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Testing for Stationarity in Panel Data Models when Disturbances are Cross-Sectionally Correlated*

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Abstract

In this paper, we investigate the effects of cross-sectional disturbance correlation on a previously suggested panel data stationarity test. We find that the previously suggested test has a serious size distortion if the disturbances to different cross sections are correlated. We suggest a new panel data test procedure that also tests the null hypothesis of stationarity. However, the test procedure that we suggest is robust against the presence of cross-sectional disturbance correlation, as well as serial correlation. Furthermore, the test has an approximate normal distribution and which makes p-values and critical values easy to obtain. By applying our test to investigate output convergence, we illustrate the adverse effects that can occur when neglecting to account for cross-sectional correlation when testing for stationarity in panel data models.

JEL Classification: C32; C33; C15
Keywords: Panel-Data Stationarity; Cross-Sectional Dependence; Output Convergence

1 Introduction

Ever since the seminal papers by Levin and Lin (1992, 1993), Quah (1994) and Im et al. (1997) the application of, and research on, panel data unit root, stationarity and cointegration have flourished. The introduction of panel data methods in traditional time-series analysis enables more powerful inference about the economic and econometric hypotheses. As with univariate time series, the panel data research has mainly focused on unit root and cointegration testing. But recently the focus has has shifted somewhat. Hadri (2000) suggests that the univariate stationarity test proposed by Kwiatkowski et al. (1992) can be

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extended to the panel data context. By appropriately standardizing the mean of stationarity tests of the individual cross sections in the panel, a test statistic with a normal limit can be constructed.

Besides the paper by Hadri (2000), Hadri and Larsson (2003), Hadri (2004), Shin and Snell (1999, 2002) and Tiffin (1999) consider the panel data stationarity test under different assumptions regarding the serial dependency of the different cross-sectional error terms. But serial correlation is not the only important issue when testing for stationarity in panel data. In many economic situations, e.g. when studying output convergence, it is unreasonable to assume that cross-sectional disturbances are independent. On the contrary, the notion of an international business cycle suggests that the disturbances that drive the time series for different cross sections should be correlated. Since the panel data stationarity tests mentioned above relies on the fact that disturbances to the time series of different cross sections are independent, inference about output convergence could be adversely affected. The problem regarding cross-sectional correlation has been recognized within the panel data unit root literature, where several suggestions regarding how to treat cross-sectional correlation have been proposed (see e.g. Bai and Ng, 2004; Chang, 2002, 2004; Jönsson, 2005; Moon and Perron, 2004; Pesaran, 2003; Phillips and Sul, 2003). In sprite of this fact, no attempt has, to our knowledge, been made to incorporate cross-sectionally correlated disturbances into the the panel data stationarity framework of Hadri (2000).[1]

In this paper, we propose a panel data stationarity test that is robust against cross-sectional correlation as well as serial correlation. We show that the test has a normal limit under the null hypothesis. Furthermore, we show by simulation that the tests works rather well also in relatively small samples. Finally, we employ the suggested panel data stationarity test to study how inferences regarding output convergence within the G7 countries can be affected when not considering cross-sectional correlation in the panel data stationarity framework.

The rest of the paper is organized as follows. In Section 2 we set out by introducing the univariate stationarity test that is underpinning the panel data test used in this paper. We then introduce the panel data stationarity test and discuss the previous contributions made in the field. In Section 3 we investigate the effect of cross-sectional correlation on the panel data stationarity test previously suggested. Finding that the existing test cannot handle cross-sectional correlation, we suggest an extension to the current test procedure, which renders a panel data stationarity test that can be used regardless of whether cross-sectional disturbance correlation is present or not. Also in Section 3, we study the size and power properties of the proposed test. In Section 4 we use the panel data stationarity tests to illustrate how inference about output convergence can be affected by cross-sectional correlation. Finally, Section 5 concludes.

In the rest of this paper, we adopt the following notation. Vectors and matrices are denoted by bold-face symbols. An element appearing in the $i{:}th$ row and the $j{:}th$ column of the matrix $\mathbf{\Sigma}$ is denoted $(\mathbf{\Sigma})_{ij}$. Finally, the subindices i and t of the type y_{it} is taken to denote cross-sectional unit and time-series observation, respectively.

[1] It can be noted that alternative approaches can be taken to account for cases where errors are cross-sectionally dependent. For example, Nyblom and Harvey (2000) and Bai and Ng (2001) deal with stationarity testing in multivariate models. However these models falls outside the intuitive framework provided by the panel data test of Hadri (2000).

2 Stationarity testing

Before we present the panel data framework, we introduce a univariate stationarity test that is fundamental to the panel data stationarity testing procedures that are to be investigated in this paper. We then go on by discussing the extensions made in the univariate test when using it to test for stationarity in panel data models. More specifically, we focus on the assumption, regarding cross-sectional dependency of the error term, that has been made in the previous panel-data stationarity tests.

2.1 Stationarity testing in univariate time series

The test that we use as point of departure in this paper is the test proposed by Kwiatkowski et al. (1992), the so called KPSS test. In the KPSS test the null hypothesis of stationarity is tested against the alternative hypothesis that the series under consideration contains a unit root. To be precise, the econometric framework used is specified by (1)-(2) below.

$$y_t = \alpha + \delta t + \xi_t + \varepsilon_t \tag{1}$$

$$\xi_t = \xi_{t-1} + \eta_t \tag{2}$$

In (1), y_t is the series that is stationary under the null hypothesis. α denotes an intercept in the series y_t, while the term δt is a time trend.[2] ξ_t in (1) is a random walk component, while ε_t is a stochastic disturbance term. The expression in (2) describes the evolvement of the random walk component. η_t in (2) is the random error that drives the random walk under the alternative hypothesis.

To derive at a test that is able to discriminate between the null and the alternative, we have to make some assumptions regarding the error processes ε_t and η_t. The assumptions that we make are presented below.[3]

Assumption 1 *The disturbance terms ε_t are iid $N(0, \sigma_\varepsilon^2)$.*

Assumption 2 *The disturbance terms η_t are iid $N(0, \sigma_\eta^2)$ and independent of ε_t.*

The null hypothesis of stationarity, when considering the econometric model presented in (1)-(2), is represented by a zero variance of the random error that drives the random walk, i.e. H_0: $\sigma_\eta^2 = 0$. The alternative hypothesis is that the series considered contains a random walk component, that is H_1: $\sigma_\eta^2 > 0$.

To derive the test statistic for testing the null of stationarity, let e_t denote the OLS residuals obtained when running a regression of y_t on a constant and, if trend stationarity is considered under the null hypothesis, a time trend. Furthermore, let $\hat{\sigma}_\varepsilon^2$ be the estimated variance of ε_t. To test the null hypothesis that a time series is stationary, H_0: $\sigma_\eta^2 = 0$, against the one-sided alternative of non-stationarity, H_1: $\sigma_\eta^2 > 0$, Kwiatkowski et al. (1992) suggest that the LM-statistic, calculated as in (3)-(5), should be used.

[2]The intercept term, α, can be interpreted as the initial value of the random walk component, ξ_0. (see Kwiatkowski et al., 1992).

[3]Throughout this paper, we make the same assumptions regarding the normality of disturbances as Hadri (2000). It can however be noted that the assumptions regarding normality of disturbances are not made by Kwiatkowski et al. (1992).

$$LM = \frac{T^{-2}\sum_{t=1}^{T} S_t^2}{\hat{\sigma}_\varepsilon^2} \tag{3}$$

$$S_t = \sum_{i=1}^{t} e_i \tag{4}$$

$$\widehat{\sigma}_\varepsilon^2 = T^{-1}\sum_{t=1}^{T} e_t^2 \tag{5}$$

Under the null hypothesis of stationarity, the test statistic in (3) converges to a functional of a Brownian bridge as in (6).

$$LM = \frac{T^{-2}\sum_{t=1}^{T} S_t^2}{\widehat{\sigma}_\varepsilon^2} \to \frac{\sigma^2 \int_0^1 V(r)^2 dr}{\sigma^2} \tag{6}$$

In (6), σ^2 denotes the long-run variance of the error term ε_t, which is equal to σ_ε^2 if ε_t is serially uncorrelated. $V(r)$ is a standard Brownian bridge.[4] If a consistent estimate of σ^2, such as $\widehat{\sigma}_\varepsilon^2$ in (5), is used in the denominator of (3), the LM statistic converges to the expected value of the standard Brownian bridge (see Kwiatkowski et al., 1992).

If the error term ε_t follows an ARMA process, while η_t still is white noise, Assumption 1 is not fulfilled. However, the test statistic in (3) could still be applied if the regularity conditions used by Phillips and Perron (1988) are fulfilled. A slight complications is that the estimator of the disturbance variance mentioned above is no longer valid. Instead, a consistent estimator of the long-run variance, in the presence of serial correlation, has to be used in the denominator of (3). Kwiatkowski et al. (1992) suggest the use of the estimator presented in (7) and (8).

$$s^2(l) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{s=1}^{l} w(s,l) \sum_{t=s+1}^{T} e_t e_{t-s} \tag{7}$$

$$w(s,l) = 1 - \frac{s}{l+1} \tag{8}$$

The weighting function in (8) is the Bartlett window used by e.g. Newey and West (1987). As long as $l \to \infty$ as $T \to \infty$, while $l/T^{1/4} \to 0$, the consistency of $s^2(l)$, and hence the Brownian bridge distribution of the test statistic in (6), is assured. Using the knowledge about the test's distribution under the null hypothesis, critical values for the stationarity test can be extracted.

The test presented above is developed to test for stationarity using one time series at a time. But if several time series were to be considered simultaneously we could increase the power of the stationarity test and hence be in better a position when testing economic and econometric hypotheses. This was recognized by Hadri (2000), who originally suggested the panel data stationarity test discussed in the next subsection.

2.2 Stationarity testing in models with panel data

The stationarity testing procedure described in Section 2.1 can be extended to test for stationarity in panel data models. One way to proceed, when dealing with panel data, was

[4]The Brownian bridge is either of the first or the second order depending on the detrending procedure used.

suggested by Hadri (2000). By considering time series for several cross sections simultaneously more powerful inference about the null hypothesis is enabled. Moreover, by applying the sequential limit theory, developed by Phillips and Moon (1999), a normal limit for the panel data stationarity test can be reached. Since, normal normal distribution is easier to work with than distributions of Brownian bridges, the framework of Hadri (2000) is very attractive. In this subsection, we describe the panel data stationarity test and discuss one of the main limitations of the test.

Suppose that we observe N different time series y_{it}, where $i \in \{1, ..., N\}$ and $t \in \{1, ..., T\}$ as in (9) and (10), instead of just one time series as in Section 2.1.

$$y_{it} = \alpha_i + \delta_i t + \xi_{it} + \varepsilon_{it} \tag{9}$$

$$\xi_{it} = \xi_{it-1} + \eta_{it} \tag{10}$$

The parameters in (9) and (10) have the same interpretations as in the previous subsection. However, a subindex i has been added to illustrate the panel structure of the data. Furthermore, suppose that we place some assumption on the disturbances of different time series.

Assumption 3 *The disturbance terms ε_{it} are iid $N(0, \sigma^2_{\varepsilon,i})$.*

Assumption 4 *The disturbance terms η_{it} are iid $N(0, \sigma^2_{\eta,i})$ and independent of ε_{jt} for all $j \in \{1, ..., N\}$.*

The assumptions above correspond to the assumptions made in the univariate model of Section 2.1. With these assumption, together with the independence assumption, presented below in Assumption 5, it is possible to derive a panel data stationarity test that has a normal limit as T and N pass to infinity sequentially. , Indeed, this was suggested by Hadri (2000).

Assumption 5 *The covariance, $cov(\varepsilon_{it}, \varepsilon_{jt}) = 0$ if $i \neq j$*

With these assumptions, the panel data stationarity test is performed by calculating the LM statistic in (3) for each and every one of the time series entering the panel. Let these test statistics be denoted LM_i. By applying the central limit theorem (CLT) of Lindberg-Levy, the standardized test statistic $\overline{LM}_{(T,N\to\infty)_{seq}}$ in (11) will converge to a standard normal distribution as $T \to \infty$ followed by $N \to \infty$.

$$\overline{LM}_{(T,N\to\infty)_{seq}} = \frac{(N^{-1}\sum_{i=1}^{N} LM_i) - E[LM]}{\sqrt{Var[LM]/N}} \tag{11}$$

The moments used to standardize the panel data stationarity test statistic in (11) are supplied by Hadri (2000) and Hadri and Larsson (2003).

Immediately when we study (11), the relevance of Assumption 5 becomes clear. If the $N \times 1$ disturbance vector $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Nt})'$ in (9) doesn't have a diagonal covariance matrix, the LM statistics of different cross-sectional units are not independent. If they are not independent, one cannot use the Lindberg-Levy CLT to achieve asymptotic normality. Instead, the asymptotic, as well as the small-sample, distribution of the panel data test statistic will depend on nuisance parameters. More specifically, the panel data stationarity test will depend on the degree of cross-sectional correlation between the disturbances of different cross-sectional units. In the next section, we investigate the effect that such a violation of Assumption 5 will have on the panel data stationarity test. We will also suggest an extension of the panel data stationarity test of Hadri (2000) that will handle the presence of cross-sectional correlation.

3 Stationarity testing when disturbances are correlated across cross sections

3.1 Effects of cross-sectional correlation.

As mentioned above, the panel data stationarity test of Hadri (2000) is derived under the assumption that the disturbances to different cross sections are independent. However, in most empirical economic applications this is not likely to be the case. Regardless of whether we consider cross sections consisting of individuals, firms, regions or countries, the time series that we study are likely to be correlated across the cross sections. Hence, one important question is what effect such a correlation will have on the panel data test of Hadri (2000).

To investigate the effects of cross-sectional correlation, assume that the noise term in (9) is correlated across cross sections, i.e. assume that the $N \times 1$ disturbance vector $\boldsymbol{\varepsilon}_t$ has the non-diagonal covariance matrix $\boldsymbol{\Sigma}$. If this is the case, Assumption 5 will be violated. As a consequence, we cannot apply a the Lindberg-Levy CLT to achieve a normal limit for the panel data stationarity test. Furthermore, the distribution of the panel data stationarity test may be affected by the presence of cross-sectional correlation.

Since cross-sectional correlation in the panel data unit root framework has been shown to cause size distortions (see e.g. O'Connell, 1998), we investigate the size properties of the panel data stationarity test. To this end, we generate data under the null hypothesis of stationarity, with different degrees of cross-sectional correlation, and study the how often the null hypothesis is incorrectly rejected. The data generating process that is used in this investigation is described in (12)-(15).

$$\mathbf{Z}_t \sim N(\mathbf{0}_{N\times 1}; \mathbf{I}_N) \tag{12}$$

$$\mathbf{P} = Chol(\boldsymbol{\Sigma}) \tag{13}$$

$$\boldsymbol{\varepsilon}_t = \mathbf{P}\mathbf{Z}_t \tag{14}$$

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\varepsilon}_t \tag{15}$$

In (13), $\mathbf{P}$ is the lower triangular Cholesky decomposition of the variance/covariance matrix $\boldsymbol{\Sigma}$. The structure of the variance/covariance matrix $\boldsymbol{\Sigma}$ will determine the cross-sectional correlation in the different time series. If $\boldsymbol{\Sigma}$ is diagonal, Assumption 5 will be fulfilled. If Σ is non-diagonal, cross-sectional correlation will be present. $\boldsymbol{\varepsilon}_t$ in (14) and (15) is a $N \times 1$ vector with possibly cross-sectionally correlated error terms. $\mathbf{y}_t$ is a $N \times 1$ vector with a times-series observations of the series under investigation. $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$ are $N \times 1$ vectors with the coefficients for the deterministic components of the cross-section time series.[5]

The effect of different degrees cross-sectional correlation is investigated by altering the elements of $\boldsymbol{\Sigma}$. Four choices of $\boldsymbol{\Sigma}$ are considered. In all the simulations, we set the diagonal elements of $\boldsymbol{\Sigma}$ to one while altering the off-diagonal elements. The off-diagonal elements are

[5]We allow for heterogeneity in the deterministic components. That is, α_i is allowed to vary across i.

set to 0.00, 0.25, 0.50 and 0.75, respectively. Since we set the variance of the error terms to unity the four different covariance cases corresponds to a correlation structure between the disturbances to different cross-sectional units of 0.00, 0.25, 0.50 and 0.75, respectively. The structure of Σ is given in (16).

$$(\mathbf{\Sigma})_{ij} \in \{0.00,\ 0.25,\ 0.50,\ 0.75\} \text{ if } i \neq j, (\mathbf{\Sigma})_{ij} = 1 \text{ if } i = j \tag{16}$$

The size of the panel data stationarity test of Section 2.2 is investigated by performing 5,000 replications of the test presented in (11). We set $\alpha_i \in U[-1, 1]$ and $\boldsymbol{\delta} = \mathbf{0}_{N\times 1}$.[6] We standardize the panel LM statistic using moments obtained from Monte Carlo simulations[7]. By using simulated moments we can isolate the effects of cross-sectional correlation, not having to pay attention to e.g. the overall finite-sample performance of the test. We then calculate the size of the test using the asymptotic critical value at the 5% significance level obtained from the normal distribution. The results from the size investigation are presented in Table 1.

As we see in Table 1, the performance of the test is rather poor as soon as the assumption of cross-sectional independency is abandoned. As expected, the test performs worse when the the degree of cross-sectional correlation increases. Furthermore, the size distortion, that arise as a consequence of cross-sectionally correlated disturbances, seems to persist in all sample sizes.

[6]To save space, we only present results for the case where the deterministic component consists of an intercept only.

[7]The moments are discussed and presented in Section 3.3

Table 1: Size distortion of the panel LM test in the presence of CSD

k=24	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.056	0.052	0.054	0.055	-	-	-	0.072	0.076	0.092	0.111	-	-	-
50	0.063	0.067	0.057	0.058	0.059	-	-	0.082	0.088	0.102	0.129	0.126	-	-
75	0.061	0.055	0.051	0.050	0.046	0.042	-	0.076	0.084	0.089	0.106	0.125	0.149	-
100	0.063	0.061	0.057	0.050	0.055	0.049	-	0.079	0.091	0.110	0.124	0.119	0.148	-
150	0.063	0.056	0.063	0.057	0.051	0.049	0.048	0.081	0.093	0.103	0.118	0.120	0.154	0.181
200	0.063	0.060	0.064	0.060	0.059	0.058	0.058	0.079	0.087	0.105	0.111	0.122	0.163	0.196
250	0.063	0.066	0.064	0.060	0.063	0.067	0.060	0.091	0.095	0.110	0.124	0.130	0.163	0.183
500	0.067	0.070	0.069	0.068	0.058	0.057	0.055	0.081	0.099	0.108	0.114	0.129	0.146	0.177
1000	0.069	0.060	0.063	0.058	0.056	0.055	0.048	0.071	0.095	0.096	0.112	0.118	0.151	0.184
	ρ=0.50							ρ=0.75						
T=25	0.103	0.133	0.157	0.175	-	-	-	0.149	0.193	0.235	0.259	-	-	-
50	0.115	0.157	0.191	0.212	0.217	-	-	0.186	0.231	0.256	0.269	0.295	-	-
75	0.114	0.148	0.168	0.189	0.202	0.230	-	0.172	0.215	0.238	0.243	0.258	0.302	-
100	0.111	0.149	0.169	0.180	0.192	0.246	-	0.170	0.208	0.233	0.255	0.259	0.279	-
150	0.111	0.144	0.164	0.185	0.197	0.222	0.262	0.157	0.193	0.219	0.236	0.237	0.280	0.295
200	0.118	0.147	0.167	0.179	0.196	0.224	0.260	0.147	0.196	0.227	0.245	0.234	0.262	0.302
250	0.119	0.140	0.178	0.194	0.186	0.238	0.263	0.161	0.191	0.212	0.240	0.241	0.265	0.287
500	0.115	0.143	0.170	0.180	0.187	0.216	0.251	0.145	0.190	0.206	0.220	0.232	0.261	0.296
1000	0.105	0.136	0.158	0.166	0.187	0.213	0.255	0.138	0.182	0.206	0.210	0.224	0.243	0.285
k=12	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.060	0.053	0.054	0.049	-	-	-	0.079	0.089	0.108	0.121	-	-	-
50	0.068	0.057	0.065	0.064	0.056	-	-	0.080	0.093	0.095	0.110	0.132	-	-
75	0.061	0.066	0.063	0.059	0.055	0.051	-	0.086	0.098	0.102	0.113	0.122	0.150	-
100	0.070	0.066	0.057	0.062	0.060	0.057	-	0.078	0.090	0.111	0.114	0.122	0.157	-
150	0.066	0.062	0.056	0.060	0.054	0.050	0.058	0.081	0.090	0.108	0.114	0.118	0.158	0.193
200	0.067	0.059	0.059	0.065	0.062	0.054	0.056	0.082	0.098	0.115	0.117	0.121	0.155	0.187
250	0.071	0.058	0.060	0.055	0.067	0.060	0.060	0.083	0.090	0.107	0.116	0.124	0.156	0.176
500	0.071	0.063	0.063	0.060	0.057	0.059	0.061	0.086	0.096	0.107	0.119	0.118	0.163	0.175
1000	0.065	0.064	0.063	0.062	0.062	0.060	0.058	0.086	0.098	0.102	0.112	0.115	0.149	0.181
	ρ=0.50							ρ=0.75						
T=25	0.115	0.149	0.177	0.202	-	-	-	0.176	0.222	0.248	0.266	-	-	-
50	0.123	0.148	0.171	0.187	0.203	-	-	0.162	0.208	0.225	0.255	0.264	-	-
75	0.111	0.154	0.169	0.192	0.196	0.225	-	0.159	0.191	0.221	0.241	0.247	0.277	-
100	0.122	0.149	0.169	0.173	0.199	0.227	-	0.158	0.200	0.225	0.220	0.236	0.281	-
150	0.112	0.137	0.164	0.170	0.182	0.224	0.247	0.153	0.186	0.206	0.223	0.238	0.271	0.283
200	0.108	0.141	0.162	0.172	0.184	0.225	0.257	0.142	0.186	0.198	0.222	0.236	0.256	0.290
250	0.115	0.137	0.153	0.179	0.192	0.215	0.253	0.145	0.184	0.198	0.226	0.222	0.260	0.284
500	0.110	0.140	0.158	0.165	0.180	0.231	0.251	0.146	0.174	0.199	0.211	0.211	0.253	0.276
1000	0.108	0.136	0.158	0.177	0.179	0.208	0.237	0.132	0.174	0.195	0.214	0.232	0.248	0.281
k=4	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.061	0.065	0.059	0.057	-	-	-	0.076	0.102	0.104	0.118	-	-	-
50	0.072	0.059	0.060	0.066	0.058	-	-	0.082	0.096	0.104	0.107	0.129	-	-
75	0.066	0.063	0.062	0.062	0.052	0.056	-	0.081	0.097	0.111	0.114	0.125	0.146	-
100	0.068	0.067	0.059	0.054	0.060	0.059	-	0.079	0.087	0.107	0.102	0.121	0.150	-
150	0.065	0.065	0.062	0.062	0.058	0.058	0.055	0.083	0.093	0.102	0.115	0.126	0.149	0.178
200	0.061	0.061	0.066	0.066	0.068	0.055	0.051	0.087	0.095	0.104	0.114	0.122	0.156	0.178
250	0.066	0.064	0.061	0.061	0.062	0.059	0.058	0.084	0.098	0.104	0.117	0.126	0.153	0.171
500	0.070	0.056	0.063	0.065	0.058	0.055	0.052	0.083	0.087	0.107	0.114	0.119	0.150	0.195
1000	0.068	0.068	0.067	0.063	0.059	0.062	0.055	0.079	0.096	0.102	0.114	0.132	0.155	0.174
	ρ=0.50							ρ=0.75						
T=25	0.111	0.156	0.171	0.191	-	-	-	0.152	0.194	0.214	0.230	-	-	-
50	0.110	0.144	0.167	0.189	0.187	-	-	0.151	0.184	0.204	0.222	0.223	-	-
75	0.106	0.140	0.154	0.173	0.182	0.214	-	0.146	0.181	0.198	0.217	0.226	0.275	-
100	0.110	0.135	0.154	0.174	0.184	0.220	-	0.145	0.170	0.204	0.211	0.225	0.253	-
150	0.107	0.139	0.164	0.171	0.170	0.214	0.242	0.142	0.176	0.197	0.206	0.226	0.251	0.284
200	0.116	0.139	0.159	0.165	0.175	0.205	0.237	0.150	0.171	0.197	0.215	0.228	0.251	0.271
250	0.118	0.141	0.158	0.171	0.184	0.206	0.240	0.139	0.183	0.192	0.205	0.228	0.250	0.280
500	0.109	0.140	0.160	0.178	0.170	0.215	0.233	0.137	0.170	0.194	0.212	0.216	0.247	0.270
1000	0.108	0.140	0.153	0.162	0.180	0.213	0.246	0.126	0.176	0.187	0.205	0.213	0.243	0.276
No lag, k=0	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.065	0.066	0.067	0.066	-	-	-	0.087	0.091	0.104	0.118	-	-	-
50	0.070	0.060	0.062	0.064	0.053	-	-	0.078	0.093	0.106	0.118	0.119	-	-
75	0.068	0.061	0.063	0.057	0.066	0.062	-	0.079	0.092	0.111	0.113	0.119	0.145	-
100	0.071	0.064	0.065	0.063	0.062	0.060	-	0.079	0.095	0.105	0.108	0.112	0.147	-
150	0.072	0.061	0.058	0.064	0.063	0.063	0.055	0.082	0.092	0.107	0.110	0.119	0.147	0.188
200	0.069	0.064	0.067	0.060	0.058	0.058	0.053	0.076	0.089	0.104	0.114	0.114	0.152	0.185
250	0.066	0.060	0.069	0.060	0.066	0.062	0.054	0.079	0.095	0.103	0.115	0.129	0.153	0.182
500	0.069	0.062	0.063	0.055	0.062	0.060	0.060	0.082	0.098	0.112	0.111	0.120	0.149	0.180
1000	0.064	0.063	0.067	0.065	0.059	0.057	0.054	0.080	0.087	0.112	0.118	0.122	0.149	0.187
	ρ=0.50							ρ=0.75						
T=25	0.101	0.141	0.169	0.169	-	-	-	0.141	0.182	0.189	0.213	-	-	-
50	0.098	0.145	0.153	0.165	0.180	-	-	0.137	0.173	0.195	0.223	0.216	-	-
75	0.109	0.143	0.151	0.169	0.172	0.217	-	0.130	0.169	0.187	0.206	0.225	0.246	-
100	0.110	0.136	0.157	0.175	0.181	0.209	-	0.128	0.177	0.202	0.199	0.211	0.256	-
150	0.107	0.130	0.149	0.165	0.182	0.222	0.236	0.134	0.168	0.195	0.217	0.214	0.250	0.274
200	0.107	0.129	0.153	0.163	0.182	0.209	0.239	0.129	0.167	0.190	0.189	0.216	0.252	0.271
250	0.106	0.128	0.156	0.161	0.183	0.208	0.234	0.131	0.161	0.190	0.205	0.225	0.249	0.267
500	0.111	0.131	0.157	0.171	0.178	0.211	0.253	0.134	0.170	0.193	0.214	0.205	0.242	0.269
1000	0.105	0.138	0.151	0.168	0.179	0.215	0.244	0.136	0.175	0.182	0.201	0.212	0.234	0.269

Given that we have panel data that is cross-sectionally correlated, we have to perform some sort of correction if we are to come to terms with the size distortions that occur. In the panel data unit root context, Im et al. (1997) and Levin et al. (2002) suggested that the problem regarding cross-sectional correlation could be reduced by subtracting the cross-sectional average form each time series observation. The same solution to the problem with cross-sectional correlation was proposed by Shin and Snell (1999) in the panel data stationarity testing framework. However, such a solution will only be viable if the cross-sectional covariation can be modelled by a time-specific factor that is common across the different cross-sectional units. Recent research (see e.g. Strauss and Yigit, 2003) has shown that the de-meaning solution cannot completely solve the problem with cross-sectionally correlated residuals.

Instead, we suggest a procedure that works under a completely general covariation structure and does not depend on the assumption that the cross-sectional correlation can be modelled by a factor model. In the next subsection we present our procedure for eliminating the size distortions that occur in the panel data stationarity test when the disturbances are correlated over cross sections.

3.2 The cross-sectionally corrected LM test

In this subsection we propose a method for correcting the size distortions that occur in the panel data stationarity test of Hadri (2000) when disturbances to different cross sections are correlated. We begin by considering the baseline case where the disturbance terms that are present under the null hypothesis are serially uncorrelated. We then go on by considering the case where the disturbances of different cross-section are serially correlated and follow a seemingly unrelated autoregressive moving-average (ARMA) process.

The intuition behind the correction that we suggest in this paper is that the correlated multivariate normal disturbances in (9) can be transformed into an independent multivariate normal disturbances under the null hypothesis. Furthermore, the transformation is valid under the null since it does not affect the stationarity properties of the series that are investigated by the stationarity test.

Consider once again the panel data model in (9) and (10). As discussed above, the panel data stationarity test of Hadri (2000) is developed for the where the noise terms of different cross sections are uncorrelated. To be able to apply the test under a wider range of conditions, we now extend the panel data model of Section 2.2 to allow for contemporaneous cross-sectional correlation in the noise terms. Our extended model is presented below in Model 1.

Model 1 *Suppose that the panel data model is given by*

$$y_{it} = \alpha_i + \delta_i t + \xi_{it} + \varepsilon_{it}$$
$$\xi_{it} = \xi_{it-1} + \eta_{it}$$

where $i \in \{1, ..., N\}$ *and* $t \in \{1, ..., T\}$ *and* ξ_{i0} *is fixed.*

The vector of noise terms in Model 1, $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$*, follows a normal distribution with mean zero and covariance matrix* $\boldsymbol{\Sigma}$*. The covariance matrix is not necessarily diagonal. The error terms that drives the random walk components under the alternative hypothesis are independent of the noise terms and normally distributed with zero mean and full-rank covariance matrix* $\boldsymbol{\Omega}$*.*

As shown in the previous subsection, the panel data stationarity test of Hadri (2000) has a serious size distortion under the null hypothesis if Model 1 is correct. The size distortions arise as a consequence of the LM statistics of different cross-sectional units being correlated. In order to obtain independent LM statistics, one could apply some sort of orthogonalization of the different LM statistics. In this paper, we propose that the orthogonalization procedure of Doornik and Hansen (1994) should be applied.[8] The orthogonalization performed, to obtain a cross-sectionally corrected LM (CSCLM) panel data stationarity test, is described below.

The Cross-Sectionally Corrected LM (CSCLM) test. The CSCLM test is performed by implementing the following steps in the environment described in Model 1:

1. Detrend the individual panel data series, $\mathbf{y}_i' = (y_{i1}, \ldots, y_{iT})$, by regressing the series either on a constant or on a constant and a trend. The residual series that are obtained are N different $T \times 1$ vectors $\mathbf{e}_i' = (e_{i1}, \ldots, e_{iT})$.
2. Calculate the correlation matrix, $\hat{\mathbf{C}}$, of the residuals. To obtain the correlation matrix, let $\overline{\mathbf{e}}_i = T^{-1}\sum_{t=1}^{T} e_{it}$ and $\check{\mathbf{e}}_t' = (e_{1t} - \overline{e}_1, \ldots, e_{Nt} - \overline{e}_N)$. Furthermore, let the variance/covariance matrix with the residuals be denoted by $\hat{\mathbf{S}}$, i.e. $(\hat{\mathbf{S}})_{ij} = T^{-1}\check{\mathbf{e}}_i'\check{\mathbf{e}}_j$, where $\check{\mathbf{e}}_i' = (\check{e}_{i1}, \ldots, \check{e}_{iT})$ for $i \in \{1, \ldots, N\}$, and let $\hat{\mathbf{V}} = diag((\hat{\mathbf{S}})_{11}^{-1/2}, \ldots, (\hat{\mathbf{S}})_{NN}^{-1/2})$ be a diagonal matrix with the inverses of the residual standard deviations in the main diagonal. From these matrices, the correlation matrix is given by $\hat{\mathbf{C}} = \hat{\mathbf{V}}\hat{\mathbf{S}}\hat{\mathbf{V}}$.
3. With the correlation matrix, $\hat{\mathbf{C}}$, at hand, perform the spectral decomposition of $\hat{\mathbf{C}}$. Let the eigenvalues and the corresponding $(N \times 1)$ eigenvectors of $\hat{\mathbf{C}}$ be denoted $\hat{\lambda}_i$ and $\hat{\mathbf{H}}_i$. Arrange the eigenvalues into a diagonal matrix, $\hat{\mathbf{\Lambda}} = diag(\hat{\lambda}_1, \ldots, \hat{\lambda}_N)$, where the N eigenvalues of $\hat{\mathbf{C}}$ appears in the main diagonal. Furthermore, sort the $N \times 1$ eigenvectors, $\hat{\mathbf{H}}_i$, into matrix $\hat{\mathbf{H}} = (\hat{\mathbf{H}}_1, \ldots, \hat{\mathbf{H}}_N)$ such that the eigenvectors appearing in the columns of $\hat{\mathbf{H}}$ corresponds to the eigenvalues appearing in the main diagonal of $\hat{\mathbf{\Lambda}}$.
4. Construct the cross-sectionally corrected (CSC) residual series $\tilde{\mathbf{e}}_t = \hat{\mathbf{H}}\hat{\mathbf{\Lambda}}^{-1/2}\hat{\mathbf{H}}'\hat{\mathbf{V}}\check{\mathbf{e}}_t$, where $\hat{\mathbf{\Lambda}}^{-1/2}$ is diagonal matrix with the inverse of the square root of the eigenvalues appearing in the main diagonal.
5. Perform the LM test in (3) on each of the cross-sectionally corrected residual series. Calculate the the panel LM statistic for the null of stationarity as in (11), using the appropriate moments.[9]

■

The null hypothesis tested by the CSCLM test is that all cross-section time series are stationary, i.e. $H_0{:}\sigma_i^2 = 0 \;\forall\; i \in \{1, \ldots, N\}$. The alternative hypothesis is formulated as $H_1{:}\sigma_i^2 > 0 \;\forall\; i \in \{1, \ldots, N\}$, which means that all time series in the panel contains a unit root.

With the CSCLM test procedure, we are able to construct a test that is valid even if the noise terms of different cross-sectional units are correlated. More specifically, if

[8] We apply this orthogonalization procedure because it is scale-invariant and invariant to the ordering of the variables (see Doornik and Hansen, 1994). However, preliminary simulations show that the Cholesky decomposition of the estimated variance/covaraince matrix can be applied without changing the results.

[9] Just as in the univariate case, the variance estimator in the denominator of (3) has to be replaced by a consistent long-run variance estimator, like the one in (7) and (8), if the noise terms are serially correlated.

Model 1 is correct, the statistic of the cross-sectionally corrected LM test presented above can be approximated by a standard normal variate under the null hypothesis as $T \to \infty$ while keeping N fixed, i.e. as $(T \to \infty, Nfixed)$. The normal approximation is valid since, if the null hypothesis if true, then the detrended series will be unbiased estimates of the true disturbances. If we let T pass to infinity while fixing N, the variance/covariance matrix of the disturbance terms can be consistently estimated. Hence, the orthogonalization procedure described in the CSCLM test will render independent residual series. This will imply that the individual LM statistics, calculated as in(3), will render LM test statistics that consists of independent Brownian bridges and, hence, don't depend on the nuisance parameters introduced by cross-sectional correlation. By standardizing the mean of these Brownian bridges using their expected value and the standard deviation of the expected value, the panel LM statistic will converge to a standard normal distribution if N is large enough. Normality will follow as a consequence of the central limit theorem of Lindberg-Levy.

However, the CSCLM test presented above relies on the assumption that the disturbances to the different series are serially uncorrelated. Under certain circumstances, this can be a rather strict assumption. Hence, the next step is to relax this assumption and let the noise processes by both cross-sectionally and serially correlated.

Assumption 6 *The vector of noise terms in Model 1, $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$, is generated by a stationary and invertible seemingly unrelated ARMA process of finite order:*

$$\boldsymbol{\Theta}(L)\boldsymbol{\varepsilon}_t = \boldsymbol{\Phi}(L)\boldsymbol{\nu}_t$$

The disturbances to the seemingly unrelated ARMA process, $\boldsymbol{\nu}_t = (\nu_{1t}, ..., \nu_{Nt})'$, are assumed to be independent over time and normally distributed with zero mean and variance/covariance matrix $\boldsymbol{\Sigma}$.

The implication of the ARMA process being seemingly unrelated is that the finite-order lag polynomial matrices $\boldsymbol{\Theta}(L)$ and $\boldsymbol{\Phi}(L)$ are diagonal. Also, since the seemingly unrelated ARMA process is invertible it has an infinite moving average representation as in (17).

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\Psi}(L)\boldsymbol{\nu}_t, \tag{17}$$

In (17), $\boldsymbol{\Psi}(L)$ is an infinite order lag polynomial matrix. Since both the polynomial lag matrices that governs the autoregressive and the moving average part of the seemingly unrelated ARMA process are diagonal, so will $\boldsymbol{\Psi}(L)$ be. However, to make the error processes workable we have to place some restrictions on the coefficients of the infinite moving average representation of the noise terms. This assumption is presented below in Assumption 7.

Assumption 7 *Let ψ_k^i be the i:th diagonal element in the lag polynomial matrix of order k. The following condition is placed upon the coefficients of the infinite order moving average representation:*

$$\sum_{k=0}^{\infty} (\psi_k^i \psi_k^j)^2 < \infty \text{ for } i, j \in \{1, ..., N\}$$

The condition placed on the coefficients of the infinite moving average representation of the seemingly unrelated ARMA process is necessary for the existence of the long-run variances and covariances of the disturbance processes to exit. If they exist, they can be consistently estimated for fixed N given that T is large.

We have now made all the assumptions necessary to present the central result for the CSCLM test with serially correlated errors. However, before we go on by studying the CSCLM test in the presence of cross-sectional and serial correlation, we first note that the orthogonalized residual series will retain the stationarity properties of the original seemingly unrelated ARMA process. That is, if the N residual series appearing as elements of the vector $\boldsymbol{e}_t$ are stationary, so is the N orthogonalized residual series of the $\tilde{\mathbf{e}}_t$. This follows from the fact that any linear combination of stationary ARMA processes is stationary as well. This observation will ensure that the stationarity properties that apply to the residual series under the null hypothesis is kept intact under the linear transformation performed in the orthogonalization procedure. Furthermore, since the orthogonalization process per definition yields cross-sectionally uncorrelated residual series, we can now apply the CSCLM test to the orthogonalized series even when disturbances are serially correlated.

Hence, in the case of serially correlated disturbances, we can note that if Model 1, Assumption 6 and Assumption 7 are correct, the statistic of the cross-sectionally corrected LM test will have an approximate standard normal distribution under the null hypothesis as $T \to \infty$ while N is fixed, i.e. as $(T \to \infty, N fixed)$.

For the test statistic of the CSCLM test to have a normal limit, it must be the case that the individual test statistics are independent. If this is true, we can utilize a CLT as above to establish that the CSCLM test statistic will have an approximate normal distribution. To this end, consider the the expected value $E[\check{\mathbf{e}}_t\check{\mathbf{e}}_t']$. From the infinite moving average representation of the seemingly unrelated ARMA process we can write this expected value as $E[\check{\mathbf{e}}_t\check{\mathbf{e}}_t'] = \sum_{k=0}^{\infty} \Psi_k \Sigma \Psi_k$. The condition placed on the infinite moving average coefficients in Assumption 7 assures that the elements of this matrix exist and are finite. As T goes to infinity, the elements of $\sum_{k=0}^{\infty} \Psi_k \Sigma \Psi_k$ are consistently estimated by replacing the population moment by the estimator for the sample moment. The spectral decomposition of the estimated varaince/covariance matrix will then imply that the orthogonalized series are independent. That is, $E[\tilde{\mathbf{e}}_t\tilde{\mathbf{e}}_t'] = \hat{\mathbf{H}}\hat{\mathbf{\Lambda}}^{-1/2}\hat{\mathbf{H}}' E[\hat{\mathbf{V}}\check{\mathbf{e}}_t\check{\mathbf{e}}_t'\hat{\mathbf{V}}]\hat{\mathbf{H}}\hat{\mathbf{\Lambda}}^{-1/2}\hat{\mathbf{H}}' = \boldsymbol{I}_N$. This follows from the fact that the expected value can be rewritten as $E[\tilde{\mathbf{e}}_t\tilde{\mathbf{e}}_t'] = \hat{\mathbf{H}}\hat{\mathbf{\Lambda}}^{-1/2}\hat{\mathbf{\Lambda}}\hat{\mathbf{\Lambda}}^{-1/2}\hat{\mathbf{H}}'$ since $E[\hat{\mathbf{V}}\check{\mathbf{e}}_t\check{\mathbf{e}}_t'\hat{\mathbf{V}}] = \hat{\mathbf{C}}$ and $\hat{\mathbf{H}}'\hat{\mathbf{C}}\hat{\mathbf{H}} = \hat{\mathbf{\Lambda}}$. The latter equality follows from the fact that $\hat{\mathbf{H}}$ contains the eigenvectors of $\hat{\mathbf{C}}$ in its columns. We can now make the simplifications $E[\tilde{\mathbf{e}}_t\tilde{\mathbf{e}}_t'] = \hat{\mathbf{H}}\mathbf{I}_N\hat{\mathbf{H}}' = \mathbf{I}_N$, which follows from the fact that $\mathbf{H}$ is orthogonal. Hence, the orthogonalization process has generated independent series. As a consequence, the individual LM test statistics in the CSCLM test will be independent and the limit $(T \to \infty, N fixed)$, together with the Lindberg-Levy CLT, will induce an approximate normal distribution for the panel test statistic. Finally, using appropriate moments for standardization will assure an approximate standard normal distribution.

The results above suggest that the CSCLM test for panel data stationarity is normally distributed under the null hypothesis. However, the way that the test is described above says nothing about how we should go around when we are to apply the test in an empirical situation. For example, we don't know what moments that should be used to standardize the test statistic. Since we have introduced an orthogonalization process in the CSCLM test, we cannot apply the moments of Hadri (2000) or Hadri and Larsson (2003). In the next subsection, we will make the CSCLM test implementable by supplying the appropriate standardizing moments. As the results of this section is asymptotic, we will also study the finite-sample performance of the CSCLM test through a Monte Carlo simulation.

3.3 Implementing the CSCLM test

In this section we describe how the CSCLM test is implemented in empirical applications. More precisely, we present the moments that are to be used in the CSCLM test procedure to standardize the test statistic in (11). We also investigate the finite-sample performance of the CSCLM test.

When we are to implement the CSCLM test, we have to make a choice regarding the estimator that is to be used when calculating the long-run variance. This choice can be divided into two separate decisions. First and foremost, we have to decide whether or not serial correlation in the noise terms is an issue or not. Second, if we decide that serial correlation should be corrected for, we must choose what lag window to apply in the estimation of the long-run variance. While the first choice can be based on theoretical considerations, the choice of lag window is somewhat arbitrary and should be guided by the believed degree of serial correlation in the residual series together with robustness considerations.[10] In this paper, we consider three different choices of lag window in (7), namely $l = int[k(T/100)^{0.25}]$ where $k \in \{4, 12, 24\}$. We also consider the case where $\widehat{\sigma}_{\varepsilon}^2 = T^{-1} \sum_{t=1}^{T} e_t^2$ can be used to estimate the variance of the error term. This case assumes that there is no serial correlation in the noise series and is referred to as the 'no lag' case.

As mentioned above, we have to find the moments that are to be used when we standardize the mean of the individual test statistics, else the CSCLM test would not be implementable. These moments depend on the specification of deterministic trends in the model, the time-series dimension chosen and on the choice of lag window. As the moments are very hard, if at all possible, to find analytically, we use numerical methods to obtain the appropriate standardizing moments. To this end, we generate data according to (12)-(15) and generate the errors following the expression in (16). In the simulations where we are concerned with an intercept only, we let $\alpha_i \in U[-1, 1]$, while $\delta_i \in U[0.1, 0.3]$ is used for the time trend. Using the different samples with generated data, we can apply the CSCLM test a number large of times for each choice of deterministic specification, sample size and lag window and then calculate the mean and the variance of the resulting test statistics.

Since our procedure is robust against cross-sectional correlation, it suffices to consider the case where the disturbance correlation is set to zero. For each of the sample sizes $T \in \{25, 50, 75, 100, 150, 200, 250, 500, 1000\}$, we generate 50,000 test statistics and extract the mean and the variance from this sample. We then perform 50 replications for each sample size and fit response surface regressions to the series with simulated moments.[11] In the response surface regressions, we include an intercept and the terms $T^{-0.5}$, T^{-1} and $T^{-1.5}$. Using these terms provides a good regression fit while still keeping the response

[10] Schwert (1989) gives some guidance toward the choice of lag window in the unit root context, which possibly can point out some direction towards a choice of lag window in the current context. An alternative to choosing a fixed lag window could be to base the choice of l on data-dependent methods as suggested by for example Newey and West (1994). However, since the moments of the individual test statistics depend on the choice of l see Jönsson (2004), this implies an infeasible amount of simulations to obtain moments to implement the CSCLM test. Hence, we consider only the three different choices of l and leave the choice of lag window open for consideration.

[11] To our knowledge, this is the first time response surface regressions are used to obtain moments in the panel data unit root, cointegration or stationarity framework. The use of response surface regressions makes small-sample moments easy to calculate, without having to rely on tabulated moments and interpolations based on such moments. Furthermore, under some circumstances asymptotic moments are readily available from the response surface regressions, where they are represented by the intercept.

surface regressions easy to handle.

In Table 2 we present the response surface estimates for the different choices of model and for different choices of lag window, k.

As seen in Table 2, the fit of the response surface models is good for all regressions. From Table 2, we can spot one extraordinary event however. When when $k = 24$ and a linear trend is present in the model, the response surface regression for the variance is badly behaved if fitted to the sample containing all the response surface observations. To resolve this problem, we run two separate response surface regressions for this case. We run one response surface regression for the case where $T < 100$ and one for the case where $T \geq 100$. When we make this division, we get an excellent fit for the response surface regression as seen in Table 2. Using the estimated coefficients, that are presented in the Table 2, the appropriate standardizing moments, that are to be used for a specific model and under a specific choice of k, can be calculated.

Next, we want to study the size properties of the CSCLM test when using the response surface regressions in Table 2. Even though we know that the CSCLM test is asymptotically normal, it is interesting to study the performance of the test for different small-sample situations. To do this, we generate 5,000 data sets according to the procedure described in (12)-(15) with the errors being governed by (16). We generate data according to the model where $\alpha_i \neq 0$, $\delta_i = 0$, i.e. where we have an intercept but not time trend.[12] To assess the size of the CSCLM test, we then use the generated data sets to calculate 5,000 test statistics for the various sample sizes, choices of k and degrees of cross-sectional correlation and study the percentage of times the null is incorrectly rejected using the 5% critical value from the normal distribution.

In Table 3 we present the size properties of the CSCLM test. As seen in the table, the CSCLM test has good size properties in the cases where T is relatively large compared to N. This is what we expect to find since the results in the current paper relies on T tending to infinity while N is fixed, which is applicable in cases where T is much larger then N. As expected, we also see from from Table 3 that the CSCLM test works well regardless of the degree of cross-sectional correlation.

Now let us consider the case where the noise term, ε_{it}, is serially correlated. If the noise term is serially correlated we cannot use $\widehat{\sigma}_\varepsilon^2 = T^{-1}\sum_{t=1}^T e_{it}^2$ as an estimator of the error variance. Instead, we have to choose the consistent estimator in (7) and let $k \in \{4, 12, 24\}$. To investigate the size properties for this case, we let the noise term ε_{it} be generated according to the autoregressive process $\varepsilon_{it} = \theta_i \varepsilon_{it-1} + \nu_{it}$, where $\theta_i \in U[0.0, 0.4]$ and the variance/covariance matrix of $\boldsymbol{\nu}_t$ is equal to $\boldsymbol{\Sigma}$. The structure of $\boldsymbol{\Sigma}$ is the same as the one described in (16).

According to the results above, the CSCLM test statistic can be approximated by a normal limit even if the noise terms ε_{it} are serially correlated. However, it can be interesting to study how fast the test statistic converges to the limiting distribution and what choice of k that is appropriate.

In Table 4 we present the size properties of the CSCLM test for the case where the noise terms are serially correlated. The first thing that we note is that the test the uses $\widehat{\sigma}_\varepsilon^2 = T^{-1}\sum_{t=1}^T e_{it}^2$ as an estimator of the error variance performs utterly bad. Also, there is a serious size distortion to the test that uses $k = 4$ in the estimation of the variance in (7). This is due to the fact that the choice $k = 4$ is insufficient to estimate the long-run variance in the CSCLM test when the autocorrelation coefficient is distributed as

[12] Results for the model with an intercept and a time trend is available upon request. However, we don't present it in the current paper to save space.

$\theta_i \in U[0.0, 0.4]$. However, the size of the CSCLM test when we use $k = 12$ or $k = 24$ looks good. The strong autocorrelation in the disturbance term seems to be captured by the variance estimators when we use a wider Bartlett window, whereas $k = 4$ is insufficient for the variance estimation. From Table 4 we also see that the CSCLM test that can incorporate the serial correlation also works good regardless of the degree of cross-sectional correlation.

Finally, we want to study the power properties of the CSCLM test to see how well the CSCLM test can discriminate between a true and a false null hypothesis. Data is one again generated as in (12)-(15) above, with the disturbance vector described by (16). However, to generate data under the alternative hypothesis, we construct a $N \times 1$ time-series vector with random walks. These random walks are added to the stationary series. We set the variance/covaraince matrix of the random walks to the identity matrix and generate $T+100$ random walk observations, letting $\xi_{i0} = 0$ and $\sigma_\eta^2 = 1$. We then discard the first 100 of these observations to reduce the influence of the initial observation. We then generate 5,000 data sets and calculate the CSCLM test statistics using these data sets.

In Table 5 and Table 6, we present the size-adjusted power of the CSCLM test for different choices of sample size, lag window and deterministic trends.[13] More specifically, in Table 5, we present the power of the CSCLM test when only an intercept is included in the model, while we present the power of the test when both an intercept and a trend is present in Table 6.

As seen from Table 5, the power of the test is good across all parameter combinations considered. However, when we consider the panel data model with both an intercept and a trend we get a somewhat different result. From Table 6 we see that the size-adjusted power of the CSCLM test falls below the significance level when k is large and T is small, i.e. we observe a small-sample bias in the test. These results are in line with what we would expect from the results previously obtained by Lee (1996) and Caner and Kilian (2001) in the univariate stationarity testing environment, and by Jönsson (2004) in the panel data stationarity context.[14] These authors have documented a fall in power of the stationarity test in the presence of a trend, while allowing for a large degree of serial correlation. Hence, there is no reason to believe that the small-sample bias of the CSCLM test is caused by the orthogonalization procedure. Instead it seems as if the bias is a general problem arising in the stationarity testing framework. An important implication is that the CSCLM test is applicable in all the situations where the panel data stationarity testing procedure is applicable.

[13]Although the size properties of the CSCLM test are good, we want to eliminate every size effect from power properties. Hence, we size-adjust the CSCLM test by extracting 5% critical values from the empirical distribution of 5,000 test statistics of the CSCLM test. We then use these critical values instead of critical values from the normal distribution.

[14]In addition, it can be noted that Moon et al. (2003) documented a similar problem, regarding power and deterministic trends, in the panel data unit root context.

Table 2: Response surface estimates

Lag window, $l = int[k(T/100)^{0.25}]$	Deterministic component	Moment	Intercept	$T^{-0.5}$	T^{-1}	$T^{-1.5}$	R^2
k=24	$\alpha_i \neq 0$, $\delta_i = 0$	Mean	0.17466	-0.34183	6.68158	-2.75772	0.9998
		Variance	0.02471	-0.23332	0.80342	-0.97795	0.9995
	$\alpha_i \neq 0$, $\delta_i \neq 0$	Mean	0.07813	-0.45787	8.38846	0.22836	0.9998
		Variance[a]	0.001750	-0.07348	0.77155	-2.00551	0.9998
		Variance[b]	0.001750	-0.00879	-0.21315	1.64240	0.9993
k=12	$\alpha_i \neq 0$, $\delta_i = 0$	Mean	0.16849	-0.06190	1.71012	1.23720	0.9993
		Variance	0.02378	-0.12094	0.04123	0.59972	0.9986
	$\alpha_i \neq 0$, $\delta_i \neq 0$	Mean	0.06764	-0.00963	1.39319	4.42050	0.9999
		Variance	0.00174	-0.00420	-0.11559	0.56114	0.9988
k=4	$\alpha_i \neq 0$, $\delta_i = 0$	Mean	0.16741	-0.02323	0.61815	-0.91157	0.9752
		Variance	0.02263	-0.02877	-0.29747	1.05645	0.9866
	$\alpha_i \neq 0$, $\delta_i \neq 0$	Mean	0.06733	-0.02048	0.78823	-1.44227	0.9953
		Variance	0.00176	-0.00180	-0.04155	0.13469	0.9900
No lag, k=0	$\alpha_i \neq 0$, $\delta_i = 0$	Mean	0.16752	-0.02776	0.43145	-0.72809	0.9101
		Variance	0.02274	-0.01662	0.10566	-0.45257	0.8359
	$\alpha_i \neq 0$, $\delta_i \neq 0$	Mean	0.06667	-0.00009	0.13725	-0.01474	0.9853
		Variance	0.00175	-0.00014	-0.00392	-0.00559	0.9026

Notes: [a] These response surface coefficients are to be used whenever $T < 100$.
[b] When $T \geqslant 100$ these parameter estimates apply.

Table 3: Size of the CSCLM test in the presence of CSD

k=24	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.043	0.027	0.018	0.007	-	-	-	0.045	0.032	0.017	0.005	-	-	-
50	0.054	0.047	0.047	0.028	0.024	-	-	0.053	0.046	0.036	0.032	0.027	-	-
75	0.051	0.044	0.040	0.033	0.024	0.007	-	0.046	0.051	0.042	0.033	0.026	0.005	-
100	0.062	0.055	0.047	0.041	0.036	0.015	-	0.061	0.061	0.045	0.045	0.035	0.016	-
150	0.066	0.059	0.047	0.049	0.038	0.031	0.006	0.058	0.050	0.050	0.048	0.041	0.025	0.006
200	0.063	0.054	0.052	0.049	0.050	0.034	0.017	0.067	0.056	0.050	0.051	0.050	0.034	0.014
250	0.067	0.064	0.056	0.058	0.051	0.043	0.023	0.066	0.064	0.060	0.058	0.051	0.044	0.030
500	0.068	0.067	0.056	0.054	0.052	0.046	0.043	0.066	0.063	0.059	0.059	0.057	0.049	0.043
1000	0.065	0.061	0.059	0.054	0.059	0.049	0.043	0.071	0.062	0.063	0.058	0.055	0.048	0.042
	ρ=0.50							ρ=0.75						
T=25	0.051	0.032	0.019	0.009	-	-	-	0.049	0.031	0.015	0.007	-	-	-
50	0.053	0.047	0.039	0.033	0.027	-	-	0.048	0.044	0.043	0.030	0.027	-	-
75	0.050	0.048	0.037	0.032	0.024	0.006	-	0.053	0.048	0.037	0.030	0.028	0.007	-
100	0.058	0.047	0.046	0.037	0.040	0.016	-	0.053	0.051	0.047	0.039	0.029	0.016	-
150	0.059	0.060	0.054	0.046	0.047	0.027	0.006	0.063	0.048	0.054	0.046	0.044	0.029	0.005
200	0.065	0.053	0.055	0.053	0.049	0.042	0.015	0.065	0.062	0.053	0.051	0.042	0.035	0.015
250	0.067	0.057	0.057	0.057	0.052	0.042	0.025	0.064	0.064	0.058	0.055	0.057	0.043	0.027
500	0.066	0.055	0.058	0.064	0.055	0.052	0.046	0.071	0.063	0.062	0.059	0.053	0.049	0.050
1000	0.073	0.063	0.059	0.052	0.055	0.045	0.042	0.067	0.064	0.055	0.061	0.054	0.052	0.041
k=12	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.043	0.027	0.012	0.003	-	-	-	0.041	0.029	0.009	0.004	-	-	-
50	0.056	0.048	0.035	0.026	0.019	-	-	0.055	0.046	0.030	0.026	0.017	-	-
75	0.057	0.051	0.041	0.030	0.024	0.004	-	0.062	0.051	0.045	0.032	0.031	0.003	-
100	0.057	0.055	0.050	0.039	0.041	0.014	-	0.061	0.054	0.045	0.038	0.035	0.013	-
150	0.064	0.058	0.056	0.055	0.044	0.030	0.005	0.070	0.056	0.052	0.048	0.045	0.028	0.004
200	0.067	0.057	0.052	0.046	0.049	0.033	0.012	0.070	0.059	0.056	0.056	0.048	0.035	0.014
250	0.062	0.058	0.059	0.053	0.046	0.038	0.019	0.067	0.069	0.060	0.056	0.052	0.045	0.022
500	0.073	0.059	0.060	0.061	0.058	0.045	0.038	0.076	0.059	0.061	0.060	0.064	0.046	0.036
1000	0.070	0.052	0.060	0.065	0.063	0.052	0.042	0.077	0.066	0.067	0.060	0.058	0.056	0.050
	ρ=0.50							ρ=0.75						
T=25	0.040	0.024	0.010	0.003	-	-	-	0.040	0.026	0.012	0.003	-	-	-
50	0.057	0.046	0.037	0.027	0.017	-	-	0.055	0.040	0.035	0.027	0.014	-	-
75	0.061	0.045	0.043	0.038	0.025	0.006	-	0.056	0.051	0.041	0.036	0.025	0.004	-
100	0.062	0.056	0.048	0.041	0.038	0.012	-	0.061	0.057	0.046	0.042	0.037	0.014	-
150	0.070	0.060	0.053	0.051	0.043	0.025	0.004	0.065	0.052	0.049	0.045	0.041	0.028	0.003
200	0.068	0.058	0.052	0.046	0.050	0.035	0.011	0.064	0.061	0.059	0.051	0.048	0.033	0.014
250	0.069	0.068	0.054	0.052	0.050	0.039	0.023	0.070	0.061	0.057	0.057	0.053	0.045	0.021
500	0.065	0.061	0.060	0.056	0.061	0.044	0.042	0.071	0.063	0.058	0.060	0.056	0.048	0.043
1000	0.070	0.060	0.061	0.059	0.061	0.048	0.048	0.068	0.066	0.065	0.063	0.057	0.051	0.043
k=4	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.049	0.021	0.003	0.000	-	-	-	0.045	0.025	0.003	0.000	-	-	-
50	0.063	0.052	0.030	0.020	0.010	-	-	0.061	0.044	0.034	0.023	0.010	-	-
75	0.066	0.049	0.040	0.031	0.029	0.002	-	0.065	0.058	0.042	0.037	0.029	0.000	-
100	0.065	0.062	0.048	0.039	0.038	0.013	-	0.070	0.058	0.049	0.037	0.037	0.012	-
150	0.069	0.069	0.054	0.051	0.041	0.024	0.001	0.061	0.060	0.054	0.050	0.043	0.026	0.001
200	0.068	0.063	0.054	0.050	0.050	0.033	0.012	0.069	0.062	0.057	0.052	0.048	0.036	0.010
250	0.066	0.060	0.057	0.052	0.058	0.040	0.018	0.066	0.064	0.045	0.049	0.057	0.037	0.018
500	0.067	0.076	0.061	0.066	0.054	0.050	0.034	0.071	0.066	0.062	0.060	0.058	0.054	0.036
1000	0.066	0.063	0.061	0.059	0.060	0.053	0.049	0.067	0.068	0.058	0.060	0.057	0.053	0.046
	ρ=0.50							ρ=0.75						
T=25	0.049	0.020	0.003	0.000	-	-	-	0.040	0.019	0.004	0.000	-	-	-
50	0.063	0.047	0.032	0.021	0.013	-	-	0.058	0.040	0.032	0.021	0.013	-	-
75	0.070	0.053	0.043	0.035	0.028	0.001	-	0.065	0.049	0.040	0.037	0.025	0.001	-
100	0.064	0.057	0.041	0.037	0.035	0.008	-	0.071	0.060	0.046	0.040	0.035	0.010	-
150	0.067	0.062	0.054	0.050	0.046	0.027	0.002	0.071	0.051	0.053	0.047	0.044	0.029	0.001
200	0.065	0.057	0.060	0.051	0.042	0.036	0.010	0.070	0.059	0.061	0.047	0.043	0.038	0.010
250	0.065	0.059	0.060	0.060	0.052	0.042	0.017	0.065	0.062	0.054	0.060	0.048	0.038	0.018
500	0.068	0.062	0.058	0.063	0.055	0.048	0.039	0.070	0.063	0.060	0.057	0.060	0.045	0.033
1000	0.069	0.062	0.067	0.063	0.056	0.051	0.049	0.067	0.071	0.061	0.064	0.062	0.053	0.043
No lag, k=0	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.052	0.025	0.000	0.000	-	-	-	0.056	0.016	0.000	0.000	-	-	-
50	0.060	0.048	0.032	0.020	0.008	-	-	0.067	0.045	0.033	0.020	0.007	-	-
75	0.065	0.057	0.044	0.035	0.027	0.001	-	0.067	0.054	0.048	0.037	0.023	0.000	-
100	0.070	0.052	0.046	0.046	0.037	0.010	-	0.065	0.065	0.050	0.047	0.043	0.009	-
150	0.071	0.066	0.061	0.047	0.048	0.031	0.001	0.070	0.054	0.058	0.049	0.045	0.025	0.001
200	0.070	0.058	0.059	0.052	0.053	0.038	0.010	0.064	0.056	0.061	0.050	0.047	0.033	0.012
250	0.067	0.064	0.063	0.057	0.050	0.039	0.017	0.066	0.058	0.056	0.056	0.052	0.040	0.019
500	0.064	0.069	0.063	0.059	0.054	0.049	0.038	0.068	0.068	0.054	0.057	0.060	0.046	0.039
1000	0.072	0.061	0.067	0.061	0.061	0.050	0.044	0.066	0.068	0.059	0.065	0.060	0.052	0.042
	ρ=0.50							ρ=0.75						
T=25	0.059	0.021	0.000	0.000	-	-	-	0.056	0.021	0.000	0.000	-	-	-
50	0.063	0.048	0.034	0.019	0.007	-	-	0.057	0.045	0.026	0.021	0.007	-	-
75	0.066	0.053	0.049	0.035	0.029	0.000	-	0.067	0.053	0.040	0.034	0.025	0.000	-
100	0.064	0.056	0.044	0.044	0.038	0.009	-	0.067	0.055	0.057	0.042	0.035	0.009	-
150	0.059	0.059	0.060	0.050	0.052	0.027	0.001	0.068	0.066	0.055	0.051	0.052	0.026	0.002
200	0.065	0.057	0.059	0.061	0.048	0.037	0.008	0.071	0.068	0.064	0.051	0.045	0.032	0.010
250	0.066	0.066	0.059	0.054	0.052	0.042	0.016	0.068	0.060	0.057	0.054	0.051	0.035	0.017
500	0.067	0.060	0.066	0.057	0.049	0.051	0.044	0.061	0.056	0.064	0.060	0.055	0.047	0.037
1000	0.065	0.063	0.066	0.055	0.059	0.057	0.045	0.069	0.065	0.060	0.065	0.062	0.054	0.044

Table 4: Size of the CSCLM test in the presence of CSD and autocorrelation

k=24	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.044	0.027	0.012	0.005	-	-	-	0.041	0.023	0.014	0.005	-	-	-
50	0.056	0.038	0.033	0.024	0.018	-	-	0.055	0.045	0.031	0.021	0.016	-	-
75	0.056	0.039	0.033	0.025	0.016	0.002	-	0.055	0.038	0.030	0.021	0.019	0.003	-
100	0.065	0.051	0.040	0.035	0.028	0.011	-	0.056	0.048	0.045	0.038	0.030	0.009	-
150	0.066	0.050	0.047	0.042	0.038	0.019	0.001	0.060	0.057	0.045	0.039	0.039	0.019	0.002
200	0.070	0.057	0.057	0.052	0.049	0.030	0.008	0.073	0.058	0.056	0.054	0.046	0.031	0.009
250	0.072	0.066	0.064	0.057	0.055	0.036	0.015	0.073	0.058	0.056	0.062	0.060	0.040	0.017
500	0.072	0.073	0.065	0.058	0.058	0.056	0.037	0.073	0.072	0.069	0.069	0.057	0.053	0.041
1000	0.072	0.070	0.063	0.067	0.060	0.061	0.045	0.079	0.068	0.060	0.067	0.060	0.055	0.045
	ρ=0.50							ρ=0.75						
T=25	0.042	0.020	0.008	0.006	-	-	-	0.041	0.024	0.012	0.004	-	-	-
50	0.057	0.043	0.031	0.023	0.014	-	-	0.061	0.042	0.029	0.021	0.016	-	-
75	0.054	0.043	0.034	0.024	0.023	0.002	-	0.055	0.044	0.035	0.024	0.018	0.003	-
100	0.060	0.044	0.046	0.036	0.032	0.011	-	0.064	0.045	0.043	0.032	0.026	0.008	-
150	0.057	0.057	0.045	0.040	0.034	0.016	0.002	0.061	0.058	0.050	0.049	0.042	0.020	0.002
200	0.064	0.064	0.057	0.048	0.049	0.028	0.010	0.071	0.063	0.054	0.051	0.052	0.035	0.007
250	0.069	0.060	0.059	0.055	0.051	0.040	0.015	0.069	0.068	0.060	0.057	0.056	0.040	0.017
500	0.070	0.067	0.064	0.066	0.060	0.051	0.036	0.076	0.062	0.065	0.069	0.069	0.054	0.040
1000	0.069	0.068	0.070	0.067	0.059	0.052	0.046	0.068	0.061	0.067	0.062	0.066	0.056	0.047
k=12	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.040	0.016	0.007	0.001	-	-	-	0.040	0.019	0.004	0.001	-	-	-
50	0.059	0.043	0.032	0.016	0.011	-	-	0.066	0.036	0.030	0.018	0.009	-	-
75	0.066	0.056	0.040	0.029	0.023	0.002	-	0.072	0.052	0.042	0.034	0.022	0.001	-
100	0.074	0.068	0.051	0.042	0.036	0.007	-	0.075	0.067	0.052	0.040	0.041	0.008	-
150	0.073	0.064	0.062	0.050	0.046	0.022	0.002	0.073	0.068	0.065	0.053	0.048	0.026	0.001
200	0.077	0.072	0.067	0.063	0.057	0.036	0.006	0.076	0.074	0.064	0.065	0.057	0.036	0.004
250	0.077	0.072	0.065	0.065	0.065	0.046	0.012	0.075	0.070	0.066	0.065	0.063	0.045	0.014
500	0.077	0.077	0.073	0.073	0.071	0.063	0.044	0.081	0.074	0.071	0.070	0.072	0.064	0.039
1000	0.077	0.071	0.065	0.074	0.067	0.072	0.059	0.073	0.074	0.075	0.073	0.072	0.070	0.065
	ρ=0.50							ρ=0.75						
T=25	0.045	0.018	0.008	0.001	-	-	-	0.045	0.018	0.006	0.001	-	-	-
50	0.062	0.050	0.028	0.017	0.010	-	-	0.059	0.043	0.032	0.019	0.009	-	-
75	0.065	0.061	0.043	0.030	0.027	0.002	-	0.061	0.055	0.047	0.031	0.022	0.001	-
100	0.069	0.059	0.046	0.043	0.032	0.005	-	0.073	0.057	0.052	0.041	0.032	0.011	-
150	0.076	0.066	0.057	0.058	0.047	0.025	0.001	0.080	0.068	0.063	0.057	0.044	0.024	0.001
200	0.075	0.071	0.064	0.060	0.056	0.032	0.006	0.074	0.070	0.066	0.054	0.059	0.034	0.006
250	0.075	0.073	0.066	0.066	0.066	0.042	0.017	0.069	0.070	0.074	0.071	0.059	0.044	0.013
500	0.075	0.077	0.080	0.071	0.079	0.060	0.045	0.077	0.075	0.074	0.067	0.066	0.068	0.050
1000	0.074	0.076	0.071	0.070	0.065	0.065	0.062	0.076	0.069	0.074	0.067	0.074	0.070	0.060
k=4	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.083	0.025	0.001	0.000	-	-	-	0.081	0.024	0.003	0.000	-	-	-
50	0.092	0.074	0.049	0.027	0.007	-	-	0.103	0.069	0.055	0.020	0.009	-	-
75	0.112	0.105	0.084	0.072	0.045	0.001	-	0.108	0.108	0.080	0.072	0.047	0.001	-
100	0.103	0.092	0.086	0.072	0.067	0.011	-	0.102	0.088	0.085	0.076	0.065	0.010	-
150	0.107	0.105	0.106	0.098	0.098	0.052	0.001	0.101	0.098	0.105	0.088	0.085	0.051	0.000
200	0.108	0.115	0.108	0.112	0.109	0.089	0.017	0.107	0.116	0.110	0.107	0.111	0.092	0.017
250	0.098	0.097	0.095	0.108	0.092	0.096	0.038	0.103	0.097	0.104	0.098	0.105	0.087	0.039
500	0.096	0.112	0.112	0.126	0.110	0.139	0.130	0.095	0.106	0.105	0.120	0.118	0.124	0.134
1000	0.098	0.099	0.098	0.103	0.107	0.117	0.126	0.087	0.088	0.094	0.108	0.106	0.111	0.126
	ρ=0.50							ρ=0.75						
T=25	0.076	0.021	0.002	0.000	-	-	-	0.075	0.023	0.001	0.000	-	-	-
50	0.090	0.072	0.053	0.026	0.008	-	-	0.094	0.076	0.050	0.019	0.009	-	-
75	0.106	0.104	0.087	0.076	0.050	0.000	-	0.109	0.097	0.090	0.070	0.050	0.000	-
100	0.103	0.096	0.088	0.077	0.061	0.010	-	0.095	0.083	0.091	0.073	0.062	0.010	-
150	0.099	0.105	0.104	0.098	0.100	0.051	0.000	0.104	0.099	0.094	0.097	0.092	0.051	0.001
200	0.104	0.113	0.121	0.116	0.113	0.091	0.016	0.098	0.112	0.116	0.111	0.106	0.094	0.017
250	0.092	0.091	0.111	0.101	0.100	0.090	0.036	0.093	0.093	0.096	0.100	0.101	0.093	0.037
500	0.097	0.112	0.111	0.114	0.114	0.129	0.120	0.105	0.104	0.115	0.116	0.110	0.121	0.128
1000	0.089	0.095	0.099	0.100	0.109	0.116	0.126	0.083	0.092	0.098	0.109	0.108	0.116	0.127
No lag, k=0	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.244	0.181	0.013	0.000	-	-	-	0.235	0.188	0.014	0.000	-	-	-
50	0.307	0.383	0.414	0.367	0.271	-	-	0.308	0.376	0.407	0.348	0.284	-	-
75	0.319	0.439	0.486	0.538	0.541	0.121	-	0.320	0.439	0.497	0.535	0.547	0.123	-
100	0.328	0.464	0.547	0.589	0.634	0.599	-	0.329	0.455	0.543	0.606	0.632	0.586	-
150	0.347	0.484	0.590	0.661	0.708	0.830	0.515	0.351	0.472	0.570	0.657	0.710	0.839	0.521
200	0.352	0.485	0.589	0.672	0.739	0.891	0.916	0.339	0.498	0.587	0.671	0.736	0.887	0.919
250	0.348	0.494	0.613	0.683	0.755	0.915	0.971	0.344	0.497	0.596	0.691	0.763	0.905	0.968
500	0.351	0.506	0.628	0.708	0.779	0.947	0.995	0.353	0.515	0.634	0.712	0.785	0.945	0.995
1000	0.358	0.513	0.625	0.707	0.792	0.957	0.999	0.341	0.527	0.614	0.718	0.792	0.949	0.998
	ρ=0.50							ρ=0.75						
T=25	0.235	0.180	0.015	0.000	-	-	-	0.241	0.181	0.013	0.000	-	-	-
50	0.311	0.387	0.397	0.371	0.283	-	-	0.307	0.373	0.397	0.358	0.282	-	-
75	0.329	0.428	0.485	0.534	0.540	0.121	-	0.323	0.420	0.506	0.533	0.550	0.137	-
100	0.331	0.457	0.533	0.592	0.634	0.604	-	0.323	0.464	0.540	0.601	0.635	0.606	-
150	0.353	0.494	0.562	0.647	0.712	0.825	0.510	0.341	0.481	0.573	0.660	0.710	0.829	0.517
200	0.335	0.480	0.595	0.679	0.748	0.891	0.910	0.328	0.483	0.603	0.681	0.743	0.886	0.914
250	0.343	0.487	0.619	0.693	0.751	0.917	0.967	0.351	0.490	0.595	0.686	0.760	0.908	0.972
500	0.362	0.506	0.613	0.709	0.785	0.950	0.995	0.347	0.506	0.613	0.713	0.785	0.943	0.995
1000	0.350	0.519	0.633	0.720	0.792	0.956	0.998	0.347	0.510	0.619	0.726	0.790	0.958	0.999

Table 5: Power of the CSCLM test, intercept only.

k=24	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.037	0.023	0.021	0.035	-	-	-	0.034	0.017	0.022	0.034	-	-	-
50	0.129	0.124	0.138	0.215	0.229	-	-	0.136	0.116	0.140	0.182	0.214	-	-
75	0.262	0.309	0.330	0.481	0.563	0.962	-	0.297	0.279	0.326	0.429	0.547	0.963	-
100	0.493	0.490	0.507	0.697	0.796	0.993	-	0.473	0.457	0.588	0.675	0.735	0.991	-
150	0.811	0.793	0.840	0.900	0.959	1.000	1.000	0.791	0.809	0.847	0.883	0.938	1.000	1.000
200	0.945	0.955	0.959	0.980	0.989	1.000	1.000	0.934	0.959	0.951	0.971	0.988	1.000	1.000
250	0.983	0.991	0.993	0.997	0.998	1.000	1.000	0.980	0.992	0.990	0.994	0.996	1.000	1.000
500	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.030	0.024	0.025	0.032	-	-	-	0.031	0.022	0.022	0.034	-	-	-
50	0.134	0.104	0.134	0.183	0.268	-	-	0.113	0.087	0.115	0.178	0.252	-	-
75	0.280	0.246	0.308	0.415	0.534	0.960	-	0.269	0.231	0.274	0.415	0.496	0.960	-
100	0.469	0.426	0.505	0.567	0.737	0.988	-	0.451	0.405	0.434	0.514	0.690	0.990	-
150	0.805	0.757	0.824	0.825	0.913	0.999	1.000	0.812	0.758	0.739	0.849	0.888	1.000	1.000
200	0.934	0.948	0.944	0.965	0.976	1.000	1.000	0.933	0.925	0.926	0.941	0.975	1.000	1.000
250	0.979	0.992	0.984	0.987	0.997	1.000	1.000	0.980	0.992	0.977	0.987	0.994	1.000	1.000
500	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
k=12	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.205	0.178	0.226	0.280	-	-	-	0.200	0.166	0.204	0.277	-	-	-
50	0.636	0.678	0.702	0.768	0.836	-	-	0.633	0.663	0.703	0.760	0.783	-	-
75	0.917	0.935	0.955	0.968	0.985	1.000	-	0.899	0.940	0.936	0.967	0.972	1.000	-
100	0.978	0.989	0.993	0.998	0.999	1.000	-	0.968	0.990	0.994	0.996	0.996	1.000	-
150	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000
200	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.210	0.158	0.183	0.289	-	-	-	0.181	0.155	0.182	0.308	-	-	-
50	0.646	0.561	0.597	0.691	0.794	-	-	0.638	0.518	0.537	0.638	0.796	-	-
75	0.909	0.913	0.910	0.947	0.965	1.000	-	0.868	0.840	0.872	0.929	0.944	1.000	-
100	0.973	0.987	0.989	0.991	0.997	1.000	-	0.974	0.979	0.977	0.982	0.992	1.000	-
150	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000	1.000
200	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
k=4	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.960	0.978	0.934	0.769	-	-	-	0.957	0.979	0.931	0.782	-	-	-
50	0.998	1.000	1.000	1.000	1.000	-	-	0.997	1.000	1.000	1.000	1.000	-	-
75	1.000	1.000	1.000	1.000	1.000	1.000	-	0.999	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.962	0.976	0.920	0.779	-	-	-	0.969	0.959	0.913	0.761	-	-	-
50	0.997	1.000	1.000	1.000	1.000	-	-	0.997	1.000	1.000	1.000	1.000	-	-
75	0.999	1.000	1.000	1.000	1.000	1.000	-	0.999	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	0.999	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
No lag, k=0	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.999	1.000	1.000	1.000	-	-	-	0.998	1.000	1.000	1.000	-	-	-
50	1.000	1.000	1.000	1.000	1.000	-	-	1.000	1.000	1.000	1.000	1.000	-	-
75	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.999	1.000	1.000	1.000	-	-	-	0.999	1.000	1.000	1.000	-	-	-
50	1.000	1.000	1.000	1.000	1.000	-	-	1.000	1.000	1.000	1.000	1.000	-	-
75	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 6: Power of the CSCLM test, intercept and trend.

k=24	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.008	0.005	0.006	0.010	-	-	-	0.014	0.008	0.006	0.010	-	-	-
50	0.007	0.002	0.001	0.001	0.000	-	-	0.006	0.001	0.001	0.002	0.000	-	-
75	0.004	0.003	0.001	0.001	0.000	0.000	-	0.006	0.002	0.001	0.000	0.000	0.000	-
100	0.013	0.004	0.001	0.000	0.001	0.001	-	0.010	0.003	0.002	0.001	0.000	0.001	-
150	0.299	0.191	0.118	0.086	0.094	0.225	0.886	0.289	0.171	0.080	0.079	0.106	0.184	0.884
200	0.678	0.620	0.527	0.503	0.498	0.801	1.000	0.701	0.606	0.502	0.508	0.521	0.749	0.999
250	0.852	0.891	0.857	0.879	0.884	0.987	1.000	0.864	0.899	0.869	0.855	0.860	0.967	1.000
500	0.998	1.000	1.000	1.000	1.000	1.000	1.000	0.994	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.013	0.005	0.008	0.010	-	-	-	0.009	0.005	0.006	0.010	-	-	-
50	0.006	0.002	0.001	0.000	0.000	-	-	0.007	0.002	0.001	0.002	0.001	-	-
75	0.006	0.002	0.001	0.000	0.001	0.000	-	0.006	0.002	0.002	0.001	0.000	0.001	-
100	0.014	0.003	0.001	0.002	0.000	0.001	-	0.011	0.003	0.001	0.001	0.001	0.000	-
150	0.293	0.130	0.085	0.049	0.074	0.231	0.903	0.278	0.131	0.043	0.048	0.045	0.187	0.924
200	0.660	0.615	0.422	0.452	0.399	0.767	0.999	0.686	0.542	0.412	0.302	0.352	0.678	0.999
250	0.882	0.869	0.825	0.850	0.750	0.959	1.000	0.844	0.891	0.823	0.737	0.709	0.926	1.000
500	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.997	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
k=12	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.007	0.004	0.003	0.007	-	-	-	0.005	0.001	0.005	0.008	-	-	-
50	0.076	0.031	0.015	0.011	0.010	-	-	0.088	0.021	0.013	0.013	0.010	-	-
75	0.538	0.488	0.445	0.342	0.332	0.592	-	0.528	0.478	0.385	0.296	0.307	0.579	-
100	0.838	0.864	0.868	0.851	0.843	0.935	-	0.843	0.886	0.843	0.845	0.835	0.932	-
150	0.977	0.999	0.999	1.000	1.000	1.000	1.000	0.981	0.998	0.999	1.000	1.000	1.000	1.000
200	0.997	1.000	1.000	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000	1.000	1.000
250	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.007	0.003	0.005	0.005	-	-	-	0.005	0.003	0.005	0.007	-	-	-
50	0.076	0.016	0.013	0.009	0.011	-	-	0.058	0.013	0.007	0.008	0.006	-	-
75	0.577	0.453	0.319	0.254	0.272	0.554	-	0.487	0.376	0.224	0.235	0.254	0.602	-
100	0.821	0.862	0.810	0.800	0.752	0.900	-	0.826	0.832	0.727	0.683	0.635	0.895	-
150	0.977	1.000	0.999	1.000	0.999	1.000	1.000	0.978	0.998	0.999	0.998	0.998	1.000	1.000
200	0.994	1.000	1.000	1.000	1.000	1.000	1.000	0.994	1.000	1.000	1.000	1.000	1.000	1.000
250	0.999	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
k=4	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.729	0.773	0.554	0.283	-	-	-	0.748	0.758	0.535	0.275	-	-	-
50	0.985	1.000	1.000	1.000	1.000	-	-	0.988	1.000	1.000	1.000	1.000	-	-
75	0.999	1.000	1.000	1.000	1.000	1.000	-	0.999	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.759	0.743	0.545	0.287	-	-	-	0.768	0.706	0.493	0.313	-	-	-
50	0.988	1.000	1.000	1.000	1.000	-	-	0.984	0.999	1.000	1.000	0.999	-	-
75	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
No lag, k=0	N=5	10	15	20	25	50	100	N=5	10	15	20	25	50	100
	ρ=0.00							ρ=0.25						
T=25	0.983	1.000	1.000	0.961	-	-	-	0.983	0.999	0.999	0.980	-	-	-
50	1.000	1.000	1.000	1.000	1.000	-	-	1.000	1.000	1.000	1.000	1.000	-	-
75	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	ρ=0.50							ρ=0.75						
T=25	0.987	1.000	1.000	0.993	-	-	-	0.993	1.000	1.000	0.999	-	-	-
50	1.000	1.000	1.000	1.000	1.000	-	-	1.000	1.000	1.000	1.000	1.000	-	-
75	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
100	1.000	1.000	1.000	1.000	1.000	1.000	-	1.000	1.000	1.000	1.000	1.000	1.000	-
150	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
250	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
500	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The main results from this section is that the panel data unit root test of Hadri (2000) has serious size distortion when disturbances are correlated across cross sections. However, we have also suggested an orthogonalization procedure that is able to restore the size properties of the panel data unit root test. Although established asymptotically, the orthogonalization procedure works well also in cases where samples are of limited size.

Next, we want to illustrate the adverse effects of neglecting cross-sectional correlation in empirical applications. This task is pursued in the next section.

4 Empirical application

In this section we illustrate the adverse effects that cross-sectional correlation can cause in the panel data stationarity framework by employing the panel data LM test of Hadri (2000) and the CSCLM test to test for output convergence.

Recently, Cheung and Pascual (2004) tested the output convergence hypothesis for the G7 countries. We follow this line of research and apply the stationarity tests discussed in the previous sections to test for convergence in real GDP per capita for six of the seven G7 countries.[15]

When testing for convergence, many hypotheses can be formulated. One hypothesis that is often adopted is the hypothesis of deterministic convergence. Deterministic convergence means that we test whether the relative GDP of a set of countries is stationary, possibly around a non-zero mean.[16] In our panel data stationarity framework, deterministic convergence implies that the demeaned relative output series should be stationary. The test presented in the previous sections can hence be employed to test the null hypothesis of deterministic convergence.

The null hypothesis of convergence amongst all countries is stated in (18). The alternative hypothesis, stated in (19), is that there is no convergence amongst any of the countries investigated.

$$H_0 : x_{it} = (Y_{it} - Y_{*t}) = I(0) \quad \forall\, i = 1, \ldots, N \tag{18}$$

$$H_1 : x_{it} = (Y_{it} - Y_{*t}) = I(1) \quad \forall\, i = 1, \ldots, N \tag{19}$$

In (18) and (19), Y_{it} is the log of real output per capita in period t for country i. Y_{*t} is the log of real output per capita of the benchmark country. Hence, x_{it} is the log of the relative output per capita when comparing country i to the benchmark country. Since we are to compare N countries to a benchmark country, we have all in all $N + 1$ countries in the sample.

The data series used to investigate the convergence hypothesis are the relative real GDP per capita for Canada, France, Italy, Japan and United Kingdom relative to USA.[17] The data series are plotted in Figure 1.

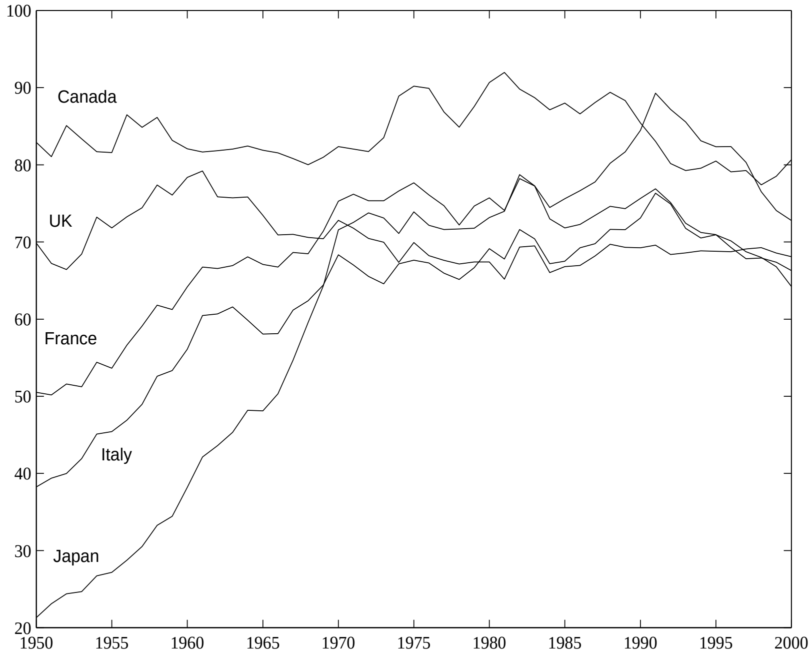
As seen in Figure 1, there seems to some convergence among the series. However, it also seems to be a structural break in the series about 1970. Up until 1970 the relative real GDP per capita seems to follow a trend stationary path, while the relative GDP seems to to level stationary from 1970 and on. Hence, when testing the notion of deterministic

[15]Germany is excluded due to measurement problems related to the unification.

[16]Li and Papell (1999) discuss different types of convergence in greater detail.

[17]The data series are collected from Heston et al. (2002).

Figure 1: Relative real GDP per capita compared to USA



convergence, the sample should be considered both over the period 1950-2000 and over 1970-2000. By considering both the entire sample and the latter subsample, the influence of structural change can be assessed.

The first step in the CSCLM test is to detrend the data. When testing deterministic convergence, as defined by Li and Papell (1999), the data series are detrended using an intercept. The estimated intercepts are presented in Table 7 for both of the different samples considered. Since it is important to know the degree of serial dependence when choosing lag window, we also give the estimated first-order autoregressive coefficient in Table 7.

As seen in Table 7, the degree of autocorrelation is strong. This implies that a large k is necessary. We choose to use $k = 24$ when we perform the stationarity tests. In Table 8 we present the test results for the LM test of Hadri (2000) and the cross-sectionally corrected LM test of Section 3.2.

From Table 8 it is evident that there is a rather large difference between the two test statistics. Moreover, if we compare the tests statistics to the 5% critical value, which is 1.64, we see that the tests give different conclusions regarding the null hypothesis of stationarity. The LM test of Hadri (2000) generates the conclusion that the null of stationarity, and hence deterministic convergence among the set of countries, should be rejected. The CSCLM test on the other hand cannot reject the same null hypothesis. Since the CSCLM test is robust against cross-sectional correlation while the LM test is not, it seems reasonable to further investigate if it indeed is the case that the reversed conclusion regarding the rejection of the null hypothesis is caused by cross-sectional correlation. To do this we calculate the cross-sectional covariance between the different countries.[18] In Table 9 we present the cross-sectional correlation of the different countries for the different samples.

The main impression when studying the covariances in Table 9 is that the issue of cross-

[18]The covariance os calculated using the residuals obtained after fitting an AR(1) model to the detrended data.

Table 7: Estimated intercept and serial correlation.

	Sample: 1950-2000	
Country	Intercept	AR(1) coefficient
Canada	4.43	0.87
France	4.22	0.92
UK	4.25	0.87
Italy	4.12	0.91
Japan	4.05	0.94
	Sample: 1970-2000	
Country	Intercept	AR(1) coefficient
Canada	4.44	0.91
France	4.30	0.94
UK	4.23	0.45
Italy	4.23	0.84
Japan	4.34	0.91

Table 8: Test results for the LM and the CSCLM tests

	Sample: 1950-2000	Sample: 1970-2000
LM[a]	1.90	2.00
CSCLM[b]	0.66	-0.59

Notes: [a]LM denotes the test of Hadri (2000).
[b]CSCLM denotes the test of Section 3.2.

sectional correlation must be considered when applying the panel data stationarity test on this data material.

To investigate the effects of disregarding the cross-sectional correlation we set up a Monte Carlo study using the information about cross-sectional as well as serial correlation obtained from Table 7 and Table 9.We generate 50,000 data sets using the variance/covariance matrix of the residuals obtained after estimating the AR(1) process for the detrended series. The data series are generated as in (20)-(22) below.

$$\boldsymbol{y}_t = \boldsymbol{\alpha} + \boldsymbol{\varepsilon}_t \tag{20}$$

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\rho}\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\nu}_t \tag{21}$$

$$\boldsymbol{\nu}_t \sim N(\mathbf{0}_{N\times 1}, \boldsymbol{\Sigma}) \tag{22}$$

As the Monte Carlo simulation is intended to capture the performance of the tests in a situation similar to the one in the empirical application, we set $N = 5$ and investigate the cases where $T = 51$ and $T = 31$. The parameters needed to generate the data series in (20)-(22) are gathered from Table 7 and Table 9.

If we generate data according to (20)-(22), it is evident that data is generated according to the null hypothesis, i.e. under the hypothesis that all series are stationary. Hence, if we can investigate the size properties of the two tests when errors are both serially and cross-sectionally correlated. If we calculate the LM statistic of Hadri (2000) and the CSCLM statistic of Section 3.2 and see how many times the null is (incorrectly) rejected, we get a

Table 9: Cross-sectional correlation of the disturbances.

	Sample: 1950-2000				
	Canada	France	UK	Italy	Japan
Canada	1.00	0.30	-0.04	0.08	-0.03
France	0.30	1.00	0.63	0.79	0.68
UK	-0.04	0.63	1.00	0.54	0.37
Italy	0.08	0.79	0.54	1.00	0.73
Japan	-0.03	0.68	0.37	0.73	1.00
	Sample: 1970-2000				
	Canada	France	UK	Italy	Japan
Canada	1.00	0.29	-0.18	0.13	-0.13
France	0.29	1.00	0.56	0.79	0.58
UK	-0.18	0.56	1.00	0.34	0.50
Italy	0.13	0.79	0.34	1.00	0.68
Japan	-0.13	0.58	0.50	0.68	1.00

picture of the size properties of the tests. The size properties at the 5% level is presented in Table 10.

Table 10: Size of the stationarity tests

	T=31	T=51
LM	0.969	0.251
CSCLM	0.034	0.041

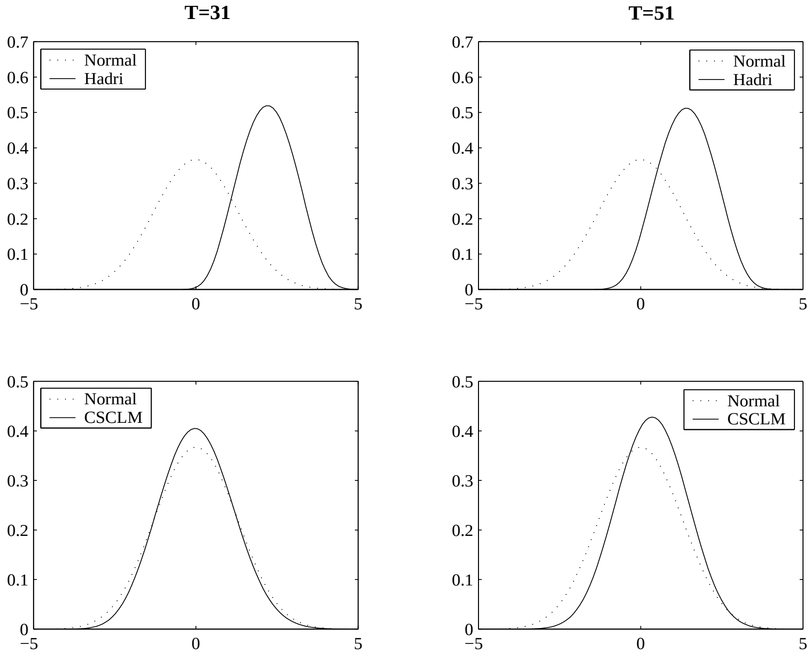
As seen in Table 10, the LM test of Hadri (2000) is severely oversized. The CSCLM test on the other hand has rather good size properties. In Figure 2, we plot the kernel distributions of the 50,000 test statistics calculated when investigating size properties of the two tests. We clearly see distributions of the LM test, presented in the top panels of Figure 2, are heavily distorted as a consequence of the cross-sectional correlation. The tests are heavily biased towards rejecting the null hypothesis although the null is true. The distributions of the CSCLM test, as depicted in the lower panels of Figure 2, are rather close to the normal distributions that apply under the null hypothesis. The discrepancies observed for the CSCLM test is due to the relative magnitude T/N. As the limit argument of the CSCLM test applies when T is much larger than N, the distribution of the CSCLM test will become increasingly larger as T increases.

The main message from Table 10 and Figure 2 is that it is important to account for cross-sectional correlation when testing for stationarity in panel data.

5 Conclusions

In this paper we study the effects of cross-sectional correlation on a previously suggested panel stationarity test. We find that the test has a serious size distortion when disturbances are correlated across cross sections. We propose a new test, the CSCLM test, that is robust against cross-sectional correlation. When studying the size properties of the so called CSCLM test, we find that the size distortions that occur when applying the LM test

Figure 2: Kernel densities for different time series dimensions.



are wiped out. The results also indicates that the CSCLM test works well in small samples and when disturbances are both cross-sectionally and serially correlated. By applying the two stationarity tests to investigate deterministic output convergence, we illustrate how conclusions regarding economic hypothesis can be adversely affected by failure to account for cross-sectional correlation.

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