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A comparative analysis of ability of mimicking portfolios in representing the background factors

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Abstract

Our aim is to give a comparative analysis of ability of different factor mimicking portfolios in representing the background factors. Our analysis contains a cross-sectional regression approach, a time-series regression approach and a portfolio approach for constructing factor mimicking portfolios. The focus of the analysis is the power of mimicking portfolios in the asset pricing models. We conclude that the time series regression approach, with the book-to-market sorted portfolios as the base assets, is the most proper alternative to construct mimicking portfolios for factors for which a time-series of factor realisation is available. To construct mimicking portfolios based on the firm characteristics we suggest a loading weighted portfolio approach.

JEL G12.

Keywords: mimicking portfolio; asset pricing; multifactor model; cross-sectional regression approach; time series regression approach

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1 Introduction

A factor mimicking portfolio is a portfolio of assets constructed to stand for a background factor. This design is usually preferred to directly using the factor when its realisations are not returns. By this approach we use only the information captured in the economic factors, which is relevant for asset returns and reduce the noise in our asset pricing model (see for example Vassalou, 2000 and Chan et al, 1998 and 1999 for building mimicking portfolios based on the macroeconomic variables). In addition, a mimicking portfolio may represent an unobservable factor when the stock sensitivities to the factor are believed to be disclosed by some firm characteristics. A well-known example of this case is in the Fama and French (1993), where the characteristics such as firm size or book-to-market ratio are supposed to reveal the loadings of the firms to some latent distress factors.

Despite the fact that mimicking portfolios are widely applied both in asset pricing analyses and portfolio valuations, there is no research on the power of the different methods of constructing these portfolios. The purpose of this paper is to shed light on the characteristics of the different factor mimicking portfolios and to give a comparative analysis of these portfolios.

There are several approaches to construct factor mimicking portfolios. A formal approach is to use cross-sectional regression of returns on factor loadings or firm characteristics to estimate the return of the mimicking portfolios, henceforth denoted by the *cross-sectional regression* approach (CSR), (see Fama, 1976). Another method that is to some extent equivalent to the regression approach is to construct a portfolio by going long on stocks with high loadings on a factor while short-selling stocks with low loadings, henceforth denoted by the *portfolio* approach (see for example Fama and French, 1993 and Chan et al, 1998 and 1999). An alternative method, initiated by Breeden et al (1989), is to define a mimicking portfolio as a projection of a factor into the asset return space using a time series regression, henceforth denoted by the *time-series regression* approach (TSR), (see for example Vassalou, 2000). This approach is only relevant when a factor realisation is available. Therefore, this method can be used to form portfolios mimicking macroeconomic variables, while it is not applicable to construct mimicking portfolios based on firm characteristics.

The first part of the paper compares the CSR approach and the portfolio approach. In the portfolio approach the common practice is to form either equally weighted portfolios (see Chan et al., 1998) or value weighted portfolios (see Fama and French, 1993). These weighting methods in contrast to the CSR approach do not consider the relative differences among loadings in portfolio formation. To deal with this problem we suggest a weighting method based on the relative distances of the loadings. Moreover, to be able to interpret the estimated coefficients of the monthly CSR as the returns of the mimicking portfolios we need to normalise the explanatory variables of the regressions. We employ two alternative approaches for normalisation. To construct mimicking portfolios, in addition to the estimated asset loadings on the world market index and several macro variables, we use some firm specific variables. We use the standard deviations of the mimicking portfolios, the estimated factor risk premia and the correlations between the portfolios to compare the portfolios mimicking each factor across different methods.

The second part of the paper studies mimicking portfolios formed based on the TSR approach. This method is applicable only for factors with available time series representation. We use therefore the world market index and several macro variables as the background factors. We analyse the sensitivity of the constructed portfolios to the choice of the based assets —the assets that are used to construct the mimicking portfolios. Our metrics of comparison is again the standard deviations and the mean excess returns of the mimicking portfolios and the correlations between the portfolios.

Finally we analyses the ability of the different approaches in representing the background factors from an asset pricing perspective. We first estimate a multifactor model for ten industrial portfolios, when the factors are represented by their original factor realisation. The risk free components of the factors, and consequently the factor risk premia, are not observed when the factors are not portfolio returns. To solve this problem we estimate the model by using a maximum likelihood approach, where the risk free part of each factor is estimated endogenously as an unknown parameter. The ability of the factor model in explaining the industry mean excess returns is assessed by a likelihood ratio test. In the next step, we estimate the multifactor model while the original factor realisations are replaced by the factor mimicking portfolios. The results of the asset pricing tests and parameter estimations are then compared across different factor representations. The purpose of this analysis is not to attain a sufficient factor model but to investigate the accuracy of the factor mimicking portfolios.

It is worth noting that it is not common practice to construct mimicking portfolio for the market factor, because the factor realisations are already portfolio returns. However, the market factor is expected to be the most import factor and therefore can be extremely valuable for comparison of the power of the different methods of constructing mimicking portfolios.

The result of the comparison of the univariate CSR approach with the portfolio approach shows a high correlation between the CSR and our suggested loading weighted portfolio approach. This depends on the fact that both approaches consider the relative distances among the loadings. The analysis of standard deviations shows that the weighting method that gives higher weights to the larger loadings results in higher standard deviations of mimicking portfolios. The value weighted approach has a very low similarity with other methods, which is due to the ad-hoc nature of this weighting method. Comparing TSR approach based on different base assets shows that the correlations between mimicking portfolios and the original series are relatively high for the world market portfolio. However, for most of the macroeconomic factors the correlations are very low. Regarding both the correlation analysis and the statistics of the mimicking portfolios we may conclude that the TSR approach is to some extent sensitive to the choice of the base assets.

In the asset pricing analysis, the model with the original factors is not rejected, while the CSR and the portfolio approaches are both found to be extremely weak in explaining the industry mean excess returns. The TSR model with size-sorted portfolios as the base assets have some significant intercepts and is therefore rejected even by the likelihood ratio test while the TSR model with book-to-market sorted base assets is very strong both in the separate *t*-tests and the joint likelihood ratio test. Analysing the parameters of the factor models shows that the effects of the different factors on mean industry excess returns seem to be to some extent different across the models. The model with the original factors and the TSR-BM model with book-to-market sorted base assets are relatively similar, particularly with respect to the effect of the market factor.

The outline of the paper is as follows: section 2 discusses the methods for the construction of the factor mimicking portfolios; test of asset pricing models is described in section 3; section 4 presents the data; the empirical results and the related analyses are given in the section 5 and section 6 concludes the paper.

2 Mimicking portfolios

The purpose is to construct a portfolio with a mean return equal to the risk premium of a background factor and with a beta equal to one against the factor (see Cochrane, 2001). There are two approaches which are in essence different: the first method is to use cross-sectional regressions (*CSR*) of the excess returns on the observed/estimated factor loading, while the other approach uses instead a time-series regression (*TSR*) of an observable background factor on the excess returns. There is also a portfolio approach that is basically related to the *CSR* method.

2.1 Cross-sectional regression approach, CSR

This method is applicable both when the background factor has a time series realisation, usually not as a portfolio return, and when the factor is not observable self but firms' loading on the factor is mirrored in some firm specific variable. To use the CSR approach we require the firms' factor loadings. The loadings are available in the latter case in the form of the firm specific variables, while in the former case we can use a time series regression of asset returns on the observed factor realisation to estimate the factor loading for each firm.

To construct the mimicking portfolios we run the following cross-sectional regression model for each period, t, t=1,....,T:

$$R_{it} = \gamma_{0t} + \sum_{k=1}^{K} \gamma_{kt} x_{ikt} + u_{it}, \tag{1}$$

where R_{it} is the return in excess of the risk free rate for asset i at time t, x_{ikt} is the value of the kth explanatory variable for asset i at t, and u_{it} is an idiosyncratic error. The values of the k:th explanatory variable in this regression are the loadings of the N assets on the factor k. The least square estimate of the γ_{kt} is the return of the mimicking portfolio for factor k. It can be written as (see Fama, 1976):

$$\gamma_{kt} = \sum_{i=1}^{N} w_{ikt} R_{it} \qquad for\ k = 0,....,K \tag{2}$$

where w_{ikt} is element $(k+1,i)$ of the $((K+1) \times N)$ matrix $\mathbf{W}_t$. This matrix is defined as:

$$\mathbf{W}_t = \left(\mathbf{X}_t'\mathbf{X}_t\right)^{-1}\mathbf{X}_t', \tag{3}$$

where $\mathbf{X}_t$ is a $(N\times(K+1))$ matrix of all explanatory variables including a vector of ones for intercept. We have the following characteristics for the weights w_{ikt}:

$$\sum_{i=1}^{N} w_{ikt} = \begin{cases} 1 & for\ k = 0 \\ 0 & for\ k = 1,...K \end{cases} \tag{4}$$

Therefore, for all $k > 0$, γ_{kt} can be interpreted as the return at time t on a zero investment portfolio with weight vector w_{kt} on N base assets. We also have

$$\sum_{i=1}^{N} w_{ikt} x_{ijt} = \begin{cases} 1 & for\ j = k \\ 0 & for\ j \neq k \end{cases} \tag{5}$$

which implies that the loading of the portfolio with weights w_{ikt} should be equal to one against the factor k and zero against the other factors. In addition, $E[\gamma_{kt}]$ gives the risk premium of the factor k.

This approach despite all its desired properties has some major drawbacks. The first problem is due to the fact that the loadings are not usually observable and should be estimated. This induces the errors in variables problem and may result in bias in estimated γ_{kt}. The second problem is when we use accounting data to represent the relative asset loadings on the factors. The reason is that the magnitude of γ_{kt} depends on the size of the corresponding x_{ikt}. Thus, γ_{kt} may be incomparable across the k factors. One possibility is to normalize explanatory variables in such a way that the x_{ikt}-values fall within the same range for all k. Chan et al. (1998) rank all the loadings and then normalised them between 0 and 2, but then the relative distances between the loadings will change. Henceforth, we denote this approach by *rank*. Our suggestion is to define x_{ikt}^{*} as:

$$x_{ikt}^{*} = 2 \times \left(1 - \frac{\max_i\left(x_{ikt}\right) - x_{ikt}}{\max_i\left(x_{ikt}\right) - \min_i\left(x_{ikt}\right)}\right), \tag{6}$$

This approach, henceforth denoted by *distance*, leaves the relative distances among the loadings unchanged. The multiplication by 2 is to have approximately the same range for x^*_{ikt} as the market beta.

The observed variables, particularly those from accounting data, are related to each other. The regression approach, in a multivariate framework, makes it possible to filter out the shared components. However, our aim is to compare the mimicking portfolios across different approaches and since the multivariate setting is not possible for some of the other approaches employed in this paper we apply only the univariate framework in this study.

2.2 Portfolio approach

An alternative approach that is essentially based on the same idea as the CSR method is the portfolio approach. This method, however, may be less subject to the errors in variables and normalising problems discussed in the previous section. Stocks are sorted according to their factor loadings. Then the stocks with low and high loadings are grouped in two different portfolios. Finally a factor mimicking portfolio is constructed by taking a long position in the portfolio with high loadings and a short position in the portfolio with low loadings on the chosen factor (*HmL*). The *HmL* portfolio, like the portfolios of the regression approach, is a zero investment strategy that is particularly sensitive to the factor *k*.

There are several alternative methods to weight the stocks in the high-loading and low-loading portfolios. The most common alternatives are to form equally weighted portfolios (e.g. Chan et al., 1998) or to form value weighted portfolios (e.g. Fama and French, 1993). These weighting methods are somewhat ad hoc and they do not satisfy the relationship between weights and loadings of the regression approach. In a univariate case, with k=1, the weight of asset i on the factor k is given by:

$$w_{ikt} = \frac{-\sum_{i=1}^{N} x_{ikt}}{N\sum_{i=1}^{N} x_{ikt}^2 - \left(\sum_{i=1}^{N} x_{ikt}\right)^2} + \frac{Nx_{ikt}}{N\sum_{i=1}^{N} x_{ikt}^2 - \left(\sum_{i=1}^{N} x_{ikt}\right)^2} \tag{7}$$

This gives a perfect correlation between w_{ikt} and x_{ikt}.[1] To maintain this link, our suggestion is to weight the stocks by their relative distance of the loadings. The weight of the asset i in the portfolio with low loading on factor k is computed as:

$$w_{ikt}^{L} = 1 - \frac{x_{ikt} - \min_i(x_{ikt})}{\max_i(x_{ikt}) - \min_i(x_{ikt})}, \tag{8}$$

and for the portfolio with high loading the weight is:

$$w_{ikt}^{H} = 1 - \frac{\max_i(x_{ikt}) - x_{ikt}}{\max_i(x_{ikt}) - \min_i(x_{ikt})}, \tag{9}$$

We then normalize the weights in order to sum to one. This method results in perfect correlation between x_{ikt} and w_{ikt} within each portfolio, which makes the method consistent with the theoretical requirements outlined above.

We use all the three alternative methods to construct low and high portfolios: weighting all stocks equally (EW), weighting all stocks by their relative market value (EW) and weighting all stocks by their relative loadings (LW). Note that for the portfolio with low loadings the stocks are ranked in reverse order.

2.3 The volatility of mimicking portfolios

One metric to assess the importance of the factors in explaining return covariation is the volatility of the factor mimicking portfolio constructed as zero investment strategy either by the CSR approach or the portfolio approach. Consider a mimicking portfolio (*mp*) constructed by going long on the high loading portfolio (*H*) and going short on the low loading portfolio (*L*):

$$R_{mp} = \sum_{h \in H} w_h R_h - \sum_{l \in L} w_l R_l, \tag{10}$$

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[1] The relationship between weights and the values of the explanatory variable can be written as:

$$w_{ikt} = a + bx_{ikt},$$

$$a = \frac{-\sum_{i=1}^{N} x_{ikt}}{N\sum_{i=1}^{N} x_{ikt}^2 - \left(\sum_{i=1}^{N} x_{ikt}\right)^2} \quad \text{and} \quad b = \frac{N}{N\sum_{i=1}^{N} x_{ikt}^2 - \left(\sum_{i=1}^{N} x_{ikt}\right)^2},$$

which yields:

$$corr(w_{ikt}, x_{ikt}) = 1$$

$$\text{var}(R_{mp}) = \sum_{i\in H}\sum_{j\in H} w_i w_j \sigma_{ij} + \sum_{i\in L}\sum_{j\in L} w_i w_j \sigma_{ij} - 2\sum_{i\in H}\sum_{j\in L} w_i w_j \sigma_{ij},$$

$$\text{var}(R_{mp}) = \sum_{i\in H+L} w_i^2 \sigma_i^2 + 2\sum_{i\in H}\sum_{j\neq i\in H} w_i w_j \sigma_{ij} + 2\sum_{i\in L}\sum_{j\neq i\in L} w_i w_j \sigma_{ij} - 2\sum_{i\in H}\sum_{j\in L} w_i w_j \sigma_{ij},$$

The first term is the sum of the all variances. The second and third terms are sum of the covariances within each group H and L. The last term is the sum of all the cross-covariances. The stronger is the factor for explaining the return covariation the higher will be within group covariances and the smaller will be the cross-covariances, which results in a larger $var(R_{mp})$.

The volatility of the mimicking portfolio can also be used to compare different weighting methods because if a factor is important we expect a large return volatility for a portfolio that goes long on assets with high factor loadings and goes short on assets with low factor loadings. To see this consider the following one-factor model:

$$R_{it} = \alpha_i + \beta_{ik} f_{kt} + \varepsilon_{it} \qquad i = 1,....,N. \tag{11}$$

$$E[\varepsilon_t] = 0, \quad E[\varepsilon_t \varepsilon_t'] = \mathbf{S}$$

The variance of a portfolio of the assets can then be written as:

$$\begin{aligned} \text{var}(R_p) &= \sum_{i}^{N}\sum_{j}^{N} w_i w_j \sigma_{ij} = \sum_{i}^{N}\sum_{j}^{N} w_i w_j \left(\beta_i \beta_j \sigma_k^2 + s_{ij}\right) \\ &= \sigma_k^2 \sum_{i}^{N}\sum_{j}^{N} w_i w_j \beta_i \beta_j + \sum_{i}^{N}\sum_{j}^{N} w_i w_j s_{ij} \end{aligned} \tag{12}$$

The first term is the part of the portfolio variance that is explained by the factor model while the second term is the unexplained part. Assuming a large number of assets and that the factor model is adequate to explain the covariations in returns the second term goes toward zero. In this case, a portfolio that gives relatively larger weights to the large betas will have a larger variance relative to a portfolio that gives the same weights to all the assets. Note that the explained part is also driven by the factor variance, σ_k^2.

2.4 Time series regression approach, TSR

One drawback of the approaches above to build the mimicking portfolios for the factors with available time series observations is that we need to estimate loadings and then construct the mimicking portfolio. This may cause bias due to the errors in variables problem. The one step time series approach that follows the maximum correlation portfolio suggested by Breeden et al. (1989) avoid this problem. This approach is not, however, applicable to build mimicking portfolio when the factor is not observable and only firms' loading on the factor are provided by some firm specific variables.

We estimate the following time series regression of factor k on N asset returns:

$$f_{kt} = \lambda_{k0} + \sum_{i=1}^{N} \lambda_{ki} R_{it} + \xi_{it}. \tag{13}$$

where f_{kt} is the factor realisation, R_{it} is the excess return on the base asset i at time t. We use the estimated λ_{ki} as the weights on asset i and form the mimicking portfolio for factor k as:

$$R_{k,t} = \sum_{i=1}^{N} \hat{\lambda}_{ki} R_{it}. \tag{14}$$

where R_{kt} is the excess return of the TSR mimicking portfolio, the maximum correlation portfolio, for factor k at time t. Note that λ_{ki} are normalised to sum to one. In equation (13), we can add some control variables, such as other macroeconomic factors to filter out the overlapping effects. According to Breeden et al. (1989), the asset betas measured relative to the maximum correlation portfolio are proportional to the betas measured using the true factor. Despite the differences in estimated betas the product of the asset beta with the factor risk premium are supposed to be the same for the original factor model and when using the mimicking portfolio (see Cochrane 2001).

3 Test of asset pricing models

In the absence of arbitrage in large economies the following linear relationship holds approximately between the expected return on a security and the risk premium associated with the factors:

$$\mu = \boldsymbol{l}\gamma_0 + \mathbf{B}\gamma, \tag{15}$$

where μ is the $(N\times1)$ vector of the expected returns on N assets, $\boldsymbol{l}$ is an $(N\times1)$ vector of ones, γ_0 is the riskfree rate of return, and $\boldsymbol{\gamma}$ is a $(K\times1)$ vector of risk premia associated with K factors. The $(N\times K)$ matrix $\mathbf{B}$ contains the factor sensitivities of the test assets. In this paper we have two different econometric approaches, which depends on the employed factor representation. The first case is when we use the original time series of the factor realization in the regression model. In the second case the mimicking portfolios represent the macroeconomic factors.

When we use the original factors in the multifactor model some of the factors are not asset portfolios and their factor risk premium cannot directly be specified. In this case, we assume that the risk free part of the factor k, θ_k, is constant over time and can be estimated as a parameter of the following factor model:

$$R_{it} = \alpha_i + \beta_{im} R_{mt} + \sum_{k=1}^{K} \beta_{ik} \left(X_{kt} - \theta_k \right) + \varepsilon_{it}, \tag{16}$$

$$R_{it} = \alpha_i - \sum_{k=1}^{K} \beta_{ik} \theta_k + \beta_{im} R_{mt} + \sum_{k=1}^{K} \beta_{ik} X_{kt} + \varepsilon_{it},$$

$$i = 1,\ldots\ldots, N, \quad \text{and} \qquad t = 1,\ldots\ldots, T$$

where R_{it} is the excess return on asset i at time t, R_{mt} is the excess return on the market portfolio, X_{kt} is realization of the macroeconomic factor k and α_i is the unexplained expected return of asset i.

In the second case the mimicking portfolios represent the macroeconomic factors. The multifactor model in equation (15) implies the following linear regression model:

$$R_{it} = \alpha_i + \sum_{k=1}^{K+1} \beta_{ik} R_{kt} + \varepsilon_{it}, \tag{17}$$

$$i = 1,\ldots\ldots, N, \quad \text{and} \qquad t = 1,\ldots\ldots, T$$

where R_{kt} is the return on factor portfolio k. The explanatory variables include mimicking portfolios for the market portfolio plus K macroeconomic factors.

Our null hypothesis in both cases is that α_i is equal to zero for all i, that is to say the factors can explain expected excess returns for all the assets. To test the model jointly for N assets we use the likelihood ratio test:

$$LR = -2(\ln L^* - \ln L) \sim \chi^2_N, \tag{18}$$

where $\ln L^*$ and $\ln L$ refer to the restricted and unrestricted loglikelihood function respectively. The likelihood function in the matrix notation is:

$$\ln L(.) = -\frac{NT}{2}\ln(2\pi) - \frac{T}{2}\det(\boldsymbol{\Sigma}) - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t'\boldsymbol{\Sigma}^{-1}\boldsymbol{\varepsilon}_t \tag{18}$$

where

$$\boldsymbol{\varepsilon}_t = \left[\varepsilon_{1t} \;\; \varepsilon_{2t} \qquad \varepsilon_{NT}\right]'.$$

are residuals from the restricted/unrestricted model for each of the models in equations (16) and (17) and Σ is the residual covariance matrix.

4 Data and choice of the factors

The data covers the period 1977 to 1997 and consists of monthly Swedish stock returns that are corrected for dividends and capital changes like splits etc. The data is collected from the database "Trust". The sample includes all shares excluding banks and financial firms on the so-called "A1-listan". The monthly data on market value of equity is collected from "Veckans Affärer”. Data for macroeconomic variables are from database Ecowin.

We use the following factors:

- Market factors: excess returns on the Morgan Stanley world index computed in SEK.

- Macroeconomic factors: Growth of the industrial production (*DIP*) is defined as the percentage change in the monthly industrial production. Slope of the yield curve (*Slope*) is the difference between the yield on ten-year government bonds and the yield on three-month treasury bills. Percentage change in the exchange rate SEK/USD (*DEX*).
- Fundamental factors: Book-to-market ratio (*BM*) is defined by dividing the book value of the equity from the annual statement to the market value of the firm at the end of December in each year. *Size* is estimated for each month as the market value of equity from the previous month.

For the market and the macroeconomic factors we estimate the loading for each firm on each of the factors by regressing the excess stock returns on the factor using the most recent 36 months historical observations before the portfolio formation month. The estimates are updated each month.

5 Analysis

5.1 CSR approach versus portfolio approach

In this section we compare the univariate CSR approach with the portfolio approach. Table 1 shows the correlations between different mimicking portfolios. The CSR-distance and CSR-rank are highly correlated with each other and with the loading weighted portfolio approach, HmL-LW. The highest correlation as expected is between the CSR-distance and HmL-LW. This depends on the fact that both approaches capture the relative distances among the loadings. The value weighted approach has very low correlations with other methods except for the variable size. This low correlation reveals the ad-hoc nature of this weighting method.

Table 2 reports the mean returns of the mimicking portfolios. Only *BM* has a mean return that is significantly different from zero for all the different methods except HmL-VW. According to the theoretical discussion in the section 2.1, we may then conclude that BM is the only priced factor with a positive significant risk premium.

Figure 1 compares the standard deviations of the mimicking portfolios of the portfolio approach. We have not included the standard deviations of the CSR mimicking portfolios. The reason is that the level of the standard deviations of these portfolios is related to the way we normalise the loadings in the CSR and may therefore be misleading. The figure shows that the HmL-LW method that gives higher weights to the larger loadings as expected results in higher standard deviations (see section 2.3). The correlation between standard deviations of the HmL-LW and HmL-EW is about 0.87 while the correlations between the HmL-VW and these two methods are around 0.30. This shows that the VW method is not appropriate to judge the relative importance of the background factors, while the two other methods may give more or less the same inferences.

5.2 TSR approach based on different base assets

In this section we analyse the sensitivities of the TSR mimicking portfolios to the choice of the base assets. We use three alternative base assets, i.e., ten size sorted portfolios, ten book-to-market sorted portfolios, and finally ten randomly sorted portfolios. Table 3 shows the correlation matrix of these mimicking portfolios and the original series for each factor. The correlations between mimicking portfolios are relatively high for the world market portfolio. For the factor DEX the correlations are also around 0.60. For these two factors the correlations between mimicking portfolios based on the BM and Size sorted portfolios are higher than the correlations based on the randomly sorted portfolios. For DIP and Slope the correlations are very low. The correlations between the mimicking portfolios and the original series are around 0.60 for the market portfolio and much lower for the other factors.

Table 4 shows the statistics of the TSR mimicking portfolios. All the portfolios mimicking the market have highly significant mean excess returns. The mean excess returns of the portfolios mimicking DEX are also all significant. The results for DIP and Slope are in line with the low correlations between the mimicking portfolios and show a relatively large variation in mean excess return and standard deviation across different mimicking portfolios.

All in all we find that the TSR approach is to some extent sensitive to the choice of the base portfolios. Although our base portfolios are formed by using the same background assets, the sorting characteristics may be important for construction of the mimicking portfolios.

5.3 Mimicking portfolios and asset pricing

In this section we analyse the ability of the mimicking portfolios in representing their background factors from an asset pricing viewpoint. As the test assets we use ten industrial portfolios. The purpose is not to find a sufficient factor model to explain the expected excess returns of the industrial portfolios but to compare the different choices of the factor mimicking portfolios. Since the results for the portfolio approach and the CSR approach are very similar we choose to not report the results for both methods. We use the HmL-LW as the preferred representative of these two groups. The reason to prefer portfolio approach to the regression approach is that the properties of the CSR mimicking portfolios may be affected to some extent by our normalising method. In addition, the CSR approach is probably more exposed to errors in variable problems comparing to the portfolio approach. The reason to prefer LW to other weighting methods of the portfolio approach is that this method is consistent with the theoretical requirements of the mimicking portfolios (as discussed in the section 2.2).

We first look at the results of the likelihood ratio test in Table 5. Using the model with the original factors results in a *p*-value equal to 0.95, which means that the factor model can explain the expected excess returns of the ten industrial portfolios. This result does not however hold when switching to the model with the HmL-LW mimicking portfolios. The *p*-value is now under the 5%, which rejects the ability of the model in explaining the industry mean excess returns. The result for TSR depends on the choice of the base assets. The model with TSR-size mimicking portfolios is rejected while that with the TSR-BM mimicking portfolios is not. To analyse the models more in details we start by presenting the estimated parameters given by each model. Table 6 shows the results for the model with the original factors. The market beta is less than but close to one for all the industries, revealing that the Swedish industries have generally lower risk than the world market. The industries exposures to the factors DIP and Slope are positive except for the *Chemical*, while the exposures to the DEX varies in sign by industries. All the factors except DEX have a positive risk premium.

The results for the models with TSR and HmL-LW mimicking portfolios are reported in Tables 8 and 9 and 10 respectively. The market betas from the model with the original factors are in general larger and those from the TSR-BM and smaller than the market betas from the TSR-size. The market betas from the HmL-VW factor model are all below the betas for three other models (see also Figure 2).

Exposure of the industrial portfolios against the factor DIP varies across the models (see also Figure 2). The results for the TSR-size, in accordance with the model with the original factors, show a positive exposure for all the industries except chemical. However since the estimated risk premium for DIP is negative by this model the effect of this factor on expected returns will be opposite to that from the model with the original factors. The model with the TSR-BM mimicking portfolios results in negative exposures for DIP except for chemical. Due to the negative risk premium, the effect of DIP on expected returns in this model has the same sign as that in the model with the original factors. The model with the HmL-LW portfolios gives some negative and some positive loadings on the factor DIP. Note that the magnitude of the factor loadings of the model with the original factors is not comparable with that from other models, except for the market factor, because the factor realisations are not asset returns. These loadings are therefore not included in Figure 2.

For the factor Slope the exposures are of the same sign for TSR-BM and the original factors while the Chemical has opposite sign in TSR-size. The risk premium of this factor is negative in both TSR models. The HmL-LW based model has also positive loadings for all the industries except Chemical and Miscellaneous, with a positive factor risk premium.

For the DEX the exposures are approximately of the same sign in the models with the original factors and TSR-size, while the exposures are mostly positive in the models with the TSR-BM and HmL-LW.

All in all the effects of the different factors on mean industry excess returns seem to be to some extent different across the models. Figure 3 illustrates the total impact of the each factor on industry excess return. For each factor and each industry this is computed by multiplying the factor risk premium with the industry factor loading. The total impact of the market factor given by the model with the original factors is very close to that from the model with TSR-BM. The effects given by TSR-size is larger for almost all of the industries, while according to the model based on the HmL-LW mimicking portfolios the market factor does not have any important effect on the mean excess returns. According to the TSR-size based model, the other factors show mostly negative effects on the mean excess returns.

Finally we plot the intercept of the factor models and their *t*-values in Figure 4 and Figure 5 respectively. The intercepts of the model with the TSR-size mimicking portfolios are larger than that of the TSR-BM mimicking portfolios for all the industries. They are also larger than the intercepts of the model with the original factors for all but one industry. The intercept of the model with HmL-LW mimicking portfolios due to the trivial impact of the market portfolio is extremely high. Accordingly, the intercepts of this model are significant at the 5% level for all industries except property. The model with TSR-size mimicking portfolios has also five significant intercepts, while the other two models, i.e. models with the original factors and TSR-BM, do not show any significant intercepts. These findings are in accordance with the results of the likelihood ratio test in Table 5, which rejects the TSR-size and HmL-LW models and does not rejects the two other models.

There is however one puzzle in the results of the likelihood ratio test, i.e. why the *p*-value of the model based on the TSR-size is lower than that of the HmL-LW based model, while according to the *t*-values we expect an opposing result. The motivation is that the weakness of the factor model according to the HmL-LW portfolios not only results in higher intercepts but at the same time gives a very large residual covariance matrix, which is due to the unexplained components of the return covariations. This larger covariance matrix increases the uncertainty of any type of the joint test. (See MacKinlay, 1995, for the poor results of the joint *F*-test when there is a missing risk factor in the model.) Figure 6 compares the magnitude of the residual covariance matrices of these two models by forming an equally weighted portfolio based on these covariance matrices. As expected the variance of the portfolios based on the HmL-LW are extremely larger than those for the TSR-size based models.

6 Conclusions

In this paper we have investigated the ability of different factor mimicking portfolios in representing the background factors. We apply several methods for constructing the mimicking portfolios, i.e. the *cross-sectional regression* approach, the *portfolio* approach and the *time-series regression* approach. We build two different CSR mimicking portfolios that differ in the way we normalise the explanatory variables. We use three different weighting methods for the portfolio approach and three different types of the based assets to construct TSR mimicking portfolios.

In addition to the analysis of the standard deviations, the estimated factor risk premia and the correlations between the mimicking portfolios we compare the mimicking portfolios using an asset pricing model. The model is meant to explain the mean excess returns of the Swedish industrial portfolios. The estimation results are compared with the results of the model when the factors are represented by their original realisation.

We find that the result of the portfolio approach is sensitive to the choice of the weighting methods. However, the cross sectional regression approach gives almost the same mimicking portfolio as the loading weighted portfolio approach. The value weighted approach has a very little similarity with other methods. One important finding is that the book-to-market is the only variable with significant mean return of the cross-sectional mimicking portfolios. We find also that the choice of the based assets is important for the estimated mimicking portfolios by the time series approach. In contrast to the CSR- and the portfolio approach all the TSR mimicking portfolios shows a highly significant mean excess return for the market.

In the asset pricing analysis, the models with the original factors and the model with time-series mimicking portfolios, with the book-to-market sorted portfolios as the base assets, outperform the other models.

As MacKinlay (1995) showed the joint *F*-test would be poor in rejecting a factor model when there is a missing risk factor. Our likelihood ratio test for the model including HmL-LW mimicking portfolios supports this hypothesis and shows that a very weak factor model not only results in higher intercepts but at the same time gives a very large residual covariance matrix. The latter is due the unexplained components of the return covariations. The larger covariance matrix increases the uncertainty of the tests and results in a higher *p*-value, which make it difficult to reject a bad factor model.

In summary, based on the results from the asset pricing tests, we conclude that the time series regression approach is a more proper way of constructing mimicking portfolios than the other alternatives investigated in this paper. For our data the book-to-market ratio, due to its relation to cross-sectional differences in mean returns, found to be an appropriate candidate to construct the base assets for the time series regression approach. However, the time series approach cannot be employed to construct mimicking portfolios based on the firm characteristics, when the original factors are not observed. In this case our suggestion is to use loading weighted portfolio approach. The motivation is that despite the close relation between this method and the theoretically motivated cross-sectional approach, the loading weighted portfolio approach suffer less than the cross-sectional approach from problems such as error in variables and normalising effects.

References

Breeden D. T., M. R. Gibbons, and R. H. Litzenberger, 1989, Empirical Tests of the Consumption-Oriented CAPM, *Journal of Finance* 44, 231-262.

Chan, L. K. C., J. Karceski, and J. Lakonishok, 1998, The risk and return from factors, *Journal of Financial and Quantitative Analysis* 33, 159-188.

Chan, L. K. C., J. Karceski, and J. Lakonishok, 1999, On portfolio optimization: Forecasting covariances and choosing the risk model, *The Review of Financial Studies* 12, 937-974.

Cochrane, J. H., 2001, *Asset pricing*, Princeton, Princeton University Press.

Fama, E. F., 1976, *Foundations of Finance*, Basic Books, New York.

Fama, E. F. and K. R. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3-56.

MacKinlay, A. C., 1995, Multifactor Models Do Not Explain Deviations from the CAPM, *Journal of Financial Economics*, 38, 3-28.

Vassalou, M., 2000, News related to future GDP growth as a risk factor in equity returns, *Journal of Financial Economics,* forthcoming.

Table 1. Correlation between mimicking portfolios constructed by the univariate CSR approach and the portfolio approach

The table reports the correlations between different factor mimicking portfolios that are constructed by the univariate CSR approach and the portfolio approach. All the loadings are estimated by the univariate regression on 36 months overlapping windows. In the *rank* approach, the explanatory variable is constructed by ranking all the loadings and normalising them between 0 and 2. In the *distance* approach, the explanatory variable is constructed by taking into account the relative distances among the loadings. HmL refers to the mimicking portfolio constructed by the portfolio approach. EW, weighting all stocks equally. VW weighting all stocks by their relative market value. LW, and weighting all stocks by their relative loadings.

Method	**Method**	Market	DIP	Slope	DEX	Size	BtoM	Average
CSR-rank	CSR-distance	0,98	0,95	0,96	0,97	0,85	0,89	0,93
	HmL-VW	0,61	0,34	0,61	0,62	0,80	0,62	0,60
	HmL-EW	0,94	0,88	0,94	0,94	0,91	0,92	0,92
	HmL-LW	0,98	0,95	0,96	0,97	0,93	0,94	0,95
CSR-distance	HmL-VW	0,59	0,21	0,51	0,58	0,90	0,54	0,56
	HmL-EW	0,90	0,76	0,86	0,88	0,76	0,75	0,82
	HmL-LW	0,99	0,97	0,98	0,99	0,94	0,98	0,97
HmL-VW	HmL-EW	0,66	0,53	0,67	0,64	0,87	0,64	0,67
	HmL-LW	0,60	0,27	0,52	0,56	0,91	0,60	0,58
HmL-VW	HmL-LW	0,91	0,79	0,89	0,89	0,91	0,82	0,87

Table 2. Statistics of the mimicking portfolios constructed by the univariate CSR approach and the portfolio approach

The table reports means, standard deviations and *t*-statistics of the factor mimicking portfolios that are constructed by the univariate CSR approach and the portfolio approach. All the loadings are estimated by the univariate regression on 36 months overlapping windows. In the *rank* approach, the explanatory variable is constructed by ranking all the loadings and normalising them between 0 and 2. In the *distance* approach, the explanatory variable is constructed by taking into account the relative distances among the loadings. HmL refers to the mimicking portfolio constructed by the portfolio approach. EW, weighting all stocks equally. VW weighting all stocks by their relative market value. LW, and weighting all stocks by their relative loadings.

	Methods	Market	DIP	Slope	DEX	Size	BM
Mean annual	CSR-rank	0,003	-0,006	0,019	0,002	-0,002	0,060
	CSR-distance	0,000	-0,013	0,032	0,007	0,009	0,098
	HmL-VW	0,009	-0,020	0,036	0,020	0,026	0,045
	HmL-EW	0,001	-0,013	0,016	0,000	0,011	0,062
	HmL-LW	0,006	-0,011	0,030	0,006	0,015	0,093
Std annual	CSR-rank	0,110	0,081	0,110	0,105	0,110	0,092
	CSR-distance	0,179	0,139	0,187	0,178	0,141	0,190
	HmL-VW	0,115	0,115	0,123	0,122	0,129	0,136
	HmL-EW	0,106	0,078	0,107	0,105	0,109	0,099
	HmL-LW	0,162	0,130	0,176	0,156	0,157	0,159
***t*-statistics**	CSR-rank	0,116	-0,31	0,70	0,08	-0,06	2,74**
	CSR-distance	0,009	-0,39	0,72	0,15	0,27	2,15*
	HmL-VW	0,327	-0,72	1,22	0,67	0,83	1,39
	HmL-EW	0,021	-0,72	0,63	0,01	0,41	2,62**
	HmL-LW	0,143	-0,36	0,72	0,16	0,40	2,46*

Table 3. Correlation between mimicking portfolios constructed by the TSR approach with different base portfolios

The table reports the correlations between the TSR mimicking portfolios and the original series. We use three alternative base assets: Ten size sorted portfolios, ten book-to-market sorted portfolios and ten randomly sorted portfolios.

		Size	*BM*	*Random*	*orig. series*
Market	*Size*	1,00			
	BM	0,87	1,00		
	Random	0,79	0,82	1,00	
	orig. series	0,59	0,61	0,53	1,00
		Size	*BM*	*Random*	*orig. series*
DIP	*Size*	1,00			
	BM	-0,23	1,00		
	Random	0,20	-0,12	1,00	
	orig. series	0,24	-0,19	0,22	1,00
		Size	*BM*	*Random*	*orig. series*
Slope	*Size*	1,00			
	BM	0,30	1,00		
	Random	0,29	0,12	1,00	
	orig. series	0,35	0,23	0,34	1,00
		Size	*BM*	*Random*	*orig. series*
DEX	*Size*	1,00			
	BM	0,67	1,00		
	Random	0,57	0,61	1,00	
	orig. series	0,35	0,37	0,36	1,00

Table 4. Statistics of the mimicking portfolios constructed by the TSR approach with different base portfolios

The table reports the means, standard deviations and *t*-statistics of the TSR mimicking portfolios and the original series. We use three alternative base assets: Ten size sorted portfolios, ten book-to-market sorted portfolios and ten randomly sorted portfolios.

		Market	
Sorting var.	Size	BM	Random
Annual mean	0,168	0,156	0,176
Annual stdev	0,251	0,225	0,238
t-test	2,81**	2,91**	3,09**
		DIP	
Sorting var.	Size	BM	Random
Annual mean	-0,068	-0,202	-0,009
Annual stdev	0,812	2,637	0,808
t-test	-0,35	-0,32	-0,05
		Slope	
Sorting var.	Size	BM	Random
Annual mean	-0,139	-0,068	0,086
Annual stdev	0,577	0,767	0,677
t-test	-1,01	-0,37	0,53
		DEX	
Sorting var.	Size	BM	Random
Annual mean	0,234	0,215	0,173
Annual stdev	0,310	0,308	0,354
t-test	3,16**	2,92**	2,05*

Table 5. Likelihood ratio test

The table reports the result of the likelihood ratio tests. The null hypothesis is that all the intercepts are equal to zero.

	Original factors	TSR-size	TSR-BM	HmL-VW
Unrestricted	3277,7	3584,4	3512,8	3291,2
Restricted	3275,7	3574,2	3507,3	3281,6
Likelihood ratio	3,88	20,58	11,16	19,23
Degree of freedom	10	10	10	10
P-value	0,95	0,02	0,35	0,04

Table 6. Multifactor asset pricing with the original factors

The table reports the estimated parameters of the unrestricted factor model with the original factors:

$$R_{it} = \alpha_i + \beta_{im} R_{mt} + \sum_{k=1}^{K} \beta_{ik} \left(X_{kt} - \theta_k \right) + \varepsilon_{it},$$

θ_k is the risk free part of the macroeconomic factors and are estimated within the model. The table also reports the risk premium associated with the factors.

Parameters of the unrestricted model with original macro variables					
Industy	Intercept	Market	DIP	Slope	DEX
Property	-0,003	0,902	0,369	0,007	0,361
Investment	0,004	0,874	0,135	0,005	0,074
Trade	0,004	0,671	0,350	0,005	-0,261
Chemical	0,005	0,879	-0,011	-0,001	-0,258
Metal	0,002	0,767	0,150	0,003	-0,289
Transport	0,005	0,624	0,173	0,006	0,142
Forest	0,005	0,826	0,084	0,003	0,303
Engineering	0,005	0,891	0,097	0,002	-0,096
Conglomerate	0,000	0,755	0,400	0,006	-0,327
Miscellaneous	0,001	0,576	0,194	0,005	-0,225
Mean		0,011	0,003	0,482	0,004
θ_k			-0,013	0,449	0,012
Risk premium		0,011	0,016	0,033	-0,009

Table 7. Multifactor asset pricing with the TSR-size mimicking portfolios

The table reports the estimated parameters of the unrestricted factor model when the factors are represented by their TSR-size mimicking portfolios:

$$R_{it} = \alpha_i + \sum_{k=1}^{K+1} \beta_{ik} R_{kt} + \varepsilon_{it},$$

The table also reports the risk premium associated with the factors.

Parameters of the model with TSR-size base Mimicking portfolios					
Industy	Intercept	Market	DIP	Slope	DEX
Property	-0,002	0,685	0,092	0,157	0,223
Investment	0,004	1,036	0,013	0,118	-0,094
Trade	0,012	0,930	0,024	0,116	-0,262
Chemical	0,007	0,856	-0,048	0,011	-0,111
Metal	0,006	1,066	0,015	0,080	-0,233
Transport	0,005	0,756	0,081	0,178	0,009
Forest	0,000	0,889	0,000	0,100	0,106
Engineering	0,005	1,085	0,001	0,027	-0,140
Conglomerate	0,009	0,970	0,041	0,127	-0,197
Miscellaneous	0,006	0,828	0,041	0,068	-0,226
Risk premium		0,014	-0,006	-0,012	0,020

Table 8. Multifactor asset pricing with the TSR-BM mimicking portfolios

The table reports the estimated parameters of the unrestricted factor model when the factors are represented by their TSR-BM mimicking portfolios:

$$R_{it} = \alpha_i + \sum_{k=1}^{K+1} \beta_{ik} R_{kt} + \varepsilon_{it},$$

The table also reports the risk premium associated with the factors.

Parameters of the model with TSR-BM base Mimicking portfolios					
Industy	Intercept	Market	DIP	Slope	DEX
Property	-0,004	0,455	-0,013	0,131	0,498
Investment	0,002	0,565	-0,009	0,102	0,359
Trade	0,010	0,393	-0,018	0,084	0,239
Chemical	0,006	1,052	0,021	-0,038	-0,134
Metal	0,004	0,632	-0,022	0,074	0,174
Transport	0,001	0,412	-0,023	0,077	0,369
Forest	-0,001	0,574	-0,025	0,060	0,382
Engineering	0,005	0,727	-0,011	0,039	0,183
Conglomerate	0,006	0,646	-0,016	0,093	0,148
Miscellaneous	0,004	0,541	-0,015	0,048	0,079
Risk premium		0,013	-0,017	-0,006	0,018

Table 9. Multifactor asset pricing with the with HmL-LW Mimicking portfolios

The table reports the estimated parameters of the unrestricted factor model when the factors are represented by their HmL-VW mimicking portfolios:

$$R_{it} = \alpha_i + \sum_{k=1}^{K+1} \beta_{ik} R_{kt} + \varepsilon_{it},$$

The table also reports the risk premium associated with the factors.

Parameters of the model with CSR Mimicking portfolios					
Industy	Intercept	Market	DIP	Slope	DEX
Property	0,009	0,331	0,386	0,585	0,362
Investment	0,015	0,372	-0,060	0,216	0,285
Trade	0,019	-0,037	0,062	0,015	0,215
Chemical	0,017	0,336	0,033	-0,105	0,072
Metal	0,015	0,252	-0,202	0,050	0,357
Transport	0,012	0,113	0,111	0,280	0,438
Forest	0,012	0,527	-0,153	0,149	0,264
Engineering	0,017	0,681	-0,102	0,034	-0,051
Conglomerate	0,017	0,101	-0,138	0,082	0,316
Miscellaneous	0,012	0,076	0,061	-0,034	0,256
Risk premium		0,0005	-0,0009	0,0025	0,0005

Figure 1. Annual standard deviations of the mimicking portfolios constructed by the univariate CSR approach and the portfolio approach

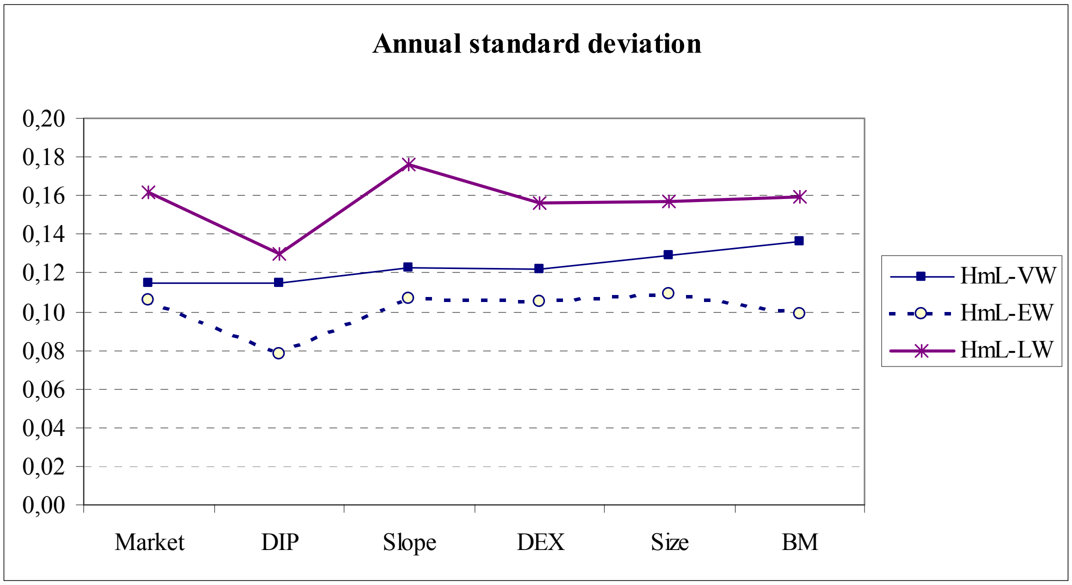

Figure 2. Factor exposure of the industrial portfolios

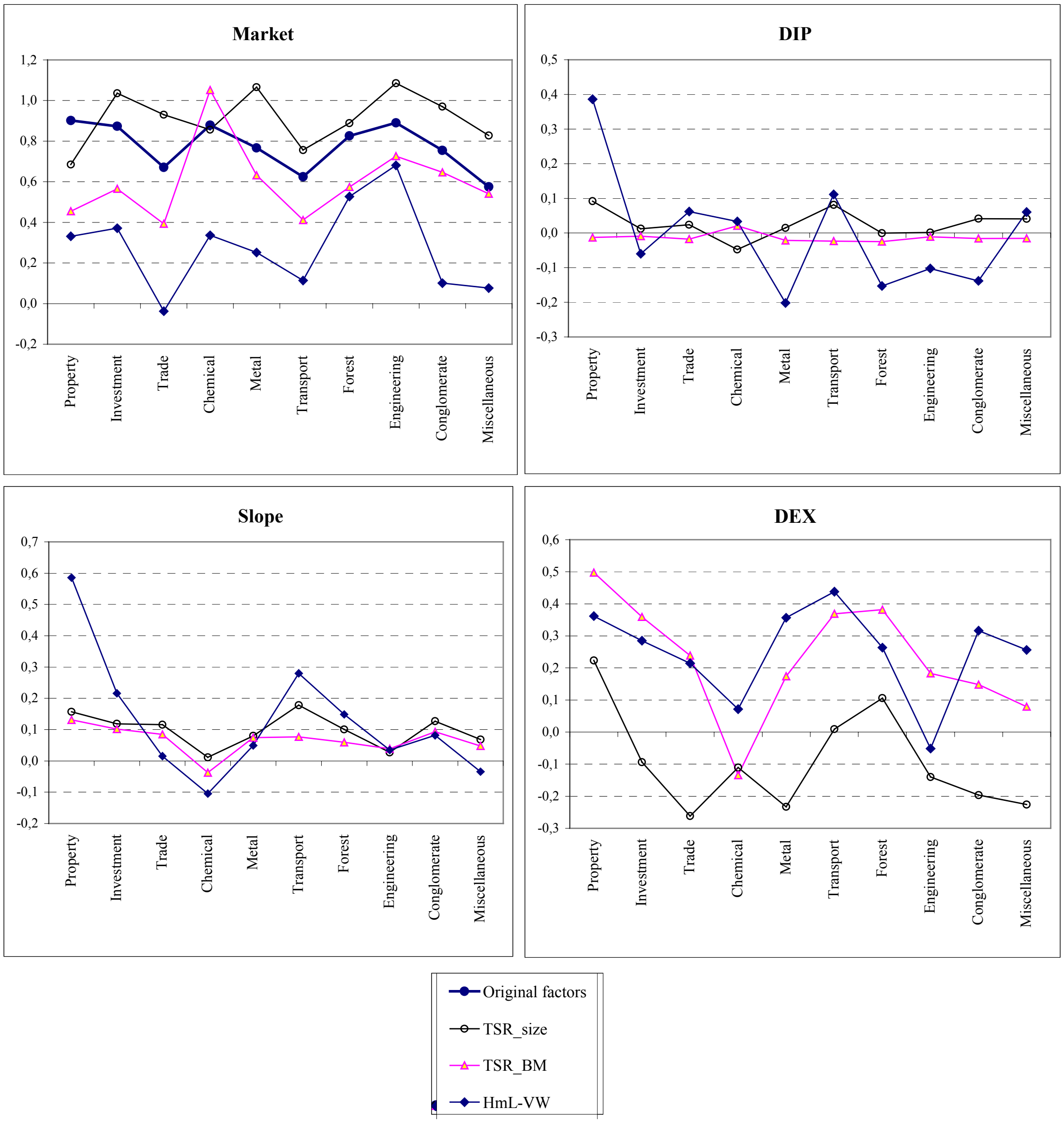

Figure 3. Impact of the factor on mean excess returns given by different factor models

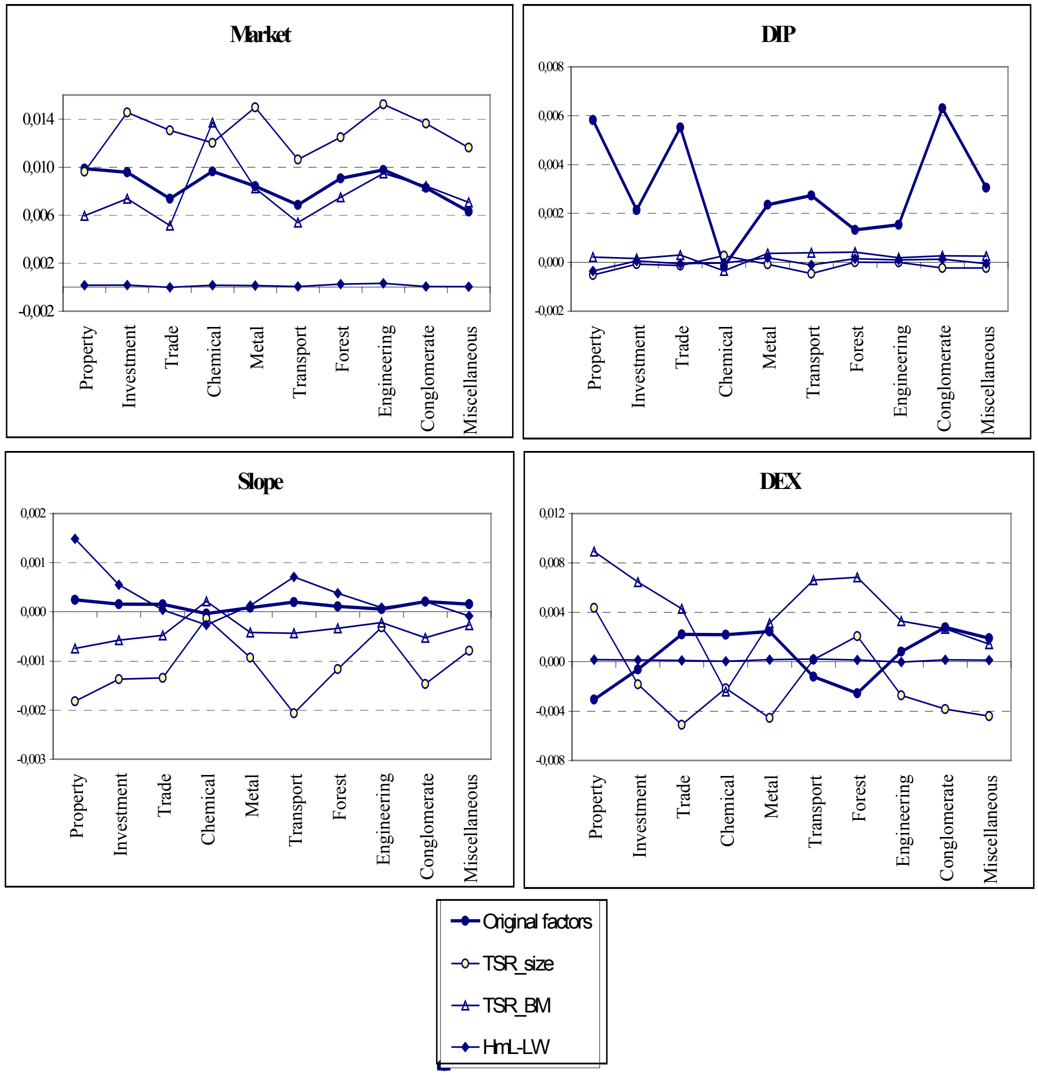

Figure 4. The intercept of the factor models

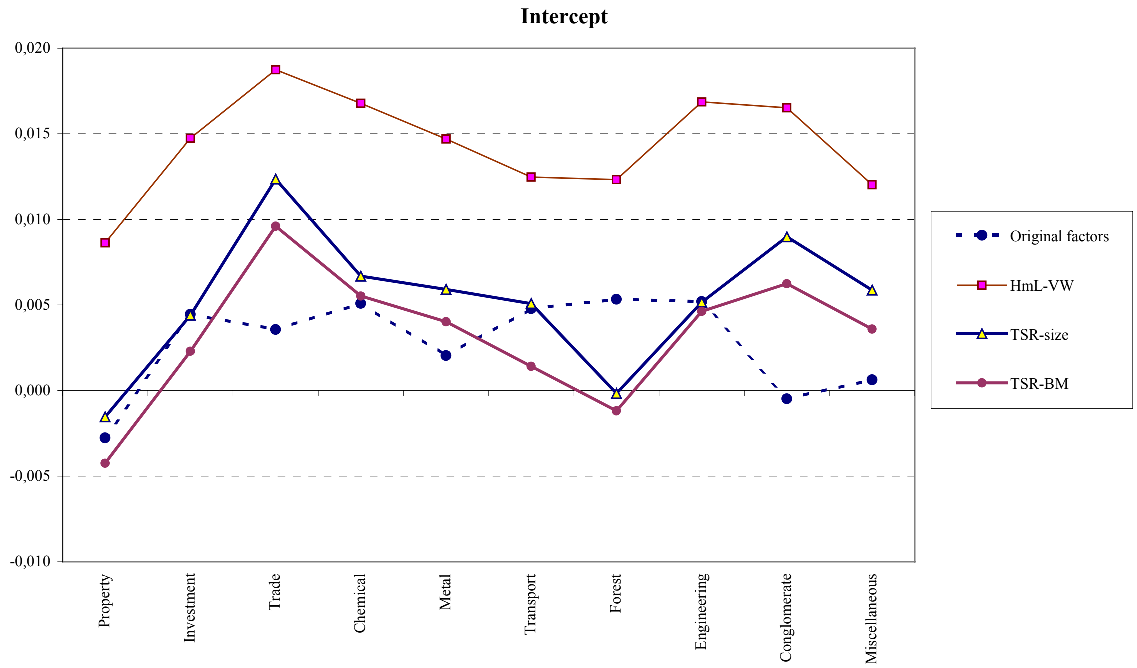

Figure 5. *t*-values of the intercepts of the factor models

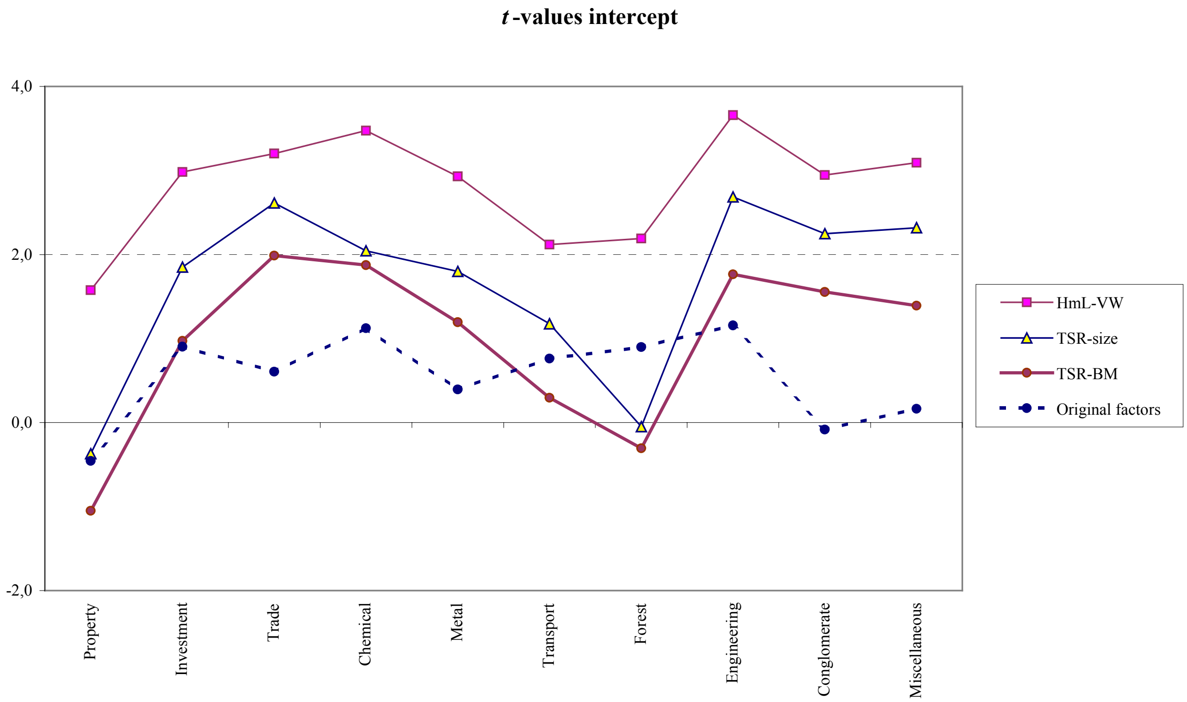

Figure 6. Variance of an equally weighted portfolio