Scattering Matrix Formulation for Substructure Characteristic Mode Analysis

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URSI GASS, Sapporo, Japan, August 21, 2023

Characteristic modes

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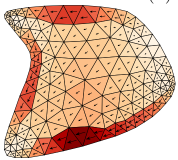

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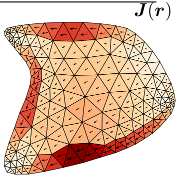

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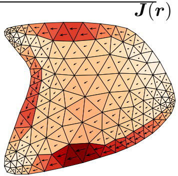

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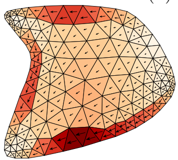

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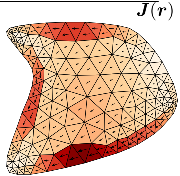

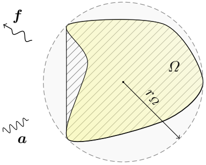

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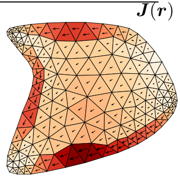

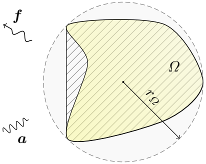

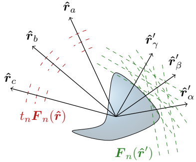

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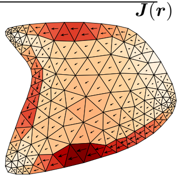
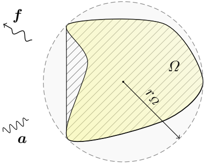
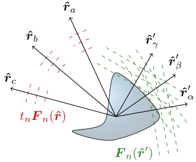
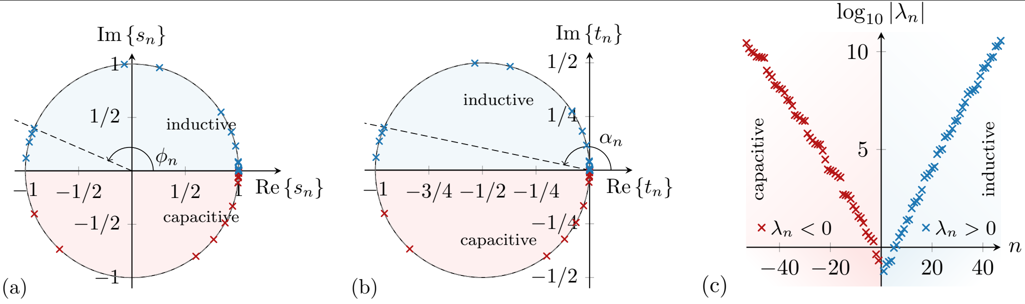






M. Gustafsson et al. "Unified theory of characteristic modes: Part I–Fundamentals". *IEEE Trans. Antennas Propag.* 70.12 (2022), pp. 11801–11813
M. Capek et al. "Characteristic Mode Decomposition Using the Scattering Dyadic in Arbitrary Full-Wave Solvers". *IEEE Trans. Antennas Propag.* 71.1 (2023), pp. 830–839

Equivalent representations for lossless objects



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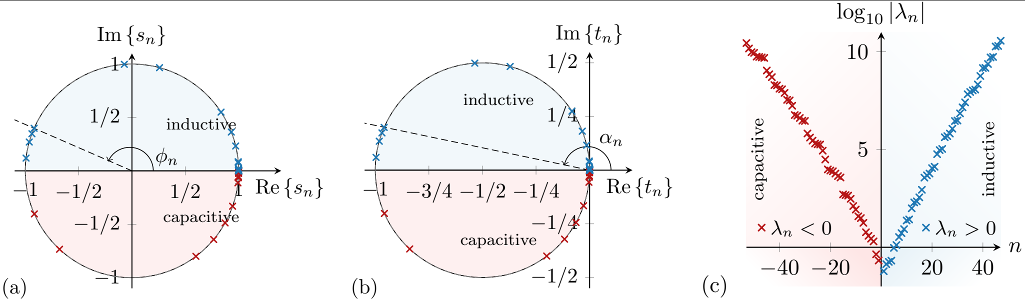

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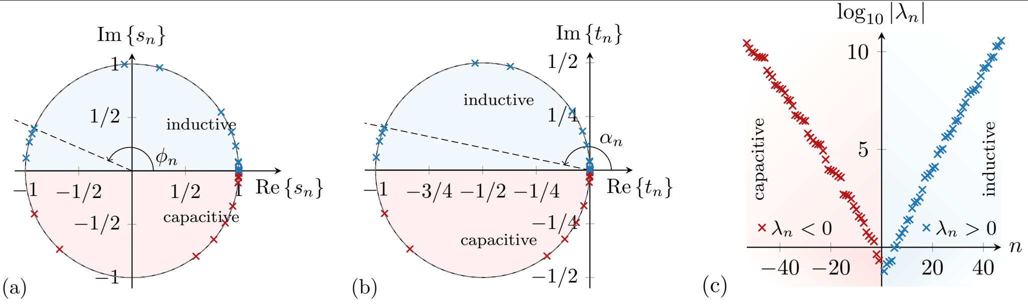

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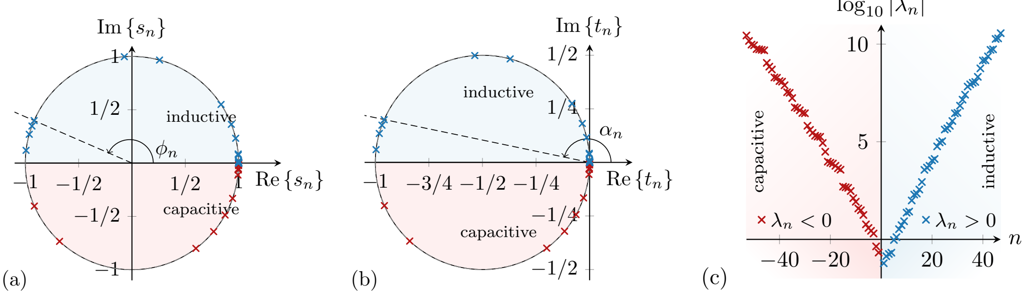

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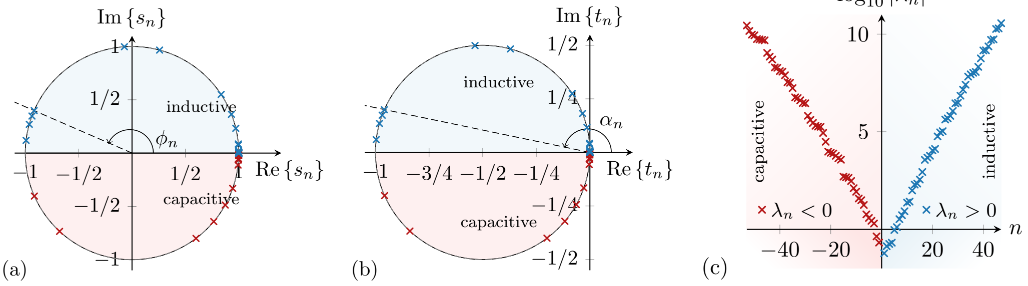

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Many equivalent representations for CM

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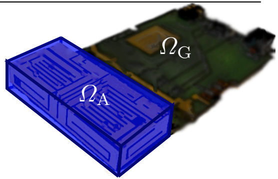

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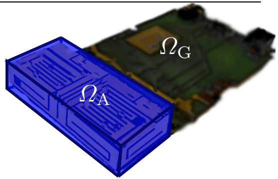

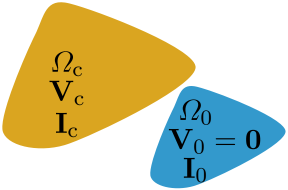

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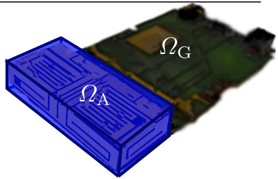

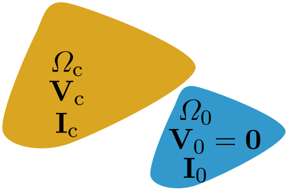

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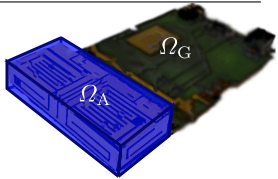

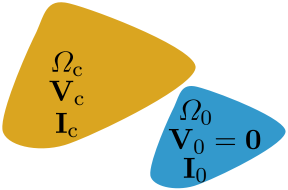

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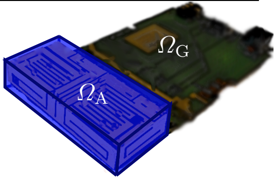

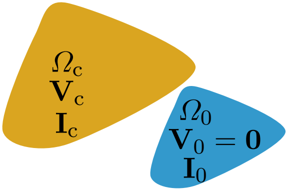

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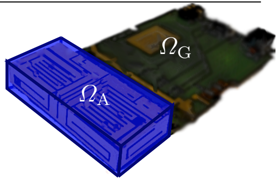

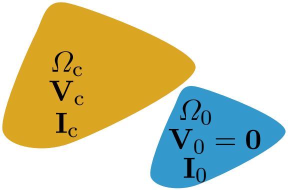

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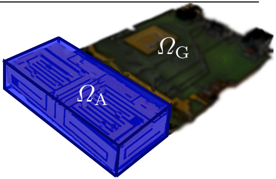

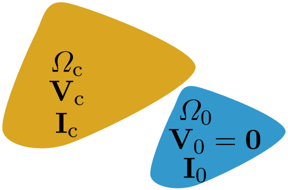

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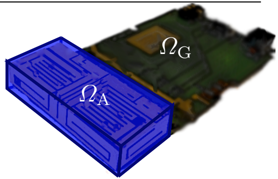

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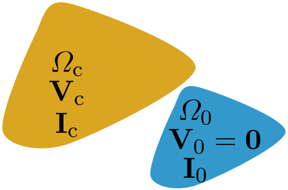

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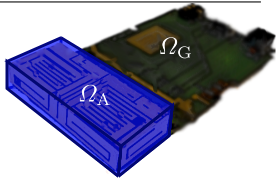
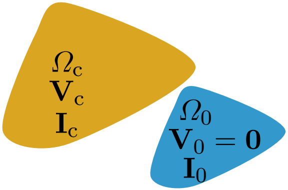


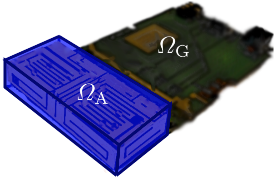


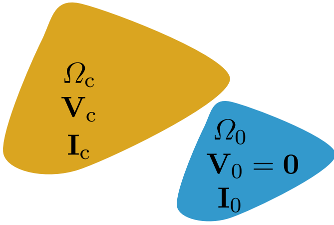
Note: decompositions based on basis function (DoF) can require a fine mesh to model connected regions

Substructure CM

- Reduced MoM system matrix

$$\widetilde{\mathbf{Z}}\mathbf{I}_\mathrm{c} = \widetilde{\mathbf{R}}\mathbf{I}_\mathrm{c} + \mathrm{j}\widetilde{\mathbf{X}}\mathbf{I}_\mathrm{c} = (\mathbf{Z}_\mathrm{cc} - \mathbf{Z}_\mathrm{c0}\mathbf{Z}_{00}^{-1}\mathbf{Z}_\mathrm{0c})\mathbf{I}_\mathrm{c} = \mathbf{V}_\mathrm{c}$$





J. Ethier and D. McNamara. "Sub-structure characteristic mode concept for antenna shape synthesis". *Electronics letters* 48.9 (2012), p. 1

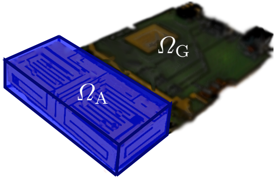
Substructure CM

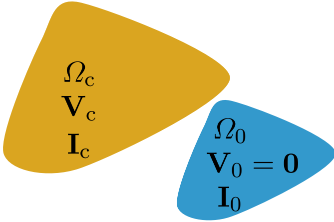
- Reduced MoM system matrix

$$\widetilde{\mathbf{Z}}\mathbf{I}_{\mathrm{c}} = \widetilde{\mathbf{R}}\mathbf{I}_{\mathrm{c}} + \mathrm{j}\widetilde{\mathbf{X}}\mathbf{I}_{\mathrm{c}} = (\mathbf{Z}_{\mathrm{cc}} - \mathbf{Z}_{\mathrm{c}0}\mathbf{Z}_{00}^{-1}\mathbf{Z}_{0\mathrm{c}})\mathbf{I}_{\mathrm{c}} = \mathbf{V}_{\mathrm{c}}$$

- Substructure CM from the generalized eigenvalue problem [EM12]

$$\widetilde{\mathbf{X}}\widetilde{\mathbf{I}}_n = \lambda_n\widetilde{\mathbf{R}}\widetilde{\mathbf{I}}_n \quad \text{or} \quad \widetilde{\mathbf{Z}}\widetilde{\mathbf{I}}_n = (1+\mathrm{j}\lambda_n)\widetilde{\mathbf{R}}\widetilde{\mathbf{I}}_n$$





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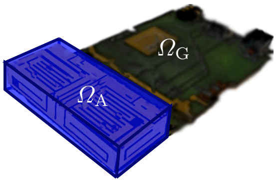

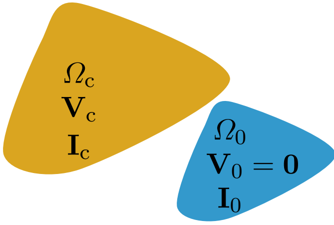

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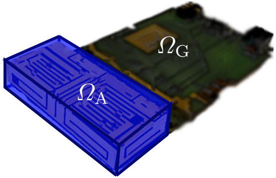
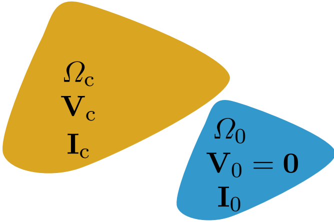
- Ω_0 is an uncontrollable background region (material), *e.g.*, a ground plane without antenna ports





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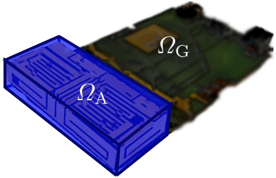

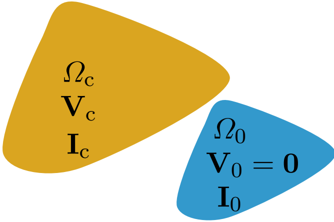

What is the corresponding scattering formulation for substructure CM?

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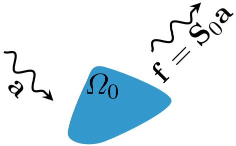
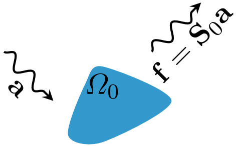
Scattering formulation for substructure CM

- Scattering matrices



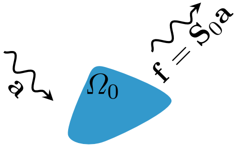
Scattering formulation for substructure CM

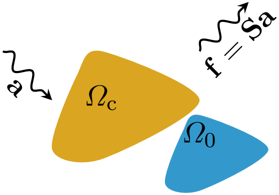
- Scattering matrices
  - $\mathbf{S}_0$ for the background (uncontrollable region)



Scattering formulation for substructure CM

- Scattering matrices
  - $\mathbf{S}_0$ for the background (uncontrollable region)
  - $\mathbf{S}$ for the composite (total) structure





Scattering formulation for substructure CM

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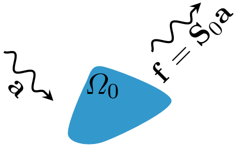
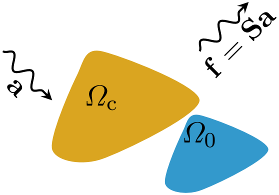
  - $\mathbf{S}$ for the composite (total) structure
- Scattering based substructure CM from

$$\mathbf{S}\mathbf{a}_n = s_n \mathbf{S}_0 \mathbf{a}_n$$

$\mathbf{a}_n$ are characteristic excitations ($\mathbf{a}_m^{\mathrm{H}}\mathbf{a}_n = \delta_{mn}$) and s_n characteristic scattering eigenvalues





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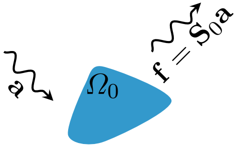

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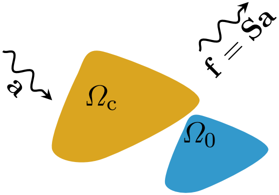
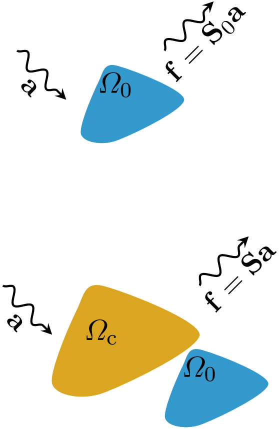


- The scattering eigenvalues s_n are related to modal significance $|t_n|$ and MoM substructure characteristic eigenvalues $\lambda_n = \operatorname{eig}(\widetilde{\mathbf{X}}, \widetilde{\mathbf{R}})$ as

$$t_n = \frac{s_n - 1}{2} \quad \text{and} \quad \lambda_n = -\operatorname{Im}\{t_n^{-1}\} = \mathrm{j}\frac{s_n + 1}{s_n - 1}$$

Interpretation scattering based substructure CM

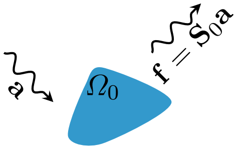
- Scaling of the scattered field from the background

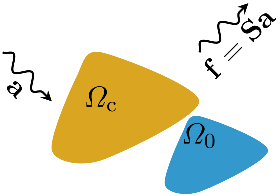


Interpretation scattering based substructure CM

- Scaling of the scattered field from the background
- Unitary scattering matrices (lossless)

$$\mathbf{S}\mathbf{a}_n = s_n\mathbf{S}_0\mathbf{a}_n \Rightarrow \mathbf{S}_0^{\mathrm{H}}\mathbf{S}\mathbf{a}_n = s_n\mathbf{a}_n$$





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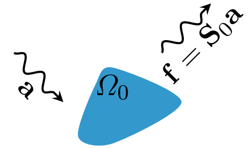
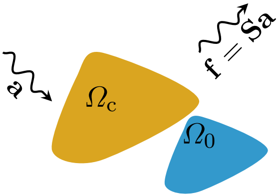


$$\mathbf{S}\mathbf{a}_n = s_n \mathbf{S}_0 \mathbf{a}_n \Rightarrow \mathbf{S}_0^{\mathrm{H}} \mathbf{S}\mathbf{a}_n = s_n \mathbf{a}_n$$

- The scattered fields can be expressed in transition matrices (or scattering dyadics) $\mathbf{S} = 2\mathbf{T} + \mathbf{1}$ and $\mathbf{S}_0 = 2\mathbf{T}_0 + \mathbf{1}$

$$\frac{1}{2}(\mathbf{S}_0^{\mathrm{H}}\mathbf{S} - \mathbf{1})\mathbf{a}_n = (2\mathbf{T}_0^{\mathrm{H}}\mathbf{T} + \mathbf{T}_0^{\mathrm{H}} + \mathbf{T})\mathbf{a}_n = t_n \mathbf{a}_n$$



Interpretation scattering based substructure CM

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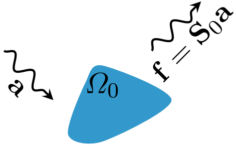

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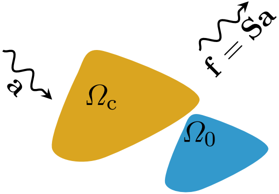

- Maximal scattering (difference between the scattered fields of the composite object $\mathbf{T}\mathbf{a}_n$ and the background $\mathbf{T}_0\mathbf{a}_n$)

$$|(\mathbf{T} - \mathbf{T}_0)\mathbf{a}_n|^2 = \mathbf{a}_n^{\mathrm{H}}(\mathbf{T}_0^{\mathrm{H}}\mathbf{T}_0 + \mathbf{T}^{\mathrm{H}}\mathbf{T} - 2\,\mathrm{Re}\{\mathbf{T}_0^{\mathrm{H}}\mathbf{T}\})\mathbf{a}_n\}$$
$$= -\,\mathrm{Re}\{\mathbf{a}_n^{\mathrm{H}}(2\mathbf{T}_0^{\mathrm{H}}\mathbf{T}+\mathbf{T}_0^{\mathrm{H}}+\mathbf{T})\mathbf{a}_n\} = -\,\mathrm{Re}\{t_n\}|\mathbf{a}_n|^2 = |t_n|^2|\mathbf{a}_n|^2$$

Proof outline

- factorizing the radiation matrix $\mathbf{R} = \mathrm{Re}\{\mathbf{Z}\} = \mathbf{U}^{\mathrm{T}}\mathbf{U}$ into spherical waves [Gus+22a] with $\mathbf{U} = \begin{bmatrix}\mathbf{U}_0 & \mathbf{U}_{\mathrm{c}}\end{bmatrix}$ and setting

$$\widetilde{\mathbf{U}} = \mathbf{U}_{\mathrm{c}} - \mathbf{U}_0\mathbf{Z}_{00}^{-1}\mathbf{Z}_{0\mathrm{c}}$$

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- reformulates the CM eigenvalue problem to

$$\widetilde{\mathbf{Z}}\widetilde{\mathbf{I}}_n = (1+\mathrm{j}\lambda_n)\widetilde{\mathbf{U}}^{\mathrm{H}}\widetilde{\mathbf{U}}\widetilde{\mathbf{I}}_n \Rightarrow -\widetilde{\mathbf{U}}\widetilde{\mathbf{Z}}^{-1}\widetilde{\mathbf{U}}^{\mathrm{H}}\mathbf{f}_n = t_n\mathbf{f}_n \qquad (*)$$

where $\mathbf{f}_n = -\widetilde{\mathbf{U}}\widetilde{\mathbf{I}}_n$ and $t_n = -1/(1+\mathrm{j}\lambda_n)$

Proof outline

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where $\mathbf{f}_n = -\widetilde{\mathbf{U}}\widetilde{\mathbf{I}}_n$ and $t_n = -1/(1+\mathrm{j}\lambda_n)$
- Transition matrices of the composite object Ω and background object Ω_1 are expressed in MoM system matrices
$$\mathbf{T} = -\mathbf{U}\mathbf{Z}^{-1}\mathbf{U}^{\mathrm{T}} = -\begin{bmatrix}\mathbf{U}_0 & \mathbf{U}_{\mathrm{c}}\end{bmatrix}\begin{bmatrix}\mathbf{Z}_{00} & \mathbf{Z}_{0\mathrm{c}}\\ \mathbf{Z}_{\mathrm{c}0} & \mathbf{Z}_{\mathrm{cc}}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{U}_0^{\mathrm{T}}\\ \mathbf{U}_{\mathrm{c}}^{\mathrm{T}}\end{bmatrix} \qquad \mathbf{T}_0 = -\mathbf{U}_0\mathbf{Z}_{00}^{-1}\mathbf{U}_0^{\mathrm{T}}$$

Proof outline

- factorizing the radiation matrix $\mathbf{R} = \mathrm{Re}\{\mathbf{Z}\} = \mathbf{U}^{\mathrm{T}}\mathbf{U}$ into spherical waves [Gus+22a] with $\mathbf{U} = \begin{bmatrix}\mathbf{U}_0 & \mathbf{U}_{\mathrm{c}}\end{bmatrix}$ and setting
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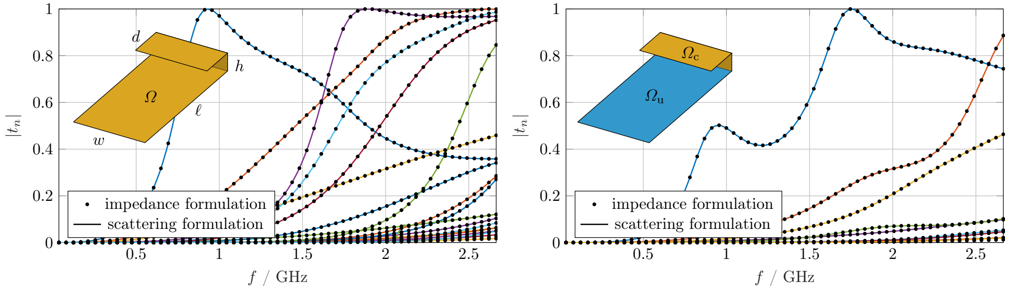
- Block inversion shows that MoM matrix (*) and scattering based substructure modes $2\mathbf{T}_0^{\mathrm{H}}\mathbf{T} + \mathbf{T}_0^{\mathrm{H}} + \mathbf{T}$ are equivalent

Numerical example

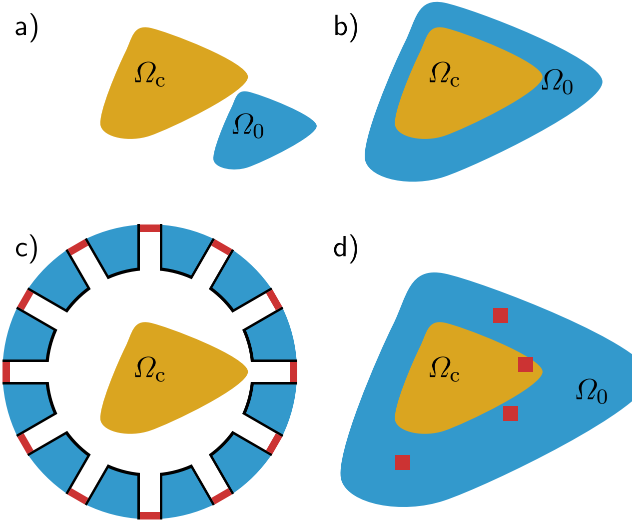


- (left) full structure (right) substructure
- Adapted from [EM12], $\ell = 120\,\mathrm{mm}$, $w = 60\,\mathrm{mm}$, $h = 15\,\mathrm{mm}$, $d = 30\,\mathrm{mm}$
- Negligible differences for such as a sheet resistance $0.01\,\Omega/\square$

Generalizations

Many possible generalizations:

- Embedded structures
- Cavities
- Combination with ports



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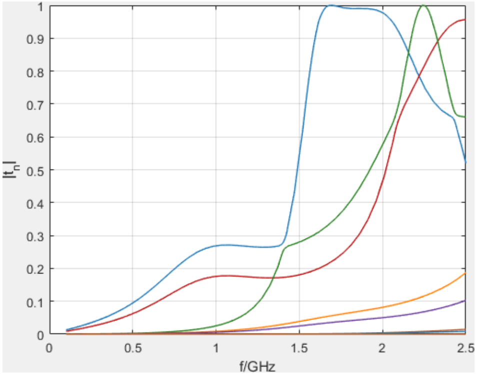
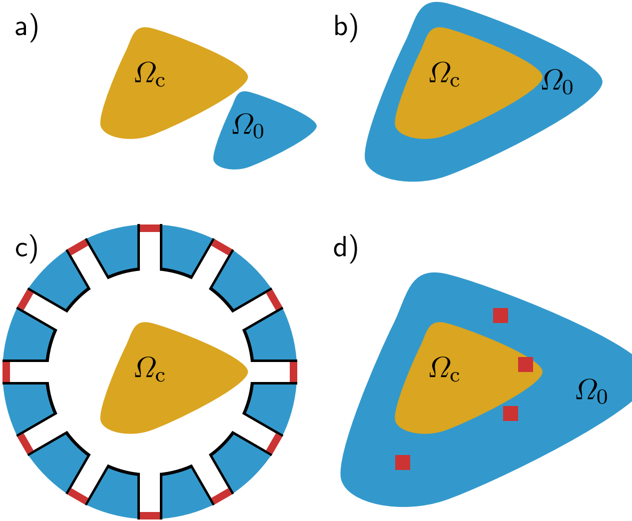
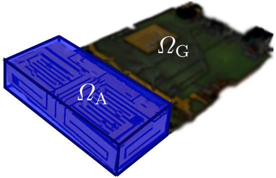
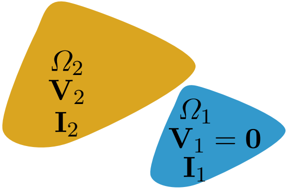




Conclusions

- Scattering based formulation of substructure CM





M. Gustafsson et al. "Unified theory of characteristic modes: Part I–Fundamentals". *IEEE Trans. Antennas Propag.* 70.12 (2022), pp. 11801–11813; M. Gustafsson et al. "Unified theory of characteristic modes: Part II–Tracking, losses, and FEM evaluation". *IEEE Trans. Antennas Propag.* 70.12 (2022), pp. 11814–11824

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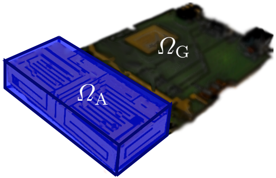
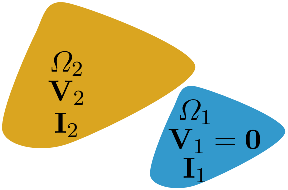
Conclusions

- Scattering based formulation of substructure CM
- Physical insight





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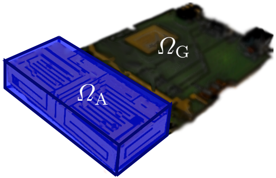
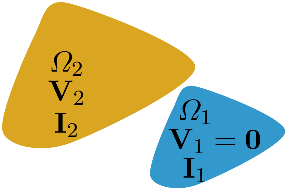
Conclusions

- Scattering based formulation of substructure CM
- Physical insight
- MoM, FEM, or FDTD to compute matrix $\mathbf{S}$ and $\mathbf{T}$





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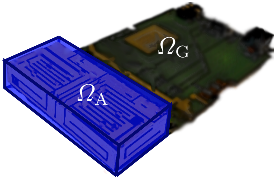

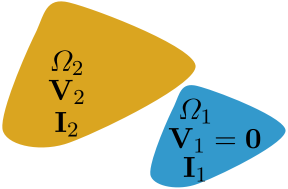

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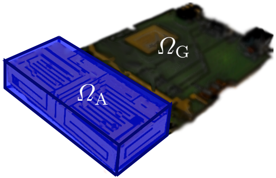

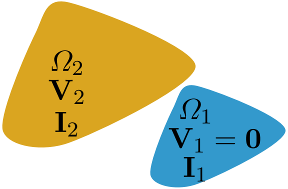

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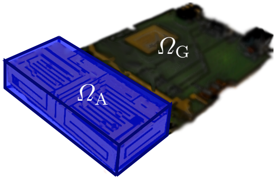

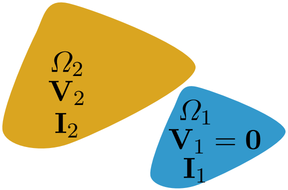

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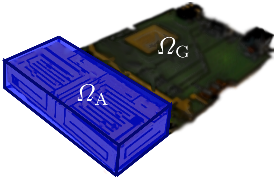

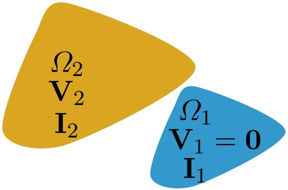

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