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1978

Document Version:
Early version, also known as pre-print

Citation for published version (APA):
Wittenmark, B. (1978). *Self-tuning algorithms for control, prediction and smoothing*. Paper presented at Institute of Mathematical Statistics Meeting on Time Series Analysis.

Total number of authors:
1

March 30, 1978

The IMS Special Topics Meeting on Time Series Analysis
Ames, Iowa, May 1-3, 1978

SELF-TUNING ALGORITHMS FOR CONTROL, PREDICTION AND SMOOTHING

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ABSTRACT

The paper will give a survey of the basic ideas behind the self-tuning regulators. This is a class of adaptive regulators which can be used to control constant or slowly time-varying unknown systems. The regulator is using the certainty equivalence hypothesis to separate the estimation and the control. The estimation is done using the method of least squares and the controller can be a minimum variance controller or a pole placement controller. Theory and applications have shown that this simple control scheme has very attractive transient as well as asymptotic properties.

The idea behind the self-tuning regulator can be extended to make it possible to make self-tuning filters for prediction and smoothing of stationary time series. Some of the theoretical results showing the properties of the predictor and the smoother will be summarized.

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*The paper was written while the author was at the Department of Electrical Engineering, University of Connecticut, Storrs, CT. The author wants to acknowledge the support from the Department.

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1. INTRODUCTION

In order to design controllers for industrial processes it is often necessary to have a model of the process and the stochastic disturbances acting on the process. The dynamics of the process and the disturbances are, however, often unknown and have to be estimated. The estimation can be done off-line or on-line. The controller can then be based on the estimated parameters. The off-line methods can not be used if it is desirable to monitor time-varying parameters.

An adaptive controller is a combination of an on-line estimation method and a design of a controller based on the performence of the closed loop system. This paper will discuss a special class of adaptive algorithms which are designed to handle constant or slowly time-varying processes. These algorithms are called self-tuning algorithms, since they automaticly tunes the parameters in the controller. Self-tuning regulators are discussed in many papers. A survey of the theory and applications can be found in Åström, Borisson, Ljung and Wittenmark (1977). In this paper we will briefly discuss the idea behind self-tuning regulators and how this idea can be extended to make self-tuning predictors and self-tuning filters for fixed lag smoothing. Self-tuning regulators have with great success been used on several industrial processes. It has turned out that the self-tuning idea is easy to use in different contexts.

A more formal description of the problems is given in Section 2. The self-tuning idea is formulated in Section 3, where also estimation and tools for analysis are discussed. Sections 4, 5 and 6 show how the self-tuning idea can be used to solve control, prediction and smoothing problems. The main properties of the different algorithms are discussed. A summary is given in Section 7 and references are found in Section 8.

2. PROBLEM FORMULATION

The problems of control, prediction and smoothing have much in common. In this section we will formulate the problems and outline the solutions if the processes are known. In the following sections we will give a unified approach to the problems if the processes are unknown.

The models

It is assumed that the processes can be described as discrete time stochastic processes

$$A(q^{-1})y(t) = B(q^{-1})u(t-k-1) + C(q^{-1})e(t) \tag{2.1}$$

where y(t) is the output at time t, u(t) is the input, e(t) is a sequence of independent equally distributed random variables with zero mean and standard deviation σ. q is the forward shift operator, i.e. $qf(t)=f(t+1)$. $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are polynomials in the shift operator.

$$A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_nq^{-n}$$
$$B(q^{-1}) = b_0 + b_1q^{-1} + \dots + b_nq^{-n}$$
$$C(q^{-1}) = 1 + c_1q^{-1} + \dots + c_nq^{-n}$$

Without loss of generality it can be assumed that the C-polynomial has all zeroes inside the unit circle. It is also assumed that the system is minimum phase, i.e. $B(q^{-1})$ has all zeroes inside the unit circle. Eq (2.1) is a general input-output decription of a system corrupted by noise. A discussion of the model can be found in Åström (1970).

The control problem

There are many different ways to formulate the control problem. We will only discuss one possiblity. Assume that the objective of the control is to eliminate the disturbances acting on the system as well as possible. It is then appropiate to chose the control signal such that the lossfunction

$$V_1 = \lim_{N\to\infty} E\left\{ \frac{1}{N}\sum_{t=1}^{N}(y(t)^2 + \mu\, u(t)^2)\right\} \tag{2.2}$$

is minimized. The calculations to find the minimizing controller are well known, see for instance Åström (1970). The calculations require the solution of a steady state Riccati equation or a spectral factorization. The problem is simplified if $\mu=0$ and the process is minimum phase. The minimum variance controller is then obtained by using a simple polynomial identity, Åström (1970). The controller is given by

$$u(t) = -\frac{G(q^{-1})}{B(q^{-1})F(q^{-1})}\, y(t) \tag{2.3}$$

where G and F are polynomials of order n-1 and k respectively and given by the identity

$$C(q^{-1}) = A(q^{-1})F(q^{-1}) + q^{-k}G(q^{-1}) \tag{2.4}$$

<u>The prediction problem</u>

Assume that B=0 in eq (2.1). This means that the output, y(t), is a stochastic process which we can not influence. Based on measurements up to and including time t we want to predict the output at time t+k. The predicted value is denoted $\hat{y}(t+k|t)$. The predictor should minimize the mean square error, i.e. minimize the lossfunction

$$V_2 = E\{(\hat{y}(t+k|t) - y(t+k))^2 \mid y(t), y(t-1), \ldots\} \tag{2.5}$$

If the system is known the optimal predictor is given by

$$\hat{y}(t+k|t) = \frac{G(q^{-1})}{C(q^{-1})} y(t) = \frac{G(q^{-1})}{A(q^{-1})F(q^{-1})} (y(t) - \hat{y}(t|t-k)) \tag{2.6}$$

where G and F are given by the identity (2.4), see Åström (1970). Again we see that the problem is easy to solve if the process is known.

<u>The smoothing problem</u>

We will now look at the problem of finding the best smoothing estimate of a signal which only can be measured corrupted by white noise. Let the desired signal be described by

$$z(t) = \frac{C(q^{-1})}{A(q^{-1})} v(t) \tag{2.7}$$

and let the measurement be

$$y(t) = z(t) + e(t) \tag{2.8}$$

The noise processes e(t) and v(t) are independent white processes with zero mean. The process y(t) can be written in the form of (2.1) with B=0 using spectral factorization. The problem can now be formulated as to find the best estimate of z(t-k) in the sense of mean square error given y(t), y(t-1),... The estimate is denoted $\hat{z}(t-k|t)$ and minimizes the lossfunction

$$V_3 = E\{(z(t-k) - \hat{z}(t-k|t))^2 \mid y(t), y(t-1), \ldots\} \tag{2.9}$$

If the parameters in (2.7) and the noise characteristics of e and v are known it is quite easy to find the optimal fixed lag smoother, see Kailath and Frost (1968) and Hagander and Wittenmark (1977). The solution is based on finding the innovation representation of (2.7) and (2.8) and to find the one step ahead prediction of z(t).

3. THE SELF-TUNING IDEA

In the previous section we saw that the minimum variance controller, the mean square error predictor and the fixed lag smoother are easy to find if the processes are known. The problem becomes, however, more complicated if the process is unknown. There are many ways to solve for instance the adaptive control problem. Surveys of stochastic adaptive control methods can be found for instance in Wittenmark (1975) and Saridis (1977). The most straight forward way is to first estimate the parameters in the process on-line. The estimated parameters are then used as if they are the true ones in order to determine the control law. This is usually called the Certainty Equivalence (CE) hypothesis, see for instance Bar-Shalom and Tse (1976).

The idea behind the self-tuning algorithms is to use the CE hypothesis and to make the estimation in the simplest possible way. It is important that both the estimation and the determination of the control signal are done iteratively in real time. This idea was used for instance in Kalman (1958) to design adaptive dead-beat controllers. This simple approach works very well in many cases. One of the interesting features with this type of algorithms is that they often work in cases when they intuitively should not work well. The problem is thus to analyze the closed loop system which is a time-varying, non-linear, stochastic system. Typical problems are if the algorithm is stable and if it will converge to the same controller, predictor or smoother that could be obtained if the system was known.

The estimation of the parameters in the process can be done in several ways. The parameters in the A, B and C polynomials can be estimated in real time using for instance the Extended Least Squares (ELS) method or the Maximum Likelihood (ML) method, Goodwin and Payne (1977). If C=1 then it is possible to use the simpler Least Squares (LS) method. Using these methods it is possible to make an explicite estimation of the parameters in the process. It is, however, also possible in several cases to avoid the estimation of the process parameters and instead directly tune the parameters in the controller, predictor or smoother. This implicite way has many attractive properties and has been used in several self-tuning algorithms.

The analysis of the self-tuning algorithms has in most cases been done using a method by Ljung. The analysis of time-varying, nonlinear, stochastic difference equations can be transformed into analysis of a set of associated deterministic differential equations. The trajectories of the differential equations can be interpreted as the average trajectories of the stochastic difference equations. The convergence properties of the algorithms can be investigated by looking at the stationary points of the differential equations and their stability. For a summary of this very useful tool see Ljung (1977a) and Ljung (1977b).

4. SELF-TUNING REGULATORS

The minimum variance controller can be interpreted as a two step procedure. First the output of the process is estimated k+1 steps ahead. Second the control signal is chosen such that the predicted value is equal to zero (or some other desired value).

A predictor for the process (2.1) can be written as

$$\hat{y}(t+k+1) = -\mathcal{A}(q^{-1})y(t) + \mathcal{B}(q^{-1})u(t) \tag{4.1}$$

where

$$\mathcal{A}(q^{-1}) = \alpha_1 + \alpha_2 q^{-1} + \ldots + \alpha_p q^{-p+1}$$

$$\mathcal{B}(q^{-1}) = \beta_1 + \beta_2 q^{-1} + \ldots + \beta_r q^{-r+1}$$

The basic self-tuning regulator now consists of two steps which are repeated at each sampling interval:

Step 1
The parameters α_i and β_i in the model (4.1) are estimated using the method of least squares.
Step 2
The estimated parameters are used to determine the control signal

$$u(t) = \frac{\mathcal{A}(q^{-1})}{\mathcal{B}(q^{-1})} y(t) \tag{4.2}$$

The algorithm and its properties are further discussed in Åström and Wittenmark (1973), Wittenmark (1973) and Åström, Borisson, Ljung and Wittenmark (1977). Only some of its properties will be repeated here.

In Åström and Wittenmark (1973) the asymptotic properties of the algorithm are analyzed. It is shown that if the estimation converges and if the regulator has sufficiently many parameters then the self- tuning regulator will converge to the optimal minimum variance controller (2.3). The algorithm also has a stabilazing property in the sense that the input and the output will be bounded with probability one, see Ljung and Wittenmark (1976). Further the convergence of the algorithm has been analyzed in Ljung (1977b), where it is shown that the convergence of the algorithm depends on if a polynomial related to the C-polynomial is strictly positive real.

The basic self-tuning regulator can only be used for mimimum phase systems. One algorithm which can handle non-minimum phase systems is described in Åström and Wittenmark (1974). This algorithm is based on solution of a steady state Riccati equation. An other algorithm for non-minimum phase systems is described in Clarke and Gawthorp (1975).

The nice theoretical properties of the self-tuning regulators have made them very attractive for industrial use. Some industrial applications are reviewed in Åström, Borisson, Ljung and Wittenmark (1977). Most of the reported applications have been feasibility studies during quite limited times. The use of a self-tuning regulator in an autopilot for a tanker is an exception, Källström, Åström, Thorell, Eriksson and Sten (1978). This autopilot has now been used for several years on different tankers. Extensive comparisons with other controllers have been done under different weather and loading conditions.

All the applications have been very promising. It has been found that the self-tuning regulator is easy to use, insensitive for the choice of the parameters in the algorithm and it gives very good control.

5. SELF-TUNING PREDICTION

Prediction of time-series is an important problem in many technical and economic situations. Prediction of time-series is discussed for instance in Box and Jenkins (1970) and Brown (1963). A time-serie can often be divided into two parts. One part which is deterministic and represents trend and seasonal variations. The second part can be regarded as random disturbances around the deterministic part. The random part can often be modelled as eq (2.1) with B=0. The deterministic as well as the random parts are usually unknown and time-varying. This makes it interesting to consider self-tuning predictors which can adjust to the variations in the generating process.

Eq (2.6) shows two different ways to write the predictor if the process is known. There are several other ways to write the predictor which all are equivalent if the system is known. The different ways to write the predictor can be used to determine different self-tuning predictors. In Holst (1977) five different self-tuning predictors are discussed and their relationships are investigated. Several simulated examples are given which illustrate the behavior of the different algorithms.

One way to see the relationship between prediction and control is to start with the far right hand side of (2.6). The preciction error or the residual is defined as

$$\varepsilon(t+k) = y(t+k) - \hat{y}(t+k|t)$$

The prediction problem can now be regarded as a control problem. The predicted value $\hat{y}(t+k|t)$ can be regarded as the control signal. The prediction is now done such that the variance of the residuals is as small as possible, compare (2.5). The residual can thus be interpreted as the output. The self-tuning regulator can now be used directly to solve the prediction problem, Wittenmark (1974).

The self-tuning predictors have properties which are very similar to those of the self-tuning regulators. For instance if the estimation converges and if the predictor has sufficiently many parameters then it will converge to the optimal predictor (2.6).

It is also possible to use auxiliar variables to imporve the prediction. The auxiliar variable can be some other measurable variable, a ramp or a seasonal profile. The auxiliar varialbes can thus be used to include the deterministic part in the predictor.

This type of adaptive predictors have been used on real data for prediction of power load (Bohlin (1976) and Holst (1977)), urban sewer flows (Beck (1977)) and natural gas consumption (Hamza, Sheirah and Wittenmark (1978)).

6. SELF-TUNING FILTERS FOR FIXED LAG SMOOTHING

In many communication applications the recieved signals are corrupted by noise. The objective of the reciever is to filter out the noise as well as possible. The filtered signal at time t can then be a function of the measurements up to and including time t. Sometimes it is possible to make substantial improvements by making a smoothing estimate. The estimate at time t-k is then based on data up to and including time t. Fixed lag smoothing is useful for instance in communication but also in monitoring of the quality of a product or of the pollution in air or water.

The stationary fixed lag smoothing problem is related to prediction. In Hagander and Wittenmark (1977) it is shown that the smoother consists of two parts. First a predictor which predicts the signal $z(t)$ one step ahead. Second the smoothing estimate is given by a filter with two inputs, the measurement and the prediction error.

It is now possible to make a self-tuning smoother for the special case when the measurement noise in (2.8) is white noise. In this case it immediately follows that the one step ahead prediction of $z(t)$ is identical to the one step ahead prediction of $y(t)$. A self-tuning predictor can then be used to find $\hat{z}(t+1|t)$. The parameters in the predictor are simple functions of the parameters in the innovation model for $y(t)$, which is needed to determine the filter in the smoother. An estimate of the innovation model is found directly without making any spectral factorization. This means that the computations in each step are very moderate and the self-tuning smoother is well suited for real time applications.

The properties of the self-tuning smoother depends on the predictor. If the predictor converges then the self-tuning smoother will converge to the optimal smoother.

7. SUMMARY

In the previous sections we have seen that the self-tuning algorithms for control, prediction and smoothing are very related. The idea behind the self-tuning algorithms is simply to make an estimation of the unknown parameters and use them as if they are the true ones. In many cases it is not necessary to make an explicite estimation of the parameters in the process model. It is instead possible to directly estimate the parameters in the controller, predictor or smoother. This will simplify the computations that have to be done at each sampling time.

There are good tools available which make it possible to analyze the asymptotic as well as the transient properties of the algorithms. It is found that the algorithms have very good theoretical properties which also are verified in simulations. This makes the self-tuning algorithms well suited for applications on real processes.

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