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Beams with notches or slits – Extensions of the Gustafsson approach

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1 Introduction

1.1 Background

In the current version of Eurocode 5 (EC5), [1], design of beams notched at the support is based on the so-called Gustafsson approach [2]. In later work (see e.g. [3]–[7]) further background, overview and discussions on this and other methods are given. Also, the influence of various geometry and material parameters on the performance of beams with notches and holes, and on the predictive capabilities of the methods are given. In other standards and handbooks, see e.g. [8], [9] and [10], alternative approaches can be found.

The case of an end-notched beam without taper, shown in Figure 1 below, defines the basic geometry parameters as given in EC5 (for non-tapered notches).

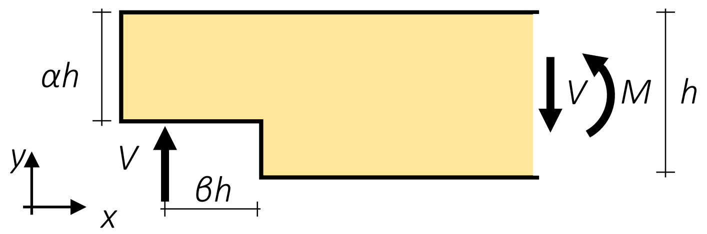

Figure 1. Definition of basic geometry parameters for end-notched beam.

As originally presented by Gustafsson, and as included in EC5, the design formulae are, of course, derived based on a number of simplifying assumptions, some of which are related to the loading conditions of the notched beam. As an example, in [2] it was assumed that any load on the beam between the support force and the re-entrant corner of the notch could be neglected. Consequently, the bending moment in the beam at the re-entrant corner, M, is assumed to be equal to the shear force, V, times the distance to the line of action of the support reaction force, x, see Figure 2. In reality other situations often occur, involving *e.g.* a notch away from the support and including distributed loading and/or including normal force, see Figure 2. In such cases the relation between bending moment and shear force at the notch is obviously different. Another situation where this is relevant is in cantilevered beam parts, see Figure 2. Also, the Gustafsson approach is limited to beams with rectangular cross-section. Finally, the EC5 approach states that the effect of the stress concentrations at a notch on the opposite side of the support need not to be taken into account, a statement that has been questioned.

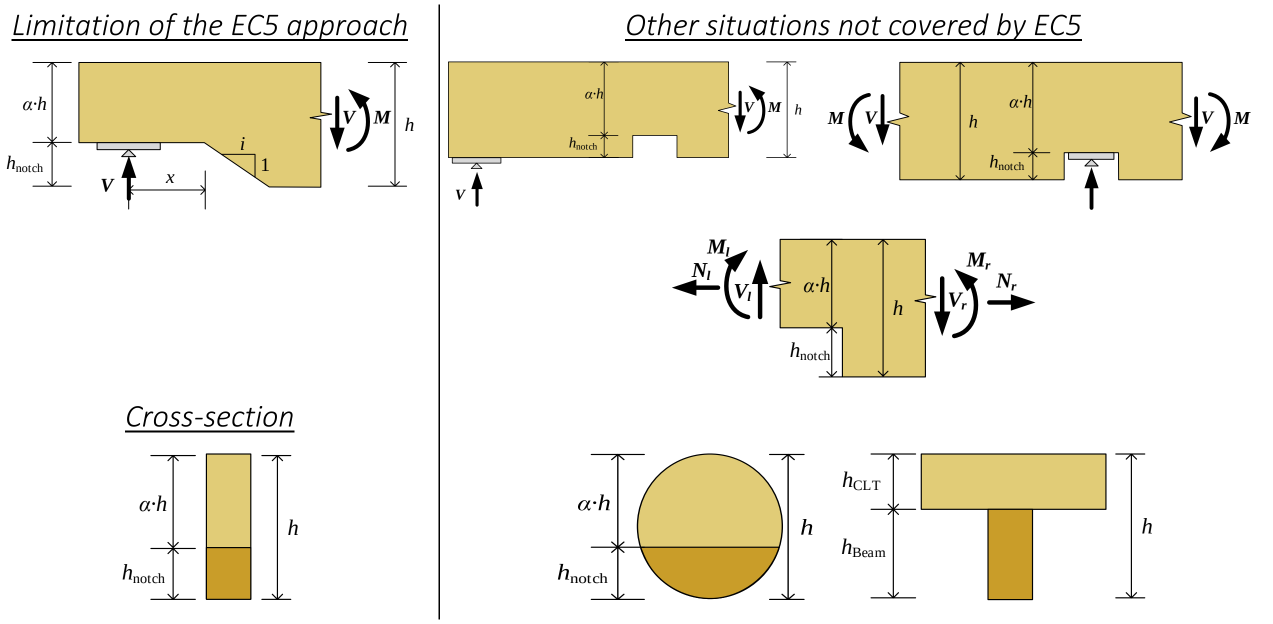

Figure 2. Limitation of the EC5 approach and examples of other situations not covered by EC5.

1.2 Aim

With the above examples in mind, it is indeed clear that it is not always straightforward to apply the current design formulae of EC5. The aim of the present paper is to discuss the basic assumptions of the Gustafsson approach, its possibilities and limitations, relative current and future versions of EC5.

2 Methods

2.1 Linear Elastic Fracture Mechanics (LEFM) and the compliance method

All analyses presented herein are performed within the framework of linear elastic fracture mechanics. This in turns means assuming linear elastic behaviour of the material and assuming load bearing capacity of components being governed only by geometry, boundary conditions, material stiffness and fracture energy.

The framework of LEFM is in general accurate in situations where the fracture process zone of the material is small in relation to other dimensions of the structure (including the size of the crack).

The load bearing capacity is determined by considering the energy balance during crack propagation. Herein, only quasi-static conditions are considered, and thus, during crack propagation, the incremental work of external forces, ∂W_{ext}, should balance the elastic strain energy increment, ∂W_e and the fracture energy needed for an incremental extension of the crack surface, giving:

$$\partial W_{ext} = \partial W_e + G_C \partial A \qquad (1)$$

where G_c is the critical energy release rate (N/m) during crack propagation, and ∂A is the increase of crack surface.

In an FE-context, assuming constant external loads during crack propagation, and assuming linear elastic behaviour, Eq. (1) can be written:

$$\alpha_{load}{}^2 \mathbf{P}_0^T (\mathbf{a}_2^0 - \mathbf{a}_1^0) = \frac{1}{2}\alpha_{load}^2 \mathbf{P}_0^T (\mathbf{a}_2^0 - \mathbf{a}_1^0) + G_c \Delta A \qquad (2)$$

with α_{load} being a load factor, $\mathbf{P}_0$ representing a reference (unit) loading, and vectors $\mathbf{a}_1^0$ and $\mathbf{a}_2^0$ representing the displacement fields of the structure for crack lengths a_1 and a_2, respectively. From Eq. (2) and assuming 2D-conditions, we then obtain:

$$\alpha_{load} = \sqrt{\frac{G_c \, \Delta A}{\frac{1}{2}\mathbf{P}_0^T (\mathbf{a}_2^0 - \mathbf{a}_1^0)}} = \sqrt{\frac{G_c \, b \, \Delta a}{\Delta W_e}} \qquad (3)$$

with b being the out-of-plane width of the crack surface, Δa being the extension of crack length and ΔW_e being the change of elastic strain energy.

2.2 Choice of critical energy release rate

In calculating the load at which crack propagation occurs, cf. Eq. (3), the critical energy release rate, G_c, is needed. Since G_c is dependent on, among other things, the mode of loading, a relevant choice of G_c needs to be done. In the work leading to the current EC5 approach, an assumption on the safe side was done, assuming G_c to be equal to its Mode 1 value (for tension perpendicular to the grain). This approach is used also here for the analytical models.

For models based on 2D solid elements, cracking is assumed to take place by propagation along the grain and for mixed mode situations the Wu criterion [11] is used:

$$\frac{K_I}{K_{IC}} + \left(\frac{K_{II}}{K_{IIC}}\right)^{2.0} = \begin{bmatrix} K_I = \sqrt{E_I G_I} \\ K_{II} = \sqrt{E_{II} G_{II}} \end{bmatrix} \Rightarrow \sqrt{\frac{G_I}{G_{IC}}} + \frac{G_{II}}{G_{IIC}} = 1.0 \qquad (4)$$

where index C indicates the respective critical quantity at pure mode of loading, and where the parameters E_I and E_{II} are defined in terms of the elastic constants according to:

$$\frac{1}{E_I}=\frac{1}{E_x}\sqrt{\frac{E_x}{2E_y}}\sqrt{\sqrt{\frac{E_x}{E_y}}+\frac{E_x}{2G_{xy}}-\upsilon_{yx}\frac{E_x}{E_y}};\quad \frac{1}{E_{II}}=\frac{1}{E_x}\sqrt{\frac{1}{2}}\sqrt{\sqrt{\frac{E_x}{E_y}}+\frac{E_x}{2G_{xy}}-\upsilon_{yx}\frac{E_x}{E_y}} \quad (5)$$

The critical energy release rate at mixed mode situations, $G_c=G_I+G_{II}$, is calculated from the ratio $k=K_{II}/K_I$. Close to the crack tip this ratio equals $k=\overline{\tau}/\overline{\sigma}$ with $\overline{\tau}$ being the shear stress along grain and $\overline{\sigma}$ the tensile stress perpendicular to grain. This ratio can be estimated by the stress values at the crack tip, or as an average value along a characteristic material length x_0 ahead of the crack tip, see [12, 13]:

$$x_0=\frac{2}{\pi}\frac{E_I G_{IC}}{f_t^2}\frac{E_x}{E_y}\left(\frac{G_{IIC}}{G_{IC}}\right)^2\frac{2}{4k^4}\left(\sqrt{1+4k^2\sqrt{\frac{E_y}{E_x}\frac{G_{IC}}{G_{IIC}}}}-1\right)^2\left(1+\frac{k^2}{\left(f_v/f_t\right)^2}\right) \quad (6)$$

2.3 FE-Models based on 2D elements

In order to allow detailed analyses, including the influence of boundary conditions and the influence of mixed-mode behaviour, models based on plane stress solid elements have been employed.

One part of the work presented includes the investigation on support conditions. Two different modelling approaches for end-notched beams have been used, *a)* assuming a stiff plate at the lower side of the beam acting as a hinged support and *b)* beam-alike boundary conditions, by introducing the support force at the free end of the beam, which in turn is modelled as a stiff cross-section, cf. Figure 3. In any case, the distance of βh is defined as the distance from the re-entrant corner to the support force.

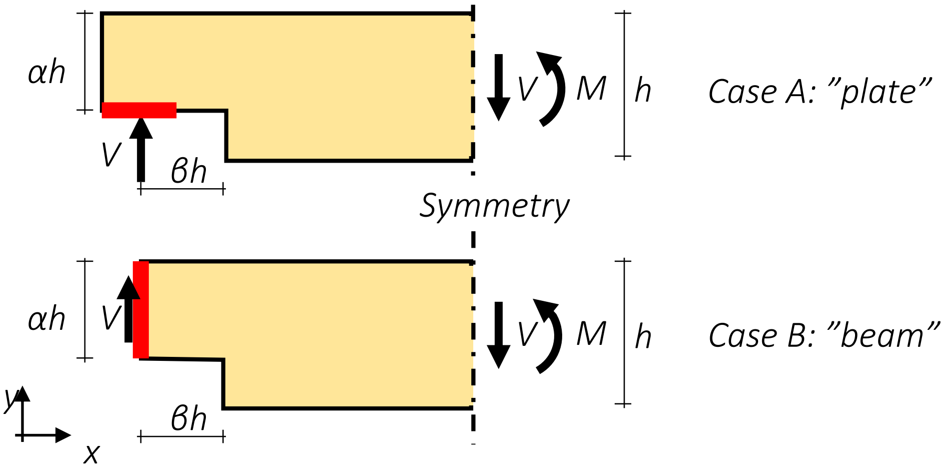

Figure 3. Support conditions. Top: Support plate. Bottom: Beam boundary conditions.

3 Common assumptions and cases analysed

3.1 Material stiffness parameters and EC5-approach

Material stiffness parameters used in the present study are according to Table 1. These parameters represent a typical glulam material. Over the years, the material and product standards have changed, making it difficult to do consistent comparisons between previous research results and code proposals and applying different analysis methods. In addition, some analysis methods make use of additional material parameters apart from those given in codes and standards. As a compromise, a typical value of longitudinal modulus of elasticity was set to 12 000 MPa, which is also in line with the values used when calibrating the EC5 formulae. Based on the values used in previous research, [12], transverse modulus of elasticity (MOE) was set to 400 MPa. To be consistent with the current design approach in EC5 for notched beams, the longitudinal shear modulus, G_{xy}, was set to the same fraction as that assumed in EC5, *i.e.* $G_{xy}=E_x/15.625$

Table 1. Material stiffness parameters.

Symbol	Quantity	Value	Unit	Remark
E_x	MOE along grain	12 000	MPa	-
E_y	MOE across grain	400	MPa	$E_x/30$ from [12]
G_{xy}	Longitudinal shear modulus	768	MPa	$E_x/15.625$ according to EC5

The design approach of EC5 is based on a theoretical expression from [2], where shear force capacity, V_f, is found to be:

$$\frac{3}{2}\frac{V_f}{b\alpha h}=1.5\frac{\sqrt{\dfrac{G_{IC}G_{xy}}{0.6}}}{\sqrt{h}\left(\sqrt{\alpha-\alpha^2}+\beta\sqrt{10(G_{xy}/E_x)}\sqrt{1/\alpha-\alpha^2}\right)}, \tag{7}$$

where geometry parameters are defined in Figure 1. With the assumption of

$$E_x/G_{xy}=15.625 \quad\Rightarrow\quad \sqrt{10G_{xy}/E_x}=0.8 \tag{8}$$

the result is

$$1.5\sqrt{\frac{G_{IC}G_{xy}}{0.6}}/f_v=k_n[\text{mm}^{1/2}]=\begin{cases}4.5 \text{ for LVL}\\ 5.0 \text{ for solid timber}\\ 6.5 \text{ for glulam}\end{cases} \tag{9}$$

With the above, the final expressions are:

$$\tau_d=\frac{1.5V_f}{b\alpha h}\le k_v f_{v,d} \tag{10}$$

with k_v given by

$$k_v = \min\begin{cases} 1 \\ k_n / \left(\sqrt{h} \left(\sqrt{\alpha - \alpha^2} + 0.8\beta \sqrt{1/\alpha - \alpha^2} \right) \right) \end{cases}, \tag{11}$$

where k_n was defined in Eq. (9) and the factor 0.8 in Eq. (8).

Adopting Eqs. (9)–(11) and expressing the results in terms of the reduction factor k_v as a function of *α, β* and beam depth, *h*, gives the curves depicted in Figure 4. By backwards calculating from Eq. (9), the equivalent energy release rate corresponding to k_n=6.5 is $G_{IC,\,6.5}$=179.7 N/m, assuming the shear strength to be f_v=3.5 MPa, a characteristic strength value typically used at the time of introducing the design approach for notched beams in EC5.

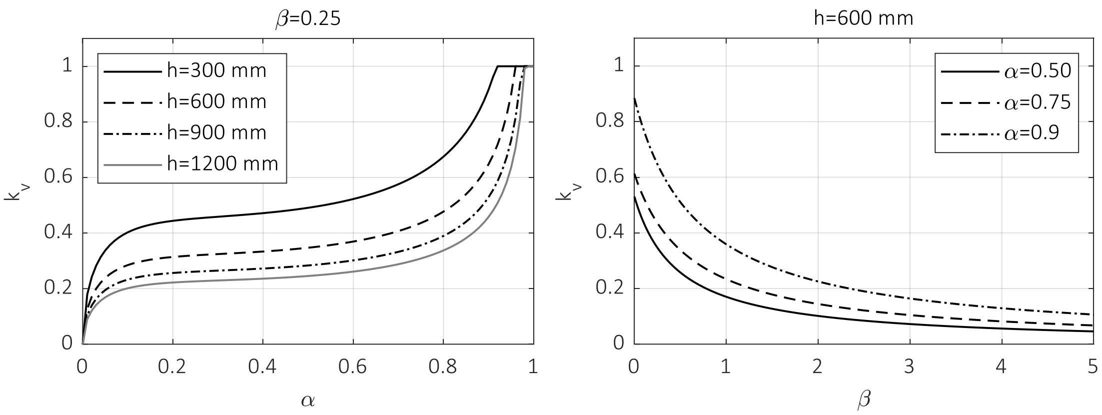

Figure 4. Strength reduction factor k_v for end-notched beams according to EC5.

For the 2D FE-analyses performed in this study, additional material parameters were needed. Apart from Poisson's ratio, also critical energy release rates for mode I and mode II, and material strength values in tension perpendicular to grain and in longitudinal shear were needed. The strength values are needed to estimate the size of the characteristic material length, x_0. The values employed are given in Table 2 and are estimated to be representative for small clear wood volumes (≈1 cm^3), a scale of relevance for the stress concentrations found around the re-entrant corner of a notch.

Table 2. Additional material parameters needed for 2D FE-analyses. Strength values represent mean values for small volumes of clear wood.

ν_{xy}	Poisson's ratio	0.3	-	
G_{IC}	Critical energy release rate, mode I	179.7	N/m	From Eq.(9)
G_{IC}	Critical energy release rate, mode II	629.0	N/m	3.5G_{IC} [12]
f_t	Tensile strength perp. grain	3	MPa	[12]
f_v	Longitudinal shear strength	9	MPa	[12]

Based on the material properties in Table 2, the characteristic material length x_0 would be varying from ca 11 mm at pure mode I (G_{IC}=179.7 N/m) to ca 23 mm at pure mode II (G_{IIC}=629.0 N/m). In the present study, the mixed mode ratio was estimated by the stress ratio at the crack tip. In preliminary analyses, also calculating the mixed mode ratio based on the average along x_0 was tested. The difference was found small, and thus only the crack tip ratio is used, for simplicity.

3.2 Case 1: Beams notched at end support

The situation of an end-notched beam is analysed in order to establish necessary mesh refinement and for verification of the EC5 design formulae.

3.3 Case 2: Beams notched at inner support

The modelling is based on assuming an inner support (mid-support) and this support representing also a symmetry section, *cf.* Figure 5.

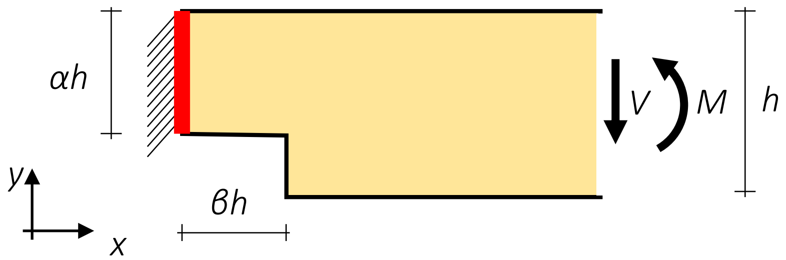

Figure 5. Beam notched at mid support modelled with a fixed end.

3.4 Case 3: General notches and slits far from support

The last case considered is a beam with a notch/slit cut at a distance from the support. The case is described in Figure 6. This situation has been analysed by investigating the geometrical parameters defining the notch, and the ratio *M/V*, the bending moment to shear, at the *centre of the notch*.

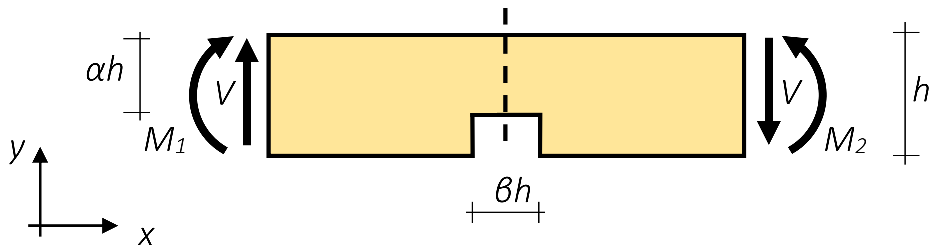

Figure 6. Beam notched away from support. Evaluation of forces refers to mid-section (dashed line).

In the original work of Gustafsson, the energy release rate, G, for such a general case of a notched or slit beam is given by:

$$G = \frac{P^2}{b^2 \alpha^2 h} \left(\sqrt{\frac{0.6\left(\alpha - \alpha^2\right)}{G_{xy}}} + \frac{x}{h} \sqrt{\frac{6\left(\frac{1}{\alpha} - \alpha^2\right)}{E_x}} \right)^2 \qquad (12)$$

where the first term in the parenthesis represents the contribution from the shear action, and the second term represents the contribution from bending action, due to the bending moment from the force, P, at the distance x. The energy release rate due to pure shear force action (V=P) is:

$$G = \frac{V^2}{b^2\alpha^2 h}\frac{0.6\left(\alpha-\alpha^2\right)}{G_{xy}} \tag{13}$$

Consequently, the resulting shear strength of a rectangular cross-section can be determined as follows:

$$\tau = \frac{1.5V}{b\alpha h} \le \frac{1.5\sqrt{\frac{G_c G_{xy}}{0.6}}}{\sqrt{h}\left(\sqrt{\alpha(1-\alpha)}\right)} \tag{14}$$

The energy release rate due to pure moment action ($M = P \cdot x$) is:

$$G = \frac{M^2}{b^2\alpha^2 h^3}\frac{6\left(\frac{1}{\alpha}-\alpha^2\right)}{E_0} \tag{15}$$

Thus, the resulting bending strength of a rectangular cross-section can be determined as follows:

$$\sigma_M = \frac{M}{\frac{b(\alpha h)^2}{6}} \le \frac{\sqrt{6G_c E_0}}{\sqrt{h}\sqrt{\left(\alpha-\alpha^4\right)}} \tag{16}$$

The combination of Eq. (14) and (16) for a notch at an arbitrary position was proposed by Gustafsson as follows:

$$\tau\frac{\sqrt{h}\left(\sqrt{\alpha(1-\alpha)}\right)}{1.5\sqrt{\frac{G_c G_{xy}}{0.6}}} + \sigma_M\frac{\sqrt{h}\sqrt{\left(\alpha-\alpha^4\right)}}{\sqrt{6G_c E_0}} \le 1 \tag{17}$$

With

$$\sigma_M = \frac{6M}{b(\alpha h)^2} \text{ and } \tau = \frac{1.5V}{b\alpha h} \tag{18}$$

This approach yields the same results as the approach for end-notched beams of the Canadian Standard CSA O.86, where the effect of the reduced stiffness at the transition between reduced and full cross-section as proposed by Gustafsson, [2], was neglected, [14]. Eq. (17) is similar to the approaches in the Australian standard AS 1720.1 and in the ”Wood Handbook” of the Forest Products Laboratory, [10].

The above approach is used herein in comparison to the 2D FE-analyses of notched/slit beam sections, by applying various ratios M/V, and by investigating various notch geometries.

4 Results

From the results of the FE models and in comparison with the analytical approach in EC5, the following aspects are evaluated:

1. General validation of the FE-models against the EC5 approach
2. Impact of the support conditions on the notch capacity
3. Impact of the notch length on the notch capacity
4. Impact of the notch position on the crack propagation load
5. Impact of the moment/shear force interaction on the notch capacity

The analyses have been performed for $300 < h < 1200$ (mm), $0.5 < \alpha < 0.9$, and $0.25 < \beta < 2.0$ for all cases. Support plate lengths were *h*/6, and in all analyses the beam width was assumed to be 100 mm.

4.1 General validation of the FE-models against the EC5 approach

4.1.1 Consistent boundary conditions and convergence study

As regards the 2D-FE approach, boundary condition Cases A and B (*cf*. Figure 3) were analysed and a convergence study was performed for Case B. Using element sizes ranging from 2.5 to 10 mm, the conclusion is that for the present study an element size of 10 mm is sufficient and still gives reasonable calculation times. Figure 7 presents the estimated critical load as function of crack propagation for element sizes 2.5–10 mm. The mesh used for the case of 10 mm element size is shown in Figure 8 and results are given in Table 3.

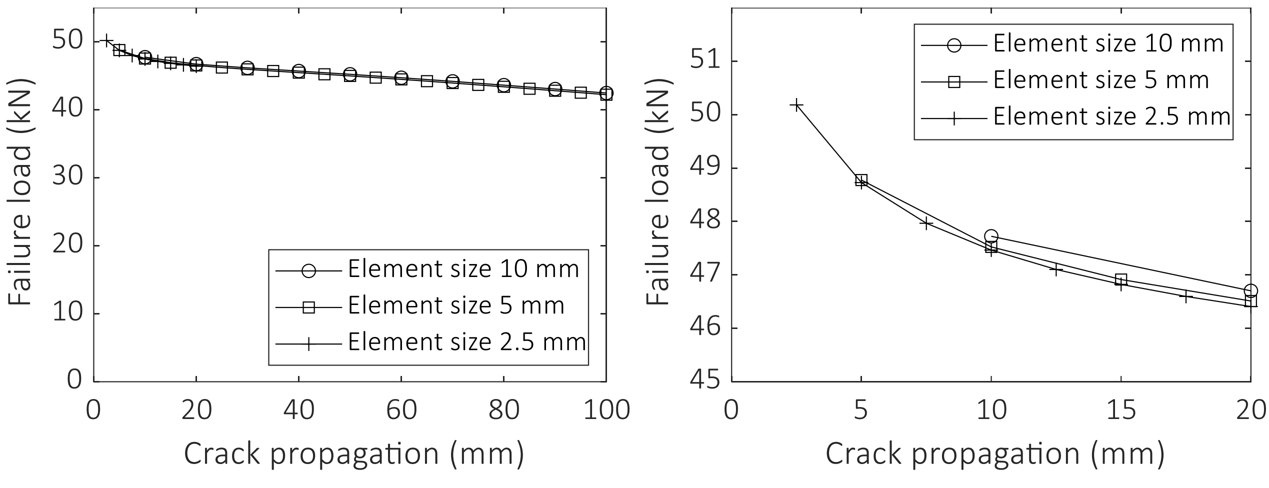

Figure 7. Convergence study of crack propagation analyses for base case (beam boundary conditions). Left: crack propagation 0–100 mm. Right: Partial zoom.

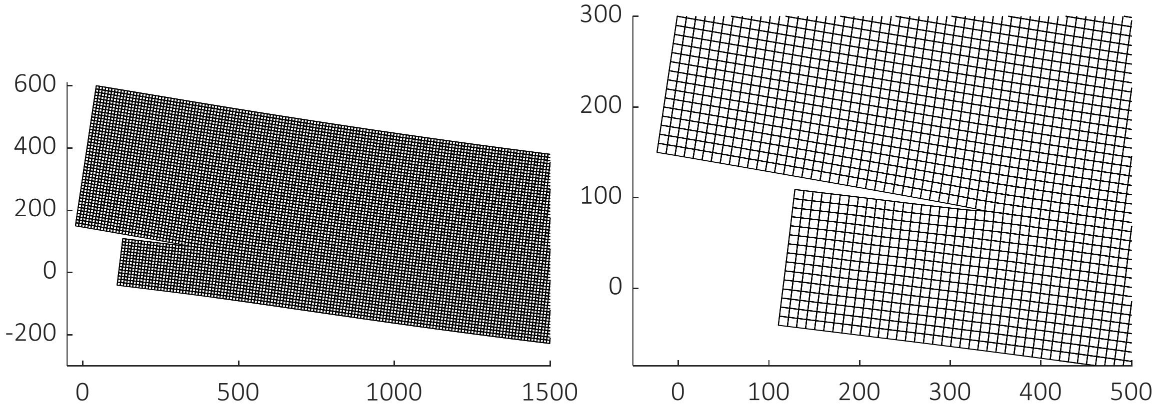

Figure 8. Example of FE-mesh used, here with 10 mm element size and 200 mm long crack. Left: Half of the model is shown (total length is 5h, in this case 3000 mm). Right: Partial zoom.

Table 3. Results for base example including influence of element size. h=600 mm, α=0.75, β=0.25. Critical load for 2D-FE models is based on crack length=20 mm and Case B boundary conditions. Note that the effective critical energy release rate is obtained as a result for 2D-FE analyses.

Approach	Critical load (kN)	Remarks
EC5	45.8	Eq.(9)–Eq.(11), k_n=6.5 (G_C=G_{IC}=179.7 N/m as input)
2D-FE	46.7	Element size 10 mm, G_C=203.3 N/m as result
2D-FE	46.5	Element size 5 mm, G_C=202.2 N/m as result
2D-FE	46.4	Element size 2.5 mm, G_C=201.9 N/m as result

The 2D-FE models in general give consistent and very similar results compared to the EC5-approach, if boundary conditions consistent with the assumptions of beam theory are applied, *i.e.* Case B of Figure 3. Thus, the predicted critical load levels and influence of material parameters and geometry parameters are very similar. As an example of this, results from the analyses of Case 1, are given in Table 4.

Table 4. Base example with results showing influence of element size and boundary conditions. h=600 mm, α=0.75, β=0.25. Critical load for 2D-FE models is based on crack length=20 mm.

Approach	Critical load (kN)	Remarks
EC5	45.8	Eq.(7)–Eq.(9), k_n=6.5 (G_C=G_{IC}=179.7 N/m)
2D-FE	46.7	Element size 10 mm, *beam boundary conditions*
2D-FE	46.5	Element size 5 mm, *beam boundary conditions*
2D-FE	46.4	Element size 2.5 mm, *beam boundary conditions*
2D-FE	31.1	Element size 5 mm, *support plate*
2D-FE	30.8	Element size 10 mm, *support plate*

The Eurocode 5 approach considers pure Timoshenko beams and disregards local effects from *e.g.* supports or load-introduction. The notch capacities using the EC5 approach and the FE-analyses of the end-notched beams with beam support conditions show similar levels, see Fig. 9. It can be observed, however, that the FE-models yield higher notch capacities for small notch heights (α>0,75).

Jockwer [4] referred this to the dominant effect of Mode 2 fracture for these smaller notches. This could only partly be confirmed by the present FE-analyses, which indeed show that the total energy release rate is more influenced by Mode 2 for large values of *α*, but this effect accounts for only around 5-10% of the increase in relation to assuming Mode 1 failure.

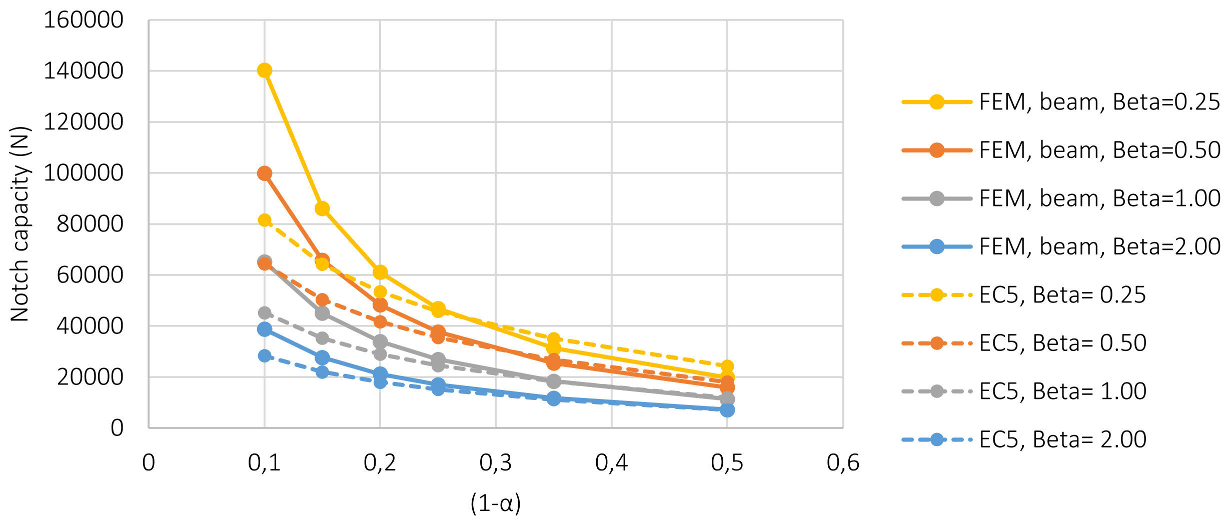

Figure 9. Notch capacities in dependency of notch ratio α according to EC5 and FE-model with beam support boundary conditions for a beam height h=600mm.

4.2 Impact of the support conditions on the notch capacity

The notch capacity for the two different boundary conditions with plate and beam supports (*cf.* Figure 3) are shown in Figure 10. It can be observed that for short notch lengths (*β*<1), the model with beam support conditions yields considerably higher notch capacities, especially for small notch heights. In contrast, for large notch length (*β*>1), both support conditions give similar notch capacities. Further, it can be observed that for plate support conditions with the smallest notch length with *β*=0.25 the notch capacity is even smaller than for the respective longer notch with *β*=0.5. This is in contradiction to the observations from the model with beam support conditions and the behaviour of the EC5 approach.

An explanation for this behaviour can be found in the interaction of the local stress concentration in compression perpendicular to the grain around the support plate and the stress concentration in tension perpendicular to the grain at the notch corner. Since the load is introduced at the lower edge of the beam, in the case of small values of *β*, the full height of the reduced part of the beam is not contributing to the force transfer. This phenomenon loses impact for longer notch length, for *β*=2 no difference between the two support conditions can be observed.

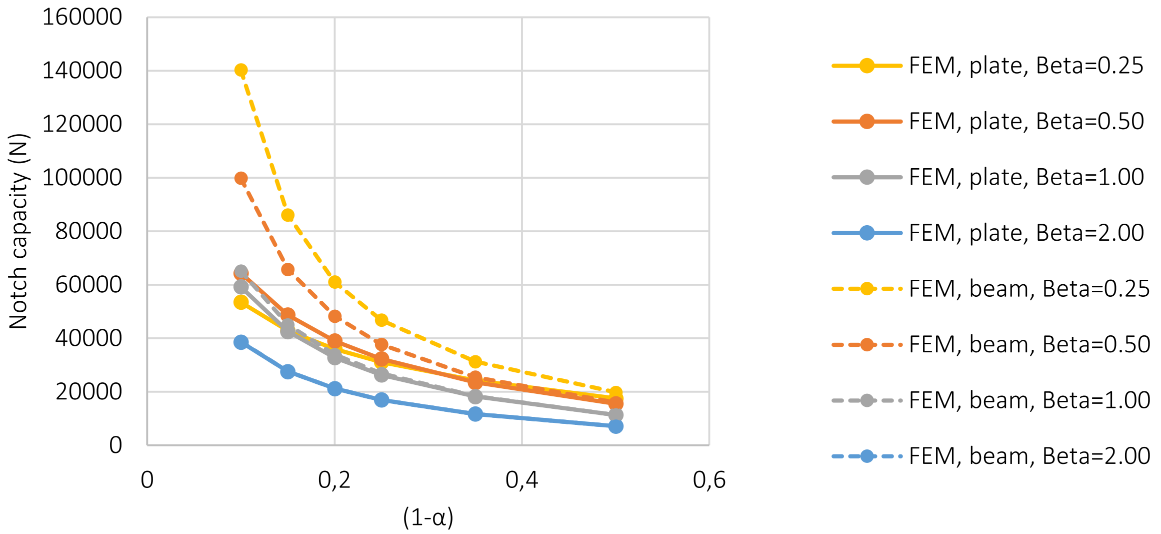

Figure 10. Notch capacities vs. reduced height (1-α) according to FE-model with plate and beam support boundary conditions for a beam height h=600mm. Note that for β=2, the curves coincide.

4.3 Impact of the notch length on the notch capacity

The comparison of the FE models with the EC5 approach shows that especially for the plate support conditions EC5 is more conservative for longer notch lengths (see Figure 11).

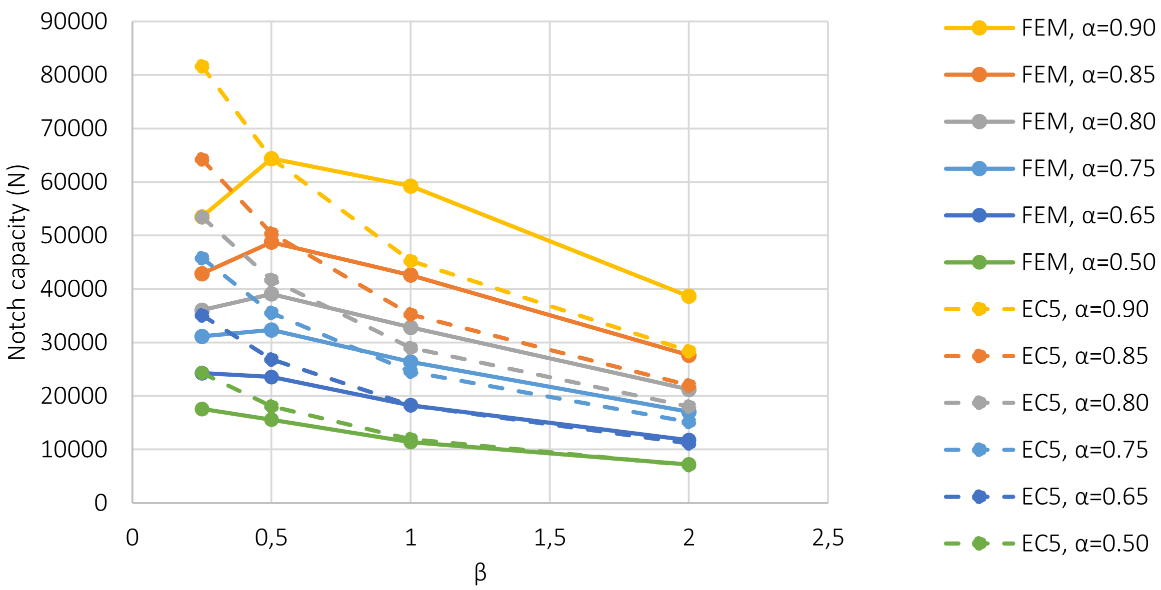

Figure 11. Notch capacities in dependency of notch ratio α according to EC5 approach and FE-model with plate support boundary conditions for a beam height h=300mm.

4.4 Impact of the moment/shear force ratio at support on the notch capacity

At a mid-support of a continuous beam both moment and shear force are acting. Typically, in timber beams ratios *M/V=1h – 4h* can be observed. In the present study, a ratio *M/V=2.5h* was assumed at support. Due to this constant ratio, it can be observed that long notches, β>1 do not influence the capacity of the beam at the support, since the predicted capacity due to crack propagation is way beyond the bending capacity of the reduced cross-section, see Figure 12. Another way of presenting these results is given in Figure 13, showing the relative shear capacity at support (*i.e.* a k_v-factor), assuming the shear strength to be 2.5 MPa and the bending strength to be 28 MPa. Here it is also seen that for small depths (h=300), the failure mode predicted is bending failure (all curves in the top left diagram of Figure 13 coincide and are straight lines).

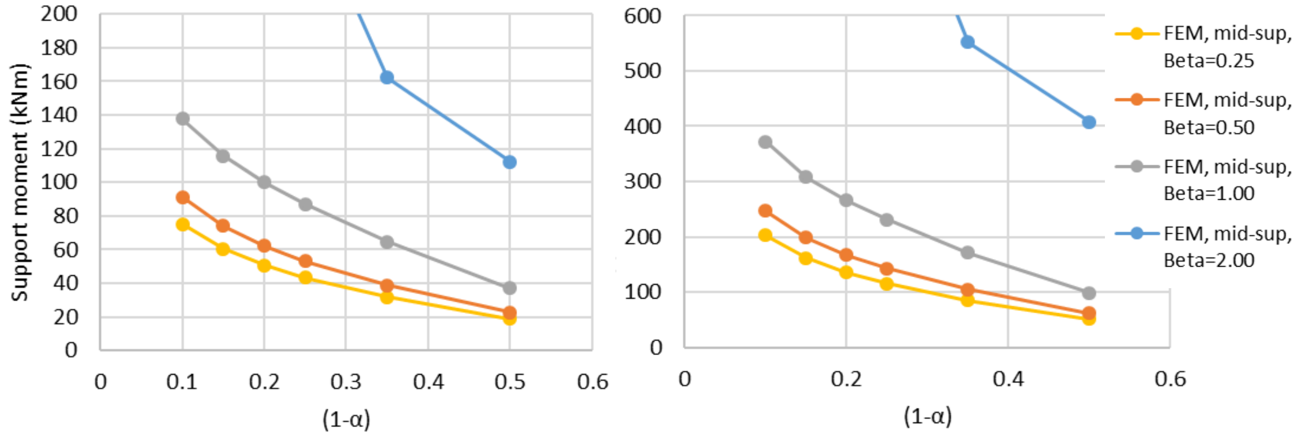

Figure 12. Notch capacities at the mid support in dependency of notch ratio α according FE-model with beam support boundary conditions. M/V=2.5h. Left: h=600 mm, Right: h=1200 mm.

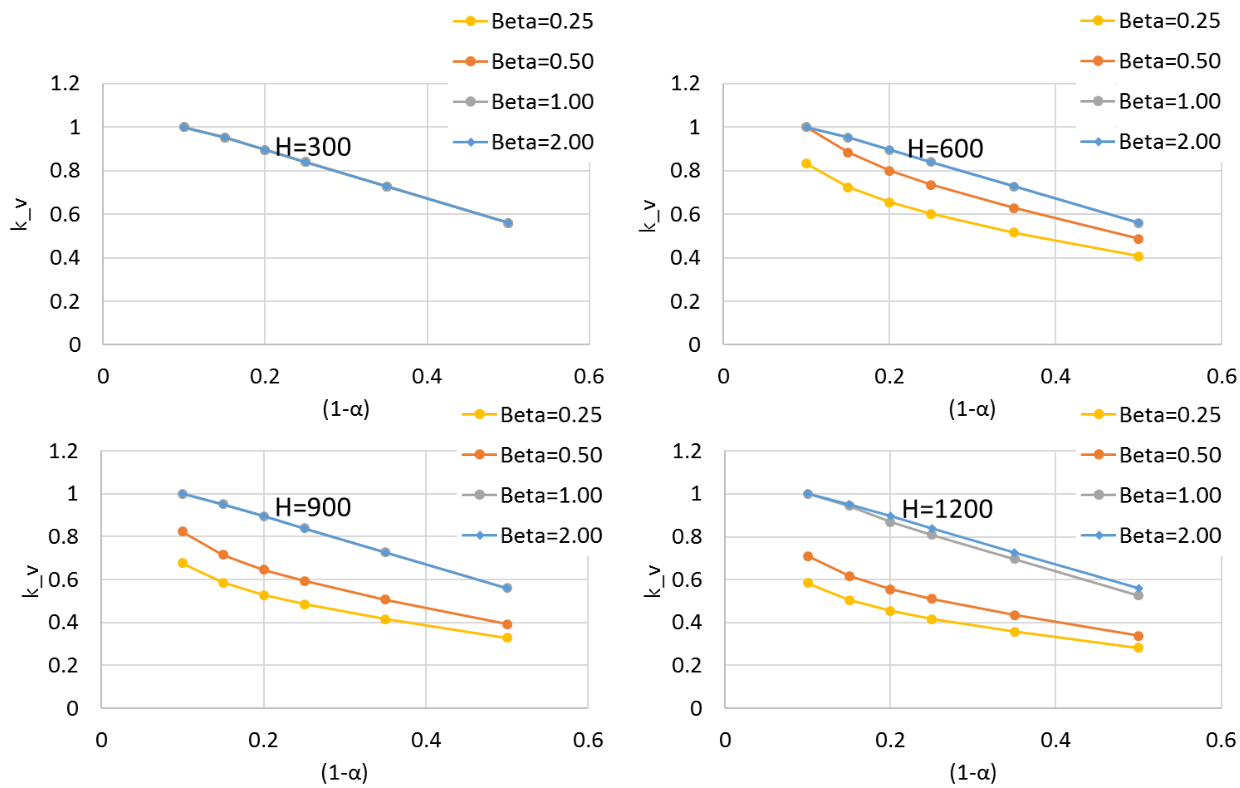

Figure 13. Relative notch capacities (k_v) at the mid support in dependency of notch ratio α. FE-model with beam support boundary conditions. M/V=2.5h. The shear strength and bending strength are set to 2.5 and 28 MPa, respectively.

4.5 General notches and slits far from support – Moment/shear force interaction

Using 2D FE-models similar to the one depicted in Figure 14, analyses of the behaviour of notches far from support were performed.

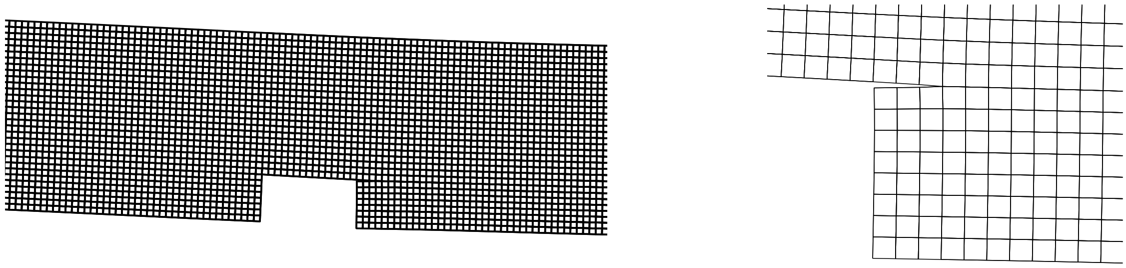

Figure 14. FE-model for analysis of beam notched far from support (left) and partial zoom (right). Here a coarser mesh than used in the analyses is shown for clarity. h=600, α =0.75, β =0.5

The main results are shown in Figure 15, depicting the linear interaction that is obtained from analyses of different combinations of moment and shear force, at the centre of the notch.

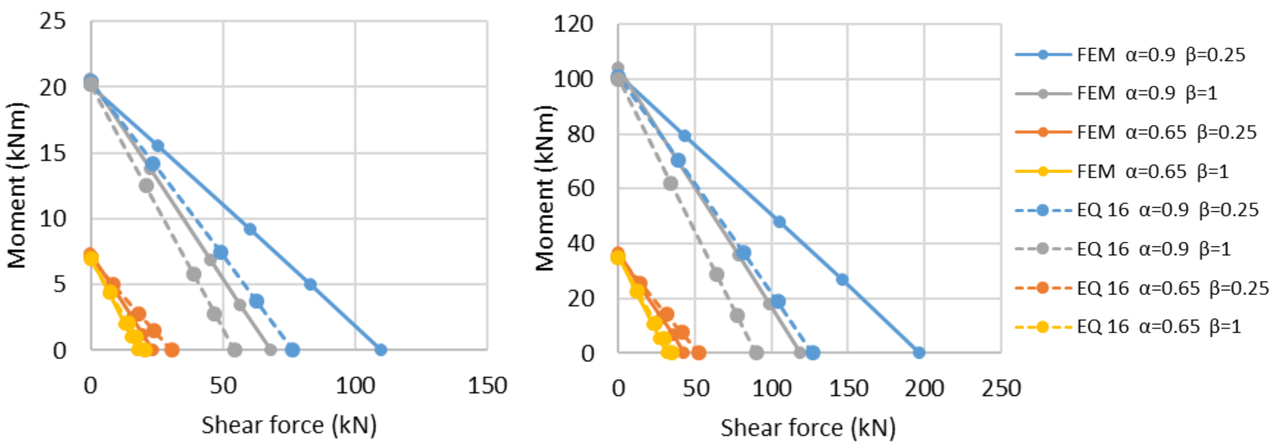

Figure 15. Results from analyses of beam with notch away from support (cf. Figure 14) Moment and shear force refer to the section in the centre of the notch; h=300 (left) h=900 (right).

5 Discussion, conclusions and outlook

5.1 Consequences for design

From the results described above the following conclusions and recommendations for design can be drawn:

The performed FE analysis on end-notched beams shows agreement with the analytical design model in Eurocode 5. This agreement is particularly good for longer notch lengths (large *β*) and large notch heights (small *α*). For smaller notch heights, the Eurocode 5 approach shows more conservative results than the comparable FE models with beam support conditions.

The support conditions have a clear effect on the notch capacity. End-notched beams with plate supports in compression perpendicular to the grain show lower notch capacities than beams with shear support along the beam end. The local and concentrated introduction of forces perpendicular to the grain in close vicinity of the notch corner reduces the notch capacity. However, currently this effect is not considered in the Eurocode 5 design approach. Based on the performed analysis a minimum notch length of β=1 could be considered in design in order to compensate for the stress interaction for plate supports. It is expected that compression perpendicular to the grain reinforcement by screws or glued-in rods and joist hangers with e.g. screws are more beneficial than pure plate support.

At mid supports in continuous beams long notches are less relevant for the beam capacity compared to the beam's shear and moment resistance at the central support. This can be related to the reduction of effective moment at the notch corner with increasing notch length. The negative moment at the mid support causes crack closure mechanisms and, consequently, causes shearing fracture (mode 2) of the notch.

At notches along the span of the beam, the FE models show a linear interaction of the effects from moment and shear force on the notch capacity. A combined analytical model, that is based on the Gustafsson approach, shows good agreement with the trends from the FEM model, but is partly more conservative for small notch heights. Such a model could be used to estimate the capacity of notches in dependency of the applied shear force and moment action at the notch corner.

5.2 Future work

Further research and more detailed analysis are needed, in particular as regards:

- the effect from the local force introduction from the supports of end-notched beams
- the discrepancy between FEM and analytical approach for small notch heights is in need of further investigation, and,
- if possible, a unified design approach of the cases studied herein, should be sought for.

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