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SYSTEM MODELLING FOR CONTROL AND OPTIMIZATION

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1. INTRODUCTION

The purpose of this paper is to give a survey of the use of models and modelling techniques in the field of Automatic Control. Although the field is highly technical, an attempt has been made to emphasize ideas and concepts and avoid technicalities. The formulation of control and optimization problems is discussed in Section 2. Examples of models which are commonly used are given in Section 3. Section 4 deals with the modelling problems. The characteristic features of the problems occurring in control and optimization are discussed together with a formalization of the identification problem. Experiences from modelling in the engineering field are also summarized.

2. FORMULATION OF CONTROL PROBLEMS

To understand modelling problems in control and optimization it is useful to know the context in which the problems occur. For this purpose a crude description of a typical control problem will first be given. The problem is illustrated using the schematic diagram in Fig. 1. The figure shows a process which is perturbed by disturbances from its environment. The process can be influenced by control signals, which are also called inputs. Certain variables of the system, called outputs, can be measured. The control problem is to find a control law, or a decision rule, such that the system has the desired performance in spite of the disturbances. The control law is a function which gives the value of the control signal to be applied at time *t* based on measurements obtained up to time *t*.

It is clear that a solution of the control problem requires knowledge of the process and its environment. A mathematical model is a convenient and compact form of expressing such a knowledge. A substantial amount of theory is available which makes it possible to determine the control law if appropriate models are available. Modelling is thus important for the solution of the control problem as stated above.

Once a candidate for a control law is found, a model of the process may be conveniently used in order to verify that the controlled process has the desired performance. In this context models are often used in combination with physical equipment.

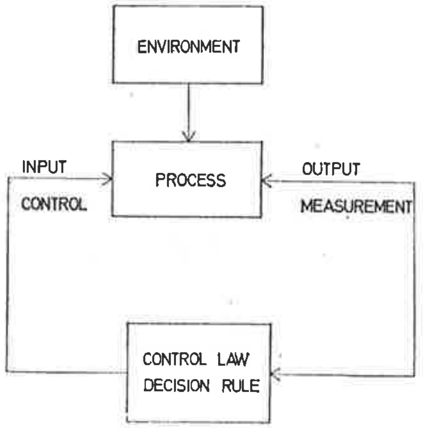

FIG. 1. Schematic diagram of a control system.

There are, however, also many other uses of models. Analysis and simulation of process models is frequently used to select control variables and measurements. Models are also used to obtain sufficient insight into the behaviour of a process to be able to make modifications so that the process will run more efficiently. Model studies are often inexpensive and convenient alternatives to pilot plant or full-scale experiments. Typical examples where models are used in these ways are design of flight-control systems and control systems for power stations.

Models are thus used in many different ways. The models used for different purposes vary significantly in complexity. It is an empirical fact that control laws, even for complex processes, can frequently be obtained from comparatively simple models. The final evaluation and testing of a control system often requires a more complicated model. For example, the design of an autopilot can be done by two models characterized by a few first-order, differential equations while the check out of such a system may be done on a simulation model having hundreds of first-order, differential equations.

The success of a control system design often depends on a significant skill and judgement in selecting models appropriate for different purposes. Too complex a model requires too much work, and too simple a model may give totally misleading results.

3. EXAMPLES OF MODELS

In this section a few examples are given of models frequently used in analyses.

Internal Descriptions

The following model has been extensively studied

$$\frac{dx(t)}{dt} = f(x(t), u(t), v(t))$$
$$y(t) = g(x(t), e(t)). \tag{1}$$

In the equations above $u(t)$ is a vector whose components represent the control variables (inputs), $v(t)$ is a vector of disturbances, which represents the influence of the environment on the process. The vector $x(t)$ is called the state vector. Its components represent the information about the past motion of the system that is necessary to describe its future motion. The measurement signals (outputs) are components of the vector $y(t)$, and $e(t)$ represents the measurement errors. The disturbance $v(t)$ and the measurement error $e(t)$ are frequently assumed to be random processes.

The inputs and the outputs are directly related to the control variables and the measured variables, which are all external to the system. The state variables are internal variables. They are not unique. A change of coordinates used when describing the system will, for instance, change the state variables and the form of the functions f and g.

Linear systems correspond to the special case of (1) when the functions f and g are linear, i.e.

$$f(x, u, v) = Fx + Gu + v \tag{2}$$
$$g(x, e) = Hx + e.$$

External Descriptions

A dynamical system can also be characterized by its input/output relation without reference to the internal variables. Linear systems can, for example, be characterized by their impulse response h, which corresponds to the input/output relation

$$y(t) = \int_{-\infty}^{t} h(t-s)u(s)\,ds. \tag{3}$$

Such systems can also be described by their transfer function G, which corresponds to the input/output relation

$$(\mathscr{L}y)(s) = G(s)(\mathscr{L}u)(s) \tag{4}$$

where $\mathscr{L}u$ denotes the Laplace transform of u.

For linear systems the relations between internal and external system descriptions are well understood. An interesting and important fact is the Kalman decomposition theorem (Kalman 1962), which implies that a linear system (1) with f and g given by (2) can be decomposed into four parts where the impulse response is uniquely determined by one part only (the completely reachable and completely observable part). This result is very important for the modeller because it means that an input/output experiment will only reflect the properties of the completely reachable and completely observable part of a system.

4. The Modelling Problem

The modelling problem is to find a model like (1) for a given process. Models for engineering processes can be obtained from first principles and process experiments. The fundamental laws expressing conservation of mass energy and momentum combined with material equations like Hooke's law and the general gas law will in some cases give the desired models. When performing experiments the process input is perturbed, and inputs and outputs are recorded. Figure 2 shows a typical result of an experiment. It is possible to obtain a model from the results of such an experiment without any reference to the physics of a process. For example, the impulse response of a process, which can be modelled by linear equations, is obtained simply by making the input a short pulse and recording the output. The value of the transfer function for a stable linear system can similarly be determined simply by choosing the input as a sinusoid and comparing the amplitudes and the phases of the input and the steady state output. Modelling based on input/output records alone, which is often called the *black box approach*, gives only an external description of the system.

Experience has shown that the black box approach often leads to surprisingly simple models, and that the method is easy to use in the sense that a model can be obtained with a reasonable effort. The black box approach does, however, suffer from serious disadvantages. *A priori* knowledge about the system is not used. With current practices it is applicable to linear systems only. The validity of the models is limited to the conditions of the experiment. A change in experimental conditions may easily lead to a different model.

The models based on physical laws, on the other hand, will have a wide range of validity. The models will frequently provide good insight into the behaviour of the system.

Modelling from physical laws also suffers from some disadvantages. The laws are not always available. The laws may have parameters whose values are not known. The procedure may be time consuming, and it is frequently difficult to make good approximations. Since many processes are described by distributed parameter processes, the lumping or aggregation problem is often present. The modelling of disturbances is frequently difficult to do based on physical laws only.

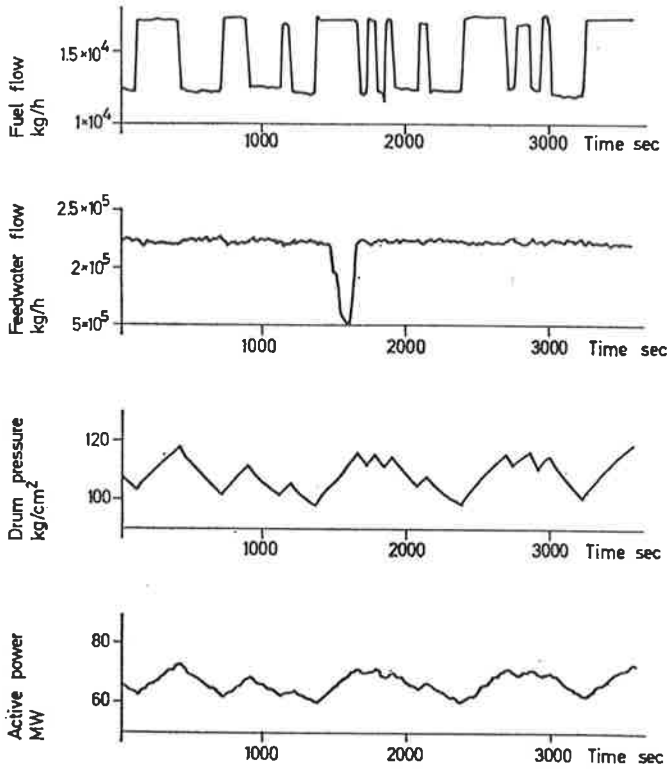

FIG. 2. Shows the result of an identification experiment. The purpose is to obtain a model for a thermal power station. The input, fuel flow, was manipulated and important process variables measured. (From Eklund, 1971.)

For the reasons given above most models used are obtained through a combination of physical laws and process experiments. It should be stressed that in this way it is possible to fully explore *a priori* information. With available process measurements the aggregation problem can often be solved. A purely statistical method for doing this is proposed by Akaike (1974).

The Identification Problem

The solution of the modelling problem is thus based on physical laws and process experiments. It is useful to formalize the modelling problem to enable us to make precise statements. The identification problem is such a formalization. It can be stated as follows:

"Given: a class of models (M), records of input/output signals from a process experiment taken under certain experimental conditions (X) and a criterion (C). Find: a model in the class which fits the experimental data as well as possible in the sense of the criterion."

The first formulation, solution and application of such a problem was given by Gauss (1809) in his famous determination of the orbit of the planet Ceres. Gauss formulated the principle of least squares as follows:

"Therefore, that will be the most probable system of values of the unknown quantities p, q, r, s, etc., in which the sum of the squares of the differences between the observed and the computed values of the functions V, V', V'', etc., is a minimum if the same degree of accuracy is to be presumed in all the observations."

The special case of the identification problem, when the models are linear and the input/output data is represented by a disturbance-free impulse response, is called the realization problem (Kalman, 1974). This problem is solved and algorithms are available.

There are many aspects of the identification problem which are not adequately understood. Parametrization of models is one example. For linear systems with one output there are no difficulties even in the stochastic case. A canonical representation of such systems is available. The parametrization problem is much more difficult for multivariable systems. For such systems there is in general no unique way to obtain a global parametrization. Even if it is possible to parametrize a class of systems in such a way that different parameter values do not give the same input/output relation, it may still not be possible to solve the identification problem because of the experimental conditions. Such cases may occur if the input is generated by feedback from the output. The experimental design is also important. Efforts have been made to select input signals which will give as accurate estimates as possible. This problem does, however, suffer from the disadvantage that a process model must be available in order to design an optimal input. The procedures can thus be used only in those cases where good *a priori* knowledge is available.

Statistical Problems

If the disturbances v and e in the model (1) are characterized as stochastic processes and if the models are characterized as a parametric class, the identification problem reduces to a parameter estimation problem. This can be exploited to select criteria or to give probabilistic interpretations of a given criterion. It is then natural to analyse the behaviour of the estimates as the length of the input/output records increases. A review of available results are given in Åström and Eykhoff (1971).

Identification Practice

Even if several important theoretical questions still remain unsolved in a general framework, the identification problem can frequently be solved in specific cases. This is now common in engineering practice. The criterion is often selected as an optimization criterion, which reduces the problem to an optimization problem. A significant development of algorithms and software has taken place. Interactive programmes which are run in conversational mode are, for instance, available. See, for example, Gustavsson *et al.* (1974).

It is, of course, difficult to summarize experience based on the solution of many specific problems. The following observations have, however, been made by many practitioners:

Determination of parameters of a nonlinear system like the one given by the equation (1) can be very time consuming. Availability of *a priori* knowledge can significantly reduce the work. The following remarks by Gauss are still valid:

> After the laws of planetary motion were discovered, the genius of Kepler was not without resources for deriving from observations the elements of motion of individual planets. Tycho Brahe, by whom practical astronomy had been carried to a degree of perfection before unknown, had observed all the planets through a long series of years with the greatest care, and with so much perseverance, that there remained to Kepler, the most worthy inheritor of such a repository, the trouble only of selecting what might seem suited to any special purpose. The mean motions of the planets already determined with great precision by means of very ancient observations diminished not a little this labour.
>
> Astronomers who, subsequently to Kepler, endeavoured to determine still more accurately the orbits of the planets with the aid of more recent or better observations, enjoyed the same or even greater facilities. For the problem was no longer to deduce elements wholly unknown, but only slightly to correct those already known, and to define them within narrower limits.

It is frequently very illuminating to solve an identification problem using one set of data and to analyse the performance of the model obtained using another set of input/output records.

5. CONCLUSION

Modelling is a standard practice in the field of automatic control. Models are used both for simulation and for design control strategies. They are obtained through a combination of physical models and process experiment. The modelling can be done in rather favourable circumstances:

the purpose of the model is well defined (design a control law with a specified criterion);

the processes considered are governed by physical laws;

experiments can be performed;

large amounts of data can be gathered and processed;

efficient algorithms and interactive software are available.

In particular, the possibility of performing experiments has in practice been shown to be extremely important in testing the validity of the models and in obtaining a suitable aggregation.

It is, of course, not easy to predict to what extent experiences of modelling in the engineering field can be transferred to global modelling. It is, however, clear that the global modeller is in a worse position than a control engineer, particularly with respect to the availability of physical laws and to the possibilities of making experiments.

I will quote from the work of the social scientist Myrdal (1967): "We do not have the constants which, for example, make it possible for a physicist to make fundamental discoveries at his desk through mathematical arguments based on a few experimentally verified universal laws. Neither can we make experiments."

I would, therefore, end by wishing my colleagues in the global modelling field the best of luck in their important and interesting work, but I would also like to introduce some caution by once more emphasizing the following line in the First Principle in Professor Moiseev's paper: "Only truly reliable data may constitute the basis of those decisions that mankind must now unavoidably take during the life time of our generation. A fundamental and careful analysis and selection of facts is needed rather than a wide dissemination of statements of good intention" (Moiseev, 1974).

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DISCUSSION

KALMAN: You have mentioned that the modelling problem in general is quite difficult. Could you say something about what problems are relatively easy and what problems are relatively difficult? What and where is the difficulty? Where are the unsolved problems?

ÅSTRÖM: One of the crucial problems is to find a suitable parametrization of a linear multivariable system. For systems of order higher than, say, ten there may also be substantial numerical difficulties in computing the estimates.

ULAM: My question was: How do you know in advance what is stable mathematically? The smallest change of parameters may produce very large changes in output after a while. After a time delay. What do you do? This is a mathematical study, isn't it? I remember discussions with Ferme during the war in Los Alamos, about the stability of the Gulf Stream flow, how stable is it?

ÅSTRÖM: For linear systems the stability can be investigated by computing the eigenvalues of the matrix F. For nonlinear problems there are, of course, no general methods available.

ULAM: Exactly. Most of these problems, I suppose, are nonlinear. In addition, the question is how well do the differential equations describe the physical system? One really has a system of partial differential equation—and one can introduce new variables through the technology of the near future.

ÅSTRÖM: When you have a regulation problem, i.e. it is attempted to maintain process variables close to a given equilibrium, linear models are often adequate.

BROCKETT: In answer to Prof. Ulam's question, a little bit more perhaps. The reason that the difficulties of the three-body problem don't come up is because they are assumed away, that is, we don't have the analogue of collisions. We don't have the delicate situation that the existence of periodic or almost-periodic solutions poses. Here, the kind of models that one can build all have a very strong kind of stability. If they don't have this property, then this modelling method probably won't work.