ADAPTIVE STABILITY AUGMENTATION

OR

HOW TO USE A PRIORI KNOWLEDGE

by

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ABSTRACT

Most adaptive controllers do not assume any a priori knowledge about the process to be controlled. In most situations there is, however, some knowledge about the process. This knowledge should be used to improve the performance compared to the "black box" approach of earlier adaptive controllers.

The a priori knowledge is in this paper used in the estimation of the process and also in the design of the controller. The use in the design is new compared to earlier approaches. The a priori knowledge is here used to design a nominal controller. The unmodelled part of the process is estimated and an adaptive stability augmentation signal is added to the nominal controller. Different types of a priori knowledge and design methods are discussed and the performances are investigated through analysis and simulations.

1. INTRODUCTION

Adaptive controllers are now more and more used in industrial applications, see the surveys by Åström (1983) and Seborg et al (1986). Most adaptive controllers are based on a "black box" approach. In many applications there is an extensive emperical knowledge about the process model. This a priori knowledge about the process model has so far only been used to a very limited extent. The knowledge has for instance been used to determine possible structures of the controller and desired closed loop responses. However, as soon as the structure of the adaptive controller has been determined all a priori knowledge is abandoned.

It is expected that the use of a priori knowledge will improve the performance compared to adaptive controllers based on a black box approach. There are some references where a priori knowledge has been used in adaptive controllers. Clary and Franklin (1984) estimate the unkown part of the process. The adaptive controller is then designed in a conventional way for the full process model. Bai and Sastry (1986) use a more general process model for the a priori knowledge. Dasgupta et al (1985) and Dasgupta and Anderson (1986) derives a parameterization for identification of physical parameters in a model. Different ways to make the parameterization is discussed in Section 2. The papers referred above mainly discuss how the a priori knowledge can be used in the estimation part of the adaptive controller. The controller design is then done based on the full process model. A different approach to the design is taken in this paper. The inspiration for the new class of adaptive controllers comes partly from design of power system stabilization equipment and design of autopilots for airplanes. These systems contain what sometimes is called stability augmentation. The idea is that a nominal control signal is augmented by an additional signal, which increases the stability properties of the closed loop system. A similar idea for linear quadratic controllers is found in Moore (1987). Characterization of all possible stabilizing controllers is discussed in Desoer et al (1980). The Adaptive Stability Augmentation (ASA) controller in this paper is based on the idea to divide the controller into a fixed nominal part and an adaptive stability augmenting part.

The design of the adaptive stability augmentation depends on the parameterization of the process and on the desired closed loop response. The parameterization issue is discussed in Section 2. The adaptive control problem is formulated in Section 3. The estimation of the unknown part of the process is discussed in Section 4. The adaptive stability augmentation controller is introduced in Section 5 for different types of parameterizations. The design principles and the properties of the closed loop system are discussed. Section 6 contains analysis and examples which show the improved performance of the new controller. Conclusions and references are found in Sections 7 and 8 respectively.

In this paper we concentrate on the design principles. We discuss the improvement in performance that can be obtained if additional information is obtained about the system. Some of the motivations for the approach in this paper are:

- Computational aspects. We don't want to resolve the full design problem at each step.
- Safety aspects. The stability augmentation can be disconnected to obtain graceful degradation of the system.
- Use available process knowledge. The a priori knowledge give an indication how the controller should look like.

The contribution of this paper relative to previous works are:

a) A new design principle for control of partially known systems. The a priori knowledge is used in the estimation as well as in the design of the controller.

b) The design and the analysis are simplified by the separation into a nominal controller and the adaptive stability augmentation.

2. A PRIORI PROCESS KNOWLEDGE

One main difficulty when using a priori knowledge is the issue of parameterization. There are several ways in which the a priori knowledge is available. Some aspects are described and discussed in this section. The extent of the knowledge depends very much on the physical process. The available a priori knowledge about a process can for instance be given in the following forms:

- The number of integrators and the dominating time constant may be known, but the process may also contain unknown or varying high frequency parts. Compare fixed body and bending mode dynamics of airplanes.
- The physical model may be well understood, but some parameters may be difficult to determine. Examples of such parameters can be friction coefficients and heat transfer coefficients.
- Parts of the dynamics may change depending on the working or loading conditions for the process. Examples are industrial robots and power systems, where the load will change the dynamics of the process.

The a priori knowledge is usually available in a continuous time form while most adaptive controllers are implemented using digital computers. This will give rise to difficulties when utilizing the physical knowledge. Another difficulty is that the a priori knowledge can be available in form of a state space model with some unknown parameters. The adaptive controllers are in general based on pulse transfer function (input/output models). The problems are highlighted through a couple of examples.

Example 2.1

Assume that the process has the following state space representation

$$\dot{x} = \begin{bmatrix} -\theta_1 & 1 \\ 0 & -\theta_2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [\theta_3 \ \theta_4]x$$

The transfer function is

$$G(s) = \frac{\theta_3 s + \theta_3\theta_2 + \theta_4}{s^2 + (\theta_1 + \theta_2)s + \theta_1\theta_2}$$

The transfer function $G(s)$ is linear in the unknown paramters for instance when $\{\theta_3,\theta_4\}$, $\{\theta_1,\theta_3)$ or $\{\theta_1,\theta_3,\theta_4\}$ are regarded as unknown. $G(s)$ is, however, nonlinear in the parameters if $\{\theta_1,\theta_2\}$ or $\{\theta_2,\theta_3\}$ are the unknown parameters. The linear in parameter assumption has in general been a very crucial assumption in the design of adaptive controllers.

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Example 2.2

Consider the RLC circuit in Figure 2.1. The state space representation is

$$\dot{x} = \begin{bmatrix} 0 & -1/C \\ 1/L & -R/L \end{bmatrix} x + \begin{bmatrix} 1/C \\ 0 \end{bmatrix} u$$

$$y = [0 \quad R]x$$

and the transfer function is

$$G(s) = \frac{\frac{R}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Let $\theta_1 = R$, $\theta_2 = 1/L$ and $\theta_3 = 1/C$ then

$$G(s) = \frac{\theta_1\theta_2\theta_3}{s^2 + \theta_1\theta_2 s + \theta_2\theta_3}$$

The coefficients of the transfer function are thus nonlinear (although of special structure) in the physical parameters R, 1/L and 1/C.

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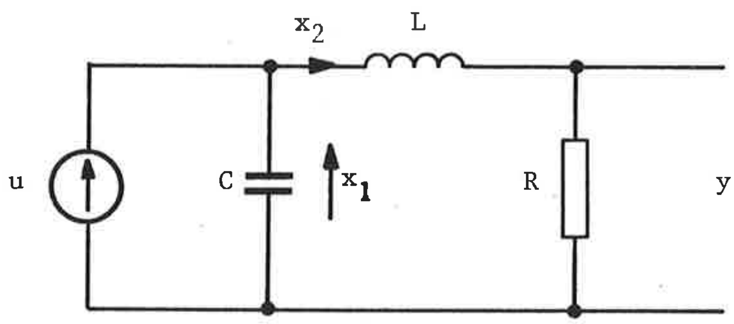

Figure 2.1 An RLC Circuit

Example 2.3

Consider the continuous time system with the transfer function

$$G(s) = \frac{1}{(1 + \theta_1 s)(1 + \theta_2 s)}$$

We may regard θ_2 as an unknown time constant, while θ_1 may be known. The pulse transfer function has the form

$$H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2}$$

where

$$b_1 = \frac{\theta_1(1 - e^{-h/\theta_1}) - \theta_2(1 - e^{-h/\theta_2})}{\theta_1 - \theta_2}$$

$$b_2 = \frac{\theta_2(1 - e^{-h/\theta_2})e^{-h/\theta_1} - \theta_1(1 - e^{-h/\theta_1})e^{-h/\theta_2}}{\theta_1 - \theta_2}$$

$$a_1 = -(e^{-h/\theta_1} + e^{-h/\theta_2})$$

$$a_2 = e^{-(1/\theta_1 + 1/\theta_2)h}$$

The pulse transfer function is thus nonlinear in both θ_1 and θ_2. Further both parameters θ_1 and θ_2 appear in <u>all</u> the coefficients of the discrete time pulse transfer function. This implies that if θ_2 is an unknown time constant then it will influence all the coefficients in the sampled data model.

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The examples above show that it is crucial how the process model is parameterized. The parameterization issue is discussed in Dasgupta et al (1985). There it is shown that a large class of physical systems with what is defined as rank one dependence has a transfer function of the form

$$G(s) = \frac{B_o(s) + \Sigma k_i B_i(s)}{A_o(s) + \Sigma k_i A_i(s)} \tag{2.1}$$

where B_i and A_i are known polynomials and k_i are multilinear functions of the unknown parameters θ_i. For instance the circuit in Example 2.2 has a rank one dependence in R, 1/L and 1/C. For a system with three unknowns, $\theta_1 - \theta_3$, we have $k_1 = \theta_1$, $k_2 = \theta_2$, $k_3 = \theta_3$, $k_4 = \theta_1\theta_2$, $k_5 = \theta_1\theta_3$, $k_6 = \theta_2\theta_3$ and $k_7 = \theta_1\theta_2\theta_3$. The models are thus quite complex even for only a few unknown parameters. In Dasgupta et al (1985) it is shown how constrained estimation of the parameters θ_i can be done for this class of systems. A parameterization similar to (2.1) is considered in Bai and Sastry (1986). They assume that all the parameters k_i are unknown parameter and not related to $k_1 = \theta_1$ and $k_2 = \theta_2$. The estimation and the design in Bai and Sastry (1986) are done for discrete time models.

Clary and Franklin (1984) consider discrete time models of the form

$$H(z) = H_k(z) \cdot H_u(z) \tag{2.2}$$

where $H_k(z)$ is a known and $H_u(z)$ is an unknown part. In the light of Example 2.2 this parameterization has some obvious problems since

$$z(G_1G_2) \neq z(G_1)\Sigma(G_2) \tag{2.3}$$

where $z(\cdot)$ denotes the z-transform (or sampling) of a transfer function. The discussion above shows some of the difficulties in making general parameterizations of a system to take advantage of a priori knowledge. The best method depends on the specific problem and which parameters that are known and unknown.

3. PROBLEM FORMULATION

The discussion of the parameterization in the previous section shows some of the problems to utilize a priori knowledge about a physical process. In this paper we take an engineering approach of the problem and look for sensible approximate ways to include the physical a priori knowledge. The motivation is that any reasonable way to include the knowledge should be better than a black box approach.

The physical knowledge is mostly available for continuous time models. This implies that the parameters should be estimated from continuous time models. Discrete time estimation of continuous time models is discussed for instance in Young (1981), Johansson (1986), Canudas de Wit (1986) and Gawthorp (1986). Another way to circumvent the problem with (2.3) is to use the δ-operator and fast sampling, see Goodwin et al (1986). The δ operator is defined by

$$\delta = \frac{q - 1}{h}$$

where q is the forward shift operator and h is the sampling interval. In Gawthrop (1980) it is shown that the limiting value

$$\lim_{h \to 0} \frac{B_h(\delta)}{A_h(\delta)} = \frac{B_o(\delta)}{A_o(\delta)}$$

is such that the coefficients in B_o and A_o are the same as the coefficients in the continuous time transfer function. This implies that the structure of the transfer function in the δ-operator is essentially the same as that of the continuous time transfer function provided that the sampling period is sufficiently short. The δ-operator formulation has the advantage that a controller in the δ-operator can be derived without sampling a continuous time model.

Motivated by the discussions in Section 2 and above we will use the following three different parameterizations in the δ-operator form:

Multiplicative disturbance dynamics:

$$H(\delta) = \frac{B_o(\delta)}{A_o(\delta)} \cdot \frac{B_1(\delta)}{A_1(\delta)} \tag{3.1}$$

Relative disturbance dynamics:

$$H(\delta) = \frac{B_o(\delta)}{A_o(\delta)} \left(1 + \frac{\Delta B(\delta)}{1 + \Delta A(\delta)}\right) \tag{3.2}$$

Coefficient disturbance dynamics:

$$H(\delta) = \frac{B_o(\delta) + \Sigma\beta_i B_i(\delta)}{A_o(\delta) + \Sigma\alpha_i A_i(\delta)} \tag{3.3}$$

We will assume that

$$H_o(\delta) = \frac{B_o(\delta)}{A_o(\delta)} \tag{3.4}$$

is known and that B_o and A_o are coprime. $H_o(\delta)$ is called the <u>nominal model</u>. $\{B_1,A_1\}$, $\{\Delta B,\Delta A\}$ and $\{\alpha_i,\beta_i\}$ are the unknown parts in the parameterizations (3.1), (3.2), and (3.3) respectively. Further it is assumed that B_i and A_i in (3.3) are known polynomials with the following degree restrictions

$$\begin{aligned} \deg A_o &= n \\ \deg A_i &< n \\ \deg B_o &\le n \\ \deg B_i &\le n \end{aligned} \tag{3.5}$$

The most structured form is obviously (3.3). The structures (3.1) and (3.2) are equivalent when

$$\frac{B_1(\delta)}{A_1(\delta)} = \frac{1 + \Delta A(\delta) + \Delta B(\delta)}{1 + \Delta A(\delta)}$$

The unknown parts of either (3.1), (3.2) or (3.3) are estimated on line. The estimation is discussed in Section 4. The total model may then be used to design a controller on line. This is the case in for instance Clary and Franklin (1984) and Bai and Sastry (1986). The design will then not be simplified by the a priori knowledge. We will regard the unknown parts as perturbations of the nominal plant $H_o(\delta)$. A nominal controller G_o for H_o is precomputed such that the closed loop response is as desired. See Figure 3.1. The design problem is now formulated as to determine the adaptive stability augmentation controller to improve

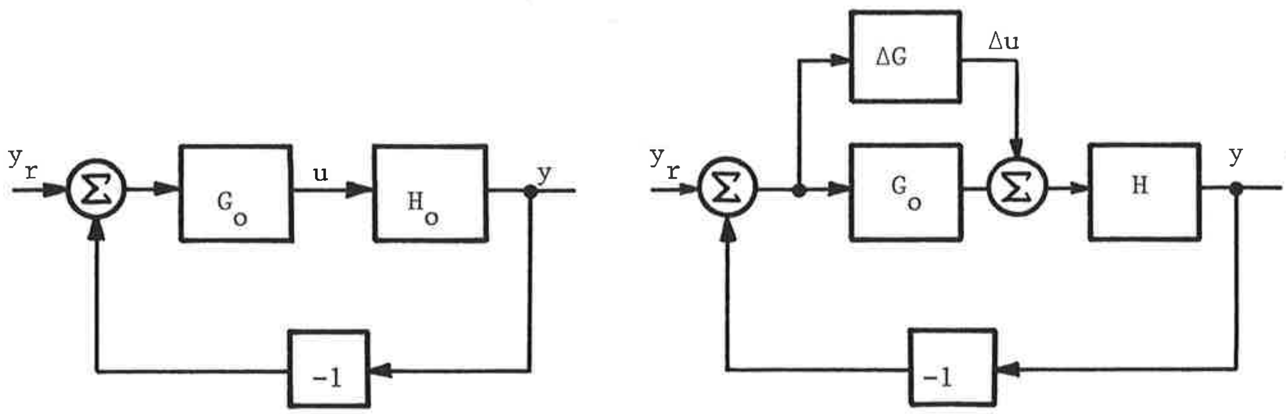

Figure 3.1 a) Control of the nominal plant.
b) Stability augmentation to improve the control of the full plant

the control of the full plant. The structure of the controller depends on H_o, H and the design specifications. Different alternatives are discussed in Section 5. The stability augmentation controller is allowed to be a function of B_o, A_o, the current estimate of the unknown part and the desired closed loop response.

The nominal controller may be designed using any feasable method. This design is only done once and not updated.

4. PARAMETER ESTIMATION

In this section we will discuss how the unknown parts of (3.1) - (3.3) can be estimated.

Consider first the multiplicative disturbance model. Following Clary and Franklin (1984) we introduce the filtered signals

$$y_o(t) = A_o(\delta)y(t) \tag{4.1}$$

$$u_o(t) = B_o(\delta)u(t) \tag{4.2}$$

The model (3.1) can now be written as

$$A_1(\delta)y_o(t) = B_1(\delta)u_o(t) \tag{4.3}$$

This parameterization is linear in the unknown parameters. The least squares method can for instance be used to estimate the unknown part of (3.1). The model (4.3) can be written as

$$y_o(t) = \varphi_m(t - 1)^T\theta \tag{4.4}$$

where

$$\varphi_m(t - 1)^T = [y_o(t-1)\ \ \delta y_o(t-1)\ \ \ldots\ \ u_o(k-1)\ \ \delta u_o(k-2)\ldots]$$

$$\theta^T = [a_{11}\ \ a_{12}\ \ \ldots\ \ b_{10}\ \ b_{11}\ \ \ldots\ ]$$

The elements in θ, a_{1i} and b_{1i}, are the unknown coefficients in the A_1

and B_1 polynomials respectively. Clary and Franklin (1984) also discuss a constrained estimation procedure. They recommend, however, the filtering approach. The filters in (4.1) and (4.2) can also be chosen as A_o/A_f and B_o/A_f where A_f is a known stable polynomial of sufficiently high order such that "near differentiation" is avoided in the estimator, see Goodwin et al (1986).

The parameterization (3.2) can be written as

$$y(t) = \frac{B_o(\delta)}{A_o(\delta)} \left(1 + \frac{\Delta B(\delta)}{1 + \Delta A(\delta)}\right) u(t)$$

$$y_o(t) - u_o(t) = \frac{\Delta B(\delta)}{1 + \Delta A(\delta)} u_o(t)$$

or

$$y_o(t) - u_o(t) + \Delta A(\delta)(y(t) - u_o(t)) = \Delta B(\delta) u_o(t)$$

This model can also be written in the form

$$y_o(t) - u_o(t) = \varphi_r(t-1)^T \theta$$

with

$$\varphi_r(t-1)^T = [y_o(t) - u_o(t) \;\; \delta(y_o(t) - u_o(t)) \; \ldots \; u_o(t) \;\; \delta u_o(t) \; \ldots \;]$$

Finally (3.3) can be rewritten as

$$A_o(\delta)y(t) + \Sigma\alpha_i A_i(\delta)y(t) = B_o(\delta)u(t) + \Sigma\beta_i B_i(\delta)u(t)$$

$$\begin{aligned} y_o(t) - u_o(t) &= - \Sigma\alpha_i y_i(t) + \Sigma\beta_i u_i(t) \\ &= \varphi_c(t-1)^T\theta \end{aligned} \tag{4.5}$$

where

$$y_i(t) = A_i(\delta)y(t)$$

$$u_i(t) = B_i(\delta)u(t)$$

$$\varphi_c(t - 1)^T = [-y_1(t) \; - y_2(t) \;\; \ldots \;\; u_1(t) \; u_2(t) \;\; \ldots \;]$$

$$\theta^T = [\alpha_1 \quad \alpha_2 \quad \ldots \quad \beta_1 \quad \beta_2 \quad \ldots \quad]$$

This is the way the estimation is done in Bai and Sastry (1986). All three parameterizations can thus be written in the common form

$$\bar{y}(t) = \varphi(t - 1)^T\theta \tag{4.6}$$

The least squares algorithm in the δ operator can be written as (see Goodwin et al (1986)).

$$\begin{aligned} \delta\hat{\theta}(t - 1) &= \frac{a(t)}{h} \frac{P(t-2)\varphi(t-1)}{1 + \varphi(t-1)^T P(t-2)\varphi(t-1)} e(t) \\ \delta P(t-2) &= - \frac{a(t)}{h} \frac{P(t-2)\varphi(t-1)\varphi(t-1)^T P(t-2)}{1 + \varphi(t-1)^T P(t-2)\varphi(t-1)} \\ e(t) &= \bar{y}(t) - \varphi(t-1)^T \hat{\theta}(t - 1) \end{aligned} \tag{4.7}$$

where $a(t)$ defines a relative dead zone function. (In the ordinary least squares estimation $a(t) = 1$). In some situations it is known that a parameter lies within some bounds. For instance the resonance frequency of a mechanical structure may be known to be within an interval or the sign of a physical parameter may be known. In such situations it is possible to use projection methods to ensure that the estimated parameter is within a specified region. See e.g. Goodwin and Sin (1984). The identifiability of the parameters of the model (4.5) is discussed in Bai and Sastry (1986). Dasgupta et al (1985) discuss the estimation of continuous time models similar to (3.3) where also the multilinear constraints between the parameters are taken into consideration.

For the adaptive stability augmentation controller we will use (4.7) to estimate the unknown parameters in any of the parameterizations (3.1) - (3.3).

5. ADAPTIVE STABILITY AUGMENTATION

Different ways to design the Adaptive Stability Augmentation (ASA) Controller will now be discussed for the different parameterizations in Section 3. The design method will depend on the physical insight and the nature of perturbation of the nominal plant. The main idea is to avoid remaking the full design at each sampling interval. We are thus looking for modifications of the nominal controller which will improve the behaviour of the closed loop system when the perturbation from the nominal plant has been estimated. Different design procedures with increasingly computational demands will be discussed in the following. We give a full spectrum of methods which can cover a wide range of applications. The method that should be used in a specific case has to be determined from the available physical insight.

Generalised dead time compensation

The first case to be considered is when the perturbation in the open loop system is allowed to be present also in the closed loop system. We will make the discussion using the multiplicative disturbance dynamics (3.1), where the unknown part multiplies the nominal plant model.
Let the nominal controller be

$$u(t) = \frac{S_o}{R_o}(y_r(t) - y(t))$$

Compare Figure 3.1.
The nominal plant and the nominal controller give the closed loop system

$$H_c^o = \frac{B_oS_o}{A_oR_o + B_oS_o} \tag{5.1}$$

It is assumed that S_o and R_o are chosen such that (5.1) gives the desired response.

One way to pose the problem is to assume that the controller should be determined such that

$$H_c = H_c^o \frac{B_1}{A_1} \tag{5.2}$$

This implies that the perturbed open loop dynamics is allowed also in the closed loop response. This can be sensible if the extra dynamics are changes in time delay or high frequency dynamics above the desired closed loop bandwidth. The specification (5.2) is inspired by the derivation of deadtime compensation, see Smith (1957). Let the controller be

$$u(t) = \frac{S}{R}(y_r(t) - y(t))$$

This gives the following equation for determination of the controller

$$\frac{B_oB_1S}{A_oA_1R + B_oB_1S} = \frac{B_oB_1S_o}{(A_oR_o + B_oS_o)A_1}$$

Simple calculations give

$$\frac{S}{R} = \frac{S_o}{R_o} \frac{1}{1 + \frac{S_o}{R_o} H_o(1 - \frac{B_1}{A_1})} \tag{5.3}$$

This controller has a nice interpretation, see Figure 5.1. It can be regarded as a feedback compensation of the nominal controller. (It thus has a different structure than suggested in Figure 3.1b.) The controller can be regarded as a generalization of Smith's deadtime compensation.

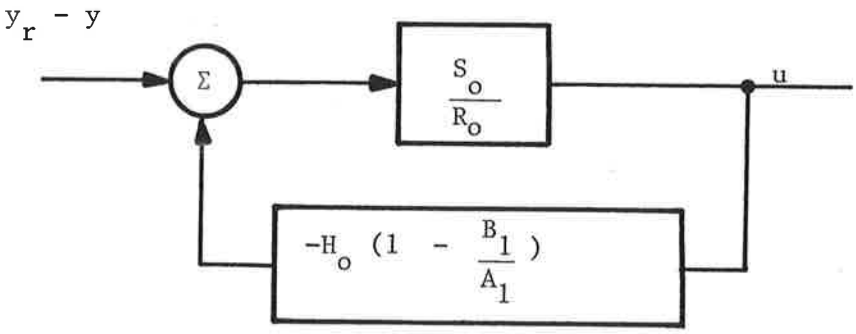

Figure 5.1 Block diagram for the generalized Smith compensator.

The controller (5.3) has the same drawback as the conventional deadtime compensator. It must be assumed that A_oA_1 is stable. See Chapter 10 in Åström and Wittenmark (1984).

The estimated perturbation B_1/A_1 can be used in the controller (5.3) to make an adaptive compensation for the disturbance of the nominal plant. If the perturbed plant is not asymptotically stable it is possible to use a method that will be presented later in this section.

Dynamic cancellation

In this case it is assumed that it is possible to cancel the dynamic of the perturbation. The method can be used for the multiplicative disturbance dynamics case (3.1) or the relative disturbance dynamics case (3.2).

Let the controller be

$$u = \frac{S_o}{R_o}\left(1 + \frac{\Delta S}{1 + \Delta R}\right)(y_r - y) \tag{5.4}$$

Assume that the specification is

$$H_c = H_c^o \tag{5.5}$$

i.e. we require that

$$\frac{\dfrac{B_oS_o}{A_oR_o}}{1 + \dfrac{B_oS_o}{A_oR_o}} = \frac{\dfrac{B_oS_o}{A_oR_o}\left(1 + \dfrac{\Delta S}{1 + \Delta R}\right)\left(1 + \dfrac{\Delta B}{1 + \Delta A}\right)}{1 + \dfrac{B_oS_o}{A_oR_o}\left(1 + \dfrac{\Delta S}{1 + \Delta R}\right)\left(1 + \dfrac{\Delta B}{1 + \Delta A}\right)}$$

Some lines of simple calculations give that the modification of the controller should be

$$\frac{\Delta S}{1 + \Delta R} = - \frac{\Delta B}{1 + \Delta A + \Delta B} \tag{5.6}$$

This modification of the nominal controller is very simple and does not require any extensive computations. The controller can easily be implemented as soon as ΔA and ΔB are estimated. The controller is, however, based on cancellation of ΔA and $1 + \Delta A + \Delta B$. These polynomials must thus be asymtotically stable, compare the generalized Smith compensator. The closed loop system is otherwise unstable.

<u>Perturbed pole placement design</u>

To avoid the cancellation of unstable polynomials we have to use another approach to the design problem. The controllers (5.3) or (5.4) and (5.6) requires very little from a computational point of view. No new design has to be done at each sampling interval. On the other side of the scale of the computational burden we have the type of controllers discussed in Clary and Franklin (1984) and Bai and Sastry (1986) where the full design problem is resolved at each sampling interval. Next we will present a design procedure that only requires moderate computations. The basic assumption is that the nominal controller is a good first approximation of the controller for the full plant. The ideas will be presented for a general class of pole placement controllers.

Consider a general linear controller of the form

$$Ru = -Sy + Ty_r \tag{5.7}$$

where R, S, and T are polynomials in the shift or δ-operators. Let the nominal controller (R_o, S_o, T_o) be designed such that the closed loop transfer function

$$\frac{T_oB_o}{A_oR_o + B_oS_o}$$

has desired properties. Let $A_oR_o + B_oS_o = A_{do}$ where A_{do} is the closed loop characteristic polynomial. This design procedure is thoroughly discussed in Åström and Wittenmark (1984). Assume that a new controller with the polynomials $R_o(1 + \Delta R)$, $S_o(1 + \Delta S)$ and $T_o(1 + \Delta T)$ has to be determined. The closed loop system now becomes

$$\frac{T_oB_o(1 + \Delta T)(1 + \Delta A + \Delta B)}{A_oR_o(1 + \Delta A)(1 + \Delta R) + B_oS_o(1 + \Delta A + \Delta B)(1 + \Delta S)} \tag{5.8}$$

The closed loop characteristic polynomial for the perturbed plant is

$$\begin{aligned}
&A_oR_o + B_oS_o + \Delta R(A_oR_o + B_oS_o + B_oS_o(\Delta A + \Delta B) + A_oR_o\Delta A) \\
&\qquad + \Delta S(B_oS_o + B_oS_o(\Delta A + \Delta B)) + \Delta A(A_oR_o + B_oS_o) + \Delta BB_oS_o \\
&= A_{do} + \Delta R(A_{do} + A_{do}\Delta A + B_oS_o\Delta B) + \Delta SB_oS_o(1 + \Delta A + \Delta B) \\
&\qquad + \Delta AA_{do} + \Delta BB_oS_o
\end{aligned}$$

Let the new desired closed loop characteristic polynomial be A_d. (We will later discuss why A_d may not be the same as A_{do}). We then get the following Diophantine equation for ΔR and ΔS

$$\Delta R(A_{do} + A_{do}\Delta A + B_oS_o\Delta B) + \Delta SB_oS_o(1 + \Delta A + \Delta B)$$

$$= A_d - A_{do} - \Delta AA_{do} - \Delta BB_oS_o \qquad (5.9)$$

Solving this equation for ΔR and ΔS is of the same complexity as solving the full design problem as discussed above. Solution of polynomial equations such as (5.9) is discussed in Kucera (1979) and Jezel (1982). We can, however, make approximations to simplify the problem. If the nominal plant is a reasonable approximation of the full plant we may disregard the $(\Delta)^2$ terms on the left hand side of (5.9). This gives the following linear equation.

$$\mathcal{S}(A_{do},\ B_oS_o)\begin{bmatrix}\Delta R\\ \Delta S\end{bmatrix} = \mathcal{C}(A_d - A_{do} - \Delta AA_{do} - \Delta BB_oS_o)$$

where $\mathcal{S}(X,Y)$ is the Sylvester matrix of the polynomials X and Y. $\mathcal{C}(X)$ is a column vector of the coefficients of the polynomial X. The approximate solution is now given by

$$\begin{bmatrix}\Delta R\\ \Delta S\end{bmatrix} = \mathcal{S}^{-1}(A_{do},B_oS_o)\ \mathcal{C}(A_d - A_{do} - \Delta AA_{do} - \Delta BB_oS_o) \qquad (5.10)$$

Notice that the inverse of $\mathcal{S}(A_{do},B_oS_o)$ is independent of ΔA and ΔB and can thus be precomputed as soon as the nominal control problem is solved. The solution (5.10) is then obtained through a matrix vector multiplication.

The controller now has the form

$$u = -\frac{S_o(1 + \Delta S)}{R_o(1 + \Delta R)} y + \frac{T_o(1 + \Delta T)}{R_o(1 + \Delta R)} y_r \tag{5.11}$$

A block diagram for the controller is shown in Figure 5.2.

Remark 5.1

The same methodology can be used for the parameterization (3.1). The details are left for the reader.

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Remark 5.2

The reason for having different desired closed loop characteristic polynomials for the nominal and the perturbed systems is the order of the open loop systems. A_o and $A_o(1 + \Delta A)$ are of different orders. For further details on the choice of orders of the different polynomials in the controller we refer to Åström and Wittenmark (1984).

Remark 5.3

The polynomial $1 + \Delta T$ can be used to introduce new zeros in the closed loop system, see (5.8). It can also be used to influence the steady state gain of the closed loop system.

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The problem that we are solving with (5.10) is

$$A_o(R_o + \Delta R) + B_o(S_o + \Delta S) = A_d - \Delta AR_o - \Delta BS_o$$

i.e. a modification of the original problem. Using the controller defined by (5.10) gives the closed loop system

$$(A_o + \Delta A)(R_o + \Delta R) + (B_o + \Delta B)(S_o + \Delta S)$$
$$= A_d + \Delta A\Delta R + \Delta B\Delta S$$

i.e. the disregarded Δ^2 terms perturbs the desired closed loop characteristic polynomial.

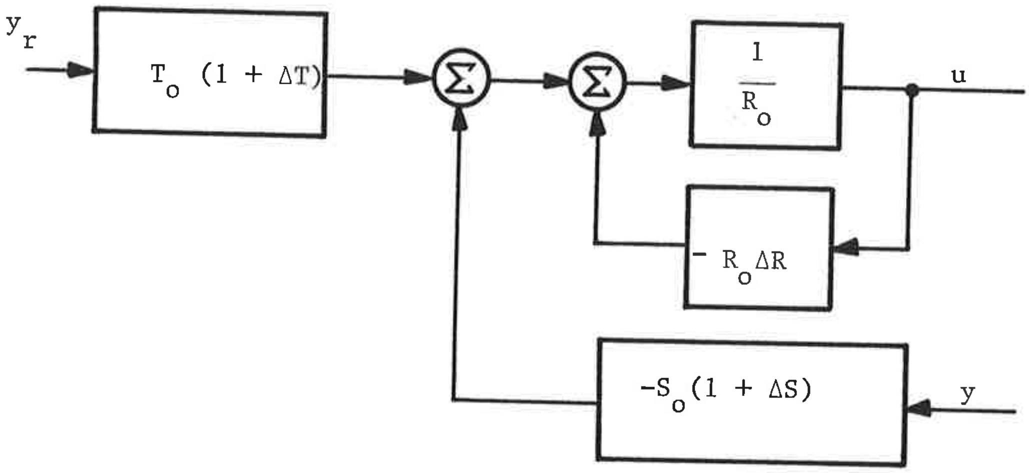

Figure 5.2 Block diagram of the controller (5.11)

The perturbed pole placement design can also be used for the coefficient disturbance dynamics (3.3). For the design of the controller we write the disturbed model as

$$H = \frac{B_o + \Delta B}{A_o + \Delta A} \tag{5.12}$$

The degree restriction (3.5) gives that $\deg(A_o + \Delta A) = \deg A_o$. The model (5.12) can be relevant to use if there are variations in some of the coefficients of the transfer function. One example may be when the damping of the system is changing. Let the controller be parameterized as

$$(R_o + \Delta R)u = -(S_o + \Delta S)y + (T_o + \Delta T)y_r \tag{5.13}$$

The closed loop system has the transfer function

$$\frac{(T_o + \Delta T)(B_o + \Delta B)}{(A_o + \Delta A)(R_o + \Delta R) + (B_o + \Delta B)(S_o + \Delta S)}$$

Simple calculations give the following equation for ΔR and ΔS

$$\Delta R(A_o + \Delta A) + \Delta S(B_o + \Delta B) = A_d - A_{do} - \Delta A R_o - \Delta B S_o$$

Disregarding the $(\Delta)^2$ terms on the left hand side we get a solution similar to (5.10)

$$\begin{bmatrix} \Delta R \\ \Delta S \end{bmatrix} = \mathcal{S}^{-1}(A_o,B_o)\, \mathcal{C}(A_d - A_{do} - \Delta A R_o - \Delta B S_o) \tag{5.14}$$

The inverse of the Sylvester matrix can also in this case be precomputed. The degree condition (3.5) implies that

$$\mathcal{S}(A_o + \Delta A,\ B_o + \Delta B) = \mathcal{S}(A_o,B_o) + \Delta\mathcal{S}$$

where $\Delta\mathcal{S}$ can be regarded as a disturbance depending on ΔA and ΔB. Assuming that $\Delta\mathcal{S}$ is small we can make the following approximation

$$(\mathcal{S}(A_o,B_o) + \Delta\mathcal{S})^{-1} \approx \mathcal{S}^{-1}(I - \Delta\mathcal{S}\mathcal{S}^{-1})$$

This gives a second approximation that can be used to calculate the controller

$$\begin{bmatrix} \Delta R \\ \Delta S \end{bmatrix} = \mathcal{S}^{-1}(A_o,B_o)(I - \Delta\mathcal{S}\mathcal{S}^{-1}(A_o,B_o))\mathcal{C}(A_d - A_{do} - \Delta AR_o - \Delta BS_o) \quad (5.15)$$

The inverse $\mathcal{S}^{-1}(A_o,B_o)$ can be precomputed, which implies that (5.15) can be implemented as three matrix vector multiplications. The computational burden is thus very moderate. The controller can be implemented analogous to Figure 5.2 with obvious modifications.

Summary of design methods

Different approaches to solve the problem of adaptive stability augmentation have been given above. The different methodologies have been examplified on some of the parametizations. The design methods can be classified into

- Generalized deadtime compensation (or the laissez-faire method) (5.3)
- Dynamic cancellation (5.6)
- Perturbed pole placement (5.10), (5.14) and (5.15)
- Redesign at each sampling interval

The solutions range from simple modifications of a nominal controller to the solution of the full design problem at each sampling interval. In the generalized deadtime compensation it is assumed that the dynamic perturbation can be allowed also in the closed loop dynamics. In (5.6) the extra dynamics is cancelled by the controller. The modifications (5.3) and (5.6) are of limited interest because of the restrictions that have to be imposed. The solutions of the type (5.10), (5.14) and (5.15) represent sensible approximations which require only a moderate amount of computations at each sampling instance. These controllers can be used to adaptively improve the closed loop performance when there is a priori knowledge about the process to be controlled.

6. EXAMPLES

The different methods presented in the previous section will now be illustrated through some examples. The system and the controller are assumed to be given in continuous time. Further it is assumed that the perturbation is known exactly, i.e. we focus on the properties of the different design principles.

Example 6.1

The nominal plant is assumed to be

$$H_o(s) = \frac{1}{(s+1)(s+2)} \tag{6.1}$$

In the first example we assume that the perturbation is only a variation in the gain. The perturbed plant is then

$$H = K\frac{B_o}{A_o} = \frac{B_o + (K-1)B_o}{A_o} = \frac{B_o + \beta B_o}{A_o} = \frac{B_o(1+\beta)}{A_o} \tag{6.2}$$

We may use dynamic cancellation since the perturbation is stable an minimum phase. This gives

$$\frac{S}{R} = \frac{S_o}{R_o}\left(1 + \frac{\Delta S}{1+\Delta R}\right) = \frac{S_o}{R_o} \cdot \frac{1}{1+\beta} \tag{6.3}$$

The closed loop system will be identical to the nominal or desired design for all values of β (or K). The perturbation thus does not need to be small. The perturbed pole placement method can also be used. (It is, however, not the best choice for the discussed perturbation.) Assume that $A_d = A_{do}$ then we should solve the equation

$$A_o \Delta R + B_o \Delta S = -\beta B_o S_o$$

Let $\Delta R = 0$, then $\Delta S = -\beta S_o$ or

$$\frac{S}{R} = \frac{S_o + \Delta S}{R_o} = \frac{S_o}{R_o} \cdot (1 - \beta) \qquad (6.4)$$

This controller is different from (6.3) because of the approximation the Δ^2 terms are omitted. Figure 6.1 shows the step responses when the nominal controller $S_o/R_o = 1/s$ is used and when (6.4) is used for $\beta = \pm 0.25$. Even if the perturbed pole placement mehtod is not the best one in this case it will maintain better control over a wider range than the nominal controller. The controller (6.4) makes the closed loop system stable for $-1 < \beta < 1$ while the nominal controller gives a stable closed loop system for $-1 < \beta < 5$. The nominal controller is thus in this case more robust (in the sense of stability) than the controller (6.4).

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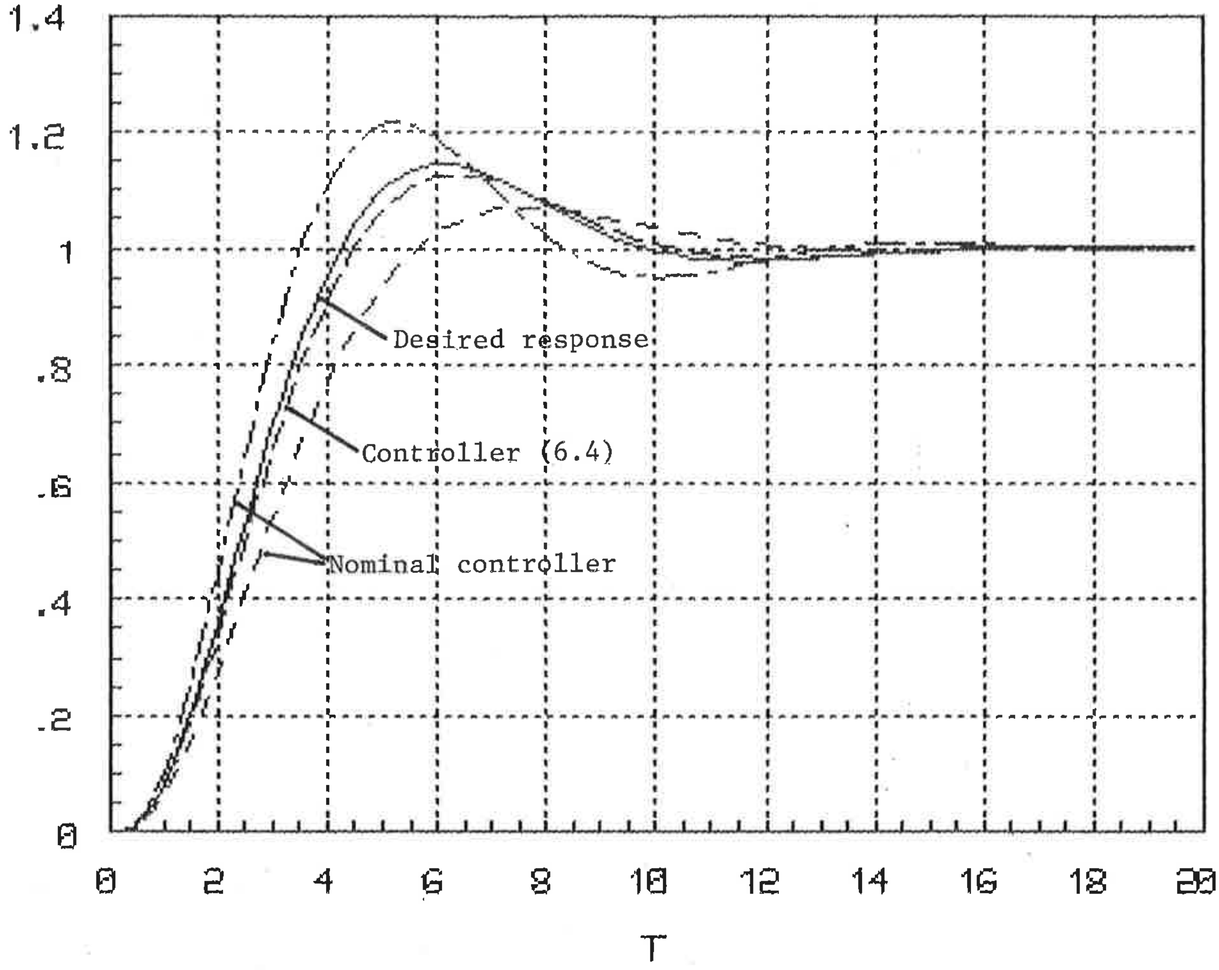

Figure 6.1 Step responses of the system (6.2) when the nominal controller $S_o/R_o = 1/s$ is used and when (6.4) is used for $\beta = \pm 0.25$.

Example 6.2

Assume now that the perturbed plant is

$$H = H_o \cdot \frac{B_1}{A_1} \tag{6.5}$$

where H_o is given be (6.1) and

$$\frac{B_1}{A_1} = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} \tag{6.6}$$

with $\omega = 1.5$ and $\zeta = 0.2$. We assume that (6.6) can be tolerated as a factor of the closed loop system. i.e. we can use the generalized deadtime compensation. Figure 6.2 shows the output and inputs when the nominal controller is used on the nominal and the perturbed plant and when (5.3) is used on the perturbed plant. The difference between the designs will decrease when ω is increased. ∇∇∇

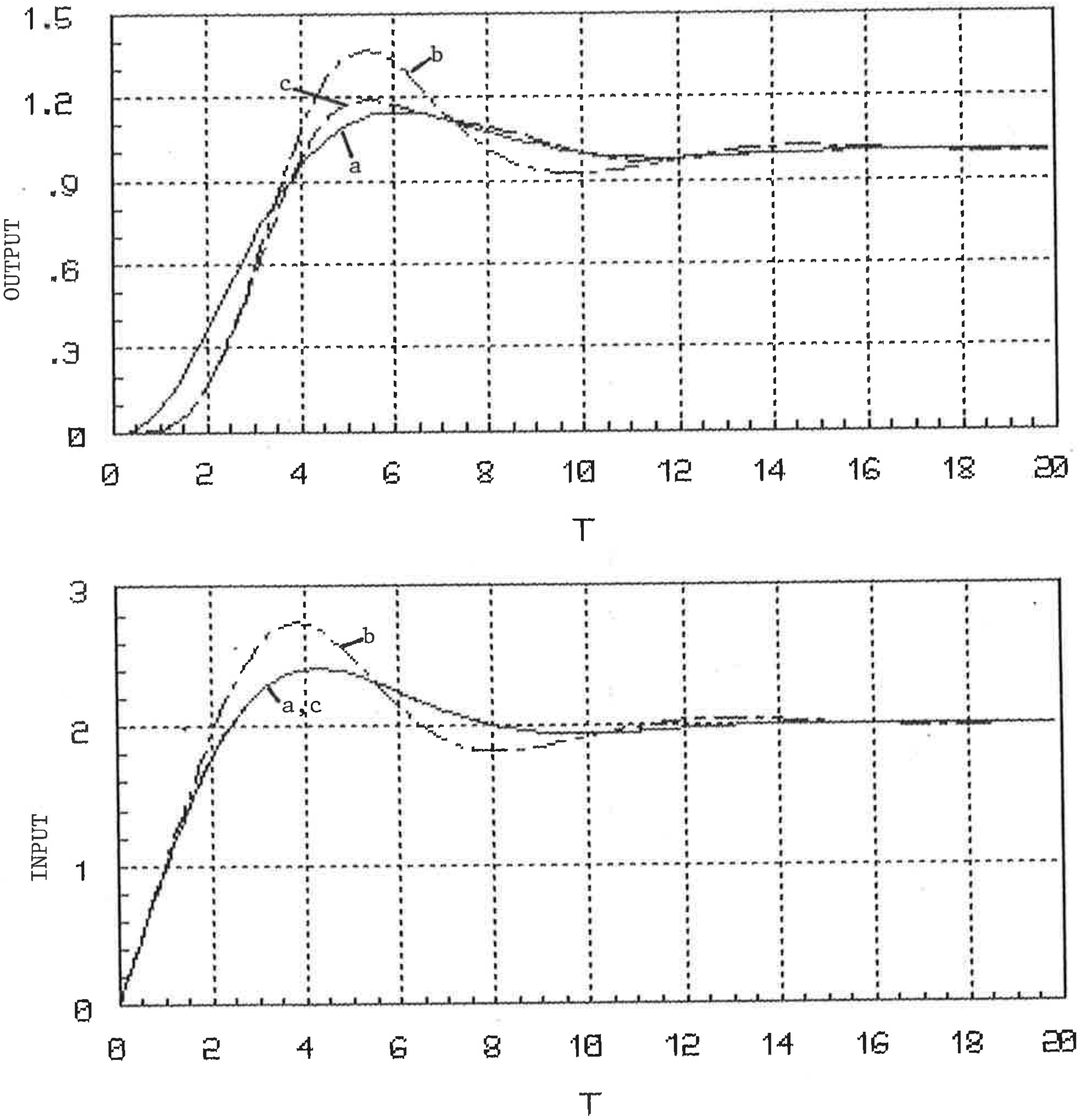

Figure 6.2 The outputs and inputs when $S_o/R_o = 1/s$ is used on a) the nominal plant (6.1) b) the perturbed plant (6.5) and c) (5.3) is used on the perturbed plant (6.5)

Example 6.3

Let the nominal model be

$$H_o(s) = \frac{1}{s(s+1)(s+2)}$$

The pole in -2 is assumed to be uncertain. The perturbed plant can be written

$$H(s) = \frac{1}{s(s+1)(s+2) + \alpha s(s+1)} \tag{6.7}$$

For $\alpha = 0$ we have the nominal plant. A pole placement controller for the nominal plant is derived such that the closed loop poles are $-1 \pm i$, $-2 \pm 2i$, -2 and -3. The observer poles are $-2 \pm 2i$ and -3. This gives the controller

$$\begin{aligned} R_o &= s^3 + 8s^2 + 28s + 50 \\ S_o &= 38s^2 + 124s + 96 \\ T_o &= 4s^3 + 28s^2 + 80s + 96 \end{aligned} \tag{6.8}$$

The perturbed poleplacement design can be used to compute ΔR and ΔS. To solve the full design problem we have to compute the inverse of $S + \Delta S$. Different approximations can be used instead by taking more terms in the series expansion

$$\begin{aligned} (S + \Delta S)^{-1} &= (I + S^{-1}\Delta S)^{-1}S^{-1} \\ &= (I - S^{-1}\Delta S + (S^{-1}\Delta S)^2 - (S^{-1}\Delta S)^3 + \ldots)S^{-1} \end{aligned} \tag{6.9}$$

Figure 6.3 and 6.4 show the step responses for different controllers.

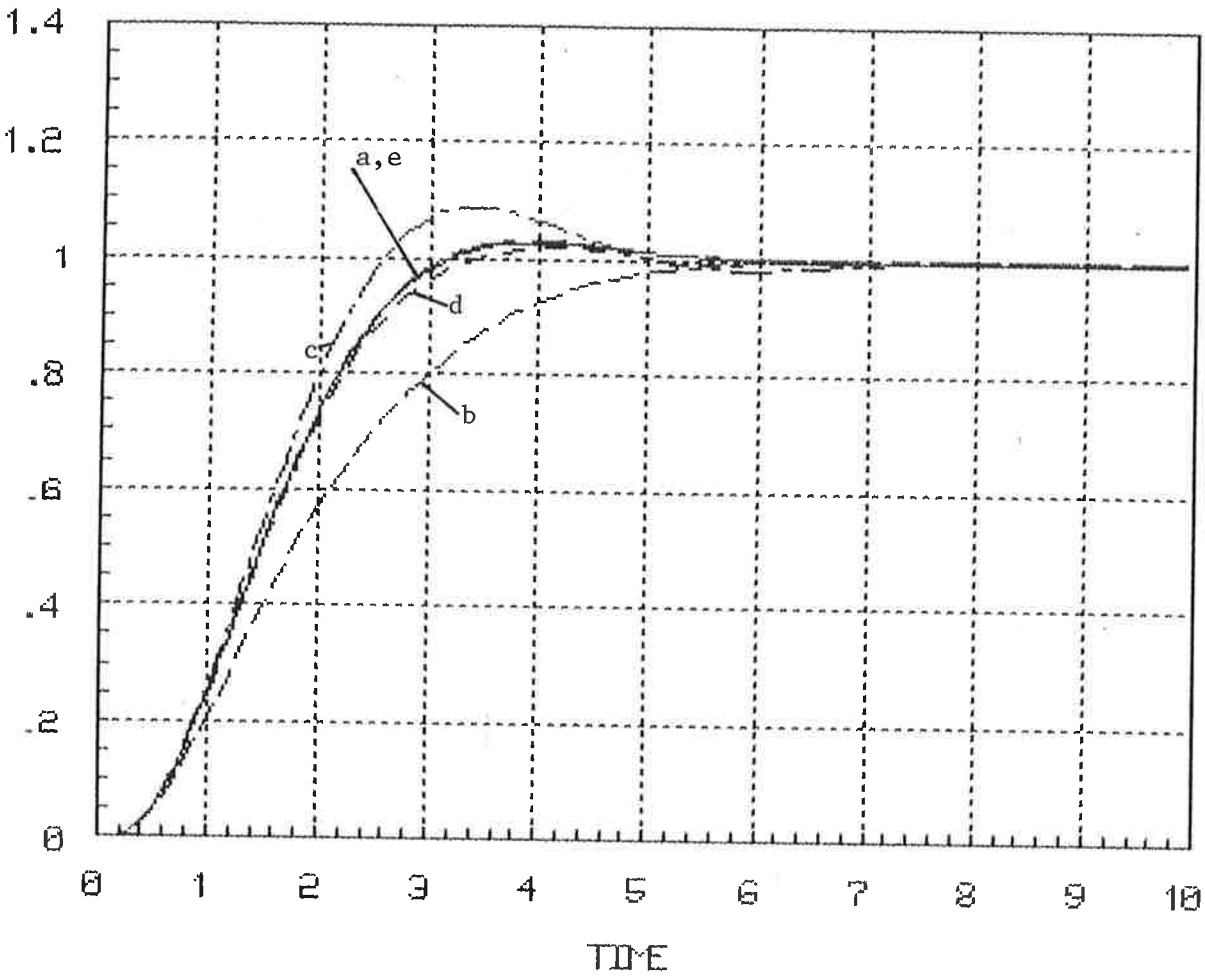

Figure 6.3 Step responses for the system (6.7) when $\alpha = 1$

a) Desired response

b) Nominal controller (6.8)

c) Perturbed poleplacement controller (5.3) with one term in (6.9)

d) Same as c but with two terms

e) Same as c but with three terms

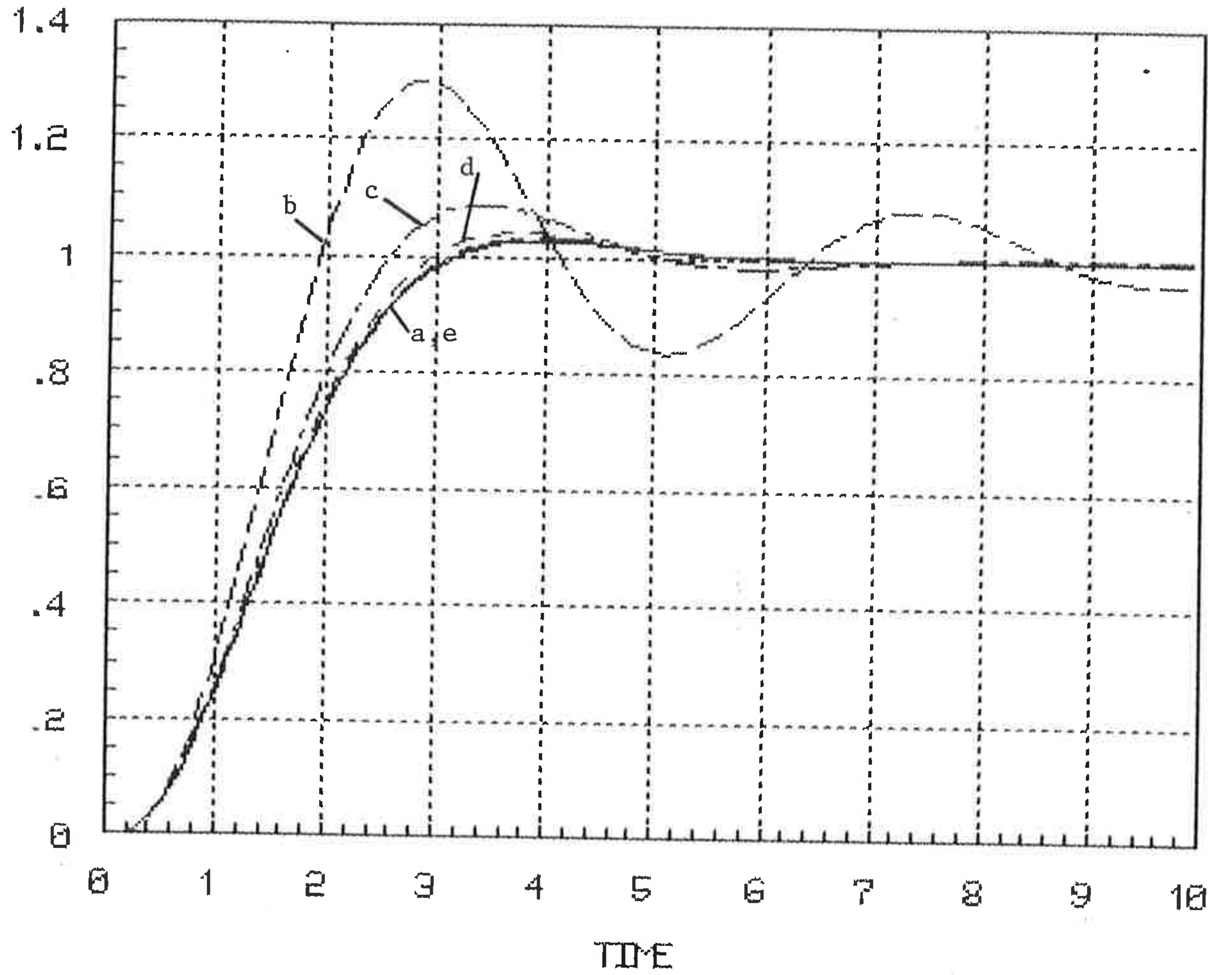

Figure 6.4 Same as Figure 6.3 but with $\alpha = -1$

The simulations show that we can get good response with the perturbed poleplacement method. The deviation from the desired response depends on ΔS and how many terms that are used in the series expansion (6.9).

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7. CONCLUSIONS

A priori knowledge about some of the dynamics of a plant is often available. Examples of this knowledge can be the number of integrators, dominating time constant, structure of the transfer function. Most adaptive controllers can't use this a priori knowledge. This paper discusses parameterizations of process models and different ways to use the a priori knowledge. It is shown how nominal controllers may be modified to compensate for disturbances in the process model. The aim has been to derive modifications that minimize the needed computations.

A fixed controller for the nominal plant can handle some deviations from the nominal plant. By making adaptive stability augmentation it is possible to improve the performance of the closed loop system. The suggested adaptive stability augmentation controllers consist of a constrained estimator which utilizes the a priori knowledge. Approximate modifications of the nominal controller is then done adaptively. This will increase the region for which the process can be controlled well compared with only using a nominal controller. The idea with adaptive stability augmentation can for some cases be regarded as an adaptive feedforward compensation.

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