Physical Bounds on Antennas with Feed Constraints

Gustafsson, Mats; Capek, Miloslav

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Mats Gustafsson* and Miloslav Capek†
*Department of Electrical and Information Technology, Lund University, 221 00 Lund, Sweden,
mats.gustafsson@eit.lth,se
†Department of Electromagnetic Field, Faculty of Electrical Engineering, Czech Technical University in Prague,
Prague, Czech Republic, miloslav.capek@fel.cvut.cz

***Abstract*—Antenna current optimization has been used to derive physical bounds on antenna parameters such as Q-factor, radiation efficiency, gain, directivity, capacity, and radiation patterns. The success of the methodology is partly due to the assumption of absolute control of the antenna current in the antenna region which in practice producing an array antenna with multiple feeds. Details of the feed such as input impedance and placement are however essential in antenna synthesis and there has so far been no successful approach to include these types of constraints. In this presentation, we illustrate how feed constraints can be included in current optimization and discuss their associated challenges.**

***Index Terms*—Antennas, numerical methods, eigenvalues and eigenfunctions, optimization methods, limitations.**

I. Introduction

The antenna current optimization method to derive physical bounds is versatile and can be used to compute bounds on combinations of parameters such as Q-factor, efficiency, directivity, and radiation patterns [1], [2], [3], [4], [5], [6], [7]. This success is partly due to the assumption of perfect control of the antenna current in the antenna region essentially producing an array antenna with multiple feeds. The details of the feed is however essential in antenna synthesis and hence it is desired to include the feed in the optimization approach.

In this presentation, we discuss how feed models can be added to the optimization problems and some challenges with their solution. The optimization problems in [1], [2], [3], [4], [5], [6] are either convex or quadratically constrained quadratic programs (QCQPs) with one to two constraints such that dual formulations can be used to solve them efficiently [8]. Unfortunately, addition of a feed adds $2N$ quadratic constraints, where N denotes the number of unknowns (current elements), making the optimization problems much harder to solve and calling to different relaxations.

II. Antenna current optimization

In order to illustrate current optimization, let us consider maximization of gain [4] $G(\hat{\boldsymbol{r}}) = 4\pi U(\hat{\boldsymbol{r}})/P_{\mathrm{acc}}$, with being $U(\hat{\boldsymbol{r}})$ the radiation intensity in the direction $\hat{\boldsymbol{r}}$ and P_{acc} being the accepted power by the antenna. Expressing the radiation intensity in the current $U(\hat{\boldsymbol{r}}) = \mathbf{I}^{\mathrm{H}}\mathbf{U}\mathbf{I}/2$ and the dissipated power $P_{\mathrm{d}} = \mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I}/2$ forms the current optimization problem

$$\begin{aligned} &\text{maximize} && \mathbf{I}^{\mathrm{H}}\mathbf{U}\mathbf{I} \\ &\text{subject to} && \mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I} = 2P_{\mathrm{d}}, \end{aligned} \tag{1}$$

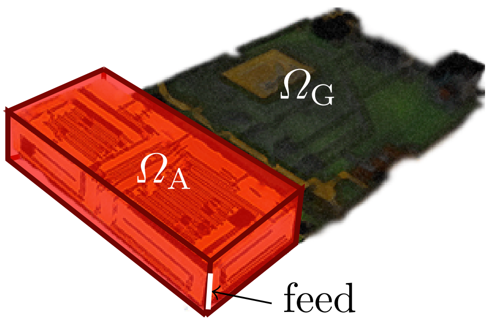

Figure 1. Example of a region $\Omega = \Omega_{\mathrm{A}} \cup \Omega_{\mathrm{G}}$ decomposed into an antenna design region Ω_{A} and a surrounding fixed region (ground plane) Ω_{G}. The desired feed point is indicted.

where the superscript $^{\mathrm{H}}$ denotes hermitian transpose. This problem is easily solved as an eigenvalue problem giving $G = 4\pi \max \operatorname{eig}(\mathbf{U},\mathbf{R})$ with $\mathbf{I}$ as the corresponding eigenvector. The power P_{d} can alternatively be interpreted as the accepted power $P_{\mathrm{d}} = P_{\mathrm{acc}} = \mathrm{Re}\{I_{\mathrm{in}}^{*}V_{\mathrm{in}}\}/2 = \mathrm{Re}\{Z_{\mathrm{in}}\}|I_{\mathrm{in}}|^2$ in feed port with input voltage V_{in}, current I_{in}, and impedance Z_{in}. Real valued input impedance $\mathrm{Im}\{Z_{\mathrm{in}}\} = X_{\mathrm{in}} = 0$ corresponds to self resonance which is expressed as $\mathbf{I}^{\mathrm{H}}\mathbf{X}\mathbf{I} = 0$ in the currents [9], [3], [4], [5] and easily included in (1) forming

$$\begin{aligned} &\text{maximize} && \mathbf{I}^{\mathrm{H}}\mathbf{U}\mathbf{I} \\ &\text{subject to} && \mathbf{I}^{\mathrm{H}}\mathbf{R}\mathbf{I} = 2P_{\mathrm{d}} \\ & && \mathbf{I}^{\mathrm{H}}\mathbf{X}\mathbf{I} = 0. \end{aligned} \tag{2}$$

The two problems (1) and (2) contain only information about the total accepted power of the port and whether the port is connected to a self resonant antenna or not. They contain no information about the position and input impedance of the port. Decomposition of the region into an antenna region Ω_{A} with the feed and a surrounding region Ω_{G}, see Fig. 1 add restrictions on the antenna performance but does not allow detailed information about position and input impedance of the feed.

III. Feed constraints

Multiplication of the current $\mathbf{I}$ solving (1) or (2) with the impedance matrix $\mathbf{Z} = \mathbf{R} + \mathrm{j}\mathbf{X}$ produces an equivalent excitation $\mathbf{V} = \mathbf{Z}\mathbf{I}$ for the optimal current. Current $\mathbf{I}$, voltage $\mathbf{V}$, and their elementwise (Hadamard) product $\mathbf{P} = \mathbf{I}^{*} \odot \mathbf{V}/2$ are in general supported everywhere in the allowed antenna region

Ω, where the superscript * denotes the complex conjugate. The product $\mathbf{P}$ can be interpreted from the power relation

$$\int_{\Omega} \boldsymbol{E}_{\text{in}}(\boldsymbol{r}) \cdot \boldsymbol{J}(\boldsymbol{r}) \,\text{dV} = \int_{\Omega} \boldsymbol{E}_{\text{in}}(\boldsymbol{r}) \cdot \sum_{n=1}^{N} I_n^* \boldsymbol{\psi}_n(\boldsymbol{r}) \,\text{dV}$$
$$= \sum_{n=1}^{N} I_n^* \int_{\Omega} \boldsymbol{E}_{\text{in}}(\boldsymbol{r}) \cdot \boldsymbol{\psi}_n(\boldsymbol{r}) \,\text{dV} = \sum_{n=1}^{N} I_n^* V_n, \quad (3)$$

where $\boldsymbol{\psi}_n(\boldsymbol{r})$ are local basis functions used to expand the current density and MoM matrices [9]. For an antenna region $\Omega_1 \subset \Omega$ fed with a local port in Ω_{f}, we have $\boldsymbol{J}(\boldsymbol{r}) = \boldsymbol{0}$ for $\boldsymbol{r} \in \Omega \setminus \Omega_1$ and $\boldsymbol{E}_{\text{in}}(\boldsymbol{r}) = \boldsymbol{0}$ for $\boldsymbol{r} \in \Omega_1 \setminus \Omega_{\text{f}}$. The corresponding product $\boldsymbol{J}^*(\boldsymbol{r}) \cdot \boldsymbol{E}_{\text{in}}(\boldsymbol{r}) \sim I_n^* V_n$ for some n vanishes hence for $\boldsymbol{r} \in \Omega \setminus \Omega_{\text{f}}$ and can be used to characterize antenna structures.

In this paper, we discuss optimization formulations to analyse optimal antennas with single- and multi-port feeds. The initial approach is based on adding constraints for the feed to the optimization problem (1). The antenna feed is modelled as a voltage gap at the feed position, see Fig. 1. We can add feed constraints to the optimization problems including feed voltage V_{in}, current I_{in}, or input impedance $Z_{\text{in}} = V_{\text{in}}/I_{\text{in}}$. The MoM impedance matrix $\mathbf{Z}$ for the region Ω can be used to model every substructure in Ω by removing rows and columns from $\mathbf{Z}$ [9], [10]. Removing columns in $\mathbf{Z}$ corresponds to zeros in the current $\mathbf{I}$ used in the matrix multiplication $\mathbf{ZI}$. Similarly, removing of rows is achieved by the Hadamard (elementwise) product with the complex conjugate of the current $\mathbf{I}^*$, *i.e.*,

$$\mathbf{I}^* \odot (\mathbf{ZI}) = \mathbf{I}^* \odot \mathbf{V}, \quad (4)$$

where the power terms $I_n^* V_n$ in (3) are recognized. Combining these N constraints with maximization of radiation intensity in (1) produces a QCQP of the form

$$\begin{aligned} &\text{maximize} && \mathbf{I}^{\text{H}}\mathbf{U}\mathbf{I} \\ &\text{subject to} && \mathbf{I}^* \odot (\mathbf{ZI}) = \mathbf{I}^* \odot \mathbf{V}. \end{aligned} \quad (5)$$

Similar problems are easily formed for efficiency, Q-factor, and other parameters by changing the object functional. Unfortunately, there is no known way to solve these type of QCQPs for large number of unknowns N but dual formulations, relaxations, and numerical techniques can produce valuable approximations of the solution [11], [8].

IV. Relaxed problems for transmission

The simplest relaxation of (5) is to sum the constraints producing

$$\sum_{n=1}^{N} \mathbf{I}^* \odot \mathbf{V} = \mathbf{I}^{\text{H}}\mathbf{V} = I_{\text{in}}^* V_{\text{in}} = Z_{\text{in}}|I_{\text{in}}|^2 = R_{\text{in}}|I_{\text{in}}|^2 \quad (6)$$

for a transmitting antenna with a single gap feed in element number f. The last equality in (6) is for self-resonant antennas with radiation resistance R_{in}. This modifies (4) to

$$\mathbf{I}^{\text{H}}\mathbf{R}\mathbf{I} = R_{\text{in}}|I_{\text{in}}|^2 = 2P_{\text{acc}} \quad \text{and } \mathbf{I}^{\text{H}}\mathbf{X}\mathbf{I} = 0 \quad (7)$$

where the conditions on dissipated power and self-resonance in (2) are recognized. The affine constraint formed by the f-th row of $\mathbf{Z}$

$$\mathbf{Z}_{f,.}\mathbf{I} = V_{\text{in}} = Z_{\text{in}} I_{\text{in}} \quad (8)$$

can also be added to (1) as the feed current cannot vanish. The resulting optimization problem can be written in many ways, *e.g.*,

$$\begin{aligned} &\text{maximize} && \mathbf{I}^{\text{H}}\mathbf{U}\mathbf{I} \\ &\text{subject to} && \mathbf{I}^{\text{H}}\mathbf{R}\mathbf{I} = 2R_{\text{in}}|I_{\text{in}}|^2 \\ &&& \mathbf{I}^{\text{H}}\mathbf{X}\mathbf{I} = 0 \\ &&& \mathbf{Z}_{f,.}\mathbf{I} = V_{\text{in}} = R_{\text{in}} I_{\text{in}} \end{aligned} \quad (9)$$

for a self-resonant antenna with radiation resistance R_{in} and given input current I_{in}. Although the added constraint in (9) tightens the bound from (2) it is still far from the bound in (5).

V. Relaxed problems for reception

Feed constraints can alternatively be analysed in reception *e.g.*, using that gain is related to the effective area from $A_{\text{eff}} = \lambda^2 G/(4\pi)$. Maximization of the received power in the port

$$\begin{aligned} &\text{maximize} && \mathbf{I}^{\text{H}}\mathbf{R}_{\text{in}}\mathbf{I} \\ &\text{subject to} && \mathbf{I}^{\text{H}}\mathbf{R}\mathbf{I} = \mathbf{I}^{\text{H}}\mathbf{V} \\ &&& \mathbf{I}^{\text{H}}\mathbf{X}\mathbf{I} = \mathbf{I}^{\text{H}}\mathbf{V}, \end{aligned} \quad (10)$$

where $\mathbf{R}_{\text{in}}$ is the matrix producing $\mathbf{I}^{\text{H}}\mathbf{R}_{\text{in}}\mathbf{I} = R_{\text{in}}|I_{\text{in}}|$ and $\mathbf{V}$ the matrix in (3) for an incident plane wave.

VI. Conclusion

The fundamental bounds with feed constraints form a new class of optimal currents. Since with additional constraints, they represent tighter bounds as compared to the optimal currents found before. In general, the problems with prescribed feeding are yet not solvable, however, the solvable subset is slowly increasing. Future research is focused on multi-port devices with both active and mutual impedances being prescribed. Final resolution of the optimization with port quantities may reduce the gap between optimal currents found with convex optimization and optimal antennas found with heuristic algorithms.

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