of Automatic Control, Lund Institute of Technology.

- Källström, C (1976a): "Simulation of Adaptive Ship Steering with Penalty on the Rudder Motion", TFRT 3133, Dept of Automatic Control, Lund Institute of Technology.
- Källström, C (1976b): "Simulation of Ship Yawing", Dept of Automatic Control, Lund Institute of Technology, CODEN: LUTFD2/(TFRT-7108)/1-092/(1976).
- Norrbin, N H (1970): "Theory and Observations on the Use of a Mathematical Model for Ship Manoeuvring in Deep and Confined Waters", Proc 8th Symp on Naval Hydrodynamics, Pasadena, California, USA. Also available as Publ No 68, The Swedish State Shipbuilding Experimental Tank, Gothenburg, Sweden.
- Wittenmark, B (1973): "A Self-tuning Regulator", TFRT 3054,

 Dept of Automatic Control, Lund Institute of

 Technology.

```
CONNECTING SYSTEM CON1
EAUTHOR, C.KALLSTROM 1976-03-24
TIME T
W1ATANK3A=E1ANOIS1A*NOI1
W2ATANK3A=E2ANOIS1A*NOI2
EE1ATANK3A=E3ANOIS1A
EE2ATANK3A=E4ANOIS1A
EE3ATANK3A=E5ANOIS1A
EE4ATANK3A=E6ANOIS1A
DELMAAUTA=DELMATANK3A
DELTAMAUTA=DELMNMTANK3A
V1XAUTR=V1MNATANK38
RAAUTA=RMNATANK38
PSIMAAUTA=PSIMATANK3A
PSIAAUTA=PSIMNATANK3A
UNAUTR=UMATANK3R
ANXAUT##NMATANK3#
DELCATANK3A=DELCOMAUTA
scc1:0.
SCC2:0.
SCC3:0.
SCC4:0.
NOI1:1.E-5
NOI2:1.E-5
END
```

CONTINUOUS SYSTEM TANK3

FAUTHOR, C.KALLSTROM 1976-03-17

F

FKOCKUMS CONVENTION OF THE RUDDER SIGN

E

INPUT DELC W1 W2 EE1 EE2 EE3 EE4

FDELC = RUDDER COMMAND ADEGA FW1 = STATE NOISE AM/(S*S)A FW2 = STATE NOISE A1/(S*S)A FEE1 = MEASUREMENT NOISE ADEGA FEE2 = MEASUREMENT NOISE AKNOTSA FEE3 = MEASUREMENT NOISE ADEG/SA FEE4 = MEASUREMENT NOISE ADEGA

OUTPUT DELM DELMN V1MN RMN PSIM PSIMN UM NM

FDELM = RUDDER ANGLE ADEGA

FDELMN= RUDDER ANGLE MEASUREMENT ADEGA

FV1MN = BOW SWAY VELOCITY MEASUREMENT AKNOTSA

FRMN = YAW RATE MEASUREMENT ADEG/SA

FPSIM = HEADING ADEGA

FPSIMN= HEADING MEASUREMENT ADEGA

FUM = FORWARD SPEED AKNOTSA

FNM = NUMBER OF PROPELLER REVOLUTIONS ARPMA

STATE DEL U V R PSI

#DEL = RUDDER ANGLE ARADA
#U = FORWARD VELOCITY AM/SA
#V = SWAY VELOCITY AM/SA
#R = YAW RATE A1/SA*100

FPSI = HEADING ARADA

DER DDEL DU DV DR DPSI

INITIAL

U=UO/CMK SGL=SQRT(G*L)

F1=(22.3-TT)/11.8 F2=(TT-10.5)/11.8 N=N0/60. U01=N*(17.25*F1+15.8*F2)/(1.46*CMK)

TS1 = 1/TS TS2 = TS1/cRG DL1 = DL/CRG

XUD=XUD1*F1+XUD2*F2
XUDL=XUD*L
XUU=(XUU1*F1+XUU2*F2)/XUDL
XVR=(XVR1*F1+XVR2*F2)/XUD
XRR=(XRR1*F1+XRR2*F2)*L/XUD
XUV=XUVVV/(G*L*XUDL)
XUUDD=(XUDD1*F1+XUDD2*F2)/XUDL
XT=X1T/XUD

YVD=YVD1*F1+YVD2*F2 YVDL=YVD*L YRU1=YRU/YVD YRUU1=YRUU/(SGL*YVD) YUV=(YUV1*F1+YUV2*F2)/YVDL YUUV=(YUUV1*F1+YUUV2*F2)/(SGL*YVDL) YVV=(YVV1*F1+YVV2*F2)/YV0L YRAV=(YRAV1*F1+YRAV2*F2)/YVD YARV=(YARV1*F1+YARV2*F2)/YVD YUUD=(YUUD1*F1+YUUD2*F2)/YVDL YTD1=YTD/YVD KTY1=KTY/YVD NRDL=NRD*L NRDLL=NRDL*L NRU=(NRU1*F1+NRU2*F2)/NRUL NRUU=(NRUU1*F1+NRUU2*F2)/(SGL*NRDL) NUV=(NUV1*F1+NUV2*F2)/NRDLL NUUV=(NUUV1*F1+NUUV2*F2)/(SGL*NRDLL) NVV = (NVV1 * F1 + NVV2 * F2) / NRDLLNRR=(NRR1*F1+NRR2*F2)/NRD NRAV=(NRAV1*F1+NRAV2*F2)/NRDL NARV=(NARV1*F1+NARV2*F2)/NRDL

XF=FW/XUD YF=FW/YVD NF=FW*LV/NRDLL

NTD1=NTD/NRDL KTN1=KTN/NRDL

JJ1=(1-W)/(N*D)
DISPL=DISP1*F1+DISP2*F2
TT1=N*N*D*D*D*D/DISPL
JJ=U01*JJ1
JJP=JJ/SQRT(1+JJ*JJ)
KKT=-0.33*JJP*JJP=0.38*JJP+0.35
TMO=KKT*(1+JJ*JJ)*TT1
LL1=CMK*L1
ALF1=ALFA/CRG

NUUD=(NUUD1*F1+NUUD2*F2)/NRDLL

OUTPUT

DELM=CRG*DEL DELMS=SC35*DELM+SC36 DELMN=DELM+D3+EE1 DMNS=SC37*DELMN+SC38 D3S=SC39*D3+SC40 VM=CMK*V VMS=SC41*VM+SC42 V1M=LL1*R/100*+VM V1MS=SC43*V1M+SC44 V1MN=V1M+D1+EE2 V1MNS=SC45 * V1MN+SC46 D1S=SC47*D1+SC48 RM=CRG*R/100. RMS=SC49*RM+SC50 RMN=RM+D2+EE3 RMNS=SC51*RMN+SC52 D2S=SC53*D2+SC54 PSIM=CKG*PSI PSIMS=SC55*PSIM+SC56 PSIMN=PSIM+EE4 PSMNS=SC57*PSIMN+SC58 UM=CMK*U UMS=SC59*UM+SC60 NM=NO

```
DYNAMICS
 RR=R/100.
 APSI=ALF1-PSI
 SINW=SIN(APSI)
 J=U*JJ1
 JP=J/SQRT(1+J*J)
 KT = \pm 0.33 \pm JP \pm JP = 0.38 \pm JP \pm 0.35
 TM = KT * (1+J*J) * TT1
 TM1=IF TM<TMO THEN TM ELSE TMO
 TMD =TM1 + DEL
 U2=U*U
 AV=ABS(V)
 AR=ABS(RR)
 RU=RR*U
 RU2=RU*U
 UV = U * V
 U2V=U*UV
 VAV=V*AV
 RAV=RR*AV
 ARV=AR*V
 U2D=U2*DEL
 DDEL1=-TS1*DEL+TS2*DELC
 DDEL=IF DDEL1<-DL1 THEN -DL1 ELSE IF DDEL1>DL1 THEN DL1 ELSE DDEL1
 DU=XUU*U2+XVR*V*RR+XRR*RR*RR*RR+XUV*UV*VAV+XUUDD*U2D*DEL+XT*TM-XF*COS(APSI)
 SL=YRU1*RU+YRUU1*RU2+YUV*UV+YUUV*U2V+YVV*VAV+YRAV*RAV
 DV=YARV*ARV-YUUD*U2D-YTD1*TMD+KTY1*TM-YF*SINW+W1/YVD+SL
 SL1=NRU*RU+NRUU*RU2+NUV*UV+NUUV*U2V+NVV*VAV+NRR*RR*AR
DR=(SL1+NRAV*RAV+NARV*ARV-NUUD*U2D-NTD1*TMD+KTN1*TM+NF*SINW+W2/NRD)*100.
DPSI=RR
G:9.80665
CMK:1.943844
CRG:57.2958
L:350.
UU:15.8
TT:22.3
TS:5.0
           £2 PUMPS
DL:2.32
XUD1:
XUD2:
XUU1:
XUU2:
XVR1:
XVR2:
XRR1:
XRR2:
XUVVV:
XUDD1:
XUDD2:
X1T:
YVD1:
YVD2:
YRU:
YRUU:
YUV1:
Anns:
YUUV1:
YUUV2:
```

Y V V 1: Y V V 2:

```
YRAV1:
YRAV2:
YARV1:
YARV2:
YUUD1:
YUUDZ:
YTD:
KTY:
NRD:
NRU1: 1 5
NRU2:
NRUU1:
NRUUZ:
NUV1:
NUV2:
NUUV1:
NUUV2:
NVV1:
NVV2:
NRR1:
NRR2:
NRAV1:
NRAV2:
NARV1:
NARV2:
NUUD1:
NUUD2:
NTD:
KIN:
FW: 0.
LV:25.
W :
D:
DISP1:
DISP2:
L1:164.35
ALFA:0.
NU:87.6
D1:1.
02:0.1
03:2.
SC35:1.
SC36:0.
sc37:1.
SC38:0.
sc39:1.
SC40:0.
SC41:1.
SC42:0.
SC43:1.
SC44:0.
SC45:1:
SC46:0.
SC47:1.
SC48:0.
SC49:1.
SC50:0.
SC51:1.
SC52:0.
sc53:1.
SC54:0.
SC55:1.
SC56:0.
```

SC57:1. SC58:0. SC59:1.

SC60:0.

END

```
C
C
C
C
C
C
6
0
```

```
SIHOW =
AN INTERACTIVE SIMULATION PROGRAM
FOR WOWLINEAR SYSTEMS
MAIN PROGRAM
AUTHOR HILDING ELMAVIST
REVISED, C.KALLSTROM 1976-03-24.
REFERENCE
______
A. ELMAVIST: SIMNON - AN INTERACTIVE SIMULATION
            PROGRAM FOR MONLINEAR SYSTEMS -
            USER'S MANUAL
DATA BASE
/PSCODE/ IPSEUD( )
        IPSEUD- PSEUDO CODE AREA
/VARTAB/ VARS( ), IPNTS( ), ITYPES( )
        VARS - IDENTIFIER TABLE
        IPNTS - ADDRESS TABLE
        1TYPES- TYPE TABLE
                 1: TIME
                 2: STATE
                 3: IMPUT
                 4: OUTPUT
                 5: INIT
                 6: DER
                 7: NEW
                8: TSAMP
                9: PAR
                10: VAR
/VALUES/ VALUE( )
        VALUE - VALUE TABLE AND LITTERAL TABLE
/SYSINF/ NASYST, ASYSTS( ), IVARS( ,2), INFSYS( ), LENTRY( ,3)
        NASYST- NUMBER OF ACTIVE SYSTEMS
        ASYSTS - SYSTEM IDENTIFIERS FOR ACTIVE SYSTEMS
        IVARS - DEFINING THE POSITION OF THE VARIABLE
                 TABLE FOR EACH SYSTEM
        INFSYS- SYSTEM TYPE
                 1: CONNECTING
                 2: CONTINUOUS
                 3: DISCRETE
                 4: CONTINUOUS (FORTRAN)
                 5: DISCRETE (FORTRAM)
        LENTRY- ENTRY POINTS FOR EACH ACTIVE SYSTEM
                 ( 1): INITIAL-SECTION
                        OR THE NUMBER OF A FORTRAN-SYSTEM
                 ( ,2): OUTPUT- OR CONNECT-SECTION
                 ( >3): DYMANICS-SECTION
/EXTCOM/ IEVAL, IERR, TYPE, SYSID, NEXTSY, NS
        TEVAL - POINTER IN VARIABLE TABLE
        IERR - ERROR INDICATOR
```

- SYSTEM TYPE FROM SUBROUTINE IDENT

TYPE

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```
'CONT' OR 'DISCR'
          SYSID - SYSTEM IDENTIFIER FROM SUBROUTINE IDENT
          MEXTSY- NUMBER OF EXTERNAL SYSTEMS
               - NUMBER OF ELEMENTS IN THE ALLOCATION AREA
          NS
 /ENTRYS/ hTRINT, NTRDER, NTRSMP
         NTRIBT- ENTRY POINT FOR INITIAL COMPUTATIONS
         NTRDER- ENTRY POINT FOR COMPUTATIONS OF DERIVATIVES
         NTRSMP- ENTRY POINT FOR SAMPLING
 /ENTRY/ LENTRY
         LENTRY- ACTUAL ENTRY POINT FOR CALCUL
 /PHTS/ HXC/NXD/KX( )/KDX( )/KXI( )/KTSAMP( )
         WXC
               - NUMBER OF STATES IN CONTINUOUS SYSTEMS
               - NUMBER OF STATES IN DISCRETE SYSTEMS
         G K vi
         KX
               - PCINTERS TO STATE VARIABLES
         KDX
               - POINTERS TO DER- AND NEW-VARIABLES
               - POINTERS TO INIT-VARIABLES
         KXI
         KTSAMP- POINTERS TO TSAMP-VARIABLES
 /COMINF/ (SEE INTRAC)
 /MACINF/ (SEE INTRAC)
 /MESSS/ MESS
         MESS - MESSAGE INDICATOR
 /SIHN/ NGSYST, OVFLO, IPLCOH, IEXIT, IWARN, ICOMPU, LDARK
         , NUCORT, LLPCOM, INIDRA
         AUSYST- TRUE IF NO SYSTEM DEFINED
         OVELO - TRUE IF OVERFLOW CHECK PERFORMED
         IPLCOM- TRUE IF PLOT-COMMAND SHOULD BE WRITTEN
         TEXTT - TRUE IF THE EDITOR IS TO MAKE
                 AUTOMATIC EXIT (SYST)
         IWARN - TRUE IF WARNINGS SHOULD BE WRITTEN
         ICOMPU- TRUE IF MESSAGE ABOUT COMPUTATIONS
                 IN OUTPUT-SECTION SHOULD BE GIVEN
         LDARK - TRUE IF NOT VISABLE LINES AT SAMPLINGS
        NOCURT- TRUE IF CONTINUATION OF THE SIMULATION
                 IS MOT POSSIBLE
        LLPCOM= TRUE IF COMMANDS SHOULD BE ECHOED ON THE LP
         INIDRA- TRUE IF INITIALIZATION OF DRAW
/PLT/ NPLT/IVADR( )/IHADR/PLTCOM( )
        MPLT - NUMBER OF PLOT-VARIABLES
        IVADR - POINTERS TO VERTICAL VARIABLES
        THADR - POINTER TO HORIZONTAL VARIABLE
        PLTCOM- BUFFER FOR PLOT-COMMAND
/STOVAR/ NSTV, IVARS( ), ISYSS( )
        HSTV - NUMBER OF VARIABLES TO BE STORED
        IVARS - POINTERS TO VARIABLE NAMES
        ISYSS - POINTERS TO SYSTEM IDENTIFIERS
/DATCOM/ FILE,DTF
        FILE - STORE FILE NAME
             - MINIMAL TIME INCREMENT
        UTF
/SHUVAR/ NSHVAR
        ASHVAR- NUMBER OF SHOWED VARIABLES SINCE AXES
/AX/ HMIN, DH, VAIN, DV
        HMIN - HORIZONTAL MINIMUM
        DH
              - HORIZONTAL VALUE PER CENTIMETER
```

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VMIN - VERTICAL MINIMUM
             - VERTICAL VALUE PER CENTIMETER
        UV
/ERRWEI/ EPS, WEIGHT( )
        EPS - ERROR BOUND
        WEIGHT- ERROR WEIGHTS
/ALG/ TALG
             * SPECIFIES INTEGRATION ALGORITHM
        IALG
                1: HAMPC
                2: RK
                3: RKFIX
/HARKS/ IMARK, MRK, THRK, DTMRK
        IMARK - TRUE IF MARKS WANTED
              - SPECIFIES WHICH MARKS
        MPK
        THRK - TIME FOR WEXT MARKS
        DIMRK - TIME DISTANCE BETWEEN MARKS
/USER/ LSTOP, LDARK, LCALUS, NRESUM, LFIRST, NOPLOT
        LSTOP - TRUE IF SIMULATION SHOULD BE STOPPED
        LUARK - TRUE IF DARK LINE
        LCALUS- TRUE IF THE SUBROUTIEN USRSUB SHOULD BE CALLED
        NRESUM- NUMBER OF DISCRETE SYSTEMS THAT HASN'T
                PRODUCED A DISCONTINUITY
        LFIRST- TRUE IF SYSTS CALLED FIRST TIMES
        ROPLOT- IF TRUE NO PLOT
/DESTIN/ ISYST, IPART
        ISYST - SYSTEM NUMBER
        IPART - PART MUMBER
/NSYSTS/ ESYST
        NSYST - NUMBER OF EXTERNAL SYSTEMS
/NALLOC/ NALL
        HALL - NUMBER OF ELEMENTS IN THE ALLOCATION AREA
/TIME/ T
               - THE SIMULATION TIME
        T
/STATES/ X( )
               STATES OF CONTINUOUS SYSTEMS
/DERS/ DX( )
              - DERIVATIVES OF THE STATES
        DX
/CMPVAR/ MODE, IASYST, ISYTYP, IERR, IVAR1, IVAR2, IVAL1, IVAL2
        L.LENTRI, LENTRZ, LENTR3
         HODE - COMPILER MODE
                 T: SYSTEM HEADING
                 2:
                 3: DECLARATIONS
                 4:
                 5: INITIAL-SECTION
                 6: OUTPUT-SECTION
                 7: DYMAMICS-SECTION
                 S: COMMECT-SECTION
                 9: END
         TASYST- INDEX FOR ACTUAL SYSTEM
         ISYTYP- SYSTEM TYPE
                 1: COMMECTING
                 2: CONTINUOUS
```

3: DISCRETE

```
IERR
                     - ERROR FLAG
                      INDEX FOR LOWER BOUND IN VARIABLE TABLE
              IVART -
              IVAR2 - INDEX FOR UPPER BOUND IN VARIABLE TABLE
              IVAL1 - POINTER IN THE VALUE TABLE
              IVAL2 - POINTER IN THE LITTERAL TABLE
                    - POINTER IN THE PSEUDO CODE AREA
              LENTRI- POINTER TO INITAL-SECTION
              LENTR2- POINTER TO OUTPUT- OR CONNECT-SECTION
              LENTR3- POINTER TO DYNAMICS-SECTION
      /RXPNT/ NXP( ,2)
                    - SPECIFIES WHICH STATES THAT BELONGS
              112
                      TO EACH DISCRETE SYSTEM
      /CUMB/ LSAMP/LSAMPS( )
              LSAMP - TRUE IF SAMPLING IS TO BE DONE
              LSAMPS- SPECIFIES WHICH SYSTEMS THAT IS TO BE SAMPLED
      /LIMITS/ MPSC.MVAR.MVAL.MX
              MPSC
                   - NUMBER OF ELEMENTS IN PSEUDO CODE AREA
                   - NUMBER OF ELEMENTS IN VARIABLE TABLE
              MVAK
              MVAL - NUMBER OF ELEMENTS IN VALUE TABLE
              IX
                   - MAXIMUM NUMBER OF STATES
     /SIMARG/ TI,TZ.DT.LCONT,LMARK
              11-
                      START TIME
              T2-
                      STOP TIME
              DT-
                      TIME INCREMENT
                     LOGICAL VARIABLE TO INDICATE IF CONTINUATION
             LCOMT-
                      OF SIMULATION IS WANTED
                     LUGICAL VARIABLE INDICATING IF MARKS IS WANTED
              LIAKK-
                      DURING THE PLOTTING
     /AKGSAV/ H1, H2, V1, V2
             H1
                    - LAST HORIZONTAL MINIMUM (AXES)
             45
                    - LAST HORIZONTAL MAXIMUM
                    - LAST VERTICAL MINIMUM
             V7
                    - LAST VERTICAL MAXIMUM
             V2
     /AXINF/ IXO,IYO,XAX,YAX
             IXO, IYO - ORIGO FOR AXES (TEKPOINTS)
             XAX, YAX - LENGTH OF AXES (CM)
    SUBROUTINE REGUIRED
             ISIM
             ESIMN
             SIMNSY
             SIMU
COMMON/ALLCOM/IDD(3)
COMMON /PSCODE/ IDUM1(150D)
COMMON /VARTAB/ IDUM2(1000).DUM2(500)
COMMON /VALUES/ DUM3(300)
COMMON /SYSINF/ IDUM4(151), DUM4(25)
COMMON /EXTCOM/ IDUM5(4), DUM5(2)
COMMON /ENTRYS/ IDUM6(3)
COMMON /ENTRY/ IDUM?
COMMON / PHIS!
                IDUM8(177)
```

COMMON /COMINE/ IDUM9(33), DUM9(41) COMMON /MACINE/ IDUM10(191), DUM10(107)

COMMON /MESSS/ IDUM11

```
100012(10)
     COMMON /SIMN/
                      IDUM13(12), DUM13(16)
     COMMON /PLT/
     COMMON /STUVAR/ INU135(101)
     COMMON /DATCOM/ DUM136(Z)
     COMBON / SHUVAR/ IDU137
 **** HCOFY ****
     COMMON/HCPCOM/DUM138(10), IDU138(30)
 ***** HCDPY ****
                      DUM14(4)
     COMMON /AX/
     common /ERRWEI/ DUM15(51)
     COMMON /ALG/
                      IDUM16
     COMMON /MARKS/
                      IDUM17(2), DUM17(2)
     COMMON /USER/
                      Inum18(6)
     (QMMON /DESTIN/ 100819(2)
     COMMON /NSYSTS/ IDM191
     COMMON /NALLOC/ IDM192
                      00120
     COMMON /TIME/
     COMMON /STATES/ DUM21(50)
     COMMON /DERS/ DUM211(50)
     COMMON / CMPVAR/ IDUM23(12)
     COMMON /NXPNT/ IDUM24(50)
                     IDUM25(26)
     COMBON /COND/
     COMMON /LIMITS/ MPSC.IDMZ61, MVAL, MX
     COMMON /SIMARG/ IDUM26(2), DUM26(3)
     common /ARGSAV/ DUMO(4)
      COMMON /AXINF/ ISUM27(2), NUM27(2)
     MPSC=1500
     MVAL=300
      X=20
      CALL LUGG (U)
      CALL LPHOL (U)
      CALL ISIMA
Ü
      FIODE=1
      CALL ESIMA (MODE)
10
C
      GOTO (1,2,3,4), MODE
      CALL LPHDL (1)
1
      CALL LUGG (1)
      STOP
C
      CALL SIMASY
2
      60 TO TU
Ç
3
      CALL SIMU
      GO TO: 19
C
      CALL LPHOL (2)
 4
          CALL LVGG(2)
      STUP
      END
```

-

```
SUBROUTINE SYSTS
C
      AUTHOR: C.KALLSTROM 1976-03-24.
      DIMENSION S(88)
      COMMON/DESTIN/ISYST, IDUM
      COMMON/ASYSTS/ASYST
      COMMON/NALLOC/NS
      CUMMON/SAVEAR/IS(9)
Ċ.
      MSYST=Z
      NS=88
C
      GO TO (1,2), ISYST
Ü.
      CALL SHUISE('NOIST', IS(1), S)
1
      RETURN
2
      CALL AUT
      RETURN
      END
```

e / 40

```
SUBROUTINE AUT
       SYSTEM DEFINITION OF AN AUTOPILOT FOR SHIP.
      AUTHOR . C. KALLSTRON 1976-03-18.
      REVISED, C.KALLSTROM 1976-04-01.
C
      SUBROUTINE REQUIRED
C
          AUTP3
C
             STUR
          IDENT
         INPUT
          OUTPUT
          TSAMP
          PAR
          PARV
          VAR
          VARV
C
      DIHENSIUN AMSO(4), AMEAS(4), AMSUM(4), SC(34), PPD(10)
C
      COMMON /DESTIN/ IDUM, IPART
Ü
      COMMON /TIME/ T
C
      COMMON /DATA/ ITIME, IDELC, MODYAW, IDEXP, ISTBU, IPORT,
          IFLAG, IPRINT, INAUT, IKX, MEAS (4),
          DELCO, DELTA(2), V1(2), R(2), PS1(2), DELG, DELGOM, DELTAS, U, AN,
          P. VEST, PSIPEF, RREF, DLIM, V(2), DELTAO, D1, D2, D3, TH(10),
          CGR, CKM, PI, PIZ, AL, ALI, A11, A12, A14, A15, A21, A22, A24, A25,
          A31,A32,A34,A35,A44,A45,B11,B21,B31,B41,AKK(32),
          TEST(4),88,PVO,RLV,AKVO,VCONST,VMIN,VMAX,VO,THO(10),
          PPO(10) resburg2, AK1, AK2, AK3, PSIMX1, PSIMX2, PSIMX3,
          EPS1Y/EPS2Y/EPS3Y/C1Y/C2Y/C3Y/AK1Y/AK2Y/AK3Y/AK4Y/
          AK5Y, AKOY, AK7Y, AK8Y, BD, BV, ALAN, DELAMP, PSIO, AKID,
          IREGK/IKAL/IKMX/IMX/IVVC/IVV/IREGV/IAKV/ILOS/IREGYT/IREG/IPID/
     ń
          ISTANAANBANCIANCZAKAIREGYAIYAWAITIYAIT3YAIT4YAIPKAIPCATPRA
         EPS(4), VV, PV, PP(55), EPSI1(2), EDEL(2), VLOS1(2),
     \star
          ENMI(2), CHIEDELTA, EPSIZ, VLOSZ, FMMZ,
         MEASUM(4), IVV1,
     业
     \pm
          X(8),PS(4),DD(3),DDOLD,YVV(4),VV2,VOV,VOV2,AKV,
          PFG1.PFG2.DELOLD.VOLO.ROLD.DAT(46).SINT.RRF.AINT2.
         AINT4, STD, STV, SL1, SL2, SL3, SL4, SL5, DUM(10),
          IRK/IREV/IMEAS(4)/IRYT/IR/NC/NAB/NP/K1/NDAT/NDAT1/
          NUTARTALRYALTIMIATINGSALTINGAALPALAJAL
      GO TO (100,200,300,400,500,600,700,800) , IPART
100
      CALL IDENT('DISCR', 'AUT')
C
      RETURN
200
      CALL INFUT (DELM/ DELM!)
      CALL INPUT(DELTA(1), DELTA')
      CALL INPUT (V1(1), V11)
      CALL INPUT(R(1), 'R')
      CALL INPUT (PSIM, 'PSIM')
      CALL IMPUT (PSII, 'PSI')
      CALL INPUT (U, U')
      CALL INPUT (AN, 'AN')
```

C

```
CALL OUTPUT(DELCO, 'DELCO')
CALL TSAMP(TS, 'TS')
CALL PAR(DT, 'DT')
CALL PAR(PREF1, PREF1')
CALL PAR(PREFZ, PREFZ')
CALL PAR(RU, 'RU')
CALL PAR(TU, TO')
CALL PAR(DLIMA DLIMA)
CALL PAR(BB, 'BB')
CALL PAR(PVU, PVU!)
CALL PAR(RLV, 'RLV')
CALL PAR (AKVO, 'AKVO')
CALL PAR(VCONST, VCONS!)
CALL PAR (VBIN, 'VBIN')
CALL PAR(VMAX, VMAX !)
CALL PAR(VU, 1VO')
CALL PAR(RL, 'RL')
CALL PAR(BU, 1801)
CALL PAR(Q2, 1921)
CALL PAR(AK1, AK1)
CALL PAR (ARZ, AKZ!)
CALL PAR(AK3, 'AK3')
CALL PAR(PSIMX1, PSIX1')
CALL PAR(PSIMX2, PSIX2)
CALL PAR(PSIMX3, PSIX3')
CALL PAR(EPS1Y, 'EPS1Y')
CALL PAR(EPSZY, 'EPSZY')
CALL PAR(EPS3Y, 'EPS3Y')
CALL FAR(C1Y, C1Y)
CALL PAR(CZY, CZY!)
CALL PAR(C3Y, C3Y)
CALL PAR (AK1Y / AK1Y )
CALL PAR (AKZY / AKZY )
CALL FAR (AK3Y, 'AK3Y')
CALL FAR (AK4Y, AK4Y)
CALL PAR (AK5Y / AK5Y1)
CALL PAR (AK6Y, AK6Y)
CALL PAR (AKTY) AKTY')
CALL PAR (AK8Y, AK8Y)
CALL PAR(BO, '60')
CALL FAR (BV, 'BV')
CALL PAR (ALAM, ALAM!)
CALL PAR(AIREGK, 'IREGK')
CALL PAR(AIKAL, 'IKAL')
CALL PAR(AIKMX, IKMX)
CALL PAR (AIMA ! IMX!)
CALL PAR(AIVVC, 'IVVC')
CALL PAR(AIVV, "IVV")
CALL PAR(ALREGV, IREGV')
 CALL PAR (ALAKV, 'IAKV')
CALL PAR(AIRYT, 'IRYT')
 CALL PAR (AIREG, 'IREG')
 CALL PAR(AIPID, 'IPID')
 CALL PAR(AIST, '1ST')
 CALL PAR(ANA/ NA')
 CALL PAR (ANB, 'NE')
 CALL PAR (ANCT / NC1')
 CALL PAR (ANCZ, 'NC2')
 CALL PARTAK, 'K')
 CALL PAR(AIREGY, 'IREGY')
 CALL PAR(AIYAW, IYAW)
```

CALL PAR (AITTY, "ITTY")

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CALL PAR (AIT3Y, 'IT3Y')
       CALL PAR(AIT4Y, 'IT4Y')
       CALL PAR(AILU, 'ILO')
 C
       CALL PARV (AMSO, 4, MSU!)
       CALL PARV (AKK, 32, 'AKK')
       CALL PARV (TEST, 4, TEST')
       CALL PARV(THO: 18, THO!)
       CALL PARV (PPO, 10, PPO!)
       CALL PARV(SC,34, SC')
 Ċ
       CALL VAR (DELCS, DELCS!)
       CALL VAR (DELES, DELES!)
       CALL VAR (VCS, VCS!)
       CALL VAR(VES, 'VES')
       CALL VAR (VIES, VIES!)
       CALL VAR (RES, RES!)
       CALL VAR(PSIES, 'PSIES')
       CALL VAR (PREFS, 'PREFS')
       CALL VAR (RREFS, 'RREFS')
       CALL VAR (VESTS, 'VESTS')
       CALL VAR (VVS, VVS!)
       CALL VAR (DUES, DOES!)
       CALL VAR(DIES . 'DIES')
       CALL VAR(DZES, DZES')
       CALL VAR(D3ES, 'D3ES')
       CALL VAR (AMODS, ! MODY!)
       CALL VAR (EDELS, 'EDELS')
       CALL VAR (PV, 'PV')
       CALL VAR(CN, 'CN')
       CALL VAR(EPSIZ, 'EPSIZ')
       CALL VAR (VLOSZ, VLOSZ')
       CALL VAR (ENM2, 'ENH2')
       CALL VAR (AIVV1, 'IVV1')
       CALL VAR (EFSIMA 'EPSIM')
       CALL VAR(SPSIE, 'SPSIM')
       CALL VAR (EDELT, 'EDELT')
       CALL VAR (SDELT, 'SDELT')
       CALL VAR(ENN, 'ENN')
       CALL VAR(VL1/ VL1')
       CALL VAR (VLZ, VLZ1)
       CALL VAK (FLL, FLL')
C
       CALL VARV (AMEAS, 4, MEAS!)
       CALL VARV(TH/10/ TH!)
       CALL VARV(EPS,4, 'EPS')
      CALL VARV(PPD, 10, PPD!)
       CALL VARV(EPSI1,2, EPI1')
      CALL VARV(EDEL, 2, 'EDEL')
      CALL VARV(VLOS1,2, VLS1)
      CALL VARV(ENM1,2, ENM1)
      CALL VARV (AMSUM, 4, MSUM!)
      RETURN
C
300
      OT=1.
      PREF1=U.
      PREF2=45.
      RG=0.1
      TU=100000.
      DLIM=35.
      H8=0.8J6
      PVU=1.
```

```
RLV=0.995
AKV0=100.
VCUNST=5.
VM1N=0.2
VEAX=12.
V0=8.
RL=0.99
80=1.
w2=0.
\timesK1=5.
AK2=200.
ak3=0.005
PSIMX1=0.35
PSIMX2=2.5
PS1MX3=2.5
EPSTY=U.
EPS2Y=0.02
EPS37=1.
C17=60.
02Y=50.
637=60.
AX1Y=5.
AK2Y=200.
AK3Y=0.005
ARAY=200.
AK5Y=200.
AKOY=S.
MR7Y=2.
ARBY=200.
ab=0.05
8V=0.05
ALAN=0.0833333
WIREGK=1.
AJKAL=1.
MIKHX=300.
MIMX=10.
AIVVC=1.
1 V V = 2 =
ALREGV=5.
AIAKV=U.
WIRYT=5.
AIREG=15.
alPid=U.
#IST=2.
ANA=4.
AN8=2.
ANC1=1.
ANC2=1.
KK=5.
KIKEGY=10.
AIYAN=2.
AITTY=50.
AITSY= Tou.
AIT4Y=300.
AIL0=1201.
00 302 1=1,4
A SQ(I)=0.
ARK(1)=-1,U546-4
AKK(Z)=5.3678-4
ARK(3)=1.173E-5
ARK(4)=1.8036-2
AKK(5)=-2.1726-5
AKK(6)=-7,6498-5
AKK(7)=-2.793E-6
```

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```
ANK(8)=2.542E-3
 MKK(9)=0.2332
 AKK(10) = 0.3541
 AKK(11)=4.458E-3
 AKK(12)=9.306E-5
 AKK(13)=-1.173E-5
 AKK(14)=7.462E-3
 AKK(15)=-9.884E-4
 AKK(16)=1.360E-6
 AKK(17)=-3.960E-2
 AKK(18)=0.1157
 AKK(19)=3.849E-3
 AKK(20)=1.717E-3
 AKK(21)=-1.692E-3
 AKK(22)=9.062E-4
 AKK(23)=2.932E=3
 AKK(24)=2.507E=6
 AKK(25)=-0.5298
 AKK(26)=1.237
 AKK(27)=0.3351
 AKK(28)=1.843E-2
 AKK(29)=-1.827E-2
 AKK (30)=9.269E=3
 AKK(31)=-5.871E-2
 AKK(32)=2.599E-5
TEST(1)=0.0070
TEST(2)=0.0084
TEST(3)=0.033
TEST(4)=0.0022
THU(1)=10.0
THU(2)=-12.9
THU(3)=3.4
THO(4)=0.
THO(5)=0.45
THO(6)=0.30
THU(7)=-5.3
THU(8)=42.8
THO(9)=0.
THÚ(10)=5.
PP0(1)=1000.
PPO(2)=1000.
PPG(3)=1000.
PP0(4)=1000.
PPU(5)=1.
PPG(6)=7.
PP0(7)=1000.
PPU(8)=1000.
PP((9)=U.
PP0(16)=0.
DO 310 I=1,33,2
SC(1)=1.
SC(I+1)=0.
RETURN
TS=T
IREGK=AIREGK+0.1
IKAL=AIRAL+U.1
IKMX=AIKMX+0.1
IMX=Almx+0.1
IVVC=AIVVC+u.1
IVV=AIVV+U.1
```

IREGV=AIREGV+O.T

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C

C C 405

```
IAKV=AlakV+U.1
      IREGYT=AIRYT+U.1
      IREG=AIREG+U.1
      IPID=AIPID+U.T
      IST=AISY+0.1
      LA = ARA+ Da 1
      NE = ANBtu.1
      NOTEANCT + Car
      1.02=ANC2+0-1
      K=4K+0-1
      IREGY=AIREGY+U.1
      IYAV=AlYAK+U.1
      ITTY=AITTY+U.1
      IT3Y=AIT3Y+U.1
      IT4Y=AIT4Y+O.1
      ILOMAILO+U.1
      DO 410 1=1.4
      MEAS(I) = AISJ(I) + J.1
410.
      ITIME==1
      よび日来を申り
      ITAUT=1
      T01=T0-0.09
      FLL=U.
      SUM1=0.
      SunZ=U.
      $ Lm 3 = 0 .
      507.4=0.
      SUM5=0.
      CGR=0.0174533
      CKH=U.514444
      PJ=3.141593
      P12=5.283165
      AL=350.
      AL1=764.35
      A11=0.99165
      A12=-0.0100810
      214--01020202207
      A15=0.U0UZ1Z358
      A21==0.5759537
      422=0.90485
      124=0.0104706
      a25=-0.00170991
      431=-6.000874514
      A32=0.0224515
      M34=U,003195418
      .35=-0.0000132723
      A44=0.51373
      A45=-0.18127
      811=-0.636212358
      821=0.00170991
      B31=0.0000132723
      841=0.18127
      ILUS=1
      IRK=0
      1PC=1
      IFKEC
      KETUKA
300
      ITIME=ITIEE+1
      PS1(1)=PSII
      IF(PSI1 .LT. U.) PSI(1)=PSII+36U.
```

```
P=0.341860*AN
        PREF=PREF1
        IF(T .GE. TO1) PREF=PREF2
        PSIREF=PREF
        IF(PSIREF .LT. U.) PSIREF=PSIREF+360.
        RREF=Ú.
        IF(1 .GE. 101) RREF=RO
 C
        CALL AUTP3
 C
        PSIE=PSI(2)
        IF(PSIE .GT. 18G.) PSIE=PSFE-36G.
       DO 508 I=1,10
       L=I*(I+1)/2
 508
       PPD(I)=PP(L)
       IF(ILO) 516,510,510
 510
       IF(ITIME-ILO) 514,512,514
 512
       FLL=U.
       SUM1=0.
       SUM2=U.
       SUM3=0.
       SUM4=0@
       SUM5=0.
 514
       FLL=FLL+1.
       SL1=PSIM-PREF
       IF(SL1 .LE. -180.) SL1=SL1+360.
       IF(SL1 .GT. 180.) SL1=SL1=360.
       SUM1=SUM1+SL1
       SUM2=SUM2+SL1*SL1
       SUM3=SUM3+DELM
       SUM4=SUM4+DELM *DELM
       SUM5=SUM5+P/(CKM*U*COS(CGR*SL1))
515
       DELCS=Sc(1) *DELCO+Sc(2)
       DELES=SC(3) *DELTA(2)+SC(4)
       VCS=SC(5) + V(1) + SC(6)
       VES=SC(7) * V(2) + SC(8)
       V1ES=SC(9)*V1(2)+SC(10)
       RES=SC(11) *R(2) +SC(12)
       PSIES=SC(13) *FSIE+SC(14)
       PREFS=SC(15)*FREF+SC(16)
       RREFS=SC(17) *FREF+SC(18)
       VESTS=SC(19) *VEST+SC(20)
       VVS=SC(21) *VV/CK++SC(22)
       DUES=SC(23) *DELTAU+SC(24)
      D1ES=SC(25) *01+Sc(26)
       D2ES=SC(27)*D2+SC(28)
      D3ES=$C(29)*03+SC(30)
      AMODS=SC(31) *FLOAT(MODYAW)+SC(32)
      EUELS=Sc(33) *EUELTA+Sc(34)
      UC 520 1=1.4
      AFEAS(I) = FEAS(I)
525
      ARSUM(I) = REASUM(I)
      AIVV1=IVV1
C
      RETURN
000
      TS=T+DT
      RETURN
```

```
C 700 RETURN C C EPSIM=SUM1/FLL VFSIM=SUM2/FLL=EPSIM*EPSIM SFSIM==1. IF(VPSIM .GE. U.) SPSIM=SQRT(VPSIM) & DELT=SUM3/FLL VDELT=SUM4/FLL=EDELT*EDELT SDELT=-1. IF(VDELT .GE. U.) SDELT=SQRT(VDELT) EAM=SUM5/FLL VL2=SUM2/FLL+ALAM*VDELT VL1=VL2+ALAM*EDELT*EDELT C KETURA C EED
```

decim

```
C
C
C
Ç
L
C
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C
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C

C

 \mathbb{C} C

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C 20

IF (MEAS(1)) 22,22,24

```
SUBROUTINE AUTP3
      AUTOPILOT FOR SHIP, INCLUDING KALMAN FILTER,
      SELF-TUNING REGULATOR AND PID-REGULATOR FOR
      STRAIGHT COURSE KEEPING, AND YAW REGULATOR.
      AUTHOR, C.KALLSTROM 1976-02-22.
      REVISED, C.KALLSTROM 1976-04-01.
      SUBROUTINE REQUIRED
            STUR
      COMMON /DATA/ ITIME, IDELC, MODYAW, IDEXP, ISTBD, IPORT,
          IFLAG, IPRINT, INAUT, IKX, MEAS (4),
         DELCG, DELTA(2), V1(2), R(2), PSI(2), DELU, DELCOM, DELTAS, U, AN,
         P, VEST, PSIREF, RREF, DLIM, V(2), DELTAO, D1, D2, D3, TH(10),
         CGR, CKM, PI, PI2, AL, AL1, A11, A12, A14, A15, A21, A22, A24, A25,
         A31,A32,A34,A35,A44,A45,B11,B21,B31,B41,AK(8,4),
         TEST(4), BB, PVC, RLV, AKVO, VCONST, VMIN, VMAX, VO, THO(10),
         PPO(10)/RL/BO/Q2/AK1/AK2/AK3/PSIMX1/PSIMX2/PSIMX3/
         EPS1Y, EPS2Y, EPS3Y, C1Y, C2Y, C3Y, AK1Y, AK2Y, AK3Y, AK4Y,
         AK5Y, AK6Y, AK7Y, AK8Y, BD, BV, ALAM, DELAMP, PSIO, AKID,
          IREGK, IKAL, IKMX, IMX, IVVC, IVV, IREGV, IAKV, ILOS, IREGYT, IREG, IPID,
          IST/NA/NB/NC1/NC2/K/IREGY/IYAW/IT1Y/IT3Y/IT4Y/IPK/IPC/IPR/
          EPS(4), VV, FV, PP(55), EPSI1(2), EDEL(2), VLOS1(2),
          ENH1(2)/CN/EDELTA/EPSI2/VLOS2/ENM2/
         MEASUM(4), IVV1,
          x(8),PS(4),DD(3),DDCLD,VVV(4),VV2,VOV,VOV2,AKV,
          PFO1, PFO2, DELOLD, VOLD, ROLD, DAT(46), SINT, RRF, AINT2,
          AINT4/STD/STV/SL1/SL2/SL3/SL4/SL5/DUM(10)/
          IRK, IREV, INEAS (4), IRYT, IR, NC, NAB, NP, K1, NDAT, NDAT1,
          NU1/N1/1KY/TTIM1/ITIM3/ITIM4/IP/I/J/L
      COMPUTE THE SWAY VELOCITY V(1).
      V(1)=V1(1)-AL1*R(1)*CGR/CKM
      IF (INAUT) 80,80,10
      INITIALIZE IF INAUT=1.
10
      IVV1=IVVC
      VVV(1)=U*CKH
      VV2=VVV(1)*VVV(1)
      VEST=U
      747(S)=VVV(1)
      VVV(3)=AN*U. 18*CKE
      VVV(4)=VCONST
      1F(VVV(1VV1) - VMIN) 10/14/14
12
      TF(VVV(IVV1)-VHAX) 18,18,16
14
      1 V V 1 = I V V 1 + 1
16
      GO TO 12
18
      VV=VVV(IVV1)
      VOV=VO/VV
      10V2=V0V*VCV
      IRK=IREGK
      1 K X = 0
      DELCOM=0.
      LELCU=Ú.
      IF (IRAL) 50,50,20
```

```
62
      X(4)=DELTA(1)*CGR
      60 TO 26
64
      \chi(4) = 0.
      IF (MEAS(3)) 28,28,30
26
      X(2)=R(1)*AL*CGR/VV
65
      GO TU 32
30
      x(2)=U.
      60 70 35
32
      1F(NEAS(2)) 34,34,36
      X(1)=V(1)*Ckn/VV
34
      60 TU 30
30
      W(1)=0.
      IF (NEAS (4)) 40,40,999
38
      \chi(5) = PSI(4) * CGR
40
      \chi(5) = 0.
      \chi(s) = 0.
      X(7) = 0.
      え(8)=0.
      50 42 I=1,4
      IMEAS(I)=0
42
      REASUR(1)=U
C
50
      J00LJ=J.
      PS(1)=PSI(1)
      PS(2)=PSI(1)
      PS(3)=PSI(1)
      PS(4)=PS1(1)
      gg(1)=U.
      55(2)=0.
      55(3)=0.
      PV=PVQ
      IREV=IREGV
C
      IRYT=IKEGYT
      PF01=PSI(1)
      09YA2=0
      IR=IREG
      NO=NE1+NC2
      ASTMATHO
      () () = Na () + p. ()
      尺寸=尺十寸
      NOAT=NAS+(K1+1)*(NC+2)-2
      BUATTERDATET
      NU1=NA+K+2
      W1=WU1+K
      J=NU1-1
      SL1=PSI(1)-PSIREF
      IF(SL1 .LE. -180.) SL1=SL1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
     00 52 I=1/J
      TJ3=(1) TAU
      j = J + j
      00 54 1=3,40
54
      DAT(I)=Q.
      DELUCD=U.
      VOLDEO.
      KULU=0.
      SINT=U.
      00 ou 1=1,10
      UO 60 J=1,I
      に=1*(1-1)/に+よ
      IF(I-J) 58,50,50
50
      #P(L)=PPU(I)
      50 TO 68
```

```
PP(L)=U.
58
50
      SUKITACO
      50 62 I=1-1U
      TH(I)=THU(1)
52
      EDELTA-D.
      ST0=1. =00
     EPSI2=U.
      VLOS2=U.
      ENM2=0.
      STV=1.-8V
      ISTBD=U
      IPORT=0
      IP=IPR
      IMAUTEU
80
      IF(IREGK) 200,200,90
90
      1F(IRK-IREGK) 180,100,200
C
      LOOP WITH SAMPLING INTERVAL IREGK.
C
100
      1 E K = 1
      IF (IKAL) 140,140,102
      KALHAN FILTER.
6
102
      SL5=CGK*DELCOM
      SL1=A11*X(1)+A12*X(2)+A14*X(4)+A15*X(5)+B11*SL5
      SL2=A21*X(1)+A22*x(2)+A24*X(4)+A25*X(5)+B21*SL5
      SL3=A31*x(1)+A32*x(2)+x(3)+A34*x(4)+A35*x(5)+B31*SL5
      SL4=A44*X(4)+A45*x(5)+B41*SL5
      x(1)=$L1
      \lambda(2) = SL2
      \chi(3) = SL3
      x(4) = SL4
C
      EPS(1)=DELTA(1)*CGR-X(4)-X(5)-X(8)
      EPS(2)=v1(1)*CKM/VV-X(1)-X(2)*AL1/AL-X(6)
      EPS(3)=R(1) *CGR*AL/VV-X(2)-X(7)
      EPS(4)=PSI(1)*CGR=X(3)
      IF(EPS(4) .LE. -PI) EPS(4)=EPS(4)+PI2
      IF(EPS(4) .GT. PI) EPS(4)=EPS(4)=PI2
C
      IF(IKX-IKHX) 104,108,108
104
      IKX=IKX+1
      DO 106 I=1/4
      IMEAS(I)=0
106
      GO TO 122
100
      DO 120 I=1.4
      IF (MEAS(1)) 112,112,118
      IF(ABS(EPS(I))-TEST(I)) 118,118,114
112
1114
      INEAS(I)=INEAS(I)+1
      出EASUM(I)=MEASUM(I)+1
      IF(IMEASTI) + IMX) 120,116,116
116
      MEAS(1)=1
118
      IPEAS(I)=0
120
      CONTINUE
C
122
      00 134 I=1,4
      IF (MEAS(I) + IMEAS(I)) 130,130,134
130
      00 132 3=1.8
      IF((J .EQ 8) .AND. (MEAS(1) .GT. 0)) GO TO 132
      IF((J .ER. 6) .AND. (MEAS(2) .GT. Q)) GO TO 132
      IF((J .ER. 7) .AND. (MEAS(3) .GT. 0)) GO TO 132
      X(J)=X(J)+AK(J-I)*EPS(I)
```

```
CONTINUE
132
134
      CONTINUE
      IF(X(3) *LT* U*) X(3)=X(3)+PI2
      IF(X(3) = 6E = P12) \times (3) = X(3) = P12
      V(2)=VV*X(1)/CK图
      R(2)=VV*X(2)/(AL*CGR)
      PS1(2)=X(3)/CGR
      DELTA(2)=(X(4)+X(5))/CGR
      V1(2)=VV*(X(1)+X(2)*AL1/AL)/CKM
      DELTAGEX(5)/CGR
      31=VV*X(6)/CKM
      DZ=VV*X(7)/(AL*CGR)
      53=X(8)/CGR
      COMPUTE THE FORWARD SPEED VV.
      IF(IREGV) 160,160,141
140
      1F(IREV-1REGV) 150,142,160
141
142
      IREV=1
      PS(1)=PSI(IVV)
      DOCCO (1) = DELTA(IVV) - DDOLD
       DOOLD=DELTA(IVV)
       SLT=FLOAT (IREGK*IREGV)
       SL2=BB*SL1*SL1*DD(2)/(AL*AL)
       SL3=1.+PV*SL2*3L2
       AKV=PV*SL2/SL3
       SE4=PS(1)-PS(4)
       IF(SL4 .LE. -180.) SL4=SL4+360.
       IF(SL4 .GT. 180.) SL4=SL4-360.
       SE5=PS(3)-PS(2)
       IF(SL5 .LE: +180.) SL5=SL5+360.
       IF(SL5 .GT. 180.) SL5=SL5-360.
       SL1=AKV
       IF (IAKV .GT. 0) SL1=AKVO
       Vy2=VV2+SL1*(SL4+3.*SL5-SL2*VV2)
       PY=(PV-AKY*AKY*SL3)/RLV
       PS(4) = PS(3)
       PS(3)=PS(2)
       PS(2)=PS(1)
       DB(2)=DB(1)
       IF(VV2) 144,143,143
       VVV(1)=SQRT(VV2)
 143
       GO TU 140
 144
       VVV(1) = -1.
       VEST=VVV(1)/CKM
 140
       VVV(2)=0*Ckn
       VVV(3)=AN*0.18*CK™
       VVV(4)=VCONST
 C
       IF(VVV(IVV1)-VHIN) 152,150,150
 148
       IF(VVV(IVVI)-VMAR) 154,154,152
 150
       IMVI=IVVI+1
 152
       60 TO 148
       AA=AAA(1AA1)
 154
       VLV=VU/VV
       1015=101*/01
       60 Tu 100
 156
       IREV=IREV+1
```

```
COMPUTE THE LOSS FUNCTIONS.
 160
       IF(ILOS) 176,166,162
 162
       1L0$=0
       Ch = ().
       00 164 1=1/2
       EPSI1(1)=0.
       ebel(I)=U.
       VLOS1(I)=U.
 164
       ENM1(I) = 0.
 166
       CM = CN + 1.
       SL1=(CN-1_)/CN
       00 168 1=1/2
       SL2=PSI(1)-PSIREF
       IF(SL2 .LE. -180.) SL2=SL2+360.
       IF(SL2 .GT. 180.) SL2=SL2-360.
       EPSI1(I)=SL1*EPSI1(I)+SL2/CN
       EDEL(I)=SL1=EDEL(I)+DELTA(I)/CN
       VLOS1(I)=SL1*VLOS1(I)+(SL2*SL2+ALAM*(DELTA(I)=EDEL(I))*
          (DELTA(I)-EDEL(I)))/CN
 168
       ETM1(I)=SL1*ENM1(I)+P/(CKM*U*COS(SL2*CGR)*CN)
C
170
       IF (IPK .GT. U) IPRINT=1
       GG TC 200
\Box
180
       IRK=IRK+1
C
200
       IF(IDEXP) 201,201,700
C
       LOOP WITH SAMPLING INTERVAL IREGYT FOR YAW TEST.
201
       IF(IRYT-IREGYT) 206,202,208
Ü
202
       IRYT=1
       SL1=PSIREF-PF01
       PF01=PSIREF
       IF (MODYAW) 204,204,300
204
       PF02=PSIREF-SL1
       IF(SL1 .LE. -180.) SL1=SL1+360.
       IF(SL1 .GT. 16J.) SL1=SL1-360.
      1F(ABS(SL1)-PSINX1) 500,500,302
200
      IRYT=IRYT+1
208
      IF (MODYAW) 500,500,300
Ü
      LOOP WITH SAMPLING INTERVAL TREGY FOR YAWING.
300
      IF(IRY-IREGY) 390,302,392
C
302
      IRY=1
      SL1=P$IREF-PF02
      PF02=PSIRFF
      IF(SL1 .LE. -180.) $L1=$L1+360.
      IF(SL1 .GT. 180.) SL1=SL1-360.
      SLZ=PS1(IYAW) - PSIREF
      IF(SL2 .LE. -180.) SL2=SL2+360.
      IF(SL2 .GT. 180.) SL2=SL2-360.
      IF (MODYAW) 309,309,304
304
      IF(ABS(SL1)-PSIMX3) 320,320,306
300
      IF (HODYAH-2) 314,314,308
308
      MODYAR=1
309
      IF(SL2) 310,310,312
310
      RREFERRER
```

```
30 TO 320
      RRF=-RREF
      60 TO 320
      IF(RRF) 316,316,318
514
      IF(SLZ) 368,320,320
310
      IF(SL2) 320,320,308
318
C
      RRF=RREF*ABS(RRF)/RRF
320
      SL3=R(IYAW)-RRF
      IF(MODYAW) 322,322,324
      IF(ABS(SL1)-PSIMX2) 338,338,326
322
      IF(MUDYAW-2) 328,332,325
324
      IF(MODYAW-4) 336,339,339
325
0
      MODYAWET
320
      (TIM)==IREGY
      ITIM1=ITIM1+IREGY
328
      IF(RKF .GE. U. .AND. SL3 .GT. -EPSTY) GO TO 330
      IF(RRF .LT. U. .AND. SL3 .LT. EPS1Y) GO TO 330 IF(ITIM1-IT1Y) 332,332,330
C
      MODYAW=2
330
       AINT2=0.
      IF(RRF .GE. O. .AND. -CZY*R(IYAW) .LT. SL2) GO TO 334
332
       IF(RRF .LT. O. AAND. -C2Y*R(IYAN) .GT. SL2) GO TO 334
       IF(MODYAW-1) 340,340,350
334
       MODYAN=3
       ITIMS=-IREGY
       ITIM3=ITIM3+IREGY
330
       IF(ABS(R(IYAH)) .LT. EPSZY) GO TO 338
       IF(RRF *GE. U. .AND. SLZ .GT. -EPS3Y) GO TO 338
       IF(RRF .LT. G. .AND. SL2 .LT. EPS3Y) GO TO 338 IF(ITIM3-IT3Y) 360,360,338
338
       MODYAHEA
       ITIM4==IREGY
       AINT4=U.
       ITI例4=ITI例4+IREGY
339
       1F(ITIM4-IT4Y) 370,370,400
Ç
       YAW PHASE 1.
C
340
       SL4=AK4Y * SL3
       SL5=ABS(C1Y*REF)
       IF(SL4 .GT. SL5) SL4=SL5
       IF(SL4 .LT. -SL5) SL4=-SL5
       DELCOM=-VOV2*SL4+EDELTA
       GQ TO 300
C
       YAW PHASE 4.
C
       DELCON = - VOV2*(AK5Y*SL3+AK6Y*AINT2) + EDELTA
35 U
       AINT2=AINT2+SL3*FLOAT(IREGY)
       60 TO 33U
 C
       YAW PHASE 3.
       SL4=AK7Y*SL2+AK8Y*R(IYAW)
 360
       SL5=ABS(C3Y*RRF)
       IF(SL4 .GT. SL5) SL4=SL5
       IF(SL4 .LT. -SL5) SL4=-SL5
       DELCOM=-VOV2*SL4
```

```
60 TO 380
       YAW PHASE 4.
 370
       DELCOM=-VUV2*(AK1Y*SL2+AK2Y*R(IYAW)+AK3Y*AINT4)
       AINT4=AINT4+SL2*FLOAT(IREGY)
 C
 Ċ
 386
       IF (DELCOM .GT. DLIM) DELCOM=DLIM
       IF (DELCON .LT. -DLIM) DELCOM=-DLIM
       60 TU 600
 C
 39U
       IRY=IRY+1
 C
 392
       60 TO 900
       INITIALIZING OF STRAIGHT COURSE KEEPING.
 C
400
       J=1,01-1
       SL1=PSI(IST)-PSIREF
       IF(SL1 .LE. -180.) SL1=SL1+360.
       IF(SL1 .GT. 180.) SL1=SL1=360.
       00 402 I=1/J
 402
       DAT(I) = SL1
       J = J + 1
       DO 4U4 I=J/46
404
       DAT(I)=U.
       DELOLD=EDELTA
       VOLD=V(1ST)
       ROLD=R(IST)
       MODYAWEJ
       SINT=C.
       GO TU 502
C
C
       LOOP WITH SAMPLING INTERVAL IREG FOR STRAIGHT COURSE KEEPING.
500
       IF(IR-IREG) 540,502,542
C
502
       1 F = 1
       SL1=PSI(IST)-PSIREF
       IF(SL1 .LE. -180.) SL1=SL1+360.
       IF(SL1 .GT. 180.) SL1=SL1-360.
       IF(IPID) 504,504,520
C
504
      SL2=V(IST)-VOLD
      SL3=R(IST)-ROLD
      VOLD=V(IST)
      ROLD=R(IST)
      DAT(1)=SL1
      IF(NC-1) 514,506,508
506
      IF(NC1) 512,512,508
508
      J=NAB+2*K+3
      DAT(J)=SL2*VV
      1F(NC-1) 514,514,510
510
      J=1,AB+3*K+5
      DAT(J)=SL3
      GG TO 514
512
      J=NAB+C*K+5
      DAT(J)=SL3
C
514
      CALL STUR(DAT, TH, PP, DUM, AL, NA, NAB, MP, K1, NDAT, NDAT1, NU1, N1)
      SL2=VUVZ*EU/(80*E0+Q2)
      DELCOM=SL2*DAT(NU1)+DELOLD
```

```
GC TO 530
      DELCOM=-VUV2*(AK1*SL1+AK2*R(IST)+AK3*SINT)
526
      SINT=SINT+SL1*FLOAT(IREG)
      IF(DELCOM .GT. DLIM) DELCOM=DLIM
530
      IF (DELCOM LIT. -DLIM) DELCOM=-DLIM
      IF(IDEXP) 531,531,800
      IF(IP10) 532,532,534
531
      DAT(NU1)=60*(DELCGM-DELGLD)/VOV2
532
      DELOLD=DELCOM
534
      60 TU 600
Ū
546
      IR=IR+1
C
      GC TU 900
542
      COMPUTE THE MEAN RUDDER COMMAND EDELTA AND THE LOSS FUNCTIONS.
Ü
      IF (NOOYAW) 604,604,602
600
      IF(MODYA = 4) 800,604,604
502
      EDELTA = EDELTA + (STD+BD) * (DELCOM-EDELTA)
004
      STU=(1.-BD)*STD/(1.-BD+STD)
      IF(MODYAW) 606,606,800
      EPSI2=EPSI2+(STV+BV)*(SL1-EPSI2)
506
      SL2=DELCOM-EDELTA
      SL3=SL1*SL1+AL4M*SL2*SL2
      VLOS2=VLOS2+(STV+BV)*(SL3-VLOS2)
      SLZ=P/(CKM*U*COS(SL1*CGR))
      ENAZ=ENAZ+(STV+8V)*(SLZ-ENAZ)
      STV = (1. -BV) *STV/(1. -BV + STV)
      GC TO SCU
      IDENTIFICATION EXPERIMENT.
C
      IF(IR-IREG) 720,702,900
700
C
702
      IR=1
      IF(ISTBD+IPORT-1) 712,704,710
      IF(ISTBD) 708,708,706
704
      DELD=DELARP
706
       60 TO 712
708
       SELS=-DELAMP
      GO TO 712
710
      DELO=O.
712
       ISTB0=U
       IFORT=U
       SL1=PSI(1) -PSIU
       IF(SL1 .LE. -180.) SL1=SL1+360.
       IF(SE1 .GT. 180.) SE1=SE1=360.
       DELCOM=DELU-AKIO*SL1
       GO TO 530
720
       IR=IR+1
       GC TO 900
C
       INDICATE RUDDER CHANGE.
C
300
       DELCO=DELCOM
       IDELC=1
       IF(IPC .GT. U) IPRINT=1
C
       IF(19R) 999,999,902
900
```

TH(NA+NS+2)=CE(2)

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SUBROUTINE STUR(DATATHAPADUMARLANAANABANPAKIANDATANDATIANUIANI)
SELFTUNING REGULATOR BASED ON LEAST SQUARES IDENTIFICATION
AND MINIMUM VARIANCE CONTROL, ADMITS FEEDFORWARD AND
EXPLOITS SYMMETRY OF P.
AUTHOR, CERALLSTROM 1976-02-18.
THE ALGORITHM IS BASED ON THE MODEL
Y(T)+X(1)*Y(T-K-1)+...+A(NA)*Y(T-K-NA)=
           80*(U(T-K-1)+6(1)*U(T-K-2)+...+B(NB)*U(T-K-NB-1))+
           C(1)*V1(T-K-1)+C(2)*V2(T-K-1)+...+C(NC)*VNC(T-K-1)+EPS(T)
AT EACH STEP THE LEAST SQUARES ESTIMATES OF THE PARAMETERS
OF THE MODEL ARE COMPUTED. THE CONTROL VARIABLE U(T) TO
BE APPLIED AT TIME T IS THEN COMPUTED FROM
US(T) = AE(1) * Y(T) + ... + AE(NA) * Y(T-NA+1)
                  -BE(1)*US(T-1)-...-BE(NB)*US(T-NB)
                  -CE(1)*V1(T)-...-CE(NC)*VNC(T)
WHERE AE BE AND CE ARE THE PARAMETER ESTIMATES
AND US THE SCALED CONTROL SIGNAL I.E. US=BO+U
WHEN USING THE ALGORITHM THE PROCESS OUTPUT Y(T) AND THE
FEEDFORWARD SIGNALS V(T) ARE READ AT TIME T AND THE CONTROL
SIGNAL U(T) TO BE APPLIED AT TIME T IS THEN COMPUTED
DAT- VECTOR OF DIMENSION NA+NB+(K+2)*(NC+2)-2 CONTAINING
         PROCESS OUTPUTS Y. SCALED CONTROL VARIABLES U
         AND FEED FORWARD SIGNALS V ORGANIZED AS FOLLOWS
                                                                                       RETURNED AS Y(T)
DAT(1) = Y(T)
                                                                                       RETURNED AS Y(T)
DAT(2)=Y(T-1)
                                                                                       RETURNED AS Y(T-1)
DAT(3)=Y(T-2)
                                                                                       RETURNED AS Y(T-K-NA+1)
DAT(NA+K+1)=Y(T+K=NA)
                                                                                       RETURNED AS US(T)
 DAT(NA+k+2)=US(T-1)
 DAT(NA+K+3)=US(T-2)
                                                                                       RETURNED AS US(T-1)
                                                                                       RETURNED AS US(T-K-NB)
DAT(NA+13+2*K+2)=US(T-K-NB-1)
DAT(MA+MB+2*K+3)=V1(T)
                                                                                       RETURNED AS US(T-K-NB-1)
                                                                                       RETURNED AS V1(T)
DAT (NA+NS+2*K+4)=V1(T-1)
                                                                                       RETURNED AS V1(T-K)
 (1) 0 A T (1) 0 A T (1) A 
DAT(NA+NB+(K+2) + (NC+1)-1)=VNC(T)
                                                                                     RETURNED AS V(NC-1)(T-K-1)
 DAT(NA+NB+(K+2)*(NC+2)-2)=VNC(T-K-1)
                                                                                   RETURNED AS VNC(T=K)
 TH- VECTOR OF DIMENSION NP=NA+NB+NC CONTAINING THE PARAMETER
        ESTIMATES ORGANIZED AS FOLLOWS
TH(1)=-AE(1)
TH(2)=-AE(2)
 TH(NA) = -AE(NA)
 TH(NA+1)=BE(1)
 TH(hA+2)=BE(2)
 TH(NA+NB)=BE(NB)
 TH(NA+NB+1)=CE(1)
```

```
TH(NA+NB+NC)=CE(NC)
        P- COVARIANCE MATRIX STORED AS FOLLOWS
        P(1) = P(1 \cdot 1)
        P(2) = P(2,1)
        P(3)=P(2,2)
        P(I*(I-1)/2+J)=P(I_J)
C
C
        P(NP*(NP*1)/2)=P(NP*NP)
C
        DUM- DUMMY VECTOR OF DIMENSION NP
        RL- BASE OF EXPONENTIAL WEIGHTING FACTOR
        NA- NUMBER OF A-PARAMETERS (MAX 10, MIN 0)
            MB- NUMBER OF B-PARAMETERS (MAX 10, MIN 0)
           HC- NUMBER OF C-PARAMETERS (MAX 10, MIN 0)
            K -NUMBER OF TIME DELAYS IN THE MODEL
               (MAX ((46-NA-NB+2)/(RC+2))-2 , MIN ())
        NAS- NATNB
        NP- NA+NB+NC (MAX 10, MIN 1)
        K1 - K + 1
        NDAT- NA8+(K1+1)*(NC+2)-2
        NDAT1- NDAT+1
C
        NU1- NA+K+2
        N1- NU1+k
        SUBROUTINE REQUIRED
              NONE
        DIMENSION DAT(46), TH(10), P(55), DUM(10)
        RES=DAT(1)-DAT(N1)
        DENOM=1.
        00 12 I=1,NP
        R=0.
        UO 10 J=1,NP
        L=I*(I-1)/2+J
        IF (J_{\alpha}GT_{\bullet}I) L=J*(J-1)/2+I
        M=K1+J
        IF (J.GT.NA) N=M+K1
        IF (J.GT.NAB) M=2*K1+(J-NAB)*(K1+1)+NAB
        R=R+P(L)*DAT(M)
10
        DUM(I) = R
        M=K1+I
        IF (I.GT.NA) N=N+K1
        IF (1,GT,NAB) M=2*K1+(I-NAB)*(K1+1)+NAB
        DENOM=DENOM+R*DAT(M)
12
        RES=RES-DAT(图)*TH(1)
C
        90 20 I=1,NP
        R=DUM(I)/DENON
        TH(I)=TH(I)+R*RES
        00 20 J=1,I
        L=I + (I-1)/2+J
20
        P(L)=(P(L)-R*OU*(J))/RL
C
        R=0.
        00 30 I=1,NP
        \Gamma = I
        IF (I.ST.NA) L=L+K1
        IF (I.GT.NAB) L=NAB+K1+(K1+1)*(I-NAB)
```

3U R=R-TH(I)*DAT(L)
C
DO 32 I=2,NDAT
L=NDAT1-I
32 DAT(L+1)=DAT(L)
DAT(NU1)=R
C
RETURN

END