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IDENTIFICATION BASED ON ASYNCHRONOUSLY SAMPLED DATA

by

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ABSTRACT

Most identification and designs method for discrete time data assume that the inputs and outputs are sampled at the same time instants. In practice, there is always a delay due to the conversion times in the analog to digital converters. In the paper it is shown how the asynchronously sampled data can be used to obtain the model corresponding to synchronously sampling. The modification is simple and can be done after the estimation. It is only required that a suitable structure is used for the estimated model and that the delay is known.

1. Introduction

When making data acquisition from a plant there is usually a time delay between the sampling of different signals. This time delay comes, for instance, from conversion times in the analog to digital converters. This delay will have little influence if it is short compared with the sampling interval and the dynamics of the process. In Sridharan et al (1985) it is pointed out that the delay can introduce bias in the estimated models if the data is used straight forwardly. In this paper we will show how to eliminate the influence of the time delay and obtain models that correspond to synchronous sampling.

The result is that the asynchronously sampled data can be used directly in any appropriate estimation routine provided that a suitable model structure is used for the estimation. The obtained model can then simply be transformed to the model that corresponds to the synchronous sampling. It is only assumed that the delay between the sampled inputs and outputs are known.

The paper is organized as follows : Section 2 defines the problem. The synchronous model is derived from the asynchronous model in Section 3. An example of the proposed method is given in Section 4.

2. Problem Formulation

It is assumed that the process is a single input single output system described by the transfer function

$$G(s) = \frac{B_c(s)}{A_c(s)} \qquad (2.1)$$

where $A_c(s)$ and $B_c(s)$ are relatively prime with $\deg B_c < \deg A_c$. It is assumed that (2.1) can be written in the minimal order state space form

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) \tag{2.2}$$

The input signal is constant over sampling periods of length h. Further it is assumed that the input is changed at the sampling instants. A zero order hold is thus used for the input signal. Sampling (2.2) gives the discrete time model

$$x(kh + h) = \Phi_h\,x(kh) + \Gamma_h u(kh)$$
$$y(kh) = C\,x(kh) \tag{2.3}$$

where

$$\Phi_t = e^{At}$$

$$\Gamma_t = \int_0^t e^{As}\,B\,ds$$

The pulse transfer function of (2.3) is

$$H(z) = C(zI - \Phi_h)^{-1}\,\Gamma_h \tag{2.4}$$

It is also possible to determine the states of (2.2) at times between the sampling periods, see for instance Åström and Wittenmark (1984) or Wittenmark (1985). Assume that $\tau < h$, then

$$x(t + \tau) = e^{A\tau} x(kh) + \int_0^{\tau} e^{As'} B u(kh + \tau - s')ds'$$

$$= \Phi_\tau x(kh) + \Gamma_\tau u(kh) \tag{2.5}$$

and

$$y(t + \tau) = C x (t + \tau) \tag{2.6}$$

These equations can now be used to specify the asynchronously sampled model. Assume that the output of the process is measured τ seconds after the input is changed, see Figure 1. It is assumed that $\tau < h$.

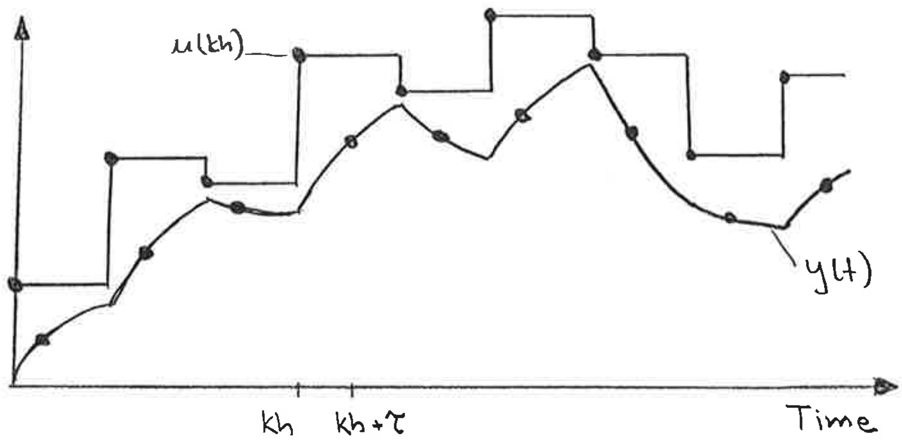

FIGURE 1 The Relation between the Input and the Output.

This implies that the estimation routine can use the following set of data

$$y_1(t_k) = y(kh + \tau)$$

$$u(t_k) = u(kh)$$

or

$$y_2(t_k) = y(kh - h + \tau)$$

$$u(t_k) = u(kh)$$

Notice that $y_1(t_{k-1}) = y_2(t_k)$. The two sequences are only delayed with respect to each other. The data set $(u(t_k), y_1(t_k))$ can be regarded as generated by the model in Figure 2.

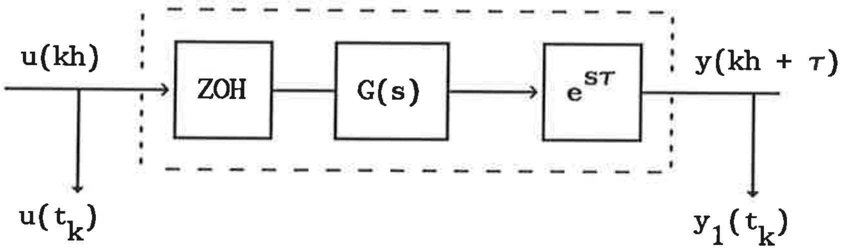

FIGURE 2 The Model for the Data Generation.

Remark

The model in Figure 2 is non-causal. A causal model will instead be obtained if $(u(t_k), y_2(t_k))$ is used. The computations will, however, be the same independent of which data set is used. The calculations in the following will be done for the set (u, y_1).

The problem can now be formulated as : Given the data set $(u(t_k), y_1(t_k))$ and the delay $\tau < h$. Determine the input/output relations G(s) and/or H(z) in (2.1) and (2.4) respectively.

3. Asynchronous and Synchronous Models

The sampled data model of the system in Figure 2 can be determined using (2.2), (2.5) and (2.6)

$$x(kh + h) = \Phi_h \, x(kh) + \Gamma_h \, u(kh)$$
$$y_1(kh) = y(kh + \tau) = C \, \Phi_\tau \, x(kh) + C \, \Gamma_\tau u(kh) \tag{3.1}$$

This fictitious system has the pulse transfer function

$$H_1(z) = C \, \Phi_\tau \, (zI - \Phi_h)^{-1} \, \Gamma_h + C \, \Gamma_\tau$$
$$= H_2(z) + C \, \Gamma_\tau \tag{3.2}$$

We can regard H_2 as being the pulse transfer function of the system

$$x(kh + h) = \Phi_h \, x(kh) + \Gamma_h \, u(kh)$$
$$\bar{y}(kh) = C \, \Phi_\tau \, x(kh) \tag{3.3}$$

The data set (u, y_1) can now be used to estimate the pulse transfer function H_1 if an appropriate structure is used in the estimation algorithm.

If the process (2.1) is of order n then the pulse transfer function (2.4) has n poles and in general $n-1$ zeros. The pulse transfer function (3.2) also has n poles and in general n zeros. The structure in the estimation routine should allow a model as in (3.2). If the input signal is sufficiently rich it is thus possible to estimate $H_1(z)$. Both $H_1(z)$ and $H_2(z)$ differ from the desired pulse transfer function $H(z)$ given by (2.4). To obtain the desired model we must modify the result of the estimation. Assume that an unbiased estimate of H_1 has been obtained. We can then eliminate the direct term to get an

$$\hat{C}\,\hat{\Phi}_{\tau}^{-1}(zI - \hat{\Phi})^{-1}\hat{\Gamma}$$

$$= C\,T^{-1}(zI - T\Phi_h T^{-1})^{-1}\,T\,\Gamma_h$$

$$= C(zI - \Phi_h)^{-1}\Gamma_h = H(z)$$

Further

$$\hat{C}\,\hat{\Phi}_{\tau}^{-1}(sI - \hat{A})^{-1}\hat{B}$$

$$= C(sI - A)^{-1}B = G(s)$$

We have thus shown that the desired input output relations can be obtained through the following procedure.

<u>Step 1</u> Estimate the input output model $H_1(z)$ from the data (u, y_1). Both the numerator and the denominator of H_1 should be of order n.

<u>Step 2</u> Eliminate the direct term to obtain H_2 defined in (3.2).

<u>Step 3</u> Make one minimal order realization of H_2 defined by $\hat{\Phi}$, $\hat{\Gamma}$ and $\hat{C}$, see (3.4).

<u>Step 4</u> Determine

$$\hat{A} = \frac{1}{h}\ln\hat{\Phi}$$
$$\hat{B} = (\hat{\Phi} - I)^{-1}\,\hat{A}\,\hat{\Gamma}$$
$$\overline{C} = \hat{C}\,\hat{\Phi}_{\tau}^{-1}$$
$$\hat{\Phi}_{\tau} = \exp(\hat{A}\tau)$$

<u>Step 5</u> Compute

$$H(z) = \overline{C}(zI - \hat{\Phi})^{-1}\,\hat{\Gamma}$$
$$G(s) = \overline{C}(sI - \hat{A})^{-1}\,\hat{B}$$

same structure as (4.1) we got the result

$$\hat{H}(z) = \frac{0.5515z^2 - 0.1529z - 0.1615}{z^3 - 1.5857z^2 + 1.4078z - 0.4090}$$

A straightforward use of the asynchronous data gives biased estimates of both the numerator and the denominator polynomials as pointed out in Sridharan et al (1985).

Further examples are given in Olsson (1986).

5. Conclusions

The paper has shown that asynchronously sampled data can be used to obtain the synchronous models if the delay between the measurements is known. The method does not require any modification of the estimation routines or the original data set. The synchronous model is obtained by simple calculations on the estimated model. The calculations require inversion of two $n \times n$ matrices where n is the order of the model. Further the logarithm and the exponential of a $n \times n$ matrix have to be calculated.

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