Bidding behaviour in interdependent markets for electricity and green certificates

Ganhammar, Kajsa

Working Paper 2023:8

Department of Economics
School of Economics and Management

Bidding Behaviour in Interdependent Markets for Electricity and Green Certificates

Kajsa Ganhammar

August 2023

LUND UNIVERSITY

Bidding behaviour in interdependent markets for electricity and green certificates*

Kajsa Ganhammar†
August 22, 2023

Abstract

Market-based climate policies have received increased attention, making it important to understand how such politically created markets affect competition in the electricity market. This paper focuses on the green certificate policy which financially supports producers of renewably sourced electricity by means of tradable certificates, and develops a simple duopoly model that incorporates both the electricity and the green certificate markets in an auction-based setting. The results suggest that, in case the subsidised technology has a higher expected marginal cost than the conventional technology, the policy can improve competition and efficiency in the electricity market. Conversely, if producers are ex-ante symmetric in their marginal costs, the advantage the policy creates enables the subsidised producer to bid higher at given cost as the probability of winning the electricity auction increases. This is harmful for competition and results in high consumer prices of electricity.

Keywords: asymmetric procurement auctions, electricity markets, green certificates, renewable energy

JEL codes: D43, D44, Q48

1 Introduction

Electricity markets around the globe are transitioning as non-renewable generation capacity is being replaced by renewable capacity at an increasing rate. Climate policy has played a significant role in fostering this process and will likely continue to be part of electricity markets in the next few decades as concern about climate change grows. In past years, there has been a call for more market-based policy designs that are in accordance with the principals of liberalised electricity markets. One such frequently used mechanism targeting renewable generation is a market for tradable green certificates with quota obligations, as has

*I am grateful to my advisors Tommy Andersson and Pär Holmberg for valuable guidance in the progress of this paper. I also thank the seminar participants at the Department of Economics at Lund University for helpful comments.

†Department of Economics, Lund University and Research Institute of Industrial Economics.
E-mail: kajsa.ganhammar@nek.lu.se

been implemented in several US states and some countries in the EU.[1] The policy functions as a financial support to producers of renewably sourced electricity who receive certificates for the 'greenness' of the electricity they feed into the grid. These can be traded at a specific market for certificates where a demand is ensured by imposing quota obligations, requiring a certain percentage of total electricity consumed to originate from renewable sources.[2] As the share of renewables increases and they become more competitive, it is plausible that green certificates will gain in importance since they leave it to the market to determine the support in terms of the price of certificates.[3]

This makes it an important concern to understand how such politically created markets affect competition in the electricity market. Specifically, the aim of this paper is to study strategic behaviour in an electricity market integrated with a green certificate market that resembles a market design based on a power exchange, where producers submit price-quantity bids and market clearance is based on a uniform pricing rule. Previous theoretical research on interactions between electricity and green certificate markets are predominantly based on assumptions of perfect competition (e.g. Amundsen and Mortensen (2001); Jensen and Skytte (2002); Böhringer and Rosendahl (2010)) or quantity (Cournot) competition (e.g. Tamás et al. (2010); Amundsen and Bergman (2012); Amundsen and Nese (2016)). To my knowledge, their interdependence in an auction setting has not been formally modelled within a theoretical framework before.

While studying this interdependence using different types of models is important on its own, as they can provide different policy insights, the paper contributes to the literature by focusing on bidding behaviour in the electricity market when incorporating the effect of a co-existing green certificate market. A question that has received limited attention in an otherwise extensive literature on the performance of green certificate schemes (see Darmani et al. (2016)). Moreover, whilst this paper considers green certificates the model can easily be altered to a setting of alternative energy permits, such as tradable emission allowances.[4]

I formulate a simple and tractable two-stage duopoly model that is solved analytically. In

[1] 29 US states and the district of Columbia have a quota obligation (renewable portfolio standard) in place with compliance markets for green certificates. An additional seven states have voluntary markets for green certificates (Greenstone and Nath, 2019). In the EU, compliance markets with a quota obligation still operate in Sweden (joint with Norway), Belgium and Poland, while voluntary trade in green certificates, in the form of guarantees of origin, have gained in popularity lately.

[2] See, for instance, Darmani et al. (2016) for a more through description of the functioning of the tradable green certificate policy.

[3] For example, European Commission (2013) recommends that feed-in tariffs are phased out and support instruments that expose renewable energy producers to market price signals, such as premiums or green certificates, are used. Many US states have also expanded or renewed their portfolio standard goals and China recently introduced a green certificate scheme to reduce feed-in tariff subsidies. (Feed-in tariffs provide a fixed payment to renewably sourced electricity producers for their generation, thereby completely shedding them from market risk. Premiums provide a fixed payment on top of the wholesale market price of electricity. Basically, my model would reduce to a premium scheme by replacing $\mathbb{E}(p_c|v_g, b_g < b_b)$ with a constant and eliminating the second-stage game as the premium is fixed rather than determined at the market like the certificate price.) Further, while this paper focuses on compliance markets for certificates with a percentage requirement in place the basics of the model should be applicable to voluntary markets as well, as may be of more relevance to countries that are far along the renewable energy transition.

[4] In the electricity auction, the marginal cost of one producer would simply be shifted by a positive, rather than negative, price expectation. Potential adjustments can also be made to the second-stage game to fit a trading mechanism for emission allowances.

the first stage, producers compete to serve load in a setup similar to that in von der Fehr and Harbord (1993), the first paper to adopt an auction-approach to model electricity markets. Variations of this model has since then been extensively used in the literature (see Fabra et al. (2006); Fabra et al. (2011); Holmberg and Wolak (2018); Fabra and Llobet (2021)). As in these studies, in my model producers generate at a constant marginal cost and are constrained to submit a single bid for the whole of their capacity. Meanwhile, I assume that producers are ex-ante asymmetric, with one producer having a cost advantage as it receives revenues on top of the wholesale electricity price from selling green certificates. Thus, the relevant cost measure that this producer should place its bids in the electricity market upon is the net marginal cost after subtracting the certificate price. Certificates are traded in a double auction in the second stage of the model, where the certificate price emerges from the ask and bid prices placed by the producer entitled to certificates and a distributor that is obligated to purchase certificates, respectively.

The results indicate that policy makers should carefully specify the price bounds on certificates, which shape price offers in the certificate market, with consideration to the degree of asymmetry in producers marginal costs. Failing to do so can harm competition in the electricity market and result in unnecessarily high consumer prices.

The reminder of the paper is organised as follows. Section 2 formally introduces the model. Section 3 derives the equilibrium ask and bid functions in the green certificate market and analyses the results. Section 4 derives the equilibrium inverse bid functions in the electricity market and illustrates the results in some numerical examples. Section 5 concludes.

2 Model

2.1 Setup

There are three risk-neutral players in the model, two producers and one distributor that demands electricity and green certificates. One producer generates electricity using a renewable technology that receives financial support in the form of tradable green certificates. The other producer is not entitled to any financial support, either because it generates electricity using a fossil-based technology or a more mature renewable technology.[5] For convenience, I refer to the producer with support as the "green" producer (indexed by g) and the other as the "brown" producer (indexed by b). The distributor is indexed by d.

Producers capacities are given by $\overline{q}_i$, $i \in \{g, b\}$, and assumed to be perfectly divisible. Electricity demand is a random variable, $D \in [\underline{D}, \overline{D}]$. It is assumed that all realisations satisfy $D \leq \min\{\overline{q}_g, \overline{q}_b\}$, such that the capacity of any single producer is always enough to serve load. In other words, both producers are nonpivotal with certainty when bids are submitted.[6] Moreover, electricity demand is perfectly inelastic up to a reservation price, P,

[5] The latter would resemble a technology specific scheme wherein only certain renewable technologies receive certificates, as opposed to a technology neutral scheme wherein all renewable technologies recieve certificates.

[6] A special feature of electricity markets is that it can happen that a producer is pivotal, that is, realised demand is larger than the total capacity of its competitors. In general, pivotal status depends on season and time of the day (hours of base and peak load). von der Fehr and Harbord (1993) and Fabra et al. (2006) derive optimal bidding strategies in both the pivotal and the nonpivotal cases when generation costs are common

and producers generate at constant marginal cost, c_i, up to capacity. These assumptions are equivalent to those in the electricity market model by von der Fehr and Harbord (1993). However, as in Holmberg and Wolak (2018), I account for information asymmetry about generation costs. Specifically, I assume that c_i is independent private information while the continuously differentiable distribution function, $F_i(c_i)$, defined uniformly on the interval $[\underline{c}_i, \overline{c}_i]$, $\underline{c}_i < \overline{c}_i$, is known to both producers.

In addition, the green producer and the distributor have a (constant) valuation of green certificates, v_j, $j \in \{g, d\}$, drawn independently from the interval $[\underline{v}, \overline{v}] \subseteq \mathbb{R}^+$, $\underline{v} < \overline{v}$. $\underline{v}$ represents a lower price bound at which the state guarantees to purchase any excess certificates unable to be sold at the market.[7] $\overline{v}$ represents an upper price bound functioning as a penalty for non-compliance that the distributor must pay the state for each missing certificate required to fill its quota obligation. Clearly, no trade at the market will occur outside these bounds, why valuations must be defined on $[\underline{v}, \overline{v}]$. The continuously differentiable distribution function of valuations, $G(v_j)$, defined uniformly on $[\underline{v}, \overline{v}]$, is common knowledge among all three players.

A simple overview of the timeline of the game is depicted in Figure 1. First, producers compete, on the basis of bids, to serve load in the electricity market. If the lowest bid is submitted by the brown producer, the game ends after the electricity market clears. If the lowest bid is submitted by the green producer, the game proceeds to the second stage where trade in certificates takes place. The succeeding two subsections describe in detail this process and specify the ex-post profits in each market.

2.2 The electricity market

In the electricity market, each producer simultaneously submits a bid, $b_i \leq P$, specifying the minimum price at which it is willing to supply the whole of its capacity. The producer that submits the lowest bid is then called to produce by the auctioneer.[8] The output assigned to this producer equals the realised demand, D.

To incorporate the effect of the green certificate policy on bidding behaviour, I initially denote by $\mathbb{E}(p_c|v_g, b_g < b_b)$ the expected certificate price conditional on v_g and that the green producer wins the electricity auction (and hence the second stage is reached). Next, I introduce the variable $x_g = c_g - \mathbb{E}(p_c|v_g, b_g < b_b)$ such that $x_g \sim \mathrm{U}[\underline{x}_g, \overline{x}_g] = [\underline{c}_g - \mathbb{E}(p_c|v_g, b_g <$

knowledge. Holmberg and Wolak (2018) extend the analysis to a setting with uncertain interdependent costs, but assume that producers are ex-ante symmetric. It is well known that asymmetries between bidders complicate the analysis of auctions considerably, and for this reason attention has been focused to finding closed-form solutions to single-unit auctions with asymmetric bidders. Indeed, I find that the differential equations characterising the pivotal case under the given setup lacks a closed-form solution or at least a tractable one even in restrictive numerical examples, why the analysis is focused to the case of nonpivotal producers. (Essentially, the difficulty emerges due to the optimisation problem containing an additional term when a producer is pivotal, since it will be called to produce regardless if it submits the low bid. A producer must therefore weigh the benefit of submitting the low bid and produce at full capacity to the benefit of submitting the high bid and serve residual demand but be paid a higher price.)

[7] A lower price bound on certificates has been used in practice, although not implemented in all markets, to protect producers from the risk of prices dropping to, or below, zero.

[8] If producers submit equal bids it is assumed, without loss of generality, that either producer is called to produce with probability 1/2.

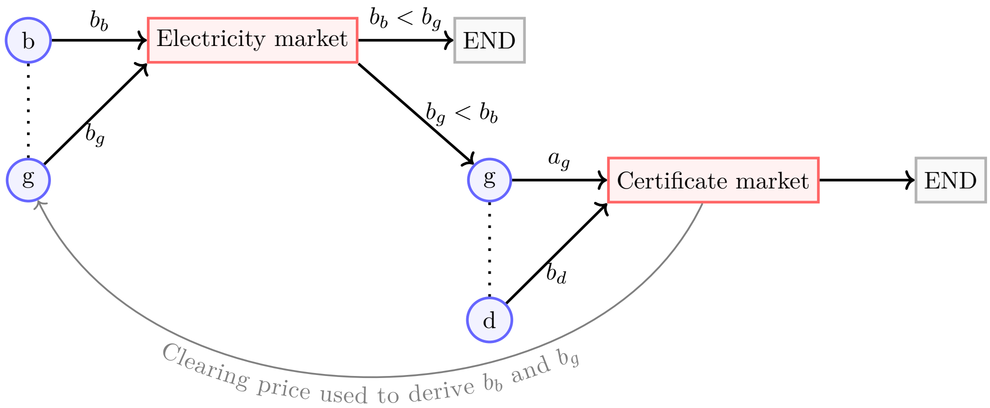

Figure 1: Game illustration. Circles represent players, red and grey rectangles represent the market mechanisms and the end of the game, respectively. b_i, $i \in \{b, g, d\}$, denote the submitted bid prices and a_g denotes the submitted ask price.

$b_b), \bar{c}_g - \mathbb{E}(p_c|v_g, b_g < b_b)]$ with associated distribution function $F_g(x_g)$, noting that the green producer should place its bid in the electricity market based on the net marginal cost after subtracting the certificate price. This specification implies that the brown producer is assumed to be perfectly informed about how the green producer values certificates, whereas both producers are imperfectly informed about how the distributor values certificates. Similarly, the distributor only knows $G(v_g)$. Asymmetric information about certificate valuations may arise due to, for instance, insufficient records on generation costs (from the distributor's perspective) and on the terms of payment to electricity end consumers or uncertainty about electricity demand, which affects certificate demand via the quota (from the producer's perspective). Thus, this assumption can be justified given that the brown producer is more informed about the green producer's generation cost than the distributor (it knows $F_g(c_g)$) and both producers have the same information about D when bids in the electricity auction are submitted.[9]

It follows that profits to each producer in the electricity market are given by:

$$\Pi_g^e = \begin{cases} (b_g - x_g)D & \text{if } b_g < b_b \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$\Pi_b^e = \begin{cases} (b_b - c_b)D & \text{if } b_b < b_g \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Most wholesale electricity markets are organised as uniform auctions where producers are paid the highest accepted bid. As can be inferred from Eqs. (1) and (2), in a duopoly where both producers are nonpivotal the uniform auction coincides with the discriminatory auction where producers are paid their own bid. Moreover, in the given setup with constant marginal costs and single bid-constraint the multi-unit electricity auction then reduces to the

[9] Allowing for information asymmetry in both c_g and v_g between producers would complicate the analysis considerably as the distribution of x_g would no longer be uniform.

first-price, single-unit, auction.

2.3 The certificate market

If the green producer was called to produce electricity, it acquires y_g certificates in proportion to its production after the electricity market clears. The financial support is thereby based on actual production rather than installed capacity, in accordance with the function of the green certificate policy. In the certificate market, the green producer submits an ask price, a_g, and the distributor simultaneously submits a bid price, b_d. The ask specifies the price at which the producer is willing to offer the entire volume of y_g certificates. Similarly, the bid specifies the price at which the distributor is willing to purchase the entire volume of y_g certificates. If $b_d \geq a_g$, trade takes place at price p_c equal to the average of the ask and the bid prices. Otherwise, no trade agreement is reached. This bargaining mechanism corresponds to the k-double auction, for the special case of $k = 1/2$, first studied by Chatterjee and Samuelson (1983) and incorporates two-sided incomplete information in its simplest form. This type of mechanism is commonly used at initial stages for trade at stock exchanges (Kadan, 2007). Hence it is appropriate to model trade in certificates, them being purely financial assets often traded at intermediated markets in reality.[10] Profits in the green certificate market are given by:

$$\Pi_g^c = \begin{cases} (p_c - v_g)y_g & \text{if } a_g \leq b_d \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$\Pi_d^c = \begin{cases} (v_d - p_c)y_g & \text{if } a_g \leq b_d \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where $p_c = \frac{a_g + b_d}{2}$.

Before proceeding to the analysis, there is one assumption that is worth clarifying. One main parameter through which policy makers can influence the certificate market is the quota obligation, that ensures a demand for certificates, by adjustment of its size. In this paper the quota is not explicitly part of the model. This is a necessary simplification under the given setup but deviates from the bulk of theoretical studies on markets for electricity and certificates, wherein the quota appears as a parameter in the models. However, these studies rely on alternative assumptions about the market micro structure and have different research objectives.[11] Rather, in this model, the effect of the quota should be implicitly captured

[10] In the general k-double auction, if $b \geq a$ trade occurs at price $kb + (1 - k)a$. The weight of the bid and the ask prices on the trade price is thereby determined by the constant $k \in [0, 1]$. Treating parties equally ($k = 1/2$) is appealing here as policy makers should value the surplus to both parties (see also Section 3.1). Specifically, whereas a high price attracts more investment in renewable capacity, it also results in high consumer prices of electricity as the cost of certificates is passed on to consumers electricity bills.

[11] For instance, Amundsen and Mortensen (2001), Jensen and Skytte (2002), Böhringer and Rosendahl (2010) and Amundsen and Nese (2016) study how the interaction between the markets for electricity and for certificates, including the size of the quota obligation, affects production and prices in a perfectly competitive setting. Amundsen and Bergman (2012), Amundsen and Nese (2016) and von der Fehr and Ropenus (2017)

by v_j. The rationale is that the quota obligation is typically defined on a quite long time interval, such as a year. Reasonably, the green producer and the distributor therefore form their valuations about certificates today based on an expectation about whether there will be a future excess or shortage of certificates to meet the quota. This expectation may depend on, for example, available information about entrance into the market, how often the green producer is expected to win the electricity auction, and future demand for electricity.[12]

Another way policy makers can influence market outcomes, as incorporated in the model, is via price bounds on certificates.

3 Equilibrium analysis of the certificate market

I solve for the Bayesian Nash Equilibrium in each market, that is, a pair of bid/offer functions that are mutual best responses. Recall that the green producer places its bid in the electricity market based on the net marginal cost. This implies that the model must be solved backwards to attain $\mathbb{E}(p_c|v_g, b_g < b_b)$ and in consequence $F_g(x_g)$. Consistently, in this section the equilibrium offer functions in the certificate market are derived. The (expected) clearing price resulting from these is then used in Section 4 to compute the equilibrium bid functions in the electricity market (see also Figure 1).

3.1 Preliminaries

Because there is a very large set of equilibria in the k-double auction game, I restrict attention to a scenario where parties use linear strategies. This is preferable as it enables me to derive closed-form expressions for the equilibrium strategies. Furthermore, the linear equilibrium of the 'split-the-difference' game (i.e., k-double auction for $k = 1/2$) has been shown to have advantageous efficiency properties (in terms of gains from trade), especially for the uniform distribution (see Myerson and Satterthwaite (1983) and Leininger et al. (1989)). There is also experimental evidence showing that parties tend to bid according to a linear strategy in this bargaining mechanism (Radner and Schotter, 1989).

The rules of the certificate bargaining game, including traders strategies, are known to the brown producer as well.

3.2 The first order conditions

I start the analysis of the certificate market by deriving the first order conditions for an equilibrium. That is, a pair of differential equations from which the linear equilibrium offer strategies are determined in the next subsection. In particular, let $A_g(v_g)$ and $B_d(v_d)$ be strictly increasing and differentiable equilibrium ask and bid functions, respectively, with

study the effects of market power in Cornout, Stackelberg and dominant firm/competitive fringe settings, respectively. Tamás et al. (2010) compare the green certificate and the feed-in tariff policies under Cournot competition. Neither of these studies account for information asymmetry between players.

[12]This is in line with the reasoning of Lemming (2003). He points out that a green producer will act in the certificate market based on expectations (not unlike in a stock market) and will bid to sell at the price it expects will be the equilibrium price. That is, the price of the certificates sold on the margin before the start of a new obligation period.

inverses $\theta_g(a_g)$ and $\theta_d(b_d)$. Suppose that the distributor bids according to $B_d(v_d)$ and the green producer asks a_g. Because of the assumption of constant valuation, it suffices to derive the optimal ask and bid prices for a single certificate. Over the region where trade occurs with positive probability, expected marginal profit to the green producer is then given by:

$$\pi_g^c(a_g, v_g) = \int_{\theta_d(a_g)}^{\overline{v}} \left(\frac{a_g + B_d(v_d)}{2} - v_g \right) g(v_d) dv_d,$$

where $g(v_d) = G'(v_d)$. Applying Leibniz' rule, we arrive at the first order condition:

$$-\theta_d'(a_g)(a_g - v_g)g(\theta_d(a_g)) + \frac{1}{2}(1 - G(\theta_d(a_g))) = 0.$$

By definition of the offer functions, the profit is maximised at ask $a_g = A_g(v_g)$ and hence $\theta_g(a_g) = v_g$. Replacing v_g with $\theta_g(a_g)$ the first order condition is thereby altered to:

$$-\theta_d'(a_g)(a_g - \theta_g(a_g))g(\theta_d(a_g)) + \frac{1}{2}(1 - G(\theta_d(a_g))) = 0. \tag{5}$$

Likewise, over the region where trade occurs with positive probability, expected marginal profit to the distributor is given by:

$$\pi_d^c(b_d, v_d) = \int_{\underline{v}}^{\theta_g(b_d)} \left(v_d - \frac{A_g(v_g) + b_d}{2} \right) g(v_g) dv_g.$$

Applying Leibniz' rule and then replacing v_d with $\theta_d(b_d)$, we arrive at the first order condition:

$$\theta_g'(b_d)(\theta_d(b_d) - b_d)g(\theta_g(b_d)) - \frac{1}{2}G(\theta_g(b_d)) = 0. \tag{6}$$

For $v_j \sim \mathrm{U}[\underline{v}, \overline{v}]$, it follows that $\theta_g(a_g)$ and $\theta_d(b_d)$ must satisfy the following pair of differential equations:

$$-2\theta_d'(a_g)(a_g - \theta_g(a_g)) + (\overline{v} - \theta_d(a_g)) = 0 \tag{7}$$

$$2\theta_g'(b_d)(\theta_d(b_d) - b_d) - (\theta_g(b_d) - \underline{v}) = 0. \tag{8}$$

3.3 The equilibrium ask and bid functions

From Theorem 3 of Chatterjee and Samuelson (1983) we know that a solution to Eqs. (7) and (8) is necessary for an equilibrium with exception for at the offer bounds when $v_d \leq A_g(\underline{v})$ and $v_g \geq B_d(\overline{v})$. Under the assumption that parties bid their valuation at these bounds (i.e., $A_g(v_g) = v_g$ whenever $v_g \geq B_d(\overline{v})$ and $B_d = v_d$ whenever $v_d \leq A_g(\underline{v})$), it follows from Theorem 3.1 of Satterthwaite and Williams (1989) that the solution also is sufficient for an equilibrium. Accordingly, in the appendix I show that the linear equilibrium in the certificate market takes the following form.

Proposition 1. Whenever $v_d \geq A_g(\underline{v})$ and $v_g \leq B_d(\overline{v})$, the equilibrium ask and bid functions in the green certificate market are given by:

$$A_g(v_g) = \frac{2}{3}v_g + \frac{3\overline{v} - \underline{v}}{12} \tag{9}$$

$$B_d(v_d) = \frac{2}{3}v_d + \frac{3\overline{v} + 7\underline{v}}{36} \tag{10}$$

and the corresponding certificate price is given by:

$$p_c = \frac{1}{3}\left(v_g + v_d + \frac{3\overline{v} + \underline{v}}{6}\right) \qquad \text{if } b_d \geq a_g.$$

Proof. The proof is in Appendix A.1.

Notice that Eqs. (9) and (10) reduce to the strategies derived in Chatterjee and Samuelson (1983) for the unit interval, $[\underline{v}, \overline{v}] = [0, 1]$, namely $A_g(v_g) = \frac{2}{3}v_g + \frac{1}{4}$ and $B_d(v_d) = \frac{2}{3}v_d + \frac{1}{12}$. As discussed in that paper, one issue with this bargaining mechanism is that it precludes sales in many circumstances when trade would be mutually beneficial, due to bid shading above and below valuation of the seller and the buyer, respectively.[13] Trade is feasible whenever $v_d \geq v_g$, and would consequently occur with probability 1/2 if both made truthful offers given the symmetry of the problem ($G(v_g) = G(v_d)$). In other words, there is a significant chance that the certificate market does not clear when parties use the optimal strategies defined by Eqs. (9) and (10). By assumption, they would then have to settle with the state at the boundary prices or wait until a new auction round.[14] The succeeding analysis of the electricity market is focused to the case that conditions on trade in the certificate market.

Nevertheless, Proposition 1 indicates that the possibility of trade can be influenced by adjustment of the upper and the lower price bounds. In particular, an increase in $\overline{v}$ has a positive effect on both offers, although the effect is 2/12 times larger on the ask price. The latter is not so reassuring when it comes to the possibility of trade. Yet, an increase in $\overline{v}$ should reasonably increase v_d too (and hence the bid price further), by making it more attractive to acquire certificates at the market rather than paying a penalty of $\overline{v}$ per missing certificate. This is good for the performance of the market mechanism as well, as it enhances the willingness to comply with the quota obligation. An increase in $\underline{v}$ should improve the likelihood of trade; a unit change in $\underline{v}$ reduces the ask price by 1/12 and increases the bid price by 7/36. Contingent on a successful bargain, Proposition 1 further shows that $\overline{v}$ has a larger impact on the trade price than $\underline{v}$. Accordingly, adjusting $\overline{v}$ is appropriate when a high certificate price is required to make the green producer competitive in the electricity market (i.e., when producers are fairly asymmetric in costs).

[13] Manipulating Eqs. (9) and (10) shows that trade occurs if and only if $v_d \geq v_g + \frac{3\overline{v} - 5\underline{v}}{12}$.

[14] As discussed in Section 2.3, the quota obligation is typically set on a relatively lengthy time interval while the model is focused to the short run dynamics.

4 Equilibrium analysis of the electricity market

4.1 Preliminaries

With the equilibrium offer strategies in the certificate market at hand, I now turn to the analysis of the electricity market. To solve for equilibrium bidding strategies in this market, I build on the results of Kaplan and Zamir (2012). They extend the analysis of Vickrey (1961) and Griesmer et al. (1967), who solve the first-price auction for the uniform distribution and the cases of symmetry and of asymmetry in one end of the support, to the general case of asymmetry in both ends of the support. The relevant adjustments of their procedure is made to fit the procurement auction setting considered here.[15]

Before continuing, some presumptions should be clarified. First, to eliminate multiple equilibria, it is assumed that a producer with zero probability of winning in equilibrium bids its cost. Second, I assume $\overline{x}_g \leq \overline{c}_b$ such that the certificate policy creates a cost advantage for at least the most inefficient green producer. This assumption is without loss of generality and of greater policy interest compared to the reversed relation. Under this assumption, if $\overline{x}_g \leq 2\underline{c}_b - \overline{c}_b$ any Nash equilibrium must include the green producer bidding $\underline{c}_b$ and thereby always winning the auction.[16] This outcome is trivial and uninteresting. For that reason, I consider only cases wherein $\overline{x}_g = \overline{c}_g - \mathbb{E}(p_c|v_g, b_g < b_b) > 2\underline{c}_b - \overline{c}_b$. This implies that there are limits to the asymmetry of producers if both are to be able to compete, and imposes constraints on the support on certificate valuations as well. Specifically, it suggests that $\overline{v}$ and $\underline{v}$ should be defined to satisfy $36(\overline{c}_g + \overline{c}_b - 2\underline{c}_b) > 15\overline{v} + 19\underline{v}$ for a non-trivial equilibrium (i.e., one in which both bidders have a positive probability of winning) to exist in the electricity market.[17]

Last, I assume that the reservation price is set equal to the smallest price where all production always participates in the auction, that is, $P = \overline{c}_b$. A similar assumption is imposed in Holmberg and Wolak (2018) for the symmetric case.

4.2 The first order conditions

I proceed in the same manner as before, starting by deriving the first order conditions for an equilibrium from which the equilibrium bidding strategies are determined in the next

[15] See also Cole and Davies (2014) for a similar application of Kaplan and Zamir (2012)'s results to a procurement auction.

[16] The proof is analogous to that in Kaplan and Zamir (2012) for the buy-auction. Let c_b^* denote the lowest cost of the brown producer for which it wins with zero probability. If $c_b^* = \underline{c}_b$, bidding $\underline{c}_b$ is optimal for the green producer; it assures winning at a positive profit for any cost-type given $\overline{x}_g \leq \overline{c}_b$ and $\overline{x}_g \leq 2\underline{c}_b - \overline{c}_b$. If $c_b^* > \underline{c}_b$, then by assumption the brown producer bids its cost for all $c_b > c_b^*$. Since the brown producer wins with some probability for all $c_b < c_b^*$, the green producer must bid c_b^* with positive probability for some x_g. For this to be part of an equilibrium, bidding c_b^* and win with a positive probability must yield a weakly larger profit than bidding $\underline{c}_b$ and win with certainty. Thus, it must hold that $(1 - F_b(c_b^*))(c_b^* - x_g) = \frac{\overline{c}_b - c_b^*}{\overline{c}_b - \underline{c}_b}(c_b^* - x_g) \geq (\underline{c}_b - x_g) = (1 - F_b(\underline{c}_b))(\underline{c}_b - x_g)$. When $\overline{x}_g \leq 2\underline{c}_b - \overline{c}_b$, the left hand side is decreasing in c_b^* and at $c_b^* = \underline{c}_b$ the left hand side equals the right hand side, providing a contradiction. See Kaplan and Zamir (2012) for a further discussion on this and the first (about multiple equilibria) presumptions.

[17] This result follows from substituting the certificate price associated with the value of v_g that just permits trade in the certificate market, i.e. $\mathbb{E}(p_c|v_g = \mathbb{E}(v_d) - \frac{3\overline{v} - 5\underline{v}}{12}, b_g < b_b) = \frac{15\overline{v} + 19\underline{v}}{36}$, into $\overline{x}_g = \overline{c}_g - \mathbb{E}(p_c|v_g, b_g < b_b) > 2\underline{c}_b - \overline{c}_b$ and rearranging.

subsection. Let $\beta_g(x_g)$ and $\beta_b(c_b)$ denote a pair of strictly increasing and differentiable equilibrium bid functions with inverses $\phi_g(b_g)$ and $\phi_b(b_b)$. Suppose that the brown producer bids according to $\beta_b(c_b)$ and the green producer bids b_g. Expected marginal profit to the green producer is then given by:

$$\pi_g^e(b_g, x_g) = (b_g - x_g)[1 - F_b(\phi_b(b_g))],$$

with first order condition:

$$\begin{aligned} 1 - F_b(\phi_b(b_g)) &= (b_g - x_g)F_b'(\phi_b(b_g))\phi_b'(b_g) \\ &= (b_g - \phi_g(b_g))F_b'(\phi_b(b_g))\phi_b'(b_g), \end{aligned}$$

where the last equality follows from substituting $\phi_g(b_g)$ in place of x_g, noting that by definition of the bid functions the profit is maximised at bid $b_g = \beta_g(x_g)$ and hence $\phi_g(b_g) = x_g$. As proved by e.g. Kaplan and Zamir (2012), the closure of the set of equilibrium bids in which a bidder has a positive probability of winning must be the same for both bidders. This means that while the equilibrium bid functions are defined on different domains, their inverses are defined on the same domain, $[\underline{b}, \overline{b}]$, in a non-trivial equilibrium. The first order condition must be satisfied for any $b_g \in [\underline{b}, \overline{b}]$, such that:

$$1 - F_b(\phi_b(b)) = (b - \phi_g(b))F_b'(\phi_b(b))\phi_b'(b),$$

replacing b_g with generic b. Likewise, expected marginal profit to the brown producer at bid b_b, when the green producer bids according to $\beta_g(x_g)$, is given by:

$$\pi_b^e(b_b, c_b) = (b_b - c_b)[1 - F_g(\phi_g(b_b))],$$

with first order condition:

$$\begin{aligned} 1 - F_g(\phi_g(b_b)) &= (b_b - c_b)F_g'(\phi_g(b_b))\phi_g'(b_b) \\ &= (b_b - \phi_b(b_b))F_g'(\phi_g(b_b))\phi_g'(b_b) \end{aligned}$$

and for any $b_b \in [\underline{b}, \overline{b}]$:

$$1 - F_g(\phi_g(b)) = (b - \phi_b(b))F_g'(\phi_g(b))\phi_g'(b),$$

replacing b_b with generic b.

For $x_g \sim \mathrm{U}[\underline{x}_g, \overline{x}_g]$ and $c_b \sim \mathrm{U}[\underline{c}_b, \overline{c}_b]$, it follows that $\phi_g(b)$ and $\phi_b(b)$ must satisfy the following pair of differential equations on the interval $[\underline{b}, \overline{b}]$:

$$\overline{x}_g - \phi_g(b) = (b - \phi_b(b))\phi_g'(b) \tag{11}$$

$$\overline{c}_b - \phi_b(b) = (b - \phi_g(b))\phi_b'(b), \tag{12}$$

with boundary conditions (proof in Appendix A.2):

$$\phi_g(\underline{b}) = \underline{x}_g \text{ and } \phi_b(\underline{b}) = \underline{c}_b \tag{B1}$$

$$\phi_b(\overline{b}) = \overline{b} \tag{B2}$$

$$\phi_g(\overline{b}) = \overline{x}_g. \tag{B3}$$

Adding Eqs. (11) and (12) together and observing that the expressions on each side can be written as a derivative after rearranging terms, gives:

$$\frac{\partial}{\partial b}(\phi_g(b)\phi_b(b)) = \frac{\partial}{\partial b}[b(\phi_g(b) + \phi_b(b) - (\overline{x}_g + \overline{c}_b))].$$

By integrating each side we have:

$$\phi_g(b)\phi_b(b) = b(\phi_g(b) + \phi_b(b) - (\overline{x}_g + \overline{c}_b)) + k_1, \tag{13}$$

where k_1 is the constant of integration. To find k_1, the common highest bid, $\overline{b}$, must first be determined.

Lemma 1. The upper bound of the bid functions, $\overline{b}$, is given by:

$$\overline{b} = \frac{\overline{x}_g + \overline{c}_b}{2}. \tag{14}$$

Proof. The proof is in Appendix A.3.

Provided a sufficiently high certificate price or small asymmetry in costs, $\overline{x}_g < \overline{c}_b$ and Lemma 1 then implies $\overline{b} < \overline{c}_b$. By assumption, the brown producer consequently bids its cost for $c_b \in (\overline{b}, \overline{c}_b]$ and loses the auction. In addition, this means that the electricity market will clear at a price less than the reservation price.

Using Lemma 1 along with boundary condition (B2) to evaluate Eq. (13) at $\overline{b}$, the integration constant is found to equal $k_1 = \frac{(\overline{x}_g + \overline{c}_b)^2}{4}$. Furthermore, using this expression the common lowest bid, $\underline{b}$, can be determined.

Lemma 2. The lower bound of the bid functions, $\underline{b}$, is given by:

$$\underline{b} = \frac{\frac{(\overline{x}_g + \overline{c}_b)^2}{4} - \underline{x}_g \underline{c}_b}{(\overline{x}_g - \underline{x}_g) + (\overline{c}_b - \underline{c}_b)}.$$

Proof. Use boundary condition (B1) to evaluate Eq. (13) at $\underline{b}$. Solving the equation for $\underline{b}$ gives the expression in the lemma. □

4.3 The equilibrium bid functions

The two differential equations can be reduced to one by solving Eq. (13) for $\phi_b(b)$ in terms of $\phi_g(b)$:

$$\phi_b(b) = \frac{b\phi_g(b) - b(\overline{x}_g + \overline{c}_b) + \frac{(\overline{x}_g+\overline{c}_b)^2}{4}}{\phi_g(b) - b}, \tag{15}$$

where $\frac{(\overline{x}_g+\overline{c}_b)^2}{4} = k_1$. Using the above, Eq. (11) can be rewritten as:

$$\phi_g'(b)\left(b(\overline{x}_g + \overline{c}_b) - b^2 - \frac{(\overline{x}_g + \overline{c}_b)^2}{4}\right) = (\phi_g(b) - b)(\overline{x}_g - \phi_g(b)). \tag{16}$$

From this expression an analytical solution can be derived. As proved by Griesmer et al. (1967), this solution also satisfies the second-order condition and hence constitutes an equilibrium. The results are summarised in the following proposition.

Proposition 2. The equilibrium inverse bid functions in the electricity market are given by:

$$\phi_g(b) = \overline{x}_g - \frac{(\overline{c}_b - \overline{x}_g)^2}{(2b - \overline{c}_b - \overline{x}_g)k_g e^{\frac{\overline{c}_b - \overline{x}_g}{\overline{c}_b + \overline{x}_g - 2b}} + 4(b - \overline{c}_b)} \tag{17}$$

$$\phi_b(b) = \overline{c}_b - \frac{(\overline{c}_b - \overline{x}_g)^2}{(2b - \overline{x}_g - \overline{c}_b)k_b e^{\frac{\overline{x}_g - \overline{c}_b}{\overline{x}_g + \overline{c}_b - 2b}} + 4(b - \overline{x}_g)}, \tag{18}$$

where

$$k_g = \frac{\frac{(\overline{c}_b - \overline{x}_g)^2}{\underline{x}_g - \overline{x}_g} - 4(\overline{c}_b - \underline{b})}{2(\overline{b} - \underline{b})} e^{\frac{\overline{x}_g - \overline{c}_b}{2(\overline{b} - \underline{b})}} \qquad < 0 \tag{19}$$

$$k_b = \frac{\frac{(\overline{c}_b - \overline{x}_g)^2}{\underline{c}_b - \overline{c}_b} - 4(\overline{x}_g - \underline{b})}{2(\overline{b} - \underline{b})} e^{\frac{\overline{c}_b - \overline{x}_g}{2(\overline{b} - \underline{b})}} \qquad < 0. \tag{20}$$

Proof. The proof is in Appendix A.4.

Evidently, these results are impractical to interpret analytically and are therefore illustrated in some numerical examples. Figure 2 depicts the inverse bid functions for bids on the interval $[\underline{b}, \overline{b}]$. In graph (a), producers are assumed to be ex-ante symmetric in their gross marginal costs with $[\underline{c}_b, \overline{c}_b] = [\underline{c}_g, \overline{c}_g] = [2, 4]$. Graph (b) corresponds to the general setting of ex-ante asymmetric producers, with $[\underline{c}_b, \overline{c}_b] = [1, 3]$ and $[\underline{c}_g, \overline{c}_g] = [2, 4]$.[18] v_g is set equal to

[18] The assumption of the green producer having a higher marginal cost than the brown producer is frequent in the literature on interactions between electricity and green certificate markets. This is plausible for the case of, e.g., one fossil-fuelled and one bio-fuelled technology (or potentially the case of two renewable technologies where it is reasonable that the marginal cost of a developing technology is higher than that of a mature technology). This should be the most policy relevant case under the given presumptions about the market

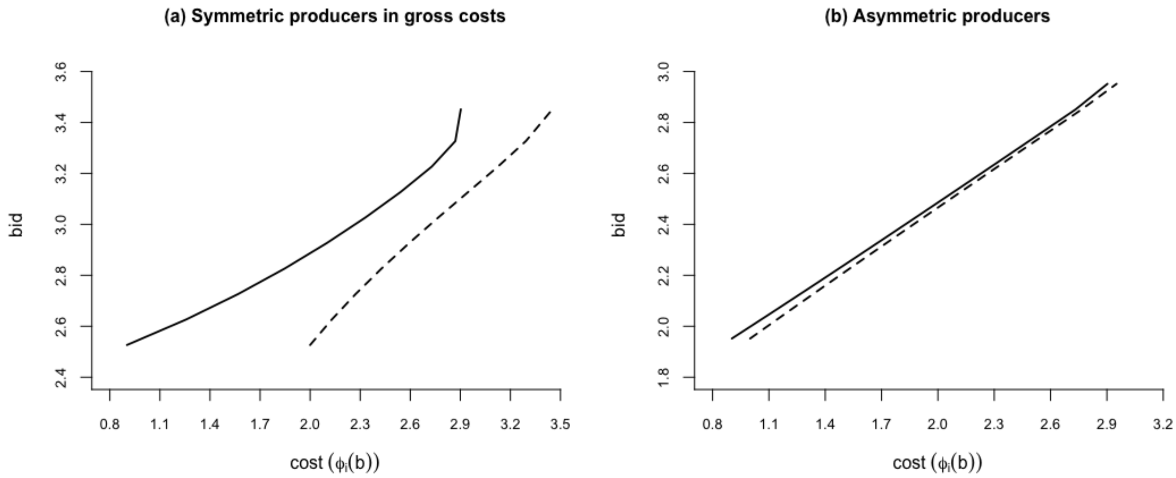

Figure 2: The solution of $\phi_g(b)$ and $\phi_b(b)$ for $b \in [\underline{b}, \overline{b}]$ and $[\underline{v}, \overline{v}] = [0.5, 2]$, when $[\underline{c}_b, \overline{c}_b] = [\underline{c}_g, \overline{c}_g] = [2, 4]$ (graph (a)) and $[\underline{c}_b, \overline{c}_b] = [1, 3]$; $[\underline{c}_g, \overline{c}_g] = [2, 4]$ (graph (b)). Solid lines are $\phi_g(b)$ and dashed lines are $\phi_b(b)$.

the valuation that just permits trade in the certificate market, that is, $v_g = \mathbb{E}(v_d) - \frac{3\overline{v}-5\underline{v}}{12}$, for $[\underline{v}, \overline{v}] = [0.5, 2]$. This yields $\mathbb{E}(p_c|v_g, b_g < b_b) = 1.1$ and $[\underline{x}_g, \overline{x}_g] = [0.9, 2.9]$. Thus, expected revenues from certificates cover about 37% of expected marginal cost to the green producer. The figure illustrates the characteristics of the bid functions established in Section 4, namely that the lowest bid is submitted by the lowest cost type of each producer, the highest winning bid is submitted by the highest cost type of the green producer, and $\overline{b}$ is lower than $\overline{c}_b = P$. This follows from the boundary conditions and bidding slightly above cost, as well as Lemma 1.

In addition, when producers are symmetric in gross marginal costs, the advantage the certificate policy creates enables the green producer to increase its bid at given cost as the probability of winning the auction increases. This forces the brown producer to bid closer to cost to be able to compete in the auction. Hence the market outcome may be inefficient should the brown producer, i.e. the high-cost producer, submit the lowest bid and be called to produce electricity. In such an equilibrium generation costs are not minimised. This is less of a concern when $\underline{v}$ and $\overline{v}$ are specified such that the asymmetry in net costs becomes small, as in graph (b), where markups are fairly equal between producers.[19] Meanwhile, Figure 3 (graph (b)) shows how this changes as $\underline{v}$ and $\overline{v}$ are adjusted upward. When the asymmetry in net costs increases and the green producer becomes more advantaged, the brown producer is compelled to bid closer to cost. Observe further that the light grey line extends below zero. This is because the certificate price exceeds $\underline{c}_g$ in this specification, resulting in a negative cost associated with the minimum bid for the green producer. Moreover, a given increase in

structure. In particular, in the case of one fossil-fuelled technology and one solar or wind power technology, it is plausible that the outcome would be the trivial equilibrium discussed in Section 4.1 since the short-run marginal cost of the latter technology is close to zero.

[19]In Figure 2, graph (a), the markups $\frac{b}{\phi_g(b)}$ and $\frac{b}{\phi_b(b)}$ are on average equal to 56% and 12%, respectively. The corresponding figures in graph (b) are 38% and 33%.

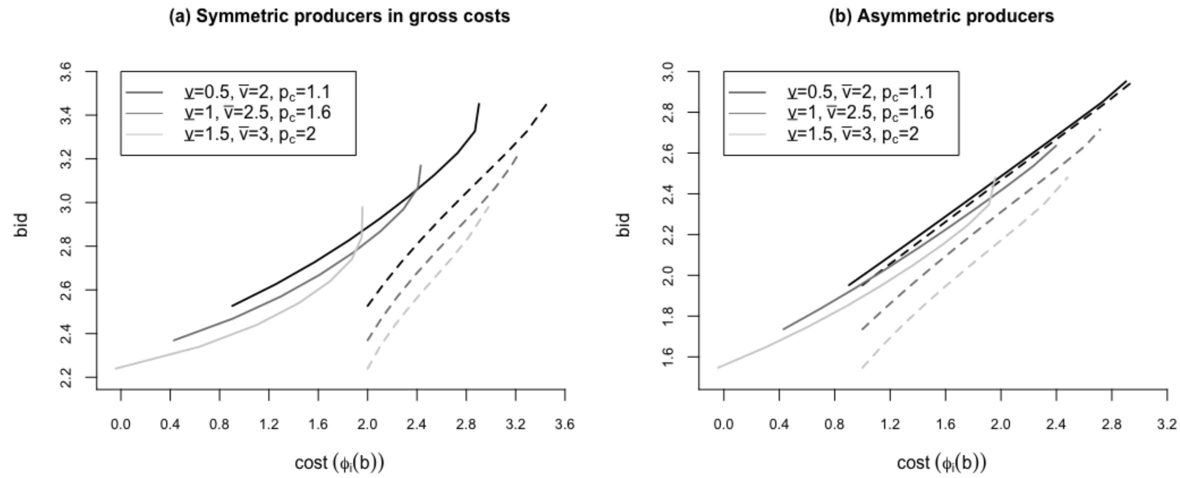

Figure 3: The solution of $\phi_g(b)$ and $\phi_b(b)$ for $b \in [\underline{b}, \overline{b}]$ when $[\underline{c}_b, \overline{c}_b] = [\underline{c}_g, \overline{c}_g] = [2, 4]$ (graph (a)) and $[\underline{c}_b, \overline{c}_b] = [1, 3]$; $[\underline{c}_g, \overline{c}_g] = [2, 4]$ (graph (b)). Solid lines are $\phi_g(b)$ and dashed lines are $\phi_b(b)$.

the certificate price results in a less than proportional decrease in the bid range in the electricity market. For instance, when $\mathbb{E}(p_c|v_g, b_g < b_b)$ raises by 0.5 units from 1.1 to 1.6 in Figure 3, $\underline{b}$ and $\overline{b}$ decline by about 0.2 units each.[20]

Overall, these examples indicate that the bounds on certificate prices should be set with careful consideration not to overcompensate the favoured technology, as it results in high markups to this producer and harms competition.

5 Concluding remarks

This paper developed a simple duopoly model of an electricity market integrated with a market for tradable green certificates. The purpose has been to investigate the impact of the certificate market on bidding behaviour in the electricity market, noting that the producer entitled to certificates should place its bids in the electricity auction based on the net marginal cost after subtracting the certificate price.

When producers are ex-ante asymmetric in their marginal costs, the results indicate that the certificate policy can be successful in reducing this asymmetry and improve competition and efficiency in the electricity market. Conversely, when producers are ex-ante symmetric in their marginal costs, the advantage the certificate policy creates enables the subsidised producer to place a higher bid at given cost as the probability of winning the electricity auction increases. This markup becomes larger as the advantage increases due to a higher certificate price, beyond a level that may be desirable to ensure the correct investment incentives in renewables. This is harmful for competition and equity between producers and consumers, as the consumer price of electricity is determined by the sum of the wholesale electricity

[20] Figure A1 in Appendix A.5 depicts a similar shape of the bid functions for alternative specifications of the range of marginal costs. Meanwhile, by a similar logic as described above, the green producer can still earn a higher markup when the asymmetry between producers is focused to one end of the support in graph (b).

and the certificate prices. Darmani et al. (2016) review research showing that excess profits, high consumer prices and a lack of equity are issues that indeed have been experienced in real-world certificate schemes. One key implication of the results is therefore that policy makers should properly define the price bounds on certificates, which shape price offers in the certificate market, such that an adequate level of support is provided depending on the degree of asymmetry in marginal costs. Nonetheless, this can be a challenging task if information on costs is poor, or if producers entitled to certificates are considerably heterogeneous in costs.

The paper builds on a stylised yet tractable model that can be extended in several ways. One limitation is that it is restricted to the case of nonpivotal producers. Though it most likely comes at the cost of obtaining a closed-form solution, further attempts in solving the pivotal case with asymmetric producers should be encouraged in future research. There are at least two reasons for this. First, the assumption of symmetry is questionable, even without the presence of climate policy, when modelling todays electricity markets where heterogeneous non-renewable and renewable technologies compete. Second, it is more likely that a producer becomes pivotal in markets with a large share of renewables, since the available capacity of these plants is contingent on fluctuating weather conditions (Holmberg and Wolak, 2018; Fabra and Llobet, 2021).

Appendix A

A.1 Proof of Proposition 1

Start by reducing the two differential equations into one by solving Eq. (8) for $\theta_d(b_d)$ in terms of $\theta_g(b_d)$:

$$\theta_d(b_d) = \frac{\theta_g(b_d) - \underline{v}}{2\theta'_g(bd)} + b_d. \tag{21}$$

Hence:

$$\begin{aligned}\theta'_d(b_d) &= 1 + \frac{1}{2}\left(\frac{\theta'_g(b_d)\theta'_g(b_d) - (\theta_g(b_d) - \underline{v})\theta''_g(b_d)}{(\theta'_g(b_d))^2}\right) \\ &= \frac{3}{2} - \frac{1}{2}\frac{(\theta_g(b_d) - \underline{v})\theta''_g(b_d)}{(\theta'_g(b_d))^2}.\end{aligned}$$

Substituting the above expressions into Eq. (7) (replacing b_d with a_g) yields:

$$(\theta_g(a_g) - a_g)\left(3 - \frac{(\theta_g(a_g) - \underline{v})\theta''_g(a_g)}{(\theta'_g(a_g))^2}\right) + \overline{v} - a_g - \frac{(\theta_g(a_g) - \underline{v})}{2\theta'_g(a_g)} = 0. \tag{22}$$

Now, assume a linear solution of the form $\theta_g(a_g) = \gamma a_g + \lambda$, such that $\theta'_g(a_g) = \gamma$ and $\theta''_g(a_g) = 0$. Substitution into Eq. (22) gives:

$$\begin{aligned}3(\gamma a_g + \lambda - a_g) + \overline{v} - a_g - \frac{(\gamma a_g + \lambda - \underline{v})}{2\gamma} &= 0 \\ \gamma a_g(6\gamma - 9) + 6\lambda\gamma + 2\overline{v}\gamma - \lambda + \underline{v} &= 0.\end{aligned}$$

It can be inferred that $\theta_g(a_g) = \gamma a_g + \lambda$ is a solution if $\gamma = \frac{3}{2}$ and $\lambda = \frac{\underline{v}-3\overline{v}}{8}$. In other words, $\theta_g(a_g) = \frac{3}{2}a_g + \frac{\underline{v}-3\overline{v}}{8}$. Eq. (9) is obtained by inverting this function.

Finally, substitute the expression for $\theta_g(a_g)$, and its derivative, into the best response function of the distributor, Eq. (22). Doing so results in: $\theta_d(b_d) = \frac{3}{2}b_d - \frac{(3\overline{v}+7\underline{v})}{24}$. Eq. (10) is obtained by inverting this function. □

A.2 Proof of boundary conditions

(B1) $\phi_g(\underline{b}) = \underline{x}_g$ and $\phi_b(\underline{b}) = \underline{c}_b$. First note that it is never optimal to bid below cost as this would result in a loss should one submit the lowest bid. Because of monotonicity of the bid functions, the lowest bid for each bidder must therefore be reached for its lowest cost.

(B2) $\phi_b(\overline{b}) = \overline{b}$. By Lemma 3.13 of Griesmer et al. (1967), $\phi_b(\overline{b}) = \min\{\overline{c}_b, \overline{b}\}$. Since there are no bids above $\overline{c}_b = P$, $\min\{\overline{c}_b, \overline{b}\} = \overline{b}$ so $\phi_b(\overline{b}) = \overline{b}$.

(B3) $\phi_g(\overline{b}) = \overline{x}_g$. Again, by Lemma 3.13 of Griesmer et al. (1967), $\phi_g(\overline{b}) = \min\{\overline{x}_g, \overline{b}\}$. Since $\overline{c}_b \geq \overline{x}_g$ by assumption, it must be $\overline{b} \geq \overline{x}_g$. Otherwise, if the green producer had a cost on $[\overline{b}, \overline{x}_g]$, it would be accepting a positive loss on this interval. Hence $\min\{\overline{x}_g, \overline{b}\} = \overline{x}_g$ so $\phi_g(\overline{b}) = \overline{x}_g$. □

A.3 Proof of Lemma 1[21]

Recall that the green producer with cost x_g solves the maximisation problem:

$$\max_{b\in[\underline{b},\overline{b}]} (b - x_g)\left(\frac{\overline{c}_b - \phi_b(b)}{\overline{c}_b - \underline{c}_b}\right).$$

The green producer with cost $x_g(\overline{b})$ must not benefit from deviating from $\overline{b}$ by bidding above or below it. Hence, using boundary condition (B2), $\phi_b(\overline{b}) = \overline{b}$, and noticing that the brown producer has a zero probability of winning for $c_b \geq c_b(\overline{b})$ and bids at cost, implying that $\phi_b(b) = b$ must hold for any $b \geq \overline{b}$, it follows that the following inequality must be satisfied:

$$(\overline{b} - x_g(\overline{b}))(\overline{c}_b - \overline{b}) \geq (b - x_g(\overline{b}))(\overline{c}_b - b) \qquad \forall\, b \geq \overline{b}.$$

Notice that the b for which the right hand side reaches its maximum is $b = \frac{x_g(\overline{b})+\overline{c}_b}{2}$. Since this holds for $b \geq \overline{b}$, we must have $\overline{b} \geq \frac{x_g(\overline{b})+\overline{c}_b}{2}$ (otherwise the green producer could gain by increasing its bid).

Furthermore, it must hold that:

$$(\overline{b} - x_g(\overline{b}))(\overline{c}_b - \overline{b}) \geq (b - x_g(\overline{b}))(\overline{c}_b - \phi_b(b)) \qquad \forall\, b \leq \overline{b}.$$

However, the brown producer optimally bids slightly above cost, meaning that $b \geq \phi(b)$. Therefore the following inequality must also be satisfied:

$$(\overline{b} - x_g(\overline{b}))(\overline{c}_b - \overline{b}) \geq (b - x_g(\overline{b}))(\overline{c}_b - b) \qquad \forall\, b \leq \overline{b}.$$

[21]Based on Kaplan and Zamir (2012), Lemma 2, and Cole and Davies (2014), Lemma 1.

Recall that the b for which the right hand side reaches its maximum is $b = \frac{x_g(\bar{b})+\bar{c}_b}{2}$. Since this holds for $b \leq \bar{b}$, we must have $\bar{b} \leq \frac{x_g(\bar{b})+\bar{c}}{2}$ (otherwise the green producer could gain by decreasing its bid). But $\bar{b} \geq \frac{x_g(\bar{b})+\bar{c}_b}{2}$, so it must be $\bar{b} = \frac{x_g(\bar{b})+\bar{c}_b}{2}$. By boundary condition (B3), $\phi_g(\bar{b}) = \overline{x}_g$, we have $\bar{b} = \frac{\overline{x}_g+\bar{c}_b}{2}$. □

A.4 Proof of Proposition 2

Although a detailed proof is already provided in Kaplan and Zamir (2012), for convenience, one that fits the setting and notation in this paper follows. Start by rewriting Eq. (16) as:

$$\phi_g'(b)(\overline{x}_g + \bar{c}_b - 2b)^2 = 4(\phi_g(b) - b)(\phi_g(b) - \overline{x}_g). \tag{23}$$

Now define $\alpha = \overline{x}_g + \bar{c}_b - 2\overline{x}_g = \bar{c}_b - \overline{x}_g$, $z = b - \overline{x}_g$ and $\Gamma(z)$ such that:

$$\phi_g(z + \overline{x}_g) = \frac{\alpha^2}{\Gamma(z)} + \overline{x}_g. \tag{24}$$

Then, $\phi_g'(z + \overline{x}_g) = -\frac{\alpha^2}{\Gamma(z)^2}\Gamma'(z)$ and Eq. (23) becomes:

$$\begin{aligned}
-\frac{\alpha^2}{\Gamma(z)^2}\Gamma'(z)(\overline{x}_g + \bar{c}_b - 2b)^2 &= 4\left(\frac{\alpha^2}{\Gamma(z)} + \overline{x}_g - b\right)\left(\frac{\alpha^2}{\Gamma(z)} + \overline{x}_g - \overline{x}_g\right) \\
\Gamma'(z)(\alpha - 2z)^2 &= 4(\Gamma(z)z - \alpha^2) \\
\Gamma'(z)(\alpha - 2z)^2 &= 4\Gamma(z)z - 16z(\alpha - z) - 4(\alpha - 2z)^2 \\
(\Gamma'(z) + 4)(\alpha - 2z)^2 &= 4z(\Gamma(z) - 4(\alpha - z)).
\end{aligned}$$

Which can be rewritten as:

$$\begin{aligned}
\frac{\Gamma'(z) + 4}{\Gamma(z) - 4(\alpha - z)} &= \frac{4z}{(\alpha - 2z)^2} \\
&= \frac{2\alpha}{(\alpha - 2z)^2} - \frac{2}{\alpha - 2z}.
\end{aligned}$$

By integrating each side we obtain:

$$\ln(\Gamma(z) - 4(\alpha - z)) = \frac{\alpha}{\alpha - 2z} + \ln(\alpha - 2z) + \ln k_g,$$

where k_g is the constant of integration. Moreover, taking the exponent of each side gives:

$$\Gamma(z) - 4(\alpha - z) = (\alpha - 2z)e^{\frac{\alpha}{\alpha - 2z}}k_g. \tag{25}$$

The lower boundary condition (B1), $\phi_g(\underline{b}) = \underline{x}_g$, determines k_g. When $b = \underline{b}$, $z = \underline{z} = \underline{b} - \overline{x}_g$ and consequently $\Gamma(\underline{z}) = \frac{\alpha^2}{\underline{x}_g - \overline{x}_g}$. Hence we have:

$$k_g = \frac{\frac{\alpha^2}{\underline{x}_g - \overline{x}_g} - 4(\alpha - (\underline{b} - \overline{x}_g))}{\alpha - 2(\underline{b} - \overline{x}_g)} e^{-\frac{\alpha}{(\alpha - 2(\underline{b} - \overline{x}_g))}}.$$

This can be rewritten as (recall $\bar{b} = \frac{\bar{x}_g + \bar{c}_b}{2}$ and $\alpha = \bar{c}_b - \bar{x}_g$):

$$k_g = \frac{\frac{(\bar{c}_b - \bar{x}_g)^2}{\underline{x}_g - \bar{x}_g} - 4(\bar{c}_b - \underline{b})}{2(\bar{b} - \underline{b})} e^{\frac{\bar{x}_g - \bar{c}_b}{2(\bar{b} - \underline{b})}},$$

which is Eq. (19).

Eq. (17) is obtained from Eqs. (24) and (25) and the definitions of α and z. Eqs. (18) and (20) are obtained from the equivalent series of steps, starting by solving Eq. (13) for $\phi_g(b)$ in terms of $\phi_b(b)$. □

A.5 Alternative numerical examples of Proposition 2

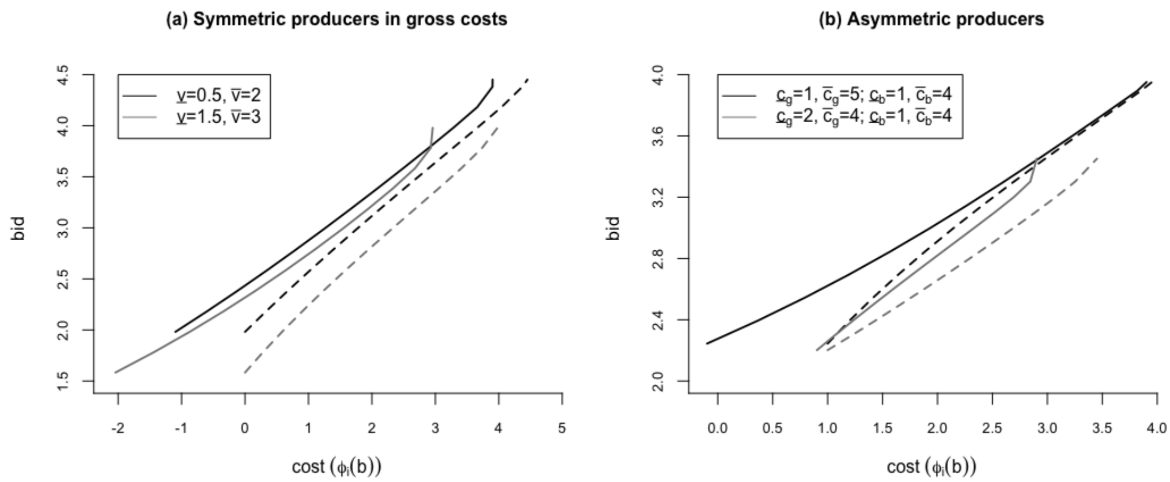

Figure A1: The solution of $\phi_g(b)$ and $\phi_b(b)$ for $b \in [\underline{b}, \bar{b}]$, solid lines are $\phi_g(b)$ and dashed lines are $\phi_b(b)$. Graph (a) illustrates the solution under larger uncertainty about marginal costs compared to Figures 2 and 3 (a), specifically $[\underline{c}_b, \bar{c}_b] = [\underline{c}_g, \bar{c}_g] = [0, 5]$. Graph (b) illustrates the solution when restricting the asymmetry to one end of the cost intervals, for $[\underline{v}, \bar{v}] = [0.5, 2]$.

References

Amundsen, E. S. and Bergman, L. (2012). Green Certificates and Market Power on the Nordic Power Market. *The Energy Journal*, 33(2):101–117.

Amundsen, E. S. and Mortensen, J. B. (2001). The Danish Green Certificate System: some simple analytical results. *Energy economics*, 23(5):489–509.

Amundsen, E. S. and Nese, G. (2016). Market Power in Interactive Environmental and Energy Markets: The Case of Green Certificates. *CESifo Working Paper Series*, (5922):1–32.

Böhringer, C. and Rosendahl, K. E. (2010). Green promotes the dirtiest: on the interaction between black and green quotas in energy markets. *Journal of Regulatory Economics*, 37(3):316–325.

Chatterjee, K. and Samuelson, W. (1983). Bargaining under Incomplete Information. *Operations Research*, 31(5):835–851.

Cole, M. T. and Davies, R. B. (2014). Foreign Bidders Going Once, Going Twice... Government Procurement Auctions with Tariffs. *Florida International University Working Paper*.

Darmani, A., Rickne, A., Hidalgo, A., and Arvidsson, N. (2016). When outcomes are the reflection of the analysis criteria: A review of the tradable green certificate assessments. *Renewable and Sustainable Energy Reviews*, 62:372–381.

European Commission (2013). Commission staff working document. European Commission guidance for the design of renewables support schemes.

Fabra, N. and Llobet, G. (2021). Auctions with Privately Known Capacities: Understanding Competition among Renewables. *CEPR Discussion Papers*, 14060:1–55.

Fabra, N., von der Fehr, N.-H. M., and de Frutos, M.-Á. (2011). Market Design and Investment Incentives. *The Economic Journal*, 121(557):1340–1360.

Fabra, N., Von der Fehr, N.-H. M., and Harbord, D. (2006). Designing Electricity Auctions. *The RAND Journal of Economics*, 37(1):23–46.

Greenstone, M. and Nath, I. (2019). Do renewable portfolio standards deliver? *University of Chicago, Becker Friedman Institute for Economics Working Paper*, 62.

Griesmer, J. H., Levitan, R. E., and Shubikt, M. (1967). Toward a study of bidding processes. Part IV – games with unknown costs. *Naval Research Logistics Quarterly*, 14(4):415–433.

Holmberg, P. and Wolak, F. A. (2018). Comparing auction designs where suppliers have uncertain costs and uncertain pivotal status. *The RAND Journal of Economics*, 49(4):995–1027.

Jensen, S. G. and Skytte, K. (2002). Interactions between the power and green certificate markets. *Energy Policy*, 30(5):425–435.

Kadan, O. (2007). Equilibrium in the two-player, k-double auction with affiliated private values. *Journal of Economic Theory*, 135(1):495–513.

Kaplan, T. R. and Zamir, S. (2012). Asymmetric first-price auctions with uniform distributions: analytic solutions to the general case. *Economic Theory*, 50(2):269–302.

Leininger, W., Linhart, P. B., and Radner, R. (1989). Equilibria of the Sealed-Bid Mechanism for Bargaining with Incomplete Information. *Journal of Economic Theory*, 48(1):63–106.

Lemming, J. (2003). Financial risks for green electricity investors and producers in a tradable green certificate market. *Energy Policy*, 31(1):21–32.

Myerson, R. B. and Satterthwaite, M. A. (1983). Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29(2):265–281.

Radner, R. and Schotter, A. (1989). The Sealed-Bid Mechanism: An Experimental Study. *Journal of Economic Theory*, 48(1):179–220.

Satterthwaite, M. A. and Williams, S. R. (1989). Bilateral Trade with the Sealed Bid k-Double Auction: Existence and Efficiency. *Journal of Economic Theory*, 48(1):107–133.

Tamás, M. M., Shrestha, S. B., and Zhou, H. (2010). Feed-in tariff and tradable green certificate in oligopoly. *Energy policy*, 38(8):4040–4047.

Vickrey, W. (1961). Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of finance*, 16(1):8–37.

von der Fehr, N.-H. M. and Harbord, D. (1993). Spot Market Competition in the UK Electricity Industry. *The Economic Journal*, 103(418):531–546.

von der Fehr, N.-H. M. and Ropenus, S. (2017). Renewable Energy Policy Instruments and Market Power. *The Scandinavian Journal of Economics*, 119(2):312–345.