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Assigning Refugees to Landlords in Sweden: Efficient Stable Maximum Matchings[*]

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Abstract

The member states of the European Union received 1.2 million first time asylum applications in 2015 (a doubling compared to 2014). Even if asylum will be granted for many of the refugees that made the journey to Europe, several obstacles for successful integration remain. This paper focuses on one of these obstacles, namely the problem of finding housing for refugees once they have been granted asylum. In particular, the focus is restricted to the situation in Sweden during 2015–2016 and it is demonstrated that market design can play an important role in a partial solution to the problem. More specifically, because almost all accommodation options are exhausted in Sweden, the paper investigates a matching system, closely related to the system adopted by the European NGO "Refugees Welcome", and proposes an easy-to-implement mechanism that finds an efficient stable maximum matching. Such matching guarantees that housing is efficiently provided to a maximum number of refugees and that no refugee prefers some landlord to their current match when, at the same time, that specific landlord prefers that refugee to his current match.

JEL Classification: C71, C78, D71, D78, F22.

Keywords: refugees, forced migration, housing markets, market design, efficient stable maximum matchings.

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1   Introduction

The European refugee crisis began in 2015 when a rising number of refugees made the journey to Europe to seek asylum. The member states of the European Union received 1.2 million first time asylum applications (more than a doubling compared to 2014).[1] Apart from the Dublin Regulation, which dictates that the member state in which an asylum seeker enters first is obliged to render asylum, there has been no systematic way to divide refugees between the member states. Obviously, this puts great pressure on member states located at the external border of the European Union, and more specifically, on Greece, Hungary and Italy.

In an attempt to reduce pressure on these three member states, the European Commission decided in September 2015 on a temporary European relocation scheme for 120,000 refugees who are in need of international protection.[2] The relocation scheme did, however, not specify which refugees should be relocated to which member states. This specific problem has attracted interest among researchers and more systematic ways to relocate refugees between European Union member states have been proposed. In two early papers, Fernández–Huertas Moraga and Rapoport (2014,2015) approached the problem as a system of tradable quotas (like, e.g., emissions control) and demonstrated that these quotas can be designed, based on matching techniques, to solve some specific refugee resettlement problems. A different matching problem has been proposed by Jones and Teytelboym (2017). In their system, member states and refugees submit their preferences about which refugees they most wish to host and which state they most wish to be hosted by, respectively, to a centralized clearing house which matches member states and refugees according to these preferences.

Even if membership quotas are settled and a centralized matching relocation system is in place, several obstacles for successful integration remain. This paper focuses on one of these obstacles, namely the problem of finding housing for refugees once they have been relocated to a European Union membership state, and, in particular, how market design can play an important role in the solution to the problem. The background to the housing problem will be described from the perspective of the situation in Sweden during 2015–2016.

In 2015, the population of Sweden was 9.9 million which accounted for around 1.4 percent of the population in Europe. Yet, 12.4 percent of the asylum seekers in the European Union in 2015 were registered in Sweden which made Sweden the state in the European Union with most asylum seekers per capita.[3] A refugee who enters Sweden is temporarily placed at a Migration Board accommodation facility in anticipation of either a deportation order or a permanent residence permit. The average waiting time for this decision was 15 months in May 2016.[4] Refugees who are granted permanent residence permits are, under Swedish law, entitled to a number of es-

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[1]Eurostat News, Release 44/2016, March 4, 2016.
[2]European Commission, Statement 15/5697, September 22, 2015.
[3]Eurostat News, Release 44/2016, March 4, 2016.
[4]Swedish Migration Board, www.migrationsverket.se/Kontakta-oss/Tid-till-beslut.html, May 13, 2016.

tablishment measures (e.g., accommodation and a monthly allowance), and their legal status is upgraded from "asylum seekers" to "refugees with a permanent residence permit".[5] The local municipality where the refugee is registered has the responsibility to find appropriate accommodation. In this process, the refugee must leave the Migration Board accommodation facility since the legal responsibility for the refugee is transferred from the state to the local municipality.

One problem in Sweden is that almost all accommodation options are exhausted. In March 2015, it was estimated that 9,300 persons with a permanent residence permit still lived in a Migration Board accommodation facility and that, at least, 14,100 residential units were needed before the end of 2016 just to accommodate those who are granted a residence permit.[6] This estimation was updated in February 2016 to at least 20,000 new residential units only in the spring of 2016 provided that there is no drastic increase in the number of asylum seekers.[7] These facts, together with a new legislation, effective from March 1, 2016, stating that all municipalities have to accept refugees puts even more pressure on some municipalities to find additional residential units. This has forced some municipalities to consider extraordinary actions. One example is the passenger ship Ocean Gala leased for use as an asylum accommodation with room for nearly 800 people in Utansjö port outside the city Härnösand in the north east of Sweden.[8] Another example is a temporary tent camp with a capacity to accommodate 1,520 asylum seekers that was scheduled to open in December 2015 on Revingehed armor training ground 20 kilometers east of the city of Lund in the south of Sweden.[9] Hence, it is urgent to find residential units for refugees, not only because they are entitled to it under Swedish law, but also because they are blocking asylum seekers from accommodation at Migration Board accommodation facilities.

A key observation, and a possible solution to the above described problem, can be found in a report from "The Swedish National Board of Housing, Building and Planning" in 2013, where it is estimated that 90 percent of the general housing shortage in Sweden can be explained by inefficient use of the existing housing stock.[10] More precisely, due to rent control, tenants tend to live in apartments which are too big for their circumstances. The question is then how this situation can be utilized. The answer may be found in a recent survey that concluded that 31 percent of the Swedish households are willing to accommodate refugees in their homes.[11] Of course, a stated willingness to accommodate a refugee and actually accommodating a refugee are

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[5]The Swedish terminology for "refugees with a permanent residence permit" is "nyanländ" but this terminology will, for convenience, be slightly abused in the remaining part of the paper as the word "refugees" will be used instead of the correct terminology "refugees with a permanent residence permit".

[6]"Nyanländas boendesituation – delrapport", The Swedish National Board of Housing, Building and Planning, Rapport 2015:10.

[7]"More than 20 000 new places needed in accommodation in the spring", Swedish Migration Board, February 19, 2016.

[8]"Migrationsverket visste inte att miljonbåten var på väg", June 15, 2016, SVT.

[9]"Första asylsökande har flyttat in i tältlägret i Revinge", December 10, 2015, Aftonbladet.

[10]"Bostadsbristen och hyressättningssystemet – ett kunskapsunderlag", Marknadsrapport, The Swedish National Board of Housing, Building and Planning, 2013.

[11]"Svenska folkets attityder till flyktingar", September 24, 2015, DN/Ipsos.

two different things, and it should also be noted that the general view on refugees in Sweden was not as positive in the spring of 2017 as it was in the fall of 2015.[12] However, there were 4,766,000 households in Sweden in January 2015[13] and if only 1 percent of the households (instead of 30 percent) are willing to accommodate a refugee, there are still 47,660 households willing to host refugees. Hence, to release the pressure on municipalities to find housing for refugees, voluntarily supplied private housing can be utilized.[14] In this way, beds that are occupied by refugees with a permanent residence permit at the Migration Board accommodation facilities can be used for asylum seekers.

In several meetings at various levels in the Swedish administration, e.g., with the State Secretary to the Minister of Housing and the Swedish Migration Board, the authors of this paper presented a version of the theoretical matching model described in this paper. As in many matching applications, the model contains two disjoint sets of agents. These are the set of "landlords" (i.e., private persons) with capacity and willingness to accommodate refugees in their private homes and the set of refugee families with permanent residence permits. However, in many market design applications, it is realistic to assume that agents can form a ranking over potential matches.[15] In, for example, the school choice problem, parents have access to information about schools in their locality and can, based on this information, form preferences over schools. In the considered refugee matching application, however, it may be difficult for landlords (refugee families) to provide preferences as there are thousands of refugee families (landlords) in the system and it is difficult to gather complete information about all these families (landlords) and even if such information is available, it is not clear how to process it. For this reason, preferences will, like in the standard kidney exchange problem (Roth et al., 2004), be induced from reported data.[16] Consequently, the preference component of the considered model is stylized and describes a simplified version of the refugee matching problem. However, these simplifications serve an important purpose as they imply that the model can be implemented as is and without any modifications. The suggested mechanism should therefore be seen as a first emergency measure to release pressure on the municipalities in their attempts to find additional residential units and not as a complete solution to the problem.

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[12]"Allmänhetens uppfattning om invandringen", March 25, 2016, Demoskop.

[13]"Antal hushåll i Sverige", 2016, Statistics Sweden.

[14]In fact, many Swedish municipalities are today actively searching for private persons that are willing to accommodate refugees in their private homes even if private persons and refugee families are matched in a non-centralized way. Examples of such municipalities include Stockholms stad, Lunds kommun, Ängleholms kommun, Nynäshamns kommun, Kristianstads kommun, Nacka kommun, Botkyrka kommun, Håbo kommun, Härryda kommun, and Lerums kommun.

[15]Examples include school choice (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003), entry-level job markets (Roth and Peranson, 1999), course allocation (Budish and Cantillon, 2012), kidney exchange (Roth et al., 2004) and cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013). For an overview of the matching and market design literature see, e.g., Roth and Sotomayor (1990) or Sönmez and Ünver (2011).

[16]In the kidney exchange problem, patient preferences over donors are typically induced based on medical data such as tissue type antibodies and blood group.

Preferences will be induced in two steps. First, landlords classify refugee families to belong to different indifference classes and have strict preferences over the indifference classes. This classification is based on mutual acceptability (discussed below) together with a few reasonable assumptions related to, e.g., monotonicity in family size (see Assumptions 1–4). Second, preferences for refugee families are based only on mutual acceptability. This type of preference structure on two-sided matching markets was recently studied by Haeringer and Iehlé (2017). They demonstrated, using data from the junior academic job market for French mathematicians, that the mutual acceptability assumption on one side of the market (in this case, the refugee side) can be made almost without loss of generality whenever interest is directed towards stable matchings. The reason for this is that this simpler preference structure is a good approximation of a strict preference ordering and that stability rules out unacceptable matches. Hence, there is also an empirical motivation for the simple type of preference structure considered in this paper.

In the model, a refugee family and a landlord find each other mutually acceptable if they have a spoken language in common and if the number of family members does not exceed the capacity of the landlord. The communication requirement is key and its importance has been stressed by politicians in, e.g., the above mentioned meetings. It is also a requirement in, e.g., the non-centralized system adopted by the European NGO "Refugees Welcome" to match refugees with private persons. It is then natural to ask: is it even possible to find private landlords that are able to communicate with refugees? Table 1 states the 15 most common native languages in Sweden in 2012 and provides a partial answer to the question. As can be seen from the table, Arabic is the third most common native language and Kurdish, Persian and Somali qualify for the list. These languages are spoken by more than 50 percent of the asylum seekers in 2015 and 2016. Given this and the observation that the number of native language speakers gives a lower bound for how many persons that actually speak the language, shows that the communication requirement is at least not totally unreasonable.

Language	Native speakers	Language	Native speakers	Language	Native speakers
Swedish	8,000,000	Polish	76,000	Norwegian	54,000
Finnish	200,000	Spanish	75,000	English	54,000
Arabic	155,000	Persian	74,000	Somali	53,000
BCSM	130,000	German	72,000	Armenian	52,000
Kurdish	84,000	Danish	57,000	Turkish	45,000

Table 1: The 15 most common native languages in Sweden in 2012 (estimated). The population of Sweden was 9,556,000 in 2012. BCSM is an abbreviation for Bosnian, Croatian, Slovenian and Macedonian. Source: Parkvall (2016).

Given the above described type of induced preferences, the objective is to find a mechanism for assigning refugee families to private landlords. In particular, interest is directed towards mechanisms that selects efficient, stable and maximum matchings. The stability axiom means

that no refugee family strictly prefers some landlord to being unmatched when, at the same time, that specific landlord strictly prefers that refugee family to his current match. Consequently, stability in the considered setting guarantees a lower welfare bound for the participating private landlords as the landlords can be made assure that if they are matched to some refugee family, there is at least no unmatched refugee family that they strictly prefer to their current match. It is also well-known that unstable mechanisms tend to die out while stable mechanisms survive the test of time (Roth, 2008). Maximality means that a maximum number of refugees are matched to landlords or, equivalently, that a maximum number of privately supplied beds are utilized. This axiom can be motivated by the above described acute shortage of residential units in Sweden. Our main results show the existence of efficient stable maximum matchings. This is surprising as in many of the existing applications, even two of the three requirements – efficiency, stability and maximum – are incompatible. The main reason for this finding is that preferences for landlords and refugee families are "correlated" by the mutual acceptability assumption (see the extended discussion in Section 3.1).

Even if a variety of problems have been investigated in different market design contexts, almost no attention has been directed towards problems related to refugee assignment. There are, however, a few papers on the local refugee matching problem, i.e., the problem of finding out where in a country that refugees should be settled once they have been granted protection. Jones and Teytelboym (2018) describe in general terms how a two-sided matching system can be constructed when assigning refugees to localities and detail how this system can be applied in order to meet the British government's commitment to resettle 20,000 Syrian refugees. Delacretaz et al. (2016) consider a two-sided matching market for the local refugee match and propose three different refugee resettlement systems that can be used by hosting countries under different circumstances.[17] The proposed solutions are all based on different versions of the Deferred Acceptance Algorithm and the Top Trading Cycles Algorithm. These mechanisms cannot be used to solve the allocation problem considered in this paper. The reason for this is that Delacretaz et al. (2016) assume that refugee families and localities have strict preferences over each other. Because private landlords (refugee families) classify refugee families (landlords) to belong to different indifference classes in the considered framework, their proposed methods cannot be adopted to solve the problem considered in this paper. Neither can our methods be used to solve their problem. Furthermore, the "coarser" type of preferences considered in this paper imply the existence of efficient stable maximum matchings. Such matchings does not generally exist in the above mentioned papers because preferences are assumed to be strict.

The remaining part of the paper is outlined as follows. Section 2 introduces the refugee assignment problem and the basic ingredients of the matching model. Results related to efficient, stable and maximum matchings as well as to manipulability and non-manipulability are stated in Section 3. Some concluding remarks are provided in Section 4. All proofs are relegated to

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[17]A recent paper by Aziz et al. (2017) investigates various notions of stability in the refugee assignment model introduced by Delacretaz et al. (2016).

Appendix A. Appendix B demonstrates that the maximum weight matching problem considered here can equivalently be formulated as a linear programming problem.

2 The Model and Basic Definitions

2.1 The Refugee Assignment Problem

Each refugee family contains a number of family members that wish to be accommodated by a landlord. The set of refugee families is given by $N = \{1, \ldots, |N|\}$ and the vector $q_N = (q_1, \ldots, q_{|N|})$ specifies the size q_i of each refugee family $i \in N$. For convenience, the term "refugee" will often be used instead of "refugee family" and it is then understood that the refugee is part of a refugee family with a specific number of family members. Moreover, the expression "size of refugee i" will often be used to indicate the number of family members in refugee family i.

Landlords are private persons supplying voluntarily parts of their homes to refugee families. Exactly how many refugees a landlord can accommodate is determined by his capacity. The set of landlords is given by $C = \{c_1, \ldots, c_{|C|}\}$ and the vector $q_C = (q_{c_1}, \ldots, q_{c_{|C|}})$ specifies the capacity q_c of each landlord $c \in C$. Landlords and refugee families speak at least one language. The set L contains the languages spoken by the refugee families and the landlords in $N \cup C$. The languages spoken by refugee family $i \in N$ are collected in the non-empty set $L(i) \subseteq L$. Landlords have strict preferences over languages they speak. Formally, a list of strict preferences $\succeq = (\succeq_{c_1}, \ldots, \succeq_{c_{|C|}})$ specifies the preferences $\succeq_c$ over $L \cup \{c\}$ for each landlord $c \in C$. It will be sometimes convenient not to separate refugees from landlords. In this case, we refer to agent v who belongs to the set $V = C \cup N$.

2.2 Induced Preferences

As already explained in the Introduction, preferences for landlords and refugee families will in this paper be induced based on the concept mutual acceptability. Even if such approximation is unlikely to represent the true preferences of the agents, it can be justified, e.g., because it may be difficult for landlords to form preferences over refugee families and vice versa given the size of the problem and the amount of available information. Furthermore, by inducing preferences, the local authorities can adopt the considered framework without making any modifications as a first emergency measure to release pressure on the municipalities in their attempts to find additional residential units.

To induce preferences of the landlords, let language ℓ be acceptable for landlord c if $\ell \succ_c c$. The set of acceptable languages for landlord c is denoted by $A(\succeq_c)$. Both language and capacity constraints play an important role in determining which refugees are acceptable for landlords and vice versa. More precisely, let $\ell_c(i) = \max_{\succeq_c} L(i) \cup \{\emptyset\}$ denote the most preferred spoken

language of refugee i from the perspective of landlord c. Then refugee i is acceptable for landlord c if and only if the most preferred language spoken by refugee i from the perspective of landlord c is acceptable for landlord c and if the size of refugee i does not exceed the capacity of landlord c, i.e., if and only if $\ell_c(i) \in A(\succeq_c)$ and $q_i \leq q_c$. By symmetry, landlord c is acceptable for refugee i if and only if refugee i is acceptable for landlord c. An agent who is not acceptable is unacceptable. Thus, landlord c and refugee i are *mutually acceptable* if and only if $A(\succeq_c) \cap L(i) \neq \emptyset$ and $q_i \leq q_c$. The following assumptions will be maintained for the remaining part of this paper.

Assumption 1. If a landlord accommodates a refugee family, then the landlord has to accommodate all members of the family.

Assumption 2. Landlords can only accommodate acceptable refugee families and refugee families can only be accommodated by acceptable landlords.

Assumption 3. Landlords can accommodate at most one refugee family.

Assumption 4. Landlords strictly prefer a larger refugee family to a smaller refugee family if both refugee families are acceptable.

The first assumption captures the humanitarian requirement that refugee families should be kept intact and, in addition, the legal requirement (specified in the Dublin Regulation) that refugee family members should not be separated. Assumption 2 has two important implications. First, participating landlords (refugee families) should not face the risk of being forced to accommodate refugee families (being accommodated by landlords) that they find unacceptable. Second, landlords should be able to communicate with the refugee families they accommodate (recall the discussion from the Introduction). Assumption 3 takes care of a potential conflict with the Swedish tax law. More precisely, if a landlord accommodates more than one family, the Swedish Tax Agency may classify the landlord's house as a "hotel"[18] meaning that the landlord formally has to operate a hotel business and, consequently, has to follow the regulations associated with this type of enterprise, pay taxes accordingly, etc.. The study of such enterprises is beyond the scope of this paper and is excluded by Assumption 3. Finally, Assumption 4 captures the idea that landlords receive a family size dependent monetary compensation for accommodating refugee families (this type of monotonic compensation schemes exist today, for example in the city of Stockholm[19]). Given that landlords only offer unused parts of their homes, it is at least not totally unrealistic to assume that larger refugee families are strictly preferred to smaller ones. One can also imagine that landlords bid their "optimal capacity" instead of their physical capacity, i.e., that landlords monotonically prefer larger refugee families to smaller ones up to the reported "optimal capacity".

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[18]See www.skatteverket.se.

[19]See www.stockholm.se/-/Nyheter/Nyanlanda/Hyr-ut-din-bostad.

Remark 1. The basic refugee assignment model considered here is a two-sided one-to-one matching model with capacities as in, e.g., Abdulkadiroğlu and Sönmez (2003), Abdulkadiroğlu et al. (2009), Balinski and Sönmez (1999) and Gale and Shapley (1962), where the number of offered beds and the number of needed beds are the capacities of the landlords and the refugee families, respectively. Because landlords offer several beds and refugee families may need multiple beds, one can view it also as a many-to-many matching model (Echenique and Oviedo, 2006; Konishi and Ünver, 2006a) with the restriction that any agent can be matched to at most one agent from the other side of the market (which follows from Assumption 3). We refer to Manlove (2013) for a typology of matching models. □

Given the notion of (mutual) acceptability and the above four assumptions, it is possible to derive an induced preference profile R for the agents in V. Let R_c denote the induced preference relation R_c for landlord $c \in C$ over the set $N \cup \{c\}$. Let also P_c and I_c denote the strict and the indifference part of the preference relation R_c, respectively. The induced (transitive) preference relation R_c for landlord c is described by (where c stands for not accommodating any refugee):[20]

- $cP_c i$ if and only if refugee i is unacceptable,

- $iI_c j$ if refugees $i, j \in N$ are unacceptable,

- $iP_c c$ if and only if refugee i is acceptable,

- $iP_c j$ if refugees $i, j \in N$ are acceptable and $q_i > q_j$,

- $iP_c j$ if refugees $i, j \in N$ are acceptable, $q_i = q_j$ and $\ell_c(i) \succ_c \ell_c(j)$,

- $iI_c j$ if refugees $i, j \in N$ are acceptable, $q_i = q_j$ and $\ell_c(i) = \ell_c(j)$.

Similarly as above, let R_i denote the induced preference relation R_i for refugee $i \in N$ over the set $C \cup \{i\}$, and let P_i and I_i denote its strict and indifference relations, respectively. The induced preference relation R_i for refugee i is based only on mutual acceptability[21] (where i stands for remaining unassigned):

- $iP_i c$ if and only if landlord c is unacceptable,

- $cI_i c'$ if landlords $c, c' \in C$ are unacceptable,

- $cP_i i$ if and only if landlord c is acceptable,

- $cI_i c'$ if landlords $c, c' \in C$ are acceptable.

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[20]Because preferences are transitive, below it suffices to specify preferences over any two acceptable refugees and any two unacceptable refugees.

[21]See the discussion related to Haeringer and Iehlé (2017) from the Introduction

Let $R = (R_v)_{v \in V}$ denote the induced (preference) profile for the agents in V. The set of all such profiles is denoted by $\mathcal{P}^V$. A profile $R \in \mathcal{P}^V$ may also be written as (R_v, R_{-v}) when the preference relation R_v of agent $v \in V$ is of particular importance.

2.3 Matchings and Mechanisms

Landlords (refugees) are either unmatched or matched to a refugee (to a landlord) under the restriction that a landlord $c \in C$ is matched to refugee $i \in N$ if and only if refugee i is matched to landlord c. Formally, a matching is a function $\mu : C \cup N \to C \cup N$ such that $\mu(c) \in N \cup \{c\}$ for all $c \in C$, $\mu(i) \in C \cup \{i\}$ for all $i \in N$, and $\mu(c) = i$ if and only if $\mu(i) = c$. Agent v is unmatched at matching μ if $\mu(v) = v$. Given a matching μ, the matched landlords and the matched refugees are collected in the sets $\mu(C) \equiv \{c \in C : \mu(c) \neq c\}$ and $\mu(N) \equiv \{i \in N : \mu(i) \neq i\}$, respectively. A matching μ is *feasible* at profile $R \in \mathcal{P}^V$ if $\mu(v)R_v v$ for all $v \in V$, i.e., if each agent is matched to an acceptable agent or remains unmatched. The set of all feasible matchings at profile $R \in \mathcal{P}^V$ is denoted by $\mathcal{A}(R)$.

Let $|\mu| = \sum_{i \in \mu(N)} q_i = \sum_{c \in \mu(C)} q_{\mu(c)}$ denote the cardinality of matching μ, i.e., the total number of matched refugee family members at matching μ. A matching $\mu \in \mathcal{A}(R)$ is *maximum* at profile $R \in \mathcal{P}^V$ if there exists no other matching $\mu' \in \mathcal{A}(R)$ such that $|\mu'| > |\mu|$. A matching $\mu \in \mathcal{A}(R)$ is *stable* at profile $R \in \mathcal{P}^V$ if there is no blocking pair, i.e., if there exist no landlord-refugee pair (c, i) such that $iP_c\mu(c)$ and $cP_i\mu(i)$. Note that given the induced preferences considered in this paper, stability means that no refugee family strictly prefers some landlord to being unmatched when, at the same time, that specific landlord strictly prefers that refugee family to his current match. A matching $\mu \in \mathcal{A}(R)$ is *(Pareto) efficient* at profile $R \in \mathcal{P}^V$ if there exists no other matching $\mu' \in \mathcal{A}(R)$ such that $\mu'(v)R_v\mu(v)$ for all $v \in V$ and $\mu'(v)P_v\mu(v)$ for some $v \in V$. An *efficient stable maximum* matching is a matching which is efficient, stable and maximum. All efficient stable maximum matchings at profile $R \in \mathcal{P}^V$ are gathered in the set $\mathcal{X}(R)$.

A *(matching) mechanism* is a function $f : \mathcal{P}^V \to \cup_{R \in \mathcal{P}^V} \mathcal{A}(R)$ choosing a feasible matching $f(R) \in \mathcal{A}(R)$ for any profile $R \in \mathcal{P}^V$. Let $f_v(R)$ denote the match for agent v at matching $f(R)$. A mechanism f is *manipulable* by agent $v \in V$ at profile $R \in \mathcal{P}^V$ if there is a profile $(R'_v, R_{-v}) \in \mathcal{P}^V$ such that $f_v(R'_v, R_{-v})P_vf_v(R)$. A mechanism f which is not manipulable by any agent $v \in V$, at any profile $R \in \mathcal{P}^V$, is *non-manipulable*.[22] A mechanism f which makes a selection from the set $\mathcal{X}(R)$ at any profile $R \in \mathcal{P}^V$ is an *efficient stable maximum mechanism*.

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[22]This paper will not consider the possibility for groups of agents to manipulate a mechanism. See Barberà et al. (2016) for a recent paper on the relation between individual manipulability and group manipulability.

3 Results

3.1 Efficient Stable Maximum Matchings

Given the induced preferences and the interest in efficient stable maximum matchings, it is first established that none of the three axioms of interest are implied by any of the other two axioms (e.g., that it it is generally not the case that a stable maximum matching is efficient).[23]

Proposition 1. The following holds on the domain $\mathcal{P}^V$:

  (i) an efficient stable matching is not necessarily maximum,

 (ii) a stable maximum matching is not necessarily efficient,

(iii) an efficient maximum matching is not necessarily stable.

It will next be demonstrated that an efficient stable maximum matching exists for any profile $R \in \mathcal{P}^V$. The proof of this result is constructive in the sense that it provides a specific method for identifying such matchings.[24] Note, however, that stable maximum matchings do not necessarily exist when either (i) refugees are allowed to express strict preferences between acceptable landlords or (ii) landlords do not necessarily prefer larger acceptable families to smaller ones. These negative findings are proved in Remark 3 in Appendix A.

To intuitively understand the reasons behind the existence of an efficient stable maximum matching, note first that there always exists a maximum matching since the set of feasible matchings is finite. Note next that preferences over languages are "correlated" in the sense that if a landlord finds a certain language acceptable, then the landlord is indifferent between any two refugee families of the same size that speak this language, and any two refugee families speaking the same language are indifferent between any two acceptable landlords speaking their language. This means that a Pareto improvement of an arbitrary maximum matching must leave the same landlords and refugees matched after the rematching and landlords must be rematched to refugee families of the same size as in the initial matching but with families that speak (weakly) more preferred languages. By such rematching procedure, an efficient maximum matching can be obtained in a finite number of steps. The only reason for why such matching may not be stable is if there exists and unmatched refugee family of the same size as a family in a matched landlord-family pair where the landlord in the pair strictly prefers the language of the unmatched family to the language of the current match. By rematching the landlord in the pair to the unmatched family, and by repeating the above procedure, a new efficient maximum matching is obtained. By continuing in this fashion, an efficient stable maximum matching will eventually be obtained. Hence, it is the "correlated preferences" that makes the described (imaginary) rematching procedure work and therefore also guarantee the existence of an efficient stable maximum matching.

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[23]Part (ii) of Proposition 1 was first established by Gale and Shapley (1962). It is restated here for completeness.

[24]A more direct proof can be found in a previous version of this paper. See Andersson and Ehlers (2016).

It will next be demonstrated that a mechanism that identifies a stable maximum matching for any profile $R \in \mathcal{P}^V$ can be formulated as a maximum weight matching problem. This technique is, in similarity with the Deferred Acceptance Algorithm (Gale and Shapley, 1962) and the Top Trading Cycles Mechanism (Shapley and Scarf, 1974), frequently adopted in the market design literature to solve various matching problems. For example, solution methods based on a maximum weight matching problem has recently been applied in problems related to kidney exchange (Biró et al., 2009), teacher assignment (Combe et al., 2016), school choice (Kesten and Ünver, 2015), and kindergarten allocation (Biró and Gudmundsson, 2017). The mechanism can also, equivalently, be formulated as a linear programming problem. Readers that are interested in the latter formulation are referred to Appendix B.

To formulate the maximum weight matching problem for a given refugee assignment problem, a bipartite graph needs to be defined and specific values must be attached to the edges in the graph. Because the structure of the graph is dependent on the given profile $R \in \mathcal{P}^V$, the bipartite graph is described by $g = (V, E, R)$ where V is a set of vertices and E a set of edges. Every vertex in the graph g corresponds to an agent in $V = C \cup N$. Moreover, there is an edge between landlord $c \in C$ and refugee $i \in N$, denoted by ci, if and only if they find each other mutually acceptable at profile R. Let $E(g)$ denote the set of edges in graph g. Note also that any matching $\mu \in \mathcal{A}(R)$ can be described as a subset of edges $E(\mu) \subseteq E(g)$ for the graph $g = (V, E, R)$, i.e., if landlord c is matched to refugee i at matching μ, then $ci \in E(\mu)$.

A weighted graph (g, w) is defined by a graph $g = (V, E, R)$ and a vector of edge weights $w \equiv (w_{ci})_{ci \in E(g)}$ where w_{ci} is the weight assigned to edge $ci \in E(g)$. Let $S(g, w, \mu) = \sum_{ci \in E(\mu)} w_{ci}$ be the sum of all edge weights at matching $\mu = E(\mu)$. A matching μ is a maximum weight matching in the weighted graph (g, w) if $S(g, w, \mu) \geq S(g, w, \mu')$ for all matchings $\mu' \in \mathcal{A}(R)$. Let $\mathcal{V}(g, w) \subseteq \mathcal{A}(R)$ denote the set of all maximum weight matchings in the weighted graph (g, w).

To identify a maximum weight matching, the vector of edge weights $w \equiv (w_{ci})_{ci \in E(g)}$ for the weighted graph (g, w) needs to be specified. Let, for this purpose, $|L_c|$ be the number of languages spoken by landlord c. To simultaneously obtain stability and efficiency, it will be important to keep track of how landlords rank the languages of the refugees that they potentially are matched to. To achieve this, let $r_c(\succeq_c, \ell)$ denote the rank of any language ℓ spoken by landlord c (for example, if landlord c only speaks Arabic and English but prefers the former to the latter according to $\succeq_c$, then Arabic is ranked as 1 and English is ranked as 2). Let now each weight w_{ci} in the vector of edge weights $w \equiv (w_{ci})_{ci \in E(g)}$ be defined by:

$$ w_{ci} = |C|q_i + 1 - \frac{r_c(\succeq_c, \ell_c(i))}{|L_c|}. \tag{1} $$

The expression $|C|q_i$ represents the part of the edge weight that is associated to family size whereas the expression $(1 - \frac{r_c(\succeq_c, \ell_c(i))}{|L_c|})$ is the part of the edge weight related to how landlord c ranks the most preferred language spoken by refugee i among all languages spoken by landlord c.

The former expression is always weakly larger than $|C|$ whereas the latter belongs to the interval $[0, 1)$. The difference between these components guarantees the selection of a maximum matching. The latter expression assures the selection of a stable matching, and the two expressions will jointly guarantee efficiency.

Theorem 1. A mechanism f that for each profile $R \in \mathcal{P}^V$ selects a matching from the set $\mathcal{V}(g, w)$ where $g = (V, E, R)$ and the vector of edge weights w is defined by equation (1) is an efficient stable maximum mechanism.

Corollary 1. For any profile $R \in \mathcal{P}^V$, there exists an efficient stable maximum matching.

A maximum weight matching can be identified in polynomial time by adopting the Hungarian method of Kuhn (1955) and Munkres (1957). Note, however, that the set of efficient stable maximum matchings $\mathcal{X}(R)$ may contain multiple matchings for a given profile $R \in \mathcal{P}^V$ and the above method only identifies one matching in the set $\mathcal{X}(R)$. The identified matching is, in a sense, identified in a lexicographic way by construction of the edge weights (1). More precisely, the edge weights guarantee that an additionally matched family member gives more weight than any weight generated by reallocating all matched refugee families among matched landlords in such a way that all matched landlords are weakly better of from the view point of their own preferences. Another way of visualizing this is that the mechanism first identifies all maximum matchings (first selection criterion) and then makes a selection from this set based on how landlords rank languages (second selection criterion). Two remarks are in order.

First, additional selection criteria may be added in order to make a "finer" selection from the set of efficient stable maximum matchings in the above described lexicographic fashion. This can be accomplished by adding "sufficiently small" constants to the edge weights (1) for each additional selection criterion. One can imagine that such additional criteria may be given to landlords that speak specific languages or landlords that live in specific geographical areas. Such additional criteria will also play an important in the next section in the discussion related to manipulability and non-manipulability (so-called consistent tie-breaking rules).

Second, in the current formulation of the problem, spoken languages are used as the second selection criterion for the mechanism. Of course, any other criteria could have been used as long as the edge weights are adjusted accordingly. As explained in the Introduction, the reason for focusing on maximum matchings and for putting such large emphasis on language is the acute shortage of housing in Sweden in combination with the input we have received related to language from various bodies and organizations.

Remark 2. From the proof of Theorem 1 it follows that if in equation (1) the vector of edge weights $w \equiv (w_{ci})_{ci \in E(g)}$ is given by $w_{ci} = q_i$, then any maximum weight matching is a maximum matching. Furthermore, if there is only one refugee speaking one language and all landlords speak this language, then any feasible matching which matches the refugee to a landlord is an efficient stable maximum matching. Hence, the Rural Hospitals' Theorem (Roth, 1986) does not hold in our setting.

3.2 Manipulability and Non-Manipulability

As already explained several times in this paper, preferences are induced and based on the concept of mutual acceptability. This also means that if landlords (refugee families) would have complete information about all refugee families (landlords) and form their true preferences based on this information, it is unlikely that the true preferences would coincide with the induced preferences considered in this paper. As a consequence, an agent that attempts to manipulate the mechanism would aim to do so based on her true preferences and not the induced ones. However, because agents do not have access to complete information, it is unlikely that they know how to form their true preferences. What they do know, however, is how preferences are induced and how the considered mechanism is designed. Consequently, the following results related to manipulability and non-manipulability are not just a theoretical curiosity, they, in fact, describe the reality for agents that do attempt to manipulate the mechanism based on the available information and design.

It is well-known that there exists no matching mechanism for two-sided matching markets which always selects a stable matching and at the same time gives the agents on both sides of the market incentives to truthfully report their preferences (Roth, 1982). However, a famous result (again by Roth, 1982) states that a mechanism selecting always a stable matching which is "optimal" for the agents on one side of the market (i.e., the male-optimal or the female-optimal stable matching) will, in general, not give the agents on that side of the market any incentives to misrepresent their preferences. This paper provides two results with a similar flavor for efficient stable maximum mechanisms. More specifically, manipulation by means of language misrepresentation is impossible for refugee families but possible for landlords. Moreover, capacity manipulation[25] is not possible for landlords but for refugee families (capacity manipulation for refugee families should be interpreted as manipulation by misrepresenting family size). To formalize these results, two clarifications are needed. First, it has not been detailed exactly how conflicts between agents are solved in the case when the set $\mathcal{X}(R)$ contains multiple matchings for a given profile $R \in \mathcal{P}^V$ and when agents are not indifferent between all matchings in this set. Second, as hinted above, the meaning of manipulability is not clear as agents report information about both languages *and* capacities.

Suppose now that the set $\mathcal{X}(R)$ contains several matchings at a given profile $R \in \mathcal{P}^V$ and, in addition, that not all agents are indifferent between all matchings in $\mathcal{X}(R)$. In this case, the mechanism must make a specific selection from the set $\mathcal{X}(R)$ based on some type of exogenously given rule. This rule can, for example, be based on a lottery that is conducted before the agents report their preferences or an exogenously given priority order. The important feature is that the rule consistently breaks ties in a way which is independent of the information reported by the agents (if the tie-braking rule is endogenously dependent on reported information, further manipulation possibilities may arise). To formalize, suppose that $f(R) = \mu$ and that matchings

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[25]For more on capacity manipulation, see, e.g., Konishi and Ünver (2006b), Ehlers (2010) or Kesten (2012).

μ and μ' belong to $\mathcal{X}(R) \cap \mathcal{X}(R')$ for some profiles $R, R' \in \mathcal{P}^V$, then it cannot be the case that $f(R') = \mu'$. That is, if the mechanism f selects matching μ over matching μ' at some profile, then it cannot be the case that the very same mechanism selects matching μ' over matching μ at some other profile whenever both matchings are efficient, stable and maximum at both these profiles. A mechanism that respects this condition is said to consistently break ties.[26] This is equivalent to requiring that the mechanism uses a priority order over all matchings and for any profile chooses the efficient stable maximum matching which is highest on this priority order.

A first observation is that it is reasonable to assume that family size can be verified by the authorities. However, if such verification is impossible, refugee families may misrepresent their family size in two different ways. First, a family of six members may, for example, claim that they are two separate families with, say, three members each. This type of size manipulation is referred to as *family split manipulation*. Such manipulation will never be successful by Assumption 1 since refugee families prefer not to be matched to any landlord rather than splitting the family. A second type of capacity or size manipulation may occur if two separate refugee families merge and pretend to be one family (i.e., some type of group manipulation). This type of size manipulation is referred to as *family merge manipulation*. Such manipulation cannot generally be avoided if interest is directed towards maximum matchings. Just imagine a situation where the landlord with maximal capacity can host six refugees and the unique largest refugee family has five members. Then if two unmatched families of, say, three members each merge and claim to be one family with six members, they will necessarily be matched to the landlord with maximal capacity. Given these conclusions, the remaining part of the analysis related to refugee manipulation focuses on language manipulation, i.e., misrepresentation of the set of spoken languages $L(i)$. For this type of manipulation, Proposition 2(iii) provides a positive result.

Proposition 2. Let f be an efficient stable maximum mechanism breaking ties consistently. Then:

(i) it is generally impossible to prevent refugee families to manipulate f by family merge manipulation,

(ii) no refugee family $i \in N$ can manipulate f by means of family split manipulation, and;

(iii) no refugee family $i \in N$ can manipulate f by misrepresenting $L(i)$ at any profile $R \in \mathcal{P}^V$.

Landlords report information related to capacities and languages even if landlords have more degrees of freedom to manipulate by language misrepresentation compared to refugee families since landlords also report a strict ranking $\succeq_c$ over the languages they speak. As it turns out, this additional degree of freedom makes it possible for landlords to manipulate any efficient stable

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[26]It is not very difficult to define a mechanism f that consistently break ties and for each profile $R \in \mathcal{P}^V$ selects a matching from the set $\mathcal{V}(g, w)$. This can be achieved by adding a "sufficiently small" weight to condition (1) where the additional weight captures the idea in the tie-breaking rule (e.g., agent-based tie-breaking).

maximum mechanism (in fact, the result is more general than this as it is obvious from the proof of Proposition 3(i) that the result holds for any maximum matching mechanism). A positive result is, however, obtained in terms of capacity manipulation. More precisely, landlords do not have the above described type of possibility to merge with other landlords as it is reasonable to assume that landlords must state their property name, address, etc. when signing up to the centralized matching system. Hence, the only type of capacity manipulation that remains for the landlords is to misrepresent q_c, i.e., the number of available beds. Such manipulation attempts are, however, fruitless as revealed below.

Proposition 3. Let f be an efficient stable maximum matching mechanism breaking ties consistently. Then:

(i) it is generally impossible to prevent landlords from manipulating the mechanism f by misrepresenting preferences over languages $\succeq_c$, but;

(ii) no landlord $c \in C$ can manipulate f by misrepresenting q_c at any profile $R \in \mathcal{P}^V$.

4 Concluding Remarks

This paper is one of the firsts to investigate a matching model related to refugee resettlement and refugee assignment. In fact, we are only aware of a handful other matching papers that have investigated this specific problem and most of them have been discussed in this paper.[27] The point of departure has been the European refugee crisis during 2015–2016 and, more specifically, the acute problem to find housing for refugees in Sweden. The presented matching model is stylized, in the sense that landlords and refugees are only allowed to submit limited information related to ability to communicate (i.e., language) and capacity/size (i.e., number of available/needed beds). In particular, and as explained in the Introduction, preferences are "one-sided" and refugee families are therefore not allowed to express their preferences over, e.g., locations.[28] Even if this is the case, we believe that the investigated model is relevant from a policy perspective because the suggested mechanism is easy-to-implement and it can be adopted as is without any modifications. The mechanism should therefore be seen as a first emergency measure to release pressure on the municipalities in their attempts to find additional residential units and, consequently, as a first step to solve an acute problem. Even if future research is needed to find alternative proposals, it is, however, clear that the investigated problem is on the highest political agenda in all member states of the European Union and it is therefore crucial that the market design community continues to investigate problems related to refugee matching and refugee resettlement.

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[27]In this rapidly growing literature, other papers that not have been discussed include, e.g., Andersson et al. (2018), Grech (2017), Trapp et al. (2018a,b), and van Basshuysen (2017).

[28]In this context, it should also be mentioned, even if it has not been discussed in the paper, that one can imagine that preferences are induced based on other criteria than communication (e.g., geographical preferences) or a combination of several variables (see, e.g., Delacretaz et al., 2016).

As also is clear from the analysis in the paper, the proposed system to match refugee families to private landlords is treated separately from the "regular" system where refugee families are placed in Migration Board accommodation facilities. The latter type of system is currently not based on sophisticated matching mechanisms in Sweden and other countries including, e.g., the US and Switzerland (Bansak et al., 2018) and the UK (Delacretaz et al., 2016). A natural future question is to investigate is if these two separate systems can be merged into a unified matching system containing both private and governmental landlords.[29] The proposed mechanism can, however, not be adopted in a unified system since it relies on the assumption that landlords at most can accommodate one refugee family and Migration Board accommodation facilities typically host multiple families. Possible pathways toward uniform systems can, e.g., be based on techniques for solving min-cost flow problems (Edmonds and Karp, 1972) or by introducing more complex integer optimization techniques. The latter approach has recently been advocated in a refugee resettlement context by Trapp et al. (2018a,b). However, it is beyond the scope of this paper to investigate these two approaches further.

Finally, we remark that even if this paper has focused exclusively on a very specific refugee matching problem, it has contributed to the matching and market design literature in a broader sense. First, the paper has provided a framework for analysing two-sided matching markets when preferences are incomplete on one side of the market (in this case, the refugee side) and therefore needs to be approximated using the concept of mutual acceptability. Such a market has recently been analyzed by Haeringer and Iehlé (2017) even if their objective is very different from ours, namely to deduce information on stable matchings from partial observation of preferences. Second, it has been demonstrated that a new class of positive non-manipulability results can be obtained on two-sided matching markets even if agents are allowed to report information in *two* dimensions. Third, the model can be seen as an extension of the matching model with a dichotomous domain that was popularized by Bogomolnaia and Moulin (2004) since agents on one side of the market are allowed to provide preferences over different indifference classes of agents on the other side or the market (note that all results presented in this paper hold for such preference structure even if preferences not are induced). The results of this paper then demonstrate that there is no conflict between efficiency, stability, and maximality on a larger domain than the dichotomous one.

Appendix A: Proofs

Proof of Proposition 1. The proposition is proved using a simple example. Let $C = \{c_1, c_2, c_3, c_4\}$ and $N = \{1, 2, 3, 4, 5\}$, and suppose that $q_v = 1$ for all $v \in V$. The induced preference profile

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[29]In, e.g., the school choice literature, it is by now well-known that systems in which admissions to private and public schools not are separated form each other (i.e., unified systems) are more efficient than independently operated admission systems. See, e.g., Doğan and Yenmez (2017), Dur and Kesten (2018), Ekmekci and Yenmez (2014), or Manjunath and Turhan (2016).

R is given by the table below (it is straightforward to verify that there exists lists $(\succeq_{c_1}, \ldots, \succeq_{c_4})$ and $(L(1), \ldots, L(5))$ which are consistent with profile R)[30].

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}	R_1	R_2	R_3	R_4	R_5
1	2	4	2	c_1, c_3	c_2, c_4	c_1, c_2	c_1, c_2, c_3	c_1
4	3	1	c_4					
5	4	c_3						
3	c_2							
c_1								

Consider next the following three matchings:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ 1 & 2 & 4 & c_4 \end{pmatrix}, \mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ 4 & 3 & 1 & 2 \end{pmatrix}, \mu'' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ 5 & 3 & 1 & 2 \end{pmatrix}.$$

The interpretation of matching μ is that landlord c_1 is matched to refugee 1, landlord c_2 is matched to refugee 2, landlord c_3 is matched to refugee 4, and that landlord c_4 as well as refugees 3 and 5 are unmatched.

In showing (i), it is easy to verify that matching μ is stable and efficient, but μ is not maximum because $|\mu| = 3 < 4 = |\mu'|$ and μ' is feasible. In showing (ii), it is easy to verify that matching μ' is stable and maximum, but μ' is not efficient because μ' can be Pareto improved if landlords c_1 and c_3 swap refugees. In showing (iii), it is easy to verify that matching μ'' is efficient and maximum, but μ'' is not stable because $(c_3, 4)$ is a blocking pair of μ''. $\square$

Remark 3. When either (i) refugees are not indifferent between acceptable landlords or (ii) landlords do not necessarily prefer larger acceptable families to smaller ones, then stable maximum matchings do not necessarily exist.

Regarding (i), in the above example it is easy to verify that for any maximum matching $\hat{\mu}$ we have $\hat{\mu}(2) = c_4$. But now any such matching contains the blocking pair $(c_2, 2)$ if refugee 2 is allowed to express the strict preference $c_2 P_2 c_4 P_2 2$ over acceptable landlords.

Regarding (ii), in the above example, suppose that refugee 5 has size 2. Then for any maximum matching $\hat{\mu}$ we have $\hat{\mu}(5) = c_1$. Now if landlord c_1 is allowed to strictly prefer refugees 1 and 4 (with size 1) over refugee 5 with size 2, then (if $\hat{\mu}$ were stable), $\hat{\mu}(1) \neq 1$ and $\hat{\mu}(4) \neq 4$. But then we must have $\hat{\mu}(1) = c_3$ and $\hat{\mu}(4) = c_2$. But then $\hat{\mu}(3) = 3$ and $\hat{\mu}$ contains the blocking pair $(c_2, 3)$. $\square$

Proof of Theorem 1. It needs to be established that any matching $\mu \in \mathcal{V}(g, w)$ is an efficient stable maximum matching for the weighted graph $(g, w) = (V, E, R, w)$ whenever $R \in \mathcal{P}^V$ and the vector of edge weights w is defined by equation (1).

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[30]Simply let $L = \{\ell_1, \ldots, \ell_5\}$ and $L(i) = \{\ell_i\}$ for all $i \in N$. Then, for instance, $\ell_4 \succ_{c_3} \ell_1 \succ_{c_3} \succ_{c_3} c_3$.

To obtain a contradiction, suppose first $\mu \in \mathcal{V}(g, w)$ but μ not is maximum. This means that there exists some other matching $\mu' \in \mathcal{A}(R)$ with $|\mu'| > |\mu|$ or, equivalently, that:

$$\sum_{c \in \mu'(C)} q_{\mu'(c)} > \sum_{c \in \mu(C)} q_{\mu(c)}. \tag{2}$$

Because $\mu \in \mathcal{V}(g, w)$, it follows that $S(g, w, \mu) \geq S(g, w, \mu')$. By definition of the edge weights in equation (1), the latter inequality can be rewritten as:

$$\sum_{c \in \mu(C)} \left(|C| q_{\mu(c)} + 1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \right) \geq \sum_{c \in \mu'(C)} \left(|C| q_{\mu'(c)} + 1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \right),$$

or, equivalently, as:

$$|C| \left(\sum_{c \in \mu(C)} q_{\mu(c)} - \sum_{c \in \mu'(C)} q_{\mu'(c)} \right) + \sum_{c \in \mu(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \right) \\ - \sum_{c \in \mu'(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \right) \geq 0. \tag{3}$$

It now follows from condition (2) that inequality (3) cannot hold unless:

$$\sum_{c \in \mu(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \right) - \sum_{c \in \mu'(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \right) \geq |C|. \tag{4}$$

Note now that $\left(1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \right) \in [0, 1)$ for all $c \in \mu(C)$ and $|\mu(C)| \leq |C|$. Consequently:

$$|C| > \sum_{c \in \mu(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \right). \tag{5}$$

It then follows from inequalities (4) and (5) that:

$$0 > \sum_{c \in \mu'(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \right).$$

But this inequality cannot hold since $\left(1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \right) \in [0, 1)$ for all $c \in \mu'(C)$. Hence, condition (4) cannot hold meaning that inequality (3) cannot hold either which is the desired contradiction. Hence, μ must be a maximum matching.

To demonstrate that μ is a stable matching, suppose that it is not. This means that there exists a landlord-refugee pair (c, i) at matching μ such that $iP_c\mu(c)$ and $cP_i\mu(i)$. By construction of preferences R_i, these conditions can only hold if $q_i \geq q_{\mu(c)}$ and $\mu(i) = i$. But then $iP_c\mu(c)$ contradicts that $\mu \in \mathcal{V}(g, w)$. To see this, consider the matching μ'' where $\mu''(c) = i$, $\mu''(i) = c$, $\mu''(\mu(c)) = \mu(c)$, and $\mu''(v) = \mu(v)$ for all $v \in V \setminus \{c, i, \mu(c)\}$. From the construction of the edge weights in equation (1), it now follows that $S(g, w, \mu'') > S(g, w, \mu)$, i.e., a contradiction to $\mu \in \mathcal{V}(g, w)$. Hence, matching μ must be stable.

Finally, to demonstrate that matching μ is efficient, suppose that μ is Pareto dominated by some other matching $\mu' \in \mathcal{A}(R)$. By construction of preferences R_c and by definition of efficiency, it follows that if $c \in \mu(C)$ then $c \in \mu'(C)$, i.e., if landlord c is matched at μ but

unmatched at μ', then the landlord c is worse off at μ' than at μ which contradicts that μ' Pareto dominates μ. Using identical arguments, it follows that if $i \in \mu(N)$ then $i \in \mu'(N)$. Because μ is a maximum matching, by the above conclusion, it then follows that $\mu(N) = \mu'(N)$ and, consequently, that $\mu(C) = \mu'(C)$. Note next that each landlord in $\mu(C)$ is matched to a refugee with the same size at both μ and μ'. This follows from Assumption 3 and the conclusion that $\mu(C) = \mu'(C)$ and $\mu(N) = \mu'(N)$ together with the assumption that μ is Pareto dominated by μ'. But this conclusion and the assumption that μ is Pareto dominated by μ' imply that each landlord $c \in \mu(C)$ weakly prefers the language spoken by refugee $\mu'(c)$ to the language spoken by refugee $\mu(c)$. That is:

$$1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \geq 1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \text{ for each } c \in \mu(C). \tag{6}$$

Note also that because $\mu(N) = \mu'(N)$ and by construction of preferences R_c, it must be the case that at least one inequality in condition (6) is strict since μ is Pareto dominated by μ' by assumption. Since $\mu \in \mathcal{V}(g, w)$, it follows that:

$$\sum_{c \in \mu(C)} \left(|C| q_{\mu(c)} + 1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \right) \geq \sum_{c \in \mu'(C)} \left(|C| q_{\mu'(c)} + 1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \right).$$

Because $\mu(C) = \mu'(C)$ and $\mu(N) = \mu'(N)$, the above inequality can be simplified to:

$$\sum_{c \in \mu(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu(c)))}{|L_c|} \right) \geq \sum_{c \in \mu(C)} \left(1 - \frac{r_c(\succeq_c, \ell_c(\mu'(c)))}{|L_c|} \right). \tag{7}$$

But this inequality cannot hold since condition (6) holds for each $c \in \mu(C)$ with at least one strict inequality. Hence, μ must be efficient. $\qquad\square$

Proof of Proposition 2. Because Parts (i) and (ii) of the proposition are proved in the main text, we only prove Part (iii). Throughout the proof, it is assumed that $R_j = R'_j$ for all $j \in V \setminus \{i\}$ and that q_i is identical at profiles R and R'. Let now $f_i(R)$ be the match of refugee family i at matching $f(R)$ and suppose, to obtain a contradiction, that refugee family i can manipulate f by reporting $L'(i) \neq L(i)$. This misrepresentation generates a profile $R' = (R'_i, R_{-i}) \in \mathcal{P}^V$. But then $f_i(R') P_i f_i(R)$ since refugee family i can manipulate the mechanism by assumption. Since f breaks ties consistently, we may assume $L'(i) = \{\ell_{f_i(R')}(i)\}$.[31]

Note next that both $f_i(R) = i$ and $f_i(R') \neq i$. This follows by construction of preferences R_i and the assumption $f_i(R') P_i f_i(R)$. From the fact that $f(R) \in \mathcal{A}(R)$ and $R_j = R'_j$ for all $j \in V \setminus \{i\}$, it then follows that $f(R) \in \mathcal{A}(R')$. Identical arguments together with the fact $f(R') \in \mathcal{A}(R')$ imply $f(R') \in \mathcal{A}(R)$. Because f is a maximum matching mechanism, for R it follows $|f(R)| \geq |f(R')|$ and for R' it follows $|f(R')| \geq |f(R)|$. Hence, $|f(R')| = |f(R)|$. But now it follows that $f(R') \in \mathcal{X}(R)$: if $f(R')$ is not stable under R, then for any blocking

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[31]If not let $L''(i) = \{\ell_{f_i(R')}(i)\}$ and $R'' = (R''_i, R_{-i})$ be the resulting profile. Then it is easy to see $f(R') \in \mathcal{X}(R'') \subseteq \mathcal{X}(R')$. Since f breaks ties consistently, we have $f(R'') = f(R')$.

pair (c, j) it must hold $j \neq i$ and (c, j) would also block $f(R')$ under R', a contradiction; and if $f(R')$ is not efficient under R, then by $L'(i) = \{\ell_{f_i(R')}(i)\}$, $f(R')$ is not efficient under R', a contradiction. Thus, $f(R') \in \mathcal{X}(R)$. Similar arguments (and using $L'(i) = \{\ell_{f_i(R')}(i)\}$) establish that $f(R) \in \mathcal{X}(R')$. Now $f(R), f(R') \in \mathcal{X}(R) \cap \mathcal{X}(R')$, which is a contradiction to the fact that f breaks ties consistently. $\qquad\square$

Proof of Proposition 3(i). This part is proved using a simple example. Let $C = \{c_1, c_2, c_3\}$ and $N = \{1, 2, 3\}$, and suppose that $q_v = 1$ for all $v \in V$, $L(i) = \{\ell(i)\}$ for all $i \in N$, and $\ell(i) \neq \ell(j)$ for all $i, j \in N$. The induced preference profile R is given by the below table (it is easy to verify that there exists lists $(\succeq_{c_1}, \succeq_{c_2}, \succeq_{c_3})$ and $(L(1), L(2), L(3))$ that are consistent with profile R).

R_{c_1}	R_{c_2}	R_{c_3}	R_1	R_2	R_3
1	2	2	c_1, c_3	c_2, c_3	c_1, c_2
3	3	1			

The set $\mathcal{V}(g, w)$ contains only the following two matchings:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 \\ 1 & 3 & 2 \end{pmatrix} \text{ and } \mu' = \begin{pmatrix} c_1 & c_2 & c_3 \\ 3 & 2 & 1 \end{pmatrix}.$$

The interpretation of matching μ is that landlord c_1 is matched to refugee 1, landlord c_2 is matched to refugee 3, and landlord c_3 is matched to refugee 2. From matching μ and μ', it follows that $f_{c_1}(R) = 3$ or $f_{c_2}(R) = 3$. The choice depends on how f breaks ties consistently.

Suppose first that $f_{c_1}(R) = 3$ and that landlord c_1 misrepresents preferences over acceptable languages by reporting $\succeq'_{c_1}$ such that $\ell(1) \succ'_{c_1} c_1$. Denote the new preference profile by (R'_{c_1}, R_{-c_1}). In this case, μ is the unique maximum matching for profile (R'_{c_1}, R_{-c_1}) and, consequently, $f(R'_{c_1}, R_{-c_1}) = \mu$. But then $f_{c_1}(R'_{c_1}, R_{-c_1}) P_{c_1} f_{c_1}(R)$, which means that landlord c_1 has a profitable deviation from R.

Suppose next that $f_{c_2}(R) = 3$ and that landlord c_2 misrepresents preferences over acceptable languages by reporting $\succeq'_{c_2}$ such that $\ell(2) \succ'_{c_2} c_2$. Denote the new preference profile by (R'_{c_1}, R_{-c_1}). In this case, μ' is the unique maximum matching for profile (R'_{c_2}, R_{-c_2}) and, consequently, $f(R'_{c_2}, R_{-c_2}) = \mu'$. But then $f_{c_2}(R'_{c_2}, R_{-c_2}) P_{c_2} f_{c_2}(R)$, which means that landlord c_2 has a profitable deviation from R. $\qquad\square$

Proof of Proposition 3(ii). Throughout, let $R_j = R'_j$ for all $j \in V \setminus \{c\}$ and that $\succeq_c$ is identical at profiles R and R'. Let now $f_c(R)$ be the match of landlord c at matching $f(R)$ and suppose, to obtain a contradiction, that landlord c can manipulate f by reporting $q'_c \neq q_c$. This misrepresentation generates a profile $R' = (R'_c, R_{-c}) \in \mathcal{P}^V$. Hence, $f_c(R') P_c f_c(R)$ since landlord c can manipulate f by assumption.

Note that landlord c cannot be matched to a refugee family with strictly more family members than q_c at matching $f(R')$, i.e., an unacceptable refugee family, and this contradicts that $f_c(R') P_c f_c(R)$ by construction of preferences R_c. Thus, $q_{f_c(R')} \leq q_c$.

Note next that landlord c cannot be matched to a refugee family with strictly fewer family members at matching $f(R')$ than at matching $f(R)$ since this contradicts that $f_c(R')P_cf_c(R)$ by construction of preferences R_c. Thus, $q_{f_c(R)} \leq q_{f_c(R')} \leq \min\{q'_c, q_c\}$. But now we have both $f(R), f(R') \in \mathcal{A}(R)$ and $f(R), f(R') \in \mathcal{A}(R')$. Because only the capacity q_c changes to q'_c from R to R', now it is straightforward that $f(R), f(R') \in \mathcal{X}(R) \cap \mathcal{X}(R')$ contradicts the fact that f breaks ties consistently. $\square$

Appendix B: The Corresponding Assignment Game[32]

As demonstrated in Section 3.1, efficient stable maximum matchings can be identified by solving an appropriately defined maximum weight matching problem. In this Appendix, it is explained how this maximum weight matching problem can be formulated as linear programming problem. Throughout the Appendix, it is assumed that a given profile $R \in \mathcal{P}^V$ is considered.

Let $\alpha(i)$ denote the set of landlords which refugee i find acceptable, and let $\alpha(c)$ denote the set of refugees which landlord c finds acceptable. Consider now for a matrix $x = (x_{ci})_{c \in C, i \in N}$ the following constraints:

$$\sum_{c \in \alpha(i)} x_{ci} \leq 1 \text{ for all } i \in N, \tag{8}$$

$$\sum_{i \in \alpha(c)} x_{ci} \leq 1 \text{ for all } c \in C, \tag{9}$$

$$x_{ci} \in \{0, 1\} \text{ for all } c \in C \text{ and all } i \in N. \tag{10}$$

The above set of constraints play the same role as the set of edges $E(g)$ for a given graph g in the sense that they can be applied to describe a feasible matching. To see this, note that any matching μ, equivalently, can be represented by a set x containing elements of type x_{ci} for $c \in C$ and $i \in N$. More precisely, if $x_{ci} = 1$ then landlord c and refugee i are matched, and if $x_{ci} = 0$ they are not matched. The linear inequalities (8) and (9) guarantee the feasibility of the matching μ, i.e., they guarantee that each agent is matched to exactly one agent that finds them mutually acceptable or remains unmatched. Consequently, the above matrix formulation x and the subset $E(\mu)$ in the graph formulation can both be used to describe a matching μ.

Recall now from Section 3.1 that $S(g, w, \mu) = \sum_{ci \in E(\mu)} w_{ci}$ is the sum of all edge weights at matching $\mu = E(\mu)$, and that a matching μ is a maximum weight matching in the weighted graph (g, w) if $S(g, w, \mu) \geq S(g, w, \mu')$ for all matchings $\mu' \in \mathcal{A}(R)$. By the above construction, it then follows that a matching $\mu \in \mathcal{A}(R)$ represented by the matrix x is a maximum weight matching if:

$$\sum_{c \in \alpha(i)} \sum_{i \in \alpha(c)} x_{ci} w_{ci} \geq \sum_{c \in \alpha(i)} \sum_{i \in \alpha(c)} x'_{ci} w_{ci},$$

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[32]We thank Referee 1 for pointing out this connection.

for any matching $\mu' \in \mathcal{A}(R)$ represented by x'. As a result, a maximum weight matching is a solution to the following linear programming problem:

$$\max \sum_{c \in \alpha(i)} \sum_{i \in \alpha(c)} x_{ci} w_{ci} \text{ subject to conditions (8), (9) and (10).}$$

This problem is sometimes referred to as the assignment game (where w_{ci} corresponds to the "value" of matching landlord c and refugee family i). Note, finally, that the (optimal) solution to the assignment game can be found by using standard linear programming techniques, see, e.g., Dantzig (1963) or Shapley and Shubik (1972).

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