Errata

- In Lemma 3.5.1, the statement and the proof of (3.10)-(3.11) are not correct. Therefore, all results on oddness and evenness do not necessarily hold in Section 3.5, 3.6, 3.7. All the results in Sections 3.5, 3.6, 3.7 involving the holomorphicity of the functions and the mapping $w_\bullet$, for $\bullet = S, P, PS$ are still correct. In particular the main result of Sections 3.5 and 3.6 (Theorem 3.6.12) is still correct.

- Page 4, line 12: "examples are [6, 26, 32]. In [25] asymptotical values of resonances are obtained for the periodic Jacobi operator with finitely supported perturbations.".

- Page 9, line 5: "mathematical setting as [36]. In particular we use the same physical assumptions and we end up getting the same semiclassical Love and Rayleigh equations.".

- Page 5, line 2 from below: "in the working documents [17,18,55]....".

- Page 9, line 9: "the P-waves and S-waves.".

- Page 9, line 13: "is the S-waves,...".

- Page 9, line 16: "Both P- and S-waves have...".

- Page 10, line 11: "interference of P-waves and vertically polarized S-waves...".

- Page 12, line 13: ", for some $\delta > 0$, which physically means..."

- Page 13, line 4: "for v_L defined as in [36, Definition 3.1], but in the isotropic case $v_L(x, \xi, Z) = \sqrt{\hat{\mu}(x, Z)}$."

- Page 13, line 10: "to only **two** independent values...".

- Page 19, line 5 from below: "the density**; see Chapter 1**) as we do...".

- Page 20, line 4 from below: "We set the quasi-momentum...".

- Page 20, line 8: "which is based on the paper [3], but also [19], who did it for the Rayleigh system.".

- Page 21, line 20:
"$D = \{u \in H^2[0,+\infty) : Vu \in L^2[0,+\infty), u'(0) + hu(0) = 0\}$".
- Page 22, line 20: "We can see that for the Rayleigh problem in [19] are used the cuts at the same points of the complex plane for q_S, although the Riemann surface seemed defined in a different way leading to q_S being an odd function, while k is an even function here.".
- Page 31, line 15: " and $f(-k_n)(-1)^n < 0$.".
- Page 39, line 12: "Let $V \in L_{1,1}$. Then, $k = 0$ is not...".
- Page 48, line 2: "the following estimates are fulfilled **for $|\xi| \to \infty$**:".
- Page 48, line 4: "Additionally, if $V' \in L^1(0,\infty)$, **for $|\xi| \to \infty$**, then".
- Page 52, line 7 from below: "We are left to prove that J_h is surjective.".
- Page 53, line 6: "$k \in \mathbb{C}_-$,".
- Page 56, line 12: "A similar idea of two frequencies reconstruction can also be seen in [19] with respect to the Weyl matrix and Jost function matrix as in Proposition V.1 and Theorem V.1."
- Page 57, line 3: "Levitan-type equation...,".
- Page 59, line 17: "The Weyl function $M(k)$ maps..."
- Page 60, line 4 from below: "can **also** be written...".
- Page 63, line 4: "$-\left(f_h(k) + hf(0,k)\right) f(0,-k) =$...".
- Page 68, line 10: "The difference between $\boldsymbol{M^+(\lambda)}$ and $M^-(\lambda)$ is".
- Page 73, line 2 from below: "...= $\frac{i}{2\pi}\int_{-\infty}^{\infty} k' e^{ik'x} \boldsymbol{dk'}$".
- Page 82, line 20: "Condition I–IV allow us to define...".
- Page 89, line 5-7: "The sentence is not correct.".
- Page 95, line 16: "$b(\phi)\,(Z,\xi)$**:=**".
- Page 97, line 2 from below: "(**3.1**)-(3.2)".

- Page 100, line 6: "A variant of Lemma 3.5.1 and Lemma 3.5.3 can also be found in [17, 18] but without complete proofs and on a different Riemann surface.
- Page 107, line 5: "A variant of Lemma 3.6.3, 3.6.4, 3.6.8 and Theorem 3.6.12 can be found in [17, 18], but the results are not the same as the analysis is done on a different Riemann surface."
- Page 108, line 5: "From the **system** of equations (3.31)–(**3.32**), we can also...".
- Page 110, line 9: "A variant of Lemma 3.7.4 can be found in [17, 18].
- Page 112, line 5 from below: "T is **the** transpose operation,...".
- Page 127, line 15: "In particular, for $|\xi| \to \infty$ it satisfies the inequality".
- Page 140, line 3 from below: "Hence, summing $c^{(03)} + c^{(21)} + c^{(12)}$, we get $\boldsymbol{c}$".
- Page 148, line 3 from below: "$\det \mathcal{F}_\Theta(w_P(\xi))$ =...".
- Page 153, line 15: "estimate is fulfilled for $\operatorname{Re} \xi_n \to \infty$:".
- Page 165, line 2: "[55] A. Iantchenko, The inverse spectral problem for the sturm-liouville operator associated to elastic rayleigh surface waves, Working document, 9th March 2021."

Author's contribution

The monograph originates from work done under supervision of the former main supervisor A. Iantchenko. It contains parts that partially overlap with working material produced by A. Iantchenko and M. de Hoop (some version of which also includes the respondent's name among the list of authors) that was previously discussed by A. Iantchenko and the respondent during the period when A. Iantchenko was the main supervisor, as part of a then ongoing collaboration. Since January 2022, J. Wittsten is the main supervisor, and the respondent has during 2022 finished the work on the monograph under the supervision of J. Wittsten and co-supervisor M. Goffeng, independently of A. Iantchenko. After the printing of the respondent's thesis, which occurred on date 12th of September 2022, A. Iantchenko and M. de Hoop uploaded a paper on arXiv (https://arxiv.org/pdf/2209.09998.pdf) dated the 20th of September 2022 containing overlaps with some results of this chapter.

The monograph consists of a preface, three chapters and an appendix.

The Preface is the respondent's contribution and is entirely written by the respondent.

The Appendix collects some important definitions and theorems, whose sources are cited.

Chapter 1 refers to the physical background and the setup of the problems considered in this monograph. The material is entirely written by the respondent. This builds upon previous ideas as in [37]; sources are correspondingly cited.

Chapter 2 is entirely the respondent's contribution. The work is based on published methods (reference [3] (or similarly [19]), [22] and [30] in the thesis) applied to a novel problem. Sources are cited accordingly.

Chapter 3 is mainly the respondent's contribution, although there are overlaps with the material produced by the former supervisor A. Iantchenko and M. de Hoop. This overlapping material is the fruit of a previous collaboration, as explained above and in the thesis.

Below the sections of Chapter 3 are discussed one by one.

Section 3.1 is the introduction and it is entirely the respondent's contribution.

Section 3.2 are just the starting equation and classical definitions of Jost solutions.

In Section 3.3 some of the discussion and the notation are based on [17, 18], while the plots and the discussion about them (end of page 90, 91 and 92) are the respondent's contribution.

Section 3.4 is entirely the respondent's contribution and the setting differs com-

pletely from the [17, 18].

Section 3.5 is the respondent's contribution, although a variant of some results (Lemma 3.5.1 and Lemma 3.5.3) can also be found in [17, 18] but without complete proofs. The results in the thesis are not the same however, as the analysis is done on a different Riemann surface.

The representation of the Jost solution in Section 3.6, as well as part of the notation and some results of Section 3.7, is based on [17, 18], although the analysis in the thesis is not the same as the problem is studied using a different Riemann surface. Specifically, a variant of Lemma 3.6.3, 3.6.4, 3.6.8, Theorem 3.6.12 and Lemma 3.7.4 can be found in [17, 18], but the results are not the same as the analysis is done on a different Riemann surface.

Section 3.8 contains a change of variable technique which appears in the papers of V.M. Markushevich (reference [39], [40], [41] and [44]) and the same notation is employed as in the paper of A. Iantchenko and M. de Hoop (reference [19]), which is cited in the thesis. The rest of section 3.8 starting from page 120 is the respondent's contribution, although results similar to Theorem 3.8.3 and Proposition 3.8.13 can be found in [19], but there they are less general, on a different Riemann surface and without proof, with some details of proof appearing in [55].

Section 3.9 is entirely the respondent's contribution.