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Published in:
International Network on Timber Engineering Research - Proceedings Meeting 48

2015

Document Version:
Publisher's PDF, also known as Version of record

A beam theory fracture mechanics approach for strength analysis of beams with a hole

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Keywords: Glulam, Hole, Fracture Mechanics, Strength Analysis.

1 Introduction

A hole in a member constitutes a sudden change in the cross section and concentrated perpendicular to grain tensile stress and shear stress appear in the vicinity of the hole, see Figure 1.1. This stress situation may for relatively low external loads lead to crack propagation in the fiber direction. Looking at the design approaches for beams with a hole in timber engineering design codes over the last decades, it can be seen that the issue has been treated in many different ways. The theoretical backgrounds on which the design approaches are based show fundamental differences and there are also major discrepancies between the strength predictions according to the different codes as well as between tests and predictions according to codes (Aicher & Höfflin 2004, Höfflin 2005, Danielsson & Gustafsson 2008, Danielsson & Gustafsson 2011).

There is at the moment no fully accepted design method based on a completely rational mechanical background. There are for example no design equations for beams with a hole in the contemporary version of Eurocode 5 (2004). However, design equations are found in the German and Austrian National Annexes to EC 5. This design approach originates from the work presented by Kolb and Epple (1985), although simplifications and empirical modifications have been added over time.

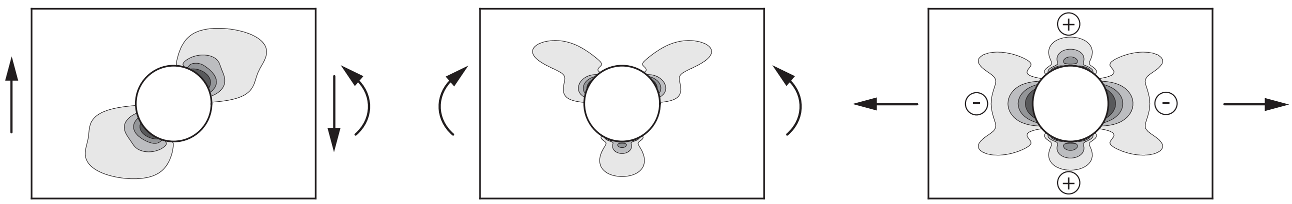

Figure 1.1. Schematic illustration of perpendicular to grain tensile stress distribution; hole placed in shear force dominated region (left), pure bending (middle) and axially loaded member (right).

For applications with concentrated perpendicular to grain tensile stress and shear stress, conventional maximum stress failure criteria are seldom of any use but instead strength analysis by means of fracture mechanics is more relevant. The EC 5 design equations for end-notched beams and dowel-type connections loaded at an angle to grain are examples of fracture mechanics based design equations.

Concerning the case of end-notched beams, an equation for the energy release rate during crack extension and the corresponding beam strength was derived by compliance analysis using beam theory almost 30 years ago (Gustafsson 1988). In its EC 5 implementation, the Gustafsson design criterion is expressed as a comparison of a nominal shear stress and the reduced shear strength, although the decisive material properties from the fracture mechanics approach are fracture energy and stiffness.

Inspired by the relative simplicity of the design equation for end-notched beams, a similar but generalized method was outlined by Gustafsson (2005). This generalized method is in the present paper in detail developed and presented for the case of beams with a hole. The application of a beam theory fracture mechanics approach for beams with a hole is more complex than for end-notched beams due to:

- the significant influence of shear makes it necessary to consider the respective contributions of modes I and II to the total energy release rate,
- the normal force acting on the cross section parts above and below the hole must be considered and
- the cross section forces and moments acting on the parts above and below the hole are statically indeterminate.

In this paper, a beam theory fracture mechanics approach for strength analysis of beams with a hole is presented. This approach is based on assumptions of an orthotropic material behavior, a beam model according to Timoshenko beam theory and a mixed mode fracture criterion based on Linear Elastic Fracture Mechanics (LEFM).

The strength analysis approach is evaluated by means of a comparison between theoretically predicted beam strengths and strengths found from experimental tests. For this purpose, test results for beams with circular holes (Höfflin 2005, Aicher & Höfflin 2006) are used as well as test results of beams with square holes (Danielsson 2008). The tests of beams with square holes are further also presented and discussed in (Danielsson & Gustafsson 2008) and (Danielsson & Gustafsson 2011).

Further evaluation is carried out by means of comparison to other strength analysis methods, including both code type methods and more general methods based on linear and nonlinear fracture mechanics approaches carried out using 2D and 3D finite element (FE) analysis.

2 Theory and model formulation

The basic concept of the present approach for analysis of beams with a hole relates to rectangular/square holes and by assumption also circular holes are analyzed. Cross section forces and moments in the beam parts above and below the hole are determined by equilibrium considering kinematic assumptions according to Timoshenko beam theory. An infinitesimally short part of the beam at the end of the hole is then considered. The forces and moment acting across a horizontal section of the infinitesimally short part of the beam, dividing it into one part above and one part below an assumed crack, are determined by equilibrium. The energy release rates for modes I and II are then obtained by using the method of work of crack closure with consideration to the deformations of the infinitesimal parts above and below the assumed crack. The beam strength with respect to cracking is then found by using a mixed mode fracture criterion.

In order to facilitate a convenient formulation, which is consistent for circular and rectangular/square holes with or without rounded corners, a number of assumptions and simplifications are introduced. These are partly based on engineering approximations and partly based on experience from experimental tests.

2.1 Basic assumptions and definition of geometry

Definitions and notation for the parameters used to describe the beam and the hole geometry are found in Figure 2.1. A circular hole with diameter $2r$ may formally be regarded as a rectangular/square hole with $a = h_d = 2r$. The parameter s defines the hole placement with respect to beam height and is given by the y-coordinate of the centre of the hole. N, V and M refer to values at the edge of the hole on its right hand side, although the figure below may seem to indicate another location.

The formulation of the present approach is general in the sense that it allows for any combination of cross sectional forces N, V and M. Depending on loading conditions and the signs of N, V and M, different areas in the hole vicinity are exposed to perpendicular to grain tensile stress and may experience cracking, see Figure 1.1. The most common case for practical engineering purposes is however believed to be transversal loading giving a combination of shear force and bending moment only.

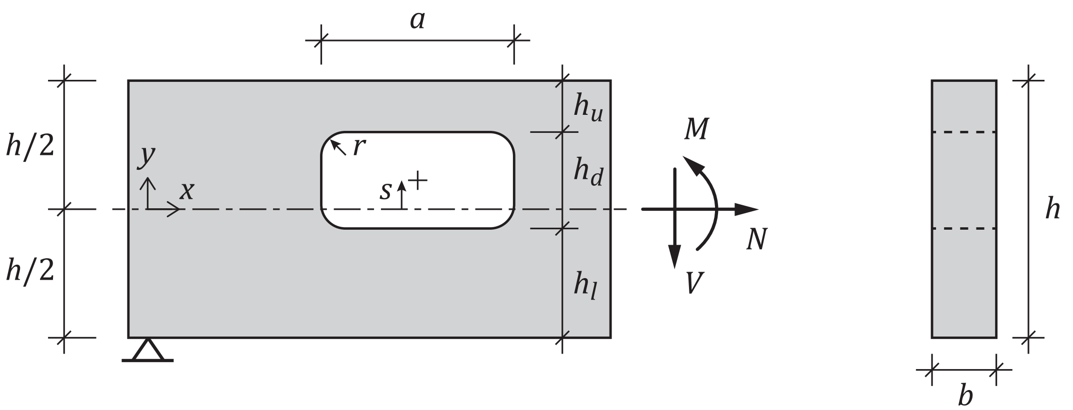

Figure 2.1. Definition of beam and hole geometry.

The location of the point of crack initiation is assumed to be known a priori and crack propagation is assumed to occur in the parallel to grain direction, which is assumed to be aligned with the beam length direction (the *x*-direction). The chosen predefined crack location is based on results from the experimental tests mentioned above of beams loaded in bending, having either a circular or a square hole. The tests with square holes having rounded corners ($r \approx 0.12h_d$) showed that cracking commonly started at the top right corner of the hole. Crack initiation commonly occurred within the rounded part of the hole corner. The tests with circular holes showed crack initiation at the hole periphery at an angle of approximately 45° with respect to the beam central axis, for holes placed centrically with respect to the beam height. Based on these findings from experimental tests, the location of the crack is in the present approach assumed to be at an angle of 45° from the beam length direction according to Figure 2.2 a), yielding a distance from the upper horizontal hole edge to the crack plane of $0.3r$.

The geometry of the beam parts above and below the hole are in the calculations simplified in the sense that these are assumed to be prismatic. As illustrated in Figure 2.2 b)-e), there are a number of feasible interpretations of the hole geometry which result in this simplification. The interpretation used for all analyses presented here is according to Figure 2.2 b). Interpretation of hole geometry according to Figures 2.2 c)-d) or some other simplification may however possibly yield equal or better agreement between model strength predictions and experimentally found beam strength.

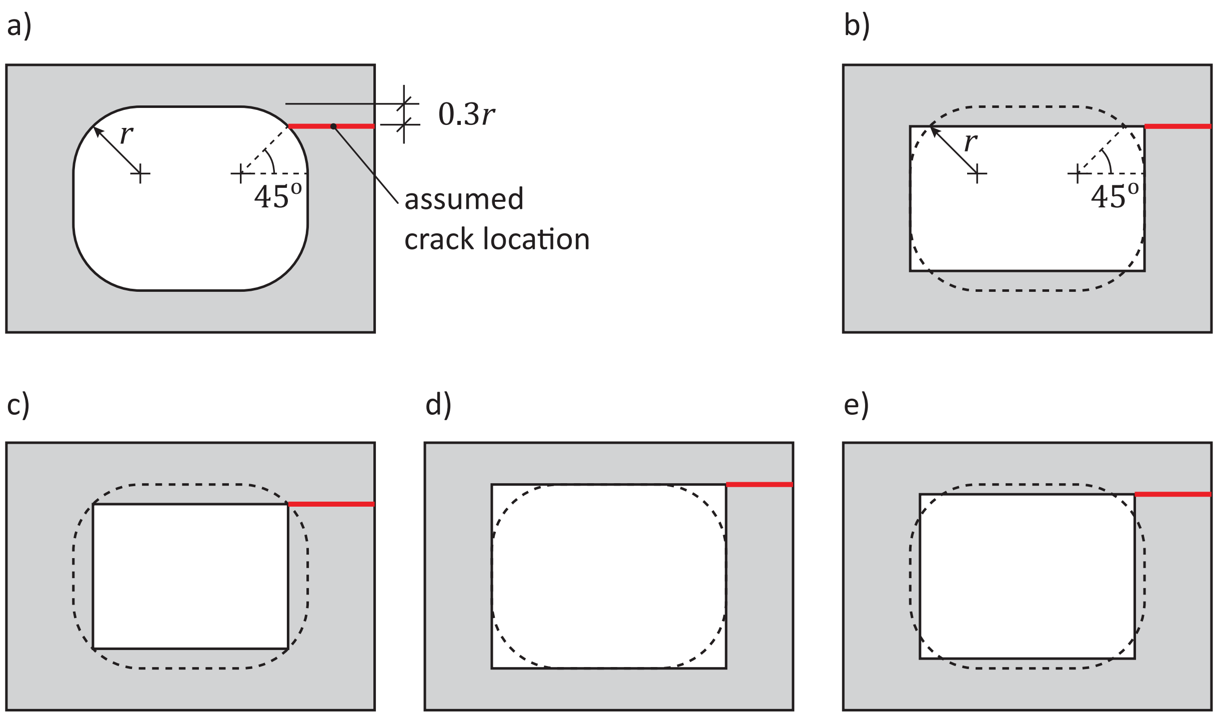

Figure 2.2. Location of crack plane and possible simplifications of the hole geometry.

2.2 Element cross sectional forces and moments

The cross sectional forces N_i, V_i and M_i (i = 1, 2) in the beam parts above (1) and below (2) the hole are determined by equilibrium considerations of the statically indeterminate system shown in Figure 2.3. Kinematic assumptions according to Timoshenko beam theory are used for beam elements 1 and 2. The vertical cross sections on the two sides of the hole are assumed to remain plane at loading. With the simplification given in Figure 2.2 b), the beam parts above and below the hole are assumed to have constant heights according to

$$h_1 = h_u + 0.3r \quad (1)$$

$$h_2 = h_l + 0.3r \quad (2)$$

To account for the elastic clamping of the beam elements in an approximate way, these are in the calculations given a length slightly longer than the actual length of the hole, $L = a + 0.5h_1$, based on numerical results presented by Petersson (1974).

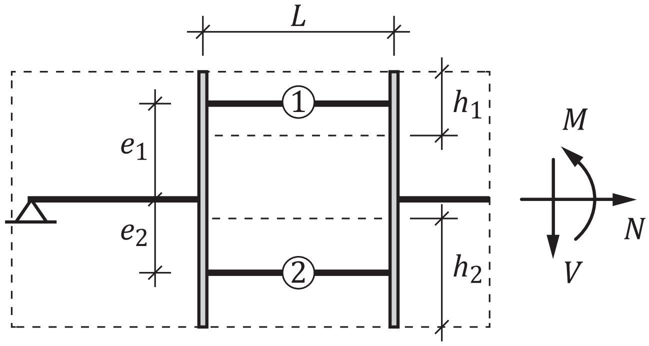

Figure 2.3. Definition of subelements 1 and 2 for beam parts above and below the hole respectively.

According to the kinematic assumptions, the infinitesimally thin cross section to the right of the hole remains plane prior to crack initiation and propagation. At crack propagation however, it is split into two cross sections according to Figure 2.4. The cross section sectional forces N_s, V_s and M_s – forcing the cross section to remain plane in the uncracked state – may be determined from equilibrium giving

$$V_s = N_1 - N_{res} \quad (3)$$

$$N_s = V_1 - V_{res} \quad (4)$$

$$M_s = M_1 - M_{res} - N_1 \frac{h_1}{2} + N_{res} \frac{h_1}{2} \quad (5)$$

where V_{res}, N_{res} and M_{res} are the resulting forces and the moment from the normal- and shear stresses acting on the right side of the cross section part above the crack plane according to

$$V_{res} = \int \tau_{xy}(y) dA = V \frac{h_1^2(3h - 2h_1)}{h^3} \quad (6)$$

$$N_{res} = \int \sigma_x(y) dA = (Nh^2 - 6Mh + 6Mh_1) \frac{h_1}{h^3} \quad (7)$$

$$M_{res} = \int \sigma_x(\bar{y}) \bar{y} dA = M \frac{h_1^3}{h^3} \quad (8)$$

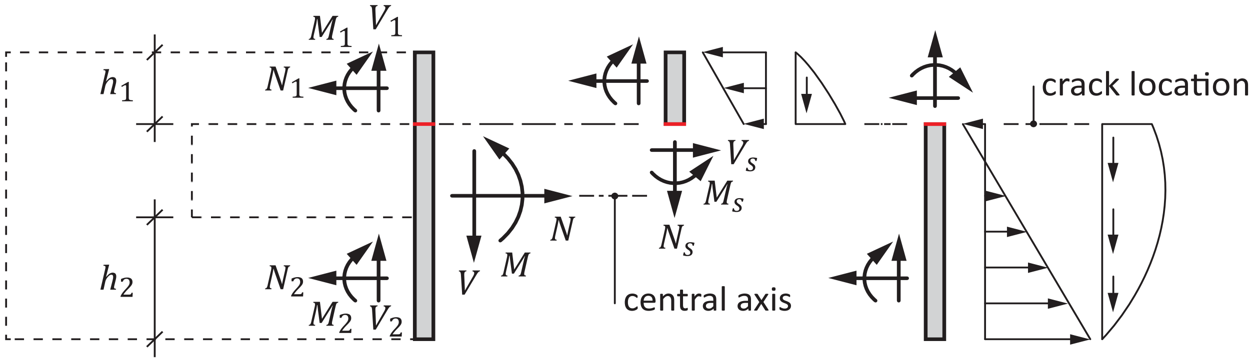

Figure 2.4. Illustration of cross section sectional forces N_s, V_s and M_s.

2.3 Relative displacements at crack propagation

At crack initiation and propagation in the beam length direction, the original beam cross section is split into two separate cross sections. The two respective cross sections are still assumed to remain plane but since they are separated, they may be deformed in different ways. The relative displacements *u* and *v* and the relative rotation θ of the two cross sections, above and below the crack plane, are illustrated in Figure 2.5. The relative displacement *v*, related to N_s, corresponds to mode I crack deformation. The relative displacement *u* and the relative rotation θ are influenced by both V_s and M_s with *u* corresponding to mode II crack deformation and θ assumed to contribute to mode I crack deformation.

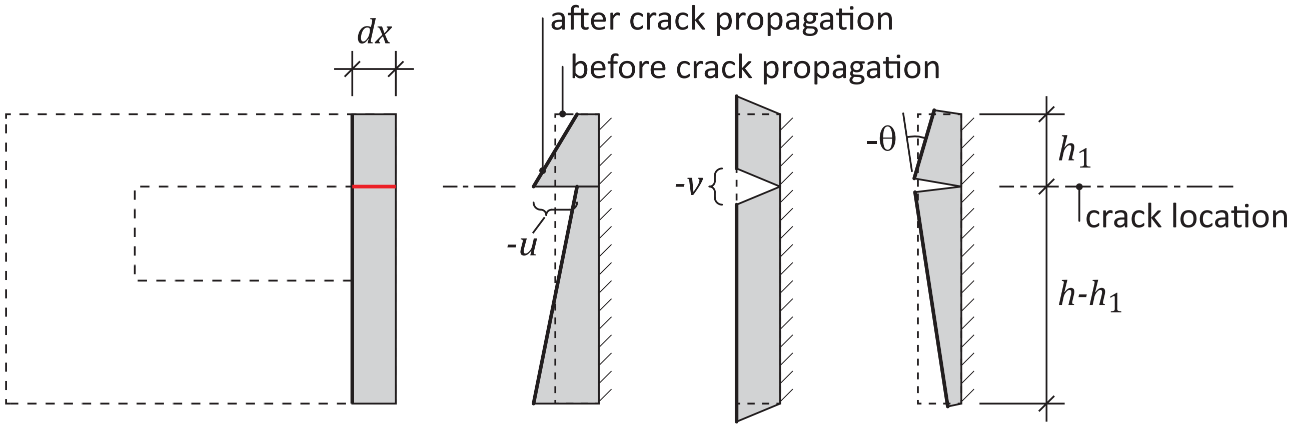

Figure 2.5. Cross section relative displacements at crack propagation.

The relations between the relative displacements *u* and *v* and the relative rotation θ and the cross section sectional forces and moment N_s, V_s and M_s may be expressed as

$$\boldsymbol{u} = \boldsymbol{C}\,\boldsymbol{F}_s \tag{9}$$

where the displacement and force vectors are given by

$$\boldsymbol{u} = [u \quad v \quad \theta]^T \tag{10}$$

$$\boldsymbol{F}_s = [V_s \quad N_s \quad M_s]^T \tag{11}$$

and the compliance matrix is given by

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & 0 & C_{13} \\ 0 & C_{22} & 0 \\ C_{31} & 0 & C_{33} \end{bmatrix} \tag{12}$$

The components of the compliance matrix are determined based on the compliance of the cross sections of infinitesimal length above and below the crack plane. Considering kinematic assumptions according to Timoshenko beam theory gives

$$C_{11} = \frac{4}{E_x b}\left(\frac{1}{h_1} + \frac{1}{h-h_1}\right) dx \tag{13}$$

$$C_{22} = \frac{1}{G_{xy} b}\left(\frac{1}{h_1} + \frac{1}{h-h_1}\right) dx \tag{14}$$

$$C_{33} = \frac{12}{E_x b}\left(\frac{1}{h_1^3} + \frac{1}{(h-h_1)^3}\right) dx \tag{15}$$

$$C_{13} = C_{31} = \frac{6}{E_x b}\left(\frac{1}{h_1^2} + \frac{1}{(h-h_1)^2}\right) dx \tag{16}$$

where E_x is the parallel to grain stiffness and G_{xy} is the shear stiffness.

2.4 Crack closure work, energy release rate and fracture criterion

According to LEFM, see e.g. textbook (Hellan 1985), crack propagation is governed by crack propagation criteria which may be formulated based on, for example, the concept of strain energy release rate or stress intensity factors. The strain energy release rate G can, according to LEFM, also be calculated in terms of *crack closure work*, i.e. the work required to completely close a propagated crack. The crack closure work W may for the present application be expressed as

$$W = \frac{1}{2}\boldsymbol{F}_s^T \boldsymbol{u} = \frac{1}{2}\boldsymbol{F}_s^T \boldsymbol{C} \boldsymbol{F}_s = \\ = \underbrace{\frac{1}{2}(C_{11}V_s + C_{13}M_s)V_s}_{W_{II,u}} + \underbrace{\frac{1}{2}C_{22}N_s^2}_{W_{I,v}} + \underbrace{\frac{1}{2}(C_{31}V_s + C_{33}M_s)M_s}_{W_{I,\theta}} \tag{17}$$

where W_I and W_{II} refer to the crack closure work related to mode I and II respectively. Indices u, v and θ refer to the relative displacements and the rotation illustrated in Figure 2.5 above. According to LEFM, the stress intensity factors K_I and K_{II} are proportional to the applied load. The principle of superposition may be used for multiple load cases giving contributions in the same mode of deformation. For the present application this means that the mode I stress intensity factor may be expressed as

$$K_I = K_{I,v} + K_{I,\theta} \tag{18}$$

The relationship between the mode I and mode II stress intensity factors K_i and the corresponding energy release rates G_i (i = I, II) is given by

$$K_i = \sqrt{E_i G_i} \tag{19}$$

where E_i (i = I, II) is a measure of the stiffness of the material with respect to the corresponding mode of deformation (Gustafsson 2002).

The energy release rate at crack extension over an area $dA = bdx$ is equal to the corresponding crack closure work giving for the present case

$$G_{I,v} bdx = W_{I,v} \quad (20)$$

$$G_{I,\theta} bdx = W_{I,\theta} \quad (21)$$

$$G_{II,u} bdx = W_{II,u} \quad (22)$$

and

$$G_I = \left(\sqrt{G_{I,v}} + \sqrt{G_{I,\theta}}\right)^2 = \frac{\left(\sqrt{W_{I,v}} + \sqrt{W_{I,\theta}}\right)^2}{bdx} \quad (23)$$

$$G_{II} = \frac{W_{II,u}}{bdx} \quad (24)$$

In order to determine the beam strength with respect to cracking, a crack propagation criterion needs to be chosen. In the present applications, with in general mixed mode of loading, the following interaction criterion has been used

$$\left(\frac{G_I}{G_{Ic}}\right)^m + \left(\frac{G_{II}}{G_{IIc}}\right)^n = 1.0 \quad (25)$$

where G_I and G_{II} are the energy release rates in mode I and II given above and where G_{Ic} and G_{IIc} are the corresponding critical energy release rates (or fracture energies). Results presented in Sections 4 and 5, relating to analysis of beams with a hole using the present approach, are based on $m = n = 0.5$ and material property parameters according to Table 1. The choice of crack propagation criterion is not obvious, and other criteria could possibly be more suitable. In case of negative values of G_I and/or G_{II}, the negative contribution is ignored and fracture in pure mode I or II is considered.

Table 1. Material property parameters used for strength analysis.

Parameter	Notation	Value
Parallel to grain stiffness	E_x	12 000 MPa
Shear stiffness	G_{xy}	600 MPa
Fracture energy, mode I	G_{Ic}	300 Nm/m^2
Fracture energy, mode II	G_{IIc}	900 Nm/m^2

3 Comparison to end-notch beam equation

The present analysis approach for beams with a hole has many features in common with the LEFM-based design approach for notched beams, originally presented by Gustafsson (1988) and with modifications included in EC 5. The present approach for strength analysis of beams with a hole may also be used for analysis of notched beams, since this is essentially only a special case in terms of the general geometry illustrated in Figure 2.1.

As mentioned in the introduction, there are however some differences in the general formulation between the two approaches. For the end-notched beam approach, no distinction between modes I and II is made in terms of the crack propagation criterion and the material is characterized by the mode I fracture energy only. Another difference relates to the effect of elastic clamping of the reduced beam parts at the section where the notch or hole corner is located, which is considered partly different in the two approaches. The expression for the nominal beam strength with respect to cracking at a right angled notch derived by Gustafsson (1988) reads

$$\frac{V_c}{A_{net}} = \frac{V_c}{b\alpha h} = \frac{\sqrt{G_c/h}}{\sqrt{0.6(\alpha-\alpha^2)/G_{xy}+\beta\sqrt{6(1/\alpha-\alpha^2)/E_x}}} \tag{26}$$

where V_c is the crack shear force, A_{net} is the net cross section area at the reduced cross section and where α and β are defined in Figure 3.1.

Illustrations of the predicted beam strengths as influenced by the normalized beam height α and the normalized notch length β according to the two approaches are shown in Figure 3.1. The comparison is based on a beam of height h = 500 mm, with $G_{Ic} = G_{IIc}$ = 300 Nm/m^2 and values of the exponent in Equation 25 as $m = n = 1$. For this choice, i.e. for $G_{Ic} = G_{IIc}$ and $m = n = 1$, is the mixed mode crack propagation criterion the same for both calculations. More or less slight differences in predicted strengths are to be expected due to the differences in the model formulations with respect to the influence of the elastic clamping of the reduced cross section.

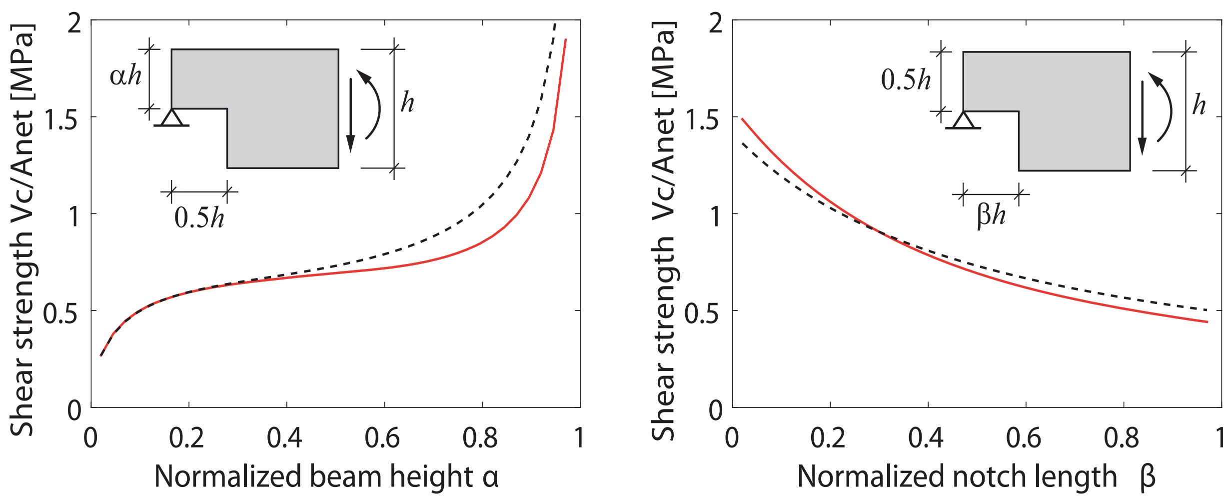

Figure 3.1. Strength according to Equation 26 (dashed black) and present approach (solid red).

4 Comparison to experimental tests

Examples of theoretically predicted strengths for beams with a hole are presented in Figures 4.1 and 4.2, where also results of experimental tests on beams with circular holes (Höfflin 2005, Aicher and Höfflin 2006) and with square holes (Danielsson 2008) are shown. Theoretical beam strengths refer to the crack propagation load predicted according the theory presented in Section 2, using Equation 25 with $m = n = 0.5$ and with material data according to Table 1. The net cross section area is defined as $A_{net} = b(h-h_d)$ and the strength from experimental tests corresponds to the load at the instant of crack propagation across the entire beam width b.

Beam and hole geometries are for these illustrations chosen based on availability of test results, yielding partly different geometries for the case of circular and square holes respectively. Results presented in Figure 4.1 relate to the influence of beam height, hole size and bending moment to shear force ratio for holes centrically placed with respect to beam height (*s* = 0). Results presented in Figure 4.2 relate to the influence of hole corner radius and hole placement with respect to beam height.

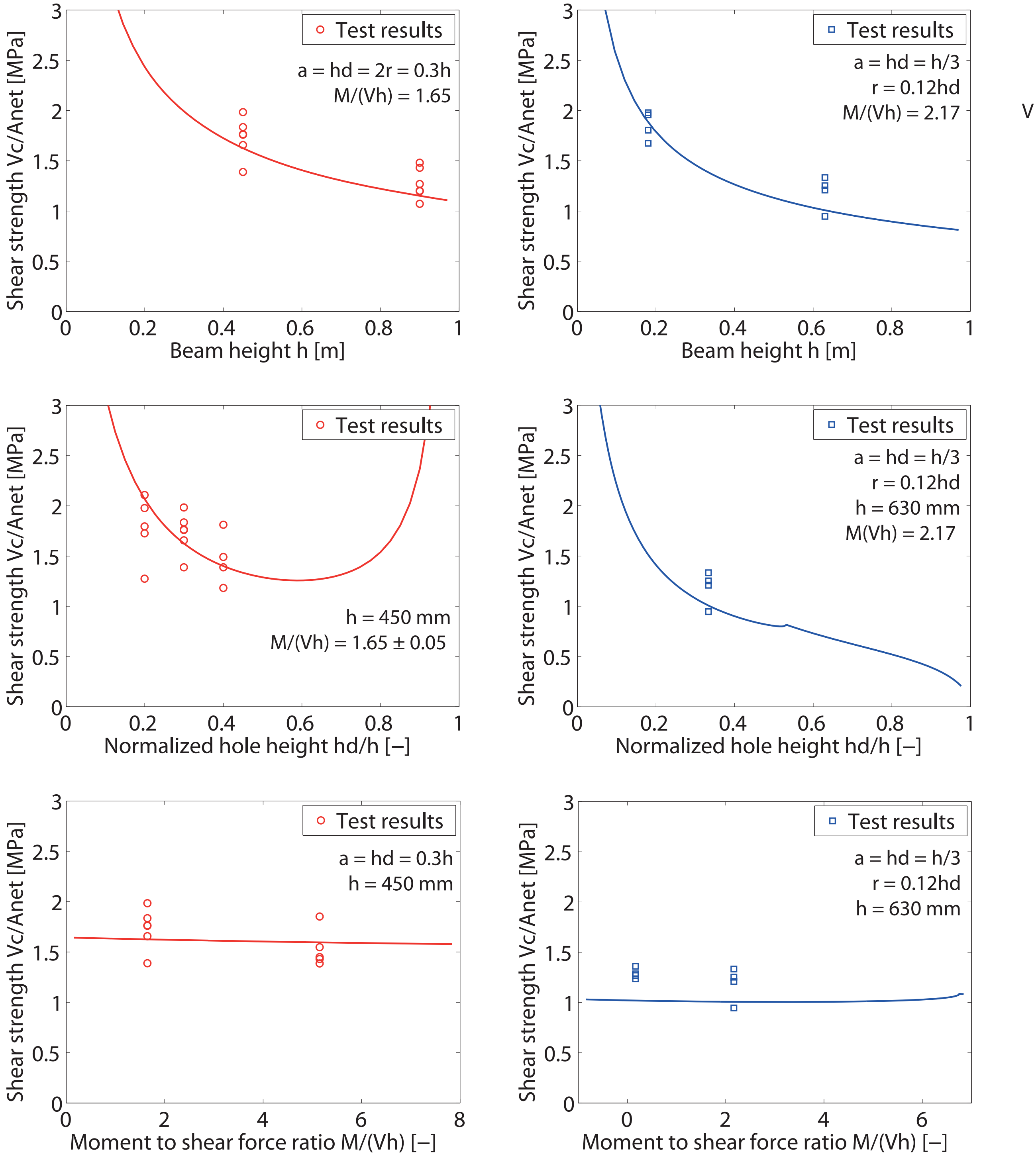

Figure 4.1. Experimentally found strength and theoretically predicted strength as influenced by beam height, hole size and bending moment to shear force ratio. Left (in red): Circular holes (Höfflin 2005, Aicher & Höfflin 2006). Right (in blue): Square holes (Danielsson 2008).

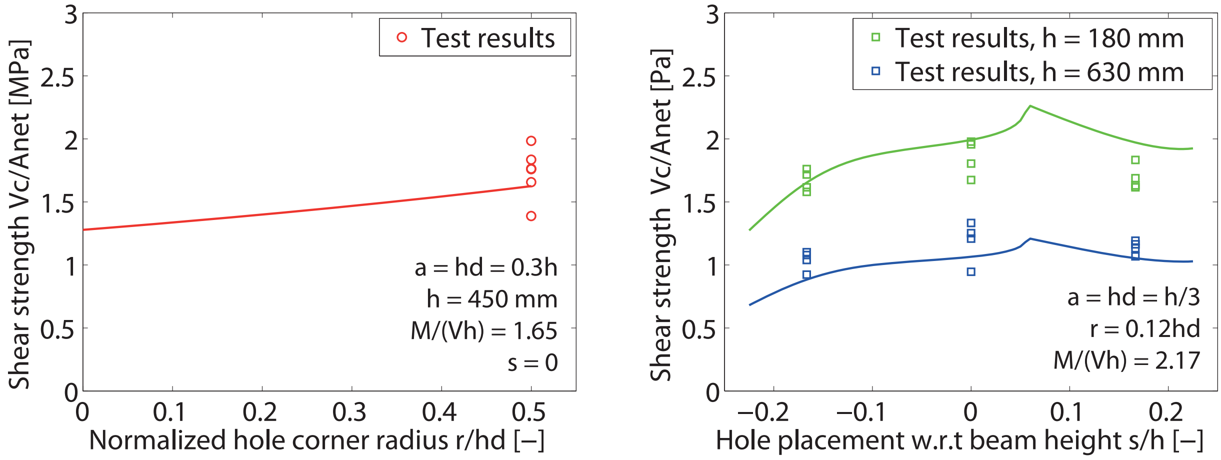

Figure 4.2. Experimentally found strength and theoretically predicted strength as influenced by hole corner radius and hole placement with respect to beam height. Left (in red): Circular holes (Höfflin 2005, Aicher & Höfflin 2006). Right (in green/blue): Square holes (Danielsson 2008).

5 Comparison of strength analysis approaches

An overview of the ratio between experimentally found beam capacity and theoretically predicted capacity according to some different approaches for strength analysis is given in Figure 5.1. The experimental tests used for this comparison consist of beams with either circular holes (Höfflin 2005, Aicher & Höfflin 2006) or square holes (Danielsson 2008) also used for comparison in Section 4.

The strength analysis methods included are:

1. PFM – A Probabilistic Fracture Mechanics method based on 2D FE-analysis and consideration of fracture ductility according to a generalized LEFM approach in combination with consideration of material variability according to Weibull theory (Danielsson & Gustafsson 2011).
2. Weibull – Classical Weibull theory based on 2D FE-analysis (Danielsson & Gustafsson 2011).
3. NLFM – A nonlinear fracture mechanics approach (a cohesive zone model) based on 3D FE-analysis (Danielsson & Gustafsson 2014).
4. Present approach – see previous sections.
5. Glulam Handbook – end-notched beam analogy approach for beams with a hole found in the old version of the Swedish Glulam Handbook (Carling 2001) and also included in a draft version of EC 5 (2002).
6. DIN EN 1995-1-1/NA – Semi-empirical approach found in German and Austrian National Annexes to EC5.
7. Aicher et al – Design approach based on Weibull theory presented by Aicher, Höfflin and Reinhardt (2007). Originally suggested to be used for circular holes only but here used also for square holes assuming $a = h_d = 2r$.

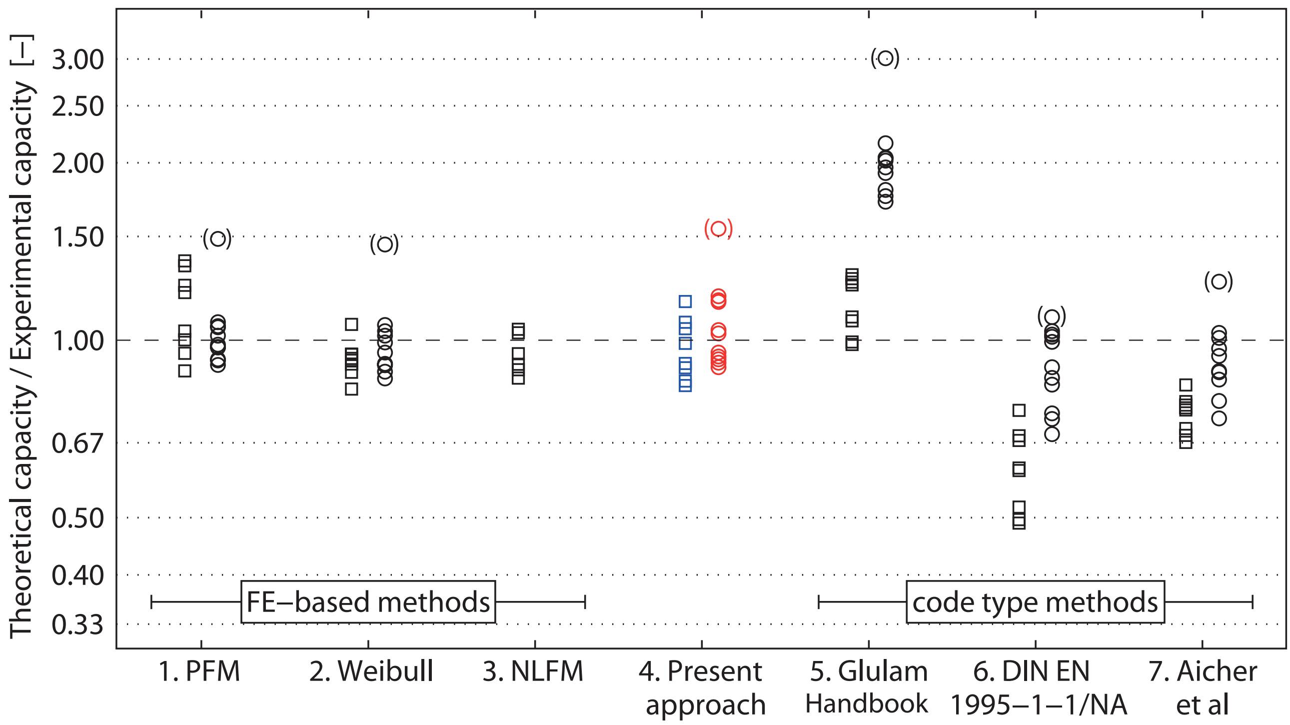

Figure 5.1. Comparison of experimentally found strengths and theoretically predicted strengths according to some different approaches for strenth analysis of beams with a hole. The test result indicated as (o) appears abnormal without known cause; see further comments in (Danielsson & Gustafsson 2011).

For the general approaches (items 1-4 above), experimentally found mean values are compared to predicted strength based on assumed mean values of material property parameters. For the NLFM approach (item 3 above), only results for the beams with square holes are presented since analysis of beams with circular holes were not included in the work presented in (Danielsson and Gustafsson 2014).

The strength according to the three code type approaches (items 5-7 above) are based on characteristic material strength values f_{vk} = 3.5 MPa and f_{t90k} = 0.5 MPa, valid for strength class GL 32h according to SS-EN 14080:2013. The predicted strengths are for these methods compared to characteristic values of the experimentally found strengths; see (Danielsson & Gustafsson 2011) for further information.

All test results included in the comparison relate to holes with larger height h_d than allowed to be used without reinforcement according to the German and Austrian National Annexes to EC 5.

6 Discussion

The presented approach is general in the sense that it allows for strength analysis including not only loading in terms of shear force and bending moment, but also axial force. Depending on relative load levels and the sign of the axial force, the assumed location of the crack (see Figure 2.2) may however need to be adjusted.

Beams with holes are in practical design situations often reinforced. The reason for this is likely related to two different factors: (i) the actual strength reduction due to the hole and (ii) the uncertainty and lack of knowledge related to strength analysis and design of beams with a hole. The cross section sectional forces N_s, V_s and M_s (see Equations 3-5) could possibly be of interest in relation to design of internal or external beam reinforcement in terms of fully threaded screws, glued-in rods or glued-on panels.

A new version of the Swedish Glulam Handbook is planned to be released during 2015, within which the end-notch beam analogy approach for beams with a hole will be removed in favour of the design approached found in the German and Austrian National Annexes to EC 5.

7 Conclusions

From the work presented in this paper, the following conclusions are drawn:

- The present approach is consistent with the LEFM-based design approach for end-notched beams found in EC 5.
- The present approach appears to be able to capture the strong beam size influence found from experimental tests of beams with a hole fairly well.
- The present approach appears fairly accurate in capturing the absolute values of the beam strength for both circular and square holes of various sizes and locations.

Although based on beam theory analysis, the present approach is not suitable for direct incorporation into timber engineering design codes of practice in its current form, because of rather complex equations. For certain applications and load configurations, more user-friendly design expressions may possibly be derived and could then serve as a base for improved design recommendations. Further work regarding verification and calibration is however needed before this can be realized.

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Discussion

The paper was presented by H Danielsson

H Blass commented that the stress distribution shown in slide 8 close to the hole is very different from reality. He questioned why the results are so good. H Danielsson agreed but stated that the beam theory is exact for normal stresses but might be different for shear stresses. PJ Gustafsson added that the solution should be exact in terms of energy release rate. As the crack propagated, the normal forces due to the moment should be exact but shear would not be true.

BJ Yeh asked how close the hole can be to the support and if there were two holes, how close can they be. H Danielsson responded the distances should be such that the support would not influence the hole. This would also apply to the multiple holes cases.

K Malo asked if there would be a limitation to the hole size. H Danielsson said that there would be no limit and the analysis would also work for notched beams.

I Smith commented that good results need accurate analysis, real structure, and accurate material properties; when results do not agree, maybe some of these are not working.

P Dietsch and H Danielsson discussed the cases of round and square holes and limitations to the model.

R Jockwer received clarifications that at this moment there is no recommendation for the size of the hole without reinforcement.