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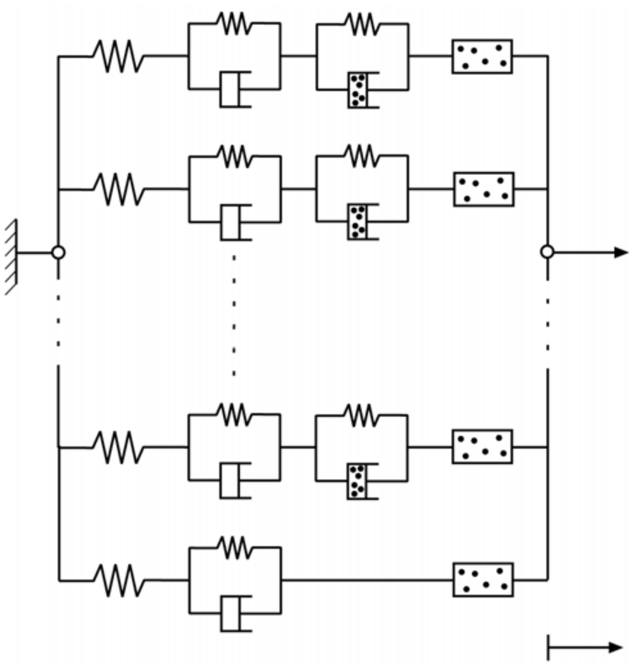

3D HOMOGENISATION OF HYGROSCOPIC ANISOTROPIC FIBRE NETWORK COMPOSITES

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Abstract

A 3D model of the behaviour of wood fibre composite materials exposed to loading and to changes in moisture content is presented. The model represents a homogenisation which, for a given fibre-orientation distribution predicts the time-dependent strain in a composite material from the properties of the fibre and matrix materials of which it consists. Homogenisation of the constituent materials is carried out under the assumption of homogenous strain, i.e. according to laminate theory. The constituents are modelled using a combination of spring, dashpot and expansion elements to simulate the viscoelastic, mechanosorptive and hygroexpansion parts of the strain. The resulting model is a differential equation having time-dependent coefficients. In a simplified case, the differential equation is solvable analytically, whereas for the complete model a numerical implementation is required. The model is investigated for different cases of moisture variation, loading and composite composition.

Keywords: Wood composites, analytical modelling, homogenisation, stiffness, viscoelasticity, mechanosorption, hygroexpansion, moisture.

INTRODUCTION

Wood-fibre-based composite materials have been used increasingly in the building industry, and to some extent in the automotive industry as well. One example of this is high pressure laminate, HPL, a material made up of layers of paper and impregnated with melamine and phenolic resin. Used in such applications as flooring and panelling, it is stiff, strong, durable and water resistant [2]. An important issue in the use of wood-based composite materials is their instability in shape in response to changes in climate, in terms both of their short- and long term performance. Accordingly, models are needed able to accurately predict the behaviour of wood-fibre composites when used in particularly demanding environments and for load carrying purposes.

General surveys of models of the time-dependent properties of composite materials are provided by Aboudi [1] and by Flügge [9]. Theoretical analyses of the linear viscoelastic behaviour of unidirectional transversal composite materials have been presented by Kaliske [13], Luciano [15], Barbero [4] and Klasztorny [14]. Studies have also been made of the hygrothermal behaviour of composites

[5]. Only few models of wood-fibre composites have been advanced, one made by Brauns [6]. A model of the elastic stiffness and hygroexpansion properties of wood-fibre composites has been presented earlier by the author [22]. Comprehensive investigations of the load- and moisture-induced straining of solid wood have been reported in doctoral dissertations of Mårtensson [17], Svensson [24] and Hanhijärvi [11]. These involve the measurement and modelling of strain in one dimension. More advanced constitutive modelling of solid wood taking account of three dimensional deformation has been performed by Ormarsson [19]. Micro-mechanical models of mechanosorption in paper have been developed by Alftan [3] for describing the phenomena shown in Figure 1. Any composite material modelling of the basic three dimensional behaviour of wood-fibre composites when exposed to load and to varying climatic conditions giving creep and mechanosorptive strain has not been found in available literature.

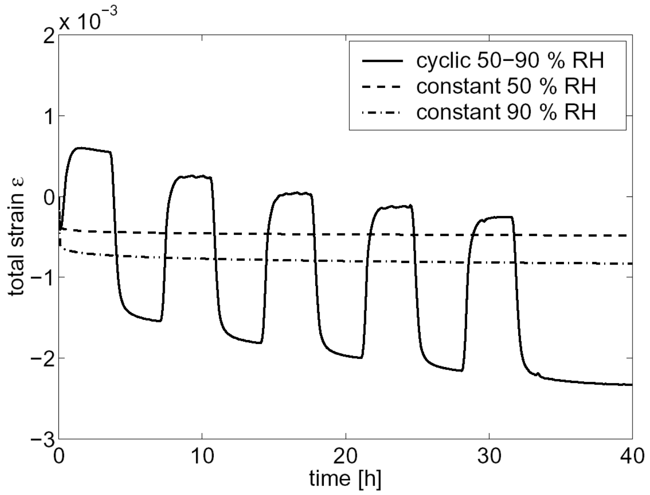

Figure 1: Example of mechanosorptive creep in paper material [8].

The aim of the model presented is to analytically predict the three-dimensional mechanical behaviour of a wood fibre network composite material under arbitrary loading and moisture conditions from the known mechanical properties of the composites constituents. Phenomenons taken into consideration by the model are elastic strain, creep, mechanosorption, hygroexpansion, a network consisting of anisotropic fibres with arbitrary orientation distribution function, proportions of the constituents and the possibility of porosity. The model is an analytical homogenisation resulting in a differential equation defining the constitutive properties of the composite material. This is achieved through rigorous calculations.

In order to make the analysis feasible a number of assumptions are required. Most important is that the homogenisation is carried out under the assumption of homogenous strain, or parallel coupling, which in composite mechanics is often referred to as laminate theory. The assumption, which gives an upper bound for the predicted stiffness of composite materials, has been shown to give a sufficiently accurate result in the case of laminated fibre network composites, such as HPL. A recent numerical study showed that the homogeneous strain assumption will overestimate the composite stiffness with approximately 15 percent for this material, using the same set of indata

[23]. The results obtained include constitutive equations for fibre network materials containing no matrix material, such as fibreboard, and for fibre composite materials.

The stress- and strain model of the constituent materials can be described by a system of springs and dashpots, the total strain being the sum of the contributions of the elastic, creep, mechanosorptive and hygroexpansion strains. It is assumed that the fibre material is orthotropic, that the matrix material neither chemically interacts with nor is absorbed by the fibre walls and that the fibre and matrix materials are completely bonded. It is likewise assumed that the matrix material is isotropic but that the composite material can be anisotropic, depending upon its fibre-orientation distribution. Porosity is taken into account by relating stress to the net cross-sectional area of the dense material. It is also assumed that the product analyzed is much larger than the micro-scale, where the homogenisation is performed, so that the composite can be regarded as a homogenous material. At the micro-scale the moisture gradient is neglected.

The homogenisation results in differential equations of varying order, depending upon the complexity assumed for the performance of the constituents. Various numerical examples presented below illustrate the performance of the composite material corresponding to particular assumptions. Numerical methods are generally required to assess the performance under a given set of conditions, although making certain material property assumptions allows analytical solutions to be found.

APPROACH OF THE MODEL

The homogenisation of the fibre network composite material is performed in three steps. First, the constitutive equations of the material in a single fibre and in the matrix material are developed in the form of two differential equations, one for each of the two constituents. Then the constitutive equation for the fibre network is calculated from the equation for the fibre material and the fibre-orientation distribution. Finally, the matrix material is taken into account in the equation for the composite material.

The choice of a constitutive model for a single fibre is based on the observed behaviour of wood and of wood-based materials and also on previously proposed models of wood materials [19, 24]. The strain increment is set equal to the sum of the elastic, creep, mechanosorptive and hygroexpansion strain increments, as shown in Figure 2. The creep part of the strain is modelled using a Kelvin element, namely a spring parallel-coupled to a dashpot. The spring prevents the fibre material from acting as a liquid as though no limits were placed on the creep strain. The mechanosorptive part is modelled by use of an element symbolically similar to the creep element, but with the strain rate in the dotted mechanosorptive dashpot made proportional to the stress and to the absolute value of the rate of change in moisture content.

The same choice of a constitutive model is employed for the matrix material, although the mechanosorptive part of the strain increment is disregarded, since no effects of that sort have been reported for thermosets or similar matrix materials like, such as for melamine-formaldehyde [10]. In addition,

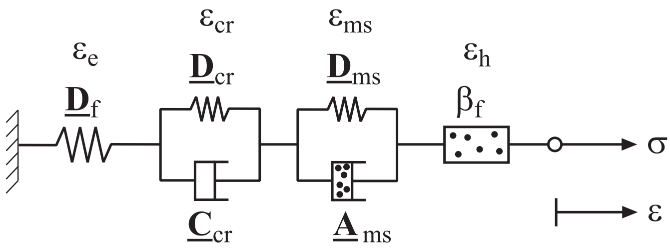

Figure 2: System of springs and dashpots describing the fibre material.

the elastic stiffness of the matrix material is assumed to be independent of the moisture content.

Certain further simplifications of the material model are necessary to make the analysis feasible. The major approximation made is to regard all time effects and moisture effects as scalar functions. This involves, firstly, all four matrices defining the creep and the mechanosorptive properties, $\mathbf{C}_{cr}, \mathbf{D}_{cr}, \mathbf{A}_{ms}$ and $\mathbf{D}_{ms}$, being assumed to be equal to the elastic stiffness matrix, $\mathbf{D}_f$, times a scalar function. Secondly, the moisture-dependent elastic stiffness matrix of the fibre material is set equal to a scalar function describing the moisture dependency times a matrix $\mathbf{D}_0$, defined as the elastic stiffness matrix at some reference level for moisture content. The same moisture dependency is assumed to be valid for the creep properties. The matrices that define the mechanosorptive material parameters are assumed to not be affected by the moisture content of the material.

The first step in development of the model from which the constitutive differential equations of the constituents are derived consists of summing strain-time derivatives of various orders involved in accordance with a scheme created for systematizing the procedure. In the second step, homogenisation from a single fibre to the fibre network is performed by integration over all the fibre directions. There are two basic alternatives for homogenizing composite materials, the one involves a homogenous strain assumption and the other a homogenous stress assumption. The homogenous strain assumption, or the parallel-coupling case, will be employed here. This means assuming that all the constituents of the composite are in the same state of strain, which although overestimating the stiffness of the composite material appears to provide a more accurate result than use of the homogenous stress assumption [21]. In the case of fibre network composite materials with long fibres bonded together the assumption is considered to give a fairly accurate homogenisation. Homogenisation is performed by integrating the fibre stresses contributing to the total stress. The fibre network equation obtained can be shown to be identical in structure with the equation describing a single fibre. This is an important result. It is based on the fact that the time effect functions chosen are scalar.

The last step in developing the model is to perform the homogenisation between the fibre network and the matrix material. This too involves making the homogenous strain assumption. The stresses and their derivatives are summed according to a scheme similar to the one described above.

The resulting constitutive equation for the composite material is a fourth-order differential equation for the strain vector, having nonlinear, matrix-valued coefficients, which in the general case are comprehensive functions of such factors as the moisture content and the material parameters.

A differential equation of lower order is obtained by making the constituents models simpler, e.g. by setting the creep spring stiffness to zero, lowering the order of the final equation by one. When modelling creep using one Kelvin element the indata, the constituents creep parameters, have to be chosen so that they give a correct description at the time scale of interest. The creep behaviour can also be described by adding further Kelvin elements in series. This increases the order of the constitutive equation by one per Kelvin element added. This way the model can be made valid at more than one time scale.

The performance of the model will be illustrated by a series of numerical examples. An analytical solution found for a simple case will be compared with the corresponding numerical solution. The numerical examples given concern predictions of the strain history of a composite under differing conditions of load and of moisture history.

THE CONSTITUTIVE EQUATION FOR A FIBRE MATERIAL

Strain Increment Components

The total strain increment of a single fibre is modelled as the sum of the increments of the elastic, creep, mechanosorptive and hygroexpansion strain contributions:

$$\underline{\dot{\boldsymbol{\varepsilon}}} = \underline{\dot{\boldsymbol{\varepsilon}}}_e + \underline{\dot{\boldsymbol{\varepsilon}}}_{cr} + \underline{\dot{\boldsymbol{\varepsilon}}}_h + \underline{\dot{\boldsymbol{\varepsilon}}}_{ms} \tag{1}$$

where $\underline{\dot{\boldsymbol{\varepsilon}}} = [\dot{\varepsilon}_x \quad \dot{\varepsilon}_y \quad \dot{\varepsilon}_z \quad \dot{\varepsilon}_{xy} \quad \dot{\varepsilon}_{xz} \quad \dot{\varepsilon}_{yz}]^T$ is the column vector containing the time derivative of all the strain components. The shear strains are defined as half the total shear angle, i.e. $\varepsilon_{xy} = (u_{x,y} + u_{y,x})/2 = \gamma_{xy}/2$, etc. Henceforth, the underlining of a matrix or a column vector and printing of the coordinate indices (x, y, z) in lower case letters will be used to indicate that the matrix or vector in question is to be evaluated in terms of the local fibre-coordinate system. Matrices and vectors appear in the global coordinate system without underlining and with the coordinate indices capitalized (X, Y, Z).

The elastic strain increment is defined in terms of the elastic stiffness matrix $\mathbf{D}_f(w)$, which is assumed to be orthotropic,

$$\underline{\dot{\boldsymbol{\varepsilon}}}_e = (\underline{\mathbf{D}}_f(w))^{-1} \underline{\dot{\boldsymbol{\sigma}}}_f \tag{2}$$

where $\underline{\dot{\boldsymbol{\sigma}}}_f = [\dot{\sigma}_x \;\; \dot{\sigma}_y \;\; \dot{\sigma}_z \;\; \dot{\sigma}_{xy} \;\; \dot{\sigma}_{xz} \;\; \dot{\sigma}_{yz}]^T$ is the column vector containing the time derivatives of all the stress components in a fibre. The stiffness matrix $\underline{\mathbf{D}}_f(w)$ can vary with the moisture content w.

The creep strain increment is modelled by use of a parallel-coupled spring and a dashpot, commonly designated as a Kelvin element. The stiffness matrix of the spring is denoted as $\underline{\mathbf{D}}_{cr}(w)$ and the creep viscosity matrix as $\underline{\mathbf{C}}_{cr}(w)$, defined by $\underline{\dot{\boldsymbol{\varepsilon}}}_{cr} = \underline{\mathbf{C}}_{cr} \, \underline{\boldsymbol{\sigma}}_{cr}$. This enables the constitutive equation of the Kelvin element to be written:

$$\underline{\boldsymbol{\sigma}}_f = \underline{\mathbf{C}}_{cr}^{-1} \underline{\dot{\boldsymbol{\varepsilon}}}_{cr} + \underline{\mathbf{D}}_{cr} \underline{\boldsymbol{\varepsilon}}_{cr} \tag{3}$$

The mechanosorptive strain increment is modelled as an element consisting of an elastic spring of stiffness $\mathbf{D}_{ms}$, coupled in parallel with a moisture-rate dashpot, the resulting strain rate being proportional both to the stress and proportional to the absolute value of the rate of change in moisture content, $\mathbf{A}_{ms}|\dot{w}|$, which yields

$$\underline{\boldsymbol{\sigma}}_f = (\underline{\mathbf{A}}_{ms}|\dot{w}|)^{-1}\dot{\underline{\boldsymbol{\varepsilon}}}_{ms} + \underline{\mathbf{D}}_{ms}\underline{\boldsymbol{\varepsilon}}_{ms} \tag{4}$$

The material parameter matrices $\underline{\mathbf{A}}_{ms}$ and $\underline{\mathbf{D}}_{ms}$ are not considered to be moisture dependent.

The hygroexpansion strain increment is denoted by a moisture-dependent hygroexpansion coefficient vector $\underline{\boldsymbol{\beta}}_f(w)$, which yields a strain increment of

$$\dot{\underline{\boldsymbol{\varepsilon}}}_h = \underline{\boldsymbol{\beta}}_f(w)\;\dot{w} \tag{5}$$

Simplifications by Means of Material Parameter Coupling

In the general case, the components of the matrices $\underline{\mathbf{D}}_f$, $\underline{\mathbf{C}}_{cr}$, $\underline{\mathbf{D}}_{cr}$, $\underline{\mathbf{A}}_{ms}$, $\underline{\mathbf{D}}_{ms}$ and of the vector $\underline{\boldsymbol{\beta}}_f$, which define the properties of the material, are all independent functions of the moisture content w, which in turn is a function of time, such that $w = w(t)$. Two simplifications are introduced here. The first is to define the influence of w on the material parameters by means of two scalar dimensionless functions, $p(w)$ and $q(w)$, only. The second simplification is to assume that the material-parameter matrices $\underline{\mathbf{D}}_f$, $\underline{\mathbf{C}}_{cr}$, $\underline{\mathbf{D}}_{cr}$, $\underline{\mathbf{A}}_{ms}$ and $\underline{\mathbf{D}}_{ms}$ are coupled with each other by means of four scalar constants, τ_f, μ_f, α_f and λ_f.

According to the first simplification the elastic stiffness is

$$\mathbf{D}_f(w) = p(w)\mathbf{D}_0 \tag{6}$$

and the hygroexpansion coefficient vector is

$$\underline{\boldsymbol{\beta}}_f(w) = q(w)\underline{\boldsymbol{\beta}}_0 \tag{7}$$

where

$$\underline{\mathbf{D}}_0 = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{yz}} \end{bmatrix}^{-1} \tag{8}$$

and

$$\underline{\boldsymbol{\beta}}_0 = [\beta_x \;\; \beta_y \;\; \beta_z \;\; 0 \;\; 0 \;\; 0]^T \tag{9}$$

the components of $\underline{\mathbf{D}}_0$ and $\underline{\boldsymbol{\beta}}_0$ being evaluated at the initial moisture content $w_0 = w(t_0)$. The moisture functions $p(w)$ and $q(w)$ are assumed to be independent and continuous. Equations (6) and (7) state that a change in moisture content over time influences the stiffness and the hygroexpansion properties to the same extent in all directions.

The second simplification results in

$$\underline{\mathbf{C}}_{cr} = \frac{1}{\tau_f}\underline{\mathbf{D}}_f^{-1} \tag{10}$$

$$\underline{\mathbf{D}}_{cr} = \frac{1}{\mu_f}\underline{\mathbf{D}}_f \tag{11}$$

$$\underline{\mathbf{A}}_{ms} = \frac{1}{\alpha_f}\underline{\mathbf{D}}_0^{-1} = \frac{p(w)}{\alpha_f}\underline{\mathbf{D}}_f^{-1} \tag{12}$$

$$\underline{\mathbf{D}}_{ms} = \frac{1}{\lambda_f}\underline{\mathbf{D}}_0 = \frac{1}{\lambda_f p(w)}\underline{\mathbf{D}}_f = \frac{1}{\lambda_f \alpha_f}\underline{\mathbf{A}}_{ms}^{-1} \tag{13}$$

The constants introduced, τ_f, μ_f, α_f and λ_f, are independent of w and thus independent of t as well. The constant τ_f has the dimension time, μ_f and λ_f are dimensionless and α_f has the same dimension as $[w]$, i.e. dimensionless, e.g. kg/kg. From this it follows that the matrix products

$$\underline{\mathbf{C}}_{cr}\underline{\mathbf{D}}_{cr} = \frac{1}{\mu_f \tau_f}\mathbf{I} \tag{14}$$

$$\underline{\mathbf{A}}_{ms}\underline{\mathbf{D}}_{ms} = \frac{1}{\lambda_f \alpha_f}\mathbf{I} \tag{15}$$

are both independent of time.

These simplifications are needed not only to make the calculations feasible, but also to keep the number of material parameters at a minimum, important since each of the parameters needs to be determined by measurement or by estimation. The simplifications made represent reasonable appearing assumptions in engineering terms.

Differential Equation for the Material in a Single Fibre

The equations (2), (3), (4) and (5) are written as a single constitutive differential equation expressed in terms of the total fibre strain $\underline{\boldsymbol{\varepsilon}}$ and the total fibre stress $\underline{\boldsymbol{\sigma}}_f$. This is achieved by adding the strains and their derivatives in accordance with equation (1), which holds for all levels of the derivatives, and by derivation of both sides of equations (2), (3), (4) and (5) up to the third-order derivative. In terms of the constants introduced, for example, equation (4) can be written as

$$\dot{\underline{\boldsymbol{\varepsilon}}}_{ms} + \frac{1}{\lambda_f \alpha_f}|\dot{w}|\underline{\boldsymbol{\varepsilon}}_{ms} = \underline{\mathbf{D}}_f^{-1}\frac{1}{\alpha_f}|\dot{w}|\underline{\boldsymbol{\sigma}}_f \tag{16}$$

Derivation of both sides of the equation yields

$$\ddot{\underline{\boldsymbol{\varepsilon}}}_{ms} + \frac{1}{\lambda_f \alpha_f}|\dot{w}|\dot{\underline{\boldsymbol{\varepsilon}}}_{ms} + \frac{1}{\lambda_f \alpha_f}\dot{\overline{|\dot{w}|}}\underline{\boldsymbol{\varepsilon}}_{ms} = \underline{\mathbf{D}}_f^{-1}\frac{1}{\alpha_f}\left(|\dot{w}|\dot{\underline{\boldsymbol{\sigma}}}_f + (\dot{\overline{|\dot{w}|}} - |\dot{w}|\frac{\dot{p}}{p})\underline{\boldsymbol{\sigma}}_f\right) \tag{17}$$

and by derivation once more,

$$
\begin{gathered}
\underline{\boldsymbol{\varepsilon}}_{ms}^{(3)} + \tfrac{1}{\lambda_f \alpha_f} |\dot{w}| \underline{\ddot{\boldsymbol{\varepsilon}}}_{ms} + 2 \tfrac{1}{\lambda_f \alpha_f} \dot{\overline{|\dot{w}|}} \underline{\dot{\boldsymbol{\varepsilon}}}_{ms} + \tfrac{1}{\lambda_f \alpha_f} \ddot{\overline{|\dot{w}|}} \underline{\boldsymbol{\varepsilon}}_{ms} = \\
\underline{\mathbf{D}}_f^{-1} \tfrac{1}{\alpha_f} \left(|\dot{w}| \underline{\ddot{\boldsymbol{\sigma}}}_f + 2 \left(\dot{\overline{|\dot{w}|}} - |\dot{w}| \tfrac{\dot{p}}{p} \right) \underline{\dot{\boldsymbol{\sigma}}}_f + \left(\ddot{\overline{|\dot{w}|}} - 2 \dot{\overline{|\dot{w}|}} \tfrac{\dot{p}}{p} - |\dot{w}| (\tfrac{\ddot{p}}{p} - 2 (\tfrac{\dot{p}}{p})^2) \right) \underline{\boldsymbol{\sigma}}_f \right)
\end{gathered}
\tag{18}
$$

since

$$
\underline{\dot{\mathbf{D}}}_f = \frac{\dot{p}}{p} \underline{\mathbf{D}}_f \quad \texttt{and} \quad \dot{\overline{\underline{\mathbf{D}}_f^{-1}}} = -\frac{\dot{p}}{p} \underline{\mathbf{D}}_f^{-1}
\tag{19}
$$

The equations and the derivatives of these for the different strains that equations (16) to (18) represent can be combined into the system of equations shown in equation (20), where the zero- to the third-order derivatives of the strains which are to be eliminated are collected in a vector having, as sought, the third-order derivative of the total strain as its the first element. The coefficients of the strains in the equations are collected in a matrix having scalar coefficients. The elements of the vector contain all six components of each of the strains. Equations (18), (17) and (16) appear here in rows 4, 8 and 11, respectively, of the equation system. The elastic equations are found in rows 2, 6 and 10, the equations for the creep element in rows 3 and 7 and the equations for the hygroexpansion in rows 5, 9 and 12. The first row corresponds to the sum of the third derivatives of the strains $\underline{\boldsymbol{\varepsilon}}^{(3)} = \underline{\boldsymbol{\varepsilon}}_e^{(3)} + \underline{\boldsymbol{\varepsilon}}_{cr}^{(3)} + \underline{\boldsymbol{\varepsilon}}_{ms}^{(3)} + \underline{\boldsymbol{\varepsilon}}_h^{(3)}$. The sum of the second derivatives is found in row 14, equation (1) being found in row 13.

$$
\begin{bmatrix}
1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{\mu_f \tau_f} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{\lambda_f \alpha_f} |\dot{w}| & 0 & 0 & 0 & 2 \frac{1}{\lambda_f \alpha_f} \dot{\overline{|\dot{w}|}} & 0 & \frac{1}{\lambda_f \alpha_f} \ddot{\overline{|\dot{w}|}} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{\mu_f \tau_f} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{\lambda_f \alpha_f} |\dot{w}| & 0 & \frac{1}{\lambda_f \alpha_f} \dot{\overline{|\dot{w}|}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{\lambda_f \alpha_f} |\dot{w}| \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\underline{\boldsymbol{\varepsilon}}^{(3)} \\
\underline{\boldsymbol{\varepsilon}}_e^{(3)} \\
\underline{\boldsymbol{\varepsilon}}_{cr}^{(3)} \\
\underline{\boldsymbol{\varepsilon}}_{ms}^{(3)} \\
\underline{\boldsymbol{\varepsilon}}_h^{(3)} \\
\underline{\ddot{\boldsymbol{\varepsilon}}}_e \\
\underline{\ddot{\boldsymbol{\varepsilon}}}_{cr} \\
\underline{\ddot{\boldsymbol{\varepsilon}}}_{ms} \\
\underline{\ddot{\boldsymbol{\varepsilon}}}_h \\
\underline{\dot{\boldsymbol{\varepsilon}}}_e \\
\underline{\dot{\boldsymbol{\varepsilon}}}_{cr} \\
\underline{\dot{\boldsymbol{\varepsilon}}}_{ms} \\
\underline{\dot{\boldsymbol{\varepsilon}}}_h \\
\underline{\boldsymbol{\varepsilon}}_{ms}
\end{bmatrix}
=
$$

$$\begin{bmatrix} \mathbf{0} \\ \underline{\mathbf{D}}_f^{-1}\underline{\boldsymbol{\sigma}}_f^{(3)} + 2\dot{\overline{\underline{\mathbf{D}}_f^{-1}}}\ddot{\underline{\boldsymbol{\sigma}}}_f + \ddot{\overline{\underline{\mathbf{D}}_f^{-1}}}\dot{\underline{\boldsymbol{\sigma}}}_f \\ \frac{1}{\tau_f}\left(\underline{\mathbf{D}}_f^{-1}\ddot{\underline{\boldsymbol{\sigma}}}_f + 2\dot{\overline{\underline{\mathbf{D}}_f^{-1}}}\dot{\underline{\boldsymbol{\sigma}}}_f + \ddot{\overline{\underline{\mathbf{D}}_f^{-1}}}\underline{\boldsymbol{\sigma}}_f\right) \\ \frac{p}{\alpha_f}\underline{\mathbf{D}}_f^{-1}\left(|\dot{w}|\ddot{\underline{\boldsymbol{\sigma}}}_f + 2\dot{\overline{|w|}}\dot{\underline{\boldsymbol{\sigma}}}_f + \ddot{\overline{|\dot{w}|}}\underline{\boldsymbol{\sigma}}_f\right) \\ \underline{\boldsymbol{\beta}}_f w^{(3)} + 2\dot{\underline{\boldsymbol{\beta}}}_f\ddot{w} + \ddot{\underline{\boldsymbol{\beta}}}_f\dot{w} \\ \underline{\mathbf{D}}_f^{-1}\ddot{\underline{\boldsymbol{\sigma}}}_f + \dot{\overline{\underline{\mathbf{D}}_f^{-1}}}\dot{\underline{\boldsymbol{\sigma}}}_f \\ \frac{1}{\tau_f}\left(\underline{\mathbf{D}}_f^{-1}\dot{\underline{\boldsymbol{\sigma}}}_f + \dot{\overline{\underline{\mathbf{D}}_f^{-1}}}\underline{\boldsymbol{\sigma}}_f\right) \\ \frac{p}{\alpha_f}\underline{\mathbf{D}}_f^{-1}\left(|\dot{w}|\dot{\underline{\boldsymbol{\sigma}}}_f + \dot{\overline{|w|}}\underline{\boldsymbol{\sigma}}_f\right) \\ \underline{\boldsymbol{\beta}}_f\ddot{w} + \dot{\underline{\boldsymbol{\beta}}}_f\dot{w} \\ \underline{\mathbf{D}}_f^{-1}\dot{\underline{\boldsymbol{\sigma}}}_f \\ \frac{p}{\alpha_f}\underline{\mathbf{D}}_f^{-1}|\dot{w}|\underline{\boldsymbol{\sigma}}_f \\ \underline{\boldsymbol{\beta}}_f\dot{w} \\ \dot{\underline{\boldsymbol{\varepsilon}}} \\ \ddot{\underline{\boldsymbol{\varepsilon}}} \end{bmatrix} \qquad (20)$$

This system of equations can be solved, using an analytical mathematical program such as Maple [16], with respect to $\underline{\boldsymbol{\varepsilon}}^{(3)}$, resulting in the following third-order differential equation in $\underline{\boldsymbol{\varepsilon}}$ and $\underline{\boldsymbol{\sigma}}_f$

$$\underline{\mathbf{D}}_f\left(\underline{\boldsymbol{\varepsilon}}^{(3)} + H_2\ddot{\underline{\boldsymbol{\varepsilon}}} + H_1\dot{\underline{\boldsymbol{\varepsilon}}} - \underline{\boldsymbol{\beta}}_f(w^{(3)} + b_2\ddot{w} + b_1\dot{w})\right) = \underline{\boldsymbol{\sigma}}_f^{(3)} + S_2\ddot{\underline{\boldsymbol{\sigma}}}_f + S_1\dot{\underline{\boldsymbol{\sigma}}}_f + S_0\boldsymbol{\sigma}_f \qquad (21)$$

where the coefficients H_i, b_i and S_i are scalar functions of the moisture content, w, and its derivatives. The explicit expressions of the coefficients are comprehensive. The coefficients are generally not constants since w can vary over time, t.

HOMGENISATION OF THE FIBRE NETWORK

Transformation of Stresses and Strains

The stresses and strains have been expressed thus far in terms of the local fibre coordinate system, the x-axis coinciding with the longitudinal direction of the fibre. They can be transformed, however, to the global coordinate system of the composite material, as shown in Figure 3, by use of the three-dimensional transformation matrix $\mathbf{T}$ according to

$$\underline{\boldsymbol{\sigma}} = \mathbf{T}\boldsymbol{\sigma}, \quad \underline{\boldsymbol{\varepsilon}} = \mathbf{T}\boldsymbol{\varepsilon} \qquad (22)$$

where the symbols that are underlined ($\underline{\boldsymbol{\sigma}}, \underline{\boldsymbol{\varepsilon}}, \underline{\mathbf{D}}_0$) and the lower-case coordinate indices denote the properties of the local coordinate system, the symbols without underlining and the indices that are capitalized denoting the global coordinates.
The three-dimensional transformation matrix, in component form, is

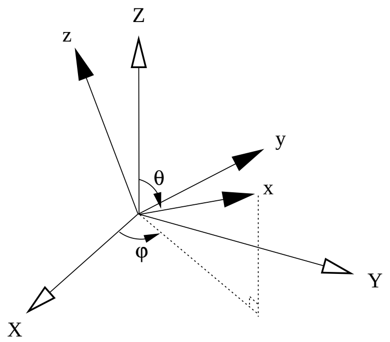

Figure 3: Coordinate transformation from local (x, y, z) to global (X, Y, Z) coordinates. The fibre is oriented in the x-direction.

$$\mathbf{T} = \begin{bmatrix} s^2m^2 & s^2n^2 & c^2 & 2s^2mn & 2csm & 2csn \\ n^2 & m^2 & 0 & -2mn & 0 & 0 \\ c^2m^2 & c^2n^2 & s^2 & 2c^2mn & -2csm & -2csn \\ -smn & smn & 0 & s(m^2-n^2) & -cn & cm \\ -csm^2 & -csn^2 & cs & -2csmn & -m(c^2-s^2) & -n(c^2-s^2) \\ cmn & -cmn & 0 & -c(m^2-n^2) & -sn & sm \end{bmatrix} \tag{23}$$

where the cosine and the sine of the rotation angles are $m = \cos\varphi$, $n = \sin\varphi$, $c = \cos\theta$ and $s = \sin\theta$ [21]. The rotation angles are defined in Figure 3. An in-plane rotation means that $\theta = \pi/2$ which gives $c = 0$ and $s = 1$. The transformation matrix is considered to be independent of time, the changes in fibre-orientation during deformation being neglected ($\dot{\mathbf{T}} = \mathbf{0}$). The transformation of a stiffness matrix is $\mathbf{D} = \mathbf{T}^{-1}\underline{\mathbf{D}}\mathbf{T}$, as given by

$$\boldsymbol{\sigma} = \mathbf{T}^{-1}\underline{\boldsymbol{\sigma}} = \mathbf{T}^{-1}\underline{\mathbf{D}}\,\underline{\boldsymbol{\varepsilon}} = \mathbf{T}^{-1}\underline{\mathbf{D}}\mathbf{T}\boldsymbol{\varepsilon} = \mathbf{D}\boldsymbol{\varepsilon} \tag{24}$$

Integration of the Fibre-Orientation Distribution

Starting from the differential equation for a single fibre, the homogenisation of a network consisting of a distribution of fibres in different directions involves the assumption of homogenous strain. Accordingly, the stress of the fibre network as a whole, $\boldsymbol{\sigma}_{fn}$, is calculated, from the stress of a single fibre, $\boldsymbol{\sigma}_f$, as the mean value

$$\boldsymbol{\sigma}_{fn} = \frac{1}{N}\sum_i^N \boldsymbol{\sigma}_f = \int_0^\pi \int_0^\pi \boldsymbol{\sigma}_f \Psi(\varphi,\theta)\sin\theta d\varphi d\theta = \int_0^\pi \int_0^\pi \mathbf{T}^{-1}\underline{\boldsymbol{\sigma}}_f \Psi(\varphi,\theta)\sin\theta d\varphi d\theta \tag{25}$$

where $\Psi(\varphi, \theta)$ is the spatial fibre-orientation distribution function containing the arguments θ and φ, which can be regarded as a probability function having the property of

$$\int_0^\pi \int_0^\pi \Psi(\varphi,\theta)\sin\theta d\varphi d\theta = 1 \tag{26}$$

In the case of a planar fibre distribution, equation (25) becomes

$$\boldsymbol{\sigma}_{fn} = \int_0^\pi \mathbf{T}^{-1}\underline{\boldsymbol{\sigma}}_f f(\varphi) d\varphi \tag{27}$$

where $f(\varphi)$ is the planar fibre-orientation distribution function having the property

$$\int_0^{\pi} f(\varphi)d\varphi = 1 \tag{28}$$

and where Ψ and f, in analogy to $\mathbf{T}$, are considered to be independent of time, i.e. $\dot{\Psi} = \dot{f} = 0$.

This allows the homogenisation of the equation for a single fibre to be developed to an equation for the entire fibre network, using the following procedure. Both sides of equation (21), which is expressed in terms of local coordinates, are multiplied by $f(\varphi)$, and from the left by the inverse transformation matrix $\mathbf{T}^{-1}$ allowing the equation to be expressed in terms of global coordinates,

$$\begin{aligned}\mathbf{T}^{-1}\underline{\mathbf{D}}_f\left(\underline{\boldsymbol{\varepsilon}}^{(3)} + H_2\ddot{\underline{\boldsymbol{\varepsilon}}} + H_1\dot{\underline{\boldsymbol{\varepsilon}}} - \underline{\boldsymbol{\beta}}_f(w^{(3)} + b_2\ddot{w} + b_1\dot{w})\right) f(\varphi) =& \\ \mathbf{T}^{-1}\left(\underline{\boldsymbol{\sigma}}_f^{(3)} + S_2\ddot{\underline{\boldsymbol{\sigma}}}_f + S_1\dot{\underline{\boldsymbol{\sigma}}}_f + S_0\boldsymbol{\sigma}_f\right) f(\varphi)&\end{aligned} \tag{29}$$

The stresses and strains and their derivatives are then transformed into global coordinates according to $\underline{\boldsymbol{\sigma}} = \mathbf{T}\boldsymbol{\sigma}$, $\underline{\boldsymbol{\varepsilon}} = \mathbf{T}\boldsymbol{\varepsilon}$ and $\dot{\underline{\boldsymbol{\sigma}}} = \mathbf{T}\dot{\boldsymbol{\sigma}}$ etc, which leads to

$$\begin{aligned}\mathbf{T}^{-1}\underline{\mathbf{D}}_f\mathbf{T}\left(\boldsymbol{\varepsilon}^{(3)} + H_2\ddot{\boldsymbol{\varepsilon}} + H_1\dot{\boldsymbol{\varepsilon}} - \mathbf{T}^{-1}\underline{\boldsymbol{\beta}}_f(w^{(3)} + b_2\ddot{w} + b_1\dot{w})\right) f(\varphi) =& \\ \mathbf{T}^{-1}\mathbf{T}\left(\boldsymbol{\sigma}_f^{(3)} + S_2\ddot{\boldsymbol{\sigma}}_f + S_1\dot{\boldsymbol{\sigma}}_f + S_0\boldsymbol{\sigma}_f\right) f(\varphi)&\end{aligned} \tag{30}$$

Finally, an integration of all fibre directions in the plane is performed on both sides of the equation.

$$\begin{aligned}\int_0^{\pi}\mathbf{T}^{-1}\underline{\mathbf{D}}_f\mathbf{T}\left(\boldsymbol{\varepsilon}^{(3)} + H_2\ddot{\boldsymbol{\varepsilon}} + H_1\dot{\boldsymbol{\varepsilon}} - \mathbf{T}^{-1}\underline{\boldsymbol{\beta}}_f(w^{(3)} + b_2\ddot{w} + b_1\dot{w})\right) f(\varphi)d\varphi =& \\ \int_0^{\pi}\mathbf{I}\left(\boldsymbol{\sigma}_f^{(3)} + S_2\ddot{\boldsymbol{\sigma}}_f + S_1\dot{\boldsymbol{\sigma}}_f + S_0\boldsymbol{\sigma}_f\right) f(\varphi)d\varphi&\end{aligned} \tag{31}$$

This integration corresponds to the homogenous strain assumption used in integrating the stress according to equation (27). Equation (31) yields in terms of the constitutive differential equation for the homogenized fibre network material,

$$\mathbf{D}_{fn}\left(\boldsymbol{\varepsilon}^{(3)} + H_2\ddot{\boldsymbol{\varepsilon}} + H_1\dot{\boldsymbol{\varepsilon}} - \boldsymbol{\beta}_{fn}(w^{(3)} + b_2\ddot{w} + b_1\dot{w})\right) = \boldsymbol{\sigma}_{fn}^{(3)} + S_2\ddot{\boldsymbol{\sigma}}_{fn} + S_1\dot{\boldsymbol{\sigma}}_{fn} + S_0\boldsymbol{\sigma}_{fn} \tag{32}$$

which introduces the new properties $\mathbf{D}_{fn}$ and $\boldsymbol{\beta}_{fn}$, defined as

$$\mathbf{D}_{fn} = \int_0^{\pi}\mathbf{T}^{-1}\underline{\mathbf{D}}_f\mathbf{T}f(\varphi)d\varphi \tag{33}$$

$$\boldsymbol{\beta}_{fn} = \mathbf{D}_{fn}^{-1}\int_0^{\pi}\mathbf{T}^{-1}f(\varphi)d\varphi\ \underline{\mathbf{D}}_f\underline{\boldsymbol{\beta}}_f \tag{34}$$

The calculated elastic stiffness matrix $\mathbf{D}_{fn}$ of the fibre network and the hygroexpansion coefficient vector $\boldsymbol{\beta}_{fn}$ agree with previous purely elastic calculations the of fibre network stiffness [21]. This is an important and surprisingly simple result, one allowing the constitutive properties of the hygroscopic network materials not containing any matrix material to be predicted on the basis of the properties of the fibre material, homogenous strain being assumed. Hygroscopic network materials which lack a matrix include wood- and cellulose-fibre materials such as fibreboard, oriented strand board and paper.

It is notable that equation (32), which describes the homogenized fibre network, is identical in its structure to equation (21), which describes the single fibre. The only difference is that the elastic stiffness matrix of the single fibre, $\mathbf{D}_f$, is replaced in equation (32) by $\mathbf{D}_{fn}$ and $\boldsymbol{\beta}_f$ is replaced by $\boldsymbol{\beta}_{fn}$. Thus, the spring and dashpot representation used for the single fibre, as shown in Figure 2, is also valid for the homogenized fibre network, $\mathbf{D}_f$ and $\boldsymbol{\beta}_f$ are replaced by $\mathbf{D}_{fn}$ and $\boldsymbol{\beta}_{fn}$. The same set of τ_f, μ_f, α_f and λ_f is associated with the fibre network. This is very useful in numerical implementation of the model, since it means there is no need of a numerical integration of the properties involved in all the fibre directions.

CONSTITUTIVE EQUATION OF THE MATRIX MATERIAL

The constitutive model of the matrix material is assumed to be of the same type as for the fibre material, except for the mechanosorptive creep effect, which is disregarded. The elastic stiffness of the matrix material, $\mathbf{D}_m$, is assumed to be isotropic and to be independent of moisture content. In line with equations (10) - (13) the creep viscosity $\mathbf{C}_{m,cr}$ and the creep stiffness matrix $\mathbf{D}_{m,cr}$ are likewise assumed to not be influenced by the moisture content. Accordingly

$$\mathbf{D}_m = \begin{bmatrix} \frac{1}{E_m} & -\frac{\nu_m}{E_m} & -\frac{\nu_m}{E_m} & 0 & 0 & 0 \\ -\frac{\nu_m}{E_m} & \frac{1}{E_m} & -\frac{\nu_m}{E_m} & 0 & 0 & 0 \\ -\frac{\nu_m}{E_m} & -\frac{\nu_m}{E_m} & \frac{1}{E_m} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_m} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_m} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_m} \end{bmatrix}^{-1} \tag{35}$$

where $G_m = E_m/(2(1+\nu_m))$ and

$$\mathbf{C}_{m,cr} = \frac{1}{\tau_m}\mathbf{D}_m^{-1} \tag{36}$$

$$\mathbf{D}_{m,cr} = \frac{1}{\mu_m}\mathbf{D}_m \tag{37}$$

The hygroexpansion is also assumed to be linear and isotropic, meaning that $\boldsymbol{\beta}_m = \beta_m[\,1\ \ 1\ \ 1\ \ 0\ \ 0\ \ 0\,]^T$, where β_m is constant. These assumptions can be shown to give the following constitutive equation for the matrix material:

$$\mathbf{D}_m\left((\dot{\boldsymbol{\varepsilon}} - \boldsymbol{\beta}_m\dot{w}) + \frac{1}{\mu_m\tau_m}(\boldsymbol{\varepsilon} - \boldsymbol{\beta}_m(w - w_0))\right) = \dot{\boldsymbol{\sigma}}_m + \frac{1}{\tau_m}\boldsymbol{\sigma}_m \tag{38}$$

where $w_0 = w(t_0)$ is the initial moisture content of the material. This equation is valid for all orientations of the coordinate system, both local and global, since the matrix material is assumed to be isotropic.

HOMOGENISATION OF THE COMPOSITE MATERIAL

As in the case of the homogenisation of single fibres to a homogenous fibre network material, the homogenisation of the fibre network and the matrix material to a composite material is performed, according to laminate theory, i.e. under the assumption of homogenous strain, by summing the stresses of the constituents and taking account of their relative volume within the composite, schematically shown in Figure 4.

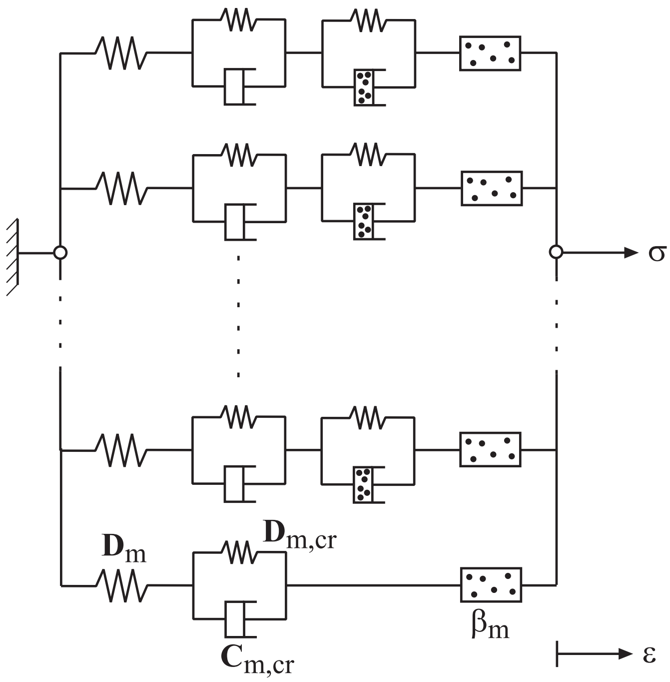

Figure 4: Homogenisation of several fibres and the matrix material with its introduced material parameters.

Since the differential equation for the fibre network (21) comprises zero- to third-order derivatives of the strain, and since the equation for the matrix material (38) comprises zero- and first-order derivatives, the resulting differential equation for the composite material comprises the first to the fourth-order derivatives of the strain. The volume fractions of the fibre phase and the matrix phase are denoted as V_f and V_m, respectively. The equation is calculated from derivatives of the equations above, combined using a scheme contained in equation (39) and analogous to equation (20), the stresses of the fibre network and of the matrix material and its derivatives being the variables contained in the column vector rather than the strains as in equation (20).

$$
\begin{bmatrix}
1 & -V_f & -V_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & S_2 & 0 & \dot{S}_2+S_1 & 0 & \dot{S}_1+S_0 & 0 & \dot{S}_0 & 0 \\
0 & 0 & 1 & 0 & \frac{1}{\tau_m} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & S_2 & 0 & S_1 & 0 & S_0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{\tau_m} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & V_f & V_m & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{\tau_m} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & V_f & V_m & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{\tau_m} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & V_f & V_m & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_f & V_m
\end{bmatrix}
\begin{bmatrix}
\boldsymbol{\sigma}^{(4)} \\ \boldsymbol{\sigma}_{fn}^{(4)} \\ \boldsymbol{\sigma}_m^{(4)} \\ \boldsymbol{\sigma}_{fn}^{(3)} \\ \boldsymbol{\sigma}_m^{(3)} \\ \ddot{\boldsymbol{\sigma}}_{fn} \\ \ddot{\boldsymbol{\sigma}}_m \\ \dot{\boldsymbol{\sigma}}_{fn} \\ \dot{\boldsymbol{\sigma}}_m \\ \boldsymbol{\sigma}_{fn} \\ \boldsymbol{\sigma}_m
\end{bmatrix}
=
\begin{bmatrix}
0 \\
\frac{d}{dt}\left(\mathbf{D}_{fn}\left(\boldsymbol{\varepsilon}^{(3)} + H_2\ddot{\boldsymbol{\varepsilon}} + H_1\dot{\boldsymbol{\varepsilon}} - \beta_f(w^{(3)} + b_2\ddot{w} + b_1\dot{w})\right)\right) \\
\mathbf{D}_m\left(\boldsymbol{\varepsilon}^{(4)} + \frac{\mu_m}{\tau_m}\boldsymbol{\varepsilon}^{(3)} - \beta_m(w^{(4)} + \frac{\mu_m}{\tau_m}w^{(3)}\right) \\
\mathbf{D}_{fn}\left(\boldsymbol{\varepsilon}^{(3)} + H_2\ddot{\boldsymbol{\varepsilon}} + H_1\dot{\boldsymbol{\varepsilon}} - \beta_f(w^{(3)} + b_2\ddot{w} + b_1\dot{w})\right) \\
\mathbf{D}_m\left(\boldsymbol{\varepsilon}^{(3)} + \frac{\mu_m}{\tau_m}\ddot{\boldsymbol{\varepsilon}} - \beta_m(w^{(3)} + \frac{\mu_m}{\tau_m}\ddot{w}\right) \\
\boldsymbol{\sigma}^{(3)} \\
\mathbf{D}_m\left(\ddot{\boldsymbol{\varepsilon}} + \frac{\mu_m}{\tau_m}\dot{\boldsymbol{\varepsilon}} - \beta_m(\ddot{w} + \frac{\mu_m}{\tau_m}\dot{w}\right) \\
\ddot{\boldsymbol{\sigma}} \\
\mathbf{D}_m\left(\dot{\boldsymbol{\varepsilon}} + \frac{\mu_m}{\tau_m}\boldsymbol{\varepsilon} - \beta_m(\dot{w} + \frac{\mu_m}{\tau_m}w\right) \\
\dot{\boldsymbol{\sigma}} \\
\boldsymbol{\sigma}
\end{bmatrix}
\tag{39}
$$

Solving this system of equations with respect to the first element in the stress vector $\boldsymbol{\sigma}^{(4)}$ results in the differential equation for the composite material

$$
\begin{aligned}
&\mathbf{D}_c\left(\boldsymbol{\varepsilon}^{(4)} + \mathbf{K}_3\boldsymbol{\varepsilon}^{(3)} + \mathbf{K}_2\ddot{\boldsymbol{\varepsilon}} + \mathbf{K}_1\ddot{\boldsymbol{\varepsilon}} + \mathbf{K}_0\dot{\boldsymbol{\varepsilon}}\right) - \\
&\quad \mathbf{D}_c\left(\boldsymbol{\beta}_c w^{(4)} + \mathbf{b}_3 w^{(3)} + \mathbf{b}_2\ddot{w} + \mathbf{b}_1\dot{w}\right) = \\
&\quad\quad \boldsymbol{\sigma}^{(4)} + L_3\boldsymbol{\sigma}^{(3)} + L_2\ddot{\boldsymbol{\sigma}} + L_1\dot{\boldsymbol{\sigma}} + L_0\boldsymbol{\sigma}
\end{aligned}
\tag{40}
$$

where

$$
\mathbf{D}_c = V_f\mathbf{D}_{fn} + V_m\mathbf{D}_m \tag{41}
$$

$$
\boldsymbol{\beta}_c = \mathbf{D}_c^{-1}(V_f\mathbf{D}_{fn}\boldsymbol{\beta}_{fn} + V_m\mathbf{D}_m\boldsymbol{\beta}_m) \tag{42}
$$

The coefficients $\mathbf{K}_i$ are matrix-valued linear combinations of $\mathbf{D}_{fn}$ and $\mathbf{D}_m$, the coefficients $\mathbf{b}_i$ are vector-valued linear combinations of $\mathbf{D}_{fn}, \mathbf{D}_m$, $\boldsymbol{\beta}_{fn}$ and $\boldsymbol{\beta}_m$, and L_i are scalar functions of the moisture content, w, and its derivatives.

The influence of the possible porosity in the material can be taken into account. The volume fraction of the porosity is denoted as V_p, such that $V_f + V_m + V_p = 1$. V_p is easily taken into account of by adding elements of zero stiffness to the model. This does not affect the equations in any other way than through the fact that $V_f + V_m < 1$.

Characteristics of the Differential Equation

Even without making any calculations, it is possible to draw certain conclusions regarding the momentary responses and the long-term behaviour predicted by the differential equation (40). The elastic response, i.e. the momentary strain a short time after loading, is calculated by setting $\boldsymbol{\varepsilon}_{cr} = \boldsymbol{\varepsilon}_{ms} = \boldsymbol{\varepsilon}_h = 0$ for both constituents, i.e. by regarding the creep and mechanosorption dashpots as rigid elements, resulting logically in

$$\boldsymbol{\varepsilon}_e = \boldsymbol{\varepsilon}(t \to 0_+) = \mathbf{D}_c \boldsymbol{\sigma}_0 \tag{43}$$

In approximating the long-term strain in the case of a constant load applied at $t = 0$, the creep and mechanosorption dashpots are set to zero stiffness, resulting in

$$\boldsymbol{\varepsilon}^{\infty} = (\mathbf{D}_c^{\infty})^{-1} \boldsymbol{\sigma}_0 \tag{44}$$

where

$$\mathbf{D}_c^{\infty} = V_f \mathbf{D}_{fn}^{\infty} + V_m \mathbf{D}_m^{\infty} = \frac{V_f \mathbf{D}_{fn}}{1 + \mu_f + \lambda_f / p} + \frac{V_m \mathbf{D}_m}{1 + \mu_m} \tag{45}$$

In the case of constant moisture content λ_f disappears from equation (45), the parameters μ_f, λ_f and μ_m determining the relationship between the momentary strain response of a load and the strain response to the same loading after a long period of time. The parameters τ_f, α_f and τ_m determine how rapidly the material will reach long-term strain, $\boldsymbol{\varepsilon}^{\infty}$.

ANALYTICAL SOLUTION OF THE SIMPLIFIED EQUATION

As already indicated, the resulting equation for the composite material is a linear fourth-order differential equation with time-dependent matrix-valued coefficients. Such an equation is virtually impossible to solve analytically, numerical methods generally being required to obtain useful results. Only in a simplified case can an analytical solution for controlling the reliability of the numerical results be achieved.

The final differential equation (40) can be simplified by neglecting the stiffness of the three elastic springs parallel-coupled with the two creep dashpots and the moisture rate dashpot, respectively. This lowers the order of the final differential equation in strain by one for each spring that is neglected, neglecting the stiffness of all the springs thus converting the fourth-order differential equation (40) into a first-order differential equation for strain. The constituents are considered in this way to behave as fluids, which in only very exceptional cases can be regarded as a satisfactory physical description of the constituent materials. This is equivalent to setting $\mu_f, \lambda_f, \mu_m >> 1$, which allows equation (40) to be simplified to

$$\ddot{\boldsymbol{\varepsilon}} + \mathbf{F}_1 \dot{\boldsymbol{\varepsilon}} = \mathbf{D}_c^{-1} \left(\ddot{\boldsymbol{\sigma}} + T_1 \dot{\boldsymbol{\sigma}} + T_0 \boldsymbol{\sigma} \right) + \boldsymbol{\beta}_c \ddot{w} + \mathbf{b}_1 \dot{w} \tag{46}$$

where

$$\mathbf{F}_1 = \mathbf{D}_c^{-1}\Big[\left(\frac{\dot{p}}{p} + \frac{\tau_m\alpha_f + \tau_m\tau_f p|\dot{w}| - \tau_m^2\tau_f p\dot{\overline{|\dot{w}|}} - \tau_m^2\tau_f\dot{p}|\dot{w}| - \tau_f\alpha_f}{\tau_m(-\tau_f\alpha_f + \tau_m\alpha_f + \tau_m\tau_f p|\dot{w}|)}\right)V_f\mathbf{D}_{fn} \tag{47}$$

$$+ \frac{\tau_m\alpha_f^2 + 2\tau_m\tau_f\alpha_f p|\dot{w}| + \tau_m\tau_f^2 p^2|\dot{w}|^2 - \tau_m\tau_f^2\alpha_f p\dot{\overline{|\dot{w}|}} - \tau_m\tau_f^2\alpha_f\dot{p}|\dot{w}| - \tau_f\alpha_f^2 - \tau_f^2\alpha_f p|\dot{w}|}{\tau_f\alpha_f(-\tau_f\alpha_f + \tau_m\alpha_f + \tau_m\tau_f p|\dot{w}|)}V_m\mathbf{D}_m\Big]$$

$$T_1 = \frac{\tau_m^2\alpha_f^2 + 2\tau_m^2\tau_f\alpha_f p|\dot{w}| + \tau_m^2\tau_f^2 p^2|\dot{w}|^2 - \tau_m^2\tau_f^2\alpha_f p\dot{\overline{|\dot{w}|}} - \tau_m^2\tau_f^2\alpha_f\dot{p}|\dot{w}| - \tau_f^2\alpha_f^2}{\tau_m\tau_f\alpha_f(-\tau_f\alpha_f + \tau_m\alpha_f + \tau_m\tau_f p|\dot{w}|)} \tag{48}$$

$$T_0 = \frac{\tau_m\alpha_f^2 + 2\tau_m\tau_f\alpha_f p|\dot{w}| + \tau_m\tau_f^2 p^2|\dot{w}|^2 - \tau_m\tau_f^2\alpha_f p\dot{\overline{|\dot{w}|}} - \tau_m\tau_f^2\alpha_f\dot{p}|\dot{w}| - \tau_f\alpha_f^2 - \tau_f^2\alpha_f p|\dot{w}|}{\tau_m\tau_f\alpha_f(-\tau_f\alpha_f + \tau_m\alpha_f + \tau_m\tau_f p|\dot{w}|)} \tag{49}$$

$$\mathbf{b}_1 = \mathbf{D}_c^{-1}\Big[\left(\frac{\dot{q}}{q} + \frac{\dot{p}}{p} + \frac{\tau_m\alpha_f + \tau_m\tau_f p|\dot{w}| - \tau_m^2\tau_f p\dot{\overline{|\dot{w}|}} - \tau_m^2\tau_f\dot{p}|\dot{w}| - \tau_f\alpha_f}{\tau_m(-\tau_f\alpha_f + \tau_m\alpha_f + \tau_m\tau_f p|\dot{w}|)}\right)V_f\mathbf{D}_{fn}\boldsymbol{\beta}_{fn} \tag{50}$$

$$+ \frac{\tau_m\alpha_f^2 + 2\tau_m\tau_f\alpha_f p|\dot{w}| + \tau_m\tau_f^2 p^2|\dot{w}|^2 - \tau_m\tau_f^2\alpha_f p\dot{\overline{|\dot{w}|}} - \tau_m\tau_f^2\alpha_f\dot{p}|\dot{w}| - \tau_f\alpha_f^2 - \tau_f^2\alpha_f p|\dot{w}|}{\tau_f\alpha_f(-\tau_f\alpha_f + \tau_m\alpha_f + \tau_m\tau_f p|\dot{w}|)}V_m\mathbf{D}_m\boldsymbol{\beta}_m\Big]$$

This first-order differential equation for strain is solved by first multiplying both sides with the integrating factor

$$\mathbf{J}(\tilde{t}) = \int_0^{\tilde{t}} \mathbf{F}_1(\hat{t})d\hat{t} \tag{51}$$

and then by integrating both sides from 0 to t two times. This yields

$$\boldsymbol{\varepsilon}(t) = \int_0^t e^{-\mathbf{J}(\tilde{t})}\int_0^{\tilde{t}} e^{\mathbf{J}(\acute{t})}\ \left[\mathbf{D}_c^{-1}\left(\ddot{\boldsymbol{\sigma}} + T_1\dot{\boldsymbol{\sigma}} + T_0\boldsymbol{\sigma}\right) + \boldsymbol{\beta}_c\ddot{w} + \mathbf{b}_1\dot{w}\ \right]\ d\acute{t}\ d\tilde{t} \tag{52}$$

In a still further simplified case of a constant moisture content of the composite material, $\dot{w} = 0$, and of a constant load beginning to act at $t = 0$, $\boldsymbol{\sigma}(t) = \theta(t)\boldsymbol{\sigma}_0$, where $\theta(t)$ is the Heaviside unit step function, the strain in equation (52) can be written out explicitly:

$$\boldsymbol{\varepsilon}(t) = \left[\left(-\mathbf{I} + \mathbf{F}_1^{-1}(\frac{1}{\tau_f} + \frac{1}{\tau_m}) - \mathbf{F}_1^{-2}\frac{1}{\tau_f\tau_m}\right)(1 - e^{-\mathbf{F}_1 t}) + \mathbf{I} + \mathbf{F}_1^{-1}\frac{t}{\tau_f\tau_m}\right]\theta(t)\ \mathbf{D}_c^{-1}\boldsymbol{\sigma}_0 \tag{53}$$

This equation can be used for verifying correct implementation of the numerical model.

NUMERICAL EXAMPLES

The numerical simulations were performed using a system of first-order differential equations and an implicit Euler scheme. It is assumed that the moisture function $q(w)$ is 1.0 and that the function $p(w)$ is either $p(w) = 1.0$ or $p(w) = 1 - 0.015(w - 0.04)$. The load is set to $\boldsymbol{\sigma}(t) = \theta(t)\boldsymbol{\sigma}_0 = \theta(t)[200\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^T$ MPa. The elastic properties of the wood-fibre material are taken from Persson [20] and the moisture dependency from Dinwoodie [7], the time dependent properties being evaluated from tests performed by Hanhijärvi [11]. The properties of melamine formaldehyde are taken from [10, 18]. All parameters defining the time-dependent behaviour ($\tau_f, \mu_f, \alpha_f, \lambda_f, \tau_m$ and μ_m) are evaluated on the basis of test result diagrams, using the least square method. The volume fractions are assumed to be $V_f = 0.62$ and $V_m = 0.38$. The fibre-orientation is assumed to be planar isotropic, all the fibres thus being aligned randomly in the plane, $f = 1/\pi$. The input values for the properties of the constituents are presented in Tables 1.

Table 1: *Assumed mechanical properties of wood fibres and matrix material.*

Wood fibre Properties						
E_x	$E_y = E_z$	$\nu_{xy} = \nu_{xz}$	ν_{yz}	$G_{xy} = G_{xz}$	G_{yz}	
[MPa]	[MPa]	[-]	[-]	[MPa]	[MPa]	
40 000	5 000	0.2	0.3	4 000	1 920	
β_x	β_y	β_z	τ_f	μ_f	α_f	λ_f
$[1/10^4]$	$[1/10^4]$	$[1/10^4]$	h	[-]	[-]	[-]
1.0	26.0	26.0	2000	0.32	0.24	0.32
Matrix Material Properties						
	E_m	ν_m	β_m	τ_m	μ_m	
	[MPa]	[-]	$[1/10^4]$	[h]	[-]	
	9 000	0.3	1.0	36	2.0	

Comparison with the Analytical Solution

Figure 5 compares, under conditions of constant moisture content, $\dot{w} = 0$, the solution to equation (46), both the numerical and the analytical solution to which is presented explicitly in equation (53), with the numerical solution of the complete model in equation (40). The numerical and the analytical solutions of the simplified equation (46) coincide both for ε_x^s and for ε_y^s if the time steps are sufficiently small. The simplified model behaves as a fluid according to

$$\dot{\boldsymbol{\varepsilon}}(t) \cong \frac{1}{\tau_f \tau_m} \mathbf{F}_1^{-1} \mathbf{D}_c^{-1} \boldsymbol{\sigma}_0 \tag{54}$$

its having a constant strain rate as $t >> \tau_f, \tau_m$.

Example of the Calculation of Mechanosorption and of Creep Deformation

Figure 6 shows long-term creep and recovery strains under conditions of constant and of varying moisture content, respectively. Two cases of constant moisture content, $w = 0.04$ and $w = 0.07$, are compared with a third case, having the same set of properties but exposed to a climate such that the moisture content of the material varies between 0.04 and 0.07. The results show the model to be able to mimic the kind of mechanosorptive effect observed in the wood and the wood-based materials, as shown in Figure 1.

Fibre-Orientation and Content

Figure 7, finally, compares the strains obtained for different fibre-orientation distributions. The strain in a composite material having a random fibre-orientation distribution, ε^{CR}, is compared

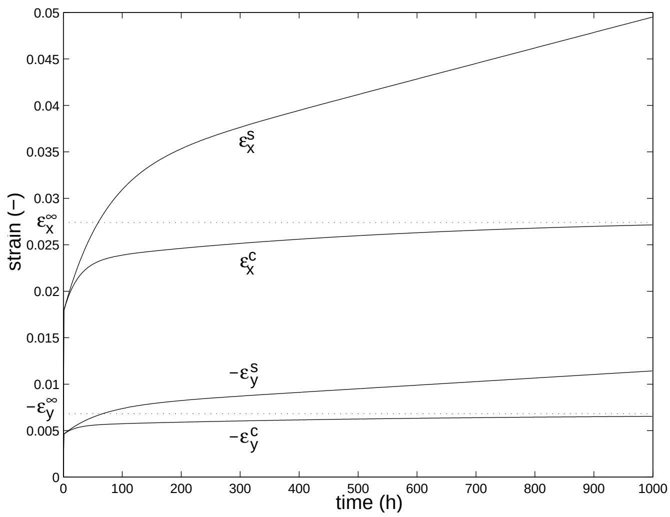

Figure 5: Simplified (ε^s) and complete (ε^c) model at constant moisture and uniaxial load.

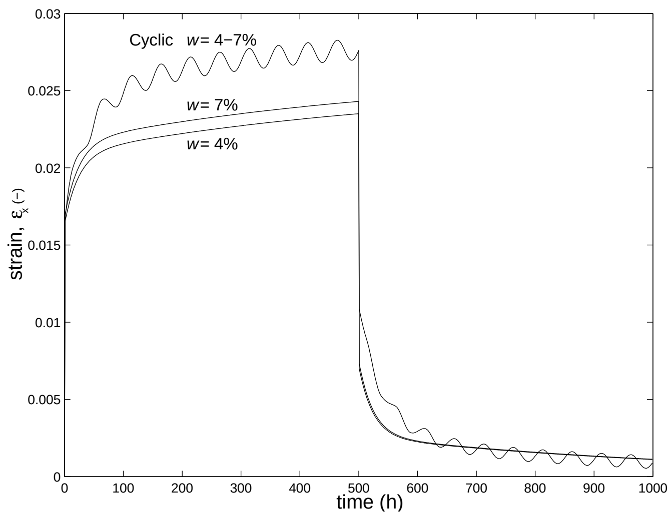

Figure 6: Strain under loading and relaxation under conditions of constant and cyclic varying moisture content.

with that in three different materials containing only fibres, one of them having a random fibre-orientation distribution, ε^{FR}; another being unidirectional, all the fibres being aligned in the loading direction, ε^{F0^o}; and a third likewise being unidirectional but all the fibres being aligned in the direction perpendicular to loading, ε^{F90^o}. The large strains obtained in the latter case reflects the poor stiffness and moisture-effect properties of wood fibres in the cross-fibre direction.

CONCLUDING REMARKS

An analytical three-dimensional model for the moisture dependent behaviour of wood-fibre network composite materials was developed. The model predicts the strain in all directions and at any particular time on the basis of the known properties of the constituents, the proportions, the

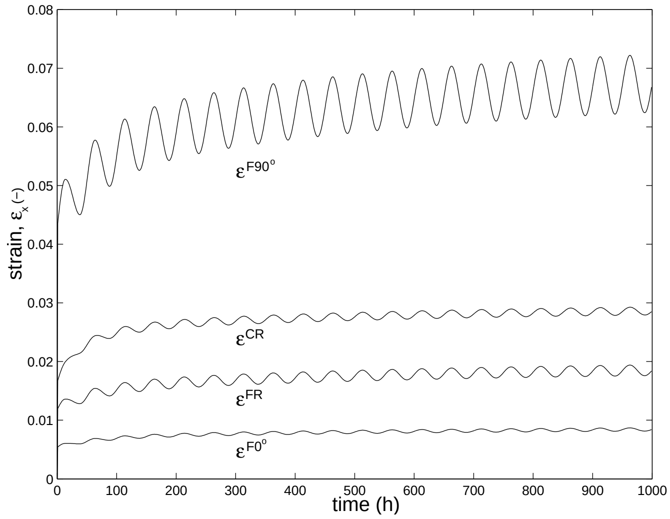

Figure 7: Comparison of unidirectional and randomly aligned fibres.

orientation distribution of the fibres, and the moisture present and the load applied to the material as a function of time. The model is presented in a complete version and a simplified version, where the simplified version is solved analytically. The simplified version of the model can be expected to give reasonably accurate results in the cases of a rapidly varying load and short term constant loading.

The model can be used for optimizing of the make of existing composite materials, for prediction of the properties of not yet existing materials and for analysis of the performance in various types of climatic conditions. The model can moreover be used for sensitivity analyzes that enable identification of critical parameters of the composite and of the course of the loading and the climatic variation.

FUTURE WORK

Before the present model can be used in practical applications two more issues should be investigated: validation and usability. Experimental verification is needed to ensure that the model gives accurate results and to identify limitations of the model. Further, the model should be applied on an industrial composite product to ensure its usability.

The model is based on certain assumptions and simplifications. In order to improve the model some of the assumptions made can be reconsidered:

- The material model can be made more advanced for example by taking into consideration non-linear creep and viscoplasticity. Now all matrices describing creep and mechanosorption are scalar functions multiplied by the stiffness matrix.
- The use of laminate theory, i.e. the assumption of homogenous strain, can be replaced by

an interpolation model between the two cases of homogenous strain and homogenous stress. An example of such interpolation is presented in [21].

- The simplification to regard all time and moisture effects as scalar functions can be generalized.

The above mentioned improvements will however lead to much more extensive calculations. A more advanced material model will also demand a larger number of material parameter values to be known.

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APPENDIX

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Notation

The following symbols are used in this paper:

$\mathbf{A}$	mechanosorption matrix
$b, \mathbf{b}$	moisture coefficients of diff. eq.
c	cosine of θ
$\mathbf{C}$	creep viscosity matrix
$\mathbf{D}$	stiffness matrix
E	Young's modulus
f	function for fibre-orientation
$\mathbf{F}$	strain coefficient of diff. eq.
G	shear modulus
H	strain coefficient of diff. eq.
$\mathbf{I}$	identity matrix
$\mathbf{J}$	integrating factor
$\mathbf{K}$	strain coefficient of diff. eq.
L	stress coefficient of diff. eq.
m	cosine of φ
n	sine of φ
p	moisture function
q	moisture function
s	sine of θ
S	stress coefficient of diff. eq.
t	time
T	stress coefficient of diff. eq.
$\mathbf{T}$	coord. transformation matrix
V	volume fraction
w	moisture content

α	mechanosorption parameter
$\boldsymbol{\beta}$	hygroexpansion coefficient vector
$\boldsymbol{\varepsilon}$	strain vector
θ	coord. transformation angle
$\theta(t)$	Heaviside unit step function
λ	mechanosorption parameter
μ	creep parameter
ν	poisson's ratio
$\boldsymbol{\sigma}$	stress vector
τ	creep parameter
φ	coord. transformation angle
Ψ	function for fibre-orientation

Superscripts

$\dot{(\ )}, \dot{\overline{(\ )}}$	time derivative of ( )
$(\ )^{(n)}$	n:th derivative of ( )
∞	long term property
c	complete model
C	composite property
F	fibre property
R	random fibre-orientation
s	simplified model

Subscripts

$\underline{(\ )}$	property in local coordinate system
0	initial value
c	composite property
cr	creep
e	elastic (strain)
f	single fibre property
fn	fibre network property
h	hygroexpansion
m	matrix material property
ms	mechanosorptive
p	porosity property