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Small Antenna Q and Gain – 80 Years of Progress

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***Abstract*—The understanding of small antennas has evolved tremendously since the initial investigations close to 80 years ago. In this presentation, we highlight some fundamental results on Q-factor, bandwidth, and gain.**

I. Introduction

Our understanding of limitations and possibilities concerning small antennas has evolved significantly since the classical publications by Wheeler (1947) [1] and Chu (1948) [2]. Today, we possess significant knowledge about the impact of size, shape, and materials on antenna performance in relation to bandwidth, efficiency, gain, directivity, and capacity. The development over these 80 years is vast, and in this paper, we focus on topics related to performance limits on Q-factors, bandwidth, and gain for passive antennas. Many of these results have been contributed by A. Yaghjian, and this paper serves as a contribution to a celebratory special session commemorating his 80th birthday.

Early fundamental results on small antennas focused on analytically solvable cases for antennas confined to canonical geometries. One of the most well-known and analyzed geometry is the spherical region (radius a) circumscribing an antenna. Chu [2] utilized a circuit model for the field outside the sphere, along with the stored energy in the lumped circuit elements, to derive the lower Q-factor bound

$$Q \geq Q_{\text{Chu}} = (ka)^{-3} + (ka)^{-1} \tag{1}$$

for omnidirectional antennas fitting within the sphere and $k = 2\pi/\lambda$ denoting the wavenumber, see also [3] for a derivation using fields and [4] for results on directivity.

There are several important questions concerning these classical results, *e.g.*, What is antenna Q, and how is it related to bandwidth? How does materials affect antennas, and can passive metamaterials be designed to enhance performance? Can one design optimal antennas? How does the shape influence antenna performance? Can small antennas have high directivity and gain? What happens for antennas close to *e.g.*, a ground plane? This paper discusses some development answering these questions, see also Fig. 1. The presentation is condensed, and for more comprehensive overviews and diverse perspectives see *e.g.*, [5]–[9].

II. Small antenna Q and gain

Antenna Q is used as a tool to investigate and quantify antenna bandwidth for small antennas [2], [8]. Fractional bandwidth B and the Q-factor are related by [10]

$$B \approx \frac{2}{Q}\frac{\Gamma_0}{\sqrt{1-\Gamma_0^2}} \quad \text{with } |\Gamma(\omega)| \leq \Gamma_0, \tag{2}$$

where Γ_0 denotes the threshold level for the reflection coefficient Γ. The same paper [10] contains a very useful

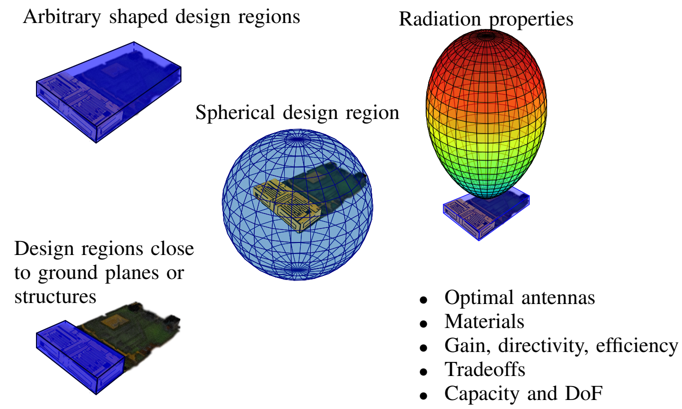

Fig. 1. Illustrations of small antenna limitation including spherical design regions, arbitrary shaped design regions, designs with desired radiation properties, and antennas designed in complex environments.

estimate of the Q-factor from the antenna input impedance $Z_{\text{in}} = R_{\text{in}} + \mathrm{j}X_{\text{in}}$ according to

$$Q_{\text{Z}'} = \big|\omega Z'_{\text{in}} + \mathrm{j}|X_{\text{in}}|\big|/(2R_{\text{in}}). \tag{3}$$

This shows that the Q-factor is related to the change of the input impedance $Z'_{\text{in}} = \frac{\partial Z_{\text{in}}}{\partial \omega}$ and generalized earlier expressions often only involving the X_{in} (reactance) derivative [10]. Both these results are derived for an antenna having a dominant single resonant. Structures with multiple resonances lack a simple relation between B and Q [11], [12], but matching theory provides estimates [7].

The Q-factor is defined as the quotient between stored and dissipated energy, which open for evaluation of Q using fields or currents. Unfortunately, the definition of stored energy is non-trivial and there exists several proposals in the literature [13]. Analytical expressions [14], numerical [15], and low-frequency approximations [16] have very good properties which are suitable for antenna analysis based on convex optimization [17]. These expressions also agree with the classical formulation based on subtraction of the far-field [18] except for a coordinate dependent term [10].

Material properties are absent from the limit (1), which raises questions whether engineered materials can affect the results. Losses naturally restrict efficiency and gain [5] but as shown in [19] temporal dispersion has a potential to increase the bandwidth [19]. Temporal dispersion and complex material properties further complicates the stored energy expressions and interpretation [20], [21]. Time varying, or active materials can enhance performance, but are beyond this paper's scope.

Antennas rarely have a spherical shape, making it important to understand the impact of different shapes. Upper bounds on the Q-factor for arbitrary shaped antennas have been derived from scattering theory [22] and energy expressions [23]–[25].

The upper bound on electric dipole radiators ($D = 3/2$) is

$$Q \geq Q_{\mathrm{lb}} = 6\pi/(k^3\gamma_{\mathrm{e}}), \tag{4}$$

which reduces to $Q_{\mathrm{lb}} \approx 1.5/(ka)^3$ for a spherical region [26]. The electric polarizability γ_{e} assesses an object's ability to separate charge in an external field. This interpretation provides physical insight and quantifies the classical intuition of higher bandwidth for 'fat' antennas, as the bandwidth (2) is proportional to the polarizability. This implies that shapes with a potentially large charge separation have a greater bandwidth potential. Extension to ultra-wideband (UWB) cases are possible [27] and larger objects can be formulated as easily solvable optimization problems [28].

Small antennas have typically dipole radiation patterns [2], and Q for different dipole combinations are investigated in [4], [24], [25]. Design of small high gain antennas are shown in [29], [30] with limits in [17], [31]

Antennas commonly interact with their environment, such as antennas in proximity of a ground plane (mobile phone) or human tissue (on or in body sensors). These environment affects antenna performance and can be analyzed using substructures with controllable regions [17], [28]. Antennas are also seldom designed for a single metric and often and tradeoffs between several metrics are required [25].

Validation of the theoretical results are essential for understanding of the tightness of the results. Spherical helix is shown to be close to optimal for electrical currents on a spherical shell [26], [32]. Several TM dipole antennas are also shown to be close to the limits for planar and cylindrical geometries [31], [33], [34]. Antenna designs reaching the Chu bound (1) is more challenging but also here several proposals exists [35], [36].

III. Conclusions

Some highlights from the 80 years development on fundamental understanding of small antennas is presented.

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