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Numerical Methods for Coupled Environmental Problems

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Introduction

Many research problems in environmental and climate science are solved with the *partitioned approach*: Specialized models for subprocesses are developed independently and coupled for the full numerical simulation.



Example: Coupled Groundwater and Surface Flows

Water management projects such as stream bed re-naturalization affect the water table. We can use numerical models to predict how the aquifer is affected by river systems, e.g., in case of floods. Collaboration with Andreas Dedner and Robert Klöfkorn.

Modeling Approach [1]



Figure 1: For example, Earth system models consist of various submodels for individual processes/subsystems

The better we model the subprocesses, the more we have to consider numerical errors developing at the interface between components. \Rightarrow We study the *coupling error in time*.

Iterative Coupling Algorithms

Simple coupling algorithms:severely restrict the convergence orderintroduce instabilities

Iterative coupling algorithms

• enable high-order solutions

pose few requirements on subsolvers (black box assumption)
confirm validity of interface boundary conditions in highly nonlinear settings



Coupling Algorithm

Dirichlet-Neumann Iteration with Relaxation

 $k \leftarrow 1, h^0$ given

```
repeat
solve (1) using (3b) \rightarrow \psi^k given h^{k-1}
```

Figure 3: Öre river in Västerbotten. (Credit: SiberianJay, CC BY-SA 4.0)

Analysis Verification

We verify our analysis with an example code, using DUNE [3] for the Richards equation and preCICE [4] for equation coupling. The experimental results are in agreement with the derived formula. ω_{opt} is strongly affected by the material parameters K and c which can span various orders of magnitude in the Richards equation.





Figure 2: The most generic, high-order coupling algorithm are waveform iterations, where the solvers exchange interpolants of their data. Adapted with permission from Benjamin Rodenberg.

Outlook

Focus: atmosphere-ocean coupling & groundwater-surface flows. *fully discrete analysis* in idealized models
verify results & test new algorithms in full complexity models
⇒ develop high-order, energy efficient, black-box coupling algorithms

Selected References

[1] P. Bastian, H. Berninger, A. Dedner *et al.*, 'Adaptive Modelling of Coupled Hydrological Processes with Application in Water Management,' in *Progress*

solve (2) using (3a)
$$\rightarrow h^{\kappa}$$
 given ψ^{κ}
relax: $h^{k} = \omega \tilde{h}^{k} + (1 - \omega)h^{k-1}$
 $k \leftarrow k + 1$
until termination

Linear Analysis [2]

• linearization: assume K, c constant • space discretization: linear finite elements • time discretization: implicit Euler method We can now express the water height at $t = t^n$ and in iteration k in terms of old data: $h^{n,k} = (\omega S + (1 - \omega))h^{n,k-1} + \omega b^{n-1},$

where $S = S(c, K, \Delta t, \Delta z)$.

Optimal Relaxation Parameter ω_{opt}

Convergence is obtained when $h^{n,k}$ does not change with increasing k. We can find an optimal value of ω which ensures convergence of the coupling algorithm in two iterations:





Figure 5: Optimal relaxation parameter ω_{opt} for varying material parameters ($\Delta t = 1/10$).

Work in Progress

How well does our 1D-0D analysis translate to the nonlinear 2D-1D case? Test case: Lake at rest, varying soil types.

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- [2] A. Monge and P. Birken, 'On the convergence rate of the Dirichlet–Neumann iteration for unsteady thermal fluid–structure interaction,' *Computational Mechanics*, vol. 62, no. 3, pp. 525–541, 1st Sep. 2018, ISSN: 1432-0924. DOI: 10.1007/s00466-017-1511-3.
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- [4] G. Chourdakis, K. Davis, B. Rodenberg *et al.*, 'preCICE v2: A sustainable and user-friendly coupling library,' *Open Research Europe*, vol. 2, no. 51, 2022. DOI: 10.12688/openreseurope.14445.2.

Code available at: github.com/valentinaschueller/richards-swe-coupling/.

 $\omega_{\rm opt} = \frac{1}{1-S}, \quad S = b^2 \alpha - \frac{a}{2}$

with

 $a = \frac{2}{3}c\Delta z + 2\frac{K\Delta t}{\Delta z}, \quad b = \frac{1}{6}c\Delta z - \frac{K\Delta t}{\Delta z},$ and $\alpha = \Delta z \sum_{i=1}^{m-1} \sin^2(j\pi\Delta z) \left(a - 2b\cos(j\pi\Delta z)\right)^{-1}.$



Figure 6: 2D setup of the coupled surface-groundwater-flow system. We obtain the 1D-0D case by omitting the x-axis.

The analysis indicates that the robustness of the coupling algorithm is very sensitive to the non-linearities in the Richards equation. \Rightarrow need to track K and c at runtime!

Future work: Study advanced coupling setups (waveform iterations, Quasi-Newton acceleration) using preCICE.