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## Eigenfrequency Optimization of Non-linear Hyperelastic Structures

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Topology optimization of nonlinear elastic structures has previously been studied in detail by e.g Buhl et al. [1], Wallin et al. [2] and Daeyoon et al. [3] to mention a few. However, most of the work on this subject has been restricted to compliance minimization, or rather displacement minimization since the load vector is constant. On the other hand, for linear elastic structures, many types of objectives are available in the literature. Topology optimization with respect to eigenfrequencies is one example of an objective that has been studied extensively for small deformations in e.g Pedersen [4] and Jianbin et al. [5], however few have investigated this type of optimization for structures undergoing finite deformations. In this work, we hence expand topology optimization formulations to a nonlinear hyperelastic material model.

One issue with performing topology optimization with respect to the eigenfrequencies for a nonlinear material model is that the stiffness depends on the displacements. This entails a different definition of the usual equations of motion for free vibrations, presented as

$$\mathbf{M}(\mathbf{z})\ddot{\mathbf{a}} + \mathbf{r}(\mathbf{a}, \mathbf{z}) = \mathbf{0} \quad (1)$$

Where  $\mathbf{M}(\mathbf{z})$  denotes the mass matrix,  $\mathbf{z}$  the design variables,  $\mathbf{a}$  the nodal displacements and  $\mathbf{r}(\mathbf{a}, \mathbf{z})$  the residual of the internal and external forces. However, by performing a Taylor series expansion around a deformed equilibrium state and identifying the stiffness matrix as the linearization of the internal force internal force vector with respect to the nodal displacements one will end up with an eigenvalue problem similar to the one in the linear case, i.e.

$$(\mathbf{K}(\mathbf{a}, \mathbf{z}) - \omega^2 \mathbf{M}(\mathbf{z}))\boldsymbol{\varphi} = \mathbf{0} \quad (2)$$

Where  $\mathbf{K}(\mathbf{a}, \mathbf{z})$  denotes the stiffness matrix (We assume dead loading),  $\omega$  the eigenfrequencies and  $\boldsymbol{\varphi}$  the corresponding eigenvectors. Hence, the main difference between the linear and the nonlinear material model from an topology optimization

perspective lies in the sensitivity analysis, since the eigenvalue problem now depends on the displacements. To find the sensitivities, we employ the adjoint approach where the implicit derivatives with respect to the displacements are eliminated by introducing adjoint vectors.

In this work we implement constraints for the eigenfrequencies such that a number of the smallest eigenfrequencies must be larger than a prescribed value while minimizing the displacement of the structure at the operating load under a volume constraint. Several similar eigenvalue formulations exist, e.g maximization of a specified structural eigenfrequency or maximization of the distances between two adjacent frequencies. However, motivated by the work of Jianbin et al. [5] we choose to look at only one possibility.

The method of moving asymptotes is used to solve the topology optimization problem. Moreover, the Helmholtz PDE-filter is utilized to introduce a minimum feature size in the design and hence generate a well-posed topology optimization problem. From the numerical examples we conclude that the magnitude of the load will influence the eigenvalues. Hence, it should be taken into account when analyzing nonlinear structures.

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