The cost-vs.-quality trade-off in make-or-buy decisions

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Abstract

The make-or-buy decision is analyzed in a simple two-task principal-agent model. There is a cost-saving-vs.-quality trade-off in effort provision, both effort and outcome having these two dimensions. The principal faces a dichotomous choice between *make*/in-house, coming with weak cost-saving incentives for the agent, and *buy*/outsourcing, coming with strong incentives; the dichotomy is due to an incomplete-contracting limitation necessitating that one party be residual claimant of cost-savings. Choosing *buy* rather than *make* leads to higher cost-saving effort and in a plausible main case to lower effort directed towards quality and lower equilibrium quality, this in spite of stronger direct quality-provision incentives. The attractiveness of make-vs.-buy is explored and shown to be aligned with its impact on quality.

JEL Classification: D23, L22, L24

Keywords: make-or-buy decision, multitask principal-agent problem, oursourcing

1 Introduction

In this note, we use a simple two-task principal-agent framework for addressing the effects of make vs. buy on costs, quality, and the strength of incentives; we also explore the relative attractiveness of make vs. buy. The aim is to provide a conceptually fruitful, yet simple, framework for assessing the tradeoffs encountered in practice by firms, organizations and government

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bodies. The two tasks are geared towards saving costs and caring for quality. The model outcome is that outsourcing will be unambiguously beneficial in terms of cost, and that this will be at the expense of quality in the main case defined by the trade-off between cost-savings and quality being genuine in terms of the agent's cost-of-effort; the reduction in quality will, moreover, result in spite of stronger direct quality incentives. We will also demonstrate, by a direct comparison of value functions, how the attractiveness of outsourcing is affected by various characteristics. In particular, a higher valuation of quality will make outsourcing less attractive if the main case mentioned applies, and vice versa.

The tenet of the results conforms with both the often-asserted claim in the context of public-service contracting that outsourcing to profit-motivated external parties undermines quality, and some recent evidence about contracting with strongly profit-motivated parties. The former point is made in a theoretical contribution by Glaeser and Shleifer (2001), and is also corroborated in empirical work surveyed by Andersson, Jordahl and Josephson (2019). Two recent contributions documenting that care providers with high-powered incentives to make profit – such as those owned by private equity firms and publicly traded companies – deliver lower-quality care than closely held private companies and non-profits are Gupta et al. (2021) and Broms, Dahlström and Nistotskaya (2021).

The results of this paper are in line with those obtained by Hart, Shleifer and Vishny (1997) in their influential analysis of government contracting using a pure incomplete-contracting framework. A key difference is that while contracts are absent in their framework, our result reflects a *contractual response* to ownership being a dichotomous choice between weak and strong incentives; this contractual response is an empirically testable implication.[1]

The key assumption made is that the cost-based performance measure is subject to an incomplete-contracting limitation that forces the principal to either pass on the full effect of cost-savings to the agent, or to make remuneration completely insensitive to cost-savings. This assumption can be justified in terms of costs being tied to an asset whose ownership can be shifted, while any sharing contract would be plagued by manipulation.[2] With this assumption,

[1] Our result is also congruent with that obtained by Corneo and Rob (2003) who show that cost-based incentives are stronger in private compared with public firms, while cooperative effort is higher in public firms. While their notion of cooperative effort is in the same spirit as quality in this model, the over-all Corneo-Rob model is driven by comparing a private firm, maximizing profit, and a public firm assumed to maximize social welfare.

[2] A fully formal elaboration of this framework is made in Andersson (2011); also, Holmström (1999) and Gibbons (2005) make reference to an element of contractual incompleteness in a principal-agent model. For a basic discussion and justification for contractual incompleteness, see Hart (1995).

the principal faces a discrete choice between *make* – i.e. owning the asset and shielding the agent from monetary incentives – and *buy* – i.e. not owning the asset and exposing the agent to strong monetary incentives. Each of the options gives rise to an optimal remuneration policy in terms of the second performance measure indicating quality, and each of the resulting incentive schemes produces a distinct outcome in terms of the agent's actual effort profile.

The point of departure for this note is thus the assumed dichotomy between *make* and *buy* in terms of direct monetary incentives. While the dichotomy is a direct consequence of contractual incompleteness, the association of the two resulting regimes with make and buy in practice is intended to reflect a stylized fact. Although some of the background for this stylized fact is anecdotal as argued by Gibbons (2005, p. 207), it has considerable backing by casual observation of remuneration of employees as compared with contractors.[3] There are also firm theoretical corroborating arguments; Acemoglu, Kremer and Mian (2008), for example, use a career-concerns framework to argue that firms, by design, "coarsify information" with weaker equilibrium effort incentives as a result. Tadelis (2002) makes a related point: The assets used by an employee are typically owned by someone else, making strong cost-based incentives hazardous due to multi-task effort-substitution incentives not to take due care of the asset.[4] Iossa and Martimort (2015) use a multitask framework to assess organization and ownership of private-public partnerships where a key policy decision is whether or not to bundle the investment in a facility with its operation.

There are also indications that incentives in other dimensions – e.g. direct incentives in terms of quality measures – are stronger (and more explicit) in the context of contracting compared with in-house provision. This difference is likely to be more clear-cut in public sector contracting due to the limited potential for reputational mechanisms; this is discussed in e.g. Domberger and Jensen (1997).

In the following we describe the basic model and the formal manifestation of the key assumption of regime choice. We then go on to the results; a brief concluding section follows.

[3] An informal account corroborating this view is provided by Williamson (1985, Ch. 6).

[4] Also, Bajari and Tadelis (2001) provide an interesting foundation – in terms of complexity and adaptation costs – for the dichotomous choice between weak in-house cost-saving incentives, and strong cost-saving incentives in contracting.

2 The Model

Basic framework We will employ a two-task specification of the Holmström-Milgrom (1991) multitask principal-agent model. A risk-neutral principal contracts with a risk-averse agent who exerts efforts, a_1 and a_2, in two dimensions; following the Introduction, we sometimes refer to the dimensions in terms of cost-saving and quality. There are two output measures, x_1 and x_2, that depend stochastically on effort according to

$$x_i = a_i + \varepsilon_i,\ i = 1, 2,$$

where ε_i is noise; the outputs too can thus be thought of in terms of costs and quality. The noise terms are assumed to be jointly normally distributed with mean zero and and a general covariance structure with $\mathrm{Var}(\varepsilon_i) = v_i$, and $\mathrm{Cov}(\varepsilon_1, \varepsilon_2) = \sigma$; some of the analysis will be done under the assumption that the errors are independent across i, i.e. $\sigma = 0$. In light of the interpretations discussed with a cost-saving dimension and a quality dimension, we assume that $v_2 > v_1$; i.e. that quality is measured less precisely than costs (the cost measure, however, being non-verifiable).

The principal offers the agent a contract that specifies monetary compensation, y, that is constrained to be linear in the performance measures,

$$y = F + m_1 x_1 + m_2 x_2.$$

The agent has preferences over monetary compensation and effort, $a = (a_1, a_2)$, according to a constant-absolute-risk-aversion utility function

$$u(y; a) = -\exp\left\{-r\left[y - c(a)\right]\right\},\ \text{where } c(a) = a_1^2 + 2\kappa a_1 a_2 + a_2^2;$$

r is risk aversion. The parameter $\kappa \in [-1, 1]$ measures the degree of substitutability between a_1 and a_2 in the agent's disutility-of-effort function; $\kappa > 0$ means that the two tasks compete for effort in the sense that the marginal cost of a_1 is increasing in a_2 and vice versa. The agent has reservation payoff u_0.

In the following, we present a number of expressions that with some remarks about underlying calculations; in the Appendix we present the full development of the model and provide proofs of the sequence of propositions following below under the following sequence of headings:

- Unconstrained solution
- Characterizing the make-vs.-buy regimes
- Regime choice

The agent's problem Starting with the final stage of the interaction, maximization by the agent yields

$$a_1^* = \frac{m_1 - \kappa m_2}{2(1-\kappa^2)};\ a_2^* = \frac{m_2 - \kappa m_1}{2(1-\kappa^2)}; \tag{1}$$

note that for $\kappa > 0$, the agent's effort devoted to each dimension of effort is decreasing in the incentive-intensity provided for the other dimension.

The principal's problem We now consider the principal's problem when both m_1 and m_2 are chosen freely. The principal values the two dimensions of realized output at β_1 and β_2 per unit,[5] and her problem is thus

$$\begin{aligned} &\max_{m_1,m_2,F} \ E\left[\beta_1 a_1 + \beta_2 a_2 - (F + m_1 x_1 + m_2 x_2)\right] \\ \text{s.t.}\ &-E\exp\left\{-r\left[F + m_1(a_1+\varepsilon_1) + m_2(a_2+\varepsilon_2) - \left(a_1^2 + 2\kappa a_1 a_2 + a_2^2\right)\right]\right\} \geq u_0, \\ \text{and } a &\in \arg\max -E\exp\left\{-r\left[F + m_1(a_1+\varepsilon_1) + m_2(a_2+\varepsilon_2) - \left(a_1^2 + 2\kappa a_1 a_2 + a_2^2\right)\right]\right\}. \end{aligned}$$

For the case of independent noise (i.e. $\sigma = 0$), the solution is (in the Appendix we state the solution for a general covariance structure)

$$m_1 = \frac{(1+2rv_2)\,\beta_1 - \kappa\beta_2}{4r^2(1-\kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}, \tag{2}$$

and

$$m_2 = \frac{-\kappa\beta_1 + (1+2rv_1)\,\beta_2}{4r^2(1-\kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \tag{3}$$

F is determined residually. Similar to the agent's tradeoff, incentives calibrate the interdependence manifest by κ – moderating each if this dependence is positive – and in addition, the strength of incentives depends intuitively (negatively) on the variances.

The original key insight of Holmström and Milgrom (1991) is that there is, in general, an interdependence between the two output dimensions, (a_1, a_2), in the sense that incentives provided for one component of the result affect inputs and results in both dimensions.[6] We will take the case with independent noise, and with a_1 and a_2 being substitutes in the agent's utility function – i.e. when $\kappa > 0$ – as the main case, and note the results for the complements case ($\kappa < 0$); this case, however, gives the effort-extraction problem a "free-lunch flavor" that seems

[5] We could have assumed that $\beta_1 = 1$ with no loss of generality, but this would have made the expressions less transparent.

[6] Similar insights were gained in a somewhat different framework by Baker (1992).

unnatural in most applications.[7] We will also at points comment on the case with interdependent errors ($\sigma \neq 0$).

Before turning to our main analysis, consider a couple of special cases:

- First, it may be worth noting that if noise (measured by v_i) or risk aversion vanishes, the incentive boils down to pure effort-substitution, and the solution is $m_1 = \beta_1 - \kappa\beta_2$ and $m_2 = \beta_2 - \kappa\beta_1$.

- Second, consider the case where a_2 has no intrinsic value to the principal so that $\beta_2 = 0$. This gives

$$m_1 = \frac{\beta_1 (2rv_2 + 1)}{4r^2(1-\kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \; m_2 = \frac{-2rv_1\kappa\beta_1}{4r^2(1-\kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1},$$

  and we see that as long as the two inputs, (a_1, a_2), are substitutes, the agent is punished for high output in the x_2-dimension.

- Finally, consider the case where the informativeness about effort of one dimension of output, say 2, grows small, i.e. when $v_2 \to \infty$. In this case

$$m_1 = \frac{2r(\beta_1 - \beta_2\kappa)}{4r^2(1-\kappa^2)v_1 + 2r}; \; m_2 = 0,$$

  and we see that the incentives provided for x_1 must be used to control both dimensions of effort; from the expression one sees e.g. that if the uninformative dimension is important enough – more precisely if $\beta_1 < \beta_2\kappa$ – output in the other dimension is punished.

The last case highlights the point that there are important circumstances under which weak incentives are desirable for "second-best reasons."

Regime choice The key assumption behind the make-vs.-buy dichotomy, is that the principal faces a dichotomous choice of cost-saving incentives, m_1. Specifically, the principal is assumed to face the choice between giving up direct cost-saving incentives, setting $m_1^0 = 0$, or providing "full cost-saving incentives" – i.e. providing no insurance – setting $m_1^1 = \beta_1$. The origin of this restriction is the assumption that money counts are non-contracible whereas the control right of the revenue stream itself can be transferred. The underpinnings of this assumption have been discussed in the Introduction.

[7] To visualize this "free-lunch flavor" the extreme case where κ approaches -1 is useful; in this case cost of effort is close to $(a_1 - a_2)^2$ where expending increasing effort in both dimensions is indeed costless.

3 Results

This paper aims at addressing two questions:

- How do incentives and effort depend on the choice between make and buy?
- How can we characterize the trade-offs characterizing the choice of regime?

Under the assumptions made about the dichotomous choice of cost-saving incentives, m_1, the remaing objects – i.e. quality incentives measured by m_2 and the agent's equilibrium effort (a_1, a_2) – can be solved for conditional on the regime choice and compared across regimes. The simplicity of the model, moreover, enables us to derive the principal's value function for each case and thus characterize the choice of regime.

Comparing solutions While the comparisons made are simple in principle, the general expression of the model is a bit unwieldy. We therefore proceed in the text by emphasizing comparisons based on the case with independent errors ($\sigma = 0$) and then making some comments about the case of dependent errors while assuming efforts in the agent's utility being separable ($\kappa = 0$).

The most immediate comparison is that between the direct quality incentives in the two regimes (recalling that cost-saving incentives are $m_1^0 = 0$ and $m_1^1 = \beta_1$). The respective optimal solutions are

$$m_2^0 = \frac{\beta_2 - \beta_1 \kappa}{2r(1-\kappa^2)v_2 + 1};$$
$$m_2^1 = \frac{\beta_2 - 2r\left(1-\kappa^2\right)\sigma\beta_1}{2r(1-\kappa^2)v_2 + 1}.$$

Note that the denominator, common for the two cases, reflects the direct risk adjustment. In the first case, m_2 is adjusted, in addition, to balance the weak incentives for cost-savings coming with $m_1 = 0$ in the *make* case; in the second case, m_2 is adjusted for the additional risk spilling over to cost-savings in the case of $\sigma > 0$ (and the correspondingly reduced risk for $\sigma < 0$).

The difference is

$$m_2^1 - m_2^0 = \frac{\kappa - 2r\left(1-\kappa^2\right)\sigma}{2r\left(1-\kappa^2\right)v_2 + 1}\beta_1, \tag{4}$$

and we see that the difference has the sign of κ for $\sigma = 0$. In particular, it is positive in the main case of the two efforts being substitutes – i.e. competing for attention – and independence between error components. We state a proposition:

Proposition 1. *Suppose that the errors are independent and that the efforts are substitutes, ($\kappa > 0$), then the incentives for quality are stronger when the principal chooses buy.*

If, under the same assumption, efforts are complements, ($\kappa < 0$), then the incentives for quality are weaker when the principal chooses buy.

In the case with $\sigma \neq 0$ and $\kappa = 0$, the difference has the opposite sign of σ.[8]

Going on to the equilibrium effort in each regime, the differences, inserting the values for m from (4) above, are

$$a_1^{1*} - a_1^{0*} = \frac{(2r(v_2 + \kappa\sigma) + 1)\beta_1}{2(2r(1-\kappa^2)v_2 + 1)}; \tag{5}$$

$$a_2^{1*} - a_2^{0*} = \frac{-r(\kappa v_2 + \sigma)\beta_1}{2r(1-\kappa^2)v_2 + 1}. \tag{6}$$

The first difference is guaranteed to be positive whenever quality is measured less precisely than cost.[9] The second difference is negative when both κ and σ are non-negative and it has the sign of $-\kappa$ for $\sigma = 0$; in particular, it is negative when the efforts are substitutes ($\kappa > 0$) and the errors are independent. We state the following proposition:

Proposition 2. *Equilibrium effort devoted to cost-savings is higher when the principal chooses buy.*

Assuming that the errors are independent ($\sigma = 0$):

- *when efforts are substitutes, $\kappa > 0$, equilibrium effort devoted to quality is lower when the principal chooses buy;*
- *when efforts are complements, ($\kappa < 0$), equilibrium effort devoted to quality is higher when the principal chooses buy.*

In the case with $\sigma \neq 0$ and $\kappa = 0$, the result is, as a matter of fact, perfectly similar in the sense that difference in quality effort has the same sign as $-\sigma$.[10]

The, arguably, most relevant prediction is most clear when $\kappa > 0$ and $\sigma = 0$, and it is that equilibrium quality is lower in the case of outsourcing, in spite of direct quality incentives being stronger.

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[8] One may note that m_2 depends on σ only in the case of outsourcing. The reason for this is that in the *make* case the only risk that hits a risk-averse party is that coming from x_2, and the dependence does not matter; in the *buy* case the risk coming with x_2 is coming with additional risk permeated by x_1 when $\sigma > 0$.

[9] This follows from $|\kappa| \leq 1$, the inequality limiting a covariance $|\sigma| \leq \sqrt{v_1 v_2} \leq (v_1 + v_2)/2$, and the assumption $v_1 \leq v_2$; note that the non-contractibility of cost-savings does not reflect a lack of measurability.

[10] This reflects the observation made previously, that incentives only depend on σ in the case of outsourcing, and that a positive dependence reduces the strength of incentives and hence effort.

Regime choice We now turn to the choice between make and buy. In formal terms we are interested in the comparative statics of the difference between the principal's value function from choosing "buy" and "make" with respect to the parameters of the model. We have:

PROPOSITION 3. *Suppose that the errors are independent. The effect of an increase in each of the parameters on the attractiveness of "buy" relative to "make" is detailed by the following list, including interpretations:*

- β_2: negative for $\kappa > 0$, positive for $\kappa < 0$; i.e., the choice of regime is perfectly aligned with the (relative) valuation of quality in the sense that an increase in β_2 makes the regime with the higher outcome in terms of quality more attractive;

- v_1: negative; i.e., a *ceteris paribus* increase in the variance in the measurement of cost favours "make", shielding the agent from this risk;

- κ: with the caveat that there may be ambiguity when κ is positive and close to 1, an increase in the absolute value of κ favors the regime that produces higher quality, i.e. "make" for $\kappa > 0$ and "buy" for $\kappa < 0$; specifically:
  - $\kappa > 0$: negative for $\kappa \in [0, \overline{\kappa}]$ for some upper bound $\overline{\kappa}$ which may be equal to one;
  - $\kappa < 0$: negative as κ grows towards $\kappa = 0$ (i.e. positive as $|\kappa|$ increases);

- v_2: an increase in the variance in the measurement of quality has the same effect as an increase in the absolute value of κ (with the same caveat), making the regime that produces better quality more attractive; i.e., for $\kappa > 0$: negative for $\kappa \in [0, \overline{\kappa}]$ for some upper bound $\overline{\kappa}$ which may be equal to one, and for $\kappa < 0$: positive.

None of the effects above are surprising, but it is worth re-emphasizing that:

- the comparative statics with respect to the direct valuation of quality, β_2, unambiguously predict the higher-quality regime becomes more attractive, and that the higher-quality regime is "make" for the – arguably more plausible – case of $\kappa > 0$; and that

- with the caveat that there is a potential ambiguity for large values of $\kappa > 0$, the implications of an increase in the absolute value of κ itself, and the variance of quality measurement, also make the higher-quality regime becomes more attractive.

The case with dependent signals ($\sigma \neq 0$) and separable effort ($\kappa = 0$) gives results that are quite similar; in particular, an increase in the value of quality, β_2, favors the regime producing higher quality, i.e., "make" when $\sigma > 0$, and "buy" when $\sigma > 0$. The calculations are available from the author.

4 Conclusions

The main conclusion from our analysis is that outsourcing leads to lower costs, while the effects on quality are likely to be negative, although this depends on a precise condition in terms of effort-substitution possibilities. While this conclusion is in line with previous work – in particular with Hart, Shleifer and Vishny (1997) – a distinguishing feature of this paper is that we establish this in a contracting framework where the effects are permeated by tangible incentive contracts. Apart from this being "realistic" in many applications, this has the benefit of producing a richer set of empirical implications.[11] In particular, in the "main case" singled out in this paper – whose relevance can likely be directly or indirectly established in many contexts – the prediction is that outsourcing will be accompanied by stronger rewards for quality, while still producing lower quality than comparable in-house arrangements.

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[11] This said, the empirical distinction between the role of quality in our agency framework and the role of quality in incomplete contracting models is not razor sharp. The empirical account of prison privatization given by Hart, Shleifer and Vishny (1997, Sec. IV), for example, is broadly consistent both with an incomlete contracting view and an effort substitution view; in particular, it seems clear from that account that the explicit conctractual regulation of quality in practice is more stringent for private prisons, while in many instances quality in the end is lower.

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Appendix

A.1 Unconstrained solution

Optimal contracts in the basic two-task model The principal's problem – with general covariance matrix and $\mathrm{Cov}(\varepsilon_1, \varepsilon_2) = \sigma$ – can be written

$$\max_{m,F} \ [(\beta_1 - m_1)a_1 + (\beta_2 - m_2)a_2 - F]$$
$$\text{s.t.} \ -\exp(-r(F + m_1 a_1 + m_2 a_2 - \frac{r}{2}m_1^2 v_1 - \frac{r}{2}m_2^2 v_2 - r m_1 m_2 \sigma - [a_1^2 + 2\kappa a_1 a_2 + a_2^2])) \geq u_0$$

and utility maximization for the agent, the (necessary and sufficient) first-order conditions for which are

$$m_1 - 2(a_1 + \kappa a_2) = 0; \ m_2 - 2(\kappa a_1 + a_2) = 0.$$

Solving for effort yields

$$a_1^* = \frac{m_1 - \kappa m_2}{2(1-\kappa^2)};\ a_2^* = \frac{m_2 - \kappa m_1}{2(1-\kappa^2)} \tag{A.1}$$

and the principal's objective function, denoted ϕ, is (with a_1^* and a_2^* inserted, $\widetilde{u} = -\ln(-u_0)/r$, and after substituting the constraint which is obviously binding)

$$\phi(m_1, m_2) = \beta_1 \frac{m_1 - \kappa m_2}{2(1-\kappa^2)} + \beta_2 \frac{m_2 - \kappa m_1}{2(1-\kappa^2)} - \frac{r}{2} m_1^2 v_1 - \frac{r}{2} m_2^2 v_2 - r m_1 m_2 \sigma$$
$$- \frac{1}{4(1-\kappa^2)^2} [(m_1 - \kappa m_2)^2 + 2\kappa(m_1 - \kappa m_2)(m_2 - \kappa m_1) + (m_2 - \kappa m_1)^2] - \widetilde{u},$$

simplifying and multiplying by $2(1-\kappa^2)$ (denoting the modified objective $\widehat{\phi}$), we have

$$\begin{aligned} \widehat{\phi}(m_1, m_2) &= \beta_1 (m_1 - \kappa m_2) + \beta_2 (m_2 - \kappa m_1) - r(1-\kappa^2) \left(m_1^2 v_1 + m_2^2 v_2 + m_1 m_2 \sigma \right) \\ & \quad - \frac{1}{1-\kappa^2} [\frac{1}{2} (m_1 - \kappa m_2)^2 + \kappa(m_1 - \kappa m_2)(m_2 - \kappa m_1) + \frac{1}{2} (m_2 - \kappa m_1)^2] - \widehat{\widetilde{u}}. \end{aligned}$$

The first-order conditions w.r.t. (m_1, m_2) are:

$$\beta_1 - \beta_2 \kappa - 2r(1-\kappa^2)(v_1 m_1 + \sigma m_2)$$
$$- \frac{1}{1-\kappa^2} \left[(m_1 - \kappa m_2) + \kappa \left[(m_2 - \kappa m_1) - \kappa(m_1 - \kappa m_2) \right] - \kappa(m_2 - \kappa m_1) \right] = 0,$$

$$\beta_2 - \beta_1 \kappa - 2r(1-\kappa^2)(v_2 m_2 + \sigma m_2)$$
$$- \frac{1}{1-\kappa^2} \left[-\kappa (m_1 - \kappa m_2) + \kappa \left[(m_1 - \kappa m_2) - \kappa(m_2 - \kappa m_1) \right] + (m_2 - \kappa m_1) \right] = 0.$$

Simplifying,

$$\beta_1 - \beta_2 \kappa = \left(2r(1-\kappa^2) v_1 + \frac{1-\kappa^2}{1-\kappa^2} \right) m_1 + \left(\frac{\kappa^3 - \kappa}{1-\kappa^2} + 2r(1-\kappa^2)\sigma \right) m_2,$$

$$\beta_2 - \beta_1 \kappa = \left(\frac{\kappa^3 - \kappa}{1-\kappa^2} + 2r(1-\kappa^2)\sigma \right) m_1 + \left(2r(1-\kappa^2) v_2 + \frac{1-\kappa^2}{1-\kappa^2} \right) m_2;$$

and simplifying further

$$\beta_1 - \beta_2 \kappa = \left(2r(1-\kappa^2) v_1 + 1 \right) m_1 + \left(2r(1-\kappa^2)\sigma - \kappa \right) m_2, \tag{A.2}$$

$$\beta_2 - \beta_1 \kappa = \left(2r(1-\kappa^2)\sigma - \kappa \right) m_1 + \left(2r(1-\kappa^2) v_2 + 1 \right) m_2. \tag{A.3}$$

The full system can be written

$$\begin{pmatrix} 2r(1-\kappa^2)v_1 + 1 & 2r(1-\kappa^2)\sigma - \kappa \\ 2r(1-\kappa^2)\sigma - \kappa & 2r(1-\kappa^2)v_2 + 1 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} \beta_1 - \beta_2 \kappa \\ \beta_2 - \beta_1 \kappa \end{pmatrix},$$

or,

$$\begin{pmatrix} 2r(1-\kappa^2)v_1 + 1 & 2r(1-\kappa^2)\sigma - \kappa \\ 2r(1-\kappa^2)\sigma - \kappa & 2r(1-\kappa^2)v_2 + 1 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 1 & -\kappa \\ -\kappa & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix},$$

or, inverting the RHS matrix,

$$\frac{1}{1-\kappa^2}\begin{pmatrix} 1 & \kappa \\ \kappa & 1 \end{pmatrix}\begin{pmatrix} 2r(1-\kappa^2)v_1+1 & 2r(1-\kappa^2)\sigma-\kappa \\ 2r(1-\kappa^2)\sigma-\kappa & 2r(1-\kappa^2)v_2+1 \end{pmatrix}\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}=\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}.$$

The LHS can be written

$$\frac{1}{1-\kappa^2}\begin{pmatrix} 1 & \kappa \\ \kappa & 1 \end{pmatrix}\begin{pmatrix} 2r(1-\kappa^2)v_1+1 & 2r(1-\kappa^2)\sigma-\kappa \\ 2r(1-\kappa^2)\sigma-\kappa & 2r(1-\kappa^2)v_2+1 \end{pmatrix}=\begin{pmatrix} 2r\left(v_1+\kappa\sigma\right)+1 & 2r\left(\kappa v_2+\sigma\right) \\ 2r\left(\kappa v_1+\sigma\right) & 2r\left(v_2+\kappa\sigma\right)+1 \end{pmatrix},$$

and the determinant is

$$D=4r^2\left(v_1v_2+v_1\sigma\kappa+v_2\sigma\kappa+\sigma^2\kappa^2\right)+2r\left(v_1+\kappa\sigma\right)+2r\left(v_2+\kappa\sigma\right)+1-4r^2\left(\kappa^2v_1v_2+v_1\sigma\kappa+v_2\sigma\kappa+\sigma^2\right),$$

or

$$D=4r^2\left(1-\kappa^2\right)\left(v_1v_2-\sigma^2\right)+2r\left(v_1+\kappa\sigma\right)+2r\left(v_2+\kappa\sigma\right)+1;$$

the determinant is positive thanks to the (absolute) upper bound on the covariance relative to variances. The solution in terms of m is thus:

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}=\frac{1}{D}\begin{pmatrix} 2r\left(v_2+\kappa\sigma\right)+1 & -2r\left(\kappa v_2+\sigma\right) \\ -2r\left(\kappa v_1+\sigma\right) & 2r\left(v_1+\kappa\sigma\right)+1 \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix},$$

or

$$m_1=\frac{\left(2r\left(v_2+\kappa\sigma\right)+1\right)\beta_1-2r\left(\kappa v_2+\sigma\right)\beta_2}{D},$$

and

$$m_2=\frac{-2r\left(\kappa v_1+\sigma\right)\beta_1+\left(2r\left(v_1+\kappa\sigma\right)+1\right)\beta_2}{D}.$$

A.2 Characterizing the make-vs.-buy regimes

Optimal contracts for each regime Now, from the first-order condition with respect to m_2 (expression A.3), we get the optimal m_2 conditional on $\overline{m}_1\in\{0,\beta_1\}$,

$$m_2(\overline{m}_1)=\frac{\beta_2-\beta_1\kappa-\left(2r(1-\kappa^2)\sigma-\kappa\right)\overline{m}_1}{(2r(1-\kappa^2)v_2+1)}; \tag{A.4}$$

specifically for the two cases,

$$m_2^0=\frac{\beta_2-\beta_1\kappa}{2r(1-\kappa^2)v_2+1)},\ m_2^1=\frac{\beta_2-2r\left(1-\kappa^2\right)\sigma\beta_1}{2r(1-\kappa^2)v_2+1},$$

and the difference is

$$m_2^1-m_2^0=\frac{\left(\kappa-2r\left(1-\kappa^2\right)\sigma\right)\beta_1}{2r\left(1-\kappa^2\right)v_2+1}$$

which has the sign of κ for $\sigma=0$, and the opposite sign of σ for $\kappa=0$.

In order to abbreviate notation, let $\Omega = 2r\left(1-\kappa^2\right)v_2 + 1 > 0$; this object will surface pervasively.

Equilibrium effort: We can now simply calculate equilibrium effort by plugging in the m's in (A.1); for the case of $\overline{m}_1 = m_1^0 = 0$ it is

$$a_1^{0*} = \frac{-\kappa\left(\beta_2 - \beta_1\kappa\right)}{2(1-\kappa^2)\Omega};\ a_2^{0*} = \frac{\beta_2 - \beta_1\kappa}{2(1-\kappa^2)\Omega}.$$

For the case of $\overline{m}_1 = m_1^1 = \beta_1$ it is

$$a_1^{1*} = \frac{1}{2\left(1-\kappa^2\right)}\left(\beta_1 - \frac{\kappa\left(\beta_2 - 2r\left(1-\kappa^2\right)\sigma\beta_1\right)}{\Omega}\right);$$
$$a_2^{1*} = \frac{1}{2(1-\kappa^2)}\left(\frac{\beta_2 - 2r\left(1-\kappa^2\right)\sigma\beta_1}{\Omega} - \kappa\beta_1\right).$$

Or, developing,

$$a_1^{1*} = \frac{\left(2r(1-\kappa^2)v_2+1\right)\beta_1 - \kappa\left(\beta_2 - 2r\left(1-\kappa^2\right)\sigma\beta_1\right)}{2\left(1-\kappa^2\right)\left(2r(1-\kappa^2)v_2+1\right)} = \frac{\left(2r(1-\kappa^2)\left(v_2+\kappa\sigma\right)+1\right)\beta_1 - \kappa\beta_2}{2\left(1-\kappa^2\right)\Omega};$$
$$a_2^{1*} = \frac{\beta_2 - 2r\left(1-\kappa^2\right)\sigma\beta_1 - \kappa\beta_1\left(2r(1-\kappa^2)v_2+1\right)}{2(1-\kappa^2)\left(2r(1-\kappa^2)v_2+1\right)} = \frac{-\left(2r\left(1-\kappa^2\right)\left(\kappa v_2+\sigma\right)+\kappa\right)\beta_1 + \beta_2}{2(1-\kappa^2)\Omega}.$$

The differences are

$$a_1^{1*} - a_1^{0*} = \frac{\left(2r(1-\kappa^2)\left(v_2+\kappa\sigma\right)+1\right)\beta_1 - \kappa\beta_2}{2\left(1-\kappa^2\right)\Omega} - \frac{-\kappa\left(\beta_2 - \beta_1\kappa\right)}{2(1-\kappa^2)\Omega} =$$
$$= \frac{\left(2r\left(1-\kappa^2\right)\left(v_2+\kappa\sigma\right) + \left(1-\kappa^2\right)\right)\beta_1}{2\left(1-\kappa^2\right)\Omega} = \frac{\left(2r\left(v_2+\kappa\sigma\right)+1\right)\beta_1}{2\Omega},$$

and

$$a_2^{1*} - a_2^{0*} = \frac{-\left(2r\left(1-\kappa^2\right)\left(\kappa v_2+\sigma\right)+\kappa\right)\beta_1 + \beta_2}{2(1-\kappa^2)\Omega} - \frac{\beta_2 - \beta_1\kappa}{2(1-\kappa^2)\Omega},$$

or, simplifying,

$$a_2^{1*} - a_2^{0*} = \frac{-2r\left(1-\kappa^2\right)\left(\kappa v_2+\sigma\right)\beta_1}{2(1-\kappa^2)\Omega} = \frac{-r\left(\kappa v_2+\sigma\right)\beta_1}{\Omega}.$$

In short,

$$a_1^{1*} - a_1^{0*} = \frac{\left(2r\left(v_2+\kappa\sigma\right)+1\right)\beta_1}{2\Omega};\ a_2^{1*} - a_2^{0*} = \frac{-r\left(\kappa v_2+\sigma\right)\beta_1}{\Omega}.$$

The specializations where either κ or σ are zero are useful:

- $\sigma = 0:$

$$a_1^{1*} - a_1^{0*} = \frac{\left(2rv_2+1\right)\beta_1}{2\Omega} > 0$$
$$a_2^{1*} - a_2^{0*} = \frac{-r\kappa v_2\beta_1}{\Omega} < 0^*$$

  where * indicates reversal when $\kappa < 0$;

- $\kappa = 0$:

$$a_1^{1*} - a_1^{0*} = \frac{\beta_1}{2} > 0$$

$$a_2^{1*} - a_2^{0*} = \frac{-r\sigma\beta_1}{2rv_2 + 1} < 0^*$$

where * indicates reversal when $\sigma < 0$.

A.3 Regime choice

In order to sort out the forces at work we make the comparisons for the case with $\kappa \neq 0$, $\sigma = 0$; as mentioned in the main text, we have made the same exercise for the case $\sigma \neq 0$, $k = 0$ with conclusions along the same lines, and σ having similar roles as κ.

Comparing value functions, $\kappa \neq 0, \sigma = 0$. The value function is

$$\phi(m_1, m_2) = \beta_1 \frac{m_1 - \kappa m_2}{2(1-\kappa^2)} + \beta_2 \frac{m_2 - \kappa m_1}{2(1-\kappa^2)} - \frac{r}{2} m_1^2 v_1 - \frac{r}{2} m_2^2 v_2 - r m_1 m_2 \sigma$$
$$- \frac{1}{4(1-\kappa^2)^2} [(m_1 - \kappa m_2)^2 + 2\kappa(m_1 - \kappa m_2)(m_2 - \kappa m_1) + (m_2 - \kappa m_1)^2] - \widetilde{u};$$

without loss of generality (for the comparison as well as the solution) we assume $\widetilde{u} = 0$. We have from (A.4) with $\sigma = 0$,

$$m_2 = \frac{\beta_2 - \kappa\beta_1 + \kappa\overline{m}_1}{2r(1-\kappa^2)v_2 + 1},$$

and the relevant comparison is between $\overline{m}_1 = 0$ and $\overline{m}_1 = \beta_1$ with respectively

$$m_2^0 = \frac{\beta_2 - \beta_1\kappa}{2r(1-\kappa^2)v_2 + 1} \text{ and } m_2^1 = \frac{\beta_2}{2r(1-\kappa^2)v_2 + 1}.$$

The case $\overline{m}_1 = 0$. The value function can be expressed

$$\phi^0 = \beta_1 \frac{-\kappa m_2}{2(1-\kappa^2)} + \beta_2 \frac{m_2}{2(1-\kappa^2)} - \frac{r}{2} m_2^2 v_2 - \frac{1}{4(1-\kappa^2)^2} [(-\kappa m_2)^2 + 2\kappa(-\kappa m_2)(m_2) + (m_2)^2],$$

and multiplying by the denominator,

$$2\left(1-\kappa^2\right)\phi^0 = \beta_1\left(-\kappa m_2\right) + \beta_2 m_2 - \frac{r}{2} 2\left(1-\kappa^2\right) m_2^2 v_2 - \frac{m_2^2}{2},$$

or

$$2\left(1-\kappa^2\right)\phi^0 = \left(\beta_2 - \kappa\beta_1\right) m_2 - \frac{1}{2}\left(2r\left(1-\kappa^2\right)v_2 + 1\right) m_2^2.$$

Inserting m_2^0, we get

$$2\left(1-\kappa^2\right)\phi^0 = \left(\beta_2 - \kappa\beta_1\right) \frac{\beta_2 - \beta_1\kappa}{2r(1-\kappa^2)v_2 + 1} - \frac{1}{2}\left(2r\left(1-\kappa^2\right)v_2 + 1\right)\left(\frac{\beta_2 - \beta_1\kappa}{2r(1-\kappa^2)v_2 + 1}\right)^2,$$

or

$$\phi^0 = \frac{1}{\left(2r(1-\kappa^2)v_2 + 1\right) 2\left(1-\kappa^2\right)} \frac{1}{2} \left(\beta_2 - \beta_1\kappa\right)^2.$$

The case $\overline{m}_1 = \beta_1$ The value function can be expressed

$$\begin{aligned}\phi^1 &= \beta_1 \frac{\beta_1 - \kappa m_2}{2(1-\kappa^2)} + \beta_2 \frac{m_2 - \kappa \beta_1}{2(1-\kappa^2)} - \frac{r}{2}\beta_1^2 v_1 - \frac{r}{2} m_2^2 v_2 - \\ & \frac{1}{4(1-\kappa^2)^2}[(\beta_1 - \kappa m_2)^2 + 2\kappa(\beta_1 - \kappa m_2)(m_2 - \kappa\beta_1) + (m_2 - \kappa\beta_1)^2],\end{aligned}$$

or, multiplying,

$$\begin{aligned}2\left(1-\kappa^2\right)\phi^1 &= \beta_1\left(\beta_1 - \kappa m_2\right) + \beta_2\left(m_2 - \kappa\beta_1\right) - \frac{r}{2}2\left(1-\kappa^2\right)\beta_1^2 v_1 - \frac{r}{2}2\left(1-\kappa^2\right)m_2^2 v_2 - \\ & \frac{1}{2(1-\kappa^2)}[(\beta_1 - \kappa m_2)^2 + 2\kappa(\beta_1 - \kappa m_2)(m_2 - \kappa\beta_1) + (m_2 - \kappa\beta_1)^2];\end{aligned}$$

inserting m_2^1, and letting $\Omega = 2r\left(1-\kappa^2\right)v_2 + 1$ (so that $m_2^1 = \beta_2/\Omega$), we get

$$\begin{aligned}2\left(1-\kappa^2\right)\phi^1 = \beta_1\frac{\beta_1\Omega - \kappa\beta_2}{\Omega} + \beta_2\frac{\beta_2 - \kappa\beta_1\Omega}{\Omega} - \frac{r}{2}2\left(1-\kappa^2\right)\beta_1^2 v_1 - \frac{r}{2}2\left(1-\kappa^2\right)\frac{\beta_2^2}{\Omega^2}v_2 \\ -\frac{1}{2(1-\kappa^2)}\left[\frac{\left(\beta_1\Omega - \kappa\beta_2\right)^2}{\Omega^2} + 2\kappa\frac{\left(\beta_1\Omega - \kappa\beta_2\right)}{\Omega}\frac{\left(\beta_2 - \kappa\beta_1\Omega\right)}{\Omega} + \frac{\left(\beta_2 - \kappa\beta_1\Omega\right)^2}{\Omega}\right],\end{aligned}$$

or,

$$\begin{aligned}2\left(1-\kappa^2\right)\Omega\phi^1 = \beta_1\left(\beta_1\Omega - \kappa\beta_2\right) + \beta_2\left(\beta_2 - \kappa\beta_1\Omega\right) - r\left(1-\kappa^2\right)\beta_1^2\Omega v_1 - \frac{1}{\Omega}r\left(1-\kappa^2\right)\beta_2^2 v_2 \\ -\frac{1}{2(1-\kappa^2)\Omega}\left[\left(\left(\beta_1\Omega - \kappa\beta_2\right)\right)^2 + 2\kappa\left(\left(\beta_1\Omega - \kappa\beta_2\right)\right)\left(\beta_2 - \kappa\beta_1\Omega\right) + \left(\beta_2 - \kappa\beta_1\Omega\right)^2\right],\end{aligned}$$

or,

$$\begin{aligned}2\left(1-\kappa^2\right)\Omega\phi^1 = \beta_2^2 - \kappa\beta_1\beta_2\left(1+\Omega\right) + \beta_1^2\Omega - r\left(1-\kappa^2\right)\beta_1^2 v_1 - \frac{1}{\Omega}r\left(1-\kappa^2\right)\beta_2^2 v_2 \\ -\frac{1}{2(1-\kappa^2)\Omega}\left[\left(\beta_1\Omega - \kappa\beta_2\right)^2 + 2\kappa\left(\beta_1\Omega - \kappa\beta_2\right)\left(\beta_2 - \kappa\beta_1\Omega\right) + \left(\beta_2 - \kappa\beta_1\Omega\right)^2\right].\end{aligned}$$

As an intermediate calculation, we simplify the bracketed expression:

$$\left(\beta_1\Omega - \kappa\beta_2\right)^2 + 2\kappa\left(\beta_1\Omega - \kappa\beta_2\right)\left(\beta_2 - \kappa\beta_1\Omega\right) + \left(\beta_2 - \kappa\beta_1\Omega\right)^2 =$$

$$\beta_1^2\Omega^2 - 2\kappa\beta_1\beta_2\Omega + \kappa^2\beta_2^2 - 2\kappa^2\beta_1^2\Omega^2 + 2\kappa\left(1+\kappa^2\right)\beta_1\beta_2\Omega - 2\kappa^2\beta_2^2 + \kappa^2\beta_1^2\Omega^2 - 2\kappa\beta_1\beta_2\Omega + \beta_2^2 =$$

$$\left(1-\kappa^2\right)\beta_1^2\Omega^2 + \left(1-\kappa^2\right)\beta_2^2 - 2\kappa\left(1-\kappa^2\right)\beta_1\beta_2\Omega.$$

Let us next resume calculating the value function after the intermediate calculation,

$$\begin{aligned}2\left(1-\kappa^2\right)\Omega\phi^1 = \beta_2^2 - \kappa\beta_1\beta_2\left(1+\Omega\right) + \beta_1^2\Omega - r\left(1-\kappa^2\right)\beta_1^2\Omega v_1 \\ -\frac{1}{2(1-\kappa^2)\Omega}\left[\left(1-\kappa^2\right)\beta_1^2\Omega^2 + \left(1-\kappa^2\right)\beta_2^2 - 2\kappa\left(1-\kappa^2\right)\beta_1\beta_2\Omega + 2r\left(1-\kappa^2\right)^2\beta_2^2 v_2\right],\end{aligned}$$

or,

$$2\left(1-\kappa^2\right)\Omega\phi^1=\beta_2^2-\kappa\beta_1\beta_2\left(1+\Omega\right)+\beta_1^2\Omega-r\left(1-\kappa^2\right)\beta_1^2\Omega v_1$$
$$-\frac{1}{2\Omega}\left[\beta_1^2\Omega^2+\beta_2^2-2\kappa\beta_1\beta_2\Omega+2r\left(1-\kappa^2\right)\beta_2^2 v_2\right],$$

or,

$$2\left(1-\kappa^2\right)\Omega\phi^1=\beta_2^2-\kappa\beta_1\beta_2\left(1+\Omega\right)+\beta_1^2\Omega-r\left(1-\kappa^2\right)\beta_1^2\Omega v_1$$
$$-\frac{1}{2\Omega}\left[\beta_1^2\Omega^2-2\kappa\beta_1\beta_2\Omega+\beta_2^2\Omega\right],$$

or,

$$2\left(1-\kappa^2\right)\Omega\phi^1=\beta_2^2-\kappa\beta_1\beta_2\left(1+\Omega\right)+\beta_1^2\Omega-r\left(1-\kappa^2\right)\beta_1^2\Omega v_1-\frac{1}{2}\left[\beta_1^2\Omega-2\kappa\beta_1\beta_2+\beta_2^2\right],$$

or,

$$2\left(1-\kappa^2\right)\Omega\phi^1=\frac{1}{2}\beta_2^2+\beta_1^2\Omega\left(1-r\left(1-\kappa^2\right)v_1-\frac{1}{2}\right)-\kappa\beta_1\beta_2\Omega,$$

or,

$$2\left(1-\kappa^2\right)\Omega\phi^1=\frac{1}{2}\beta_2^2+\frac{1}{2}\beta_1^2\Omega\left(1-2r\left(1-\kappa^2\right)v_1\right)-\kappa\beta_1\beta_2\Omega.$$

Comparison To compare, we recapitulate the value functions,

$$2\left(1-\kappa^2\right)\Omega\phi^0=\frac{1}{2}\left(\beta_2-\beta_1\kappa\right)^2=\frac{1}{2}\left(\beta_2^2+\kappa^2\beta_1^2-2\kappa\beta_1\beta_2\right),$$

and

$$2\left(1-\kappa^2\right)\Omega\phi^1=\frac{1}{2}\beta_2^2+\frac{1}{2}\beta_1^2\Omega\left(1-2r\left(1-\kappa^2\right)v_1\right)-\kappa\beta_1\beta_2\Omega.$$

The difference (the prime denoting that we still have a multiplying factor for simplification) is

$$\Delta'=\frac{1}{2}\beta_1^2\Omega\left(1-\frac{\kappa^2}{\Omega}-2r\left(1-\kappa^2\right)v_1\right)+\kappa\beta_1\beta_2\left(1-\Omega\right),$$

or,

$$\Delta'=\frac{1}{2}\beta_1^2\Omega\left(1-\kappa^2+\kappa^2\frac{\Omega-1}{\Omega}-2r\left(1-\kappa^2\right)v_1\right)-2r\left(1-\kappa^2\right)v_2\kappa\beta_1\beta_2,$$

or,

$$\Delta'=\frac{1}{2}\beta_1^2\Omega\left(1-\kappa^2+\kappa^2\frac{2r(1-\kappa^2)v_2}{\Omega}-2r\left(1-\kappa^2\right)v_1\right)-2r\left(1-\kappa^2\right)v_2\kappa\beta_1\beta_2,$$

or,

$$2\left(1-\kappa^2\right)\Omega\Delta\phi=2\left(1-\kappa^2\right)\left[\frac{1}{2}\beta_1^2\Omega\frac{(1-2rv_1)}{2}+\kappa^2\frac{rv_2}{\Omega}-rv_2\kappa\beta_1\beta_2\right].$$

Returning to the original value function, we have

$$\Delta\phi = \left[\frac{1}{2}\beta_1^2\frac{(1-2rv_1)}{2} + \kappa^2\frac{rv_2}{2\Omega^2} - \frac{rv_2\kappa\beta_1\beta_2}{\Omega}\right],$$

or with $\beta_1 = 1$, $\Omega = 2r\left(1-\kappa^2\right)v_2 + 1$,

$$\Delta\phi = \left[\frac{(1-2rv_1)}{4} + \kappa^2\frac{rv_2}{2(2r\left(1-\kappa^2\right)v_2+1)^2} - \frac{rv_2\kappa\beta_2}{2r\left(1-\kappa^2\right)v_2+1}\right],$$

or,

$$\Delta\phi = \frac{1}{4} - \frac{rv_1}{2} + \frac{\kappa rv_2}{(2\left(1-\kappa^2\right)v_2+1)}\left[\frac{\kappa rv_2}{2(2r\left(1-\kappa^2\right)v_2+1)} - \beta_2\right].$$

or,

$$\Delta\phi = \frac{1}{4} - \frac{rv_1}{2} + \frac{\kappa}{(2\left(1-\kappa^2\right)+1/rv_2)}\left[\frac{\kappa}{2(2\left(1-\kappa^2\right)+1/rv_2)} - \beta_2\right].$$

The comparative statics can be read quite straightforwardly from this; the effects of an increase in the respective paramaters are stated below:

- β_2: negative for $\kappa > 0$, positive for $\kappa < 0$;
- v_1: negative;
- κ:
  - $\kappa > 0$: negative for $\kappa \in [0, \overline{\kappa}]$ for some upper bound which may be equal to one;
  - $\kappa < 0$: negative as κ grows towards $\kappa = 0$ (i.e. positive as $|\kappa|$ increases);
- v_2:
  - $\kappa > 0$: negative for $\kappa \in [0, \overline{\kappa}]$ for some upper bound which may be equal to one;
  - $\kappa < 0$: positive;
- r:
  - $\kappa > 0$: negative for $\kappa \in [0, \overline{\kappa}]$ for some upper bound which may be equal to one;
  - $\kappa < 0$: ambiguos.