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Abstract

In most real-life binary referendums, there are several alternatives that potentially can challenge the status quo alternative. Depending on which alternative that is selected, the voters are also differently likely to caste their vote on it. The fact that there are several potential challenger alternatives also means that there may exist Condorcet cycles that only can be identified by taking into account the alternatives that not are listed on the ballot. We analyse such "hidden" cycles in a simple theoretical framework where Condorcet cycles cannot exist, but may emerge when taking into account that voters often experience a reluctance to abandon the status quo alternative. Necessary and sufficient conditions for the existence of hidden Condorcet cycles are derived and a Monte Carlo simulation finds (in different scenarios) that the probability is roughly one percent.

Keywords: binary referendum, hidden Condorcet cycles, non-trivial referendums, Monte Carlo study.

1 Introduction

Ever since the seminal work by Marquis de Condorcet in the late 18th century, so-called Condorcet cycles has been one of the centerpieces in voting theory. These cycles are easiest described using three alternatives, say x_1, x_2 and x_3, where the electorate prefers x_1 to x_2, x_2 to x_3, and x_3 to x_1. This type of cyclicity constitutes a democratic dilemma, commonly referred to as the Condorcet Paradox, as a majority of the electorate are dissatisfied with the outcome of the vote, independently of which of the three alternatives that is coined as the winner. However, Condorcet cycles can normally not be observed due to lack of information. That is, to identify them,

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the outcome of a set of pairwise majority votes must be known, but in real-life situations voters typically only cast one vote.

To better understand the prevalence of Condorcet cycles, researchers have attempted to calculate the probability of their existence and, when possible, to empirically investigate their occurrence. The findings are not conclusive and depends on a number of different factors, e.g., the number of voters and alternatives, and the probability (both in simulated and empirical environments) varies between zero and ten percent (see, e.g., Black, 1958; May, 1971; Gehrlein, 2006; Gehrlein and Fishburn, 1976; Gehrlein and Lepelley, 2011; Tideman, 2006; van Deemen, 2014). The democratic dilemma, associated with Condorcet cycles, has also inspired a large theoretical literature that aims to characterize preference domains under which cycles cannot exist (see, e.g., Black, 1958; Downs, 1957; Gaertner, 2001; Inada, 1964; McKelvey, 1976; Sen and Pattanaik, 1969).

Most real-life referendums are binary, meaning that a single alternative represented as, e.g., a new policy or scenario, competes against a status quo alternative. Multi-alternative referendums offer a wider range of policy alternatives, but are much rarer in practice. Between 2000 and 2019, there were only nineteen multi-alternative referendums in the world (Wagenaar, 2020).[1] The fact that a referendum is binary does, however, not mean that Condorcet cycles are non-existing even if their presence require that there are at least three alternatives. More precisely, while the status quo alternative is fixed, there may be several potential challenger alternatives even if only one of them is allowed to contest the status quo alternative in the actual binary referendum.

To make the latter point clear, consider the The United Kingdom European Union membership referendum, commonly referred to as the Brexit referendum, on 23 June, 2016, where the electorate was asked whether the country should remain a member of, or leave, the European Union. Clearly, the status quo alternative to "remain" was fixed, but there was more room to define the "exit alternative." One plausible alternative is to make a "hard exit" and leave both the European Union and the European economic area. A more "soft exit" is to leave the European Union, but stay in the European economic area. Depending which alternative that is chosen, the voters may be differently likely to vote for an "exit," and it is also in this selection process that hidden Condorcet cycles may emerge. They are hidden because they cannot be observed by only observing the outcome of the referendum. Instead, alternatives that not were on the ballot have to be taken into consideration to identify the cycle. In the Brexit referendum, it is in fact not unlikely that a Condorcet cycle existed and, based on several voter polls, it has been suggested that the voters would prefer a soft exit to remain, remain to a hard exit, but the latter alternative to a soft exit (Eggers, 2021, uses two Brexit polls and shows how diagrams, among other things,

[1]For empirical analysis of multi-alternative referendums and agenda setting, see, e.g., Wagenaar (2020) and Wagenaar and Hendriks (2021).

can be used to identify Condorcet cycles).[2,3]

To analyze hidden Condorcet cycles, this paper considers the simplest possible theoretical framework with finite sets of voters and alternatives, but where only two alternatives enter the binary referendum. Voters are endowed with symmetric single-peaked preferences. In this framework, it is well-known that there cannot be any Condorcet cycles (see, e.g., Black, 1958; Downs, 1957, or Remark 1). However, binary referendums, where the electorate decides whether to abandon the status que alternative or not, are often irreversible (at least for a long period of time, see, e.g., Moldovanu and Rosar, 2021) and it has then been observed in the literature that some voters may experience a reluctance to abandon the status quo alternative. For example, because of a status quo bias (Fernandez and Rodrik, 1991; Samuelson and Zeckhauser, 1988), voter loyalty and the benefits of the doubt (Feld and Grofman, 1991; Sloss, 1973), or simply because of various costs (Buchanan and Tullock, 1962; Hinich and Ordeshook, 1969; Tullock, 1967). To capture such voter frictions, or status quo bias, the voters are partitioned into two types; the ones that never would abandon the status quo alternatives and the ones that may consider doing it, but only if the challenger alternative deviates sufficiently much from the status quo. When such frictions are present, Condorcet cycles may exist, even if they are hidden phenomenons in the actual referendum. This also means that an agenda setter may become powerful in the usual sense, i.e., choosing the agenda may be equivalent to choosing the outcome. Note, however, that the considered model is not about political competition in the Downsian sense, but rather a tool to illustrate that voter frictions may reintroduce Condorcet cycles even in the simplest possible theoretical framework where such cycles cannot exist without the frictions.

Focus will be directed towards referendums that are interesting from an analytical perspective, so-called non-trivial referendums. These are the referendums where the status quo alternative not wins against each of the potential challenger alternatives or not looses against all potential challenger alternatives. The main theoretical contribution of this paper is to provide a necessary and sufficient condition for the existence of hidden Condorcet cycles in the considered framework. A Monte Carlo simulation evaluates the proportion of non-trivial referendums and investigates the probability that Condorcet cycles exists. The simulation study shows that almost all referendums are non-trivial, and that the probability that a Condorcet cycle exists is roughly one percent. The latter finding is robust under different assumptions on, e.g., on the number of voters, alternatives, and peak distributions.

The rest of the paper is outlined as follows. Section 2 introduces the basic model and some key definitions. The theoretical findings and the Monte Carlo simulation can be found in Sections 3 and 4, respectively. Some conclusions and final remarks are provided in Section 5.

[2]See the article "Deal>Remain>No-deal>Deal: Brexit and the Condorcet Paradox," available at `https://blogs.lse.ac.uk/politicsandpolicy/brexit-condorcet/`.

[3]It is easy to find similar examples. For example a yes or a no vote to death penalty needs to specify under which circumstances the penalty is applicable (e.g., for first-degree murder or a more restrictive policy), and a yes or no vote to extend the presidential term must also specify the extension in number of years (e.g., an extension by one or two years).

2 The Model and Basic Definitions

This section introduces the simplest possible model that captures the essential features of binary referendums where voters may experience emotional switching costs when abandoning the status quo alternative, noting that all results presented in the paper hold under more general assumptions (throughout the paper, remarks related to generalizations are delegated to footnotes).

The voters are gathered in the finite set $N = \{1, \ldots, n\}$ for some odd number n. The alternatives are represented by integer numbers, collected in the finite set $X = \{x_0, \ldots, x_m\}$, where x_0 represents the status quo alternative.[4] It is convenient to think about the index j as a measure of the "policy distance" between alternative x_j and the status quo alternative. So, the higher index j an alternative have, the more it deviates from x_0. For example, in a referendum about leaving the EU, it may be instructive to regard the status quo alternative as remain in the EU, and alternatives x_1, x_2 and x_3 as renegotiate a new deal within the EU, leave the EU but stay in European economic area, and leave both the EU and the European economic area, respectively.

Each voter $i \in N$ have a weak preference relation over the finite set of alternatives X denoted by R_i, with corresponding strict and indifference relations P_i and I_i, respectively. The preference relation R_i have a unique maximum element denoted by x_i^*, henceforth referred to as the peak of voter i. The peaks are gathered in the vector $x^* = (x_1^*, \ldots, x_n^*)$, and are distributed on the interval X according to the discrete function $G(x^*)$. There is an ordering $>$ of the alternatives in X such that, for every voter $i \in N$:

$$x_k < x_j \leq x_i^* \text{ implies that } x_j P_i x_k,$$
$$x_k > x_j \geq x_i^* \text{ implies that } x_j P_i x_k.$$

That is, when voter i compares two distinct alternatives that both are either to "the right'" or to "the left" of her peak x_i^*, she strictly prefers whichever alternative that is closest to x_i^*. The preference relation R_i is also assumed to be symmetric around its peak, i.e., when voter i evaluates two distinct alternatives on different sides of the peak, she strictly prefers whichever alternative that is closest to her peak and can, therefore, only be indifferent between two distinct alternatives if their distances to the peak are identical. In case a voter needs to break ties, it is assumed that the voter always selects the alternative with the lowest index.[5] The vector $R = (R_1, \ldots, R_n)$ contains the preference relations of the voters.

A fraction of all voters, called the type-S voters (S stands for "status quo"), have their peaks at the status quo alternative x_0. Voters that don't have their peaks at alternative x_0 are referred to

[4]The fact that the status quo alternative has index 0 is not crucial for any of the results, but this assumption simplifies the analysis since it can be restricted to only focus on alternatives that located to "the right" of the status quo alternative.

[5]All theoretical results presented in this paper hold even if this assumption is relaxed and some other tie-breaker is applied, but the assumption will make the analysis more to the point as some cases need not be analysed. See footnote 6 for some further remarks.

as type-A voters (A stands for "alternative"). Each type-A voter i have preferences R_i on X, but with a positive friction that captures some type of emotional switching cost when abandoning the status quo alternative (see the Introduction section for references). Thus, the status quo alternative have a special meaning for all type-A voters in the sense that they are not willing to abandon it for a "sufficiently small" policy change. The friction f_i for a type-A voter i belongs to the set $\{x_1, \ldots, x_m\}$. If $f_i = x_l$, voter i strictly prefers the status quo alternative x_0 to any of the alternatives in the set $\{x_1, \ldots, x_l\}$.[6] Since all type-S voters have their peaks at the status quo alternative, their frictions are, without loss of generality, set to x_0. Note, however, that when a voter evaluates two alternatives in $X^0 = X \setminus \{x_0\}$, the friction is irrelevant as it is only "active" when an alternative is evaluated against the status quo alternative. The frictions of the voters are gathered in the vector $F = (f_1, \ldots, f_n)$. A profile $D = (R, F)$ is a complete description of voter preferences with frictions.

A binary referendum is a majority vote between the status quo alternative and some other alternative in the set X^0.[7] In general, the profile D determines the outcome of a majority vote between any two alternatives in X, where $x_j D x_k$ means that alternative x_j wins a majority vote against alternative x_k. For any two alternatives $x_j, x_k \in X^0$, let:

$$\text{mean}(x_j, x_k) = G(\lfloor 0.5 \times (x_j + x_k) \rfloor).$$

If $j < k$ and $\text{mean}(x_j, x_k) \geq 0.5$, then alternative x_j wins the majority vote against alternative x_k since at least fifty percent of the peaks are more closely located to x_j than to x_k.[8]

For a given profile D, an alternative x_j is a Condorcet winner if $x_j D x_k$ for any distinct alternative $x_k \in X$. There exists a Condorcet cycle for a given profile D if $x_j D x_{k_1}, \ldots, x_{k_l} D x_{k_{l+1}}$, and $x_{k_{l+1}} D x_j$ for $x_j, x_{k_1}, \ldots, x_{k_l}, x_{k_{l+1}} \in X$ and some integer $l \in \{1, \ldots, m-1\}$.

Remark 1. There are two assumptions that make the above described model deviate from the classical framework of political competition (Black, 1958; Downs, 1957). Namely, that some political candidate must represent the status quo alternative and that voters have frictions. If both these assumptions are dropped, the model reduces to the classical frameworks of political competition. Among other things, this implies that the peak of the median voter $\overline{x}$ is the Condorcet

[6] For simplicity, it is assumed that each type-A voter have a positive friction. All theoretical results presented in this paper hold under a weaker assumption, namely for profiles where the set of voters containing all type-S voters and all type-A voters with a positive friction constitutes a majority.

[7] The assumption that there is a binary referendum captures most real-life referendums, even if there are exceptions, see, e.g., Wagenaar (2020) or Wagenaar and Hendriks (2021). Furthermore, it is likely that the alternative in the referendum must deviate "sufficiently much" from the status quo alternative to motivate a referendum in the first place, i.e., that the index j of the challenger alternative x_j must be sufficiently high.

[8] The notation $\lfloor \cdot \rfloor$ means that the number is rounded down to the closest integer, which may be needed if x_j is odd and x_k is even, or vice versa. This will not affect any of the conclusions since the alternatives in X^0 are represented by integers, so the discrete distribution function G is not defined for any of the numbers in the open interval (x_j, x_k). Note also that the inequality is weak since an agent that is indifferent between any two alternatives, without loss of generality, is assumed to select the alternative with the lowest index.

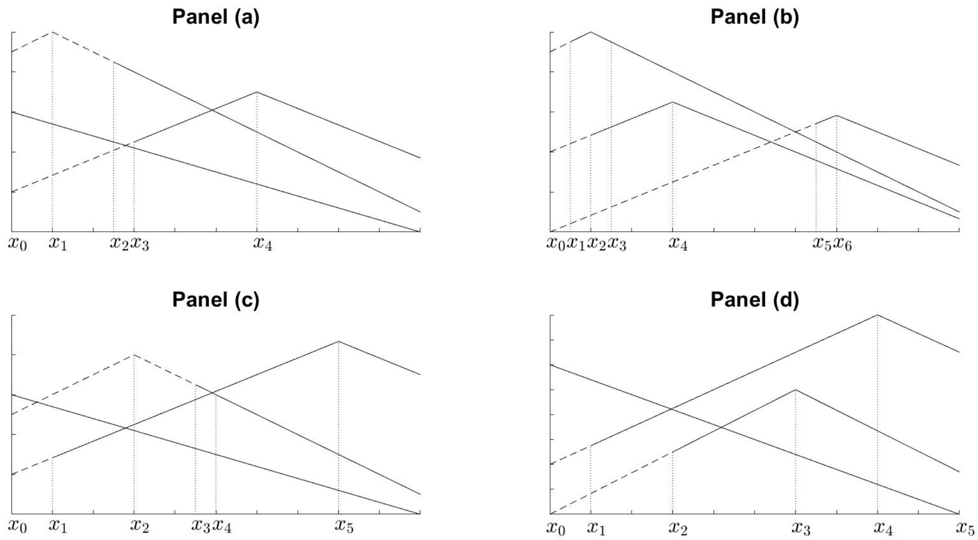

Figure 1: Panels (a)–(d) in the figure illustrate the four leading examples considered in this paper.

winner and that there are no Condorcet cycles.[9] □

3 Theoretical Findings

Throughout this section, the insights from the four panels of Figure 1 will be used to illustrate the main theoretical findings. Each panel in the figure describes the preferences of three voters.[10] In panel (a), there is one type-S voter and two type-A voters. The latter two voters have their peaks at x_1 and x_4 with frictions at x_2 and x_3, respectively. The part of R_i where the frictions are "active" when the status quo alternative is evaluated is represented by a dashed line. Note that the median peak $\overline{x}$ is "inactive" in panels (a) and (c) in the sense that the frictions make the median peak loose a majority vote against the status quo alternative. If the main findings only would hold under in such a restrictive situation, they wouldn't be very interesting. However, as will be established later, all results presented in this paper holds independently of if the median peak is "inactive" or not.

Restriction will be directed towards profiles D where the binary referendum is non-trivial (the Monte Carlo simulation in Section 4 evaluates the frequency of such referendums). Informally,

[9]To see that there are no Condorcet cycles in this case, recall that any voter who faces two alternatives will always prefer the alternative that is "closest" to her peak. Consequently, if x_iDx_j and x_jDx_k, then a majority of the voters find that their peaks are closer to x_i than to x_j and that their peaks are closer to x_j than to x_k. But then, it follows directly that a majority of the voters also finds that their peaks is closer to x_i than to x_k, so there cannot be any Condorcet cycles.

[10]Only the alternatives that are relevant for the analysis are marked in the figure, but we assume that they are finite in number even if all results presented in this paper also qualitatively hold for a continuum of alternatives.

these are the referendums where the status quo alternative not wins against each of the potential challenger alternatives or not looses against all potential challenger alternatives. Formally:

Definition 1. For a given profile D, a binary referendum is non-trivial if there are alternatives $x_j, x_k \in X^0$ such that (i) $x_0 D x_j$ and (ii) $x_k D x_0$.

A referendum based on the profile illustrated in panel (a) in Figure 1 is not non-trivial since the status quo alternative wins over any other alternative in X^0. This follows since the type-S voter always votes for x_0. Furthermore, the voter with the peak at x_1 not only prefers x_0 to x_2 and x_0 to any of the alternatives to the left of alternative x_2 because of her friction, but the voter also prefers x_0 to any of the alternatives to the right of x_2. The profiles illustrated in panels (b)–(d) in Figure 1, however, induces non-trivial referendums. To see this, note first that the status quo alternative x_0 wins over alternative x_1 in panels (b)–(d), but the status quo alternative x_0 loses to alternatives x_3, x_4 and x_3 in panels (b), (c) and (d), respectively. Consequently, a binary referendum is non-trivial only for some profiles.

The following result shows that there exists a special alternative, called x_r, that satisfies Definition 1(i) and, in addition, is most "closely located" to any alternative that satisfies Definition 1(ii) without being located to "the right" of any such alternative.

Lemma 1. For any profile D where the referendum is non-trivial, there exists some alternative $x_r \in X^0$ with a highest index r such that $x_0 D x_r$, and $r < k$ for any alternative x_k where $x_k D x_0$.

Proof. The fact that there is an alternative $x_j \in X^0$ where $x_0 D x_j$ follows immediately since the referendum is non-trivial and voters have frictions. In particular, this is true for $j = 1$ since all type-A voters have frictions (see also footnote 6), which doesn't exclude that $j > 1$. Define now k as the lowest index where $x_k D x_0$ and note that $k > 1$ by the previous conclusions. But then since $x_0 P x_j$ for $j = 1$, there must be an alternative $x_r \in X^0$ with a highest index r such that $x_0 P x_r$, and $r < k$ for any alternative x_k where $x_k D x_0$. □

In panels (b), (c) and (d) in Figure 1, x_r is given by x_2, x_3 and x_2, respectively. Note also from panels (b) and (c) that x_r can be located either to "the left" or "the right" of the median peak $\overline{x}$.

In the classical framework, where voters don't have frictions, for any two alternatives $x_j \neq \overline{x}$ and $x'_k \neq \overline{x}$, it always hold that $\overline{x} P x_j$ and $\overline{x} P x_k$. Consequently, the peak of the median voter $\overline{x}$ is always the Condorcet winner and there are no Condorcet cycles (see Remark 1). When voter frictions are introduced, the next result reveals that the latter conclusion need not hold (even if the median peak obviously may be the Condorcet winner for some profiles D even when frictions are present).

Lemma 2. For profiles D where the referendum is non-trivial, the median peak $\overline{x}$ need not be a Condorcet winner. This conclusion holds independently of if $\overline{x} > x_r$ or $\overline{x} < x_r$.

Proof. To prove the the lemma, it suffices to find one example of a non-trivial referendum with an alternative x_k such that $x_k D\overline{x}$. The result is proved using Figure 1 where it already has been established that the referendums in panels (b)–(d) are non-trivial.

Panel (b) shows that the median peak $\overline{x}$ need not be a Condorcet winner in the case when $\overline{x} > x_r$. Note first that $\overline{x} = x_4$ and $x_r = x_2$, and consider a majority vote between alternatives x_0 and x_4. Because the voters with peaks at x_2 and x_6, vote for the status quo alternative, it follows that $x_0 D x_4$.

Panel (c) shows that the median peak $\overline{x}$ need not be a Condorcet winner in the case when $\overline{x} < x_r$. To see this, note first that $\overline{x} = x_2$ and $x_j^r = x_3$, and consider a majority vote between alternatives x_0 and x_2. Because the voters with peaks at x_0 and x_2, vote for the status quo alternative, it follows that $x_0 D x_2$. □

Lemma 2 shows that the median peak need not play the same decisive role when voters have frictions as it does in the classical framework of political competition, previously discussed in Remark 1. As will become clear shortly, this insight also means that Condorcet cycles may be present (which, obviously doesn't mean that they always are present; there is, for example, no Condorcet cycle in panel (b) of Figure 1). But even when Condorcet cycles are present, the challenger alternative need not be selected in such a way that the Condorcet cycle is problematic from a democratic perspective, in the sense that a majority of the voter prefer another alternative than the winning. To make this point clear, consider Figure 1(d) where there is a Condorcet Cycle since $x_0 D x_2$, $x_2 D x_5$, and $x_5 D x_0$.[11] An agenda setter that nominates the challenger alternative x_5 will, consequently, win the referendum against the status quo alternative x_0. In other words, an agenda setter can successfully push the winning alternative away from the median peak. But as the observant reader will see, any of the alternatives x_2, x_3, and x_4 win a majority vote against alternative x_5, and any of the alternatives x_3, x_4 and x_5 win a majority vote against the status quo alternative x_0. Therefore, the Condorcet cycle can be "neutralized" if the challenger alternative is appropriately selected. More precisely, in this specific example, alternative x_3 turns out to be the Condorcet winner, so even if there exists a Condorcet cycle, it need not be problematic from a democratic perspective if the agenda setter, for some reason, decides to nominate alternative x_3 to challenge the status quo alternative.

To neutralize Condorcet cycles (if they exist), the distribution of peaks as well as the distribution of frictions must be known. In any real-life referendum, an agenda setter is unlikely to possess that type of information, so the process for the agenda setter to nominate the challenger alternative must somehow be approximated, e.g., based on various investigations and polls.[12] But

[11] This follows since the voters with peaks at x_0 and x_3 vote for x_0 in a majority vote against x_2. Furthermore, alternative x_2 wins a majority vote over alternative x_5 (the voters with peaks at x_0 and x_3 vote for alternative x_2), and alternative x_5 wins a majority vote over alternative x_0 (the voters with peaks at x_3 and x_4 votes for alternative x_5).

[12] Romer and Rosenthal (1978, 1979) assumes that the agenda setter have complete information about the preferences of the voters in their model with a status quo alternative. As observed by Banks (1990), it is more realistic

there is always a positive probability that such information is misinterpreted or biased in some way and that the agenda setter therefore enters the referendum with a "non-optimal alternative" from a voting maximizing perspective. This is particularly true as it has been established in Lemma 2 that the median peak is not as informative when frictions are present as in the classical framework without frictions. So, if the agenda setter enters the referendum with a "non-optimal alternative," the Condorcet cycles may in fact not be neutralized and therefore also problematic from a democratic perspective. For that reason, it is important to characterize under which circumstances Condorcet cycles exists.

Theorem 1. For any profile D where the referendum is non-trivial, there exists a Condorcet cycle if and only if (i) $\overline{x} \leq x_r$ or (ii) $\overline{x} > x_r$ and $\text{mean}(x_r, x_s) \geq 0.5$ where x_s is the alternative with the highest index $s \in \{r+1, \ldots, m\}$ such that $x_s D x_0$.

Proof. Note first that there exists profiles D such that the referendum is non-trivial in both cases (i) and (ii), so both cases are relevant (see Figure 1 and the analysis of it). In the remaining part of the proof, such profile D is fixed.

We start by proving that if condition (i) or (ii) holds, then there is a Condorcet cycle. Because the referendum is non-trivial, there exists an alternative that wins a majority vote over the status quo alternative. This observation together with Lemma 1 imply that there exists an alternative x_s with a highest index such that $x_s D x_0$. So, it needs only to be established that there is an alternative $\hat{x} \in X^0$ such that $x_0 D \hat{x}$ and $\hat{x} D x_s$. From Lemma 1, it follows that there exists some alternative $x_r \in X^0$ with a highest index $r < k$ such that $x_0 P x_r$ for any alternative x_k where $x_k D x_0$. By setting $\hat{x} = x_r$, it follows that $x_0 P \hat{x}$, so it remains only to prove that $\hat{x} D x_s$. Note next that the latter condition holds if $\text{mean}(\hat{x}, x_s) \geq 0.5$ since $\hat{x}, x_s \in X^0$ (recall that the frictions doesn't play any role for the alternatives in X^0, so the classical results hold, see Remark 1). But this condition holds in case (i) since $\overline{x} \leq x_r$ and $r < s$, and in case (ii) by the assumption in the statement of the theorem as $\hat{x} = x_r$.

We next prove that if there is a Condorcet cycle, then condition must (i) or (ii) hold. Suppose that there is a Condorcet cycle, but that neither of the two conditions hold. The latter means that $\overline{x} > x_r$ and $\text{mean}(x_r, x_s) < 0.5$. Note next that by definition of x_r, for any alternative x_k where $x_k D x_0$, it must be the case that $k > r$. Consequently, because there is a Condorcet cycle by assumption, there must be alternatives $x_j, x_k \in X^0$ such that $x_0 D x_j$ and $x_j D x_k$ where $k > r$. Since, $\text{mean}(x_r, x_s) < 0.5$, it follows that $x_s D x_r$. Hence, $x_k D x_j$ for any $k = r+1, \ldots, s$ and any $j = 1, \ldots, r$. But this means that a Condorcet cycle cannot exist, which contradicts our assumptions. □

As observed in the above, there is no Condorcet cycle in Figure 1(b), and the reason is that condition (ii) in Theorem 1 is violated. This follows since $\overline{x} = x_4$, $x_r = x_3$, and $x_s = x_3$, so $\overline{x} > x_r$ and $\text{mean}(x_r, x_s) = 0.33 < 0.5$.

to assume that the agenda setter have incomplete information. For an analysis of this case, see, e.g., Banks (1993, 1990) and Morton (1988).

4 Monte Carlo Simulations

A Monte Carlo simulation will be employed to evaluate the probability that a hidden Condorcet cycle exists. The results are presented for averages over 1,000 simulations for each of the 600 considered instances (i.e., $5 \times 3 \times 4 \times 2 \times 5 = 600$) based on the variables in Table 1. The variables "number of voters" and "number of alternatives" are self-explained, but the others must be clarified. "Type-S voter probability" represents the probability that a given voter is of type-S, which is determined by a random draw based on the instance specific probability. The distribution of peaks G and the distribution of frictions F for a given number of voters are determined by random draws from discrete Poisson distributions with mean λ_G and λ_F, respectively, based on the variables "mean peak percentage" and "mean friction percentage." For example, if there are 20 alternatives and both "mean peak percentage" and "mean friction percentage" are given by 0.5, then $\lambda_G = 20 \times 0.5 = 10$ and $\lambda_F = \lambda_G \times 0.5 = 10 \times 0.5 = 5$.[13]

Variable	Notation	Considered values
Number of voters	n	11, 101, 1,001, 10,001, 100,001
Number of alternatives	m	6, 10, 20
Type-S voter probability	π_S	0.10, 0.20, 0.30, 0.40
Mean peak percentage	μ_{x^*}	0.50, 0.75
Friction mean percentage	μ_F	0.25, 0.50, 0.75, 1.00, 1.25

Table 1: Variables and values for the Monte Carlo simulation. There are 600 instances in total, and for a given number of voters, there are 120 instances.

For a given number of voters, all 120 instances (i.e., $3 \times 4 \times 2 \times 5 = 120$) were evaluated based on averages over 1,000 simulations. Almost all of the simulated referendums turned out to be non-trivial, see the summary statistics related to non-trivialness in the upper half of Table 1. This is also exactly what can be expected from real-life referendums since there is little value in organizing them if there is wide consensus among the voters that the status quo alternative is preferred to all possible challenger alternatives. In fact, when the number of voters are 10,001 or 100,001, the simulation results reveal that all referendums are non-trivial, and when the number of voters is 1,001 only five out of the 120,000 simulated referendums turned out not to be non-trivial. With a smaller number of voters, there are naturally fewer non-trivial referendums since random draws are more likely to be "skewed" in some direction. A more detailed visualization related to non-trivialness for the case when $n \in \{11, 101\}$ can be found in Figure 2. But even in these cases, the referendums are almost always non-trivial independently of values of the

[13] There are two minor complications. First, the discrete Poisson distribution is defined on the half-open interval $[0, \infty)$, and the randomly generated peak and friction for any voter may (with a small probability) be larger than the number of alternatives. In such cases, the randomly generated numbers was set to "number of alternatives." Second, λ_G and λ_F need not be an integer. In such cases, the parameters was rounded to the closest integer.

variables. Most exceptions can be found when there are many type-S voters ($\pi_S \in \{0.3, 0.4\}$) and when the median peak is located far to "the right" ($\mu_{x^*} = 0.75$). This is not very unexpected as it is exactly for these cases where a majority of the voters are likely to support the status quo alternative independently which challenger alternative that is chosen.

Number of voters	11	101	1,001	10,001	100,001
Minimum proportion of non-trivial referendums	0.610	0.824	0.997	1.000	1.000
Mean proportion of non-trivial referendums	0.905	0.989	1.000	1.000	1.000
Maximum proportion of non-trivial referendums	1.000	1.000	1.000	1.000	1.000
Minimum number of Condorcet cycles	1.182	0.373	0.000	0.000	0.000
Mean number of Condorcet cycles	7.410	7.616	6.815	6.591	6.491
Maximum number of Condorcet cycles	7.410	7.616	6.815	6.591	6.491

Table 2: Summary statistics of non-trivial referendums and Condorcet cycles based on averages for the 120 instances (1,000 simulations per instance) for a given number of voters.

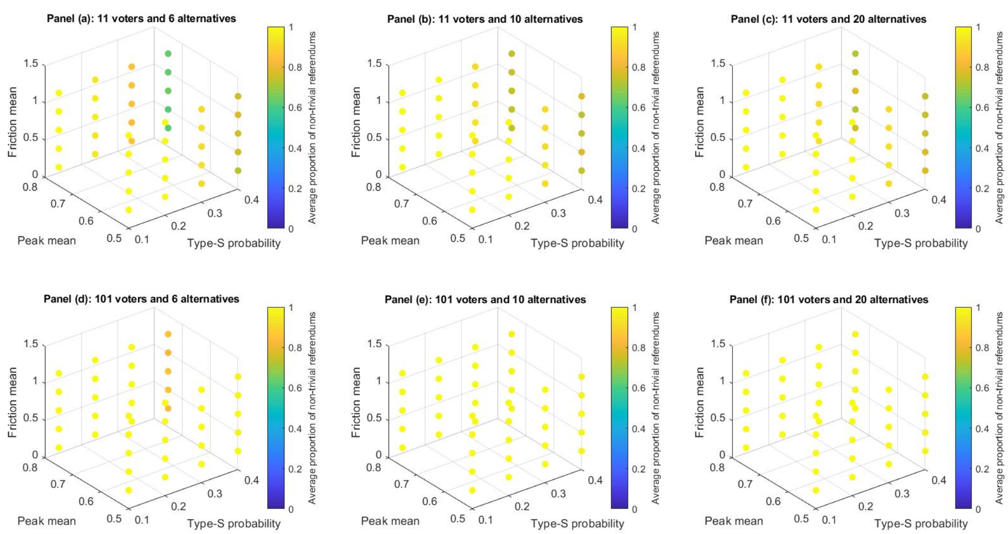

Figure 2: Panels (a)–(f) illustrate the average proportion of non-trivial referendums for the 240 instances specified in Table 1 when $n \in \{11, 101\}$.

The lower half of Table 2 displays the average number of Condorcet cycles based on the 120 instances for a given number of voters. A general take-away from the table is that it consistently seems to be around seven cycles per 1,000 simulations, implying that the probability of a Condorcet cycle is roughly 0.7 percent (the largest number of Condorcet cycles found in any of the 600,000 simulations was 97). A more detailed visualization can be found in Figure 3. As expected, the average number of Condorcet cycles grows with the number alternatives (see the five panels furthest to the right in the figure), i.e., the more alternatives, the more possible cycles. The more centered the median peak is (i.e., $\mu_{x^*} = 0.5$) and the larger proportion of type S-voters (i.e., $\pi_S \in \{0.3, 0.4\}$), the higher number of Condorcet cycles. In non-trivial referendums, this is anticipated since this reflects situations where there is an alternative that is defeated by the status quo alternative, but is preferred to some alternative to "the far right" that beats the status quo.

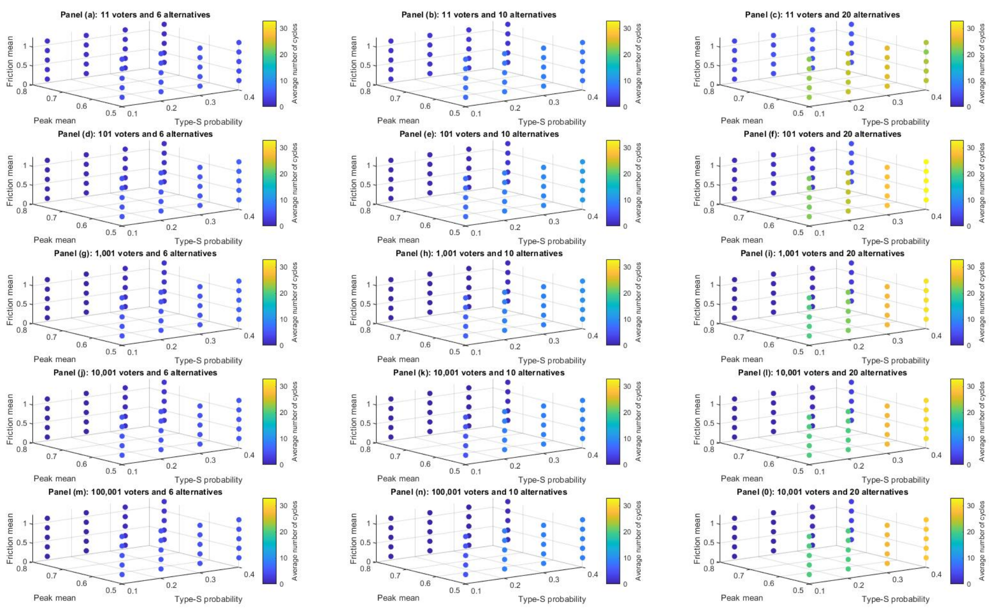

Figure 3: Panels (a)–(o) illustrate the average number of Condorcet cycles for the 600 instances specified in Table 1.

5 Conclusions

The analysis in this paper has been based on the observations that the challenger alternative not necessarily is unique in binary referendums and depending which alternative that is selected, the voters may be differently likely to cast their votes on it. The main point has been to illustrate that Condorcet cycles may exits even in the simplest possible framework if voters experience some type of reluctance to abandon the status quo alternative, say a status quo bias, voter loyalty, or simply some benefits of the doubt. These cycles are hidden to the electorate since information to even determine their existence rarely is collected (even if the case can be made for the Brexit referendum).

Further work on the empirical side includes a thorough investigation of various polls and voter investigations to see if hidden Condorcet cycles can be identified in real-life binary referendums (see,e.g., Eggers, 2021). On the theoretical side, future work includes more general models and careful modelling of various incomplete information settings In this paper, no such information structure has been imposed on the model, simply because it is not needed to make the main points related to status quo bias and hidden Condorcet cycles.

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